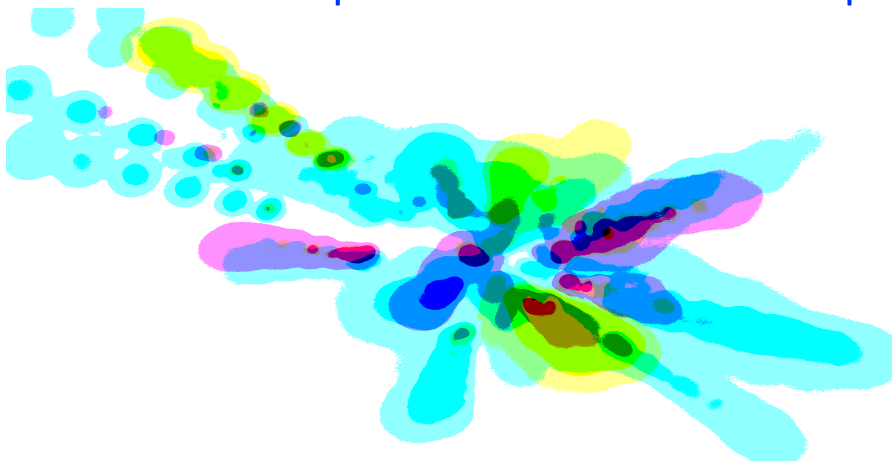
# The chiral anomaly and the proton's spin: Axion-like dynamics and spin diffusion via topological transitions



Raju Venugopalan

(BNL)

EIC Seminarium, Poland, April 19, 2021

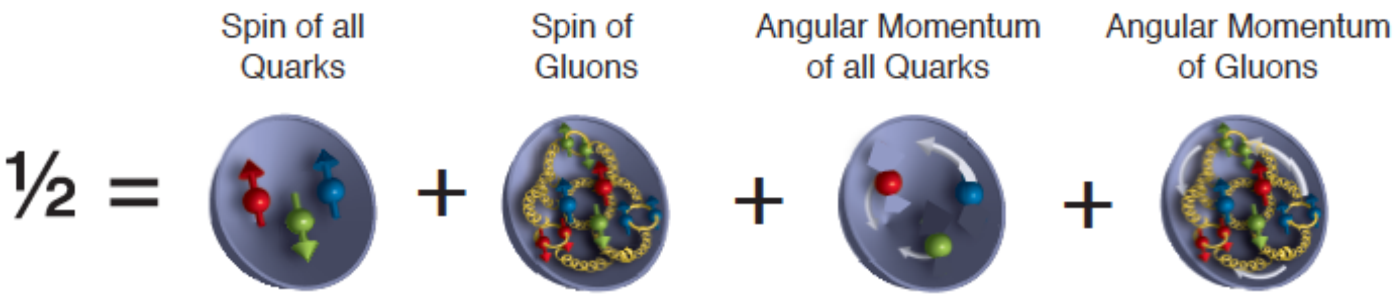
Work* in collaboration with Andrey Tarasov (The OSU and CFNS)

- The first paper is published in PRD: https://arxiv.org/abs/2008.08104
  and another is nearing completion…

# Talk outline

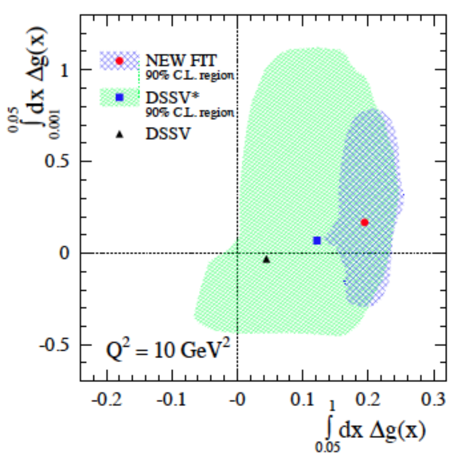
- The proton spin puzzle: new tools motivate reexamining an old problem
- Quark helicity and the chiral anomaly: probing the topology of the QCD vacuum
- Using worldlines to uncover triangles inside boxes
- $g_1$ and the $U_A(1)$ problem: a key assist from the prodigal Goldstone
- An axion-like effective action at high energies
- Interplay of gluon saturation and spin:  spin diffusion via sphaleron transitions

# The proton's spin puzzle: a many-body picture

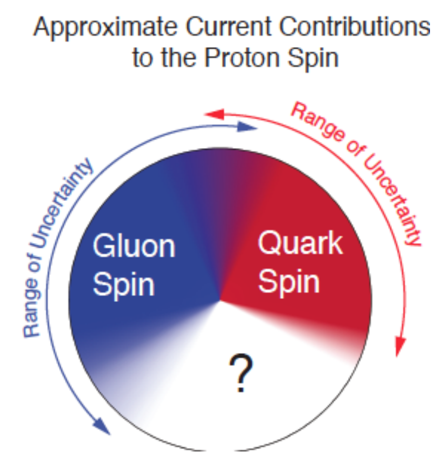



Fixed target DIS experiments showed that quarks carry only about 30% of the proton's spin

"Spin crisis": failure of the quark model ("Ellis-Jaffe sum rule") picture of relativistic "constituent" quarks
-quark helicity ($\Delta\Sigma$) much smaller than quark model expectations

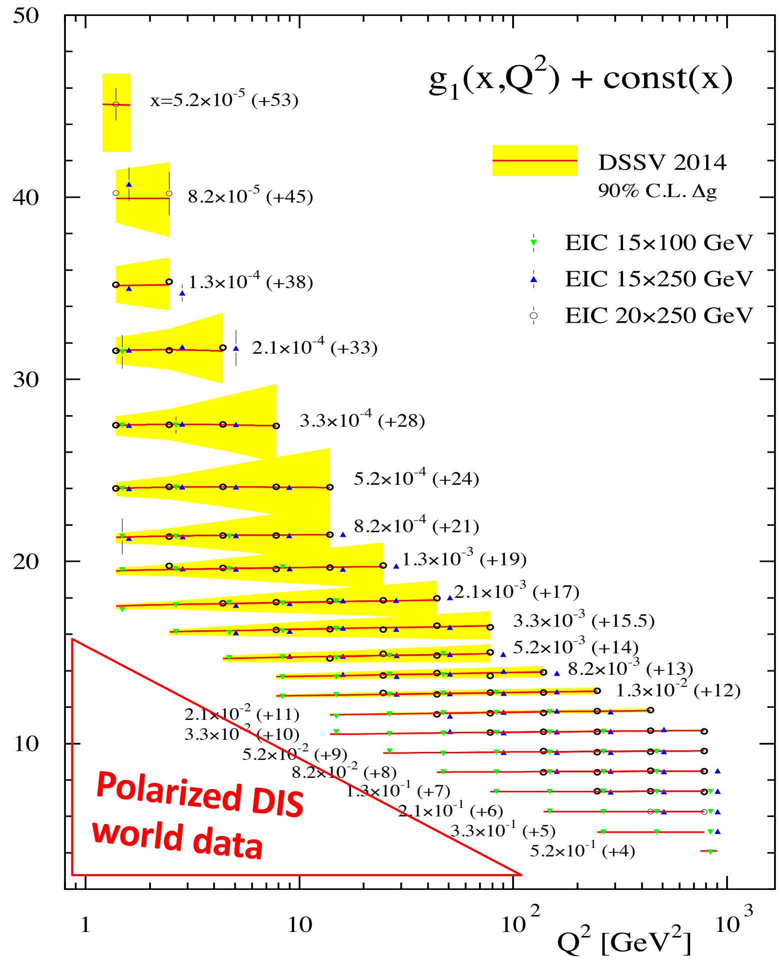



Evidence for gluon spin ($\Delta G$) from RHIC
but large uncertainties from small x

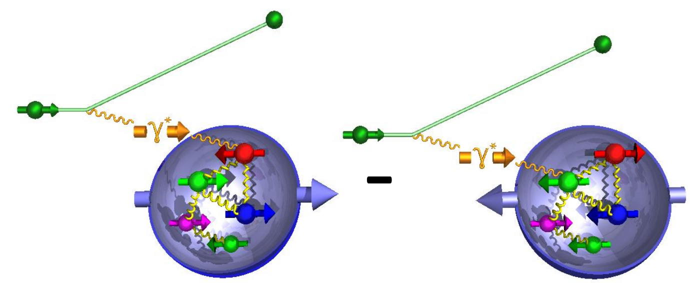



D. De Florian, R. Sassot, M. Stratmann, W. Vogelsang, PRL 113 (2014)

# Resolving the proton's spin puzzle: the $g_1$ structure function

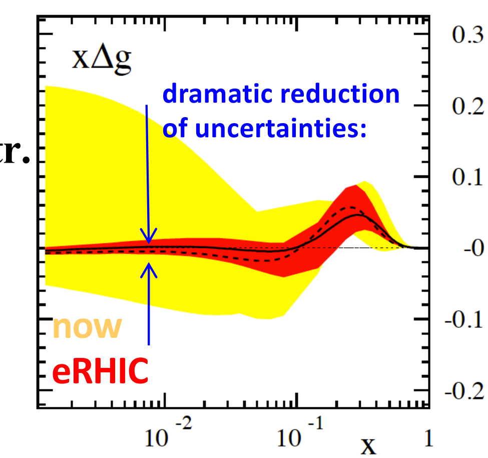



$g_1$ extracted
from longitudinal spin
asymmetry



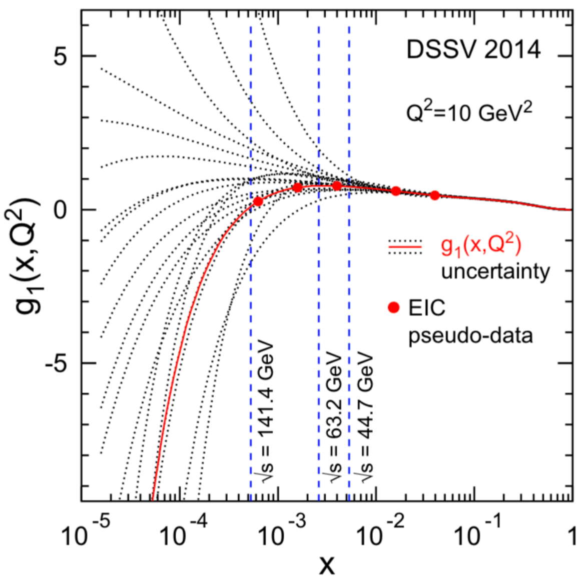

$$\Delta\Sigma(Q^2) \propto \int_0^1 dx\, g_1(x,Q^2) \rightarrow \text{quark contribution}$$

