About Radiative Corrections

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H. Spiesberger

PRISMA^{+} Cluster of Excellence,
Institut für Physik,

Johannes Gutenberg-Universität Mainz

Overview

- About some general properties of radiative corrections
- Remarks about radiative corrections for DVCS
- To Do

- Emission of real photons
  experimentally often not distinguished from non-radiative processes:
  soft photons, collinear photons
  ➜ ”radiative corrections”
- Virtual corrections: loop diagrams
  needed to cancel infrared divergences (Bloch-Nordsieck)

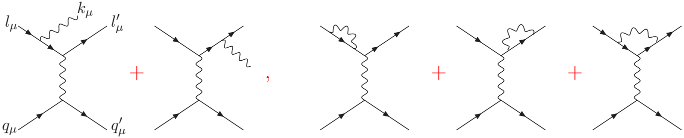
- Radiation from the electron
  model independent (universal)
- Radiation from the hadronic initial/final state
  parton model: radiation from quarks
  part of the nucleon structure
- Interference of electronic and hadronic radiation
  2γ exchange
  new structure
- vacuum polarization
  universal
- purely weak corrections

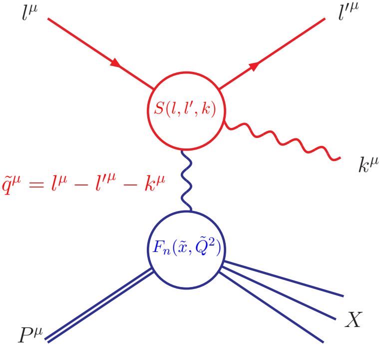
Note: for NC-scattering straightforward separation
IR divergences: need to combine real and virtual radiation

Leptonic radiation

Feynman diagrams for leptonic radiation at $O(\alpha)$ (NC)

for *eq* scattering:





radiative leptonic tensor $S_{\mu\nu}(l, l', k)$ is

- gauge invariant
- infrared finite
- universal

(includes Born + loops: $\delta^{(4)}(k^\mu)$)

Observed cross section:

Convolution of true cross section $\otimes$ radiator functions

$$\mathrm{d}\sigma^{\mathrm{obs}}(P,q) = \int \frac{d^3k}{2k^0} \sum_n R_n(l,l',k)\, \mathrm{d}\hat{\sigma}_n^{\mathrm{true}}(P,q-k)$$

Shifted kinematics

observed momentum transfer: $Q^2 = -q^2$, $q^\mu = l^\mu - l'^\mu$,
➜ true, shifted momentum transfer: $\tilde{Q}^2 = -(q-k)^2$
➜ correction factor: enhancement by $Q^2/\tilde{Q}^2$ ➜ radiative tail

➜ expect strong dependence on experimental prescriptions for measuring kinematic variables

➜ need full Monte-Carlo modelling

Can be extended to include higher-order effects: multi-photon emission, soft-photon exponentiation, e^+e^--pair creation with partial fractioning, write: $R_n(l, l', k) = \frac{J}{k \cdot l} + \frac{F}{k \cdot l'} + \frac{C}{\tilde{Q}^2} + \ldots$

- initial state radiation, $k \cdot l$ small for $\sphericalangle(e_{in}, \gamma) \to 0$
- final state radiation, $k \cdot l'$ small for $\sphericalangle(e_{out}, \gamma) \to 0$
- Compton peak, $\tilde{Q}^2$ small for $p_T(e_{out}) \simeq p_T(\gamma)$

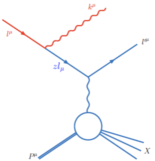
ISR, FSR: narrow peaks, width $\simeq \sqrt{m_l/E_l}$: collinear or mass singularities

upon angular integration: large logarithm $\propto \frac{\alpha}{\pi} \log \frac{Q^2}{m_e^2} \simeq 10\,\%$

Note: additional large logarithms from experimental cuts $\propto \log \frac{\Delta E}{E_{max}}$

A technical remark about numerical integration:
Do partial fractioning, choose denominator as integration variable

e.g., for initial-state radiation: assume $k^\mu = (1 - z) l^\mu$
➜ Radiator function



$$R_{\text{ISR}} = \frac{\alpha}{2\pi} \frac{1 + z^2}{1 - z} \log \frac{Q^2}{m_e^2}$$

