Quarkonium production near threshold

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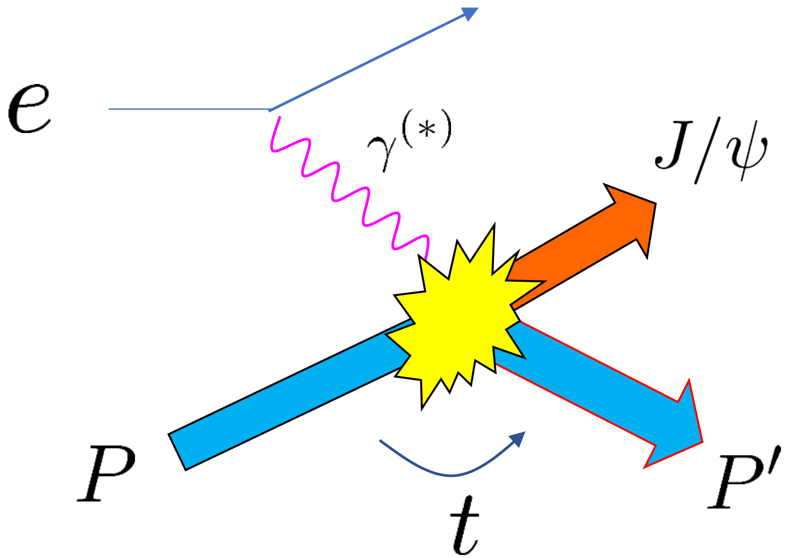
Outline

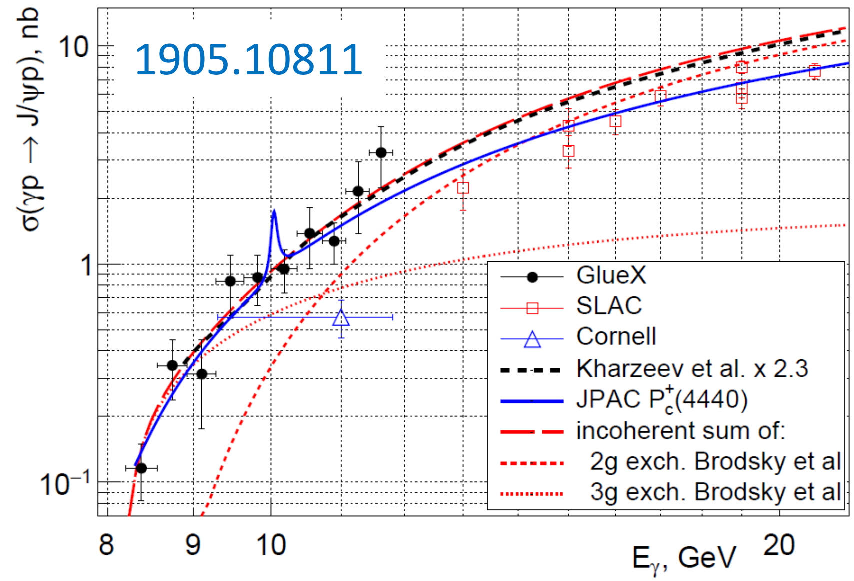
- Near-threshold quarkonium production
- Gravitational form factors
- Photo-production
- Lepto-production
- Recent theory developments

Refs. YH, Yang, 1808.02163
YH, Rajan, Yang, 1906.00894
Boussarie, YH, 2004.12715
YH, Strikman, 2102.12631

+ a flurry of theory papers recently!

Photo-production of J/ψ , Υ near threshold



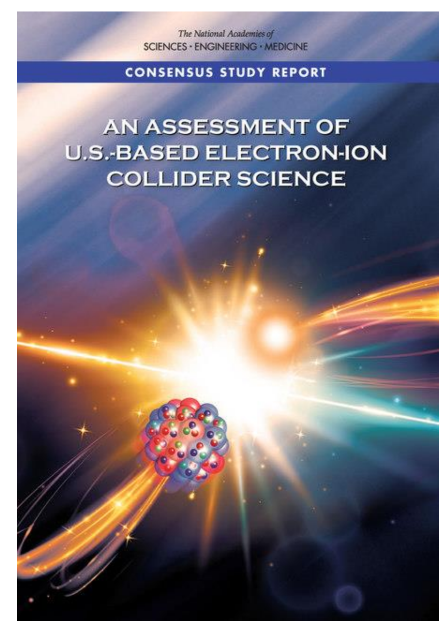


Ongoing experiments at Jlab (GlueX collaboration)
Future measurement at EIC, EIcC?

One of the main motivations:

Insight into the origin of proton mass

Is the connection clear, can be made precise?
What else can one learn from this process?
Is lepto-production $Q^2 \neq 0$ also useful?



Kinematics ($Q^2 = 0$)

CoM energy $W^2 = (P+q)^2$ very low, barely enough to produce a J/ψ

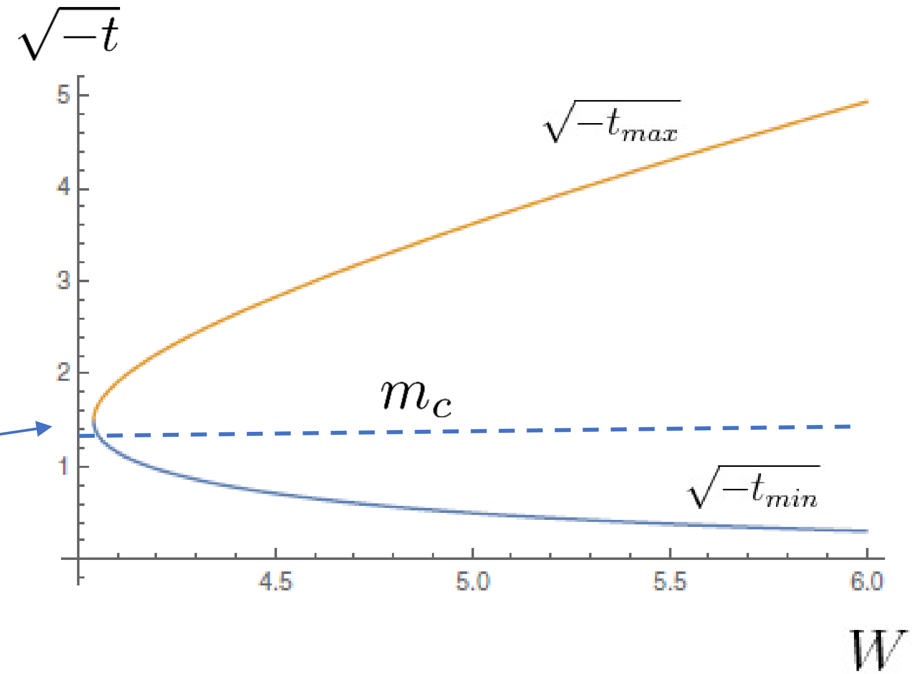
$W_{th} = M_p + M_{J/\psi} \approx 4.04\text{GeV}$ or $E_\gamma \approx 8.2\ \text{GeV}$ in the proton rest frame