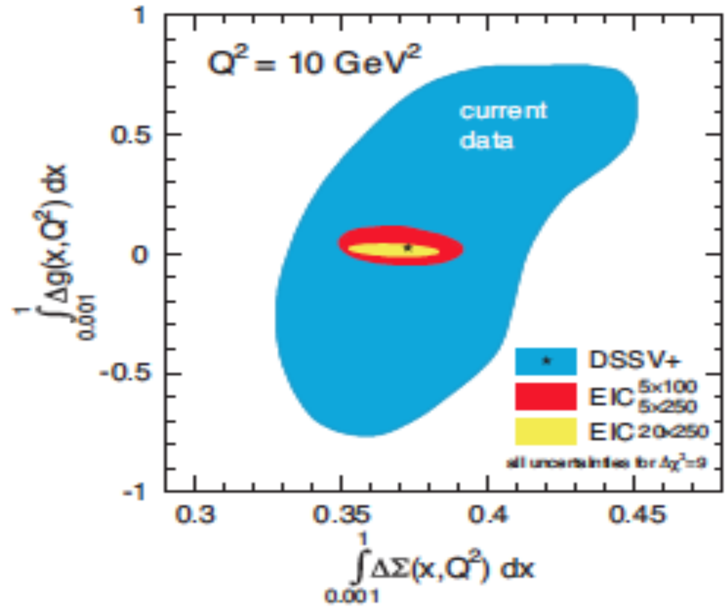
$$\frac{dg_1}{d\log(Q^2)} \overset{?}{\sim} -\Delta g(x,Q^2) \rightarrow \textbf{gluon contr.}$$



# Resolving the proton's spin puzzle: the $g_1$ structure function

Aschenauer et al., Rep. Prog. Phys. 82, 024301 (2019)

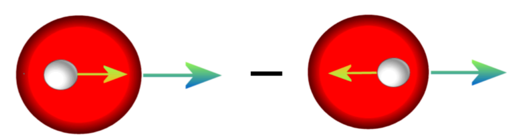



In the parton model, at leading twist

$$g_1(x_B,Q^2)=\frac{1}{2}\sum_f e_f^2\left(\Delta q_f(x_B,Q^2)+\Delta\bar{q}_f(x_B,Q^2)\right)$$

$\Delta q(x) =$



Most generally, $g_1(x,Q^2)=\dfrac{1}{8\lambda}\epsilon_T^{\mu\nu}\tilde{W}_{\mu\nu}(q,P,S)$

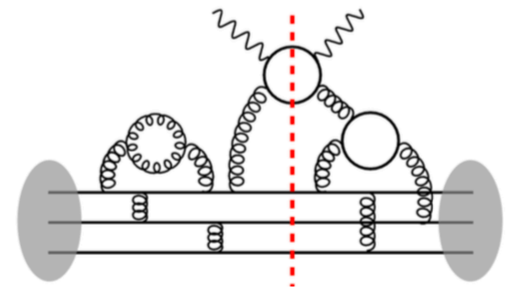
where $\widetilde{W}^{\mu\nu}$ is the antisymmetric part of $W^{\mu\nu}$

$S^\mu = \frac{2\lambda}{m_P}P^\mu$ and $\lambda = \pm 1/2$

# Quark helicity and the chiral anomaly: probing the topology of the QCD vacuum

**Reviews:**
**S. D. Bass, RMP, hep-ph/0411005**
**G. M. Shore, hep-ph/0701171**

# $g_1$ structure function: formal definitions



Hadron tensor $W^{\mu\nu}$ = $\mathrm{Im}\,\frac{i}{\pi}\int d^4\mathbf{x}\, e^{i\mathbf{q}\cdot\mathbf{x}}\langle P,S|\mathbb{T}\,\hat{j}^{\mu}(\mathbf{x})\hat{j}^{\nu}(0)|P,S\rangle$

with $j^{\mu} = \sum_f e_f \bar{\psi}_f \gamma^{\mu}\psi_f$

Most generally, $g_1(x,Q^2) = \frac{1}{8\lambda}\epsilon_T^{\mu\nu}\tilde{W}_{\mu\nu}(q,P,S)$

where $\widetilde{W}^{\mu\nu}$ is the antisymmetric part of $W^{\mu\nu}$

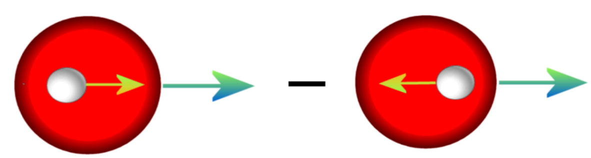
$S^{\mu} = \frac{2\lambda}{m_P}P^{\mu}$ and $\lambda = \pm 1/2$

Generalized parton model ("leading twist"): $g_1(x_B,Q^2) = \frac{1}{2}\sum_f e_f^2\left(\Delta q_f(x_B,Q^2) + \Delta\bar{q}_f(x_B,Q^2)\right)$

Where the quark helicity pdf is defined to be

$$\Delta q(x) = \frac{1}{4\pi}\int dy^-\, \mathrm{e}^{-iy^- xP^+}\langle P,S\,|\,\bar{\psi}\left(0,y^-,\mathbf{0}_\perp\right)\gamma^+\gamma_5\,\psi(0)\,|\,P,S\,\rangle$$

$$\Delta q(x) =$$



# Iso-singlet axial vector current

In general, the first moment of $g_1$: $$\int_0^1 g_1(x,Q^2) = \frac{1}{18}\left(\underbrace{3F+D}+2\,\Sigma(Q^2)\right)$$

*Combination of triplet axial vector current (gives $g_A$) measured in $\beta$ decay and octet axial vector current measured in hyperon decays*

with the iso-singlet quark helicity given by $$\Delta\Sigma(Q^2) = \sum_q^{N_f}\int_0^1 dx\left(\Delta q(x,Q^2)+\Delta\bar{q}(x,Q^2)\right)$$

In the parton model picture, this mixes, under evolution, with other isospin blind moment $\Delta G = \int_0^1 dx\, \Delta g(x,Q^2)$

$$\frac{d}{d\ln Q^2}\begin{pmatrix}\Delta\Sigma \\ \Delta G\end{pmatrix} = \begin{pmatrix}\Delta P_{\Sigma\Sigma} & 2N_f\Delta P_{qG} \\ \Delta P_{Gq} & \Delta P_{GG}\end{pmatrix}\begin{pmatrix}\Delta\Sigma \\ \Delta G\end{pmatrix}$$

Splitting functions known to high loop order

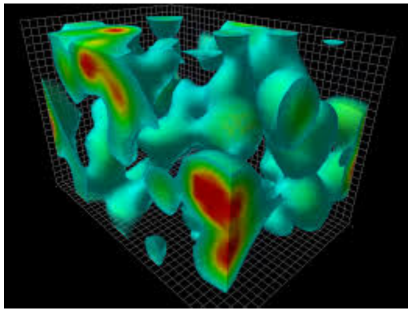
**Moch, Rogal, Vermaseren, Vogt**
**Talk by Vogelsang at POETIC IX, LBNL; DeFlorian, Vogelsang (2019)**

# Iso-singlet axial vector current and the chiral anomaly

$$\int_0^1 g_1(x,Q^2) = \frac{1}{18}\left(3F + D + 2\,\Sigma(Q^2)\right) \;\Rightarrow\; S^\mu\,\Delta\Sigma = \langle P,S\,|\,\bar\psi\,\gamma^\mu\gamma_5\,\psi\,|\,P,S\rangle \equiv \langle P,S\,|\,j_5^\mu\,|\,P,S\rangle$$

$U_A(1)$ violation from the chiral anomaly:

$$\partial_\mu J_5^\mu = 2n_f\partial_\mu K^\mu + \sum_{i=1}^{n_f} 2im_i\bar q_i\gamma_5 q_i$$



Divergence of C-S current ∝ topological charge density

where the Chern-Simons current $K_\mu = \frac{g^2}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}\left[A_a^\nu\left(\partial^\rho A_a^\sigma - \frac{1}{3}g f_{abc}A_b^\rho A_c^\sigma\right)\right]$

