New Physics in Neutrino Oscillations and role of tau neutrinos

Jacobo López-Pavón

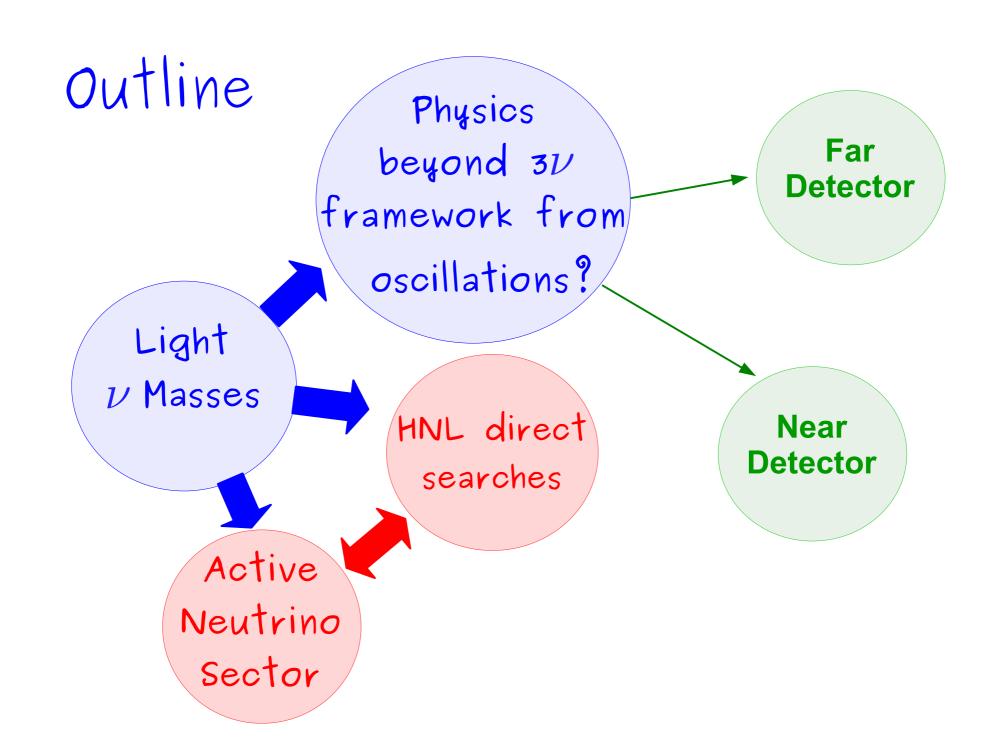
NuTau 2021 - Tau Neutrinos from GeV to EeV 28 September 2021











