Electroweak effective field theory from massive scattering amplitudes

Teppei Kitahara
Nagoya University

RIKEN seminar

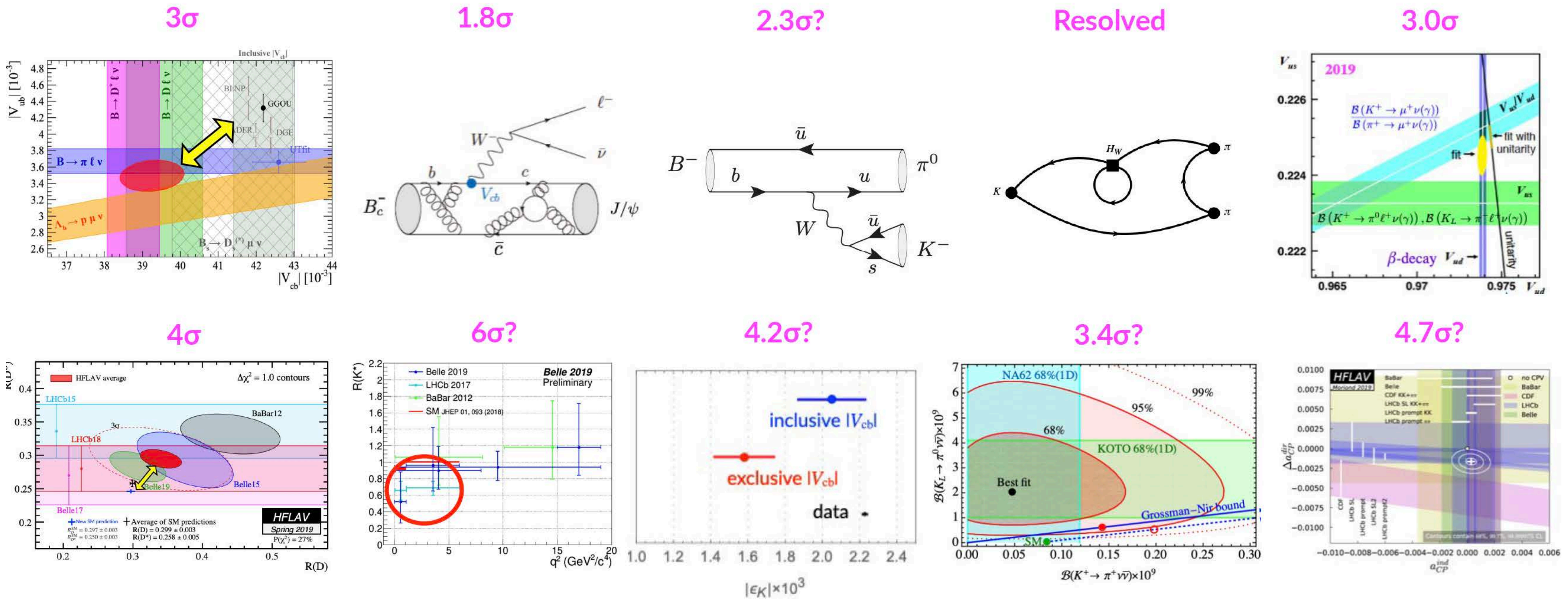
RIKEN BNL,
December 3, 2020, online talk

Before going to the main part…

- My main research aria is **flavor physics**

  The latest my review talk about the flavor anomaly is available here

  "Review of current flavor anomalies in precision measurements of mesons"



- This talk is basically no relation to any flavor physics, **so far**

Based on

Novel formalism

[1709.04891]
Nima Arkani-Hamed, Tzu-Chen Huang, Yu-tin Huang

Technion, scattering amplitudes group

[1809.09644]
Yael Shadmi, Yaniv Weiss

[1909.10551]
Gauthier Durieux, **TK**, Yael Shadmi, Yaniv Weiss

[2008.09652]
Gauthier Durieux, **TK**, Camila S. Machado, Yael Shadmi, Yaniv Weiss

Effective field theory

- Effective field theory (EFT) can be generally constructed by assuming field contents and Lorentz, global and gauge symmetries, *e.g.*, SMEFT, HEFT, HQET, SCET, …
- EFT is bottom-up and natural approach (when one does not discover any new resonance)
- Problems:
  - Find nice operator basis: operator redundancy via field redefinitions and EOMs

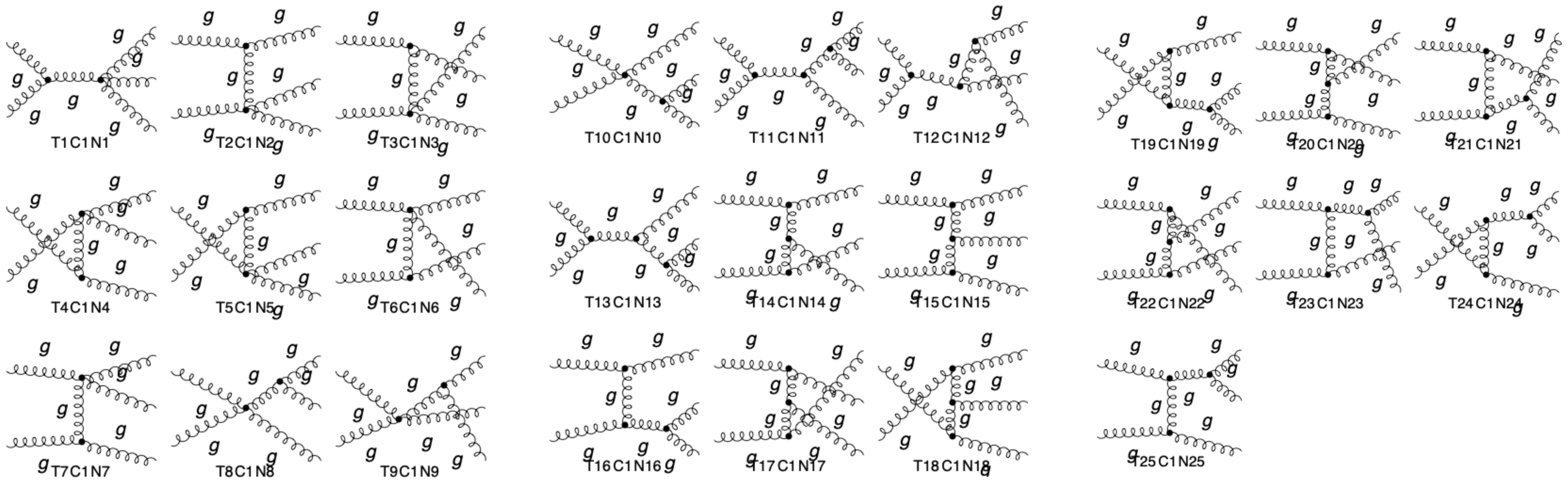
    e.g., Warsaw basis (dimension-six SMEFT) [Grzadkowski, Iskrzynski, Misiak, Rosiek '10]
  - Gauge redundancy (requires gauge-fixing, ghost d.o.f.), which is canceled out at amplitude level (after the complicated calculations)

Scattering amplitudes

- **Scattering amplitude (on-shell amplitude, modern amplitude method, or spinor-helicity formalism)** is an alternative way to EFTs (will explain after next slide)
- Scattering amplitudes can be bootstrapped from Lorentz symmetry, locality and unitarity
- Advantages:
  - No operator redundancy. No gauge redundancies. Gauge invariance is manifest
  - Bypassing Lagrangian, operators, and Feynman rules/diagrams
  - Extremely simple results compared to Feynman methods (next slide)

Five-point pure QCD amplitudes

- For example, let us compare $gg \to ggg$ amplitudes



There are 25 Feynman diagrams.
We must calculate everything.

