

Diffraction in DIS

With prospects at EIC

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March 29 , 2021

Outline

- Introduction: diffraction in optics
- Diffraction in hadronic collisions. Example: elastic vector meson production
- Inclusive diffraction
- Prospects for EIC

EIC Yellow Report, 2103.05419

EIC White paper 1212.1701

Armesto, Newman, Slominski, Stasto 1901.09076

W. Slominski, 2nd Yellow Report Workshop, Pavia, 2021

Note: I will focus on ep/eA. Not pp.

Diffraction in optics



PHYSICO-MATHESIS DE LVMINE. COLORIBVS. ET IRIDE.

ALIISQVE ADNEXIS

LIBRI DVO.

In quorum Primo afferuntur Noua Experimenta, & Rationes
ab ijs deductæ pro Substantialitate Luminis.

In Secundo autem dissoluntur Argumenta in Primo adducta,
& probabiliter sustineri posse docetur Sententia,
Peripatetica de Accidentalitate Luminis.

QVA OCCASIONE

De huiusmodi incognita Luminis Diffusione, de Reflexione, Refractione, ac Dis-
fractatione Modis & Causis, de Visibus, deque Speciebus Intentionalibus
Visibilibus & Audibilibus, ac de Substantiali Magnitudinis effluuiis omnia
corpora penetrantia, non pauca sicut digna perscrutantur,
& specialia etiam argumenta impugnantur Atomistæ.

AVCTORE

P. FRANCISCO MARIA GRIMALDO
SOCIETATIS IESV.

OPVS POSTHVMVM.



BONONIÆ. MDCCLXV.

Ex Typographia Hieronimi Victorii Senarij. Superincum permissa.
Impressio Hieronymi Senarij Bibliopulæ Bononiensis.



Francesco Maria Grimaldi
1618-1663

Jesuit priest from Bologna

‘Light propagates and diffuses not only directly, refractively and reflectively, but also, somehow, in a fourth manner, that is DIFFRACTIVELY.’

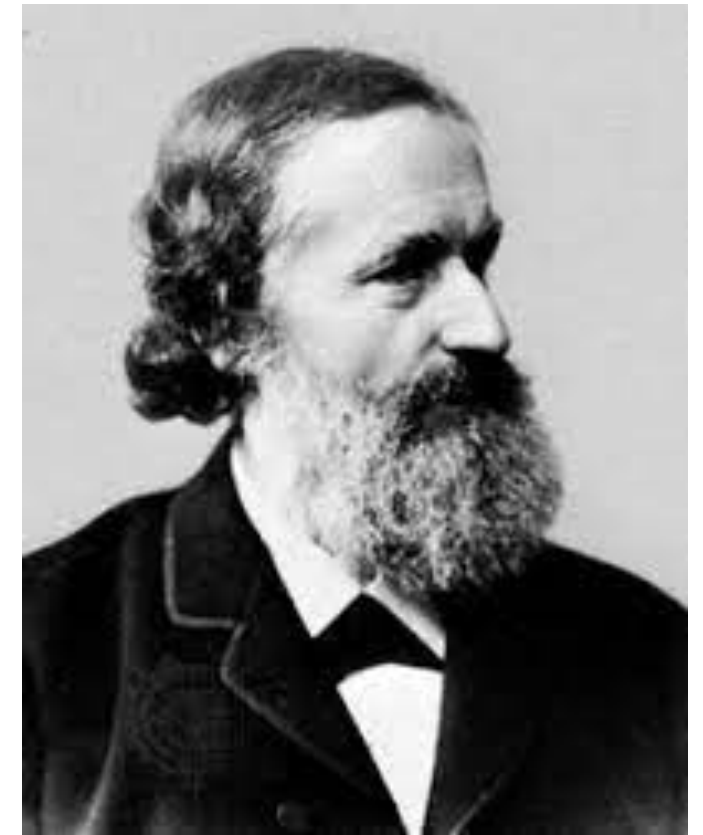
Theory of diffraction



Christiaan Huygens
1629-1695



Augustin Fresnel
1788-1827



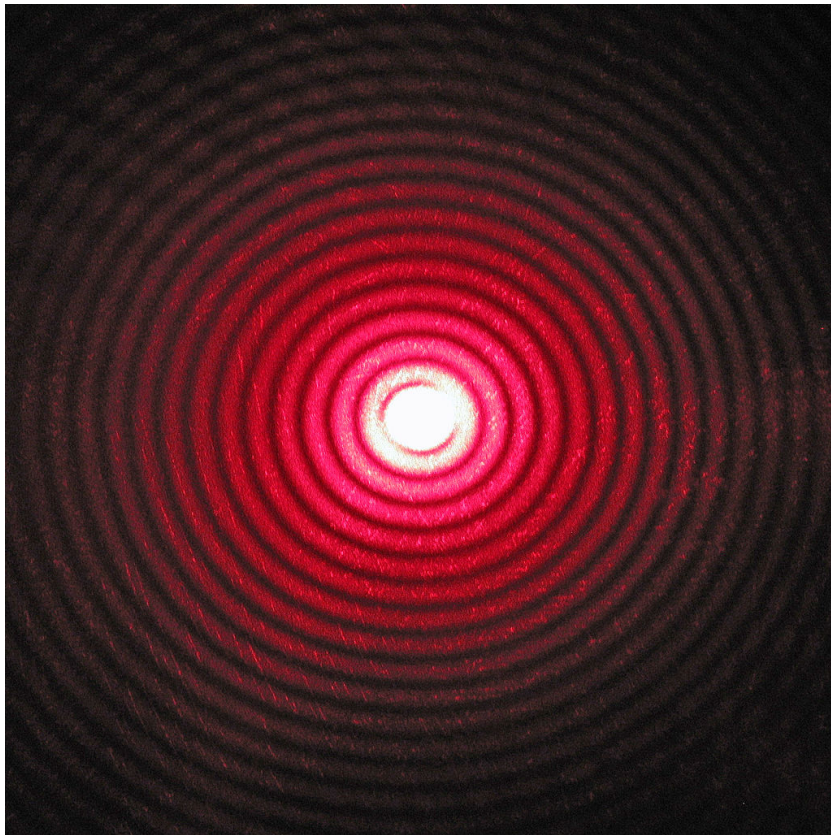
Gustav Kirchhoff
1824-1887

Geometrical optics: applicable in the limit when the wavelength is infinitely small

Diffraction phenomena: deviation from geometrical optics due to finite wavelength

Diffraction : occurs when a wave (for example light) encounters an obstacle or an opening. Most pronounced when the dimensions of obstacle/opening are comparable to wavelength

Laser light passing through a circular aperture



Source: Wikipedia
Author: Wisky

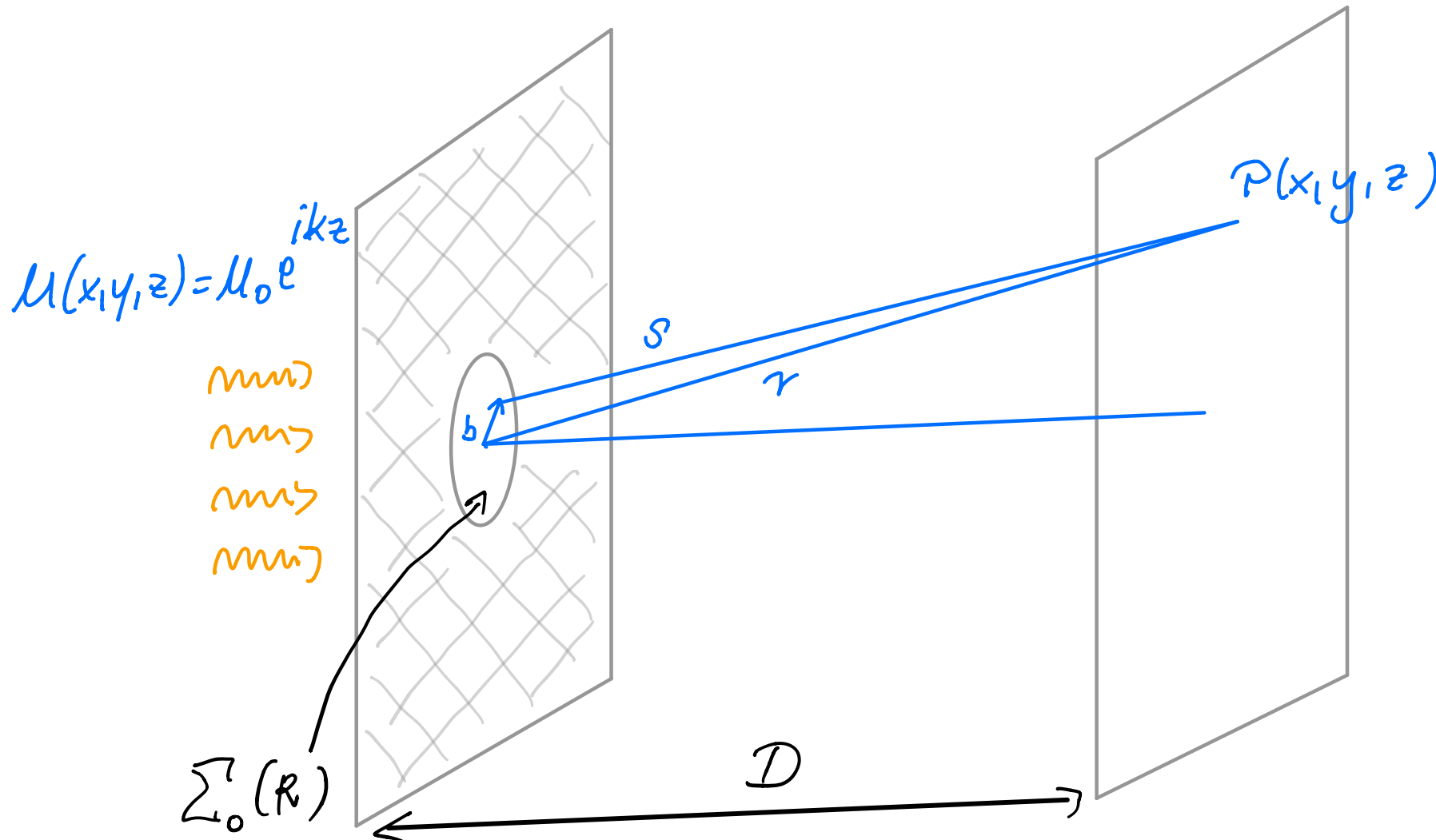
Water waves passing through small entrance



Source: Wikipedia
Author: Verbcatcher

**In quantum theory:
hadronic and nuclear diffractive scattering**

Kirchhoff theory



$$P \equiv (x, y, z)$$

$$k = 2\pi / \lambda$$

Wave number

U Amplitude

$$\phi(x, y, z, t) = U(x, y, z)e^{-i\omega t}$$

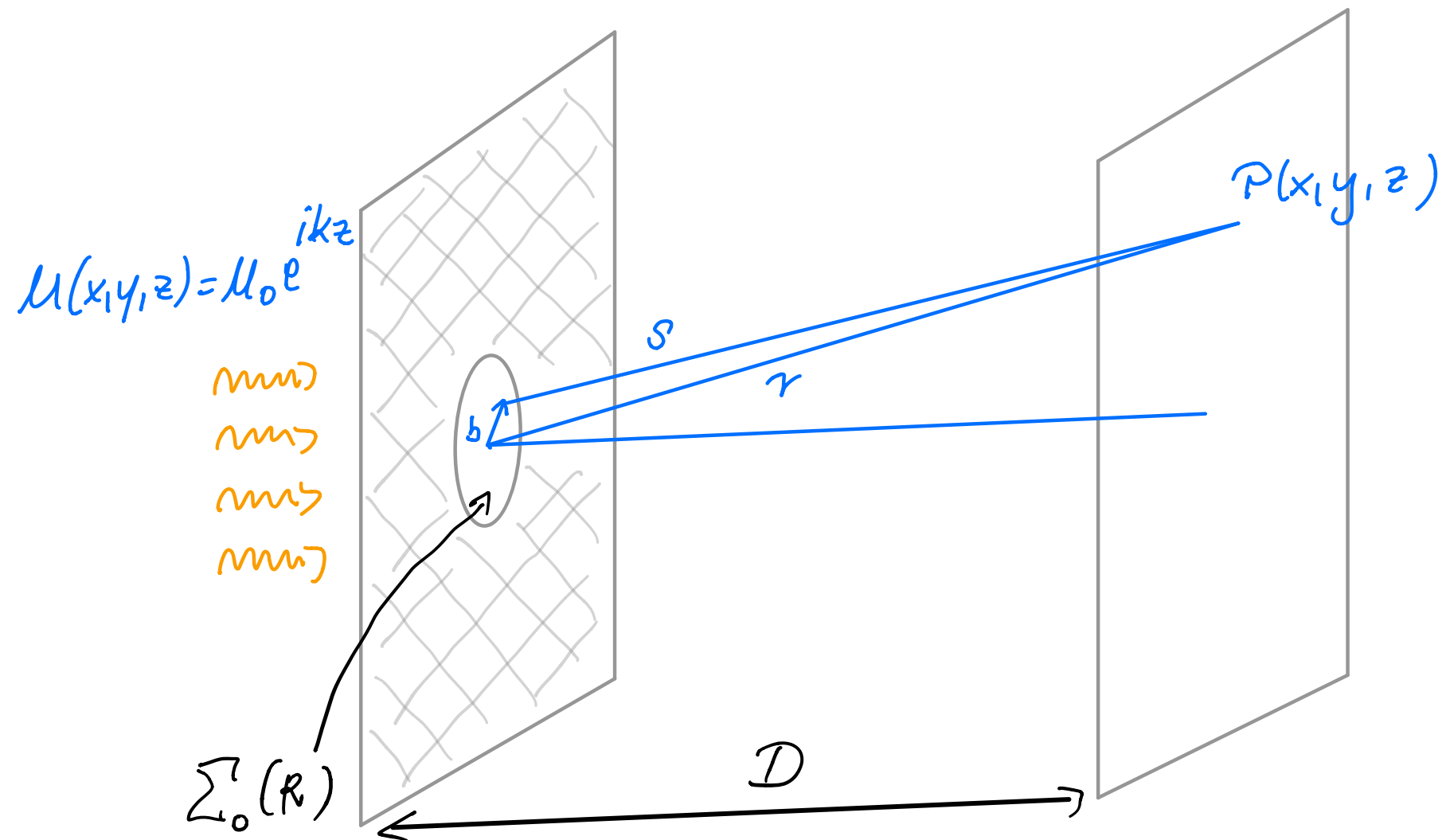
$$(\nabla^2 + k^2)U = 0$$

Helmholtz equation

Short wave length limit ($kR > 1$)

$$U(x, y, z) = -\frac{ik}{2\pi} U_0 \int_{\Sigma_0} d^2\mathbf{b} \frac{e^{iks}}{s}$$

Fresnel-Kirchhoff
integral



Geometrical optics:

$$kR^2 / D \gg 1$$

Fresnel diffraction:

$$kR^2 / D \approx 1$$

Near field

Fraunhofer diffraction:

$$kR^2 / D \ll 1$$

Far field

Relevant for hadronic physics

Fraunhofer diffraction

For the hole in the screen:

$$U(x, y, z) = -\frac{ik}{2\pi} U_0 \frac{e^{ikr}}{r} \int d^2\mathbf{b} \Gamma(\mathbf{b}) e^{-i\mathbf{q}\mathbf{b}}$$

$$\mathbf{q} \approx \mathbf{k}' - \mathbf{k}$$

Momentum transfer (2 dimensional vector)

\mathbf{k} Incoming wave vector

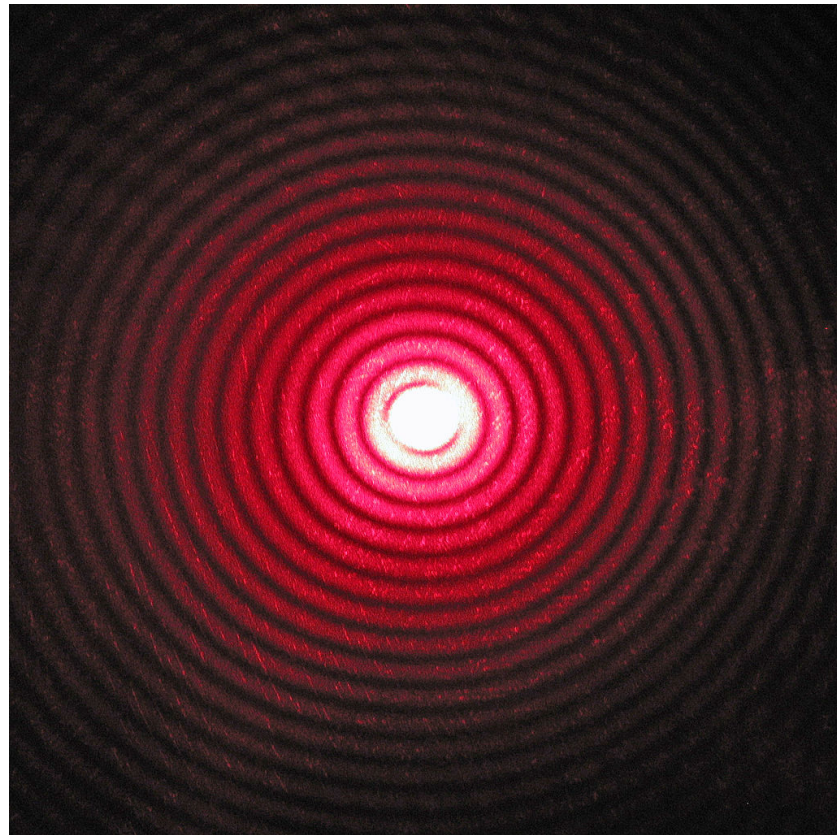
\mathbf{k}' Outgoing wave vector

$\Gamma(\mathbf{b})$ Profile function

For the hole:

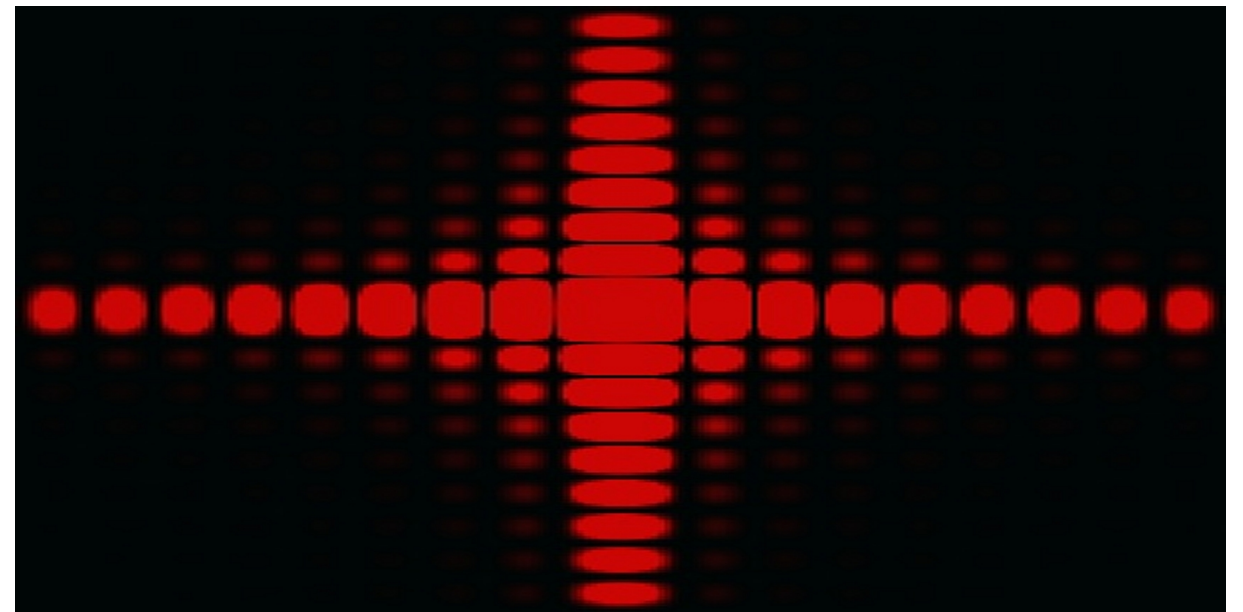
$$\Gamma(\mathbf{b}) = \begin{cases} 1, & \text{on } \Sigma_0 \\ 0, & \text{outside } \Sigma_0 \end{cases}$$

Diffraction patterns



Source: Wikipedia
Author: Wisky

Circular aperture



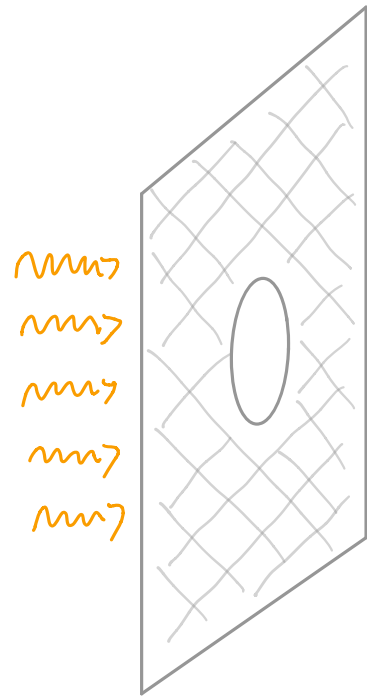
Source: Wikipedia
Author: Epzcaw

Rectangular aperture

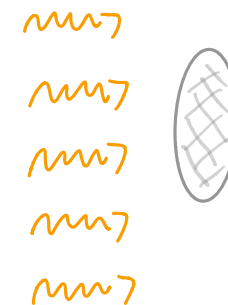
In Fraunhofer regime the diffraction pattern (far field) is a Fourier transform of the apertured field.