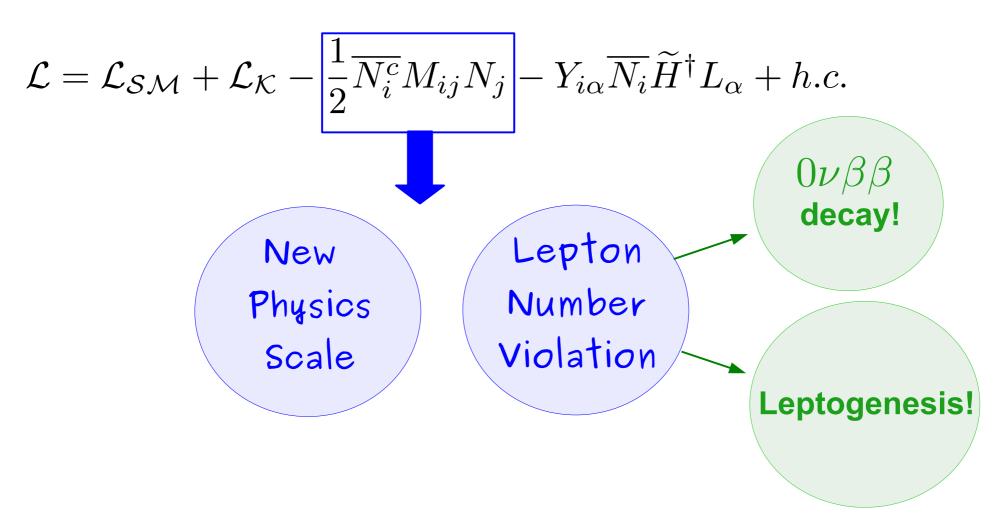
Minimal model: Seesaw Model

• Simplest extension of SM able to account for neutrino masses. Consists in the addition of heavy fermion singlets (N_R) to the SM field content:

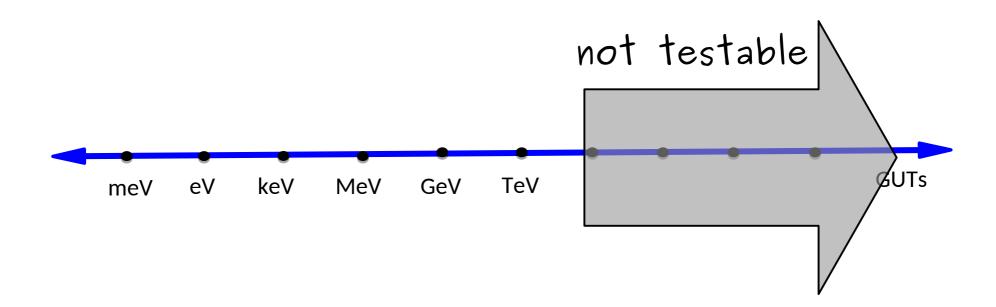
$$\mathcal{L} = \mathcal{L}_{\mathcal{SM}} + \mathcal{L}_{\mathcal{K}} - \boxed{rac{1}{2} \overline{N_i^c} M_{ij} N_j - Y_{ilpha} \overline{N_i} \widetilde{H}^\dagger L_lpha} + h.c.$$
 Light Neutrino Masses $m_
u = rac{v^2}{2} Y^T M^{-1} Y$

Minimal model: Seesaw Model

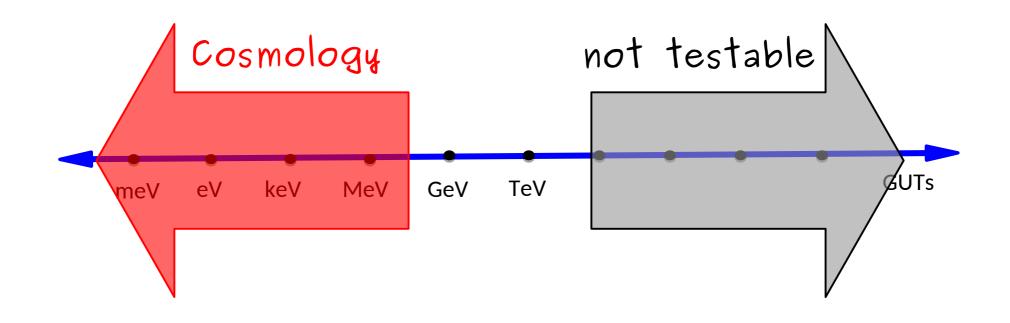
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The New Physics Scale