# of Feynman diagram in $gg \to ng$

n =	2	3	4	5	6	7	8
#	4	25	220	2485	34300	559405	10525900

[Parke, Taylor '85]

- By scattering amplitudes [Mangano, Parke, Xu '88; Mangano, Parke '91]

$$\mathcal{M}_5 = \mathcal{M}_5\left(1_g^-, 2_g^-, 3_g^+, 4_g^+, 5_g^+\right) + \mathcal{M}_5\left(1_g^+, 2_g^+, 3_g^-, 4_g^-, 5_g^-\right) + \text{perm.}$$ ← Other polarizations vanish

$$\mathcal{M}_5\left(1_g^-, 2_g^-, 3_g^+, 4_g^+, 5_g^+\right) = i g_s^3 \sum_{\text{perm}'} \text{tr}(\lambda^{a_1}\lambda^{a_2}\lambda^{a_3}\lambda^{a_4}\lambda^{a_5}) \frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 51\rangle}$$ Extremely simple!

permutation up to cyclic

On-shell approach to the SMEFT

- Derive anomalous dimension matrix (one- and two-loop levels)

  [Cheung, Shen '15; Bern, Parra-Martinez, Sawyer '19, '20; Elias Miro, Ingoldby, Riembau '20; Jiang, Ma, Shu '20]

- Derive non-interference theorem for the new physics operators

  [Azatov, Contino, Machado, Riva '16; Craig, Jiang, Li, Sutherland '20, Jiang, Shu, Xiao, Zheng '20; Gu, Wang '20]

- Enumeration of independent massless operators (consistent with Hilbert series approach)

  [Shadmi, Weiss '18; Ma, Shu, Xiao '19; Falkowski '19; Durieux, Machado '19; Durieux, TK, Machado, Shadmi, Weiss '20]

  Hilbert series [Henning, Lu, Melia, Murayama '15, '17]

- Investigate the electroweak symmetry (relations from $SU(2)_L \times U(1)_Y$ SSB) using massive scattering amplitudes

  This talk

  [Christensen, Field '18; Aoude, Machado '19; Christensen, Field, Moore, Pinto '19; Durieux, TK, Shadmi, Weiss '19; Bachu, Yelleshpur '19]

Spinor-helicity formalism (massless scattering amplitudes) (1/2)

reviews *e.g.*, [Elvang, Huang '13, Dixon '13; Schwartz '14]

- Massless particle is an irreducible representations of the Poincaré group; particle $i = |p_i, h_i\rangle$

  $h = \pm 1/2, \pm 1$ is particle's helicity

- Massless *n*-pt amplitudes are given by $M_n(p_1^{h_1}, p_2^{h_2}, \ldots, p_n^{h_n})$ (all particles are incoming)

- Little-group (LG) is subgroup of the Lorentz group, which leaves p_i invariant; $p_i \to p_i$

- In $D = 4$, SO(2) $\simeq$ U(1) LG for massless particle

- Massless amplitudes are scaled by their helicities $\{h_1, h_2, \ldots\}$ under U(1) LG transformation

  Little group scaling; $M_n(p_1^{h_1}, \ldots, p_n^{h_n}) \to e^{2i\xi \sum h_i} M_n(p_1^{h_1}, \ldots, p_n^{h_n})$

Spinor-helicity formalism (massless scattering amplitudes) (2/2)

Lorentz group irreducible representation

	symbol	(A, B) $\hat{A}, \hat{B} = \frac{1}{2}(\hat{J} \pm i\hat{K})$	spinor-helicity formalism	
undotted spinor	$\lambda_{i,\alpha} = u_-(p_i), \bar{v}_-(p_i)$	**2**: (1/2, 0)	$\lvert i\rangle_\alpha \to e^{-i\xi}\lvert i\rangle_\alpha$ (under LG)	$\langle ij\rangle = -\langle ji\rangle$
dotted spinor	$\tilde{\lambda}_i^{\dot{\alpha}} = u_+(p_i), \bar{v}_+(p_i)$	**2***: (0, 1/2)	$\lvert i]^{\dot{\alpha}} \to e^{+i\xi}\lvert i]^{\dot{\alpha}}$ (under LG)	$\langle ii\rangle = [ii] = 0$
4-vector	p_i^μ	**2**×**2***: (1/2, 1/2)	$p_{i,\alpha\dot{\alpha}} = p_i^\mu \sigma_{\mu,\alpha\dot{\alpha}} = \lvert i\rangle_\alpha [i\rvert_{\dot{\alpha}}$	$\det p_{i,\alpha\dot{\alpha}} = p_i^2 = 0$
polarization vector	$\varepsilon_i^{\mu,\pm}$	constrained 4-vector $p_i \cdot \varepsilon_i^\pm = 0,\ \ \varepsilon_i^\pm \cdot (\varepsilon_i^\pm)^* = -1$ $\sum_{\lambda=\pm} \varepsilon_i^{\mu,\lambda}(\varepsilon_i^{\nu,\lambda})^* = -\eta^{\mu\nu}$	$\varepsilon_{i,\alpha\dot{\alpha}}^+ = \varepsilon_i^{\mu,+}\sigma_{\mu,\alpha\dot{\alpha}} = \sqrt{2}\frac{\lvert\zeta\rangle_\alpha [i\rvert_{\dot{\alpha}}}{\langle i\zeta\rangle}$ $\varepsilon_{i,\alpha\dot{\alpha}}^- = \varepsilon_i^{\mu,-}\sigma_{\mu,\alpha\dot{\alpha}} = \sqrt{2}\frac{\lvert i\rangle_\alpha [\zeta\rvert_{\dot{\alpha}}}{[i\zeta]}$	auxiliary spinor ζ corresponding gauge redundancy
⋮	⋮		⋮	

Little group scaling is powerful

- For example, let us reconsider $gg \to ggg$ amplitudes
- Little group scaling; $M_n(p_1^{h_1}, \ldots, p_n^{h_n}) \to e^{2i\xi \sum h_i} M_n(p_1^{h_1}, \ldots, p_n^{h_n})$

$$\mathcal{M}_5\left(1_g^-, 2_g^-, 3_g^+, 4_g^+, 5_g^+\right) \to e^{2i\xi} \mathcal{M}_5\left(1_g^-, 2_g^-, 3_g^+, 4_g^+, 5_g^+\right)$$

- Amplitude dimension: dim[M_5] = -1 (with all couplings are dimensionless),

  + momentum conservation: $\sum p_i = 0$

Form is unique ➡ $\mathcal{M}_5\left(1_g^-, 2_g^-, 3_g^+, 4_g^+, 5_g^+\right) \propto \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \langle 51 \rangle}$ $\quad |i\rangle_\alpha \to e^{-i\xi} |i\rangle_\alpha$ (under LG)