Diffraction off an obstacle

Babinet's principle:



S1 Screen with a hole



S2 Obstacle with the same size and shape as a hole

Waves diffracted by a hole S1 and obstacle S2 must combine to reconstruct the incident wave front.

Diffraction patterns away from incident direction are the same for screen with hole and complementary obstacle.

Diffraction off an obstacle



$$U(x, y, z) = U_{\text{inc}} + U_{\text{scat}}$$

$$U(x, y, z) = U_0(e^{ikz} + f(\mathbf{q})\frac{e^{ikr}}{r})$$

Scattering amplitude

$$f(\mathbf{q}) = \frac{ik}{2\pi} \int d^2\mathbf{b} \Gamma(\mathbf{b}) e^{-i\mathbf{q}\cdot\mathbf{b}}$$

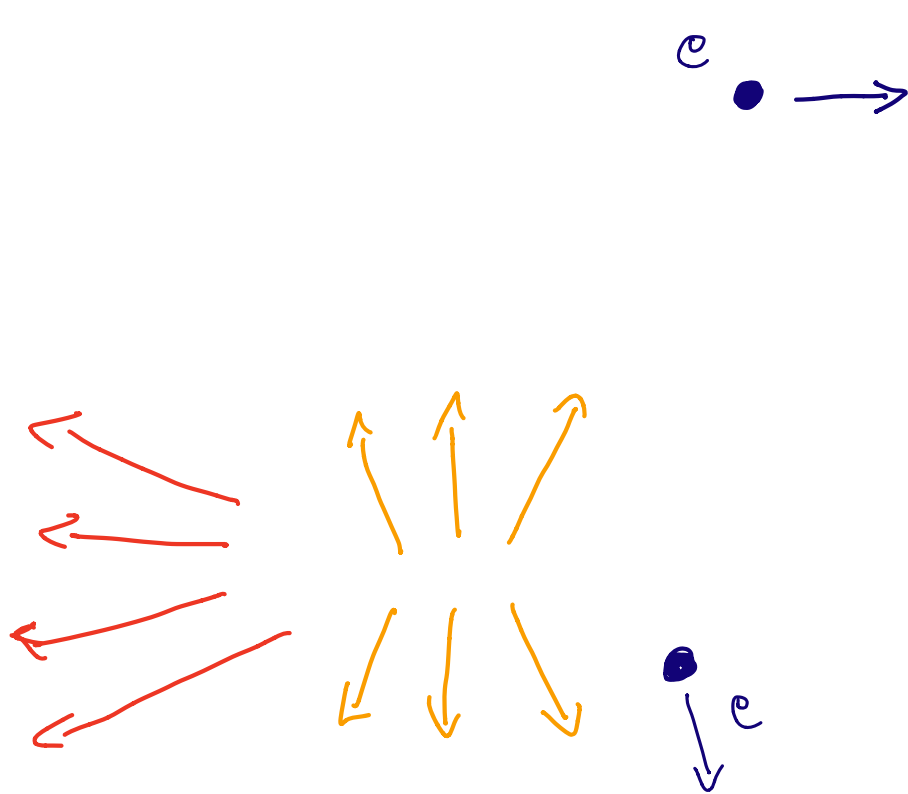
$$\Gamma(\mathbf{b}) = \frac{1}{2\pi ik} \int d^2\mathbf{q} f(\mathbf{q}) e^{i\mathbf{q}\cdot\mathbf{b}}$$

**Scattering amplitude is Fourier transform of the profile function.
Profile function is inverse Fourier transform of the scattering amplitude.**

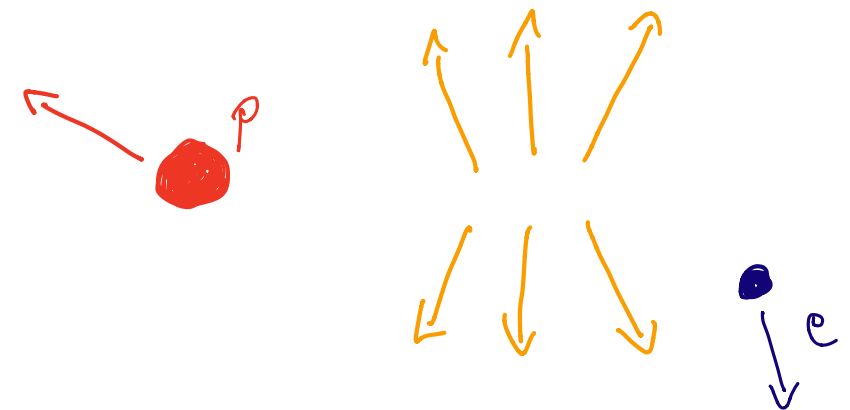
Diffraction in hadronic / nuclear physics

In quantum physics: propagation and interaction of particles as an absorption of the various components of their wavefunction

Electron - hadron(nucleus) scattering (like at EIC)



Proton is fragmented



Target(proton) is intact
No activity in vicinity

Scattering at ep collider HERA

HERA: (1992-2007)

27.5 GeV electrons/positrons

820/920 GeV protons

318 GeV CoM energy

Lumi: $10^{31} \text{ cm}^{-2} \text{ s}^{-1}$

Electrons, positrons and protons

Physics:

Structure functions

Parton density functions

Established growth of gluon with decreasing Bjorken x

Measurement of coupling constant

Diffraction

Jets, heavy quarks

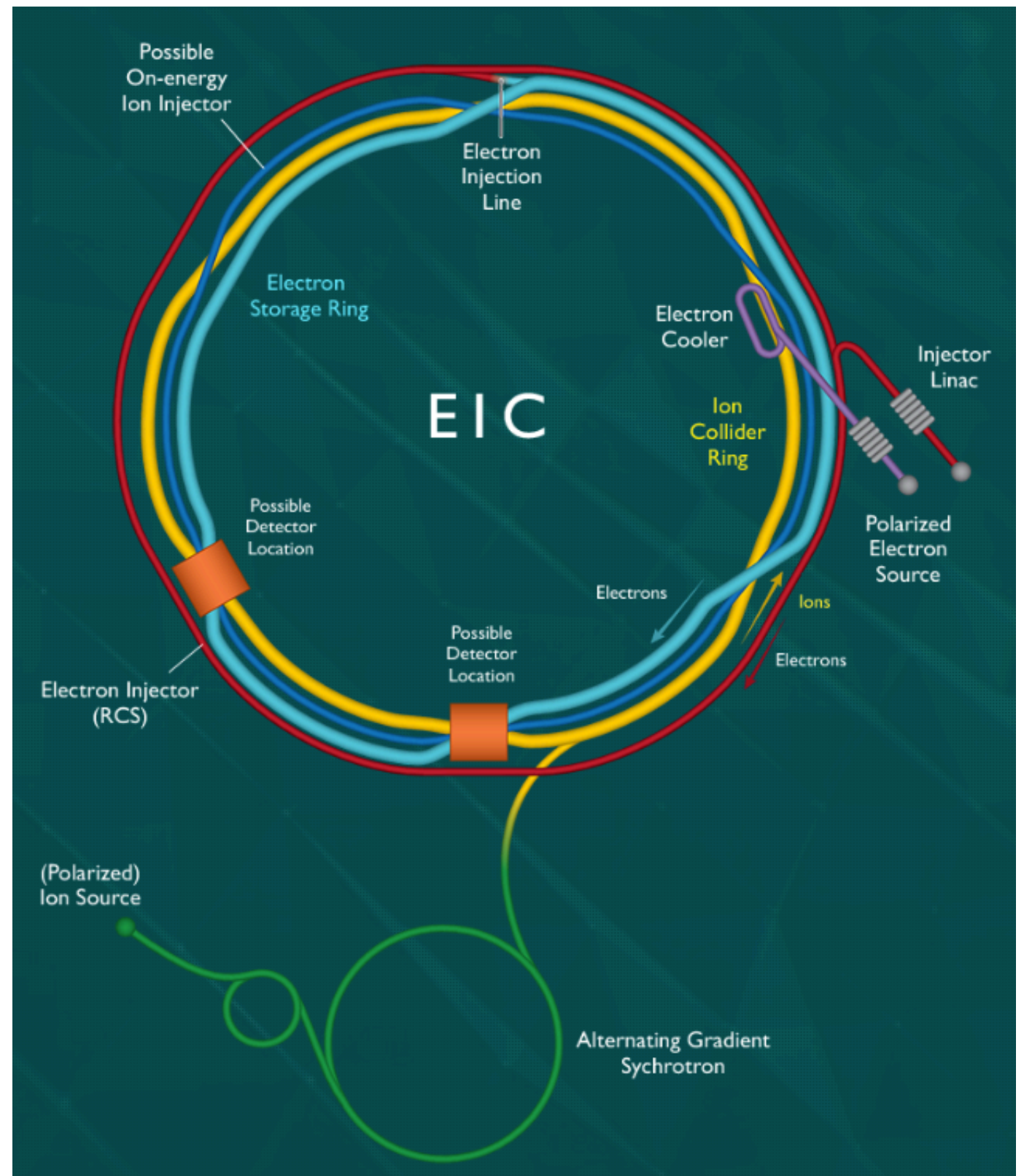
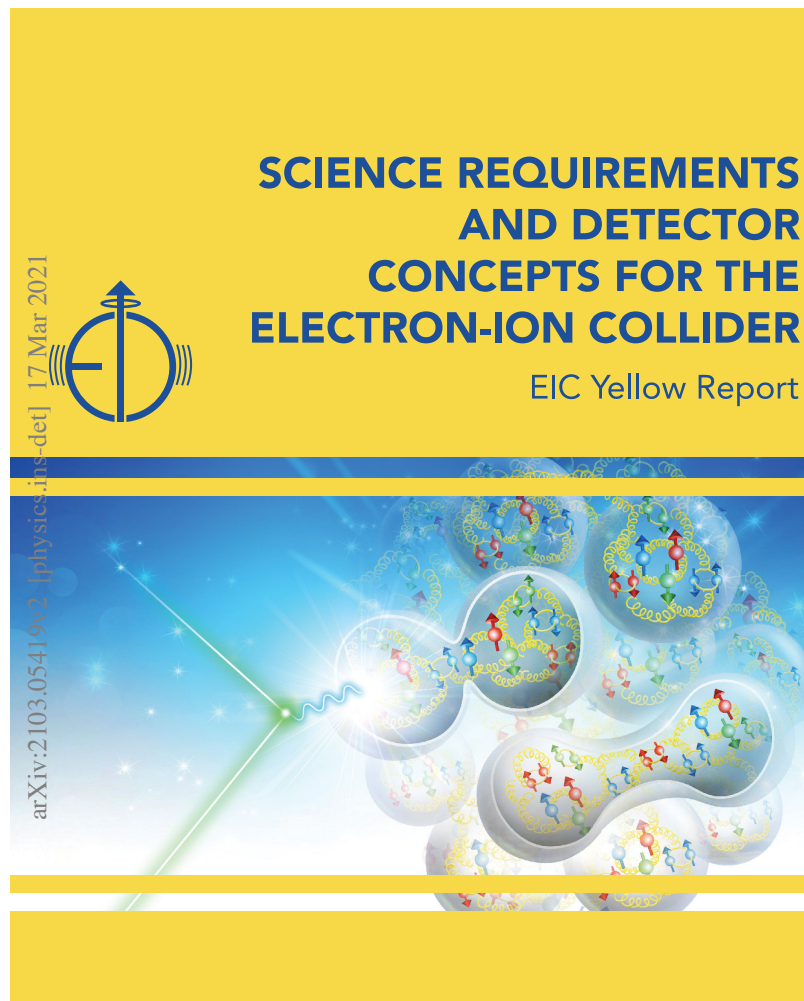
BSM searches



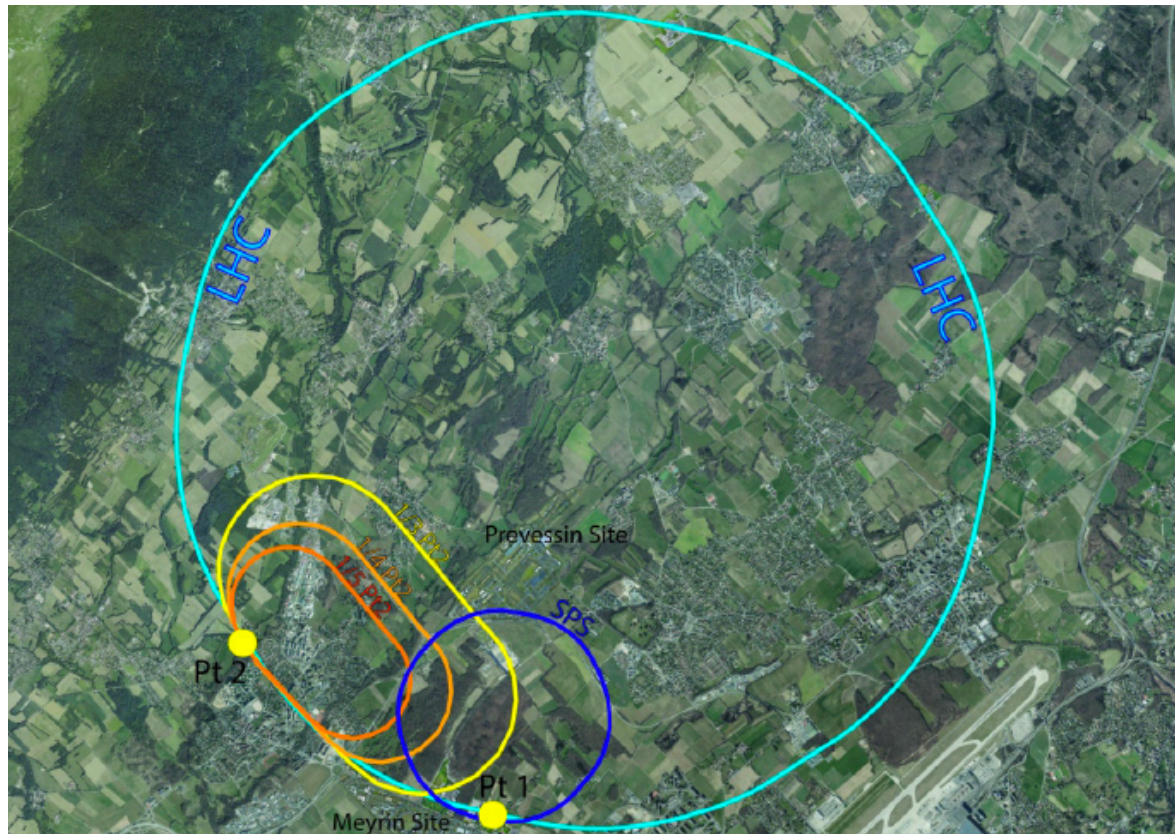
Low luminosity/limited statistics, no nuclei, no polarized beams

Future DIS machines EIC

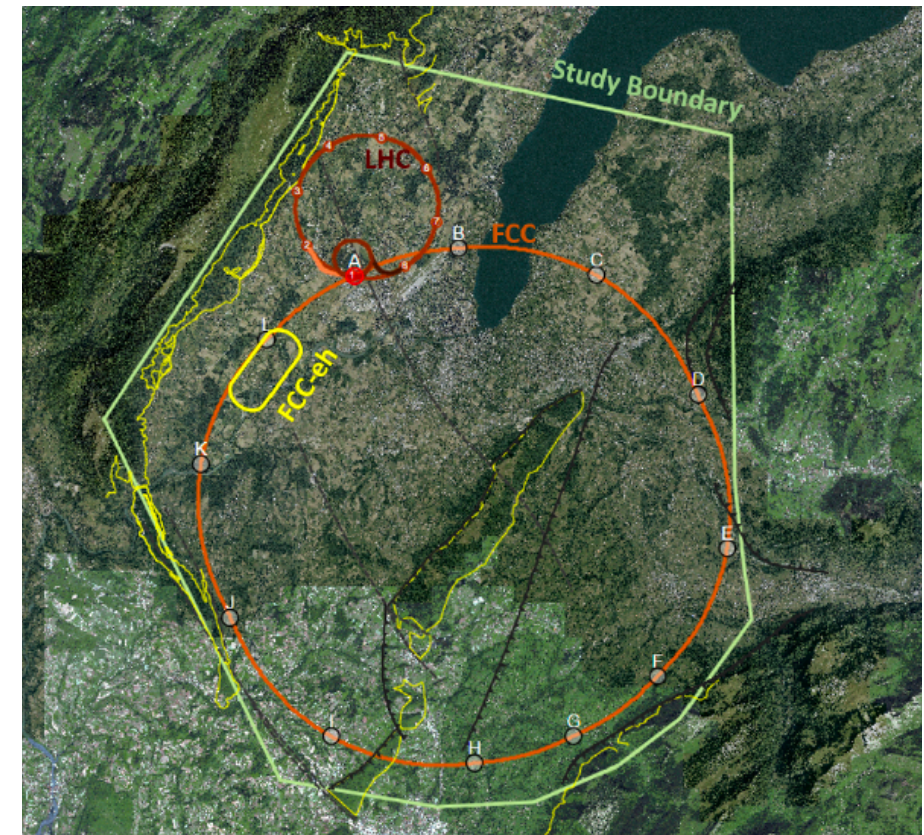
EIC: 5-20 GeV electrons
20-140 GeV CoM energy
Lumi: $10^{34} \text{ cm}^{-2} \text{ s}^{-1}$
Polarized e,p,d, ^3He
Wide range of nuclei



Future DIS machines LHeC, FCC-eh at CERN



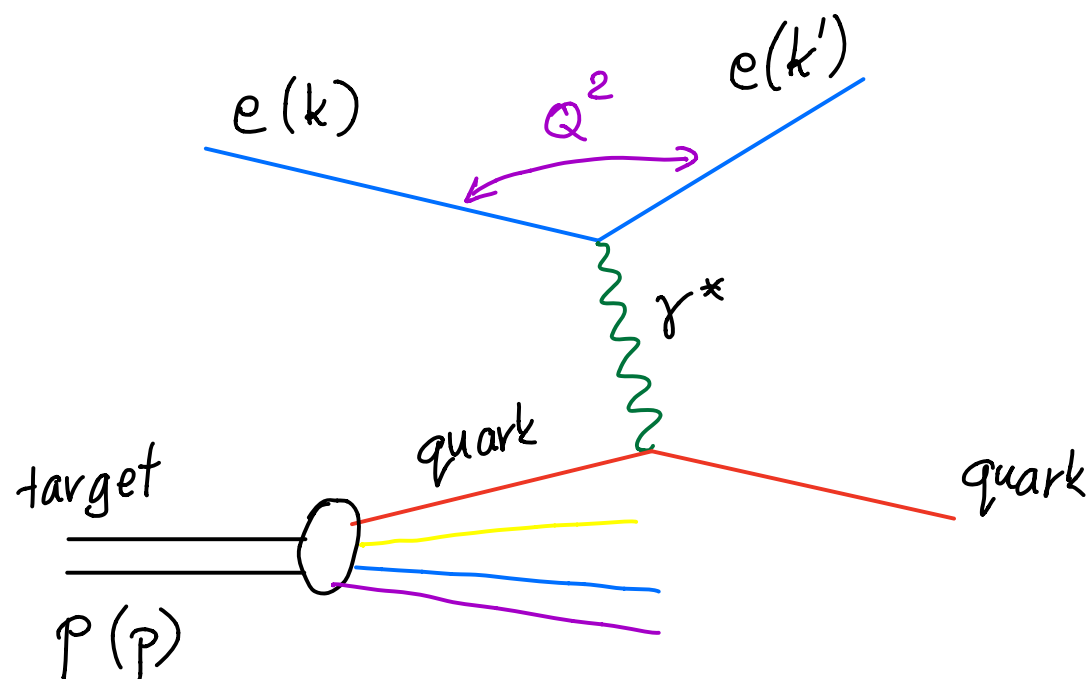
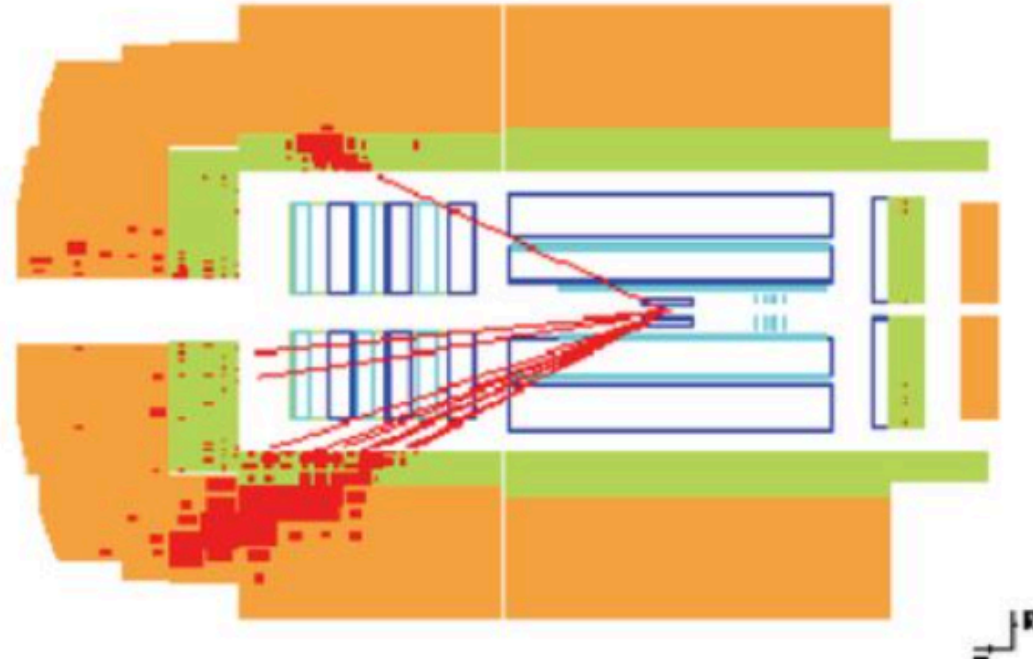
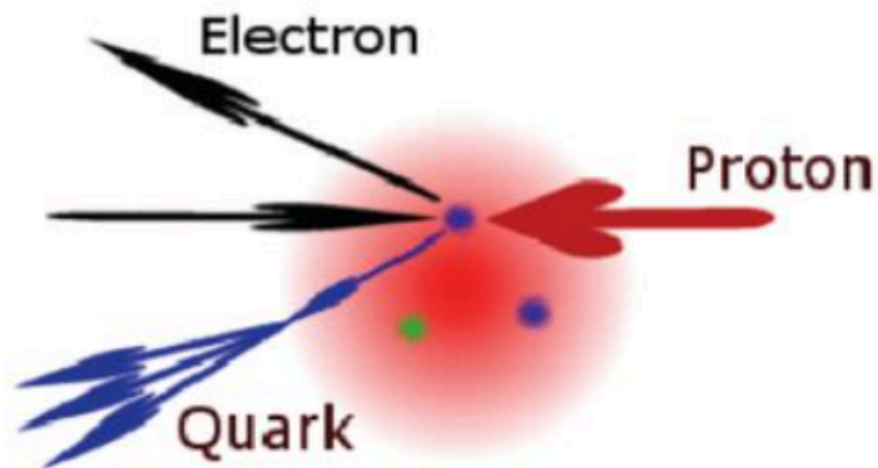
LHeC: 60(50) GeV
electrons x LHC protons
and ions
1.3 TeV CoM energy for ep
812 GeV CoM for ePb
Lumi: $10^{34} \text{ cm}^{-2} \text{ s}^{-1}$
Simultaneous running with
ATLAS and CMS in HL-
LHC period