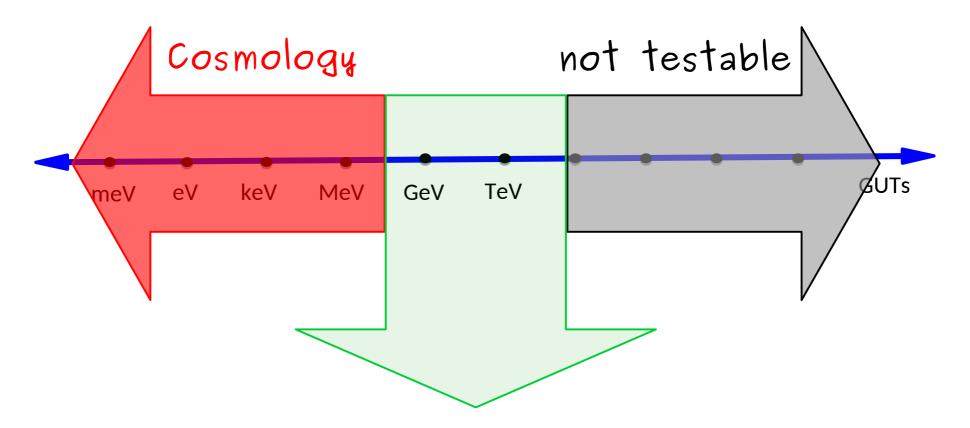


The New Physics Scale



P. Hernandez, M. Kekic, JLP 1311.2614 1406.2961

The New Physics Scale

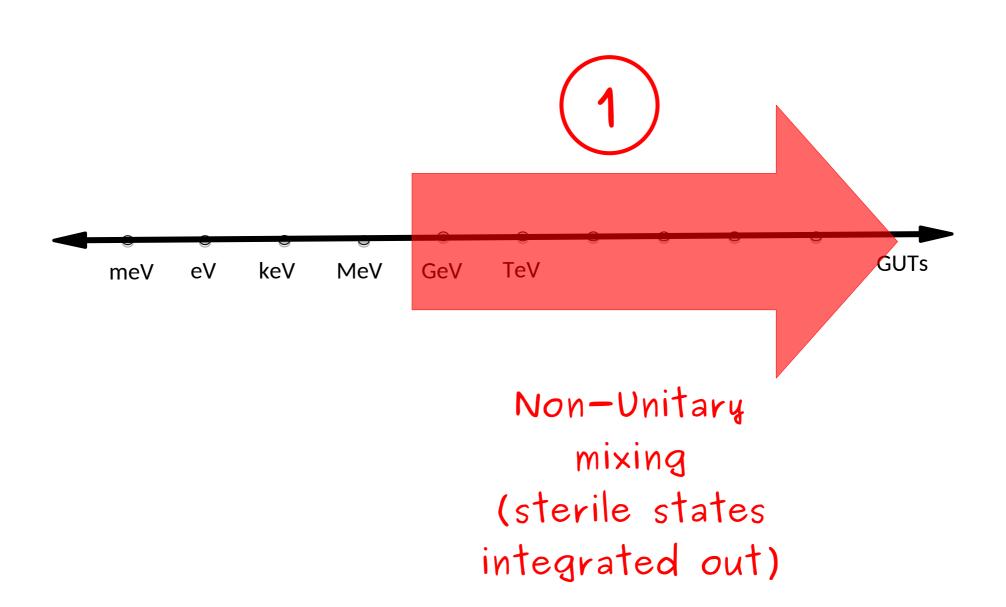


 $0\nu\beta\beta$ decay, CLFV, Colliders, direct searches...