One can also derive

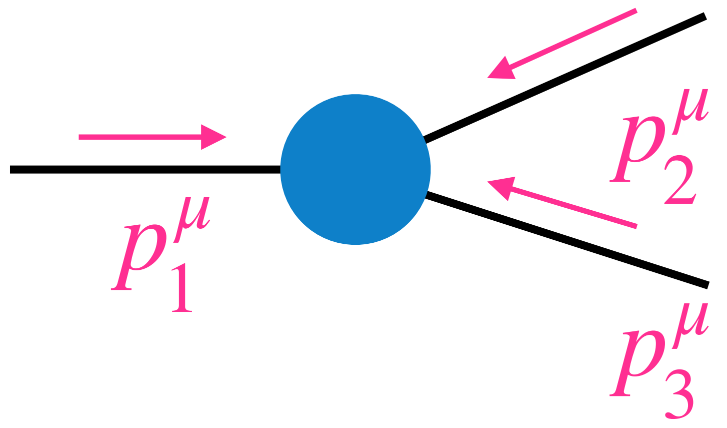
$$\mathcal{M}_5\left(1_g^+, 2_g^+, 3_g^+, 4_g^+, 5_g^+\right) = 0$$

$$\mathcal{M}_5\left(1_g^-, 2_g^+, 3_g^+, 4_g^+, 5_g^+\right) = 0$$

(For tree-level analysis)

Three-point amplitudes in scattering amplitudes (1/3)

- Let us consider massless three point:



- On-shell amplitude leads to

$$p_1^2 = p_2^2 = p_3^2 = 0, \quad p_1^\mu + p_2^\mu + p_3^\mu = 0$$

- Then

$$(p_1 + p_2)^2 = 2p_1 \cdot p_2 = (-p_3)^2 = 0$$

One can reach ➡ $p_i \cdot p_j = 0$ These momenta are very constrained

Three-point amplitudes in scattering amplitudes (2/3)

- Spinor-helicity formalism leads to (Note: $p_{i,\alpha\dot{\alpha}} = p_i^\mu \sigma_{\mu,\alpha\dot{\alpha}} = |i\rangle_\alpha [i|_{\dot{\alpha}}$)

$$\langle 12\rangle[21] = \mathrm{tr}(p_1^{\dot{\alpha}\alpha} p_{2,\alpha\dot{\alpha}}) = p_1^\mu p_2^\nu \mathrm{Tr}(\bar{\sigma}_\mu \sigma_\nu) = p_1^\mu p_2^\nu \times 2g_{\mu\nu} = 2p_1 \cdot p_2$$

One can reach ➡ $\langle 12\rangle[12] = \langle 13\rangle[13] = \langle 23\rangle[23] = 0$

- This condition corresponds to two different solutions

$|1\rangle \propto |2\rangle \propto |3\rangle$ (then $\langle ij\rangle = 0$) amplitudes is described by $[ij]$

Or $|1] \propto |2] \propto |3]$ (then $[ij] = 0$) amplitudes is described by $\langle ij\rangle$

- When the momenta are real, then $\langle ij\rangle^\dagger = [ji] = -[ij]; \rightarrow \langle ij\rangle = [ij] = 0$

Complex momentum is required for non-vanishing amplitudes

Three-point amplitudes in scattering amplitudes (3/3)

- Little group scaling and dimensional analysis (dim[M_3] = +1) can totally determine the form

$$\mathcal{M}(1^{h_1}, 2^{h_2}, 3^{h_3}) = g \begin{cases} [12]^{h_1+h_2-h_3}[23]^{h_2+h_3-h_1}[31]^{h_3+h_1-h_2} & \text{for } h_1+h_2+h_3>0 \\ \langle 12\rangle^{-h_1-h_2+h_3}\langle 23\rangle^{-h_2-h_3+h_1}\langle 31\rangle^{-h_3-h_1+h_2} & \text{for } h_1+h_2+h_3<0 \end{cases}$$

- For instance, scalar and fermions/vectors

$$\mathcal{M}(1^+_{\psi^c}, 2^+_{\psi}, 3_h) = y_\psi [12], \qquad \mathcal{M}(1^+_{\gamma}, 2^+_{\gamma}, 3_h) = [12]^2/M$$

- Fermions and vector $\mathcal{M}(1^+_{\psi^c}, 2^-_{\psi}, 3^+_g) = g_s\, T^3_{12}\, [13]^2/[12]$

- Three gluons, three gravitons

$$\mathcal{M}(1^+_g, 2^+_g, 3^-_g) = g_s f^{123}\frac{[12]^3}{[23][31]}, \qquad \mathcal{M}(1^+_G, 2^+_G, 3^-_G) = \frac{1}{M_p}\left(\frac{[12]^3}{[23][31]}\right)^2$$

"double copy (KLT)"
gravity = (YM)^2
[Kawai, Lewellen, Tye '86]

massless → **massive**

[Kleiss, Stirling ’85; Dittmaier ’98; Cohen, Elvang, Kiermaier ‘10]

↓ formalize/generalize for any mass and spin particles

[1709.04891]
Arkani-Hamed, Huang, Huang

Massive-spinor formalism (1/4) [Arkani-Hamed, Huang, Huang '17]

$$\det p_{i,\alpha\dot\alpha} = \det p_i\cdot\sigma = \overbrace{\begin{vmatrix} p_i^0+p_i^3 & p_i^1-ip_i^2 \\ p_i^1+ip_i^2 & p_i^0-p_i^3\end{vmatrix}}^{p_{i,\alpha\dot\alpha}} = (p_i^0)^2-(p_i^1)^2-(p_i^2)^2-(p_i^3)^2$$

$$= p_i^2 = 0 \longrightarrow = m^2 > 0$$

$p_{i,\alpha\dot\alpha}$: rank 1 → product of two vectors

$$p_{i,\alpha\dot\alpha} = |i\rangle_\alpha[i|_{\dot\alpha}$$

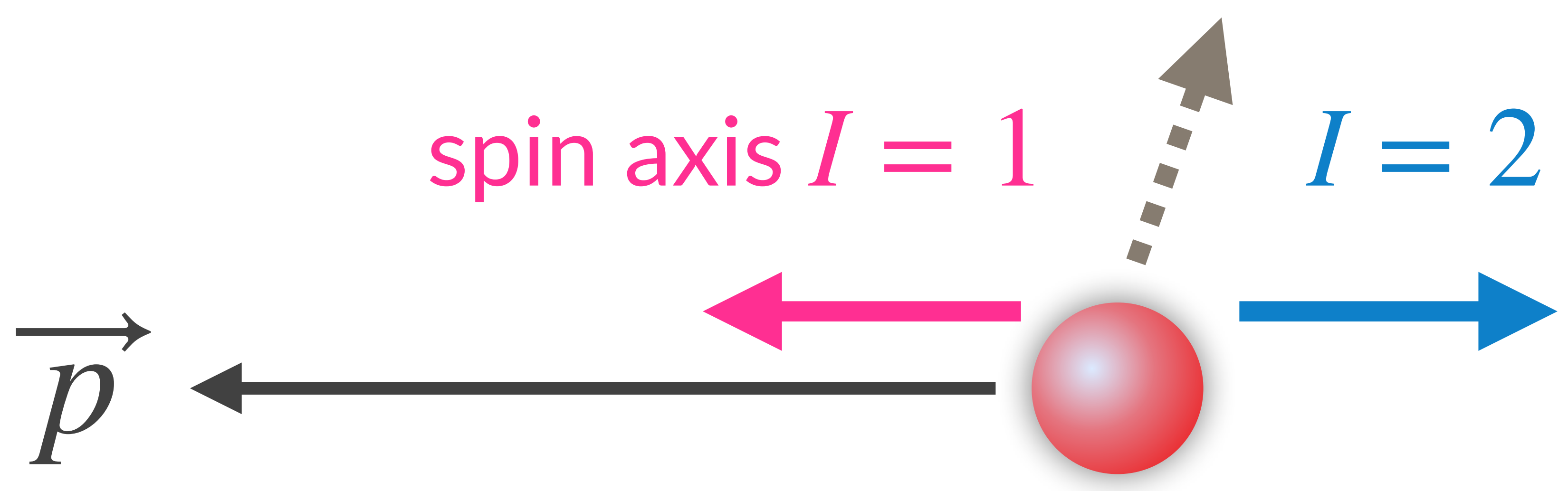
rank 2 → sum of two products of two vectors

$$p_{i,\alpha\dot\alpha} = |i^1\rangle_\alpha[i_1|_{\dot\alpha} + |i^2\rangle_\alpha[i_2|_{\dot\alpha} \equiv \sum_{I=1,2}|\mathbf{i}^I\rangle_\alpha[\mathbf{i}_I|_{\dot\alpha}$$