For massless quarks, conserve $J_5^\mu - 2\,n_f\,K^\mu$

$$\Delta\Gamma(Q^2) = -\frac{\alpha_s(Q^2)}{2\pi}N_f\Delta g(Q^2)$$

So perhaps then the "real" ΔΣ is

$$\Sigma(Q^2) = \tilde\Sigma(Q^2) - \frac{\alpha_s(Q^2)}{2\pi}N_f\Delta g(Q^2)$$

Offers a possible explanation of empirical small ΔΣ
(in addition to flavor SU(3) violation)

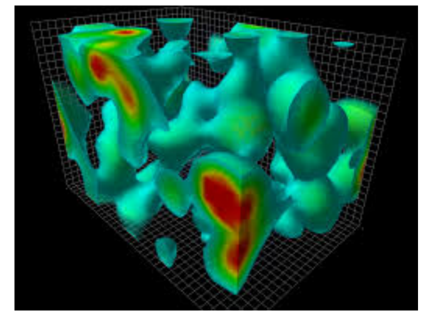
ca., 1988
Efremov, Teryaev; Altarelli, Ross ; Carlitz, Collins, Mueller

# Iso-singlet axial vector current and the chiral anomaly

$$\int_0^1 g_1(x,Q^2) = \frac{1}{18}\left(3F + D + 2\,\Sigma(Q^2)\right) \Rightarrow S^\mu\,\Delta\Sigma = \langle P,S\,|\,\bar\psi\,\gamma^\mu\gamma_5\,\psi\,|\,P,S\rangle \equiv \langle P,S\,|\,j_5^\mu\,|\,P,S\rangle$$

$U_A(1)$ violation from the chiral anomaly
-famous anomaly equation:

$$\partial_\mu J_5^\mu = 2n_f\partial_\mu K^\mu + \sum_{i=1}^{n_f} 2im_i\bar q_i\gamma_5 q_i$$



Divergence of C-S current ∝ topological charge density

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Problem: Identification of CS charge with ΔG is intrinsically ambiguous
… *the latter is gauge invariant, the former is not*

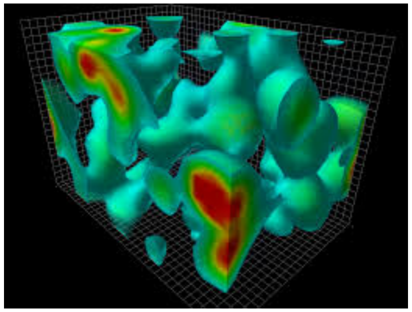
$$K_\mu \to K_\mu \;+\; i\frac{g}{8\pi^2}\epsilon_{\mu\nu\alpha\beta}\partial^\nu\left(U^\dagger\partial^\alpha U A^\beta\right)$$
$$+\;\frac{1}{24\pi^2}\epsilon_{\mu\nu\alpha\beta}\left[(U^\dagger\partial^\nu U)(U^\dagger\partial^\alpha U)(U^\dagger\partial^\beta U)\right]$$

"Large gauge transformation"
- deep consequence of topology

Jaffe, Manohar (1990)

R. Jaffe: identification of $K^\mu$ with ΔG
a source of much confusion
in the literature (Varenna lectures, 2007)

# Iso-singlet axial vector current and the chiral anomaly



However, identifying CS – charge with ΔG "works" remarkably well …
(Vogelsang, POETIC IX)

$$\frac{d}{d\ln Q^2}\begin{pmatrix}\Delta\Sigma \\ \Delta\Gamma\end{pmatrix} = \begin{pmatrix}\Delta P_{\Sigma\Sigma}(a_s) & 0 \\ -\frac{1}{2N_f}\Delta P_{\Sigma\Sigma}(a_s) & 0\end{pmatrix}\begin{pmatrix}\Delta\Sigma \\ \Delta\Gamma\end{pmatrix}$$

$\propto \Delta G$

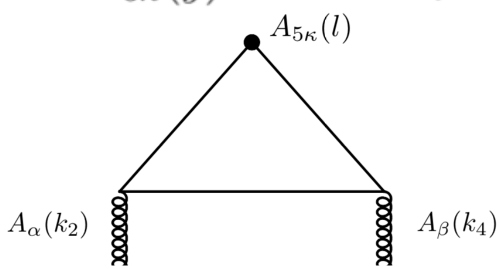
NNLO splitting function computations confirm this

At LO, Altarelli, Lampe (1990)
NNLO: Vogt, Moch, Rogal, Vermaseren, arXiv:0807.1238

This suggests that ΔΣ only mixes with itself; likewise ΔG as defined, only depends on ΔΣ with the same splitting function

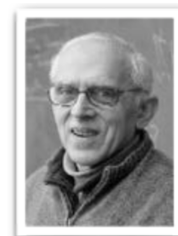
# The Adler-Bell-Jackiw chiral (triangle) anomaly

$$\langle P', S | J_5^\kappa | P, S \rangle = \int d^4y \frac{\partial}{\partial A_{5\kappa}(y)} \Gamma[A, A_5]\Big|_{A_5=0} e^{ily} \equiv \Gamma_5^\kappa[l]$$

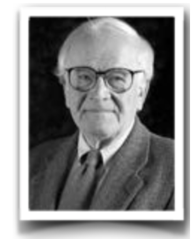



$$= \frac{1}{4\pi^2} \frac{l^\kappa}{l^2} \int \frac{d^4k_2}{(2\pi)^4} \int \frac{d^4k_4}{(2\pi)^4} \, \mathrm{Tr_c} F_{\alpha\beta}(k_2) \tilde{F}^{\alpha\beta}(k_4) \, (2\pi)^4 \delta^4(l + k_2 + k_4)$$

Famous infrared pole of anomaly. One loop exact: Adler-Bardeen theorem


Steven Adler

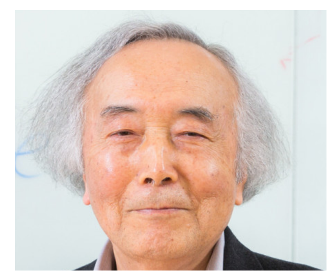

John S. Bell


Roman Jackiw


William A. Bardeen

Key insight from Fujikawa: Anomaly arises from the non-invariance of the path integral measure under chiral ($\gamma_5$) rotations

$$e^{iW} = \int \mathcal{D}A\mathcal{D}\bar{q}\mathcal{D}q \;\; \exp\left[ i \int dx (\mathcal{L}_{\rm QCD} + V_5^{\mu a} J_{\mu 5}^a + V^{\mu a} J_\mu^a + \theta Q + S_5^a \phi_5^a + S^a \phi^a) \right]$$


Kazuo Fujikawa

# The resolution of the $U_A(1)$ problem regularizes the triangle

UA(1) problem: why is there no isosinglet Goldstone boson
or why is the $\eta'$ so massive ?

tHooft (1976); Witten; Veneziano (1979)

R. L. Jaffe

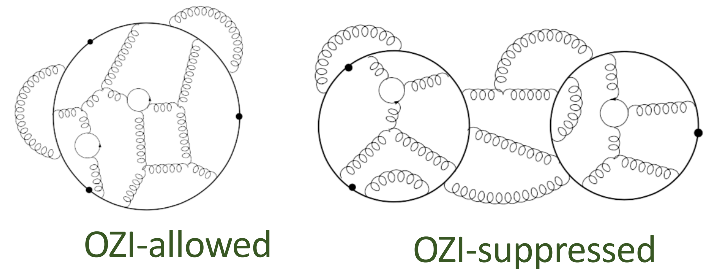
A. Manohar

> The authors of refs. [12, 13] suggest that the triangle diagram provides a *local* probe of the gluon distribution in the target. If this were true, $\Delta\Gamma$ would be protected from infrared problems and the calculation would be reliable in the usual sense. However, we believe there are strong arguments that the triangle is not local in the sense required. It is therefore not necessarily protected from infrared effects, in particular from the non-perturbative effects which give the $\eta'$ a mass★.
>
> Jaffe,Manohar (1990)

QCD factorization is deeply problematic for $g_1$

# Alternative picture: topological charge screening of spin

Shore, Veneziano, PLB (1990); NPB (1992)
Narison, Shore, Veneziano, hep-ph/9812333




$J_\mu^5$ is anomalous: employ chiral Ward identities +extended PCAC

Wess-Zumino action for connected QCD graphs coupled to external sources

Example: Witten-Veneziano formula $m_{\eta'}^2 = \frac{2\,n_f}{f_\pi^2}\chi_{\rm YM}(0) + O((\frac{n_f}{N_c})^2)$ → 0 when $N_c \to \infty$

where the topological susceptibility $\chi_{\rm YM}(l^2) = i\int dx\, e^{il\cdot x}\langle 0|T(\Omega(x)\,\Omega(0))|0\rangle$

with $\Omega(x) = \frac{\alpha_S}{8\pi}\,{\rm Tr}\left(F_{\mu\nu}\tilde{F}^{\mu\nu}\right)$ the topological charge density

In this picture, $\Sigma(Q^2) = \frac{1}{3m_N}\Delta C_1^S(\alpha_S)\left(g_{QNN}\chi(0) + g_{\eta' NN}\sqrt{\chi'(0)}\right)$

G. Veneziano

In chiral limit $\chi(0) \to 0$, $\Delta\Sigma$ "controlled" by the slope $\chi'$ at $p^2$=0
– estimated to be small by Veneziano et al. – explains small value of $\Delta\Sigma$

