Jets in Deep Inelastic Scattering

Krzysztof Kutak
Resources

• Previous lectures by: K. Cichy, K. Golec-Biernat, A. Kusina

• „Basics of QCD”, „Jet Physics” Gavin Salam

• Talks by F. Ringer, M. Arratia, B. Page, A. Quintero. DIS 2021, BNL workshop on Jets 2020, QCD Evolution 2021, weekly YR working group meetings

• „Lectures on Sudakov form factors”, B. Xiao, QCD master class

• „Quarks and leptons: An Introductory Course in Modern Particle Physics”, Halzen, Martin

• „Jets at hadron colliders”, Prog.Part.Nucl.Phys. 89 (2016) S. Sapeta
Plan of the lecture

- Facts about QCD.
- Formation of jets.
- Jet definition.
- Jet algorithm.
- Deep Inelastic Scattering.
- Collinear factorization.
- DESY and selection of recent HERA results.
- Non-collinear factorizations
- EIC and jet observables.
- Low x – dipole and WW gluon density.
- Summary.

Comment:
I am not going to be very careful about historical aspects of DIS and chronology.
Facts about QCD – QCD Lagrangian

The QCD Lagrangian reads

\[ \mathcal{L}_{\text{QCD}} = \bar{\psi}_i (i \gamma^\mu \partial_\mu - m) \psi_i - \frac{1}{4} F_{a \mu \nu}^\rho F_{\mu \nu}^a - g_s \bar{\psi}_i \lambda_i \psi_j \gamma^\mu A^a_{\mu} \]

Describes propagation of quarks

Describes propagation and self interactions of gluons

Describes interaction of quarks and gluons

Quarks have electric charge so they couple to photon. This is important for electron hadron interactions.
**Facts about QCD – QCD Lagrangian**

The QCD lagrangian reads

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- Describes propagation of quarks
- Describes propagation and self interactions of gluons
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Quarks have electric charge so they couple to photon. This is important for electron hadron interactions.
Facts about QCD – asymptotic freedom and confinement

At short distances: partons are weekly coupled: asymptotic freedom

At long distances: confinement i.e. energy is minimized when new hadrons are produced

\[ V_{QCD} = -\frac{4\alpha_s}{3r} + kr \]
Facts about QCD – coupling constant

\[ r \sim \frac{1}{Q} \]

momentum scale

https://particlesandfriends.wordpress.com/2016/10/13/couplings-and-asymptotic-freedom/
**Facts about QCD – QCD coupling constant**

- Perturbative partons at large scales
- Confining dynamics at small scales

\[
Q^2 = \frac{(p_2 + p_4)^2}{2}
\]

\[
\alpha_s(Q^2) = \frac{12\pi}{(33 - 2n_f) \log \frac{Q^2}{\Lambda_{QCD}^2}}
\]

S. Bethke, J Phys G 26, R27
Facts about QCD – QCD coupling constant

\[ Q^2 = \frac{(p_2^2 + p_4^2)}{2} \]

- Perturbative partons at large scales
- Confining dynamics at small scales
- This gives rise to sprays of hadrons at final states jets produced in high energy collision

\[ \alpha_s(Q^2) = \frac{12\pi}{(33 - 2n_f) \log \frac{Q^2}{\Lambda_{QCD}^2}} \]
Formation of jets – e+e- annihilation

Initial state as well as final state is color neutral.

- We have proliferation of partons that go to the final state.
- The partons tend to be collimated.
- The collection of color carrying partons turns into color neutral hadrons seen by the detectors.
- Short distance physics is connected to long distance physics.
- Jets are proxy to what is happening in the hard collision. You try to estimate what has happened at the short distance scale.
- You cluster together perturbative partons to form hadrons. Jets are fundamentally ambiguous. There is a tension: partons carry color, hadrons are color neutral.
Jet definition – jets in $e^+ e^-$

Theory definition by Sterman and Weinberg. We define event as jet event if we can find two cones of opening angle $\delta$ that contain all of the energy of the event, excluding at most a fraction of the total energy.

Pair of Sterman-Weinberg jets

Jets in experiments are defined as a collimated distribution of hadrons with total energy $E$ within the jet cone size

$$R = \sqrt{\Delta \eta^2 + \Delta \phi^2}$$


$E_1 + E_2 + E_3 < \epsilon E$
Formation of jets – why do jets form?