- In $D=4$, SO(3) $\simeq$ SU(2) LG for massive particles; leaves $p_{i,\alpha\dot\alpha}$ invariant; $p_{i,\alpha\dot\alpha}\to p_{i,\alpha\dot\alpha}$
- Amplitudes are transformed by SU(2) LGs (for massive external particles)
- **Bold spinors** $|\mathbf{i}^I\rangle, |\mathbf{i}^I]$ carry the SU(2) LG index $I=1,2$

Massive-spinor formalism (2/4)

- One can use the SU(2) LG rotation for the spin-quantization axis
- Convenient choice (for any spin particles):



Arbitrary spin polarization can be given by two opposite spin states

$$\begin{pmatrix} a \\ b \end{pmatrix} = a \begin{pmatrix} 1 \\ 0 \end{pmatrix} + b \begin{pmatrix} 0 \\ 1 \end{pmatrix} = a|+_z\rangle + b|-_z\rangle$$

- In this choice, in high energy limit, $I=1$ $(I=2)$ spinor corresponds to positive (negative) helicities
- Any choice of spin-quantization axis is possible in general ("SU(2) LG covariant")

Massive-spinor formalism (3/4)

	symbol	massive-spinor formalism
undotted spinor	$\lambda^s_{i,\alpha} = P_L u^I(p_i), \bar{v}^I(p_i) P_L$	$\lvert \mathbf{i}^I \rangle_\alpha \to W^I_J \lvert \mathbf{i}^J \rangle_\alpha$ (under LG)
dotted spinor	$\tilde{\lambda}^{s,\dot{\alpha}}_i = P_R u^I(p_i), \bar{v}^I(p_i) P_R$	$\lvert \mathbf{i}^I]^{\dot{\alpha}} \to \left(W^{-1}\right)^I_J \lvert \mathbf{i}^J]^{\dot{\alpha}}$ (under LG)
4-vector	p^μ_i	$p_{i,\alpha\dot{\alpha}} = p^\mu_i \sigma_{\mu,\alpha\dot{\alpha}} = \sum_{I=1,2} \lvert \mathbf{i}^I \rangle_\alpha [\mathbf{i}_I \rvert_{\dot{\alpha}}$
polarization vector	$\varepsilon^{\mu,\pm,L}_i$	$\varepsilon^{IJ}_{i,\alpha\dot{\alpha}} = \varepsilon^{\mu,\pm,L}_i \sigma_{\mu,\alpha\dot{\alpha}} = \sqrt{2} \dfrac{\lvert \mathbf{i}^I \rangle_\alpha [\mathbf{i}^J \rvert_{\dot{\alpha}}}{m}$
⋮	⋮	⋮

- $\langle \mathbf{i}^I \mathbf{j}^J \rangle = -\langle \mathbf{j}^J \mathbf{i}^I \rangle$
- $\langle \mathbf{i}^I \mathbf{i}^J \rangle = [\mathbf{i}^I \mathbf{i}^J] = 0$
- $\det p_{i,\alpha\dot{\alpha}} = p_i^2 = m^2$
- no auxiliary spinor

Massive-spinor formalism (4/4)

- Equations of motion (EOM) $\sim$ "chirality flip"

$$\bar{p}_i|\mathbf{i}^I\rangle = m|\mathbf{i}^I] ,\quad p_i|\mathbf{i}^I] = m|\mathbf{i}^I\rangle ,\quad \langle\mathbf{i}^I|p_i = -m[\mathbf{i}^I| ,\quad [\mathbf{i}^I|\bar{p}_i = -m\langle\mathbf{i}^I|$$

- Massive polarization vectors [Gauthier Durieux, **TK**, Yael Shadmi, Yaniv Weiss '19]

$$\varepsilon^{IJ}_{i,\alpha\dot\alpha} = \sqrt{2}\frac{|\mathbf{i}^I\rangle_\alpha[\mathbf{i}^J|_{\dot\alpha}}{m}\quad \begin{cases} \varepsilon^{+}_{i,\alpha\dot\alpha} = \varepsilon^{11}_{i,\alpha\dot\alpha} = \sqrt{2}\frac{|\mathbf{i}^1\rangle_\alpha[\mathbf{i}^1|_{\dot\alpha}}{m} \xrightarrow{m\to 0} \sqrt{2}\frac{|\zeta\rangle_\alpha[i|_{\dot\alpha}}{\langle i\zeta\rangle} = \varepsilon^{+}_{i,\alpha\dot\alpha} \\ \varepsilon^{-}_{i,\alpha\dot\alpha} = \varepsilon^{22}_{i,\alpha\dot\alpha} = \sqrt{2}\frac{|\mathbf{i}^2\rangle_\alpha[\mathbf{i}^2|_{\dot\alpha}}{m} \longrightarrow \sqrt{2}\frac{|i\rangle_\alpha[\zeta|_{\dot\alpha}}{[i\zeta]} = \varepsilon^{-}_{i,\alpha\dot\alpha} \\ \varepsilon^{L}_{i,\alpha\dot\alpha} = \varepsilon^{\overset{\sqcap}{12}}_{i,\alpha\dot\alpha} = \frac{|\mathbf{i}^1\rangle_\alpha[\mathbf{i}^2|_{\dot\alpha} + |\mathbf{i}^2\rangle_\alpha[\mathbf{i}^1|_{\dot\alpha}}{m} \longrightarrow \sim\frac{p_{i,\alpha\dot\alpha}}{m} = \mathcal{O}\left(\frac{E}{m}\right) \end{cases}$$

massless polarizations

well-known energy growth

Factor $1/\sqrt{2}$ (in *L* mode) corresponds to Clebsch-Gordan; we modify the original formalism

$$\Rightarrow\quad p_i\cdot\varepsilon^{\pm}_i = 0,\qquad \varepsilon^{\pm,L}_i\cdot(\varepsilon^{\pm,L}_i)^* = -1\,,\qquad \sum_{\lambda=\pm,L}\varepsilon^{\mu,\lambda}_i(\varepsilon^{\nu,\lambda}_i)^* = -\left(\eta^{\mu\nu} - \frac{p_{i,\mu}p_{i,\nu}}{m^2}\right)$$

corresponds to "unitary gauge"

Our several results

Our strategy

- Spectrum: different masses + massless photon

$$\psi\,(\psi^c), Z, W^{\pm}, h \quad +\gamma$$

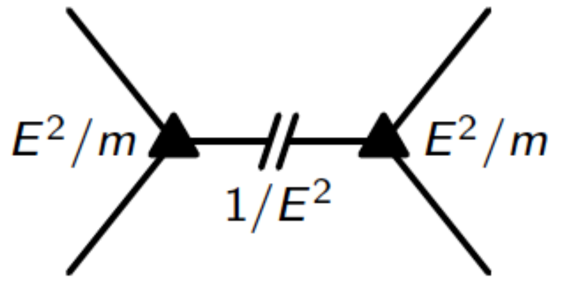
- We do not impose $SU(2)_L\times U(1)_Y$ symmetry, but impose only $U(1)_{EM}$