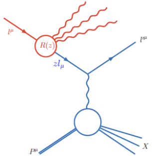
$(+\delta(1 - z)$ from loops $\to$ +-distribution $1/(1 - z)_+)$

$$\mathrm{d}\sigma_{\text{ISR}} = \int \frac{\mathrm{d}z}{z} R_{\text{ISR}}(z)\, \mathrm{d}\sigma_{\text{Born}}(l^\mu \to z l^\mu)$$

(similar for final-state radiation)

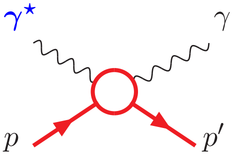
Can be extended to include multi-photon emission:

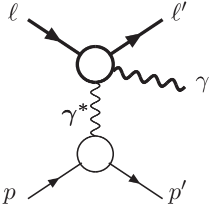
$$R^{(2)}_{\text{ISR}}(z) = \int_z^1 \frac{\mathrm{d}z'}{z'} R^{(1)}_{\text{ISR}}(z') R^{(1)}_{\text{ISR}}(z/z') + \dots$$

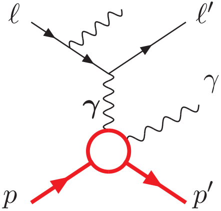


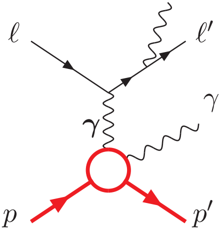
Solution of evolution equations like DGLAP
Known at $O(\alpha^2)$ (complete) and partially at $O(\alpha^3)$

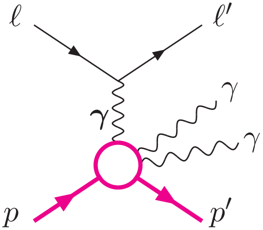
Leading order





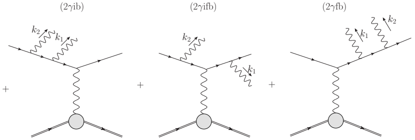




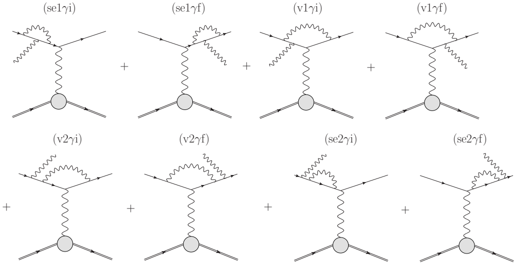


Leptonic corrections for VCS
(+ loops)

VCS $+1\gamma$



2-photon radiation



1-loop corrected 1-photon radiation

VCS at LO, $e(p_-) + p(p) \to e'(p'_-) + p'(p') + \gamma(k_1)$

$$\sigma_{\rm LO} = \int_{x_{\rm min}}^{x_{\rm max}} dx \int_{Q^2_{\rm min}}^{Q^2_{\rm max}} dQ^2 \int_{t_{1,\rm min}}^{t_{1,\rm max}} dt_1 \int_{\phi_{\rm min}}^{\phi_{\rm max}} d\phi \; \hat{\sigma}_{\rm LO}(x, Q^2, t_1, \phi)$$

$$\begin{aligned} x_{\rm min} &= \text{input}\,, & x_{\rm max} &= \text{input}\,, \\ Q^2_{\rm min} &= \text{input}\,, & Q^2_{\rm max} &= \min\left(xs,\ \text{input}\right)\,, \\ t_{1,\rm min} &= \max\left(t_1^-,\ \text{input}\right)\,, & t_{1,\rm max} &= \min\left(t_1^+,\ \text{input}\right)\,, \\ \phi_{\rm min} &= 0\,, & \phi_{\rm max} &= 2\pi\,. \end{aligned}$$

$$t_1^\pm(r=0) = -\frac{Q^2}{1 + 4\tau\frac{1-x}{x}} \left(1 + 2\tau\frac{1-x}{x^2}\left(1 \pm \sqrt{1 + \frac{x^2}{\tau}}\right)\right)\,.$$