Total γp cross section

$$\sigma_{tot}(W) = \int_{t_{min}}^{t_{max}} dt \frac{d\sigma}{dt}$$

Momentum transfer large at the threshold,

$$t_{\min} = -\frac{MM_\psi^2}{M+M_\psi} \approx -(1.5\ \text{GeV})^2$$



as large as the charm quark mass!

Υ cannot be produced at JLab energy → EIC, EIcC, RHIC

Threshold photo-production at RHIC

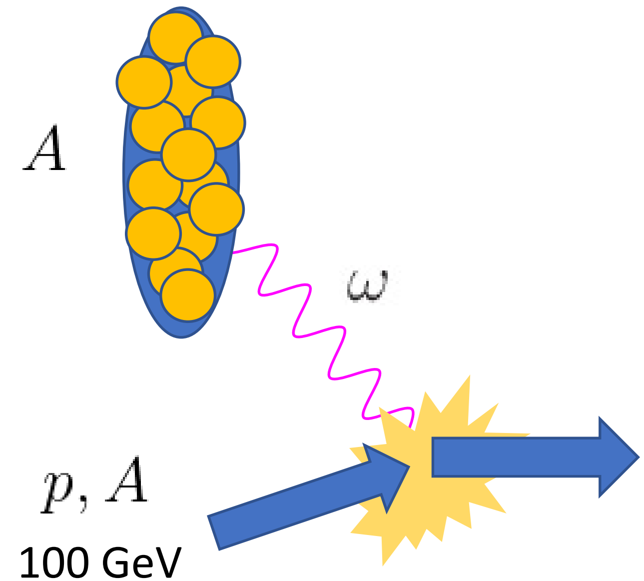
YH, Rajan, Yang (2019)

RHIC, ultra-peripheral pA collisions (UPC)

Cross section enhanced by $Z^2 = 6241$

Typical γp energy at 200 GeV runs

$\sqrt{(P+q)^2} \sim 28\,\mathrm{GeV}$ enough to produce Υ



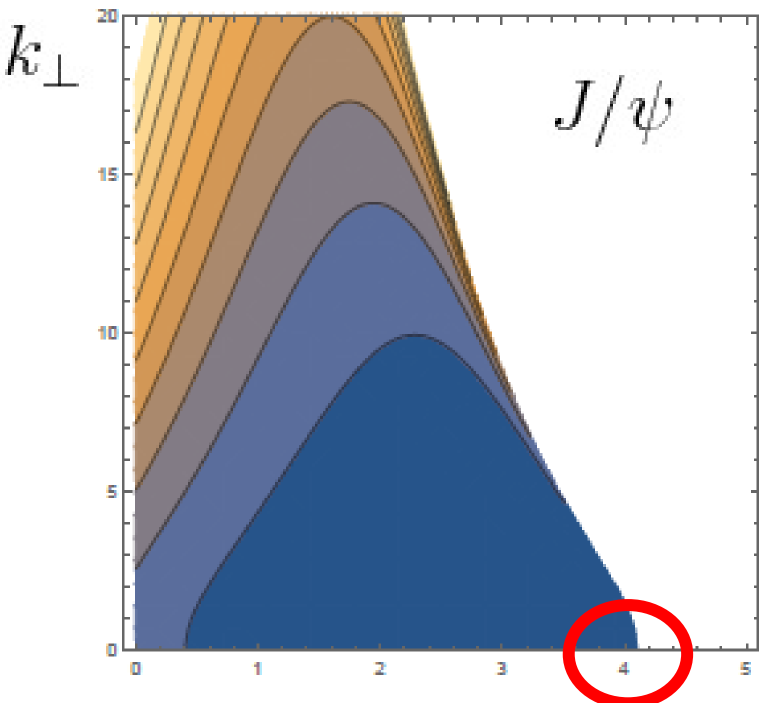
Photon energy ω distribution

$$\frac{dN}{d\omega} = \frac{2Z^2\alpha_{em}}{\pi\omega}\left[\zeta K_0(\zeta)K_1(\zeta) - \frac{\zeta^2}{2}(K_1^2(\zeta) - K_0^2(\zeta)\right] \qquad \zeta = \omega\frac{R_p+R_A}{\gamma}$$

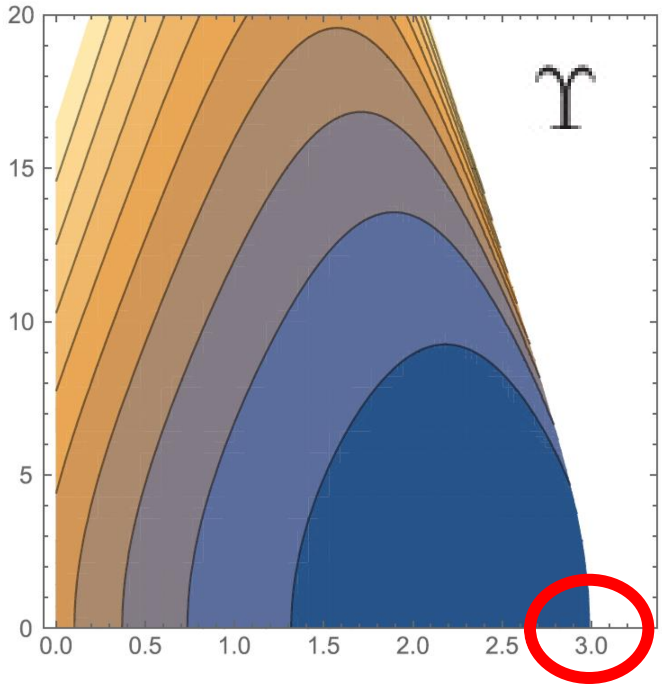
Threshold region $\omega > \dfrac{M_Q(2M+M_Q)}{2(E_P+P)} \approx \begin{cases} 0.27\ \mathrm{GeV} & (\Upsilon) \\ 0.039\ \mathrm{GeV} & (J/\psi) \end{cases}$