FCC-ep: 60(50) GeV
electrons x 50 TeV protons
from FCC, lead beams 19.7
TeV/per nucleon
3.5 TeV CoM energy for ep
2.2 TeV CoM for ePb
Lumi: $10^{34} \text{ cm}^{-2} \text{ s}^{-1}$

Scattering at ep collider HERA

Non-diffractive DIS event



$$Q^2 = -q^2 = -(k - k')^2 \quad \text{4 momentum transfer at a lepton vertex}$$

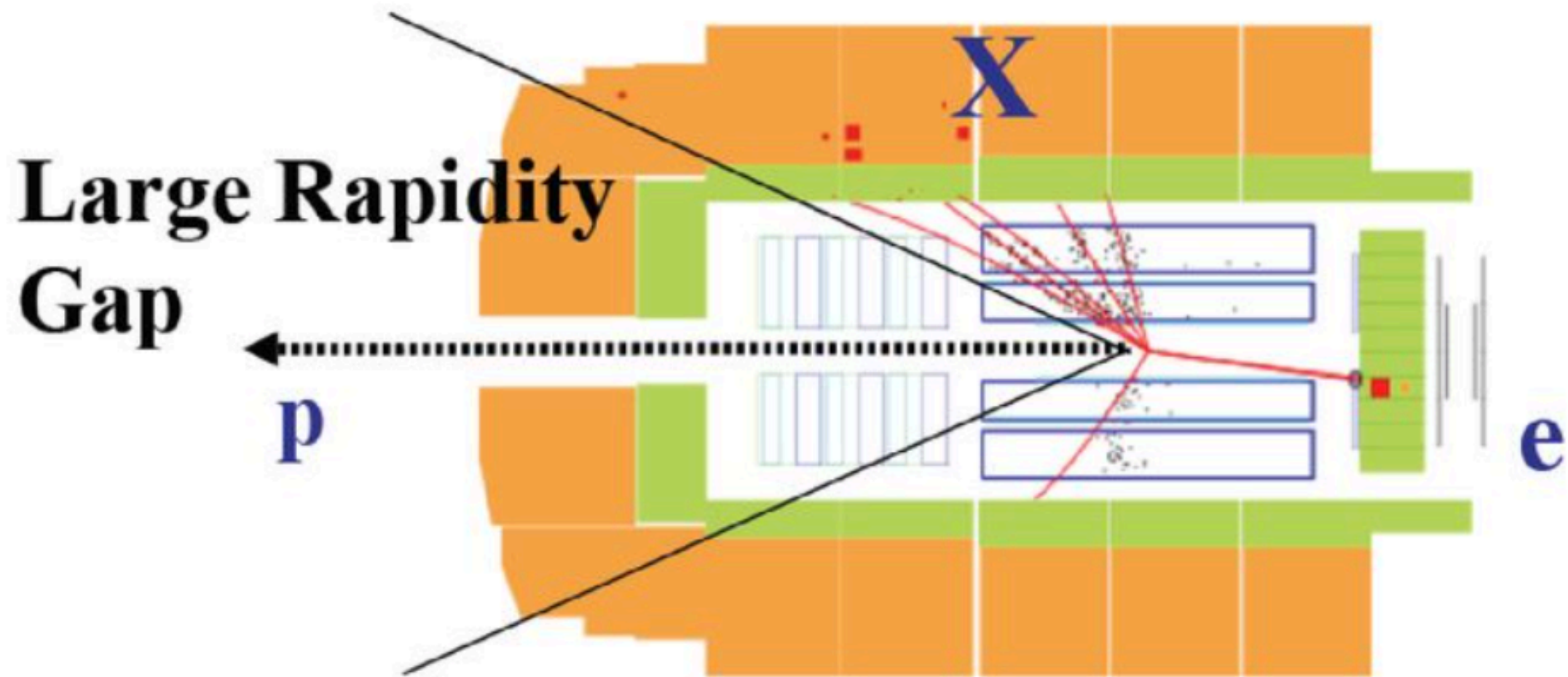
$$W^2 = (q + p)^2 \quad \text{Photon-proton cms energy squared}$$

$$s = (k + p)^2 \quad \text{Electron-proton cms energy squared}$$

$$x = Q^2 / 2p \cdot q \quad \text{Longitudinal momentum fraction of the target carried by the struck quark (in infinite momentum frame)}$$

$$y = q \cdot p / k \cdot p \quad \text{Inelasticity}$$

Diffraction at HERA



10% events at HERA were of diffractive type

Large portion of the detector void of any particle activity: **rapidity gap**

Proton stays intact despite undergoing violent collision with a 50 TeV electron (in its rest frame)

Rapidity: recap

$$p^\mu = (E, \vec{p}_T, p_z) \qquad y = \frac{1}{2} \ln \frac{E + p_z}{E - p_z} \qquad \frac{E}{p_z} = \tanh y$$

Under boosts in z direction rapidity transforms additively

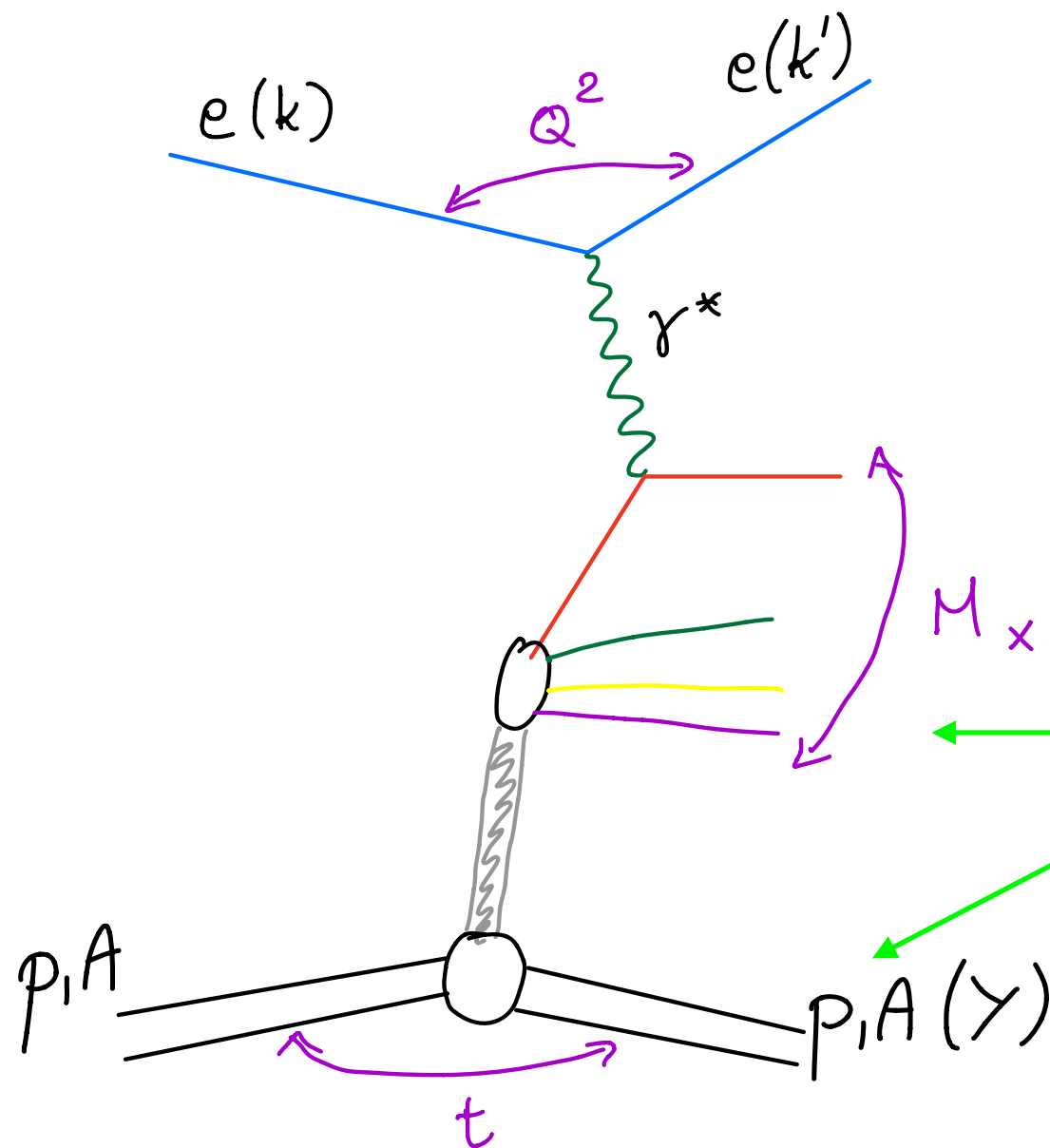
$$\begin{pmatrix} p_z \\ E \end{pmatrix} = \begin{pmatrix} \cosh \phi & \sinh \phi \\ \sinh \phi & \cosh \phi \end{pmatrix} \begin{pmatrix} p'_z \\ E' \end{pmatrix}$$

Then $y = y' + \phi$

Pseudorapidity $\eta = -\ln \tan \frac{\theta}{2}$ Angle between 3-momentum and z-axis

When $m \ll \vec{p}_T$ then $y \rightarrow \eta$

Diffraction in electron - proton(nucleus)



t Momentum transfer at the proton vertex (target recoil)

X Diffractive system with mass M_x

Rapidity gap

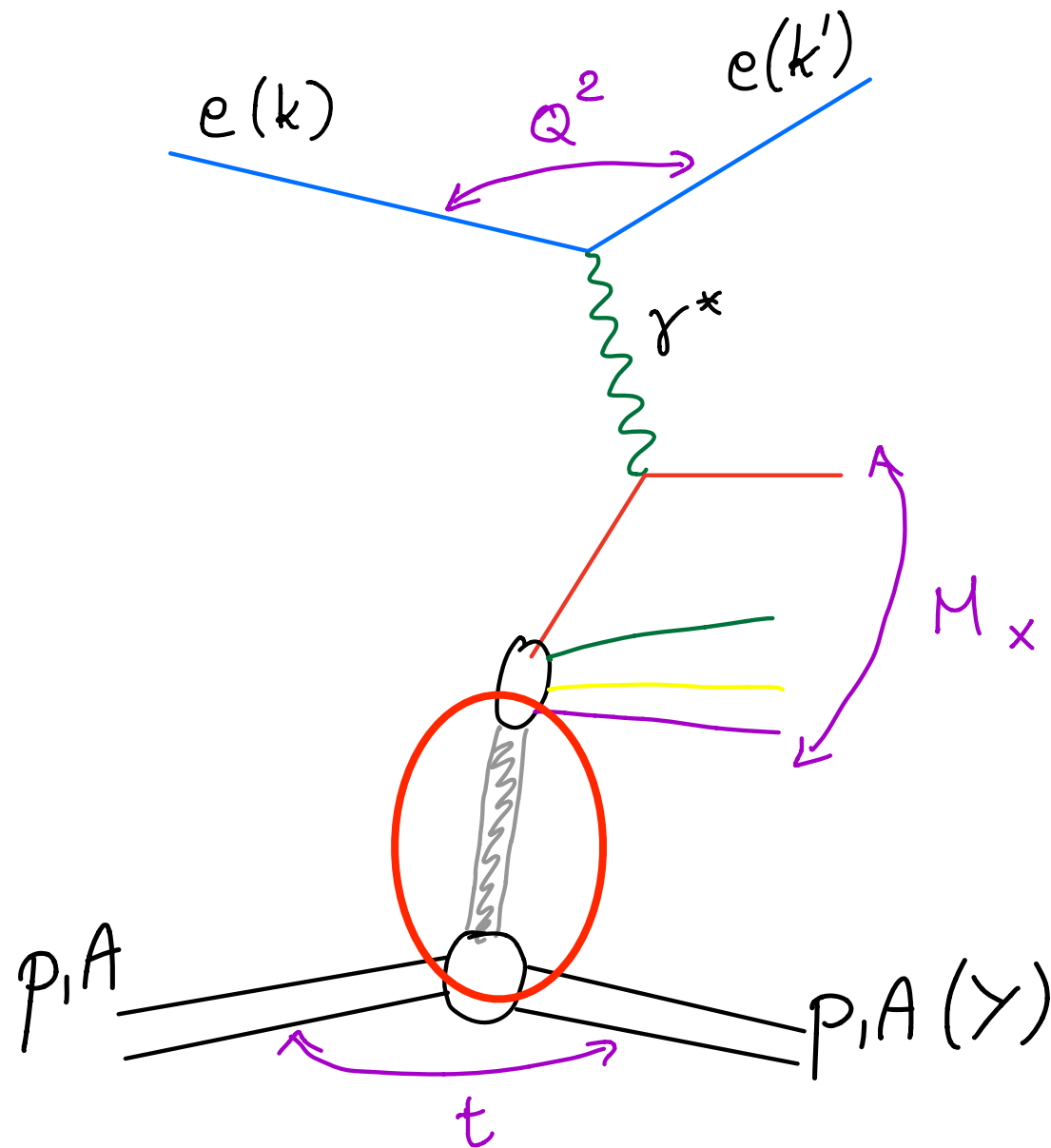
Target is scattered elastically: elastic scattering

It can also dissociate into a state Y with the same quantum numbers, but still separated from the rest of particles

In order for the rapidity gap to exist it needs to be mediated by the *colorless* exchange

Diffraction: a reaction characterized by a large rapidity gap in the final state

Diffraction and the Pomeron



In order for the rapidity gap to exist it needs to be mediated by the *colorless diffractive exchange*

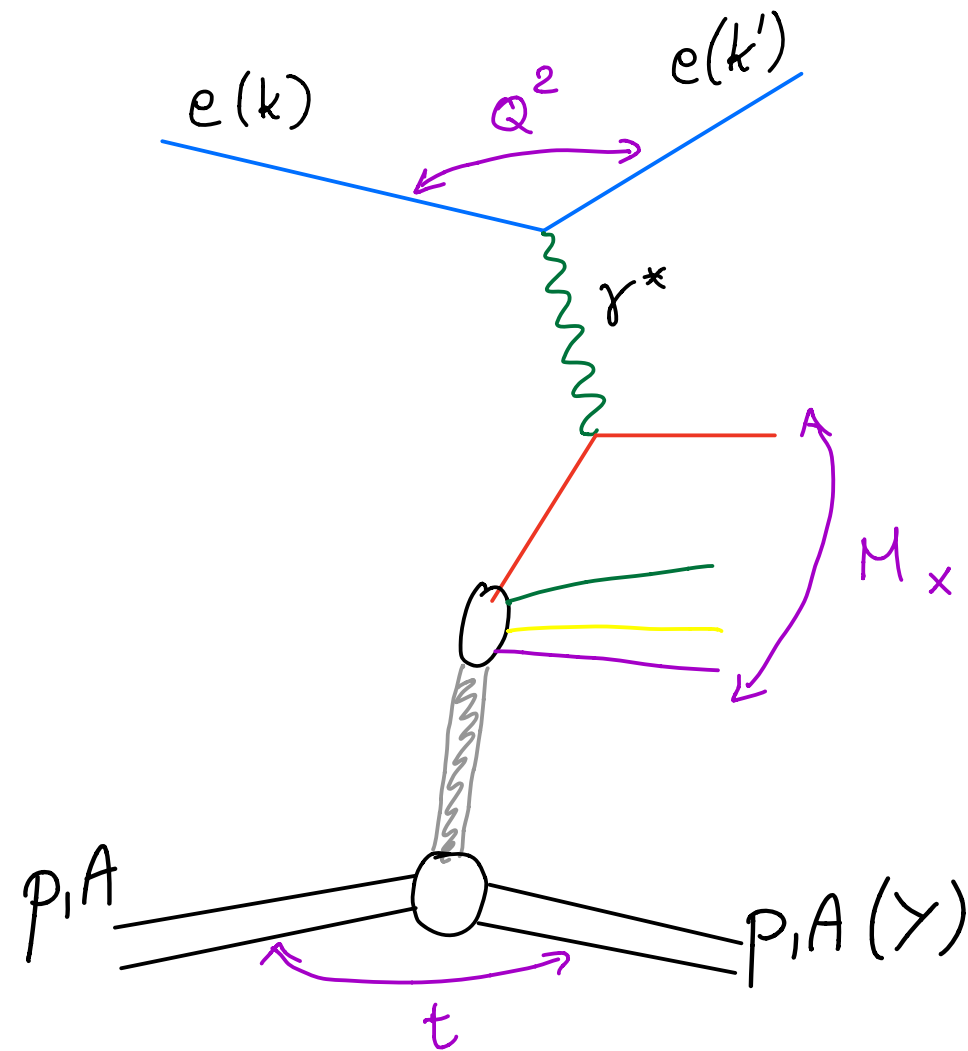
But what is this *diffractive exchange* ?

Usually referred to as the *Pomeron*.
Quantum numbers of the vacuum

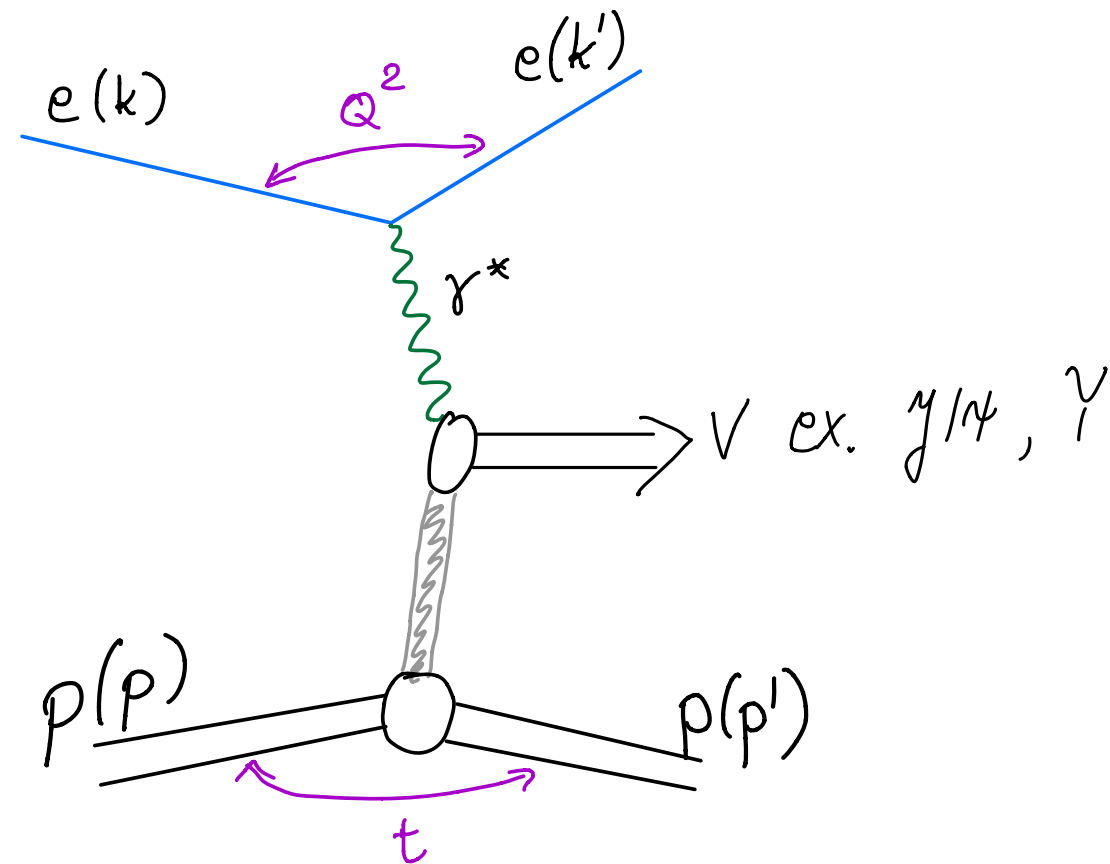
Modeled as a composite system of gluons and/or quarks.

