Electromagnetic currents induced by color fields

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May-September 2014: BNL as a brain circulation fellow
October 2014-Present: University of Heidelberg
Outline

- Introduction:
  photon puzzle and photon production in glasma

- Electromagnetic current:
  an important building block to compute photon spectra

- EM current in uniform color electric fields
  - Abelianization
  - SU(2) vs. SU(3)
  - Color direction dependence

- Inhomogeneous color fields
Photon puzzle

Direct photon excess

Hydrodynamic models fail to describe simultaneously photon yield, temperature and v2.

Large photon v2

photons production in pre-equilibrium?

Geometrical scaling

C.K-Boesing, L.McLerran (2014)
Quark production in glasma

Glasma gauge fields produce quarks

Quarks are accelerated or kicked by the gauge fields

Chemical and thermal equilibration?

can be computed by real-time lattice simulations with the classical(-statistical) approximation of gauge fields

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Chemical and thermal equilibration?

During these processes, quarks can emit photons.

Can we see glasma by photons?
Photon production formula

In thermal equilibrium,

\[
\omega \frac{dR}{d^3 k} = -\frac{g^{\mu\nu}}{2(2\pi)^3} \int d^4 x \, e^{ik \cdot x} \langle J_\mu(0) J_\nu(x) \rangle_\beta
\]

Extension to non-equilibrium....

One of characteristic features of a non-equilibrium state is nonzero current expectation.

\[
\langle J_\mu(x) \rangle = e \langle \overline{\psi}(x) \gamma_\mu \psi(x) \rangle \neq 0
\]

Gives the same order contribution in \( \alpha \) as the connected one-loop.

McLerran and Toimela (85), Weldon (90), Gale and Kapsta (91)
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Abelianization of color fields

\[ F_{\mu\nu}^a(x) = F_{\mu\nu}(x) n^a \]

\[ A_\mu^a(x) = A_\mu(x) n^a \]

Diagonalize \( n^a T^a \)

\[ D_\mu \psi = [\partial_\mu + ig A_\mu n^a T^a] \psi \rightarrow \left[ \partial_\mu + ig A_\mu \begin{pmatrix} w_1 & w_2 & w_3 \end{pmatrix} \right] \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} \]

U(1) theory with effective coupling \( \omega_c g \)
Abelianization of color fields

\[ F_{\mu\nu}^a(x) = F_{\mu\nu}(x)n^a \quad A_\mu^a(x) = A_\mu(x)n^a \]

constant vector in color space

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U(1) theory with effective coupling \( w_c g \)

- An important difference between SU(2) and SU(3)

SU(2) is rank 1: \( U^\dagger n^a T^a U = T^3 \)

SU(3) is rank 2: \( U^\dagger n^a T^a U = T^3 \cos \theta - T^8 \sin \theta \)

color direction parameter
Abelianization of SU(3) fields

\[ U^\dagger n^a T^a U = T^3 \cos \theta - T^8 \sin \theta = \frac{1}{\sqrt{3}} \begin{pmatrix} \cos(\theta + \frac{\pi}{6}) \\ \cos(\theta + \frac{5\pi}{6}) \\ \sin \theta \end{pmatrix} \]

Relation between \( \theta \) and \( n^a \)

\[ \sin^2 3\theta = 3(d^{abc} n^a n^b n^c)^2 \]

gauge invariant quantity (Casimir invariant) characterizing the color direction

The color direction can be parametrized in a gauge-invariant way.

Physical observables can depend on it.
Quark production in SU(2) uniform electric fields

\[ D_\mu \psi = [\partial_\mu + igA_\mu n^a T^a] \psi \rightarrow \left[ \partial_\mu + igA_\mu \left( \begin{array}{cc} 1/2 \\ -1/2 \end{array} \right) \right] \left( \begin{array}{c} \psi_1 \\ \psi_2 \end{array} \right) \]

The diagonalized effective couplings are always 1/2 and -1/2.

**Uniform and constant electric field** \( E^\alpha_z = E_0 n^a \)

strong field classical limit \( gE_0 = \text{const}, g \to 0 \)  
no gauge field fluctuations  
no backreaction

The distribution functions of produced quarks

The distributions of anti-particle is given by \( p \leftrightarrow -p \)
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Cancellation of EM current in SU(2) fields

The contributions from color 1 and 2 are cancelled out.

\[ J_z^{\text{EM}} \simeq 4e_f \sum_c \int \frac{d^3p}{(2\pi)^3} \frac{p_z}{\omega_p} f_c(t, p) \]

In the case of the Schwinger mechanism,

\[ f_c(t, p) \simeq e^{-\pi \frac{m^2 + p_z^2}{|w_0E|}} \theta(0 < p_z < w_0E) \]
Quark production in SU(3) uniform electric fields

Uniform and constant electric field $E_z^a = E_0 n^a$

$\theta = 0^\circ$

$\theta = 30^\circ$

The distribution functions of produced quarks

N.T. (2010)
Non-cancellation of EM current in SU(3) fields

\[ J_z^{EM} / \left( e \left( gE_0 \right)^{3/2} \right) \]

\[ J_z^{EM} \sim \frac{4e_\gamma}{(2\pi)^3} \sum_c w_c |w_c| e^{-\frac{\pi m^2}{\left| w_c \right| gE_0}} \left( gE_0 \right)^2 t \]
Inhomogeneous color electric fields: SU(2)

initial energy density of gauge field
uniform in z
randomly distributed in the transverse plane
a scale $Q$

a snapshot of the EM current

time-evolution of space-averaged energy density
Inhomogeneous color electric fields: \textbf{SU}(3)

initial energy density of gauge field
uniform in z
randomly distributed in the transverse plane
a scale \(Q\)

\[
|J_z^{\text{EM}}|/(eQ^3)
\]

a snapshot of the EM current

time-evolution of space-averaged energy density
Summary and Outlook

- Investigated EM currents induced by color fields as a first step to study photon production in glasma.
- In SU(2) uniform fields, EM currents are not at all induced because of the cancellation between two color components.
- In SU(3) uniform fields, the cancellation is not perfect. The EM currents exist depending on the color direction of the background field.
- In inhomogeneous color fields, SU(2) and SU(3) give quantitatively different results.

- Quark production in glasma
- Effects of gauge field fluctuations and backreaction
- Photon production