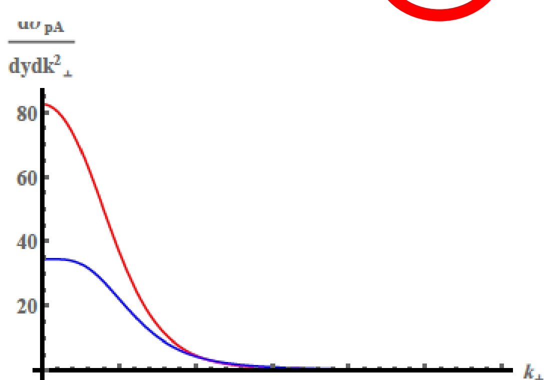
Where to search at RHIC

Quarkonium (and its decay products) to be found at very forward rapidity, very low kt.



rapidity





Measurable after the completion of STAR forward upgrades?

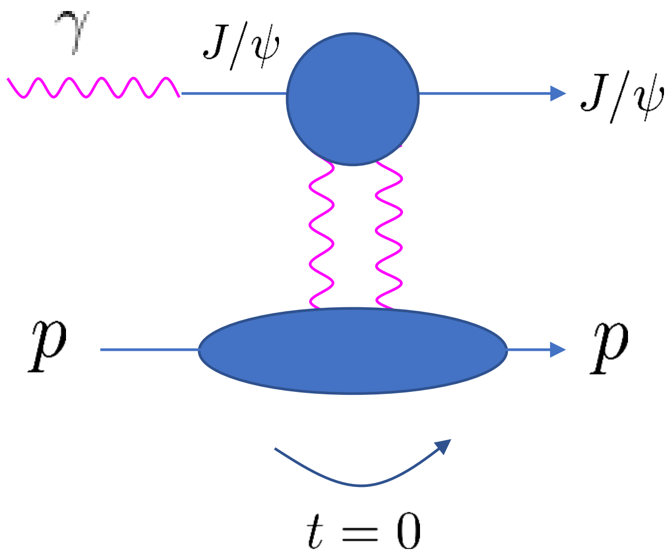
Theory approach 1: Vector meson dominance + heavy quark OPE

Kharzeev, Satz, Syamtomov, Zinovjev (1998)

Use VMD to reduce $\gamma p \to J/\psi p$ to forward $J/\psi p \to J/\psi p$ ($t = 0$)

Interaction between J/ψ and proton via local gluonic operators in the limit $m_c \to \infty$

Peskin (1979), Luke, Manohar, Savage (1992)



$$\mathcal{L}_{\text{int}} = \sum_v \frac{1}{\Lambda_Q^3} \left(P_v^\dagger P_v - V_{\mu v}^\dagger V_v^\mu \right) \left(c_E \mathcal{O}_E + c_B \mathcal{O}_B \right)$$

$$\mathcal{O}_E = T_{\text{gluon}}^{\mu\nu} v_\mu v_\nu + \frac{2\pi}{b_Q \alpha_s(\Lambda_Q)} T_\alpha^\alpha ,$$

$$\mathcal{O}_B = T_{\text{gluon}}^{\mu\nu} v_\mu v_\nu - \frac{2\pi}{b_Q \alpha_s(\Lambda_Q)} T_\alpha^\alpha .$$

$\left. \frac{d\sigma}{dt} \right|_{t=0}$ sensitive to $\langle P | F^2 | P \rangle$ QCD trace anomaly (gluon condensate)

Theory approach 2: Gravitational form factors

In VMD approach, no discussion of t-dependence.
But there's a wealth of physics in t-dependence!

t-dependence must be power-law, not exponential.
Amplitude related to `2-gluon form factors' Frankfurt, Strikman (2002)

More precisely, gravitational form factors YH, Yang (2018); YH, Boussarie (2020)

Very roughly,
$$\frac{d\sigma}{dt} \sim A^2(t) \sim \frac{1}{(1-t/m^2)^4}$$

Warning:

People often use this simple form, but the actual relation is much more complicated YH, Yang (2018); YH, Boussarie (2020)

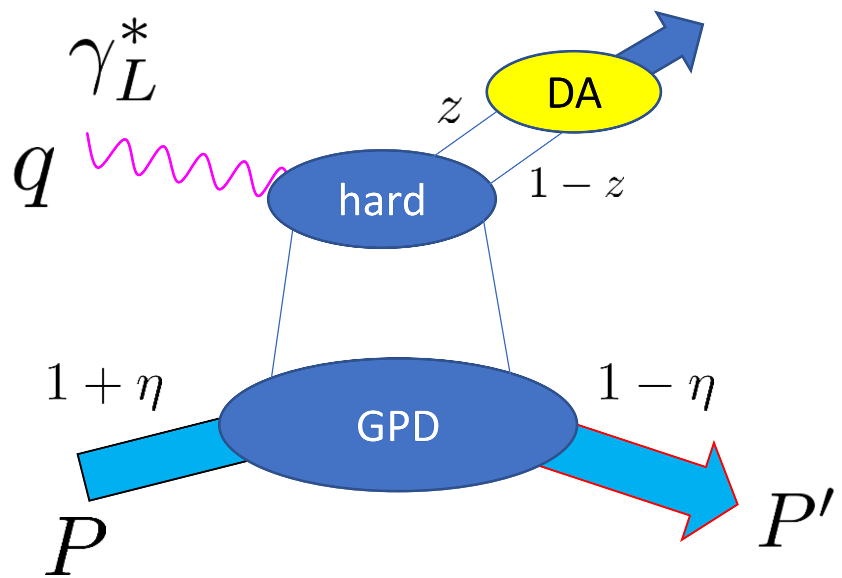
See also a recent objection Sun, Tong, Yuan (2021)

Theory approach 3: QCD factorization at $\begin{matrix} Q^2 \to \infty \\ M_{QQ} \to \infty \end{matrix}$?