Studying diffractive processes can shed light onto properties of this intriguing object.

Example : elastic vector meson production



Final state contains only vector meson, scattered lepton and proton



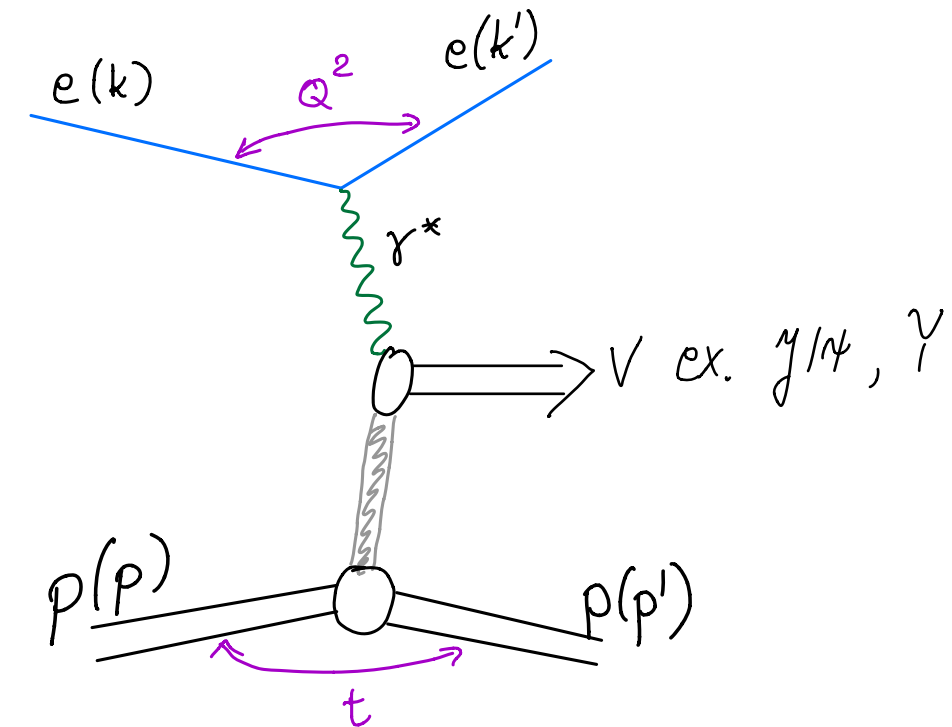
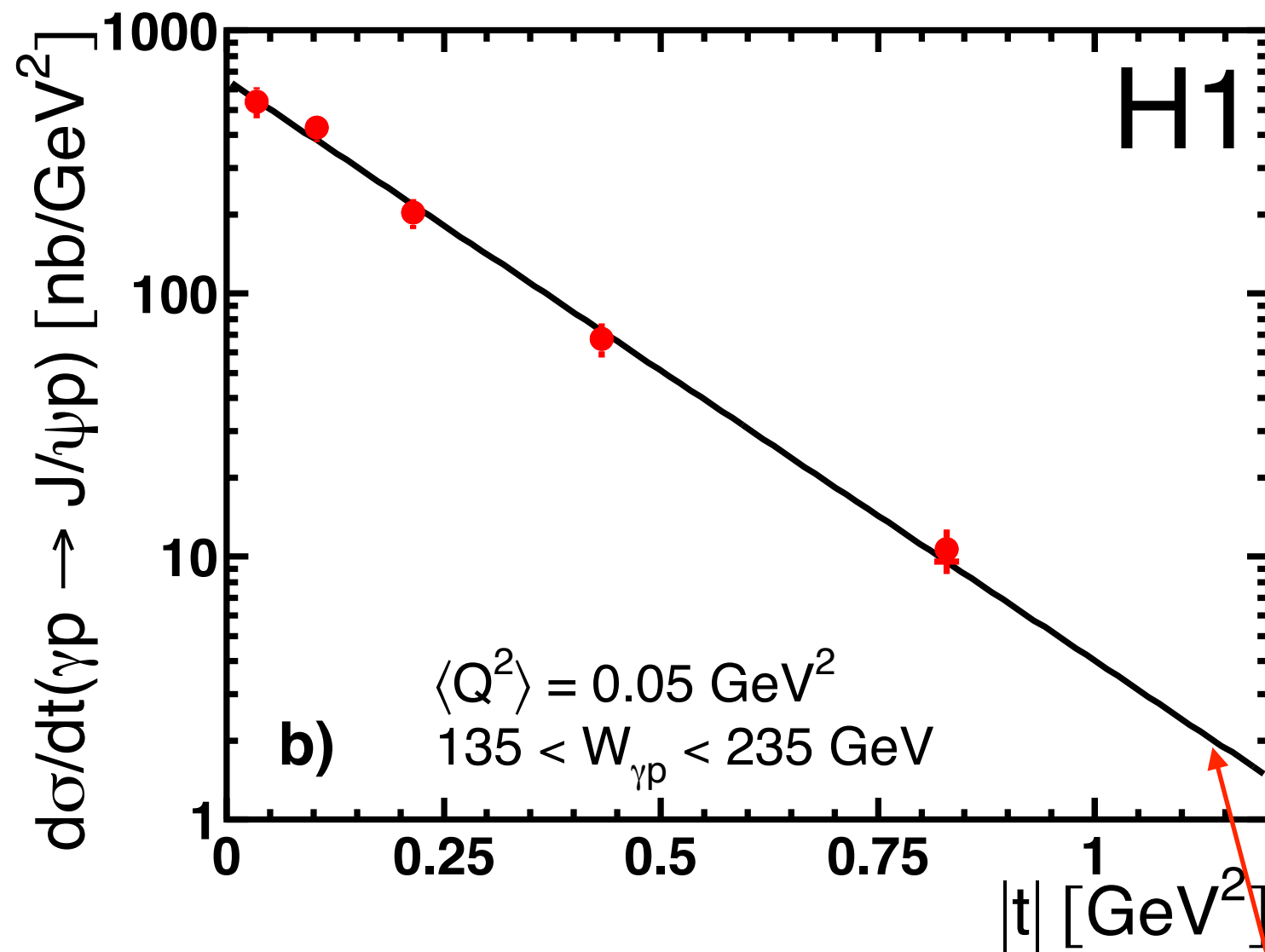
J/ ψ vector meson: charm -anti charm system

Upsilon vector meson: bottom - anti bottom system

$$m = 3.09 \text{ GeV}$$

$$m = 9.46 \text{ GeV}$$

Elastic vector meson production

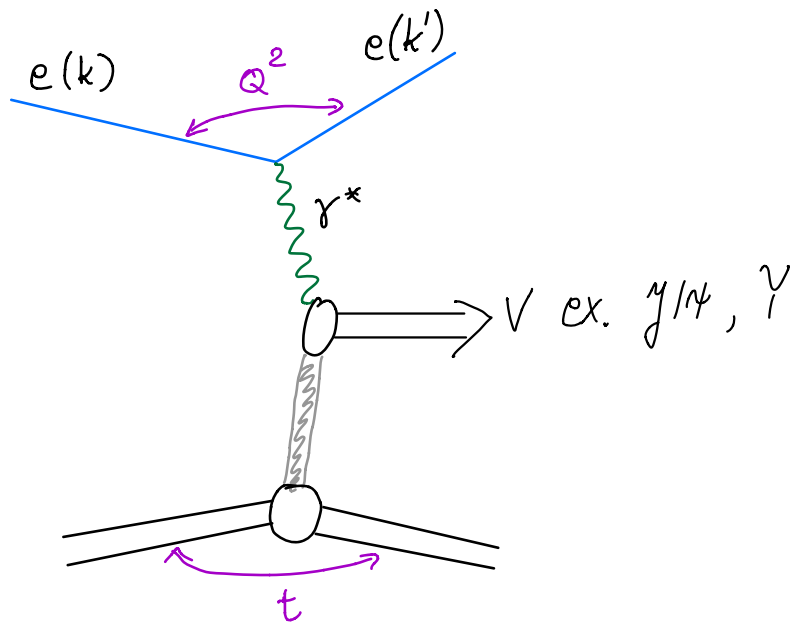


$$t = (p - p')^2 < 0$$

momentum transfer at the proton vertex

$$\frac{d\sigma}{dt} \sim e^{bt}$$

Exponential fit



$$\frac{d\sigma}{dt} = \frac{1}{16\pi} |\mathcal{M}(\Delta)|^2$$

$$\mathcal{M}(\Delta) = \langle \psi_{\gamma^*} | A(\Delta) | \psi_V \rangle$$

Momentum transfer $t = -\Delta^2$

\mathcal{M}

amplitude for vector meson process

A

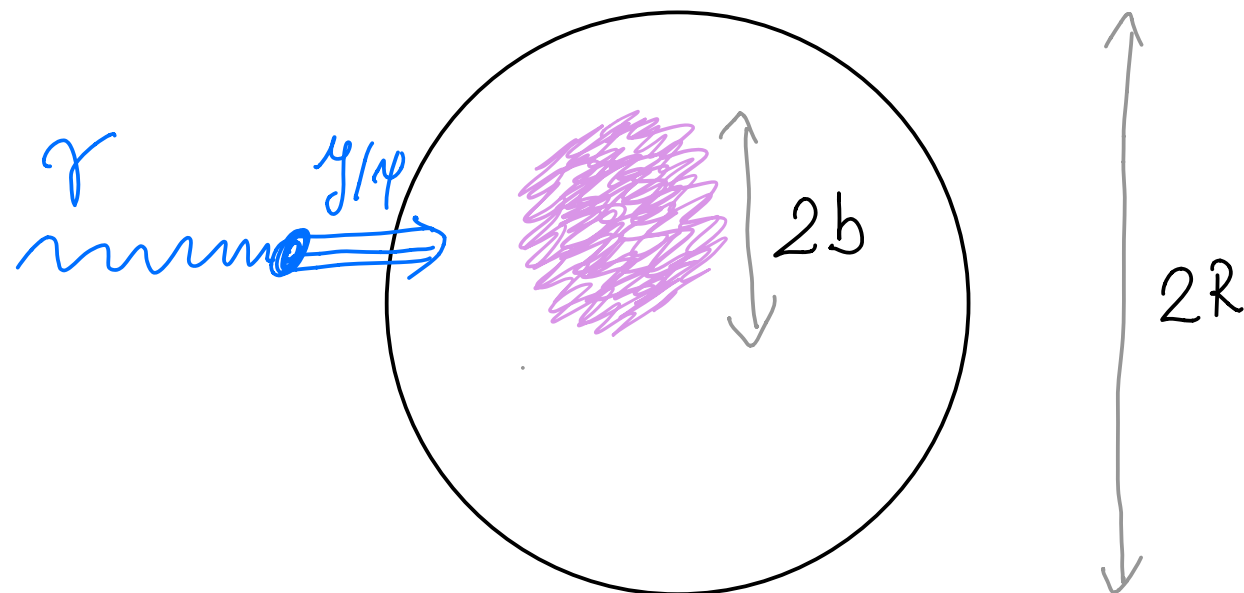
elementary (quark dipole) amplitude

$$A(\Delta) = \int d^2\mathbf{b} e^{i\Delta \cdot \mathbf{b}} \tilde{A}(b)$$

$$\langle \tilde{A}(b) \rangle = \frac{\langle \psi_{\gamma^*} | \tilde{A} | \psi_V \rangle}{\langle \psi_{\gamma^*} | \psi_V \rangle} = \frac{\int d^2\Delta e^{-i\Delta b} \sqrt{\frac{d\sigma}{dt}}}{\pi^{3/2} \langle \psi_{\gamma^*} | \psi_V \rangle}$$

t-dependence of the elastic cross section provides information about the profile of the target

Diffractive elastic vector meson production as a way to study nucleon structure



Radius measured in diffractive scattering of vector mesons

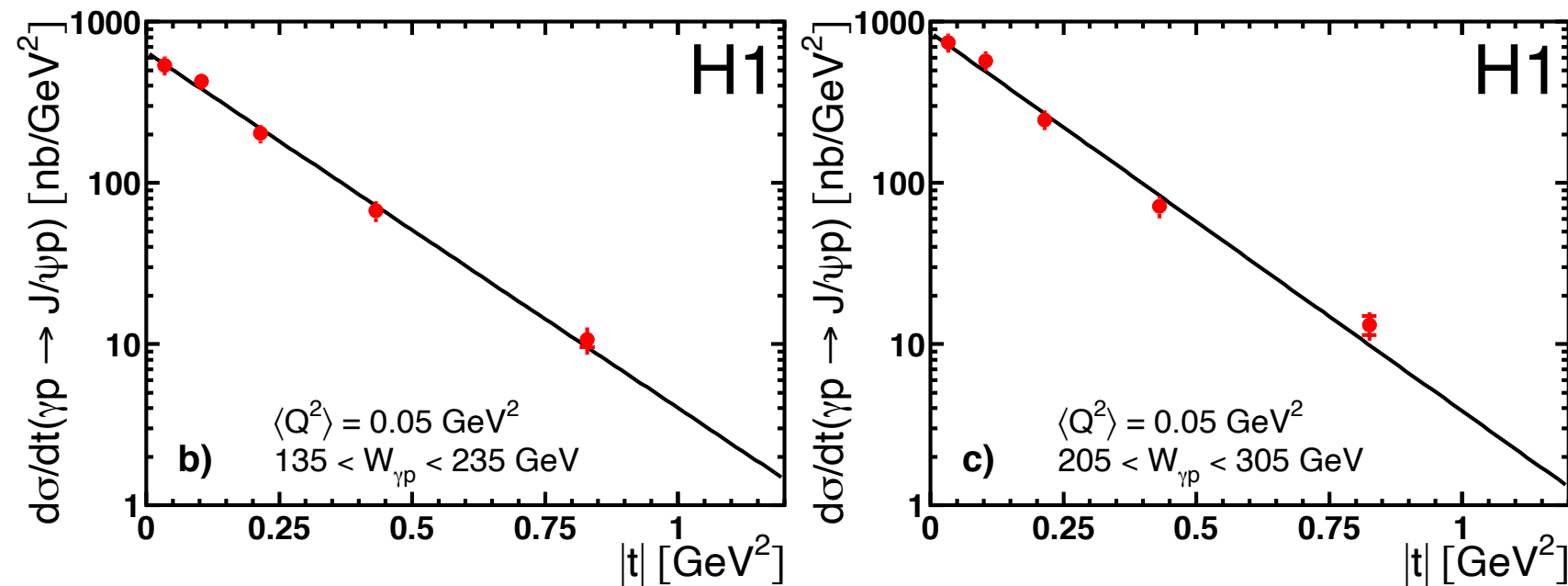
$$b \approx 0.5 \div 0.6 \text{ fm}$$

Proton charge radius

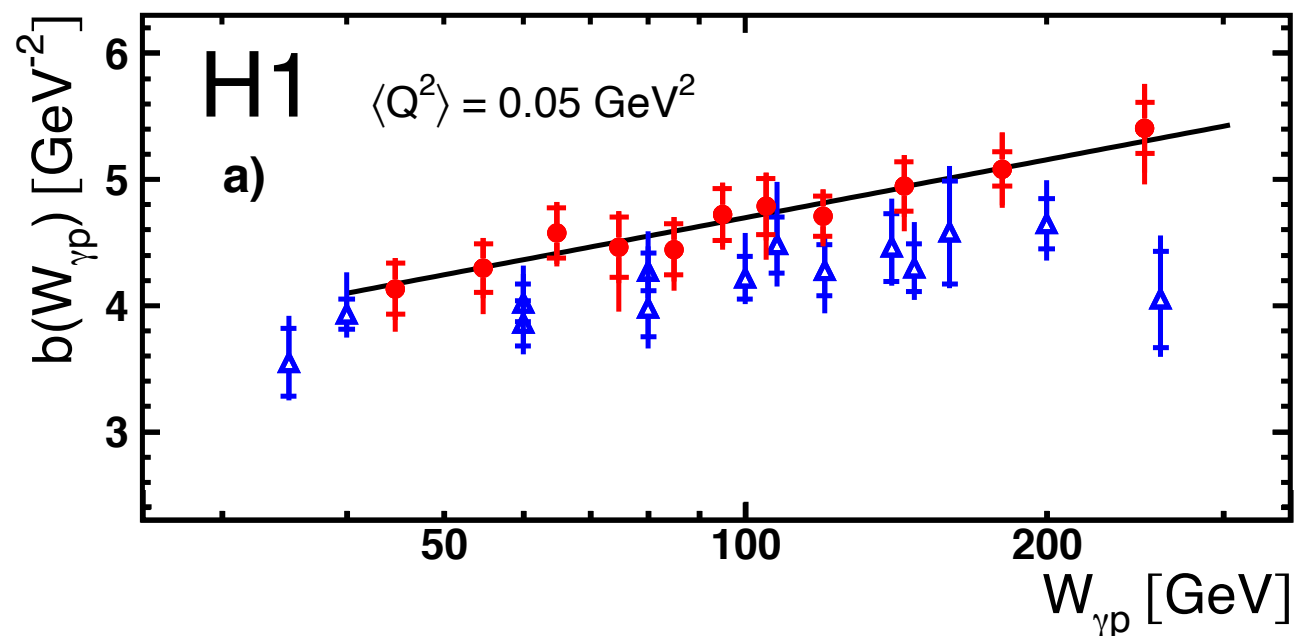
$$R \approx 0.84 \div 0.87 \text{ fm}$$

Experiments on elastic VM production suggest gluons are concentrated in smaller regions than quarks