Our perspective: We agree with this picture and argue that it applies not just to the first moment $\Delta\Sigma$ but to $g_1$ in both the Bjorken and Regge asymptotics of QCD

# Low energy dynamics of $\eta'$ in QCD

For $N_f$=3, dynamical variables of effective theory are massless modes in limit $N_C \to \infty$ $and$ $m \to 0$

Symmetry group is G= $U_R(3) \times U_L(3)$

Spontaneous symmetry breaking: $U_R(3) \times U_L(3) \to U_V(3)$

The nine parameters of its coset space correspond to the nine pseudoscalar Goldstone bosons – including the prodigal $\eta' \to \eta_0$

Relative to the "standard" SU(3) framework, where $\det U(x) = e^{i\eta_0(x)}$ and $\eta_0$ transforms as

$\eta_0'$= $\eta_0$- i $\mathrm{Ln} \det V_R + i \, \mathrm{Ln} \det V_L$

For non-zero quark masses, expansion in # of derivatives, powers of m and 1/$N_c$

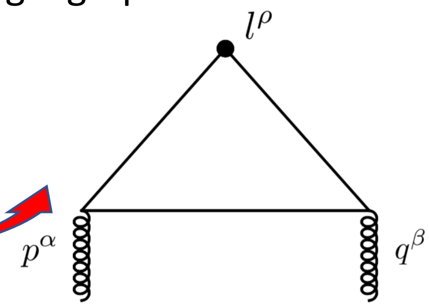
Wess-Zumino-Witten terms for the SU(3) and U(1) sectors correspond to the "un-natural parity"part of the effective Lagrangian

**Leutwyler, hep-ph/9601234**
**Herrera-Siklody et al, hep-ph/9610549**
**Kaiser, Leutwyler, hep-ph/0007101**

# The resolution of the $U_A(1)$ problem regularizes the triangle

We begin to glimpse the key role of the $U_A(1)$ "problem" from the structure of the triangle graph in the off-forward limit ($l^\mu \to 0$, $t = l^2 \to 0$) of the matrix element of $< P'$,S| $J^5_\mu$|P, S $>$

$$\langle P', S | J^\mu_5 | P, S \rangle = G_A(t) S_\mu + l \cdot S\, l_\mu G_P(t)$$




triangle has infrared pole proportional to $\frac{l^\mu}{l^2} F\tilde{F}$

This perturbative/nonperturbative interplay gives a highly non-trivial result:

$$\langle P', S | J^\mu_5 | P, S \rangle \to \frac{l \cdot S\, l^\mu}{l^2} \kappa(t) + \left( S^\mu - \frac{l \cdot S\, l^\mu}{l^2} \right) \lambda(t) \qquad \kappa(0) = \lambda(0) \propto F\tilde{F}$$

**Jaffe. Manohar (1990)**

The infrared pole in in the axial form factor $G_A(t)$ must be canceled by a pole in its pseudoscalar counterpart $G_P(t)$

– this primodial isosinglet $\eta'$ mixes with the topological charge density to generate the physical $\eta'$

**Topological mass generation in the Schwinger model: Dvali, Jackiw, Pi, arXiv:hep-th/0511175**

# Thinking properly about anomalies with worldlines

**Review: Schubert, Phys. Repts. (2001)**
**N. Mueller, RV: 1701.03331.1702.01233,1901.10492**
**Tarasov, RV: 1903.11624, 2008.08104 and in preparation**

The worldline formulation of QFT is equivalent to the string amplitude formalism of Bern and Kosower, as shown by Strassler - provides a powerful "first quantized"intuition

**Bern,Kosower, NPB 379 (1992) 145;**
**Bern, TASI lectures, hep-ph/9304249**
**Strassler, NPB 385 (1992) 145**

# World-line formalism: vector and axial vector fields

$$S[A, A_5] = \int d^4x\, \bar{\psi}\, (i\partial\!\!\!/ + A\!\!\!/ + \gamma_5 A\!\!\!/_5)\, \psi$$

One loop effective action $\Gamma[A, A_5] = \log\det(\chi)$ with $\chi = i\partial\!\!\!/ + A\!\!\!/ + \gamma_5 A\!\!\!/_5$

**Berezin, Marinov (1976);**
**Brink et al; Barducci et al;**
**Balachandran et al (1976-77);**
**Strassler (1992)**
**D'Hoker, Gagne (1996)**
**Review: Schubert (2001)**

"Schwinger trick" to obtain heat kernel representation of $\chi$:
*sandwich* between $0+1-\mathrm{D}$ boson (x, p)
and fermion (Grassmann) coherent states

Wigner-Weyl formalism: $\hat{a}_i^{\pm} \equiv \frac{1}{2}(\gamma_i \pm i\gamma_{i+2})$ i=0,1 $\hat{a}_i^{-}|\theta\rangle = \theta_i|\theta\rangle$ $\hat{a}_i^{+}|\theta^*\rangle = \theta_i^*|\theta^*\rangle$

Grassmann Majorana fermions: $\psi_i \equiv \frac{1}{\sqrt{2}}(\theta_i + \theta_i^*)$ $\psi_{i+2} \equiv \frac{i}{\sqrt{2}}(\theta_i - \theta_i^*)$

# World-line formalism: vector and axial vector fields

$$S[A, A_5] = \int d^4x\, \bar{\psi}\, (i\partial\!\!\!/ + A\!\!\!/ + \gamma_5 A\!\!\!/_5)\, \psi$$

One loop effective action $\Gamma[A, A_5] = \log\det(\chi)$ with $\chi = i\partial\!\!\!/ + A\!\!\!/ + \gamma_5 A\!\!\!/_5$

$$\Gamma[A, A_5] = \Gamma_R + i\Gamma_I$$

→ *Phase of the determinant..*
*An elegant way to represent the chiral anomaly which*
*Fundamentally arises from non-invariance of path*
*integral measure under a chiral rotation*

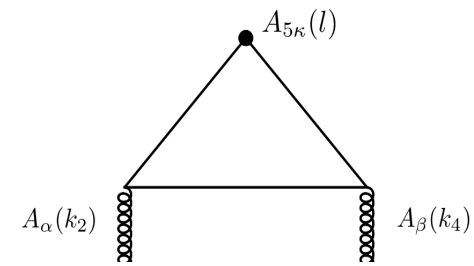
*Remarkably, this phase can be re-expressed in a form that is nearly identical to* $\Gamma_R$ !

Useful mnemonic: *Odd* powers of $\gamma_5$ $A_5$ contribute to $\Gamma_I$ and *Even* powers to $\Gamma_R$

**D'Hoker, Gagne, hep-ph/9512080**
**See also Mondragon, Nellen, Schmidt, Schubert,**
**hep-th/9502125**

# The triangle anomaly in the worldline formalism

The axial vector couplings project out the imaginary part of the effective action




Point particle Bose and Grassmann path integrals

$$\Gamma[A, A_5] = -\frac{1}{2}\mathrm{Tr}_c \int_0^\infty \frac{dT}{T} \int \mathcal{D}x \int_{AP} \mathcal{D}\psi$$

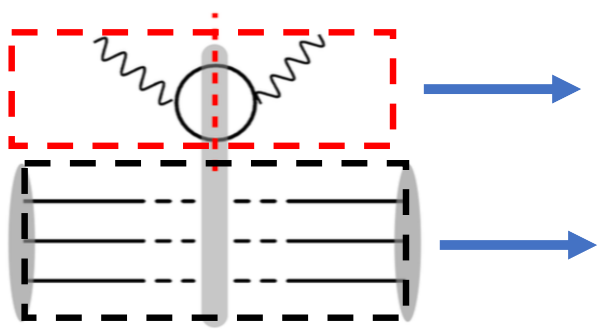
$$\times \exp\Big\{ -\int_0^T d\tau \Big( \frac{1}{4}\dot{x}^2 + \frac{1}{2}\psi_\mu \dot{\psi}^\mu + ig\dot{x}^\mu A_\mu - ig\psi^\mu\psi^\nu F_{\mu\nu} - 2i\psi_5 \dot{x}^\mu \psi_\mu \psi_\nu A_5^\nu + i\psi_5 \partial_\mu A_5^\mu + (D-2)A_5^2 \Big) \Big\}$$

Wilson line ($ig\dot{x}^\mu A_\mu$) — Spin precession ($ig\psi^\mu\psi^\nu F_{\mu\nu}$) — "re-exponentiated" axial vector couplings

$$\langle P', S | J_5^\kappa | P, S \rangle = \int d^4y \frac{\partial}{\partial A_{5\kappa}(y)} \Gamma[A, A_5]\Big|_{A_5=0} e^{ily} \equiv \Gamma_5^\kappa[l]$$

$$= \frac{1}{4\pi^2} \frac{l^\kappa}{l^2} \int \frac{d^4k_2}{(2\pi)^4} \int \frac{d^4k_4}{(2\pi)^4} \; \mathrm{Tr}_c F_{\alpha\beta}(k_2) \tilde{F}^{\alpha\beta}(k_4) \; (2\pi)^4 \delta^4(l + k_2 + k_4)$$