For a longitudinal virtual photon, factorization theorem exists
in the (generalized) Bjorken limit   Collins, Frankfurt, Strikman (1997)

$$Q^2 \to \infty, \; 2P \cdot q \to \infty$$

with $x_B = \frac{Q^2}{2P \cdot q}$ fixed



What if

$$W^2 = (P+q)^2 = m_N^2 + 2P \cdot q - Q^2$$

is fixed near the threshold?

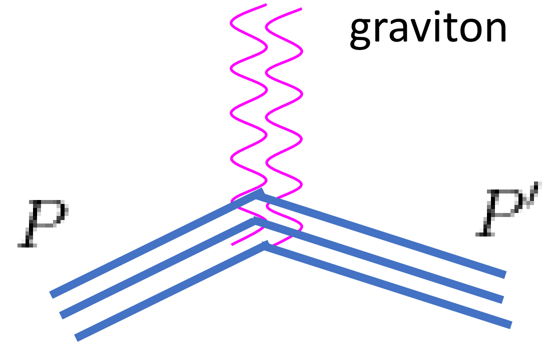
Can we apply GPD factorization approach to the threshold region?

YH, Strikman (2021)
Guo, Ji, Liu (2021)

Nucleon gravitational form factors

Ji (1995)

$$\langle P'|T^{\mu\nu}_{q,g}|P\rangle = \bar{u}(P')\left[A_{q,g}\gamma^{(\mu}\bar{P}^{\nu)} + B_{q,g}\frac{\bar{P}^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha}{2M} + D_{q,g}\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{4M} + \bar{C}_{q,g}Mg^{\mu\nu}\right]u(P)$$



$$\Delta = P' - P$$

$$t = \Delta^2$$

All the form factors are interesting and measurable!

$A_{q,g}$ Momentum fraction

$B_{q,g}$ Ji sum rule

$D_{q,g}$ pressure

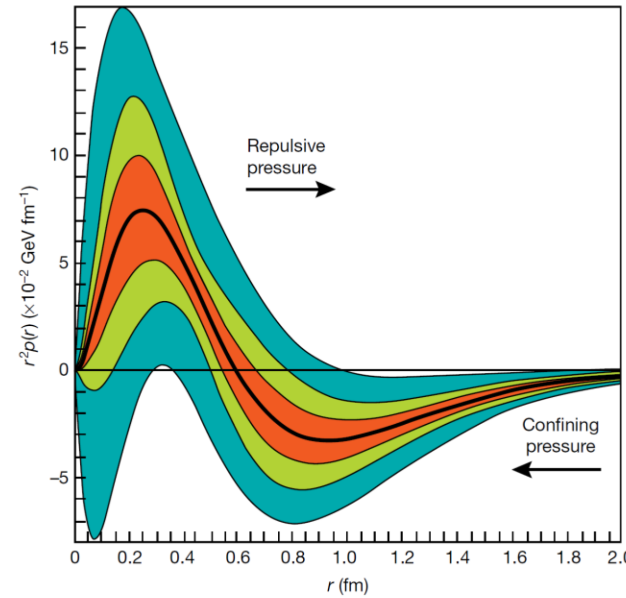
$\bar{C}_{q,g}$ trace anomaly

$$\frac{1}{2} = J_q + J_g$$

$$J_{q,g} = \frac{1}{2}(A_{q,g} + B_{q,g})$$

D-term: the last global unknown

Burkert, Elouadrhiri, Girod (2018)



$D(t=0)$ is a conserved charge of the nucleon, just like mass and spin!

May be interpreted as `internal force' after Fourier transforming

$$D = D_u + D_d + D_s + D_g + \cdots$$

$D_{u,d}$ from DVCS, related to the subtraction constant in the dispersion relation for the Compton form factor

Teryaev (2005)

$$\mathrm{Re}\mathcal{H}_q(\xi,t) \;=\; \frac{1}{\pi}\int_{-1}^{1} dx \mathrm{P}\frac{\mathrm{Im}\mathcal{H}_q(x,t)}{\xi - x} + 2\int_{-1}^{1} dz \frac{D_q(z,t)}{1-z} \qquad \int_{-1}^{1} dz z D_q(z,t) = D_q(t)$$

Need a significant lever-arm in Q^2 to disentangle different moments → EIC

D_g → $J/\psi, \Upsilon$ threshold production

D_s → ϕ threshold production

$\bar{C}_{q,g}$ related to the QCD trace anomaly, origin of mass

YH, Rajan, Tanaka (2018)

in $\overline{\mathrm{MS}}$ scheme, 2-loop result

$$\bar{C}_q^R(\mu) = -\frac{1}{4}\left(\frac{n_f}{4C_F+n_f}+\frac{2n_f}{3\beta_0}\right)+\frac{1}{4}\left(\frac{2n_f}{3\beta_0}+1\right)\frac{\langle P|\left(m\bar{\psi}\psi\right)_R|P\rangle}{2M^2}$$
$$-\frac{4C_F A_q^R(\mu_0)+n_f\left(A_q^R(\mu_0)-1\right)}{4(4C_F+n_f)}\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\frac{8C_F+2n_f}{3\beta_0}}$$
$$+\frac{\alpha_s(\mu)}{4\pi}\left[\frac{n_f\left(-\frac{34C_A}{27}-\frac{49C_F}{27}\right)}{4\beta_0}+\frac{\beta_1 n_f}{6\beta_0^2}\right.$$
$$\left.+\frac{1}{4}\left(\frac{n_f\left(\frac{34C_A}{27}+\frac{157C_F}{27}\right)}{\beta_0}+\frac{4C_F}{3}-\frac{2\beta_1 n_f}{3\beta_0^2}\right)\frac{\langle P|\left(m\bar{\psi}\psi\right)_R|P\rangle}{2M^2}\right]+\cdots,$$

nucleon sigma-term

Can be traded for gluon condensate $\langle P|F^2|P\rangle$

$$\simeq -0.146 - 0.25\left(A_q^R(\mu_0)-0.36\right)\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\frac{50}{81}} - 0.01\alpha_s(\mu)$$
$$+\left(0.306+0.08\alpha_s(\mu)\right)\frac{\langle P|\left(m\bar{\psi}\psi\right)_R|P\rangle}{2M^2},$$