Growth of the target size with energy



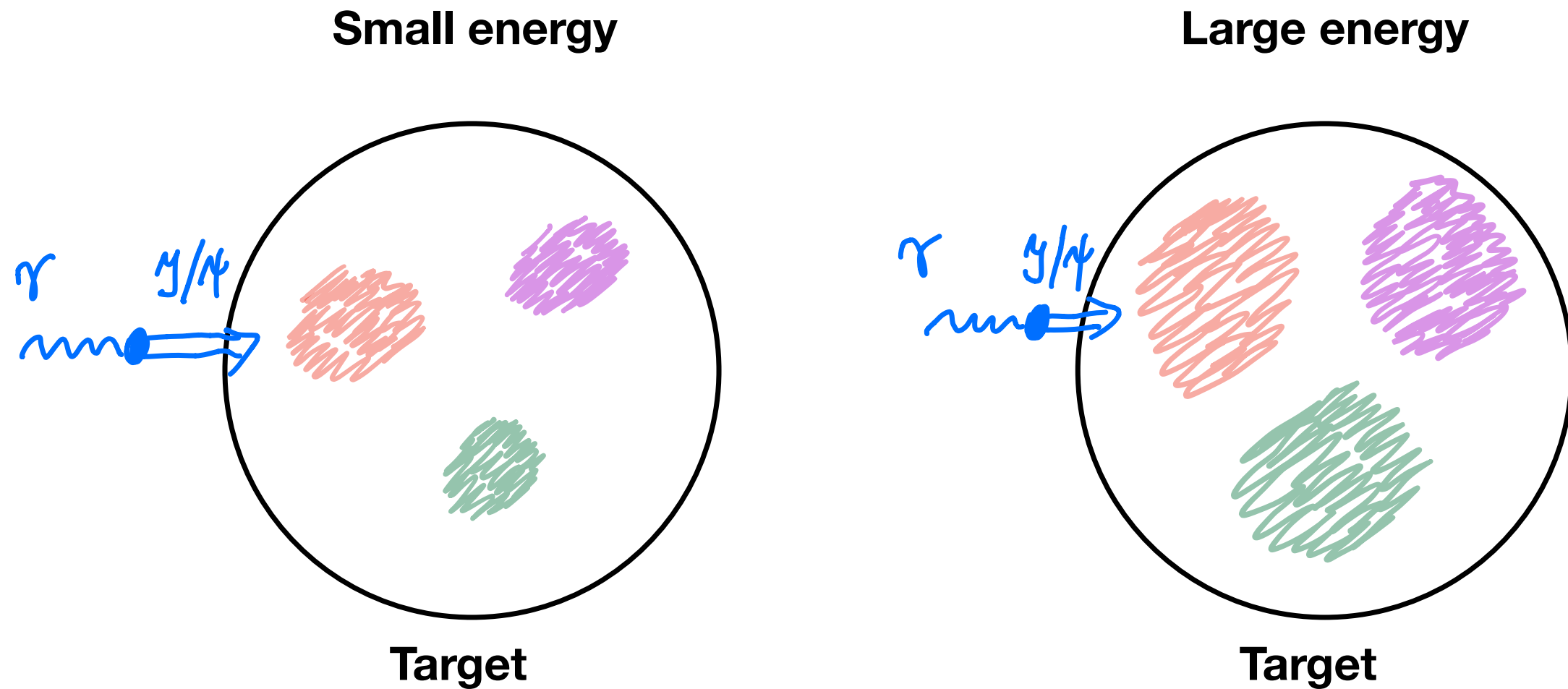
$$\frac{d\sigma}{dt} \sim e^{bt}$$



The slope grows with energy

$$b(W) = b_0 + 4\alpha' \ln(W_{\gamma p}/W_0)$$

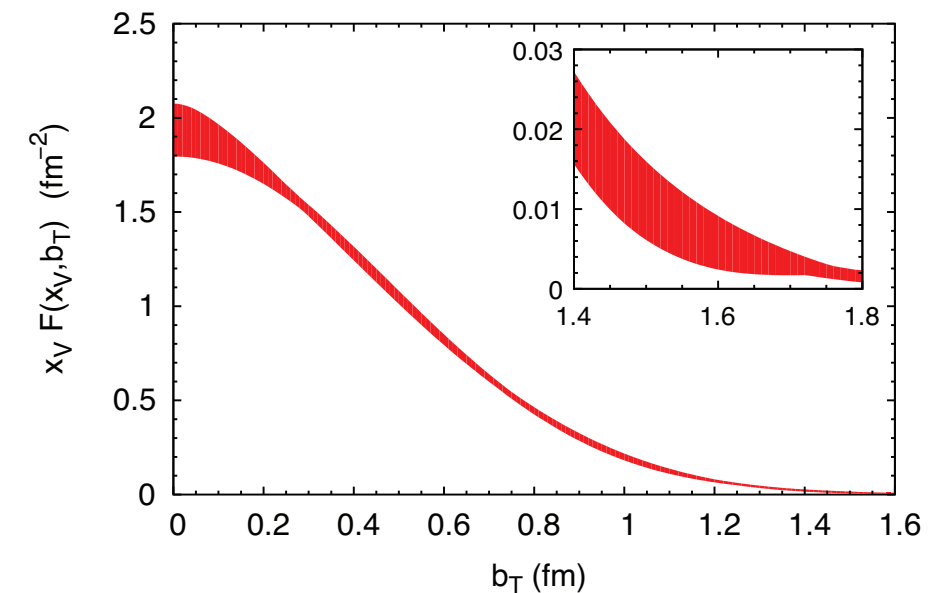
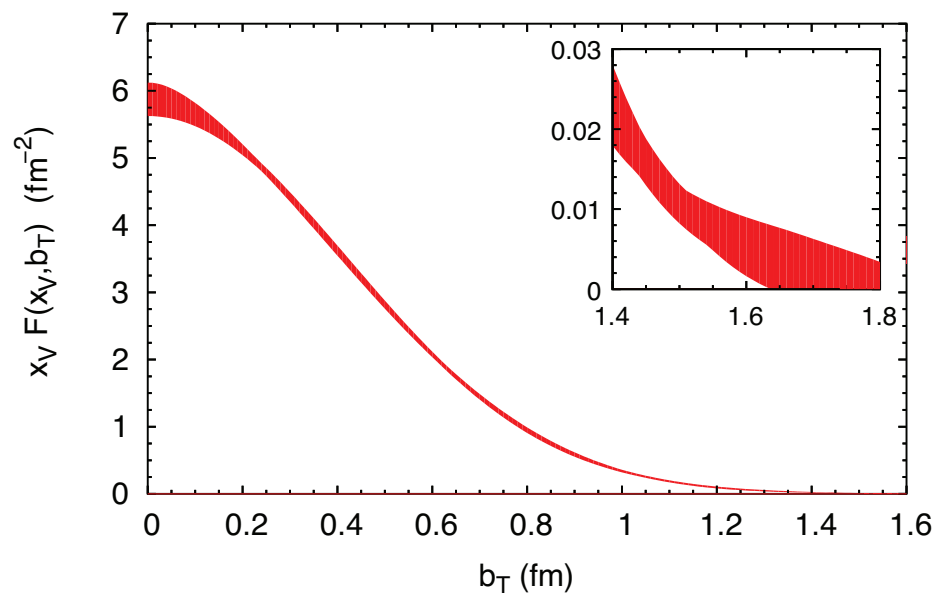
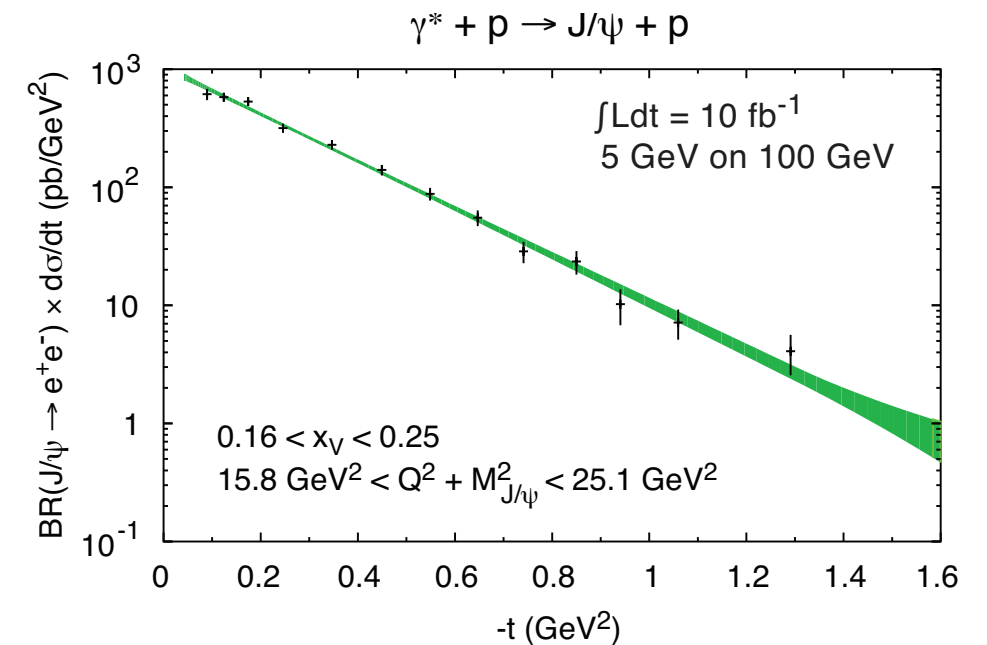
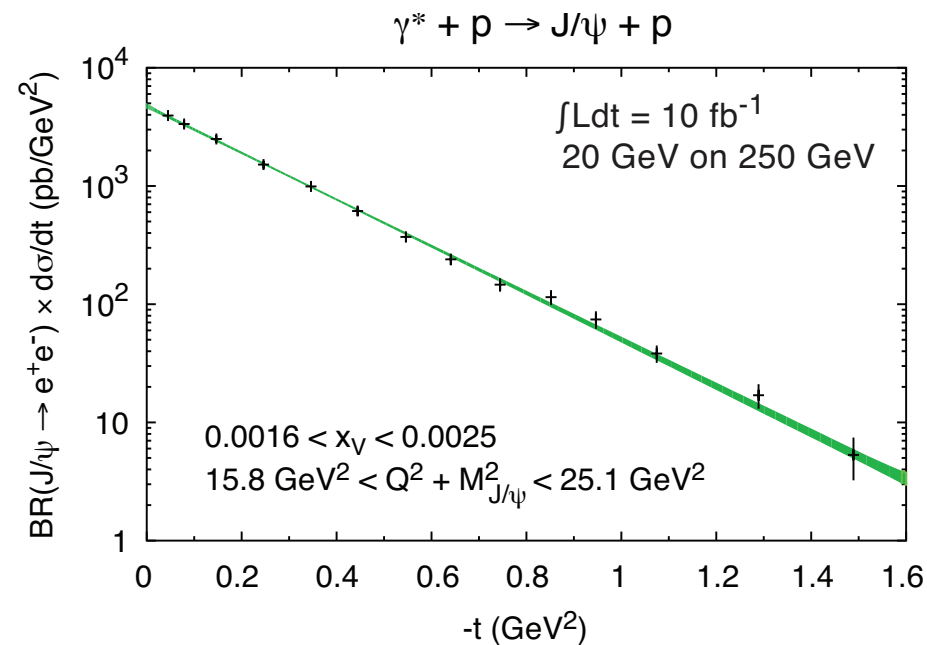
Growth of the target size with energy



The target size, i.e. the region of gluon density probed by the vector meson increases with energy

Elastic vector meson production at EIC

Fourier transform



EIC, White paper

EIC: lower energy than HERA, different kinematics.
Very high statistics, high precision

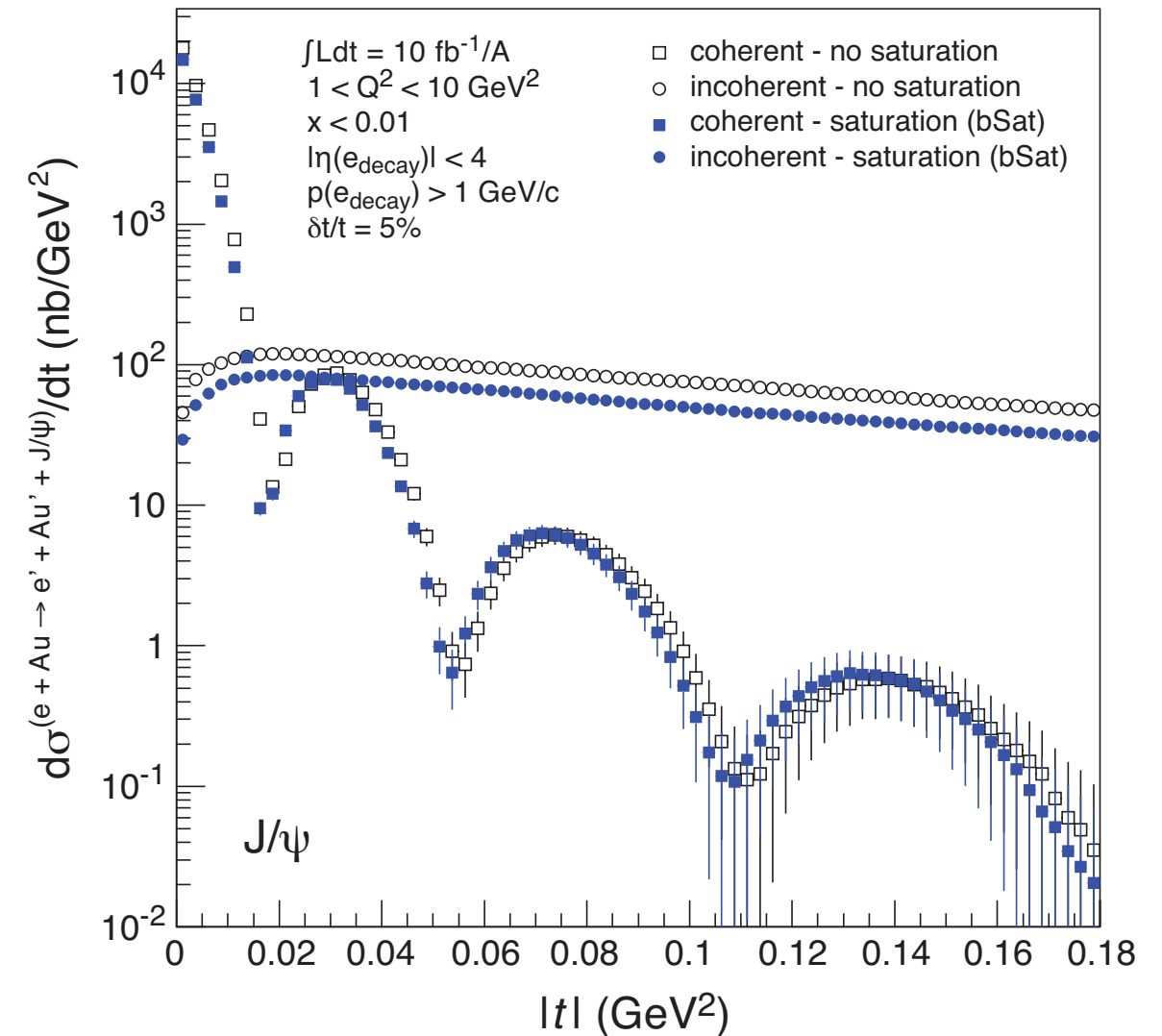
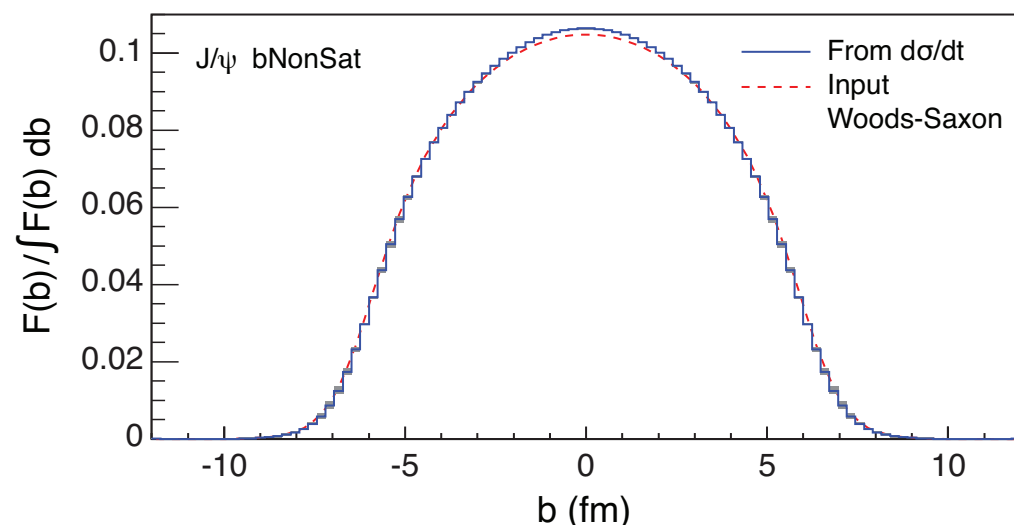
Elastic vector meson production at EIC: Nuclei

EIC, White paper

Nuclear target: Au

$$e + Au \rightarrow e + Au + J/\psi$$

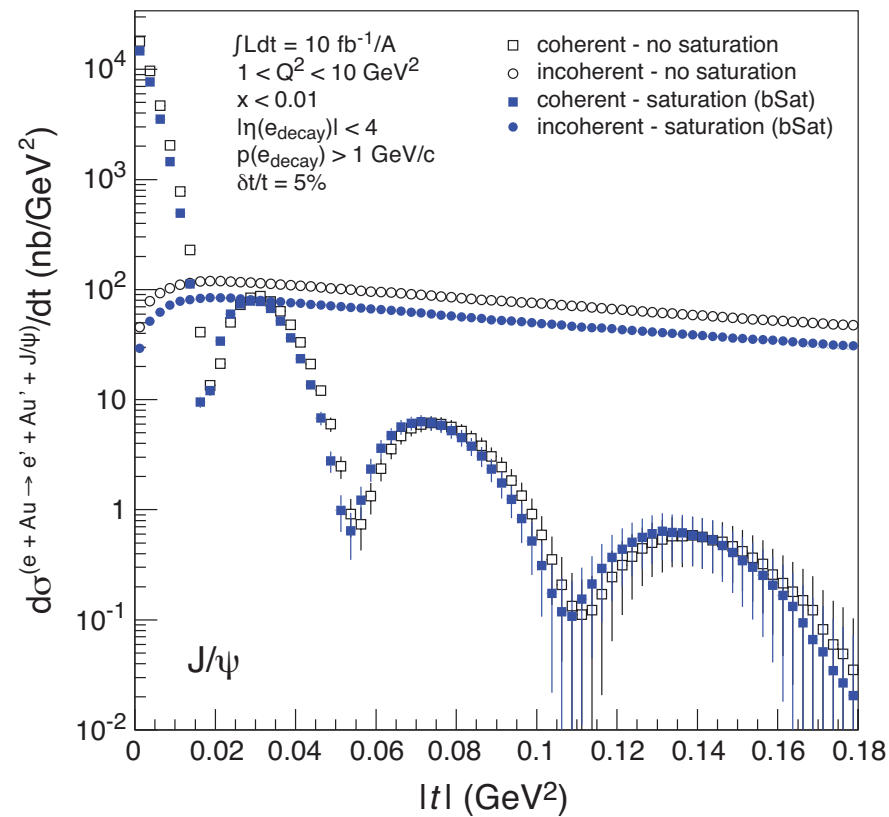
Characteristic 'dips' in t-distribution



$$F(b) = \int_0^\infty \frac{q dq}{2\pi} J_0(qb) \sqrt{\frac{d\sigma_{\text{coherent}}}{dt}}$$

$$t = -q^2$$

Coherent vs incoherent

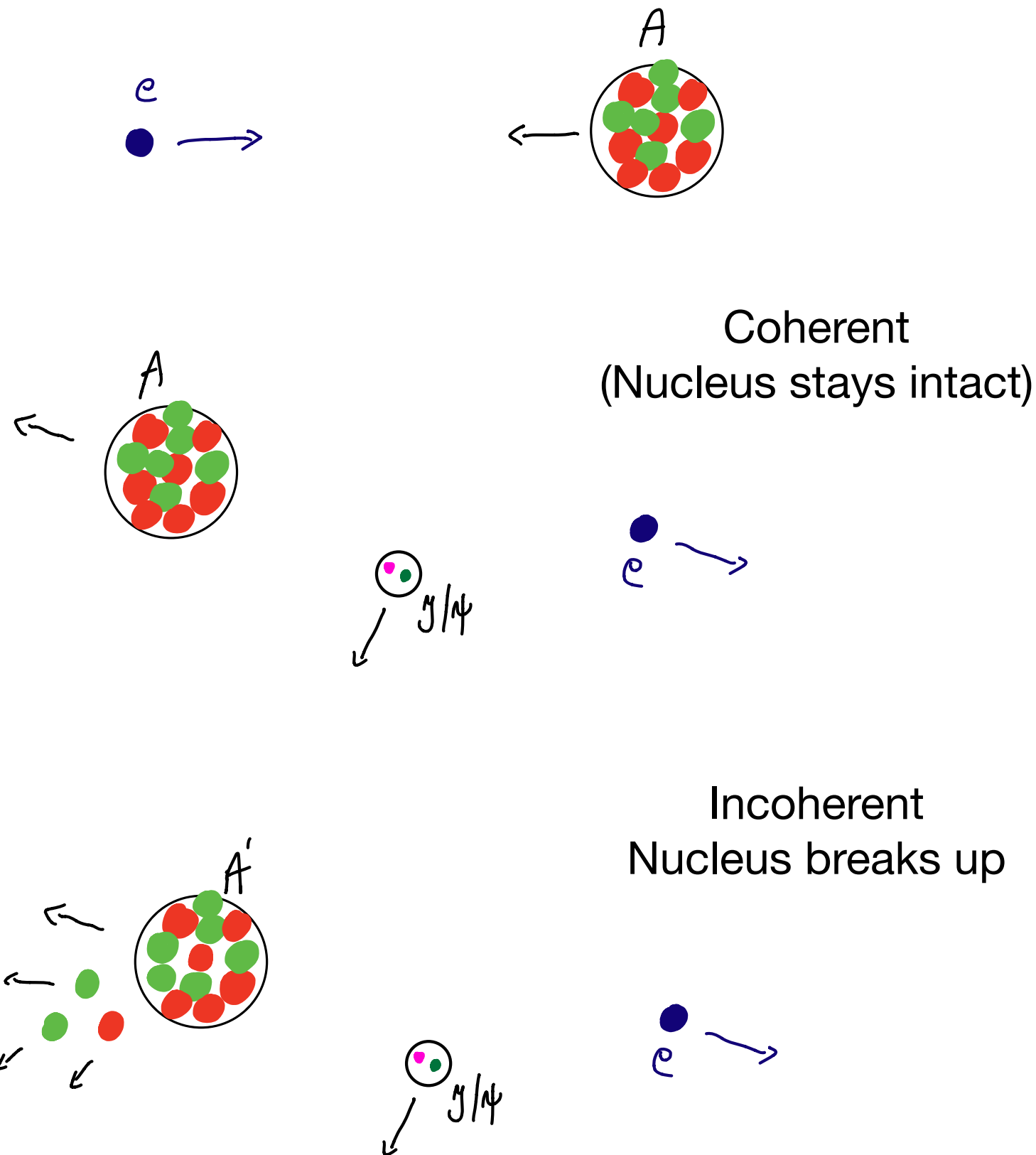


Coherent:

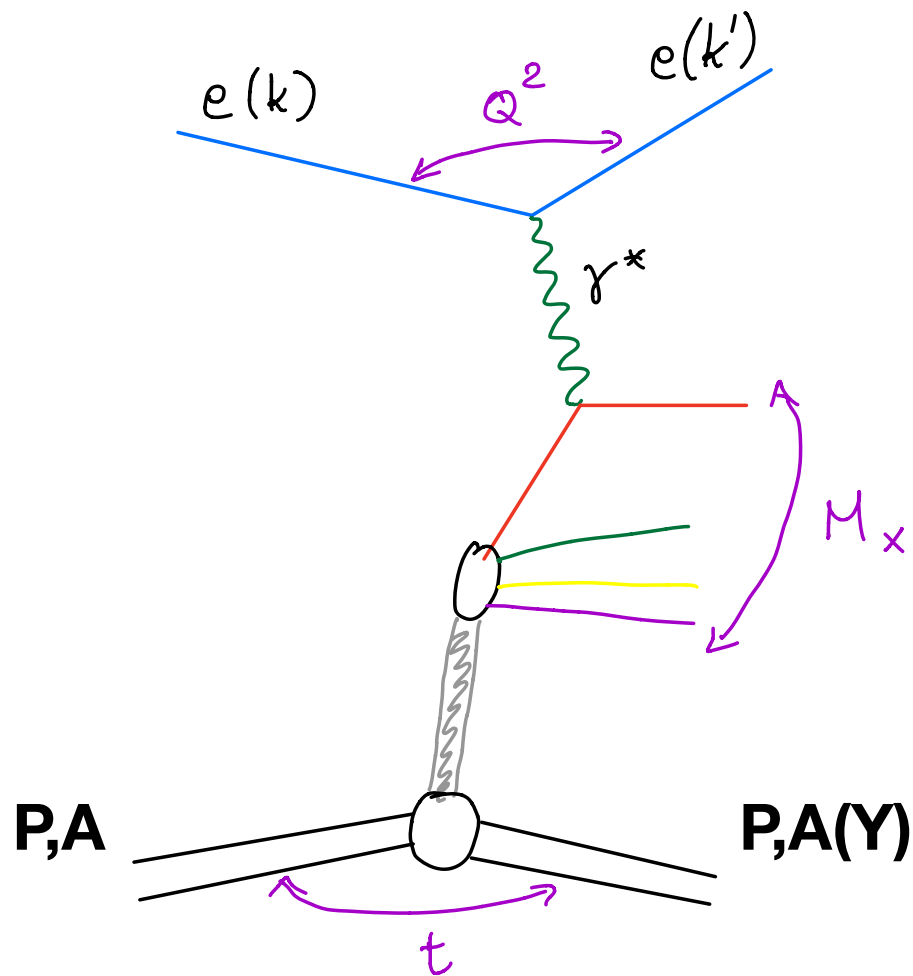
Depends on the shape of the source, average distribution

Incoherent:

Provides information about the fluctuations or lumpiness of the source



Inclusive Diffraction in e p(A)



t

4-momentum transfer squared

$$x_{Bj} = \frac{Q^2}{2p \cdot q}$$

Bjorken x

$$Q^2 = -q^2$$

(minus) photon virtuality

$$\xi \equiv x_{IP} = \frac{Q^2 + M_X^2 - t}{Q^2 + W^2}$$

momentum fraction of the Pomeron w.r.t hadron

$$\beta = \frac{Q^2}{Q^2 + M_X^2 - t}$$

momentum fraction of parton w.r.t Pomeron

$$x_{Bj} = x_{IP} \beta$$

Rapidity gap

$$\ln 1/x_{IP}$$

Inclusive diffraction: system X can contain anything (jets, heavy quarks),
Can learn about structure of the diffractive exchange

Diffractive structure functions

$$\frac{d^3 \sigma^D}{dx_{IP} dx dQ^2} = \frac{2\pi \alpha_{\text{em}}^2}{x Q^4} Y_+ \sigma_r^{D(3)}(x_{IP}, x, Q^2)$$

3 variables

$$Y_+ = 1 + (1 - y)^2$$

Reduced diffractive cross section depends on two structure functions :

$$\sigma_r^{D(3)} = F_2^{D(3)} - \frac{y^2}{Y_+} F_L^{D(3)}$$

Transverse photon
polarization and
longitudinal
polarization

For y not too close to unity we have:

$$\sigma_r^{D(3)} \simeq F_2^{D(3)}$$

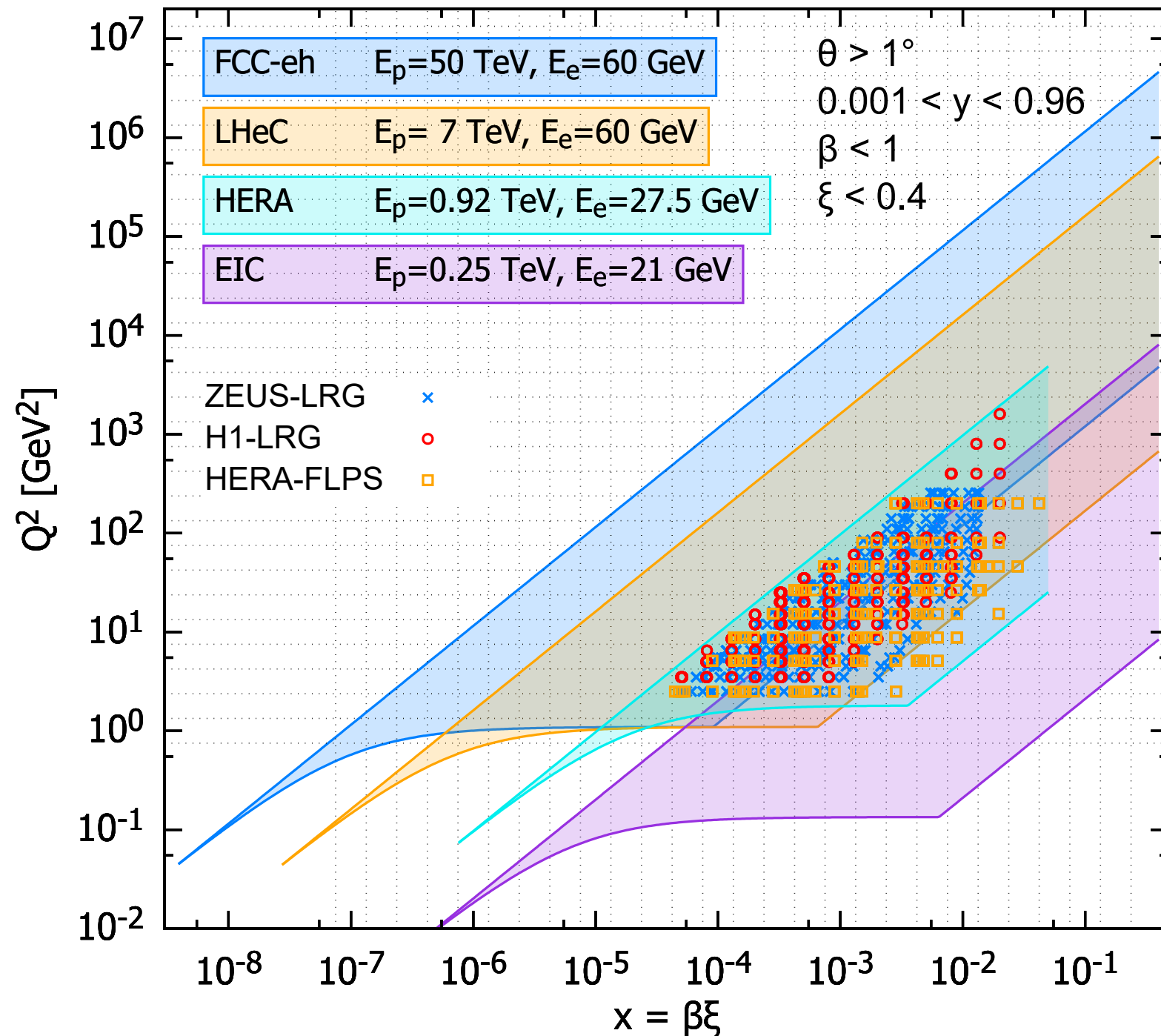
Integrated vs unintegrated structure functions over t :

$$F_{T,L}^{D(3)}(x, Q^2, x_{IP}) = \int_{-\infty}^0 dt F_{T,L}^{D(4)}(x, Q^2, x_{IP}, t)$$

$$F_2^{D(4)} = F_T^{D(4)} + F_L^{D(4)}$$

4 variables

Phase space of HERA and future colliders

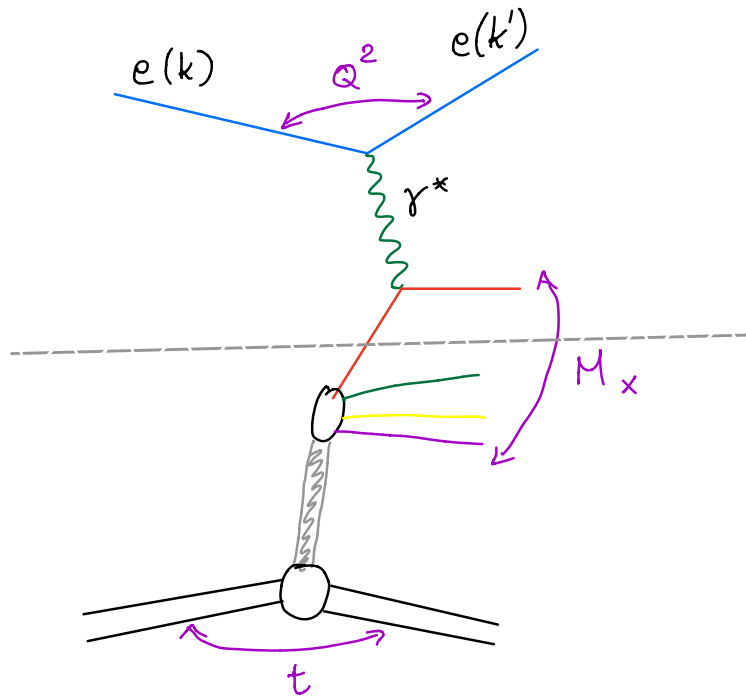


EIC: large x , up by factor 10,
small / moderate Q^2

LHeC: small x ,
 x_{\min} down by factor 20,
wide range of Q^2 , up by
factor 100

FCC-eh: very small x ,
down by factor 200,
 Q^2 up by factor 1000

Collinear factorization in diffraction



Collins

Collinear factorization in diffractive DIS

$$d\sigma^{ep \rightarrow eXY}(\beta, \xi, Q^2, t) = \sum_i \int_{\beta}^1 dz \, d\hat{\sigma}^{ei}\left(\frac{\beta}{z}, Q^2\right) f_i^D(z, \xi, Q^2, t)$$

- Diffractive cross section can be factorized into the perturbatively calculable partonic cross sections and diffractive parton distributions (DPDFs).
- Partonic cross sections are the same as for the inclusive DIS.
- The DPDFs represent the probability distributions for partons *i* in the proton under the constraint that the proton is scattered into the system Y with a specified 4-momentum.
- Factorization should be valid for sufficiently(?) large Q^2 (and fixed t and x_{IP}).

DPDF parametrization

Regge factorization (additional assumption)

$$f_i^D(x, Q^2, x_{IP}, t) = f_{IP/p}(x_{IP}, t) f_i(\beta = x/x_{IP}, Q^2)$$

Pomeron flux is parametrized as

$$f_{IP/p}(x_{IP}, t) = A_{IP} \frac{e^{B_{IP}t}}{x_{IP}^{2\alpha_{IP}(t)} - 1}$$

$$\alpha_{IP}(t) = \alpha_{IP}(0) + \alpha'_{IP}t$$

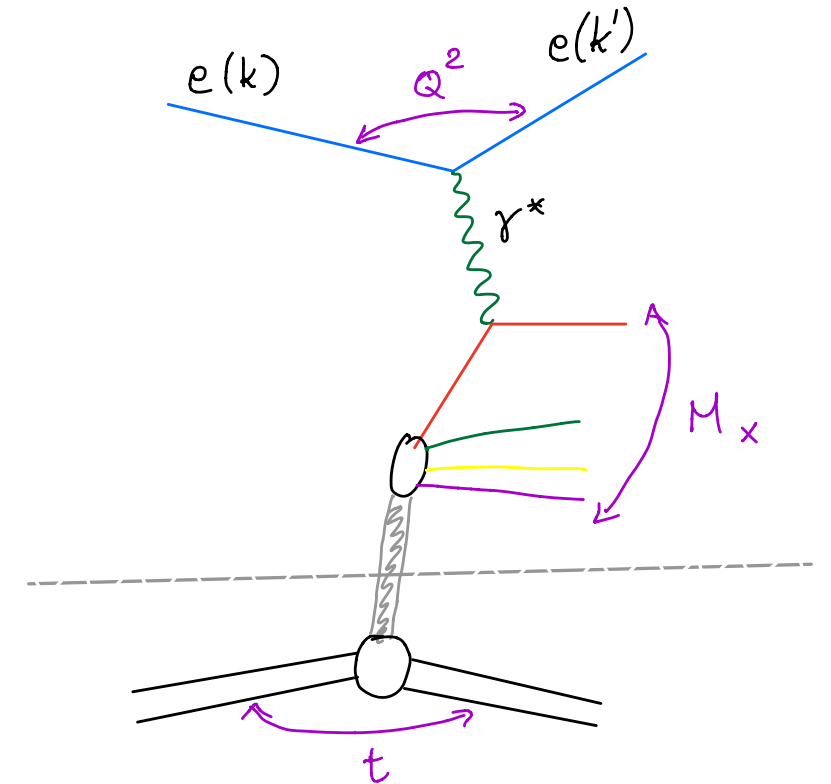
parton distributions in the Pomeron, evolved with DGLAP evolution and initial conditions

$$f_k(z) = A_k z^{B_k} (1 - z)^{C_k}$$

where $k=g,d$. Light quarks equal $u=d=s$.

For good description of the data usually subleading Reggeons are included

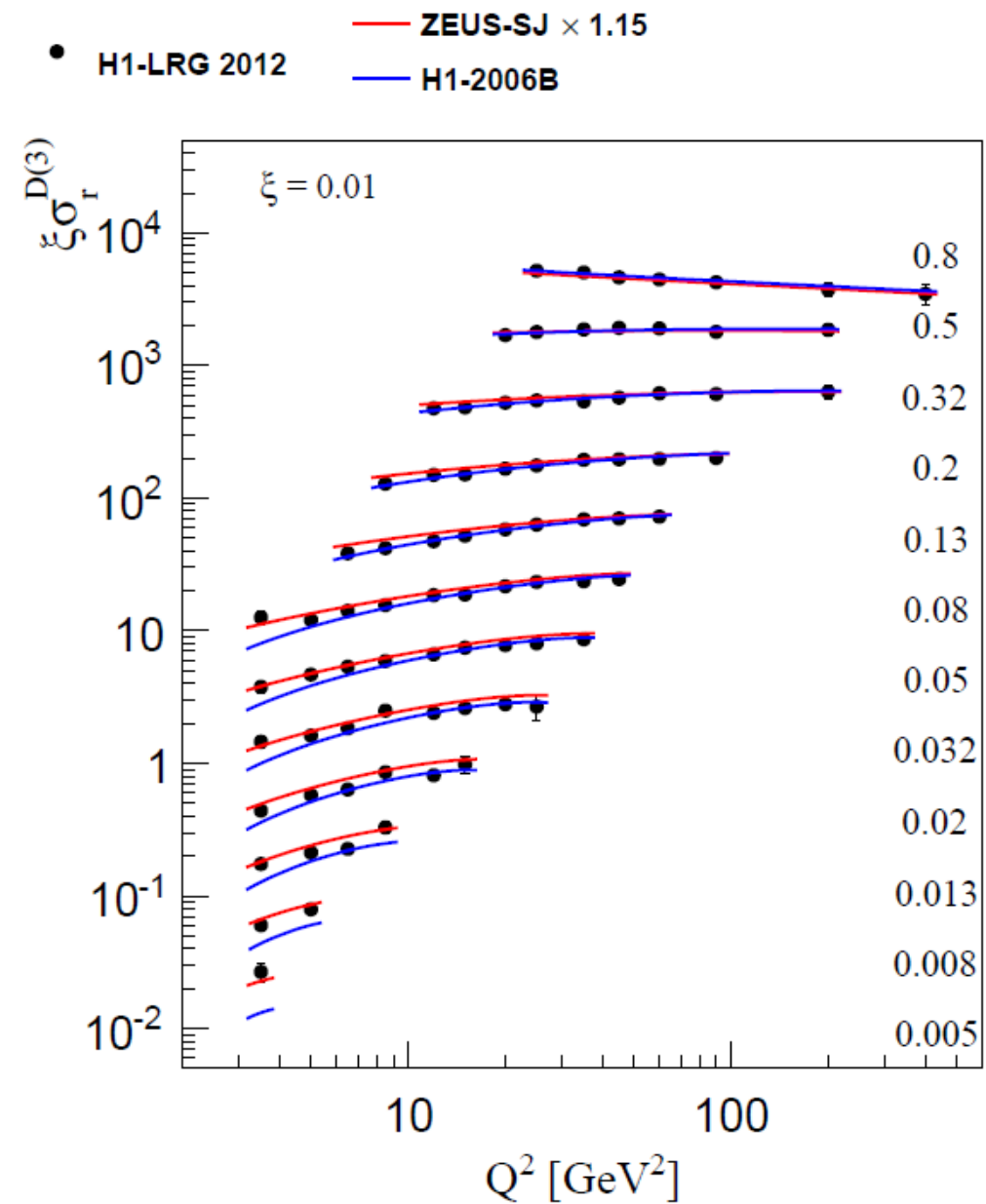
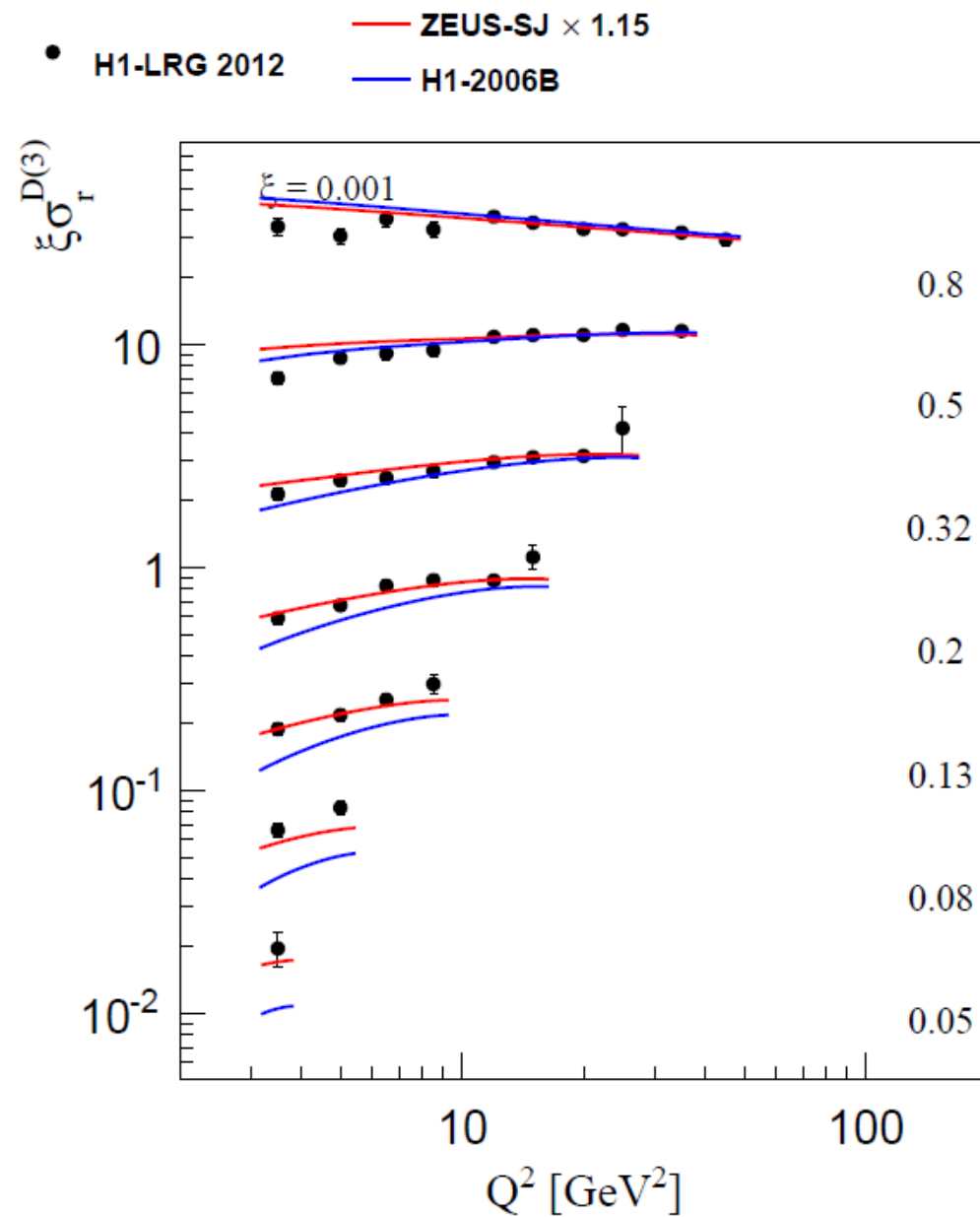
$$f_i^D(x, Q^2, x_{IP}, t) = f_{IP/p}(x_{IP}, t) f_i(\beta, Q^2) + n_{IR} f_{IR/p}(x_{IP}, t) f_i^{IR}(\beta, Q^2)$$