- [LGs ⊂ Lorentz ⊂ Poincaré] + [locality] [Arkani-Hamed, Huang, Huang '17]

  + [perturbative unitarity ⊂ unitarity] [Gauthier Durieux, **TK**, Yael Shadmi, Yaniv Weiss '19]

  - For three-pt amplitudes,

    E^2/m has to be forbidden;



$\sim E^2/m^2$ unacceptable energy growth

  - For full four-pt amplitudes, E/m has to be forbidden

Note that: there is no longitudinal mode in massless scattering amplitudes

Three-point: hhZ

- Result (LGs + locality): [Durieux, **TK**, Shadmi, Weiss '19]

SU(2) LG indices I, J are implicit

$$\mathcal{M}_3(\mathbf{1}_h, \mathbf{2}_h, \mathbf{3}_Z) \propto \langle \mathbf{3}(\mathbf{1}-\mathbf{2})\mathbf{3}] \quad = \langle \mathbf{3}|(p_1 - p_2)|\mathbf{3}] \text{ (notation)}$$

The scalars $\mathbf{1}$ and $\mathbf{2}$ have to be asymmetric:
when the scalars $\mathbf{1}$ and $\mathbf{2}$ are identical, this amplitude must vanish at the all order

One-line proof to "why $\rho^0 \to 2\pi^0$ is forbidden in our world"

A good application of massive scattering amplitude!

$\rho(770)^0$ **decays**

Γ_6	$\pi^+\pi^-$	(100)%
Γ_7	$\pi^+\pi^-\gamma$	$(9.9 \pm 1.6) \times 10^{-3}$
Γ_8	$\pi^0\gamma$	$(4.7 \pm 0.6) \times 10^{-4}$
Γ_9	$\eta\gamma$	$(3.00 \pm 0.21) \times 10^{-4}$
Γ_{10}	$\pi^0\pi^0\gamma$	$(4.5 \pm 0.8) \times 10^{-5}$
Γ_{11}	$\mu^+\mu^-$	$(4.55 \pm 0.28) \times 10^{-5}$
Γ_{12}	e^+e^-	$(4.72 \pm 0.05) \times 10^{-5}$

Three-point: $\psi^c \psi Z$ (1/3)

- Result (LGs + locality + good massless limit): [Durieux, **TK**, Shadmi, Weiss '19]

$$\mathcal{M}(\mathbf{1}_{\psi^c}, \mathbf{2}_{\psi}, \mathbf{3}_Z) = \frac{c^{RRR}_{\psi^c\psi Z}}{\bar{\Lambda}}[\mathbf{13}][\mathbf{23}] + \frac{c^{LR0}_{\psi^c\psi Z}}{m_Z}\langle\mathbf{13}\rangle[\mathbf{23}] + \frac{c^{RL0}_{\psi^c\psi Z}}{m_Z}[\mathbf{13}]\langle\mathbf{23}\rangle + \frac{c^{LLL}_{\psi^c\psi Z}}{\bar{\Lambda}}\langle\mathbf{13}\rangle\langle\mathbf{23}\rangle$$

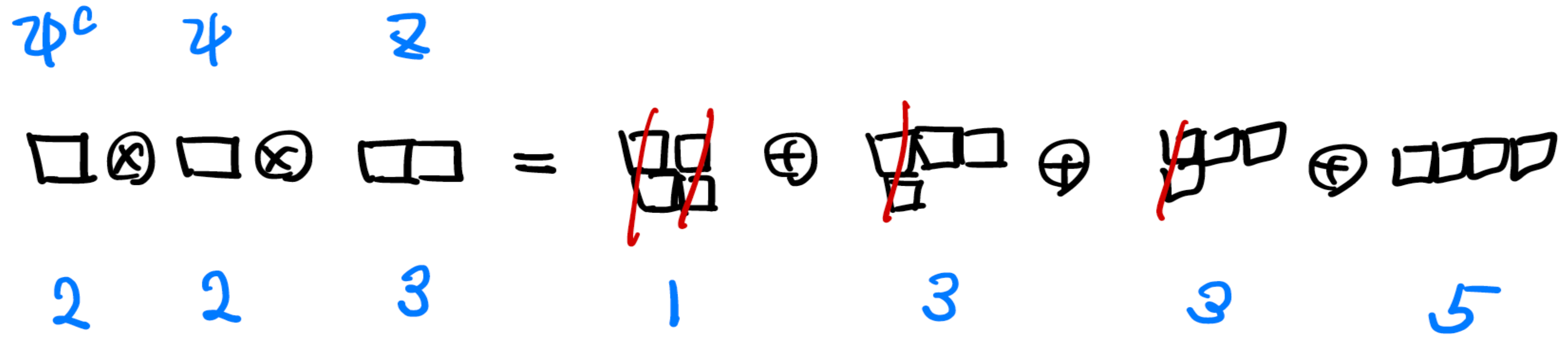
We observed 4 spinor structures

- Angular momentum conservation (in three-pt amplitudes) [Costa, Penedones, Poland, Rychkov '11]

# of irreps of sum of three spins = # of independent spinors in three-pt amplitudes

⟶ 4 combinations is expected $2 \otimes 2 \otimes 3 = 1 \oplus 3 \oplus 3 \oplus 5$

decomposition by the Young tableau



Three-point: $\psi^c \psi Z$ (2/3)

- High-energy limit ($E \to \infty$ with $E/\bar{\Lambda}$ fixed):

$$\mathcal{M}(\mathbf{1}_{\psi^c}, \mathbf{2}_{\psi}, \mathbf{3}_Z) = \frac{c^{RRR}_{\psi^c\psi Z}}{\bar{\Lambda}}[\mathbf{13}][\mathbf{23}] + \frac{c^{LR0}_{\psi^c\psi Z}}{m_Z}\langle\mathbf{13}\rangle[\mathbf{23}] + \frac{c^{RL0}_{\psi^c\psi Z}}{m_Z}[\mathbf{13}]\langle\mathbf{23}\rangle + \frac{c^{LLL}_{\psi^c\psi Z}}{\bar{\Lambda}}\langle\mathbf{13}\rangle\langle\mathbf{23}\rangle$$

IR

high energy

$-m_Z$ $\quad m_Z \quad \frac{m_\psi}{\sqrt{2}} \quad -\frac{m_\psi}{\sqrt{2}} \quad \frac{m_\psi}{\sqrt{2}} \quad m_Z \quad -m_Z$

helicity	$+++$	$-+-$	$-++$	$--0$	$++0$	$+--$	$+-+$	$---$	UV
	$[13][23]$	$\frac{\langle 13\rangle^2}{\langle 12\rangle}$	$\frac{[23]^2}{[12]}$	$\langle 12\rangle$	$[12]$	$\frac{\langle 23\rangle^2}{\langle 12\rangle}$	$\frac{[13]^2}{[12]}$	$\langle 13\rangle\langle 23\rangle$	

2 non-renormalizable dipole

6 renormalizable amplitudes

We can obtain 8 (=6+2) high-energy amplitudes **from 4 independent coefficients**

Conversely, the **bold form** is called **"IR unification"** [Arkani-Hamed, Huang, Huang '17]

Three-point: $\psi^c \psi Z$ (3/3)