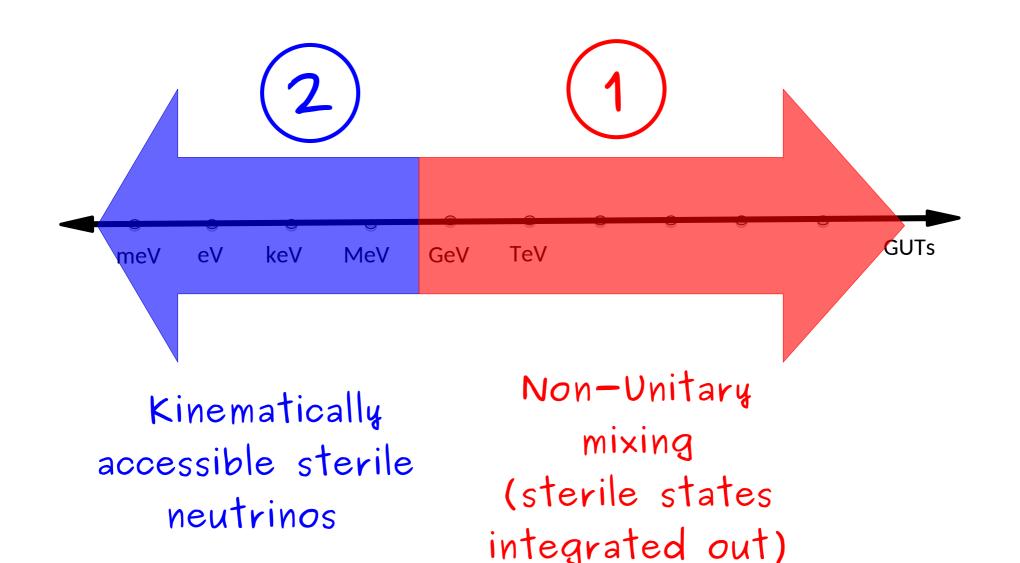
See talks by Laura Field, Vladimir Gligorov, Yuval Grossman, Felix Kling, Albert De Roeck Are Long Baseline
Neutrino Oscillation experiments
sensitive to
New Physics
Beyond 3v framework



Neutrino Oscillations vs NP scale



Neutrino Oscillations vs NP scale



Both limits can be studied in a unified & model independent way

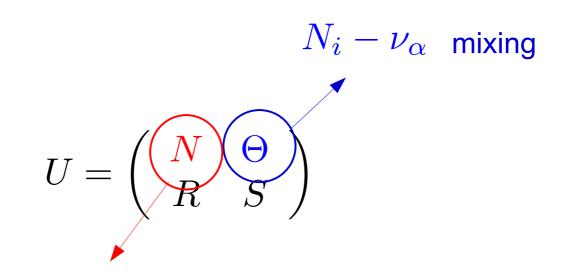
Model Independent Approach

$$U = \left(\begin{array}{cc} N & \Theta \\ R & S \end{array}\right)$$

Model Independent Approach

$$N_i -
u_lpha$$
 mixing
$$U = \left(egin{array}{c} N & \Theta \\ R & S \end{array} \right)$$

Model Independent Approach



Deviation from unitarity of the PMNS matrix

General Parameterizations

· Triangular parameterization

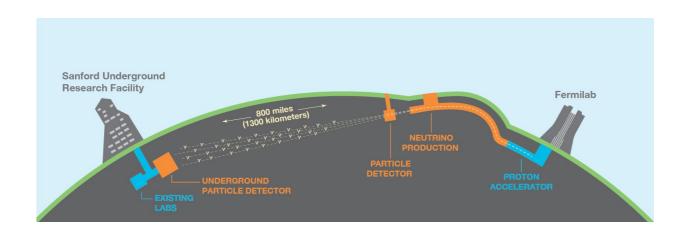
$$N = (I + T)U$$

Deviation from unitarity

$$T = \begin{pmatrix} \alpha_{ee} & 0 & 0 \\ \alpha_{\mu e} & \alpha_{\mu \mu} & 0 \\ \alpha_{\tau e} & \alpha_{\tau \mu} & \alpha_{\tau \tau} \end{pmatrix}$$

Unitary matrix
(standard unitary PMNS
matrix
up to small corrections)

Far Detector vs Near detector



$$N_{\nu_{\alpha} \to \nu_{\beta}} \sim \frac{\Phi_{\alpha}(E)}{L^2} P_{\alpha\beta}(L/E) \sigma_{\beta}(E) \epsilon_{\beta}(E)$$

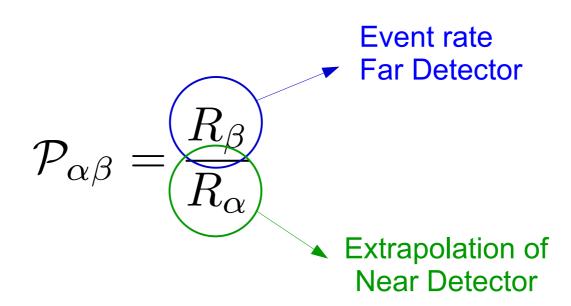
- Sources of systematics
 - Cross sections
- Neutrino flux