Diffractive fits

$$\xi = x_{IP}$$

Example of the DGLAP fit to the diffractive data

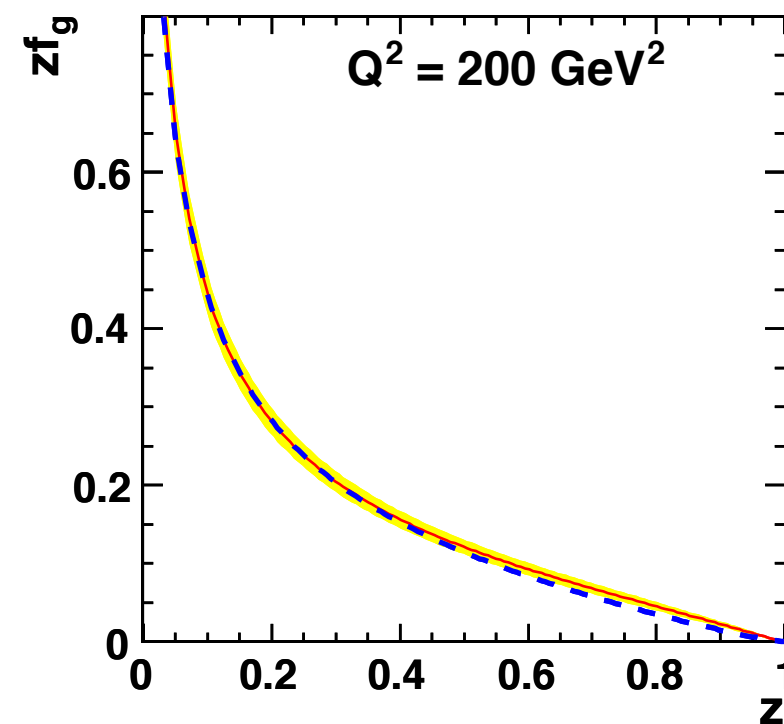
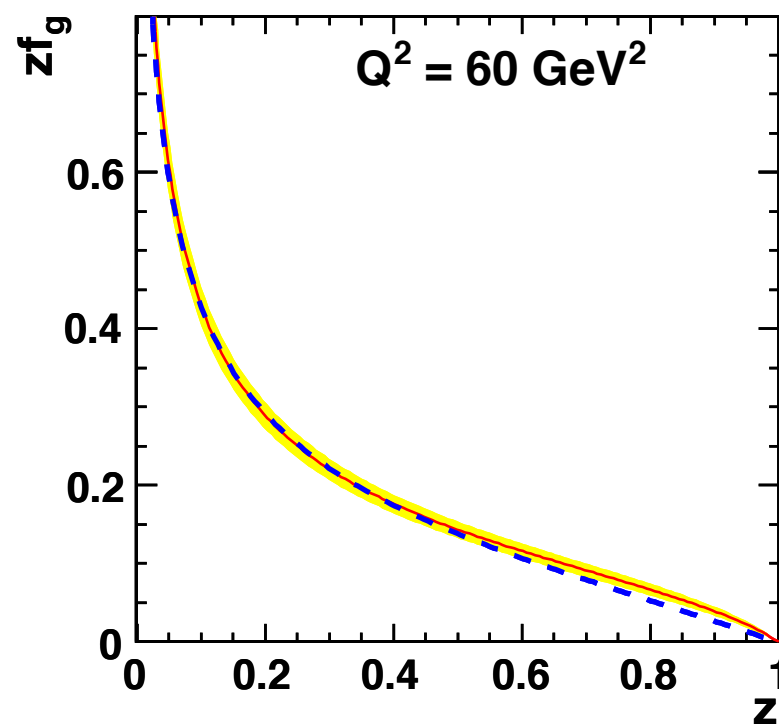
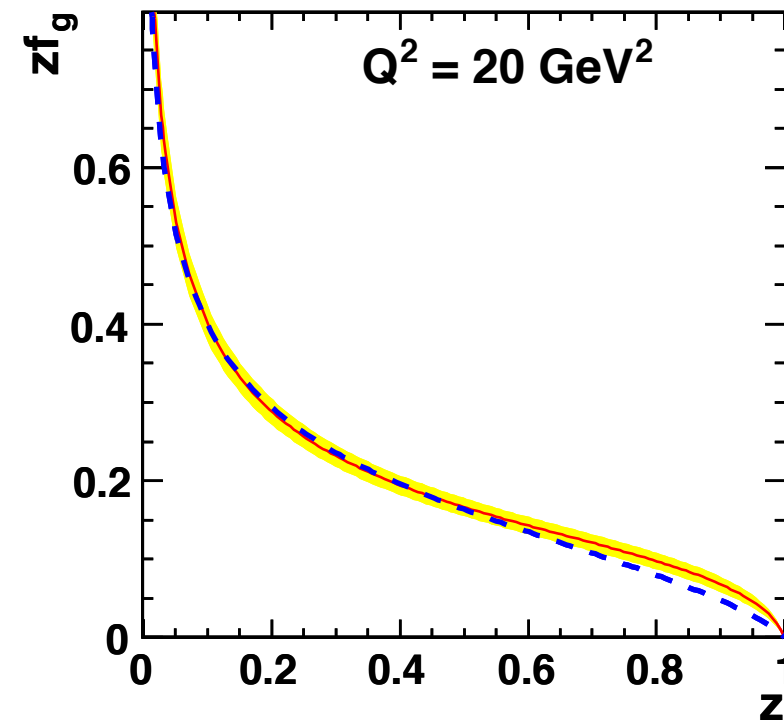
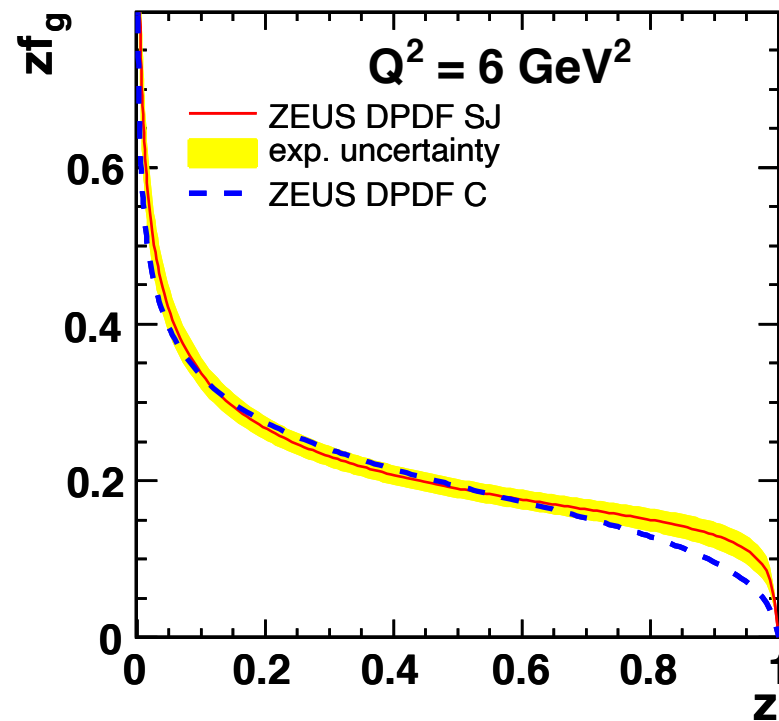


Comparison of H1-2006B and ZEUS-SJ fits to the H1-LRG 2012 data
ZEUS-SJ fit seems to better describe the data in the low β region

DPDFs

Gluon

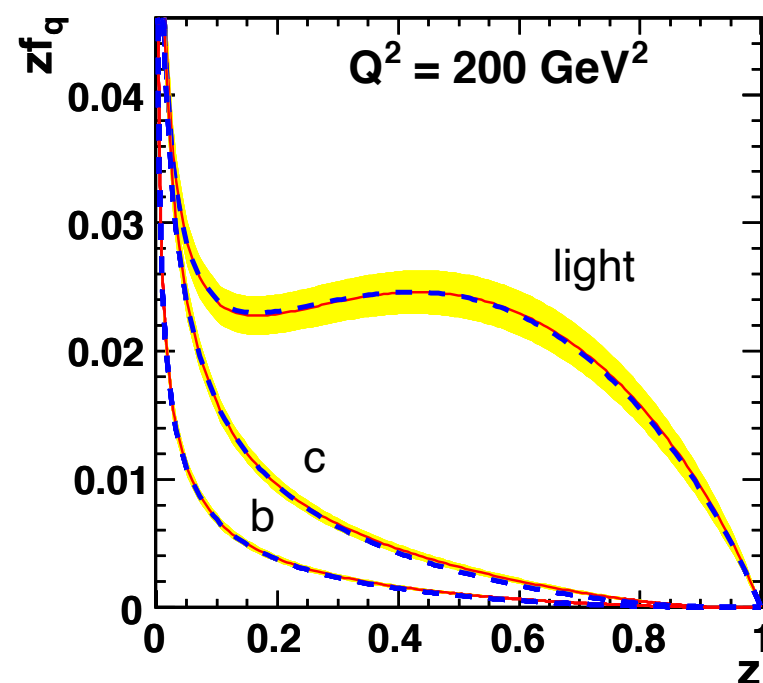
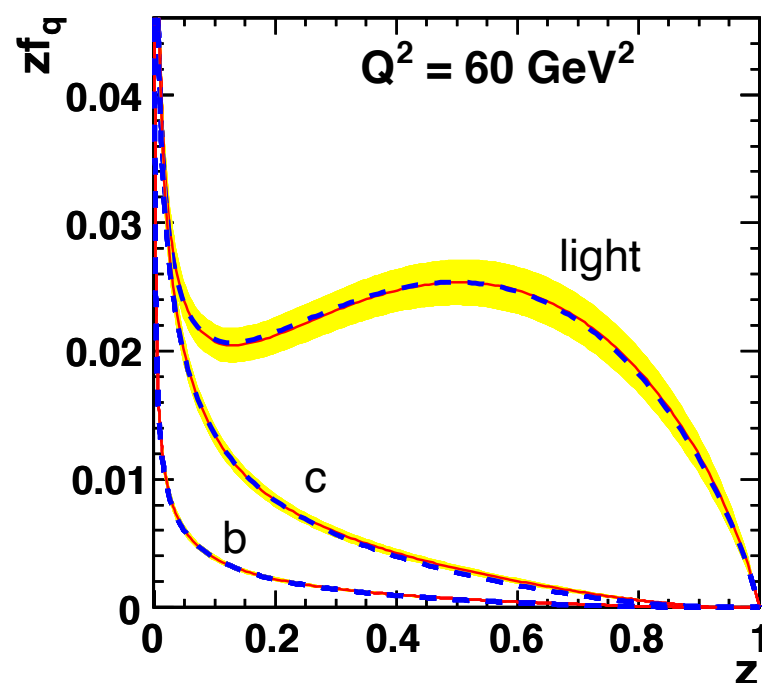
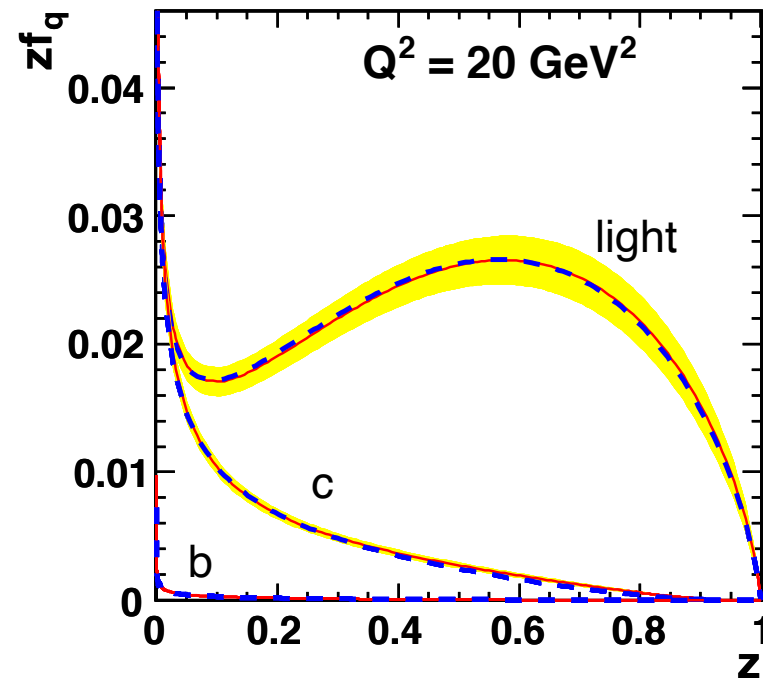
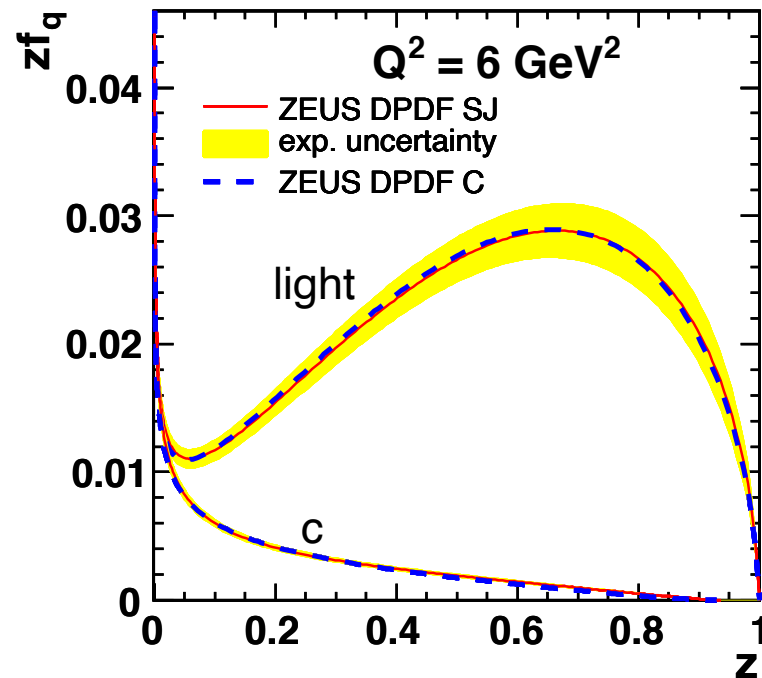
$$f_i^D(x, Q^2, x_{IP}, t) = f_{IP/p}(x_{IP}, t) f_i(\beta = x/x_{IP}, Q^2)$$



DPDFs

Quark

$$f_i^D(x, Q^2, x_{IP}, t) = f_{IP/p}(x_{IP}, t) f_i(\beta = x/x_{IP}, Q^2)$$



Inclusive diffraction possibilities at EIC

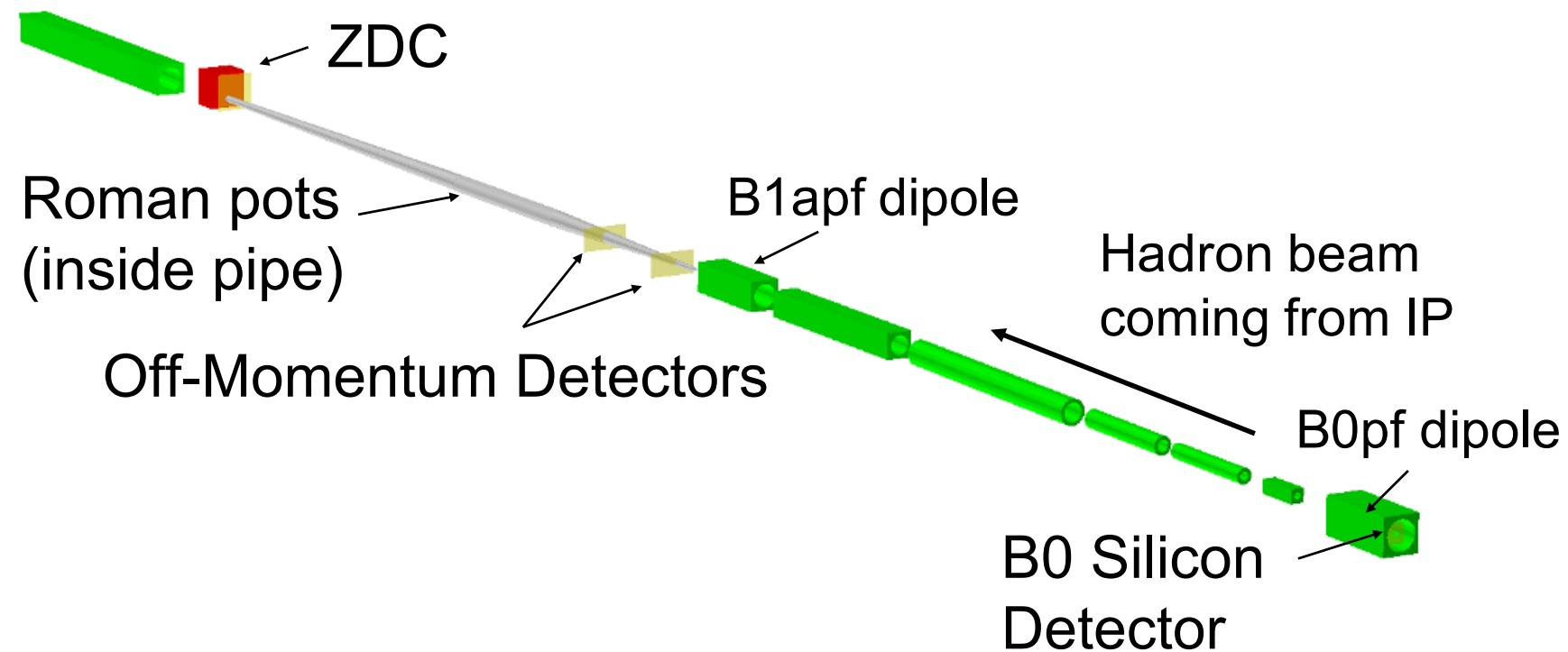
Physics with protons

- Precision measurement of $\sigma_{\text{red}}^{D(3)}, \sigma_{\text{red}}^{D(4)}$
- Access to large $\xi = x_{IP}$
- Possibility of disentangling Pomeron and Reggeon components
- Precise measurements of diffractive parton densities at large z
- Measurements of the diffractive longitudinal structure function F_L^D

Physics with nuclei

- Measurements of $\sigma_{\text{red}}^{D(3)}, \sigma_{\text{red}}^{D(4)}$
- First extraction of the nuclear diffractive parton distribution functions
- Simultaneous measurements of shadowing and diffraction in the same experimental setup

Forward instrumentation at EIC



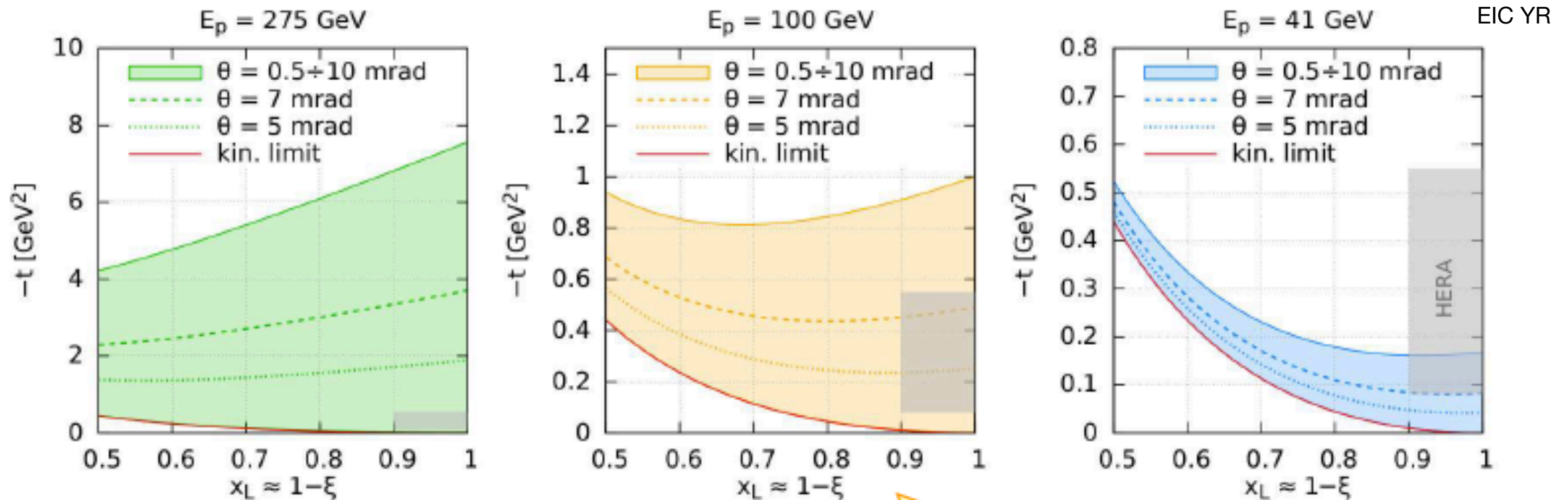
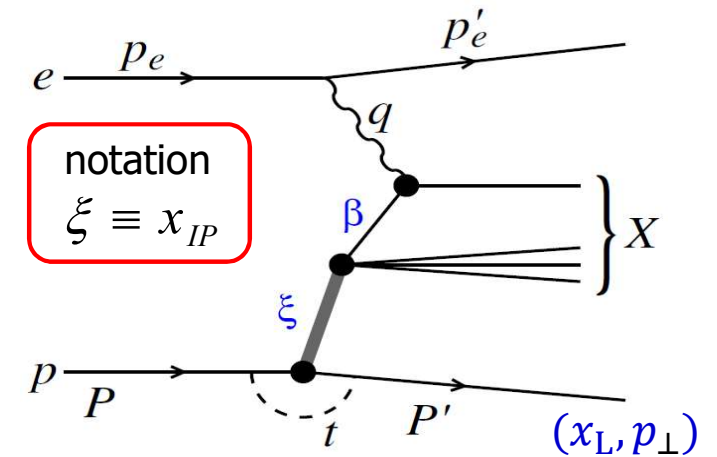
Detector	(x,z) Position [m]	Dimensions	θ [mrad]	Notes
ZDC	(0.96, 37.5)	(60cm, 60cm, 2m)	$\theta < 5.5$	~ 4.0 mrad at $\phi = \pi$
Roman Pots (2 stations)	(0.85, 26.0) (0.94, 28.0)	(25cm, 10cm, n/a)	$0.0 < \theta < 5.5$	10σ cut.
Off-Momentum Detector	(0.8, 22.5), (0.85, 24.5)	(30cm, 30cm, n/a)	$0.0 < \theta < 5.0$	$0.4 < x_L < 0.6$
B0 Spectrometer	($x = 0.19$, $5.4 < z < 6.4$)	(26cm, 27cm, n/a)	$5.5 < \theta < 13.0$	~ 20 mrad at $\phi=0$