- Focus on the longitudinal mode

$$\mathcal{M}(1^-_{\psi^c}, 2^-_{\psi}, 3^0_Z) \to +\langle 12\rangle\, (c^{LR0}_{\psi^c\psi Z} - c^{RL0}_{\psi^c\psi Z})\, m_\psi/\sqrt{2}m_Z\,,$$

$$\mathcal{M}(1^+_{\psi^c}, 2^+_{\psi}, 3^0_Z) \to -[12]\, (c^{LR0}_{\psi^c\psi Z} - c^{RL0}_{\psi^c\psi Z})\, m_\psi/\sqrt{2}m_Z\,,$$

We observed a relative sign, which corresponds to "pseudo-scalar coupling γ_5"

(consistent with NG boson equivalence theorem)

- For $c^{LR0} \neq c^{RL0}$, m_ψ must tend to vanish when $m_Z \to 0$: fermion mass has the same origin as Z

  (reproduce an expectation from spontaneous electroweak symmetry breaking)

- For $c^{LR0} = c^{RL0}$ (vector fermion case), the longitudinal component vanishes, and one can take $m_Z \to 0$ with finite m_ψ

Three-point: W^+W^-Z (1/4)

- Result (LGs + locality): 11 spinor structures [Arkani-Hamed, Huang, Huang '17]

  → Schouten identity, and momentum conservation → 8 spinor structures

  $|\mathbf{i}\rangle\langle\mathbf{jk}\rangle + |\mathbf{j}\rangle\langle\mathbf{ki}\rangle + |\mathbf{k}\rangle\langle\mathbf{ij}\rangle = 0 \qquad p_1 + p_2 + p_3 = 0$

- Furthermore, we observe a non-trivial massive spinor identity [Durieux, **TK**, Shadmi, Weiss '19, + Machado '20]

$$m_1\langle\mathbf{12}\rangle\langle\mathbf{13}\rangle[\mathbf{23}] + m_2\langle\mathbf{12}\rangle[\mathbf{13}]\langle\mathbf{23}\rangle + m_3[\mathbf{12}]\langle\mathbf{13}\rangle\langle\mathbf{23}\rangle = m_1[\mathbf{12}][\mathbf{13}]\langle\mathbf{23}\rangle + m_2[\mathbf{12}]\langle\mathbf{13}\rangle[\mathbf{23}] + m_3\langle\mathbf{12}\rangle[\mathbf{13}][\mathbf{23}]$$

  → 7 spinor structures (final)

- Angular momentum conservation:

  # of irreps of sum of three spins = # of independent spinors in three-pt amplitudes

  → 7 combinations is expected $\quad 3 \otimes 3 \otimes 3 = 1 \oplus 3 \oplus 3 \oplus 3 \oplus 5 \oplus 5 \oplus 7$

- 7 form factors for general WWZ coupling [Hagiwara, Peccei, Zepenfeld, Hikasa '86]

Three-point: W^+W^-Z (2/4)

- + perturbative unitarity [Durieux, **TK**, Shadmi, Weiss '19]

  $\bar{\Lambda}$ dependence of 7 spin structures is fully determined

  c_{WWZ} : dimensionless

$$\mathcal{M}(\mathbf{1}_{W^+}, \mathbf{2}_{W^-}, \mathbf{3}_Z) = 2\frac{c_{WWZ}}{m_Z m_W}\left(\frac{m_Z}{m_W}\langle\mathbf{12}\rangle[\mathbf{13}][\mathbf{23}] + [\mathbf{12}]\langle\mathbf{13}\rangle[\mathbf{23}] + [\mathbf{12}][\mathbf{13}]\langle\mathbf{23}\rangle\right)$$
$$+ \frac{c_{WWZ}^{[L0]0}}{m_Z\bar{\Lambda}}\langle\mathbf{12}\rangle\left(\langle\mathbf{13}\rangle[\mathbf{23}] - [\mathbf{13}]\langle\mathbf{23}\rangle\right) + \frac{c_{WWZ}^{\{L0\}0}}{m_Z\bar{\Lambda}}\langle\mathbf{12}\rangle\left(\langle\mathbf{13}\rangle[\mathbf{23}] + [\mathbf{13}]\langle\mathbf{23}\rangle\right)$$
$$+ \frac{c_{WWZ}^{[R0]0}}{m_Z\bar{\Lambda}}[\mathbf{12}]\left(\langle\mathbf{13}\rangle[\mathbf{23}] - [\mathbf{13}]\langle\mathbf{23}\rangle\right) + \frac{c_{WWZ}^{\{R0\}0}}{m_Z\bar{\Lambda}}[\mathbf{12}]\left(\langle\mathbf{13}\rangle[\mathbf{23}] + [\mathbf{13}]\langle\mathbf{23}\rangle\right)$$
$$+ \frac{c_{WWZ}^{RRR}}{\bar{\Lambda}^2}[\mathbf{12}][\mathbf{13}][\mathbf{23}] + \frac{c_{WWZ}^{LLL}}{\bar{\Lambda}^2}\langle\mathbf{12}\rangle\langle\mathbf{13}\rangle\langle\mathbf{23}\rangle\,.$$

non-trivial single renormalizable structure

$\mathbf{1} \leftrightarrow \mathbf{2}$ symmetric: $\not{C}$

- $m_Z \to 0$ limit provides $M_3(\mathbf{1}_{W^+}, \mathbf{2}_{W^-}, 3_\gamma^\pm)$ with 5 spin structures

  consistent with angular momentum analysis: $3 \otimes 3 \otimes 2 = 2 \oplus 2 \oplus 4 \oplus 4 \oplus 6$

Three-point: W^+W^-Z (3/4)

- Furthermore, we match the massive scattering amplitudes onto the SMEFT in the broken phase.

compare our massive amplitudes to the SMEFT

$$\mathcal{L}_{\rm SMEFT} = \mathcal{L}_{\rm SM} + \sum_i \frac{C_i}{\Lambda^2}\mathcal{O}_i$$

Warsaw basis (dimension-six SMEFT)
[Grzadkowski, Iskrzynski, Misiak, Rosiek '10]

Warsaw basis in the broken phase
[Dedes, Materkowska, Paraskevas, Rosiek, Suxho '17]

result of 7 coefficients

$$2c_{WWZ} = -\sqrt{2}\frac{\bar{g}^2}{\sqrt{\bar{g}^2+\bar{g}'^2}} + \sqrt{2}\frac{\bar{g}^3\bar{g}'}{(\bar{g}^2+\bar{g}'^2)^{3/2}}\frac{v^2}{\Lambda^2}C_{\varphi WB}\,,$$