$$\sigma_{\rm rad} = \int_{x_{\rm min}}^{x_{\rm max}} dx \int_{Q^2_{\rm min}}^{Q^2_{\rm max}} dQ^2 \int_0^{R^2_{\rm max}} dR^2 \int_{t_{1,\rm min}}^{t_{1,\rm max}} dt_1 \int_{\phi_{\rm min}}^{\phi_{\rm max}} d\phi$$

$$\times \int_{c_{1,\rm min}}^{c_{1,\rm max}} d\cos\theta_1 \int_{\phi_{1,\rm min}}^{\phi_{1,\rm max}} d\phi_1 \; \hat\sigma_{\rm rad}(x, Q^2, t_1, \phi; R^2, \theta_1, \phi_1)$$

$$x_{\rm min} = \text{input}\,, \qquad x_{\rm max} = \text{input}\,,$$

$$Q^2_{\rm min} = \text{input}\,, \qquad Q^2_{\rm max} = \min\left(xs,\ \text{input}\right)\,,$$

$$R^2_{\rm max} = Q^2 \left(A_r + \sqrt{A_r^2 - \left(\frac{1-x}{x}\right)^2} \right)\,, \qquad A_r = \frac{1}{x\tau_x} + \frac{1-x}{x} \cdot \frac{\tau_x + x}{\tau_x + 2x}\,,$$

$$t_{1,\rm min} = \max\left(t_1^-,\ \text{input}\right)\,, \qquad t_{1,\rm max} = \min\left(t_1^+,\ \text{input}\right)\,,$$

$$\phi_{\rm min} = 0\,, \qquad \phi_{\rm max} = 2\pi\,,$$

$$\cos\theta_{2,\rm min} = -1\,, \qquad \cos\theta_{2,\rm max} = +1\,,$$

$$\phi_{2,\rm min} = 0\,, \qquad \phi_{2,\rm max} = 2\pi\,.$$

$$t_1^\pm = -\frac{Q^2}{1 + 4\tau\frac{1-x}{x}} \left(1 - r + \frac{2\tau}{x}\left(\frac{1-x}{x} - r\right) \pm \sqrt{B_t} \right)\,, \qquad B_t = \frac{4\tau}{x^2}(\tau + x^2)\left(r^2 - 2rA_r + \left(\frac{1-x}{x}\right)^2 \right)\,.$$

$$e(p_-) + p(p) \to e'(p'_-) + p'(p') + \gamma(k_1) + \gamma(k_2)$$

Propgator denominators of the diagrams for radiation

$$\begin{aligned}
D_5 &= 2p'_- \cdot k_1 \\
D_6 &= 2p'_- \cdot k_2 \\
D_{56} &= D_5 + D_6 + 2k_1 \cdot k_2 \\
E_5 &= -2p_- \cdot k_1 \\
E_6 &= -2p_- \cdot k_2 \\
E_{56} &= E_5 + E_6 + 2k_1 \cdot k_2
\end{aligned}$$

Partial fractioning
Subtractions to remove infrared and collinear divergences
e.g. following dipole subtraction scheme of S. Dittmaier

HERACLES and DJANGOH: for inclusive DIS
QCD-based event generation, valid at large Q^2: parton model

- Complete QED and electroweak corrections at $O(\alpha)$ for NC and CC scattering, polarized lepton, polarized nucleon
- Interface to LEPTO, JETSET. Jets, parton showers, hadronic final state. SOPHIA for low-mass hadronic final states

Fortran code. Well tested, but less efficient for exclusive processes

POLARES: second-order corrections for elastic *ep*
Includes $O(\alpha)$ corrections for $ep \to ep\gamma$
C++ code, to be published (with R. Bucoveanu)

To Do

- Define objects to be implemented in the PARTONS framework

  ➜ non-radiative and radiative processes, kinematics
- First step: leading log approximation

  ➜ Use radiator functions
- Beyond the collinear approximation

  ➜ Implement radiative leptonic tensor

  Work out decomposition in terms of structure functions
- Radiative corrections for DVCS

  ➜ Finish project with M. Hentschinski and M. Stratmann (?)

  Use POLARES for cross-check