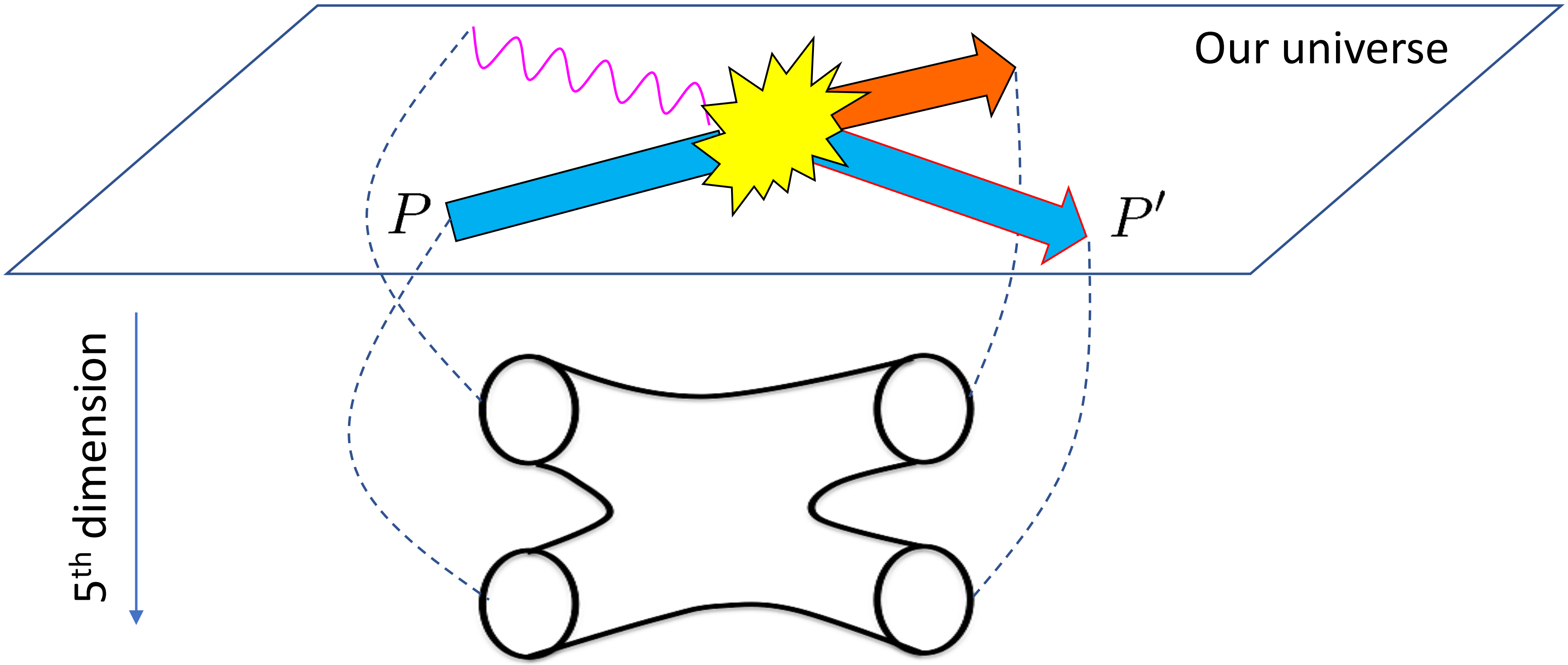
Asymptotic value in the chiral limit $(n_f = 3)$

3-loop result available Tanaka (2018)

Theory approach 4: AdS/CFT

YH, Yang (2018)
Mamo, Zahed (2019)

Scattering of hadrons in QCD(-like theories)
$\approx$ scattering of closed strings in asymptotically AdS_5



$$\langle P|\epsilon\cdot J(0)|P'k\rangle \approx -\frac{2\kappa^2}{f_\psi R^3}\int_0^{z_m} dz \frac{\delta S_{D7}(q,k,z)}{\delta g_{\mu\nu}}\frac{z^2R^2}{4}\langle P|T^{gTT}_{\mu\nu}|P'\rangle$$ ← graviton exch.

$$+\frac{2\kappa^2}{f_\psi R^3}\frac{3}{8}\int_0^{z_m} dz \frac{\delta S_{D7}(q,k,z)}{\delta \phi}\frac{z^4}{4}\langle P|\frac{1}{4}F^{\mu\nu}_a F^a_{\mu\nu}|P'\rangle$$ ← dilaton

Fitting the GlueX data

YH, Rajan, Yang (2019)

$$\langle p'|F^2|p\rangle = \left[K_g(A_g+4\bar{C}_g)+K_q(A_q+4\bar{C}_q)+(K_gB_g+K_qB_q-3K_gD_g-3K_qD_q)\frac{\Delta^2}{4m_N^2}\right]m_N\bar{u}(p')u(p),$$

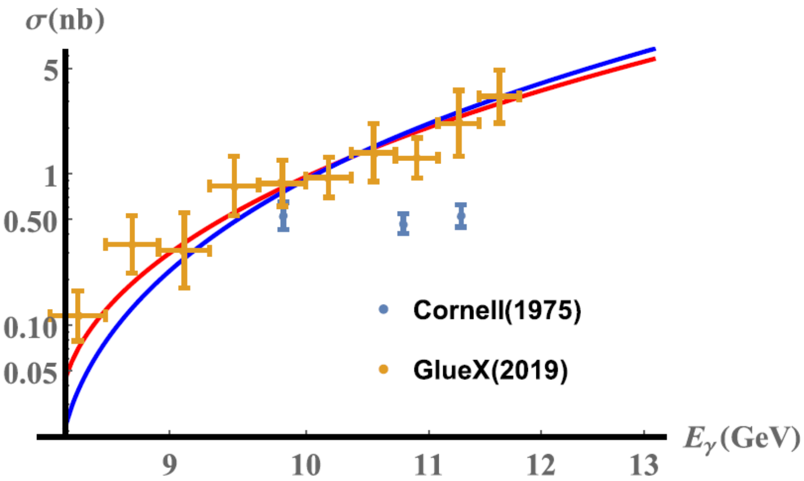
Assume dipole form factor for $A_g, \bar{C}_g$, tripole for D_g