# g1 in polarized DIS and the UA(1) problem: how the prodigal Goldstone provides a key assist

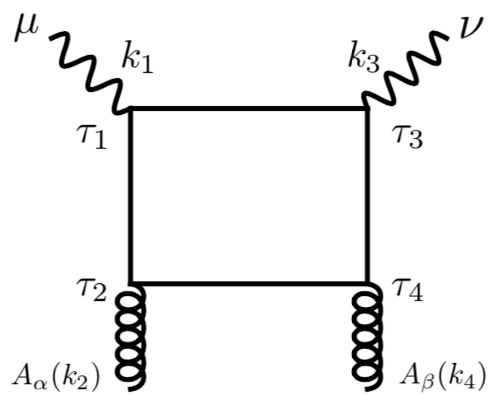


Polarization tensor from world-line Hamiltonian

Gauge (gluon) background

# The box diagram for polarized DIS ($g_1(x,Q^2)$)

**DIS with worldlines,**
**Tarasov, RV, 1903.11624**
**2008.08104**



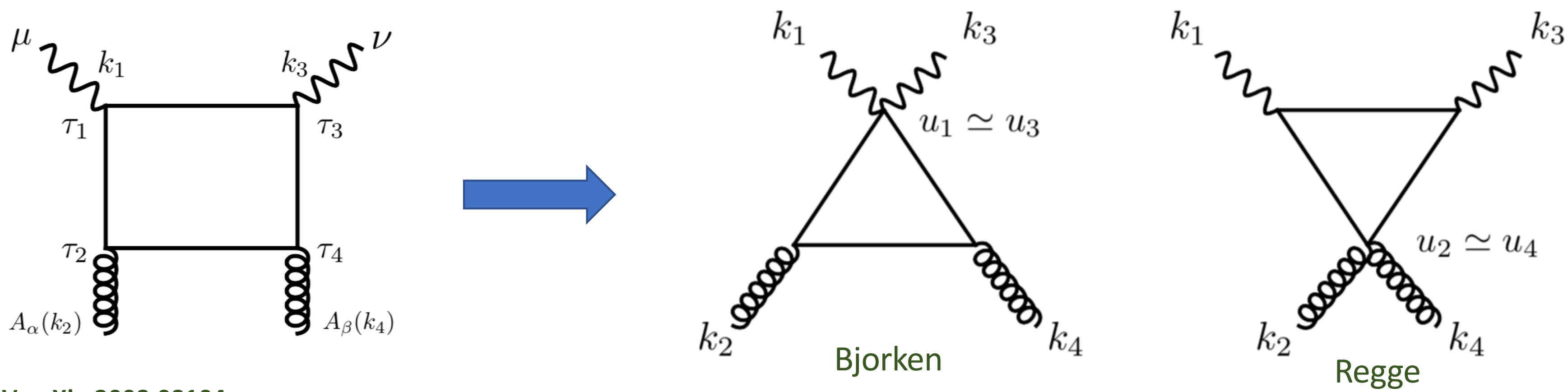

$$\Gamma_A^{\mu\nu}[k_1,k_3] = \int \frac{d^4k_2}{(2\pi)^4} \int \frac{d^4k_4}{(2\pi)^4}\, \Gamma_A^{\mu\nu\alpha\beta}[k_1,k_3,k_2,k_4]\ \mathrm{Tr_c}(\tilde{A}_\alpha(k_2)\tilde{A}_\beta(k_4))$$

Polarization tensor (antisymmetric piece)

Box diagram

We can compute the box explicitly in both the Bjorken limit of QCD ($Q^2 \to \infty, s \to \infty, x = \text{fixed}$) and the Regge limit ($x \to 0, s \to \infty, Q^2 = \text{fixed}$). The latter result is new

# Finding triangles in boxes in Bjorken and Regge asymptotics




Tarasov, RV, arXiv:2008.08104

Remarkably, box diagram for $g_1(x_B,Q^2)$ has same structure in both limits, dominated by the triangle anomaly !

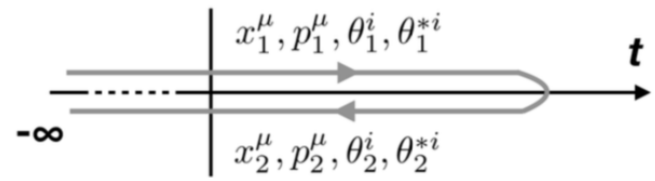
$$S^\mu g_1(x_B,Q^2)\Big|_{Q^2\to\infty} = \sum_f e_f^2 \frac{\alpha_s}{i\pi M_N}\int_{x_B}^1 \frac{dx}{x}\left(1-\frac{x_B}{x}\right)\int \frac{d\xi}{2\pi}e^{-i\xi x}\lim_{l_\mu\to 0}\frac{l^\mu}{l^2}\langle P',S|\mathrm{Tr}_c F_{\alpha\beta}(\xi n)\tilde{F}^{\alpha\beta}(0)|P,S\rangle + \text{non-pole } \frac{\Lambda^2_{QCD}}{Q^2} \ll 1$$

$$S^\mu g_1(x_B,Q^2)|_{x_B\to 0} = \sum_f e_f^2 \frac{\alpha_s}{i\pi M_N}\int_{x_B}^1 \frac{dx}{x}\int \frac{d\xi}{2\pi}e^{-i\xi x}\lim_{l_\mu\to 0}\frac{l^\mu}{l^2}\langle P',S|\mathrm{Tr}_c F_{\alpha\beta}(\xi n)\tilde{F}^{\alpha\beta}(0)|P,S\rangle + \text{non-pole } \frac{x_B}{x} \ll 1$$

Hence $g_1$ is topological in both asymptotic limits of QCD…its relation to $\Delta g(x, Q^2)$ is unclear

# Computing the worldline matrix element

$$\langle \mathcal{O} \rangle = \mathrm{Tr}\{\mathcal{O}\mathcal{R}\} \;\Longrightarrow\; \langle \mathcal{O} \rangle = \int \mathcal{D}A\; \mathcal{O}[A]\; \mathcal{R}[A]\; e^{iS_{YM}[A]}$$




$$\mathcal{R}_q[A] = \mathrm{tr}_c \int d^4 z_{\max} \int d^4 z_{\min}\; e^{iP^+(z^-_{\max} - z^-_{\min})}\; e^{-iP_\perp(z_{\perp\max} - z_{\perp\min})}$$

$$\times \int_0^\infty dT \int_{z_{\min}}^{z_{\max}} \mathcal{D}z \int \mathcal{D}\psi\; \mathcal{R}^c_{\mathrm{q\ init.}} \otimes \mathcal{R}^f_{\mathrm{q\ init.}}\; \exp\left[ -\int_0^T d\tau \left( \frac{1}{4}\dot{z}^2 + \frac{1}{2}\psi\dot{\psi} + igA\dot{z} - ig\psi F\psi \right) \right]$$

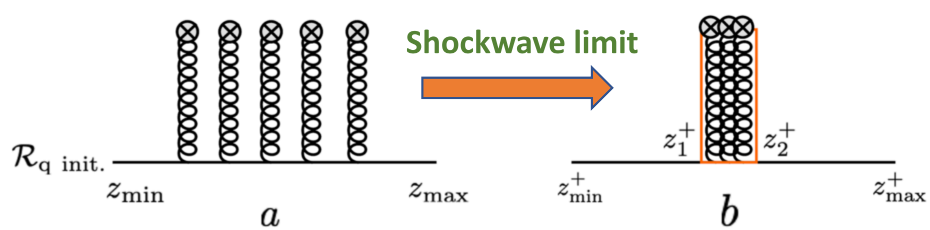
**Berezin,Marinov (1976)**
**Tarasov,RV**

$$\frac{1}{4}\left(1 + \lambda\psi_5\right)\left(1 + 2\psi^-\psi^+\right)$$

Wilson line

Spin precession

**See also**
**Bakker,Leader,Trueman,**
**Hep-ph/0406139**

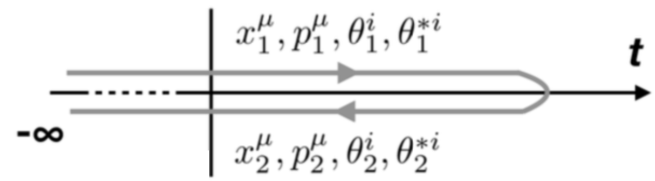



Spin precession and smearing of width of shock wave become important to obtain helicity flip contributions

*Most importantly, the density matrix has an anomalous contribution that couples to the topological charge*

# Computing the worldline matrix element

$$\langle \mathcal{O}\rangle = \mathrm{Tr}\{\mathcal{O}\mathcal{R}\} \quad \Longrightarrow \quad \langle \mathcal{O}\rangle = \int \mathcal{D}A\; \mathcal{O}[A]\; \mathcal{R}[A]\; e^{iS_{YM}[A]}$$

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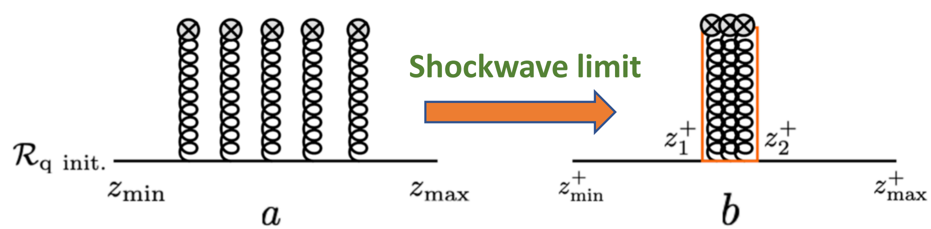



A subtle technical point: In perturbative computations, only the leading sub-eikonal spin precession contribution is retained. This is wrong…

… because the anomaly contribution, naively sub-sub-eikonal, trumps spin precession
- since for any finite P⁺, the $l^2 \to 0$ pole from the anomaly dominates. The eikonal expansion breaks down!

