Hadron lepton spin correlation & polarimetry

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Can the spin observable A_{SL} for elastic p$\uparrow$–e$\rightarrow$ act as a polarimeter ?

Would the correlation A_{tl} also work for elastic ^{3}He$\uparrow$–e$\rightarrow$ collisions ?

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- The polarization of protons or ^{3}He scattering on electrons follows from QED
- Measure asymmetry for elastic collisions on longitudinally polarized electrons
- The polarization direction of the light ion needs to be in the scattering plane
- A_{SL}(100 GeV p) $\approx -50\%$ when static electrons are scattered at 5 ± 1 mrad
- Similar analyzing power for 67 GeV/N He-3 when e^{-} measured near 5 mrad
- Scattering angle of electrons $\approx (1 + m_e E/M)/E$, where E is energy/mass
- Absolute polarimeter of a p or ^{3}He beam if inelastic events can be understood
- The π^{0} and excitations of the proton need to be excluded to ensure elasticity
- Study breakup of ^{3}He to a proton and deuteron at 5.5 MeV (no excited state)

SPIN CORRELATION ASYMMETRY

The spin observable for transversely polarized proton or ^{3}He ions scattering off longitudinally polarized electrons is

$$A_{\mathrm{tl}} = \frac{-2\,(\phi_1 - \phi_3)\,\phi_6}{|\phi_1|^2 + |\phi_2|^2 + |\phi_3|^2 + |\phi_4|^2 + 2\,|\phi_5|^2 + 2\,|\phi_6|^2}$$

for the real one photon exchange helicity amplitudes ϕ_i given below for ions of mass $m_A = M$ and charge $q = Ze$ elastically scattering off electrons of mass $m_B = m_e$.

The size of this analyzing power for polarized protons of energy E GeV scattering off longitudinally polarized electrons peaks at the following electron angle, in general agreement with Gakh et al (2011)[1].

$$\sqrt{M^2 + 2m_e E}\,\left(1 + (m_e \mu E / Z m_p\, s)^2\right)/2E$$

ANOMALOUS MAGNETIC MOMENT

A study of the electromagnetic current matrix element leads to the factor

$$\left(\frac{\mu}{m_{\rm p}} - \frac{Z}{m}\right)$$

for a fermion of mass m and charge $q = Ze$ with initial and final p_μ, p'_μ

$$\bar{u}' \left\{ (p' + p)^\mu F_1 \; - \; \tfrac{1}{2} \left[\gamma^\mu, \gamma^\nu \right] (p' - p)_\nu \, G_{\rm M} \right\} u \, / \, 2\, m$$

where the electromagnetic form factors $F_1(t)$ and $G_{\rm M}(t)/(2m)$ with

$$t = (p' - p)_\mu (p' - p)^\mu ,$$

have static values equal to the charge and magnetic moment of the fermion

$$F_1(0) \; = \; q \, , \qquad \frac{G_{\rm M}(0)}{2\, m} \; = \; \mu' = \mu \, \frac{e}{2\, m_{\rm p}}$$

noting that μ' is normally quoted as μ in nuclear magnetons.

An alternative expression for the current uses $F_1(t)$ and $F_2(t)$

$$\bar{u}' \left\{ \gamma^\mu F_1 - \tfrac{1}{2} [\gamma^\mu, \gamma^\nu] (p' - p)_\nu F_2 \right\} u$$

in a normalisation where the electromagnetic form factors are related by

$$G_{\mathrm{M}}(t) = F_1(t) + 2mF_2(t)$$

$$G_{\mathrm{E}}(t) = 2mF_1(t) + tF_2(t)$$

so that a fermion with charge $q = Ze$ has anomalous magnetic moment

$$F_2(0) = \mu' - \frac{q}{2m} = \frac{e}{2} \left(\frac{\mu}{m_p} - \frac{Z}{m} \right) .$$

The Dirac magnetic moment is $q/2m$, or Zm_p/m , in nuclear magnetons and $m_{\mathrm{h}} = 2808.3914$ MeV$/c^2$

One photon exchange amplitudes are useful for helion proton polarimetry.