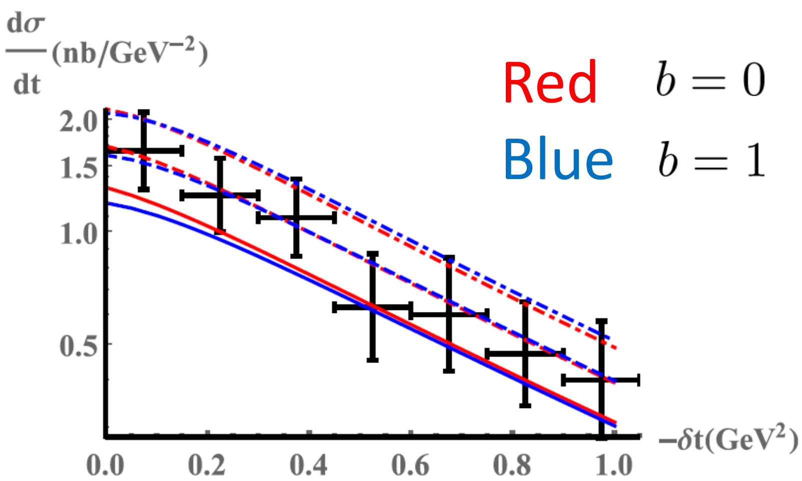
$$A_g(t)=\frac{A_g(0)}{(1-t/m_A^2)^2},\qquad A_q(t)=\frac{1-A_g(0)}{(1-t/m_A^2)^2},$$

$$D_g(t)=\frac{D_g(0)}{(1-t/m_C^2)^3},\qquad \bar{C}_g(t)=\frac{\bar{C}_g(0)}{(1-t/m_A^2)^2}.$$

Impact of gluon condensate

$$\langle P|\frac{\beta}{2g}F^2|P\rangle = 2M^2(1-b)$$



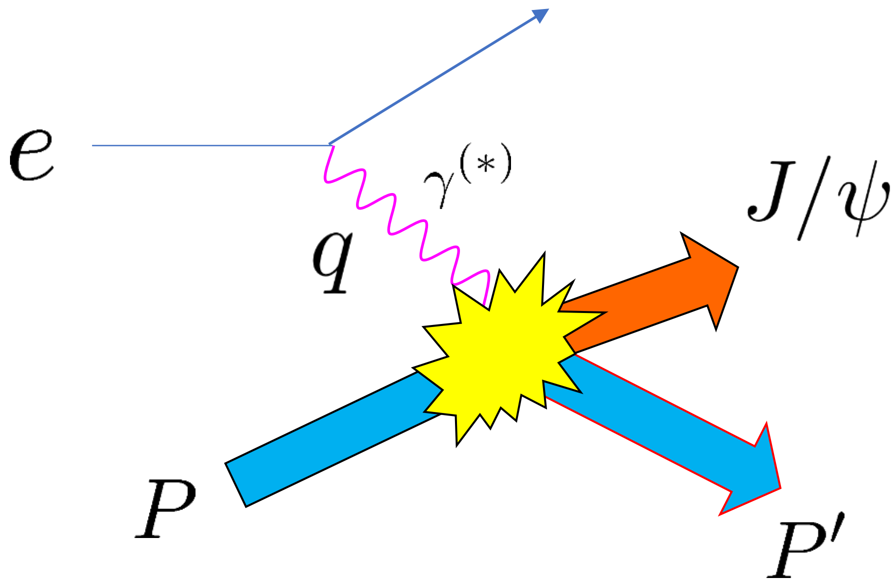


Red $b=0$

Blue $b=1$

Threshold leptoproduction at high-Q^2

Boussarie, YH (2020)



Better chance to use perturbative approaches

$Q^2 < 10\mathrm{GeV}^2$ at JLab.

ideal for EIC low energy runs.

Threshold production can occur even when $S_{ep},\ Q^2$ are large

$$W^2 = y(S_{ep} - m_N^2) + m_N^2 - Q^2$$

Momentum transfer larger $\quad |t_{th}| = \dfrac{m_N(m_V^2 + Q^2)}{m_N + m_V}$

Relation to the ep cross section $\quad \dfrac{d\sigma}{dW dQ^2} = \dfrac{\alpha_{em}}{4\pi} \dfrac{W(W^2 - m_N^2)}{(P\cdot\ell)^2 Q^2 (1-\epsilon)} \int dt \dfrac{d\sigma}{dt}$

OPE approach

Boussarie, YH (2020)

Use the LSZ reduction formula to relate $\langle p'k|J^\nu_{em}(0)|p\rangle$ to the charm current correlator

$$ee_f\epsilon^*_\mu(k)i\int d^4xd^4ye^{ik\cdot x-iq\cdot y}\langle p'|\mathrm{T}\{\bar{c}\gamma^\mu c(x)J^\nu_{em}(y)\}|p\rangle$$
$$\approx\frac{g_{\gamma J/\psi}}{k^2-M^2+iM\Gamma}\int d^4ye^{-iq\cdot y}\langle p'k|J^\nu_{em}(y)|p\rangle,$$

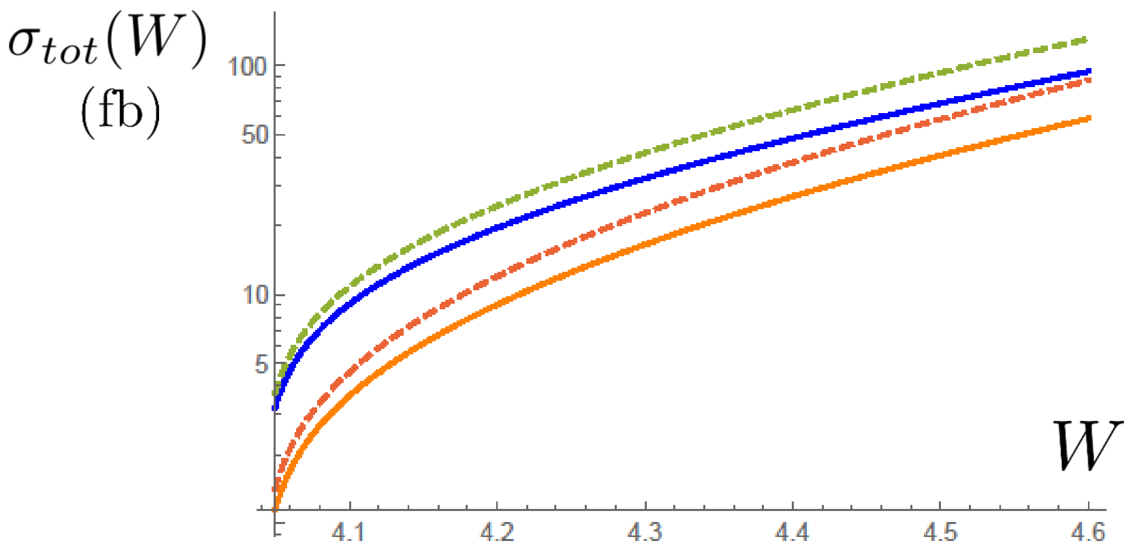
Do the OPE at $Q^2\gg|t|,M^2_{QQ}$ and keep two-gluon operators.