- Near detector measurements reduce far detector systematic uncertainties
- New Physics at near detector (strongly affected by systematic uncertainties)

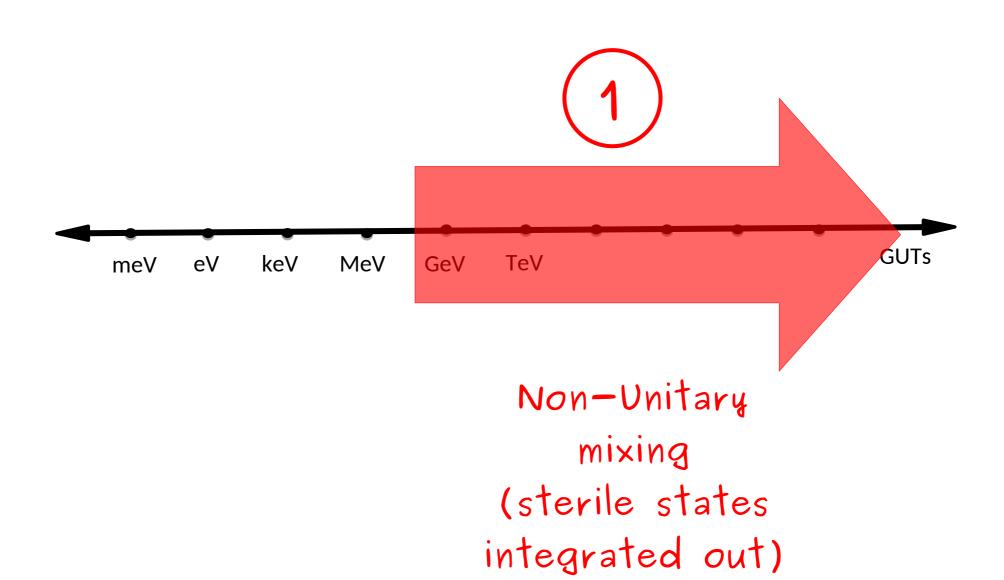
Far Detector

Far Detector

· What is measured in neutrino oscillation experiments



1) Non-Unitary Mixing

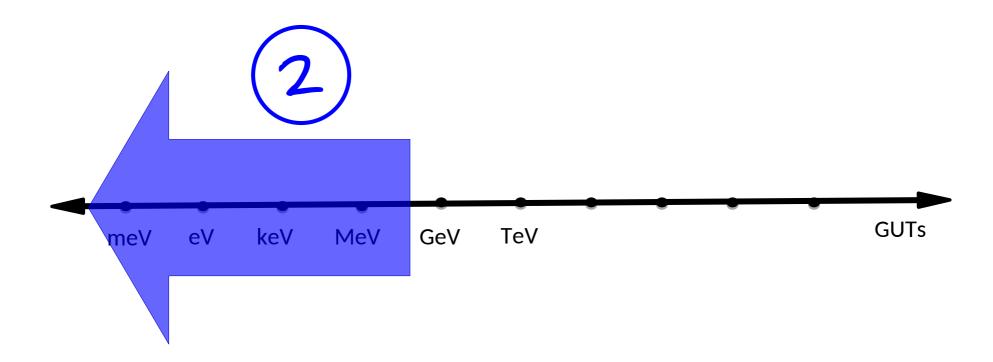


1 Non-Unitary Mixing

· What is measured in neutrino oscillation experiments

$$\mathcal{P}_{\alpha\beta} = \frac{\left| (N \exp(-iHL)N^{\dagger})_{\beta\alpha} \right|^2}{((NN^{\dagger})_{\alpha\alpha})^2}.$$