If the velocities of particles A and B in the centre of momentum frame are

$$\beta_{\mathrm{A}} = \left(1 + m_{\mathrm{A}}^2/k^2\right)^{-1/2} \qquad \beta_{\mathrm{B}} = \left(1 + m_{\mathrm{B}}^2/k^2\right)^{-1/2}$$

and the momentum of each particle in that particular frame is given by

$$4k^2 = s - 2m_{\mathrm{A}}^2 - 2m_{\mathrm{B}}^2 + \left(m_{\mathrm{A}}^2 - m_{\mathrm{B}}^2\right)^2/s$$

The one photon amplitudes for the scattering of particles A and B are

$$\phi_1^\gamma + \phi_3^\gamma \;=\; -G_\mathrm{M}^\mathrm{A}\, G_\mathrm{M}^\mathrm{B} + \left(4 + \frac{t}{k^2}\right)\left(m_\mathrm{A} m_\mathrm{B}\, {F_2}^\mathrm{A} {F_2}^\mathrm{B} \;+\; \frac{s - m_\mathrm{A}^2 - m_\mathrm{B}^2}{2\,t}\, {F_1}^\mathrm{A} {F_1}^\mathrm{B}\right)$$

$$\phi_1^\gamma - \phi_3^\gamma \;=\; -G_\mathrm{M}^\mathrm{A}\, G_\mathrm{M}^\mathrm{B}\,,$$

$$\phi_2^\gamma - \phi_4^\gamma \;=\; \frac{s^2 - \left(m_\mathrm{A}^2 - m_\mathrm{B}^2\right)^2}{4\,s}\left(4 + \frac{t}{k^2}\right) {F_2}^\mathrm{A} {F_2}^\mathrm{B} + \left(\frac{m_\mathrm{A}}{k} {F_1}^\mathrm{A} - 2k {F_2}^\mathrm{A}\right) \times$$

$$\phi_2^\gamma + \phi_4^\gamma \;=\; 0\,, \qquad\qquad \times \left(\frac{m_\mathrm{B}}{k} {F_1}^\mathrm{B} - 2k {F_2}^\mathrm{B}\right)$$

$$\phi_5^\gamma \;=\; +\sqrt{-\frac{1}{t} - \frac{1}{4k^2}}\left(\frac{m_\mathrm{B}}{\beta_\mathrm{A}} {F_1}^\mathrm{A} {F_1}^\mathrm{B} - 2k\sqrt{s}\, {F_1}^\mathrm{A} {F_2}^\mathrm{B} + \frac{m_\mathrm{A} t}{\beta_\mathrm{B}} {F_2}^\mathrm{A} {F_2}^\mathrm{B}\right)$$

$$\phi_6^\gamma \;=\; -\sqrt{-\frac{1}{t} - \frac{1}{4k^2}}\left(\frac{m_\mathrm{A}}{\beta_\mathrm{B}} {F_1}^\mathrm{A} {F_1}^\mathrm{B} - 2k\sqrt{s}\, {F_1}^\mathrm{B} {F_2}^\mathrm{A} + \frac{m_\mathrm{B} t}{\beta_\mathrm{A}} {F_2}^\mathrm{A} {F_2}^\mathrm{B}\right)$$

The differential cross section is (O'Brien & NB, Czech J Phys, 2006)

$$\frac{d\sigma}{dt} = \frac{\pi}{k^2 s}\left[\frac{1}{4\,t^2}\left(G_{\mathrm{E}}^{\mathrm{A}}\,G_{\mathrm{E}}^{\mathrm{B}}\right)^2 + \frac{1}{2}\left(G_{\mathrm{M}}^{\mathrm{A}}\,G_{\mathrm{M}}^{\mathrm{B}}\right)^2 + \right.$$

$$\left. + \frac{\left(m_{\mathrm{A}}^2 - m_{\mathrm{B}}^2\right)^2 - su}{t^2}\left(\frac{{G_{\mathrm{E}}^{\mathrm{A}}}^2 - t\,{G_{\mathrm{M}}^{\mathrm{A}}}^2}{4\,m_{\mathrm{A}}^2 - t}\right)\left(\frac{{G_{\mathrm{E}}^{\mathrm{B}}}^2 - t\,{G_{\mathrm{M}}^{\mathrm{B}}}^2}{4\,m_{\mathrm{B}}^2 - t}\right)\right]$$

a generalisation of the Rosenbluth formula for lepton proton collisions

where the expression above results from a spin sum of helicity amplitudes

$$\frac{d\sigma}{dt} = \frac{\pi}{2k^2 s}\left(|\phi_1|^2 + |\phi_2|^2 + |\phi_3|^2 + |\phi_4|^2 + 2\,|\phi_5|^2 + 2\,|\phi_6|^2\right) .$$

References

[1] G. I. Gakh, A. Dbeyssi, D. Marchand, E. Tomasi-Gustafsson and V. V. Bytev, Phys. Rev. C **84**, 015212 (2011) doi:10.1103/PhysRevC.84.015212 [arXiv:1103.2540 [nucl-th]].