# The role of pseudoscalar fields in the worldline formalism

We **did not** previously write down the most general form of the **imaginary part** of the worldline effective action :

D'Hoker, Gagne,hep-th/9508131

$$W_{\Im}[\Phi,\Pi,A] = \frac{1}{8}\int_{-1}^{1} d\alpha \int_0^\infty dT\,\mathcal{N}\int \mathcal{D}x\mathcal{D}\psi\,\mathrm{Tr_c}\,\mathcal{J}(0)\,\mathcal{P}e^{-\int_0^T d\tau \mathcal{L}_\alpha}$$

with $$\mathcal{L}_\alpha = \frac{\dot{x}^2}{2\mathcal{E}} + \frac{1}{2}\psi_A\dot{\psi}_A - i\dot{x}\cdot A + \frac{i}{2}\mathcal{E}\psi_\mu F_{\mu\nu}\psi_\nu + \frac{1}{2}\mathcal{E}\alpha^2\Phi^2 + \frac{1}{2}\mathcal{E}\Pi^2$$

$$+\boxed{i\,\varepsilon\,\psi_\mu\psi_5 D_\mu\Pi} + \alpha\,\varepsilon\,\psi_5\psi_6[\Pi,\Phi]$$

and $\mathcal{J}(0) \propto \psi_5\psi_6\{\Pi,\Phi\}$

*where $\Phi$ is the chiral condensate*

Expanding out the worldline Lagrangian, the first nontrivial contribution to $W_I$ is the Wess-Zumino-Witten term !

This U(3) WZW term can be re-written as $\propto \eta_0 F\tilde{F}$ Leutwyler-Kaiser (2000)

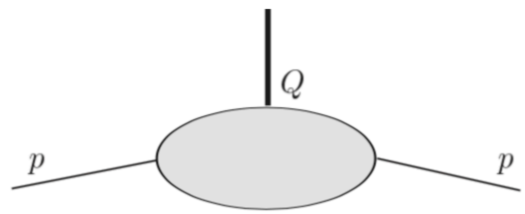
We can show explicitly how one recovers the functional Ward identities for axial-vector, pseudo-scalar and scalar fie in this worldline framework Tarasov, RV (in preparation)

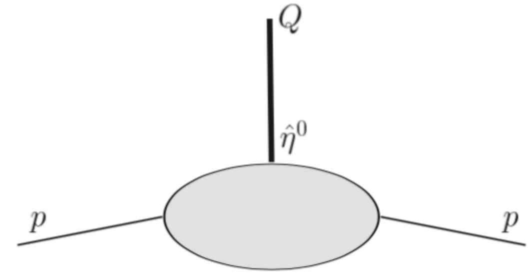
# Topological charge screening of spin

Topological charge $\Omega = \frac{\alpha_S}{8\pi}\mathrm{Tr}\left(F\tilde{F}\right)$

Topological susceptibility $\chi(t) = i\int d^4x\, e^{il\cdot x}\,\langle 0|T\left(\Omega(x)\Omega(0)\right)|0\rangle$

**Shore, Veneziano, PLB (1990); NPB (1992)**
**Narison, Shore, Veneziano, hep-ph/9812333**




$$\Sigma(Q^2) = \frac{1}{3m_N}\Delta C_1^S(\alpha_S)\left(g_{QNN}\chi(0) + g_{\eta' NN}\sqrt{\chi'(0)}\right)$$

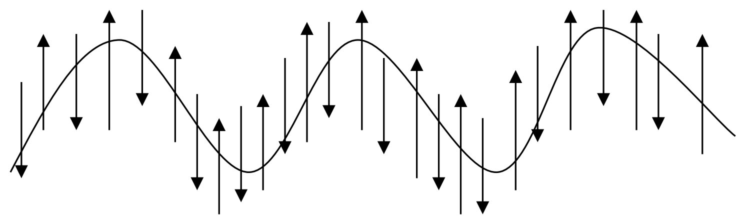
$\approx 0$

Narison, Shore and Veneziano (see hep-ph/0701171) argue that the result is dominated by the derivative of the topological susceptibility ($\chi'$) in the QCD vacuum

- which they compute (using QCD sum rules) to be in agreement with the HERMES and COMPASS data

Lattice computations: $\chi$QCD collaboration, arXiv: 1806.08366v2

# Conjecture: "Axion-like" effective action for Regge limit

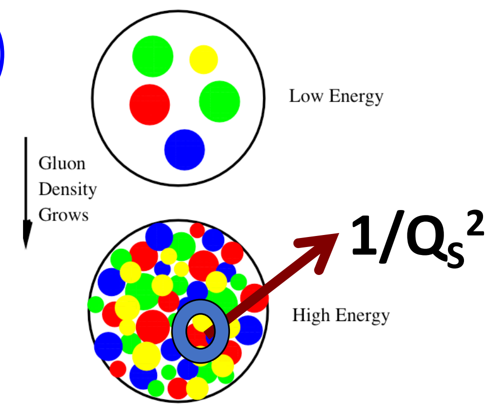


At small x (large "Ioffe" times), the background field couples to a large # world-line trajectories: from their worldline density matrix, construct the *effective action for an ensemble of spinning, colored partons*

Effective action has a piece coupling large x color degrees of freedom to small x gauge fields:

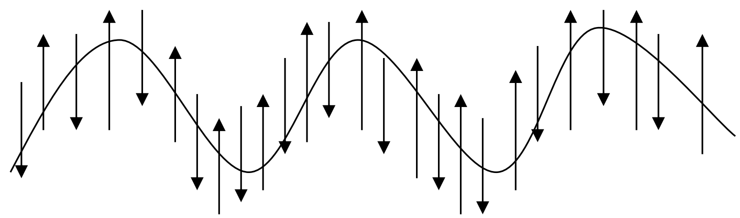
$$g_1(x_B, Q^2) \propto \int [D\rho]\, W_Y^P[\rho] \times \int [dA]\, \Omega(X)\, \exp\left(iS_{\rm YM}[A] + \frac{i}{N_c}{\rm Tr}_c\left(\rho\, U_{-\infty,\infty}\right)\right)$$

This is the Color Glass Condensate effective action whose dynamics is controlled by a dimensionful saturation scale $Q_S(x)$




Gelis,Iancu,Jalilian-Marian,Venugopalan, arXiv:1002.0333

# Conjecture: "Axion-like" effective action for Regge limit



Full effective action including coupling to spin probed by the triangle

$$g_1(x_B, Q^2) \propto \int [D\rho]\, W_Y^P[\rho] \int [D\eta_0]\, \tilde{W}_Y^{P,S}[\eta_0]$$

Coupling of color and spin to proton

$$\times \int [dA]\, \Omega(X)\, \exp\left(iS_{\mathrm{YM}}[A] + \frac{i}{N_c}\mathrm{Tr}_c\left(\rho\, U_{-\infty,\infty}\right)\right)$$

YM+eikonal color coupling

$$\times \exp\left(\int d^4X \left(-\frac{\Omega^2}{2\chi_{\mathrm{YM}}} - \sqrt{\frac{N_c}{2}}\Omega\,\eta_0 + \frac{1}{2}F^2\eta_0\partial^2\eta_0\right)\right)$$

Axion-like effective action

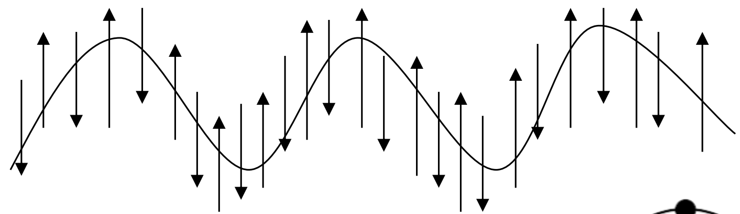
Can equivalently be written as: $\exp\left(-\int d^4X\left(F^2\eta_0\partial^2\eta_0 + (\eta_0+\theta)\Omega + \chi_{\mathrm{YM}}(\eta_0+\theta)^2\right)\right)$

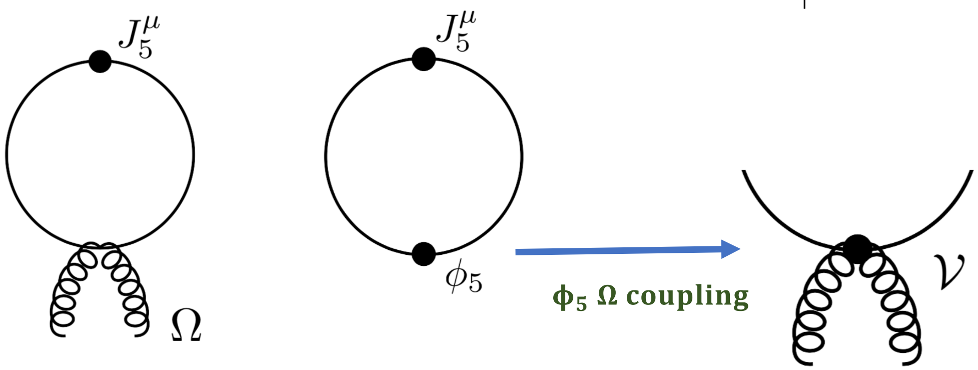
**Veneziano, Mod. Phys. Lett. (1989)**
**Hatsuda, PLB (1990)**