$$i\int d^4re^{ir\cdot q}\bar{c}\gamma^\mu c(0)\bar{c}\gamma^\nu c(-r)$$
$$\approx-\frac{\alpha_s(\mu_R)}{3\pi q^2}\left[2\ln(-q^2/\mu_R^2)\left\{\left(g^{\mu\alpha}-\frac{q^\mu q^\alpha}{q^2}\right)\left(g^{\nu\beta}-\frac{q^\nu q^\beta}{q^2}\right)+\frac{q^\alpha q^\beta}{q^2}\left(g^{\mu\nu}-\frac{q^\mu q^\nu}{q^2}\right)\right\}\hat{T}^g_{\alpha\beta}(0)\right.$$
$$\left.-2\frac{q^\alpha q^\beta}{q^2}\left(g^{\mu\nu}-\frac{q^\mu q^\nu}{q^2}\right)\hat{T}^g_{\alpha\beta}(0)+3\frac{q_\alpha q_\beta}{q^2}F^{\mu\alpha}F^{\nu\beta}(0)\right],$$

gluon EMT (traceless part)

sensitive to the trace anomaly

J/ψ $Q^2 = 64\,\mathrm{GeV}^2$ $\sqrt{S_{ep}} = 20\,\mathrm{GeV}$ (plots revised from 2004.12715)