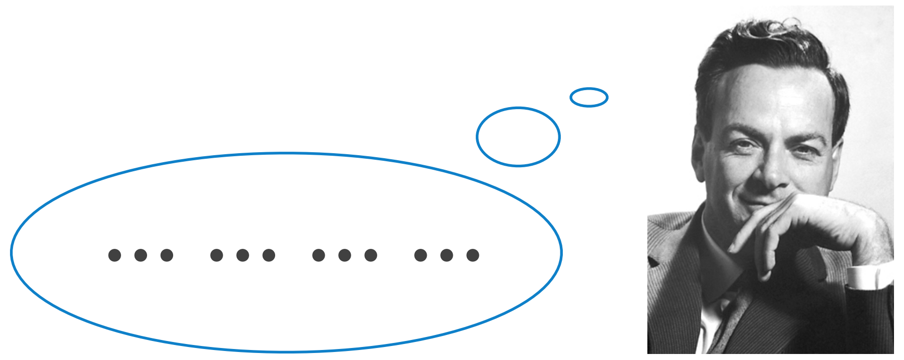
$$c_{WWZ}^{[L0]0} = c_{WWZ}^{[R0]0} = 0\,,$$

$$\frac{c_{WWZ}^{\{R0\}0}}{m_Z\bar{\Lambda}} = -\frac{1}{\sqrt{2}m_W m_Z}\frac{\bar{g}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}\frac{v^2}{\Lambda^2}\left(C_{\varphi WB}+iC_{\varphi\tilde{W}B}\right)\,,$$

$$c_{WWZ}^{\{L0\}0} = \left(c_{WWZ}^{\{R0\}0}\right)^*\,,$$

$$\frac{c_{WWZ}^{RRR}}{\bar{\Lambda}^2} = -3\sqrt{2}\frac{\bar{g}}{\sqrt{\bar{g}^2+\bar{g}'^2}}\frac{1}{\Lambda^2}\left(C_W+iC_{\tilde{W}}\right)\,,$$

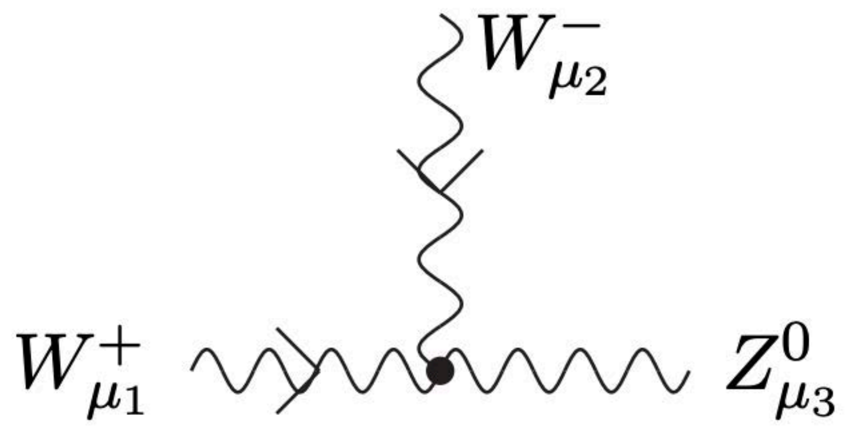
$$c_{WWZ}^{LLL} = \left(c_{WWZ}^{RRR}\right)^*\,.$$



Three-point: W^+W^-Z (4/4)

- One example

dimension-six operator: $\varepsilon^{IJK}W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$



Feynman rule for this operator

$$-\frac{6i\bar{g}}{\sqrt{\bar{g}^2+\bar{g}'^2}}\frac{C_W}{\Lambda^2}\left(p_3^{\mu_1}p_1^{\mu_2}p_2^{\mu_3}-p_2^{\mu_1}p_3^{\mu_2}p_1^{\mu_3}+\eta_{\mu_1\mu_2}\left(p_1^{\mu_3}p_2\cdot p_3-p_2^{\mu_3}p_1\cdot p_3\right)\right.$$
$$\left.+\eta_{\mu_2\mu_3}\left(p_2^{\mu_1}p_1\cdot p_3-p_3^{\mu_1}p_1\cdot p_2\right)+\eta_{\mu_3\mu_1}\left(p_3^{\mu_2}p_1\cdot p_2-p_1^{\mu_2}p_2\cdot p_3\right)\right)$$

massive-spinor formalism

$$-3\sqrt{2}\frac{\bar{g}}{\sqrt{\bar{g}^2+\bar{g}'^2}}\frac{C_W}{\Lambda^2}$$
$$\times\left([\mathbf{12}][\mathbf{13}][\mathbf{23}]+\langle\mathbf{12}\rangle\langle\mathbf{13}\rangle\langle\mathbf{23}\rangle\right)$$

@Wikipedia

All EW three-points are bootstrapped and mapped

[Durieux, **TK**, Shadmi, Weiss '19]

LGs
+
Locality
+
Unitarity
+
$U(1)_{EM}$

bootstrap (our work) →

$\psi^c \psi Z$ (example 2)
$\psi^c \psi \gamma$
$\psi^c \psi' W$
$\psi^c \psi h$
$ZZh,\ WWh$
$\gamma\gamma h,\ \gamma Zh$
$hhZ,\ hh\gamma$ (example 1)
hhh
WWZ (example 3)
$WW\gamma$
$ZZZ,\ ZZ\gamma,\ Z\gamma\gamma,\ \gamma\gamma\gamma$

← map (our work)

[Dedes, Materkowska, Paraskevas, Rosiek, Suxho '17]

broken phase
$v \neq 0$
canonically normalized

← Warsaw basis

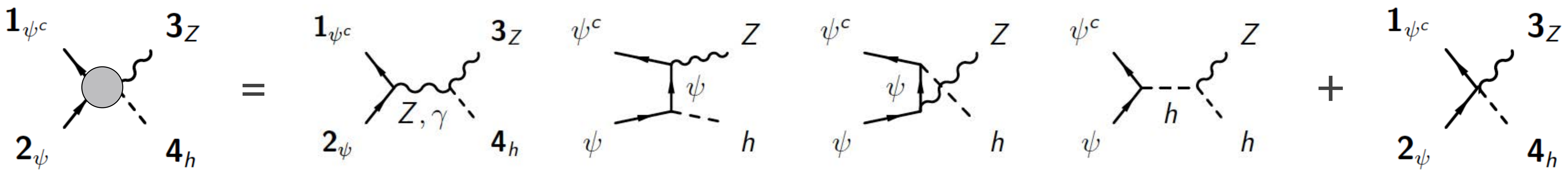
$$\mathcal{L}_{\rm SMEFT} = \mathcal{L}_{\rm SM} + \sum_i \frac{C_i}{\Lambda^2}\mathcal{O}_i$$

They are important building blocks for the SMEFT computations with on-shell techniques

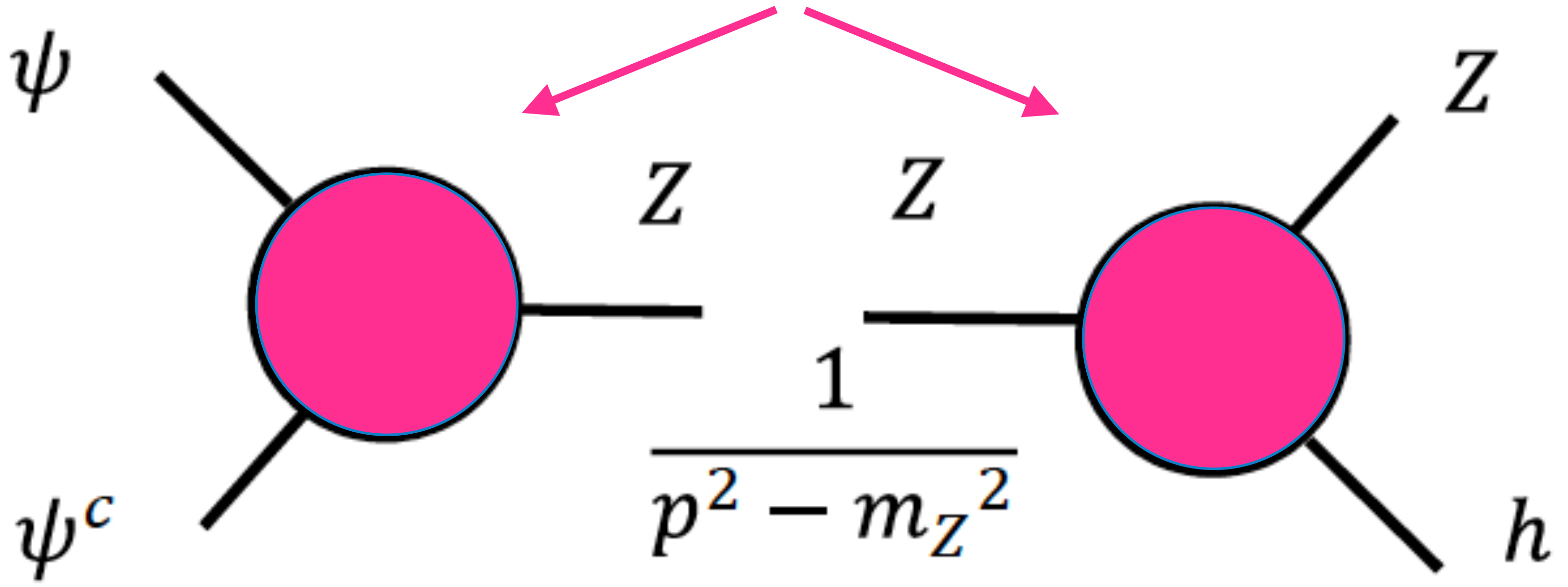
Four-point: $\psi^c \psi Z h$ (1/3)