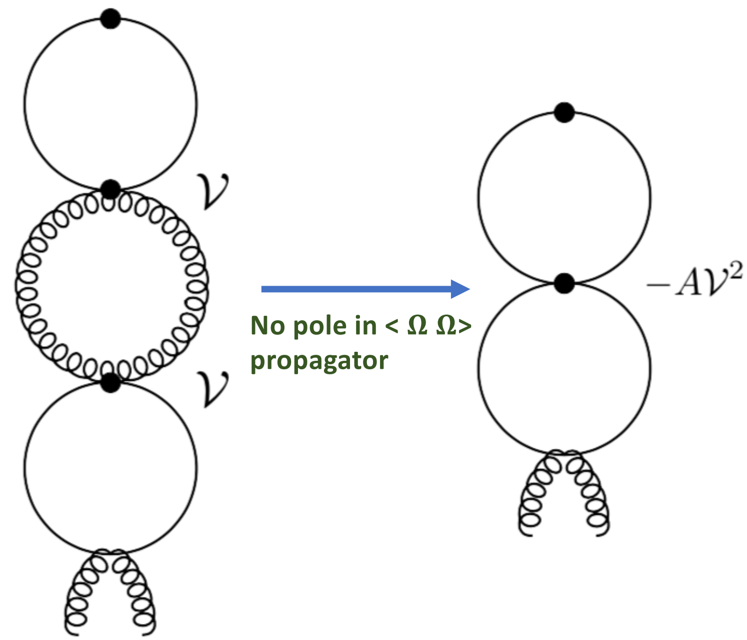
With this form of the effective action, we can show explicitly how the pole of the anomaly is canceled by $\eta_0$ exchange

**Tarasov, RV, in preparation**

# Conjecture: "Axion-like" effective action for Regge limit






$$\int d^4x e^{ilx}\langle P|J_5^\mu(x)|P\rangle = -i\frac{l^\mu}{l^2}2N_f\left[1+\frac{m_{\eta'}^2}{l^2-m_{\eta'}^2}\right]\langle P|\Omega(l)|P\rangle$$

Resummed expression is finite when $l^2 \to 0$

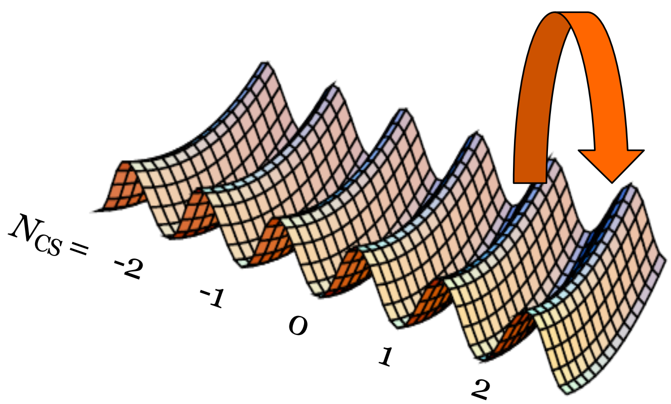
Ladder resummation generates a massive $\eta'$

# Spin diffusion via sphaleron transitions in topologically disordered media

Two scales – the height of the barrier given by $m_{\eta'}^2 = 2n_f \frac{\chi_{\rm YM}}{F^2}$

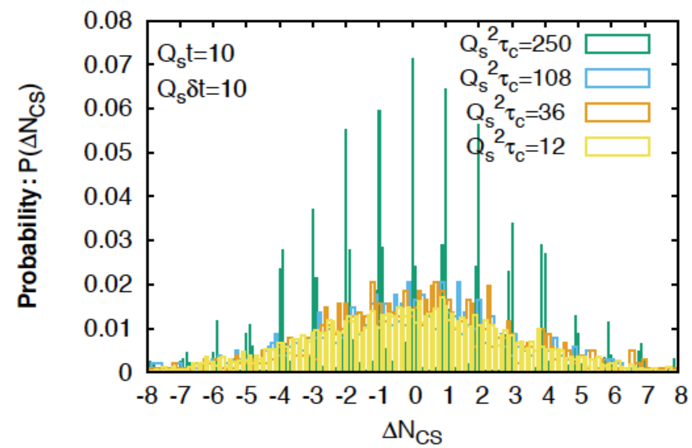
- the gluon saturation scale $Q_S$

When $Q_S^2 >> m_{\eta'}^2$ over the barrier gauge configurations dominate over instanton configurations




Over the barrier (sphaleron) transitions between different topological sectors of QCD vacuum…
characterized by integer valued Chern-Simons #

Topological transitions in overoccupied gauge fields




Mace, Schlichting, RV: PRD (2016) 1601.07342

Axion-like dynamics in a hot QCD plasma - McLerran,Mottola,Shaposhnikov (1990)

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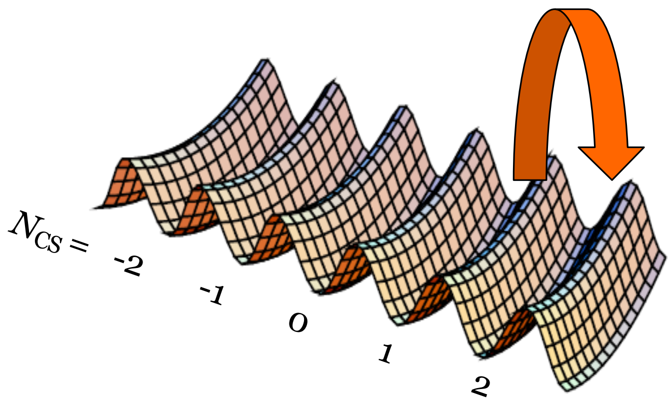
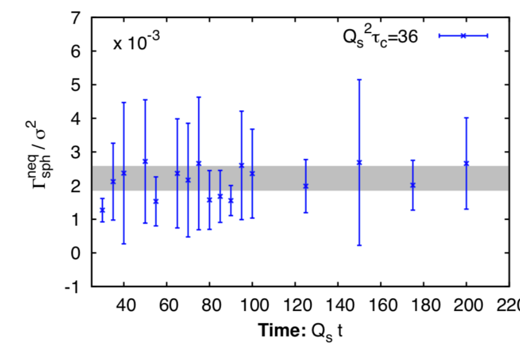
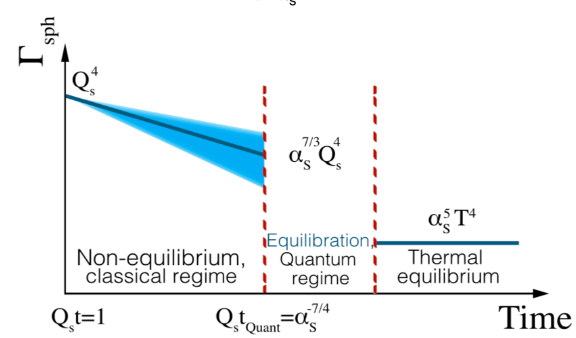



**Sphaleron transition rate off-equilibrium**

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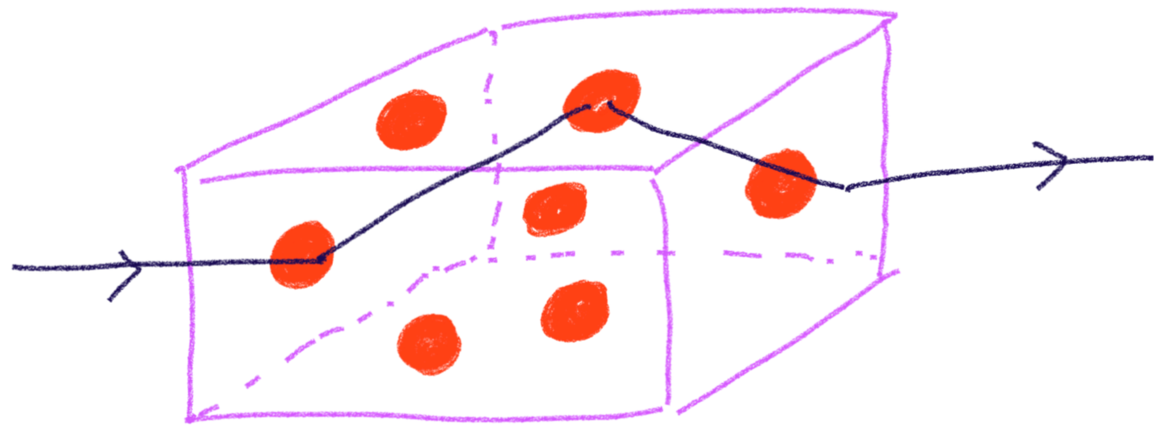
# Spin diffusion via sphaleron transitions in topologically disordered media



- Dense gluon blob of size $1/\sigma$ given by $\Gamma^{Y}_{\mathrm{sphaleron}} = \# \sigma^2$ & carrying topological charge