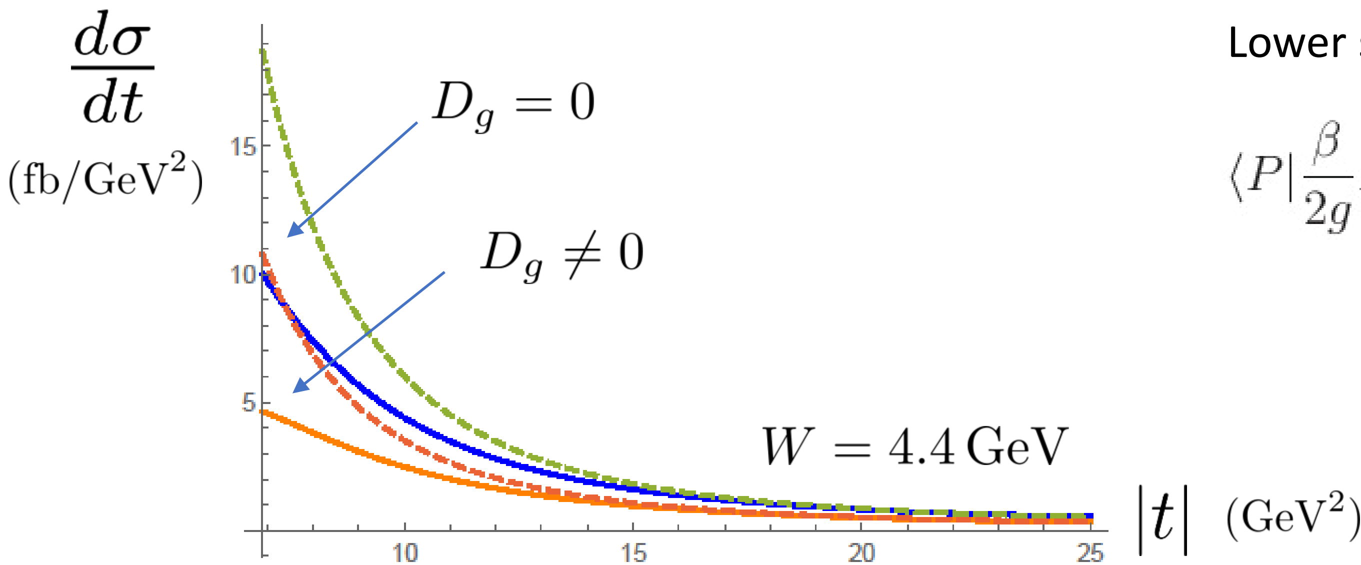
Dashed curves: without gluon D-term

Solid curves: with gluon D-term

Upper solid $b = 1$

Lower solid $b = 0$

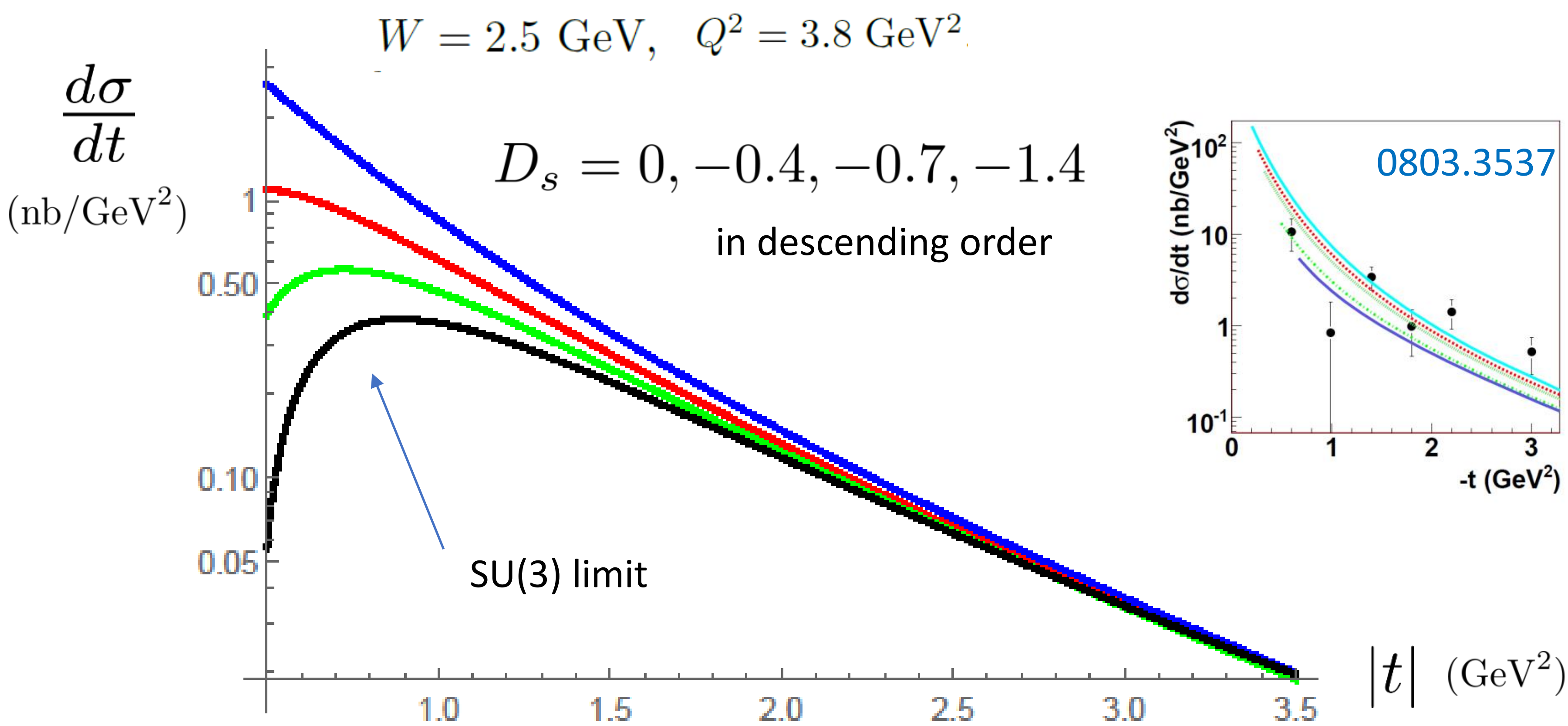
$$\langle P | \frac{\beta}{2g} F^2 | P \rangle = 2M^2(1 - b)$$



Strangeness D-term from ϕ -leptoproduction

YH, Strikman (2021)

Keep $T_s^{\mu\nu}$ in OPE



Possible flattening or bump in the $|t| < 1\,\text{GeV}^2$ region due to the strangeness D-term!

Can we make threshold business precision physics?

In my paper with Renaud, we only included dimension 4 operators

$T_g^{\mu\nu}$ twist-2, spin-2

F^2 twist-4 ← Nonpertubatively enhanced by $1/\alpha_s$
due to trace anomaly

.

$$\langle P|T^\mu_\mu|P\rangle \sim \langle P|\alpha_s F^2|P\rangle = M_N^2$$

What about twist-2, higher-spin operators? They are parametrically of the same order.

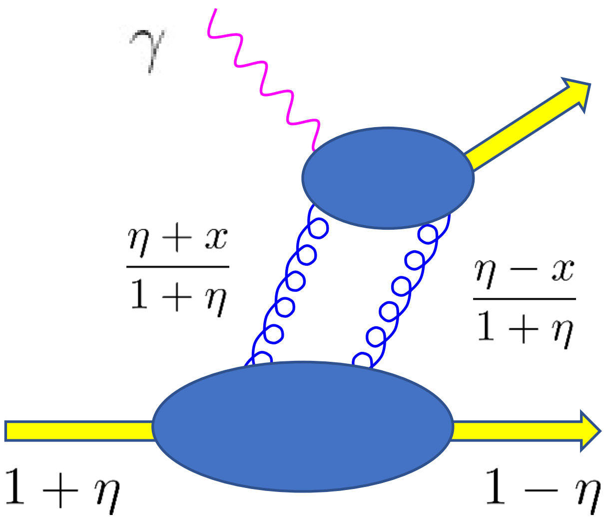
$$F^{+\alpha}(D^+)^{j-2}F^+_{\alpha} \qquad j > 2$$

→ Use large leverage in Q^2 to disentangle different spin contributions

Exactly the same problem arises when trying to extract $D_{u,d}$ in DVCS

GPD factorization for threshold production?

In GPD-type approach, all twist-2 operators are automatically summed (but not twist-4 F^2)



Amplitude proportional to Compton form factor

$$\int_{-1}^{1}\frac{dx}{x}\left(\frac{1}{\eta-x-i\epsilon}-\frac{1}{\eta+x-i\epsilon}\right)H_g(x,\eta,t)$$

Gluon GPD

Skewness $\eta=\frac{P^+-P'^+}{P'^++P^+}$

At large- Q^2 , threshold region corresponds to

$$x_B\approx\eta\approx 1$$

Proton makes a full stop!

Spin-two (energy momentum tensor) dominance

YH, Strikman, 2102.12631
Guo, Ji, Liu, 2103.11506

If (and only if) $\eta \approx 1$, one can Taylor expand.

$$\frac{1}{1-x} = 1 + x + x^2 + \cdots$$ ← convergence radius is 1

$$\int_{-1}^{1} \frac{dx}{x}\left(\frac{1}{\eta - x - i\epsilon} - \frac{1}{\eta + x - i\epsilon}\right) H_g(x,\eta,t) \approx 2\int dx (1 + x^2 + x^4 + \cdots) H_g(x,\eta,t)$$

spin=2 (energy momentum tensor) → 1; spin=4 → x^2; spin-6 → x^4

When $Q^2 \to \infty$, $M_{QQ} \to \infty$, use the asymptotic form $H_g(x,\eta=1) \approx (1-x^2)^2$

all spins $$\int dx \frac{H_g(x,\eta=1,t)}{1-x^2} \sim \int_0^1 dx \frac{(1-x^2)^2}{1-x^2} = \frac{2}{3}$$

spin-2 only $$\int_0^1 dx (1-x^2)^2 = \frac{8}{15}$$ ← 80% of the total! (100% in AdS/CFT)

Conclusions

Threshold quarkonium production is a new, exciting frontier.
Can be studied at Jlab, RHIC, EICs…

Connection to GPD at $\eta = 1$. Energy momentum tensor rules!

Unique opportunity to probe gluon gravitational form factors,
D-term and trace anomaly.

Recently significant progress towards 1st-principle calculations