It is not obvious from Lagrangian that jets are dominant final state products of the collision.

\[ M_{q\bar{q}} = -\bar{u}(p_1) i e_q \gamma^\nu \nu(p_2) \]

\[ M_{q\bar{q}g} = \bar{u}(p_1) i g_s \hat{t}^A \frac{i}{p_1 + k} i e_q \gamma^\mu \nu(p_2) - \bar{u}(p_1) i e_q \gamma^\mu \frac{i g_s \hat{t}^A \nu(p_2)}{p_2 + k} \]

\[ |M_{q\bar{q}g}^2| \simeq |M_{q\bar{q}}^2| C_F g_s^2 \frac{2 p_1 \cdot p_2}{(p_1 \cdot k)(p_2 \cdot k)} \]

It diverges for \( E \to 0 \) – infrared (soft) divergence

It diverges for \( \theta \to 0 \) and \( \theta \to \pi \) – collinear divergence - relevant for jets
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\[-\bar{u}(p_1)i e_q \gamma_\mu \frac{ig_s \ell^A \nu(p_2)}{p_2^2 + k^2}\]

\[ |M_{q\bar{q}g}^2| \simeq |M_{q\bar{q}}^2|C_F g_s^2 \frac{2p_1 \cdot p_2}{(p_1 \cdot k)(p_2 \cdot k)} \]

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\[ \frac{1}{E^2(1 - \cos^2 \theta)} \]

invariant mass approaches 0
Jet algorithm

Which particles do you put together into a same jet?

How do you recombine their momenta
(4-momentum sum is the obvious choice, right?)

You cluster together perturbative partons to form hadrons

One needs to use jet definition technically expressed as jet algorithm

\[
\text{partons} \quad \{p_i\} \quad \rightarrow \quad \{k_i\} \quad \text{jets}
\]

Clusters of partons are the same as clusters of hadrons up to corrections.

Used jet algorithms: Cone, SISCones, kT, anti-kT, and new developments for jets in DIS

Good algorithm should give unambiguous answer.

Adding soft or collinear emission should not modify the clustering of partons into jet.

Jet algorithm should be infrared safe. Let us illustrate the problem...
Jet algorithm

From
Gavin Salam
Jet algorithm

LO partons  NLO partons  parton shower  hadron level

jet 1  jet 2  jet 1  jet 2  jet 1  jet 2  jet 1  jet 2

2 clear jets

From
Gavin Salam
Jet algorithm

LO partons  NLO partons  parton shower  hadron level

jet 1    jet 2    jet 1    jet 2    jet 1    jet 2    jet 1    jet 2

2 clear jets  3 jets?

From
Gavin Salam
Jet algorithm

A good algorithm should give an unambiguous answer. It should be collinear safe.

From Gavin Salam
Deep inelastic scattering

$p_e = E_e (1,0,0,1)$

$E_\gamma \sim \sqrt{Q^2} \sim k$

$p_p = E_p (1,0,0,-1)$

$E_{e^-} \sim 27 \text{ GeV}$

$E_P \sim 820 \text{ GeV}$

$\sqrt{s} \sim 300 \text{ GeV}$

From the uncertainty principle we get $\Delta x \Delta k \sim 1$

The final state depends on the virtuality of photon $Q^2 = -q^2$

$Q^2 \ll M_p^2$ proton behaves like an elementary pointlike particle

$Q^2 \sim M_p^2$ proton stays intact but the observables depend on its structure

$Q^2 \gg M_p^2$ proton gets destructed and new particles are created - DIS

Parameters from DESY
Collinear factorization

\[ p = zP \]

Momentum of proton

\[ P = E_p(1, 0, 0, -1) \]

Momentum of parton

\[ p_j = z_j P \]

\[ q^2 = (k - k')^2 \]

\[ \sigma = \sum_j \mathcal{O} \otimes f_j(x/x_p, Q^2) \]

cross section

matrix element defining observable

\[ f_j/P \]

probability of finding parton \( j \) in the proton
**DESY**

**Circumference:** 6.3 km

**Proton Beam:**
- Injection Energy: 40 GeV
- Lumi-Energy: (820) 920 GeV

**Electron Beam:**
- Injection Energy: 12 GeV
- Lumi Energy: 27.5 GeV

**Magnetic field p-ring:** 5.1 Tesla

at I=5500 A for 920 GeV

From: desy.de
Examples for jet production in DIS

Semi-inclusive DIS

Boson-gluon fusion

QCD Compton

QCD Compton

From Miguel Arratia, QCD Evolution 2021
Production of jet in DIS
Example from HERA: NLO inclusive jet production in DIS

Measurements motivated to test and further improve QCD calculations.

Notice: three different jet algorithms were used
Other results from HERA: NLO inclusive jet production in DIS


More on isolated photons plus jets in PHP (arXiv:1405.7127, JHEP 2014 (23)).