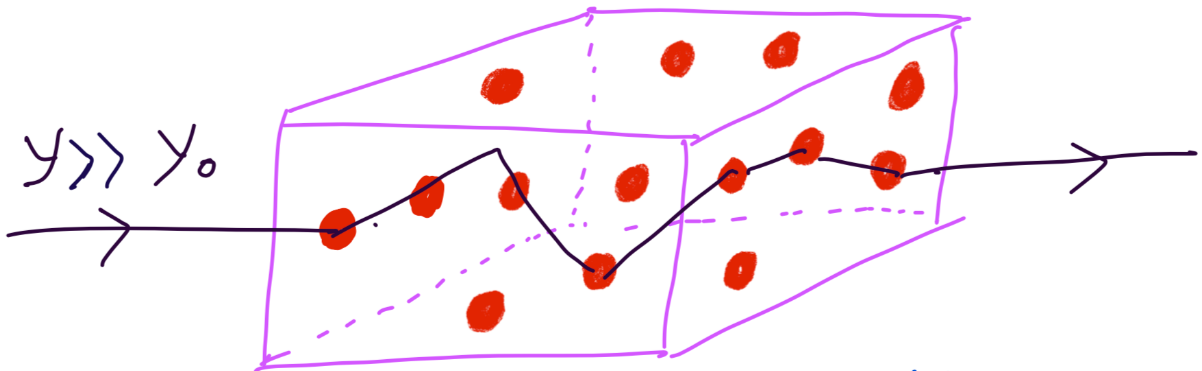
---

Atiyah-Singer index theorem

Helicity flip for massless quarks given by $n_L - n_R = n_f \nu$,

where $\nu$ is the topological charge and $\Gamma^{Y}_{\mathrm{sphaleron}} \propto \langle \nu^2 \rangle$

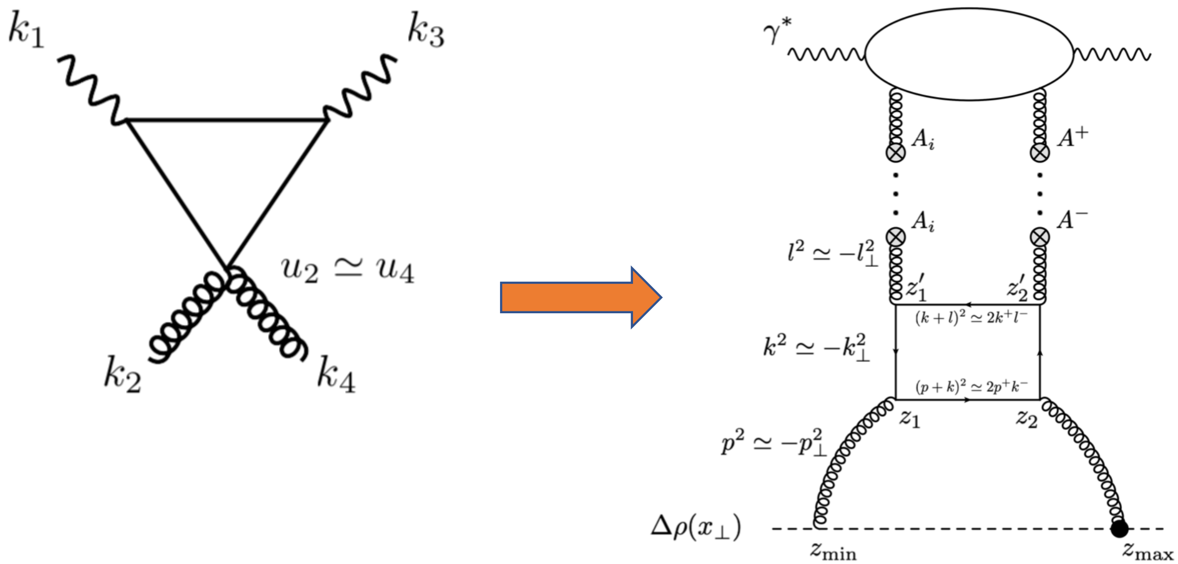
# Spin diffusion via sphaleron transitions in topologically disordered media




As x decreases ($y >> y_0$), the blobs become smaller ($Q_s(y) > Q_s(y_0)$) and denser with more topological charge

Expect very rapid quenching of $g_1$ at small $x_B$: interplay between QCD evolution of the topological charge and the saturation scale

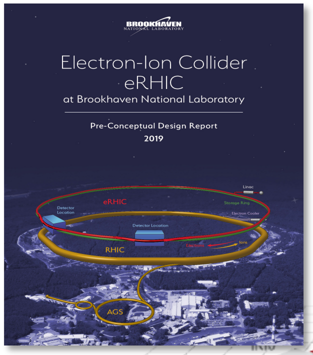
# QCD evolution to small x



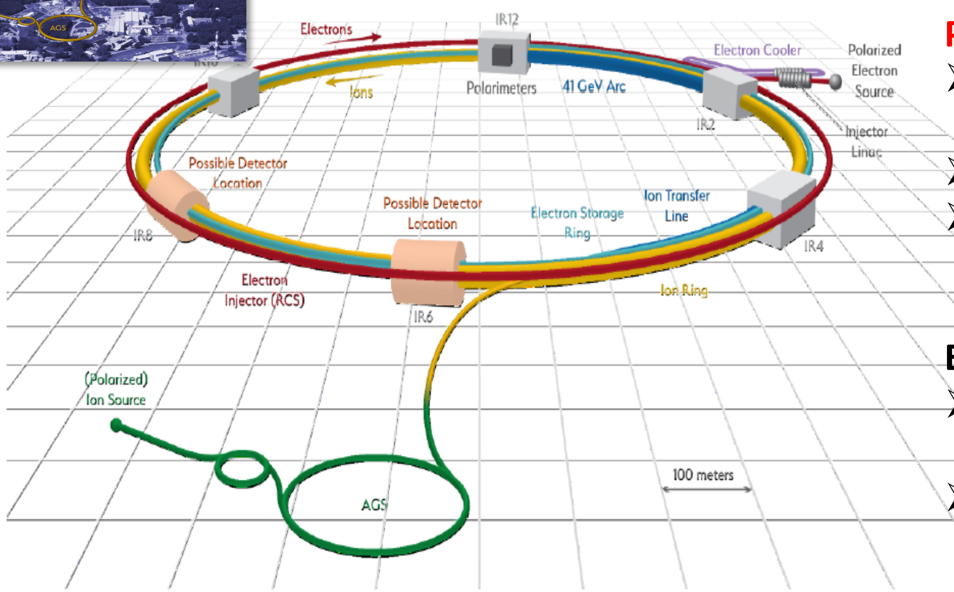

**Kirschner Lipatov (1980's);**
**Bartels, Ermolaev, Ryskin (1990's);**
**Kovchegov, Pitonyak, Sievert (2015-),**
**Kovchegov,Cougoulic (2019);**
**Boussarie, Hatta, Yuan (2019)**

Renormalizing the axion effective action at small x, one should recover the triangle structure in the "box diagram"

Fascinating interplay between gluon saturation - controlled by the saturation scale $\mathbf{Q_S}$ and spin diffusion controlled by the YM topological susceptibility $\chi_{YM}$




# A powerful new femtoscope: The Electron-Ion Collider




❖ Electron storage ring with frequent injection of fresh polarized electron bunches
❖ Hadron storage ring with strong cooling or frequent injection of hadron bunches

**Polarized protons up to 275 GeV; Nuclei up to ~ Z/A*275 GeV/n**

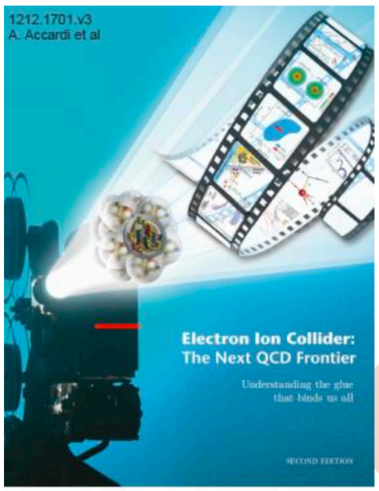
- Existing RHIC complex: Storage (Yellow), injectors (source, booster, AGS)
- Need few modifications
- RHIC beam parameters fairly close to those required for EIC@BNL

**Electrons up to 18 GeV**

- Storage ring, provides the range sqrt(s) = 20-140 GeV. Beam current limited by RF power of 10 MW
- Electron beam with variable spin pattern (s) accelerated in on-energy, spin transparent injector (Rapid-Cycling-Synchrotron) with 1-2 Hz cycle frequency
- Polarized e-source and a 400 MeV s-band injector LINAC in the existing tunnel

**Design optimized to reach $10^{34}$ $cm^{-2}sec^{-1}$**

# Outlook: these ideas can be tested at the EIC !




White paper
arXiv:1212.1701

**Conformal Anomaly**

**Chiral Anomaly**

Origin of nucleon mass

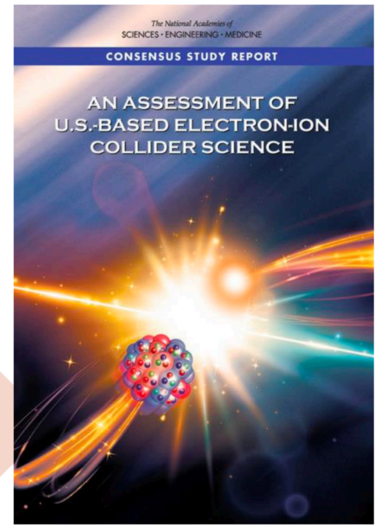
Origin of nucleon spin

Gluon saturation

Nucleon tomography

**Color Glass Condensate**

**Confined many-body parton dynamics**




NAS report
July 2018

Precision probes of the strong interplay between perturbative many-body parton dynamics and non-perturbative structure ("the ether") of the QCD vacuum

Thank you for your attention !