"factorizable" contribution

"non-factorizable" contribution
(contact term)



LG analysis for thee-point amplitudes



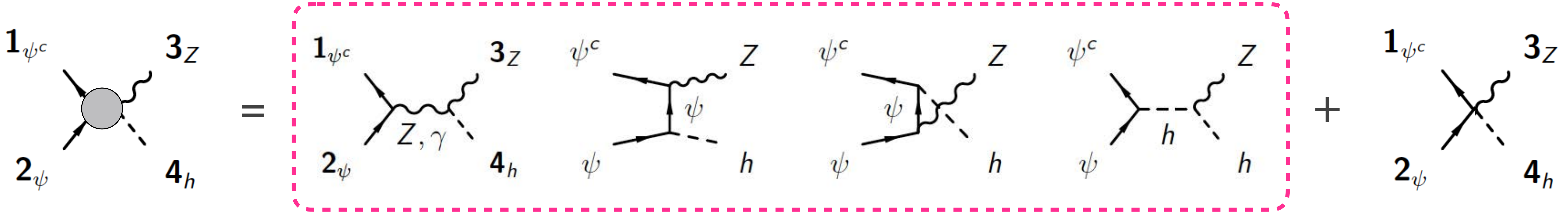
gluing

LG analysis for four-point amplitudes

Four-point: $\psi^c \psi Z h$ (2/3)

"factorizable" contribution

"non-factorizable" contribution
(contact term)



+ perturbative unitarity requires [Durieux, **TK**, Shadmi, Weiss '19]

$$(--00): \quad -\langle 12\rangle\,(c^{RL0}_{\psi^c\psi Z} - c^{LR0}_{\psi^c\psi Z})\,(c^{00}_{ZZh}\, m_\psi/2m_Z - c^{LL}_{\psi^c\psi h})/\sqrt{2}m_Z \quad = 0 + \mathcal{O}(m/\bar{\Lambda})$$

$$(++00): \quad +[12]\,(c^{RL0}_{\psi^c\psi Z} - c^{LR0}_{\psi^c\psi Z})\,(c^{00}_{ZZh}\, m_\psi/2m_Z - c^{RR}_{\psi^c\psi h})/\sqrt{2}m_Z \quad = 0 + \mathcal{O}(m/\bar{\Lambda})$$

"Unitarity sum rules" **would be reproduced**

[Gunion, Haber, Wodka '91; Grinstein, Murphy, Pirtskhalava, Uttayarat '14; Nagai, Tanahashi, Tsumura '15]

either vector-like fermion: $c^{RL0}_{\psi^c\psi Z} = c^{LR0}_{\psi^c\psi Z}$

or Higgs mechanism: $c^{RR}_{\psi^c\psi h} = c^{00}_{ZZh} m_\psi/2m_Z = c^{LL}_{\psi^c\psi h}$

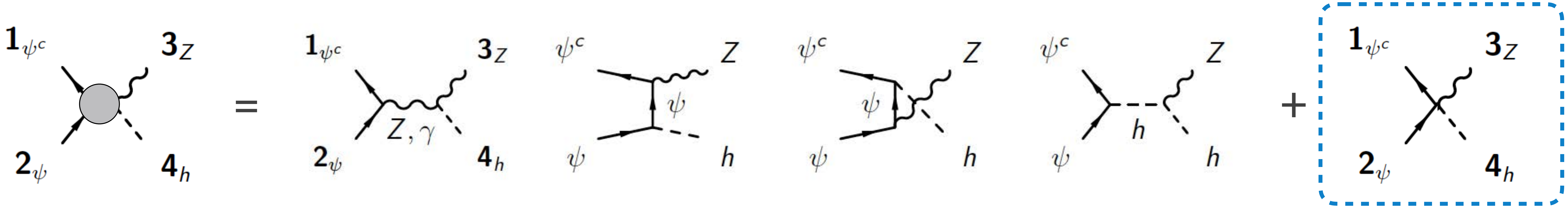
up to $\mathcal{O}(m/\bar{\Lambda})$

consistent with study for $t\bar{t}Zh$ amplitude [Maltoni, Mantani, Mimasu '19]

Four-point: $\psi^c \psi Z h$ (3/3)

"factorizable" contribution

"non-factorizable" contribution
(contact term)



- Result (LGs + locality etc.): 14 spinor structures

- Furthermore, we observe a non-trivial massive spinor identity [Durieux, **TK**, Shadmi, Weiss '19,+ Machado '20]

$$[\mathbf{12}]\langle\mathbf{3123}\rangle = 2\,[\mathbf{12}]\,\langle\mathbf{3}\,\{\mathbf{1}\,(p_2\cdot p_3) - \mathbf{2}\,(p_1\cdot p_3)\}\,\mathbf{3}]/m_3$$
$$- 2(p_1\cdot p_2)[\mathbf{13}][\mathbf{23}] - m_1[\mathbf{321}\rangle[\mathbf{23}] - m_2[\mathbf{312}\rangle[\mathbf{13}].$$

⟶ 12 spinor structures (final) ⟶ soft Higgs limit — 4 spinor structures

all terms are non-renormalizable

$p_h \to 0;\ p_1 + p_2 + p_3 \to 0$ — reproduce $\psi^c \psi Z$ result

Outlook

- Map EW four-point amplitudes onto the SMEFT
- Renormalization group evolution, running coupling in massive scattering amplitudes?
- An application: infrared photon/gluon corrections?

  [Soft Matters, or the Recursions with Massive Spinors, Falkowski, Machado '20]

- γ_5? anomalous triangle diagram?
- New direction? Monopole scattering amplitudes

  [Scattering Amplitudes for Monopoles: Pairwise Little Group and Pairwise Helicity, Csaki, Hong, Shirman, Telem, Terning, Waterbury ‘20]

Conclusions

- The powerful scattering amplitude approach can avoid gauge redundancy and operator redundancy
- We clarified a few details in the massive-spinor formalism, and bootstrapped all the EW three-point amplitudes, as well as the four-point amplitudes
- We mapped all EW three-point amplitudes onto the SMEFT
- We observed the emergence of the EW relations from the perturbative unitarity

- We paved the way for the SMEFT computations in the on-shell formalism

Backup