Non-collinear factorizations

Transverse Momentum Dependent factorization, Color Glass Condensate,
Improved Transverse Momentum Dependent factorization, Soft Collinear Effective Theory

\[ p = zP \]

\[ q^2 = (k - k')^2 \]

Momentum of proton \( P = E_p(1, 0, 0, -1) \)

Momentum of parton \( p = zP + k_T \)

\[ \sigma_O = \sum_j \mathcal{O}(k_T) \otimes \mathcal{S}(Q^2) \otimes \mathcal{F}_j(x/x_p, Q^2, k_T) \]

Possible additional factors – depend on exact type of factorization

Possible that this TMD obeys nonlinear evolution equation

Probability of finding parton \( j \) in the proton.

Cross section

Matrix element defining observable

Imbalance
Why jets are useful?

For jets we have

$$\sigma_{\mathcal{O}} = \sum_{j} \mathcal{O}(k_T) \otimes S(Q^2) \otimes \mathcal{F}_j(x/x_p, Q^2, k_T)$$

For individual particles we have

$$\sigma_{\mathcal{O}} = \sum_{j} \mathcal{O}(k_T) \otimes S(Q^2) \otimes D \otimes \mathcal{F}_j(x/x_p, Q^2, k_T)$$

- jets avoid nonperturbative TMD fragmentation functions
- modern jet substructure techniques offer new methods for precise QCD calculations and to control nonperturbative effects
- one can minimize hadronization effects or study TMD evolution
Electron Ion Collider (EIC)

Will use the existing RHIC complex at Brookhaven National Laboratory

- Variable center of mass energies: 20-140 GeV
- Ion beams: from d to Au, Pb, U
- High luminosity: \(10^{33} - 10^{34}\) cm\(^{-2}\) s\(^{-1}\)
- Highly polarized (70%) electron and nucleon beams

The future Electron-Ion Collider (EIC) [1]
will produce the first jets in polarized and nuclear DIS,
which will enable a rich jet program

Complementary to semi-inclusive DIS observables, jets
avoid nonperturbative TMD fragmentation functions.
Electron Ion Collider – science program with jets and results relevant for EIC jet studies

Recent publications
• The spin of the proton, PDFs

Hinderer, Schlegel, Vogelsang `15, `17, Abelof, Boughezal, Liu, Petriello `16, Boughezal, Petriello, Xing `18, Aschenauer, Chu, Page `19, Borsa, Florian, Pedron `20, Arratia, Furletova, Hobbs, Olness, Sekula `20

• 3D nucleon/nucleus tomography


• Saturation

Hatta, Xiao, Yuan `17, Salazar, Schenke `19, Roy, Venugopalan `19, Kang, Liu `19
P. Kotko, K. Kutak, A. Stasto, M. Strikman, S. Sapeta `17
T. Altinoluk, R. Boussarie, C. Marquet, P. Taels `20
T. Altinoluk, C. Marquet, P. Taels `20

• Hadronization and quarks and gluons in the nucleus

Klasen, Kovarik `18, Aschenauer, Lee, Page, FR `19, Qin, Wang, Zhang `19, Arratia, Song, FR, Jacak `19, Li et al. `20
For most HERA studies

Background
\[ \sim 0 \ p_T \ \text{in Breit frame} \]

\[ 2x_{Bj} p + q = 0 \]

Signal (gluon PDFs, \( \alpha_s \))
High \( p_T \) in Breit frame
At the EIC we expect

Signal: quark TMDs
\sim 0 \ p_T \text{ in Breit frame}

Signal: gluons
High $p_T$ in Breit frame

\[2x_{Bj} \vec{p} + \vec{q} = 0\]
Jets algorithm for EIC

The HERA jet measurements in DIS targeted gluon initiated processes by requiring large transverse momentum in the Breit frame. This suppresses the Born configuration, which has recently been postulated as key to probe transverse-momentum dependent (TMD) PDF.

It is important for separation the beam remnant from forward jets. is longitudinally invariant along the Breit frame beam axis but yet captures the struck-quark jet.