Excellent capabilities in the forward region for t measurement of the scattered proton

Possibilities at EIC: proton tagging

$$\xi \equiv x_{IP} = \frac{Q^2 + M_X^2 - t}{Q^2 + W^2}$$

- EIC can tackle large $\xi \cong 1 - x_L$ regions beyond HERA
- The coverage depends on the angular acceptance



$$t = -\frac{p_{\perp}^2}{x_L} - \frac{(1 - x_L)^2}{x_L} m_p^2 - \frac{1 - x_L}{x_L} (M_Y^2 - m_p^2)$$

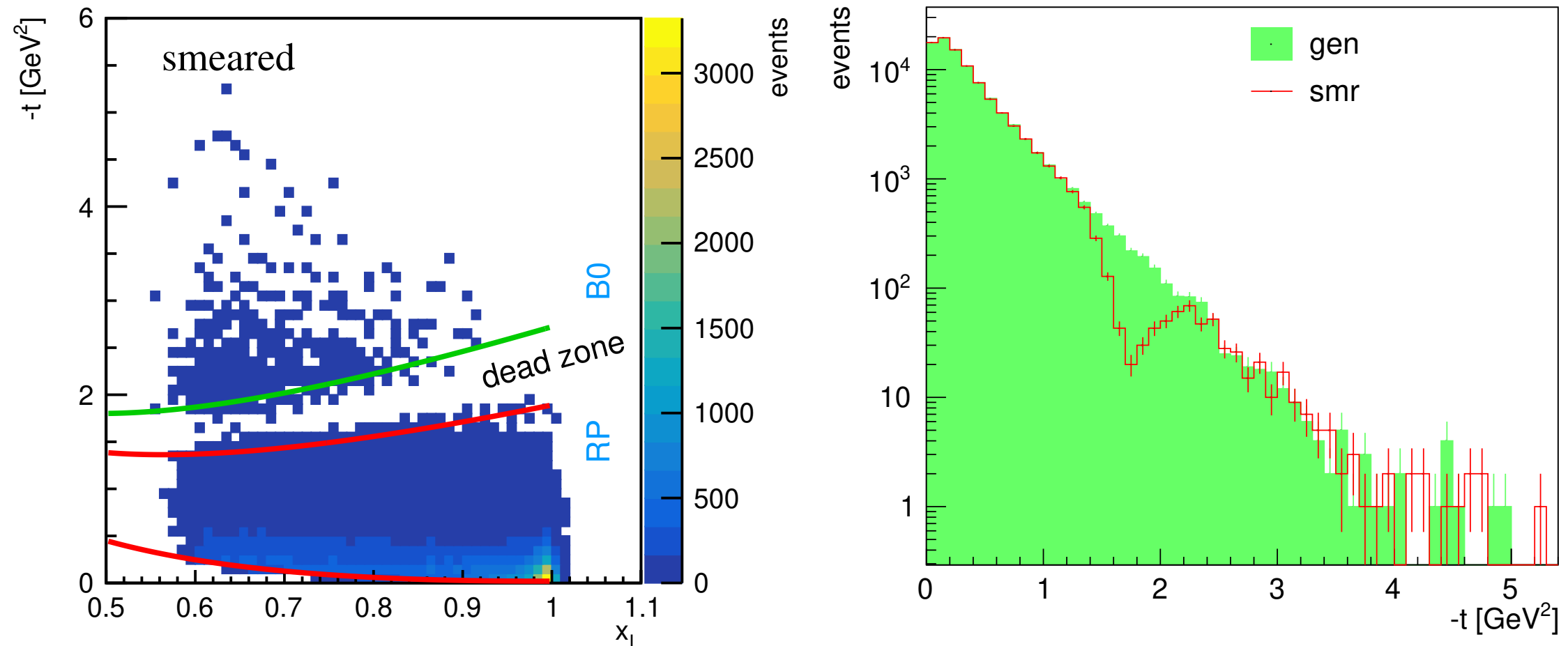
x_L, p_{\perp}, θ measured in
LAB = collinear(e, p) frame

$|t|$ measured up to $\sim 2 \text{ GeV}^2$, is very interesting,
e.g. for determination of the t -dependence of the secondary exchange.

Inclusive diffraction in ep: extended range in t and x_L

RAPGAP MC simulation

EIC YR



Gap in acceptance between Roman pots and B0 detectors, at relatively large value of t

Gap moves to lower t for smaller energies

Could be mitigated in the design with two EIC detectors, and different forward instrumentation in the second case.

Inclusive diffraction in ep: secondary exchange

Regge factorization works at low

At higher values need additional term : Reggeon terms

$$f_i^D(x, Q^2, x_{IP}, t) = f_{IP/p}(x_{IP}, t) f_i(\beta, Q^2) + n_{IR} f_{IR/p}(x_{IP}, t) f_i^{IR}(\beta, Q^2)$$

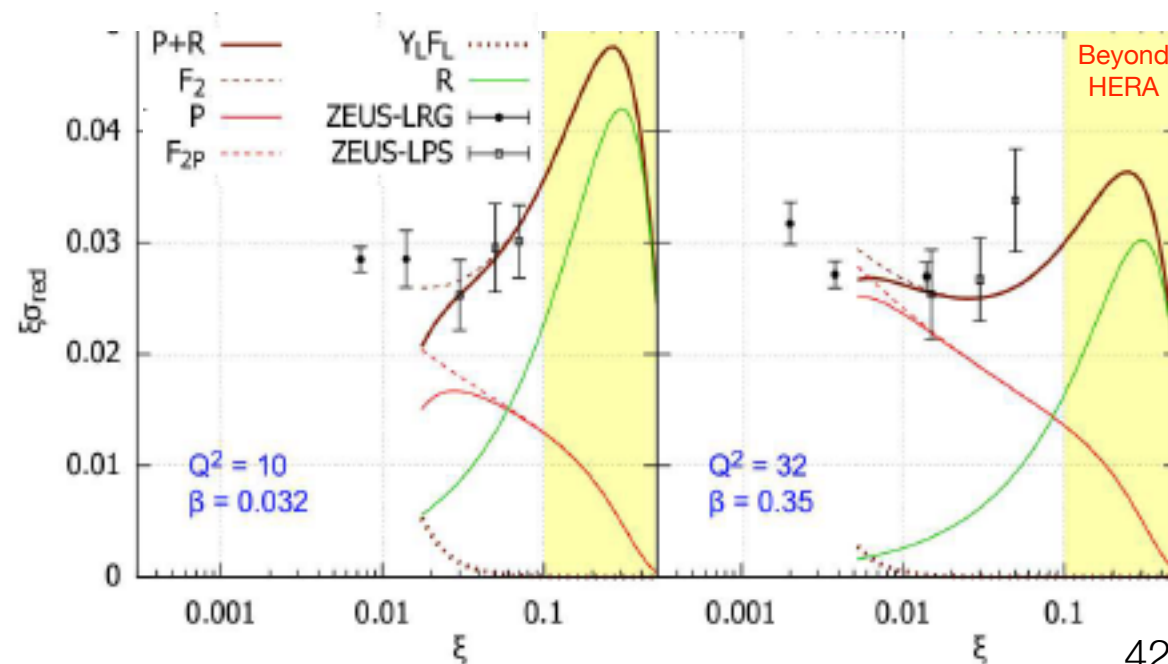
Pomeron flux is parametrized as

$$f_{IP/p}(x_{IP}, t) = A_{IP} \frac{e^{B_{IP}t}}{x_{IP}^{2\alpha_{IP}(t)} - 1}$$

$$\xi \phi_P(\xi, t) \propto \xi^{-0.22} e^{-7|t|}$$

$$\xi \phi_R(\xi, t) \propto \xi^{0.6+1.8|t|} e^{-2|t|}$$

$$\alpha_{IP}(t) = \alpha_{IP}(0) + \alpha'_{IP}t$$

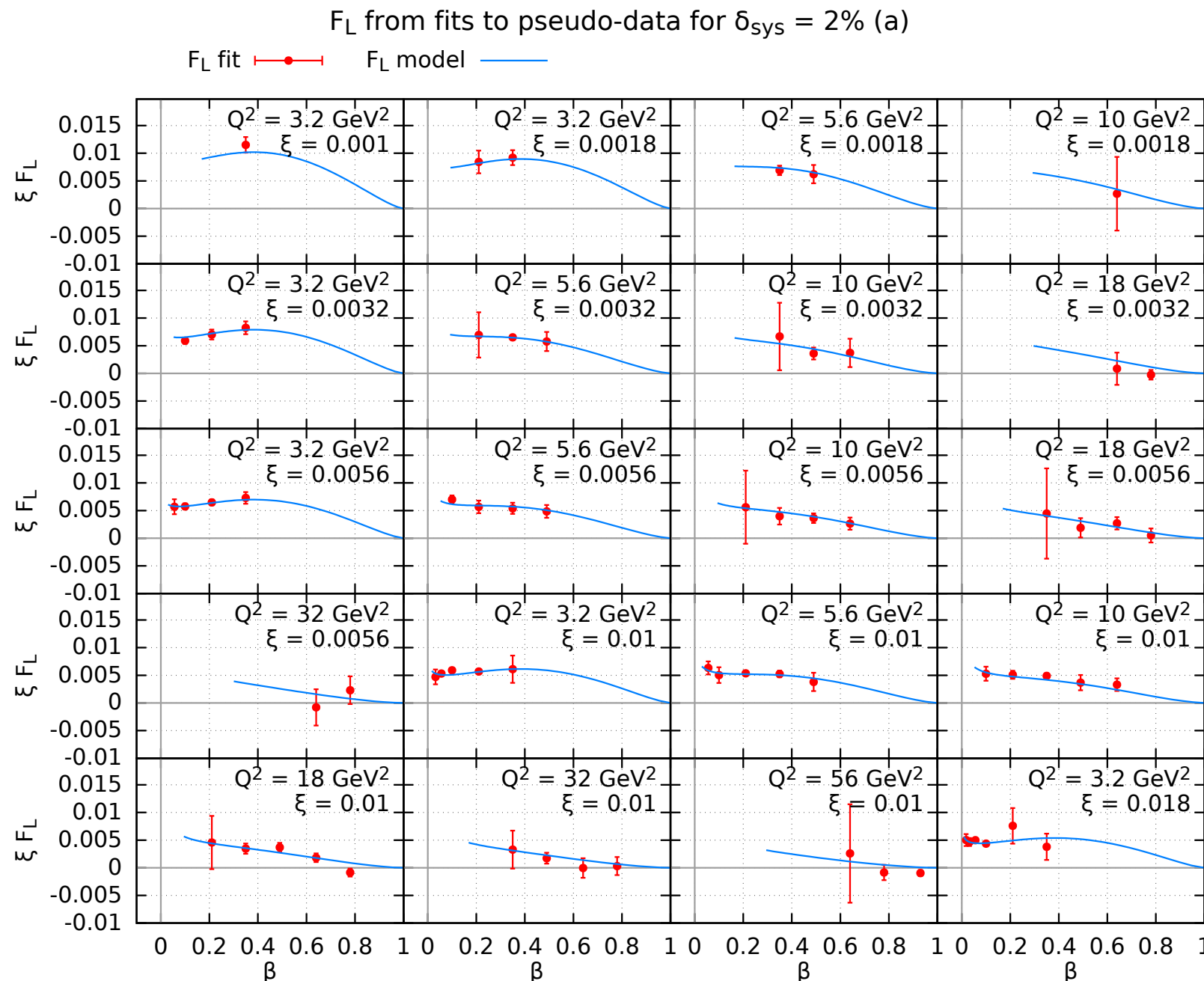


- ☐ \mathcal{R} contribution grows with ξ
- ☐ High ξ required for the determination of subleading "Reggeon" term
- ☐ Significant F_L component, ~ 30 times higher than at HERA
- ☐ However, some intermediate beam energy settings needed for F_L measurements

Inclusive diffraction in ep: F_L^D

The pseudo-data are fitted to $\sigma_{\text{red}} = F_2 - Y_L(y)F_L$ $Y_L(y) = \frac{y^2}{1 + (1 - y)^2}$

F_2 and F_L are free parameters in the fit



Systematic error 2%

18 beam setups

469 bins selected such that they are common to at least four beam setups

Nuclear cross sections: EIC

Frankfurt, Guzey, Strikman model: high and low shadowing model predictions

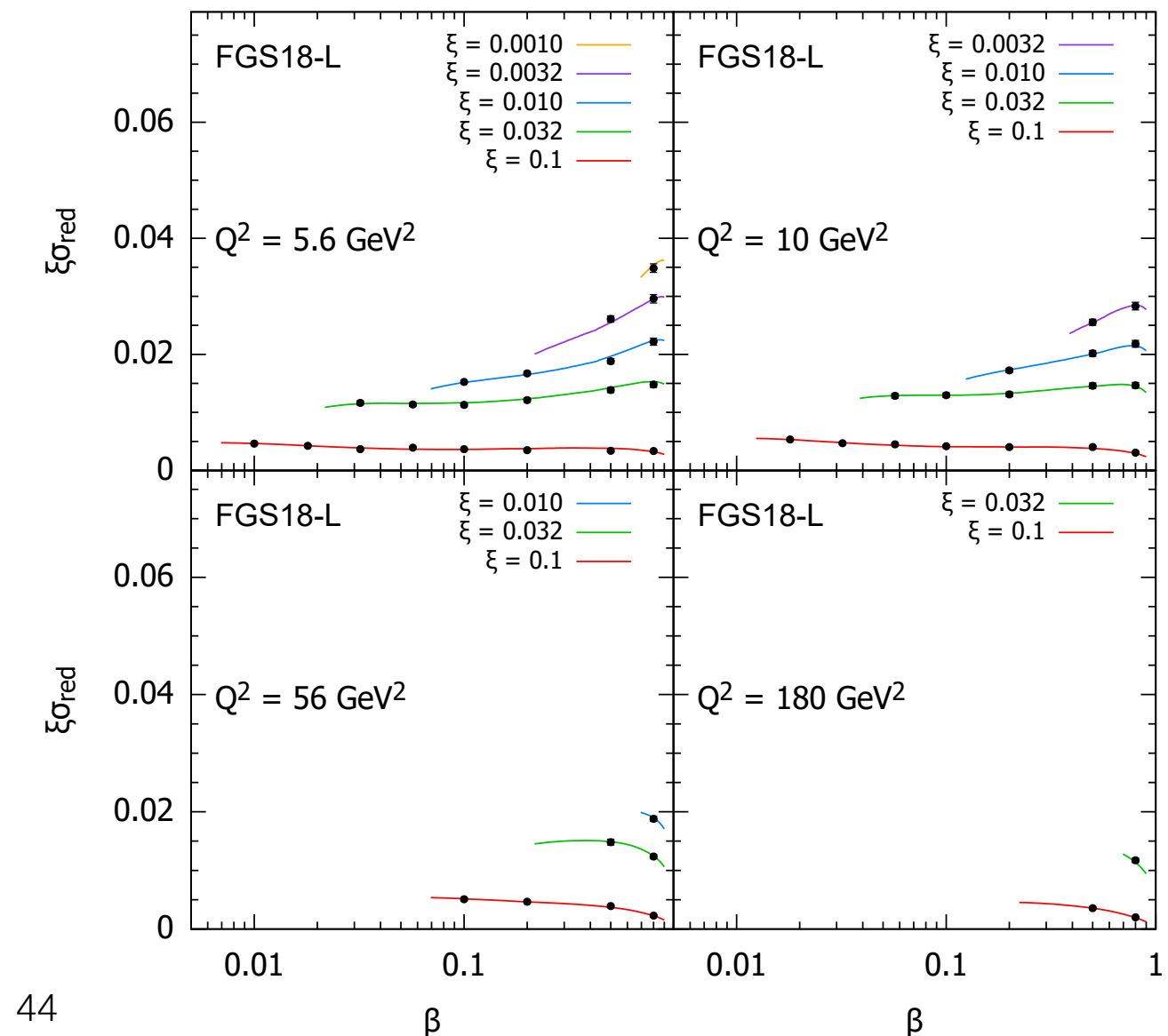
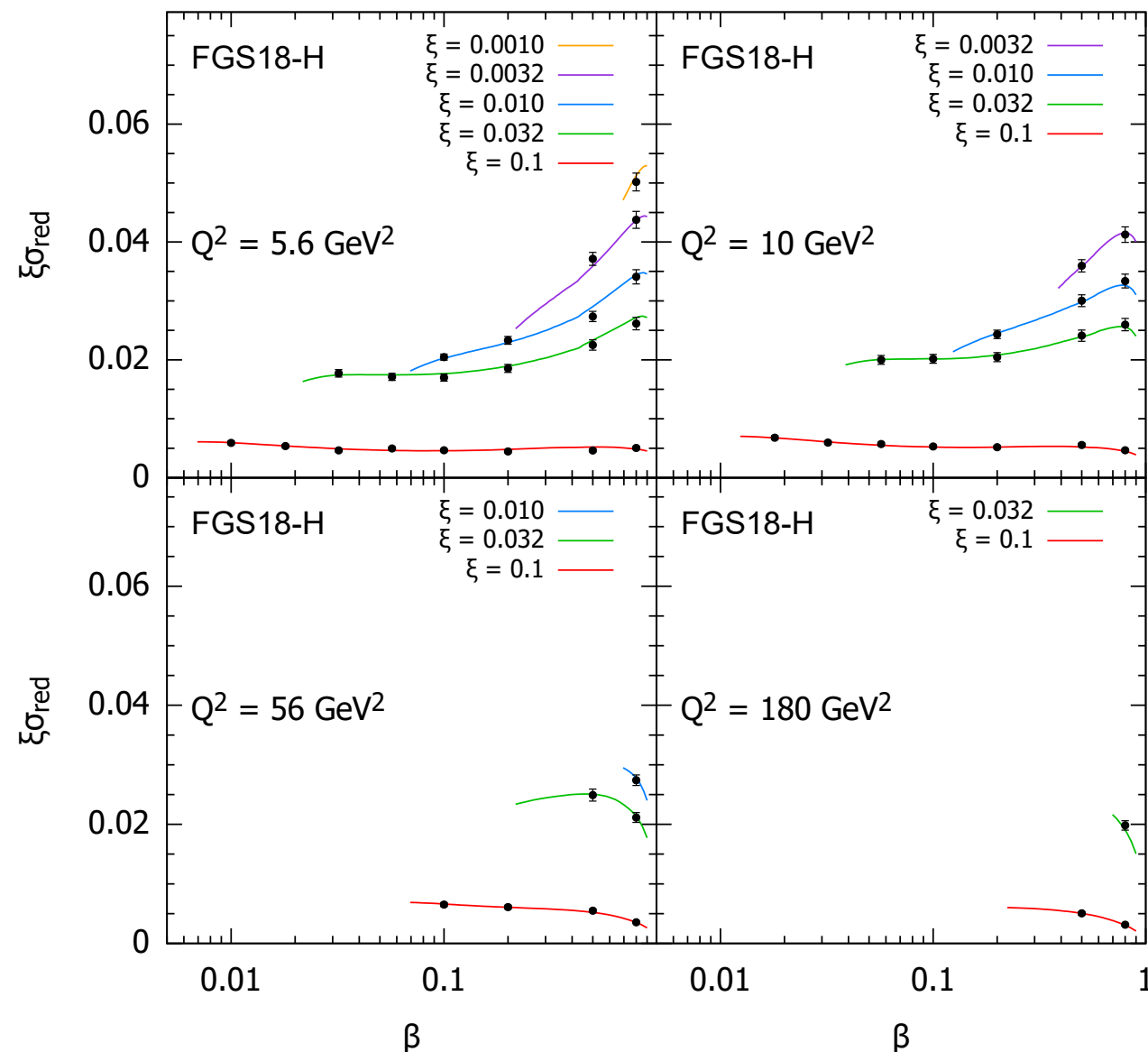
Pseudodata simulated under the same assumptions: 5% systematics, luminosity 2 fb^{-1}

Illustration of the possible reach in (Q^2, β, ξ) kinematics

Reduced cross section

e-Au $E_{\text{Au}}/A = 100 \text{ GeV}$, $E_e = 21 \text{ GeV}$, $L = 2 \text{ fb}^{-1}$

e-Au $E_{\text{Au}}/A = 100 \text{ GeV}$, $E_e = 21 \text{ GeV}$, $L = 2 \text{ fb}^{-1}$



Summary

- Diffraction observed in hadronic collisions (ep,pp ...)
- Provides important information about nucleon/nuclear structure
- Sensitive to the nature of the Pomeron
- EIC can provide precise measurements on:
 - Reduced cross sections, particularly with t-dependence and with nuclei
 - Exclusive processes, i.e. with vector mesons in the final state
 - Longitudinal diffractive structure function
 - Explore relation between the diffraction and nuclear shadowing
 - Many more...