New longitudinally-invariant algorithm that is asymmetric in the backward and forward directions, Centauro.
Distribution in imbalance – EIC

$\vec{q}_T = |\vec{p}_T^e + \vec{p}_T^{\text{jet}}|$

i.e. transverse momentum of quark in hadron
Lepton-Jet Correlations in Deep Inelastic Scattering

- Access small Bjorken-$x$ (HERA kinematic region)
- Study $Q^2$ dependence on TMD evolution.
- Study parton radiation effects.
- Test $p$QCD and MC generators.

\[
\frac{d^5 \sigma(\ell p \to \ell' J)}{d y_{\ell} d^2 k_{\ell \perp} d^2 q_{\perp}} = \sigma_0 \int d^2 k_{\perp} d^2 \lambda_{\perp} x f_q(x, k_{\perp}, \zeta_c, \mu_F) \\
\times H_{TMD}(Q, \mu_F) S_J(\lambda_{\perp}, \mu_F) \\
\times \delta^{(2)}(q_{\perp} - k_{\perp} - \lambda_{\perp}).
\]

Imbalance $q_T = |\vec{P}^e_T + \vec{P}^{\text{jet}}_T|$ from A. Quintero, DIS 2021
Lepton-Jet Correlations in Deep Inelastic Scattering

Imbalance

\[ q_T = | \mathbf{p}_T^e + \mathbf{p}_T^{\text{jet}} | \]
Gluon densities in DIS and low $x$ – dipole density

Contributes to semi-inclusive DIS and $F_2$ at low $x$

This gluon density can be used to introduce unintegrated quark density at low $x$. 

$S(x_g, r) \propto \langle Tr(U^\dagger(x_1)U(x_2)) \rangle$

that target is homogenous and large as compared to dipole size we can express it in terms of dipole size

$F_{dip}(k_T) \propto F.T. S(r)$

$r \equiv x_1 - x_2$
Gluon densities in DIS and low $x$ – Weizsacker-Williams density

Contributes to 2,3,… jets

$$Q \propto \langle Tr(U^\dagger(x_1)U(x_2)U^\dagger(x_3)U(x_4)) \rangle$$

After introducing large $N_c$ approximation and assuming that target is homogenous and large we can express Quadrupol in terms of one transverse variable

$$\mathcal{F}_{WW}(x_g, k_T) \propto F.T.\tilde{Q}(x_g, r)$$
In collinear physics at LO for $2 \rightarrow 2$ we get delta function since the colliding partons do not carry transverse momentum. Adding more jet we get some improvement $2 \rightarrow 3, \ 2 \rightarrow 4$. The unobserved partons can be soft and can introduce large logs. Note: $k_t$ factorization also smears the delta function but takes into account also low $x$ effects.

$\frac{1}{\sigma} \frac{d\sigma}{d\Delta \phi}$

From Bowen Xiao lecture at QCD 2019 master class

$p_t \gg k_t$

$\Delta \phi$

$\ln^2 \frac{p_t^2}{k_t^2}$

leading jet

imbalance between leading jet and associated jet – in forward jet scenario this can be linked to $k_t$ of incoming parton

$L$ needs to be resummed
Decorelations and WW gluon in DIS

This process allows to probe the Weizsacker-Williams TMD

\[ d\sigma_{\gamma^* A \rightarrow 2j + X} \propto \int \frac{dx}{x} d^2 k_T F_{gg}^{(3)}(x, k_T; \mu) M_{\gamma^* g^* \rightarrow 2j} \]

A. Mueller, B-W. Xiao, F. Yuan, 2013

The Weizsacker-Williams TMD with Sudakov resummation to account for soft emissions

\[ S_{\text{Sud}}^{g \rightarrow q\bar{q}}(\mu, b_T) = \frac{\alpha_s N_c}{4\pi} \ln^2 \frac{\mu^2 b_T^2}{4e^{-2\gamma_E}} \]

From S. Sapeta

Related studies for dijet/dihadron at EIC

A. Dumitru, V. Skokov, 2018

In progress, P. Kotko, K. Kutak, Sapeta, A. van Hameren E. Żarow

Decorelations and WW gluon in DIS

This process allows to probe the Weizsacker-Williams TMD

The Weizsacker-Williams TMD with Sudakov resummation to account for soft emissions

Particles

\[ d\sigma_{\gamma^* A \to 2j + X} \propto \int \frac{dx}{x} d^2 k_T \mathcal{F}^{(3)}_{gg} (x, k_T; \mu) \mathcal{M}_{\gamma^* g \to 2j} \]

Jets

\[ S_{\text{Sud}}^{q\bar{q}} (\mu, b_T) = \frac{\alpha_s N_c}{4\pi} \ln^2 \frac{\mu^2 b_T^2}{4e^{-2\gamma_E}} \]

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From S. Sapeta
Summary and further comments

- Jets are relevant to understand better QCD
- EIC offers new opportunities in studies of QCD using jets
- There are many more results that I did not have time to talk about
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From F. Ringer Jet Observables at the Electron-Ion Collider 2020, BNL