(2) Kinematically accessible Sterile ν



Kinematically accessible sterile neutrinos

1. The light-heavy oscillations averaged out at the near detector.

Identical to the heavy non-unitarity case

- 1. The light-heavy oscillations averaged out at the near detector.
- Identical to the heavy non-unitarity case
- 2. The light-heavy oscillations have not yet developed at the near detector. No normalization factor

DUNE: $0.1 \,\mathrm{eV}^2 \lesssim \Delta m^2 \lesssim 1 \,\mathrm{eV}^2$

(2) Kinematically accessible Sterile ν

- 1. The light-heavy oscillations averaged out at the near detector.
- Identical to the heavy non-unitarity case
- 2. The light-heavy oscillations have not yet developed at the near detector. No normalization factor
- The oscillation frequency dictated by the light-heavy frequency matches the near detector distance.
- Oscillations could be observed at the near detector

See also talks by Katarzyna Grzelak, Miriama Rajaoalisoa and Carlos Arguelles

(2) Kinematically accessible Sterile ν

1. The light-heavy oscillations averaged out at the near detector.

Identical to the heavy non-unitarity case

2. The light-heavy oscillations have not yet developed at the near detector.

No normalization factor

Low Scale Non-Unitarity

Present Bounds

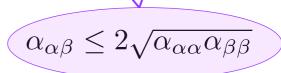
	High-scale Non-Unitarity	$_{\hspace*{-0.1cm} \bullet} \hspace*{-0.1cm} u_i$
	(m > EW)	$\sim \sim \sim (N^\dagger N)_{ij}$
α_{ee}	$1.3 \cdot 10^{-3}$	ν_i
$lpha_{\mu\mu}$	$2.2\cdot 10^{-4}$	l_{α}
$\alpha_{ au au}$	$2.8\cdot 10^{-3}$	$W \sim N_{\alpha i}$
$ lpha_{\mu e} $	$6.8 \cdot 10^{-4} \; (2.4 \cdot 10^{-5})$	EW & CLFV ν_i
$ \alpha_{ au e} $	$2.7\cdot 10^{-3}$	precision data
$ lpha_{ au\mu} $	$1.2\cdot 10^{-3}$	

Fernandez-Martinez, Hernandez-Garcia, JLP 1605.08774 Blennow, Coloma, Fernandez-Martinez, Hernandez-Garcia, JLP 1609.08637

Present Bounds

	High-scale Non-Unitarity	Low-scale Non-Unitarity
	(m > EW)	$\Delta m^2 \gtrsim 100 \text{ eV}^2 \Delta m^2 \sim 0.1 - 1 \text{ eV}^2$
α_{ee}	$1.3 \cdot 10^{-3}$	$2.4 \cdot 10^{-2}$ BUGEY $1.0 \cdot 10^{-2}$ BUGEY
$\alpha_{\mu\mu}$	$2.2 \cdot 10^{-4}$	$2.2\cdot 10^{-2}$ SK $1.4\cdot 10^{-2}$ MINOS
$\alpha_{ au au}$	$2.8 \cdot 10^{-3}$	$1.0 \cdot 10^{-1}$ SK $1.0 \cdot 10^{-1}$ SK
$ lpha_{\mu e} $	$6.8 \cdot 10^{-4} \ (2.4 \cdot 10^{-5})$	$2.5 \cdot 10^{-2}$ NOMAD $1.7 \cdot 10^{-2}$
$ lpha_{ au e} $	$2.7 \cdot 10^{-3}$	$6.9 \cdot 10^{-2}$ $4.5 \cdot 10^{-2}$
$ lpha_{ au\mu} $	$1.2 \cdot 10^{-3}$	$1.2 \cdot 10^{-2}$ NOMAD $5.3 \cdot 10^{-2}$

Fernandez-Martinez, Hernandez-Garcia, JLP 1605.08774 Blennow, Coloma, Fernandez-Martinez, Hernandez-Garcia, JLP 1609.08637



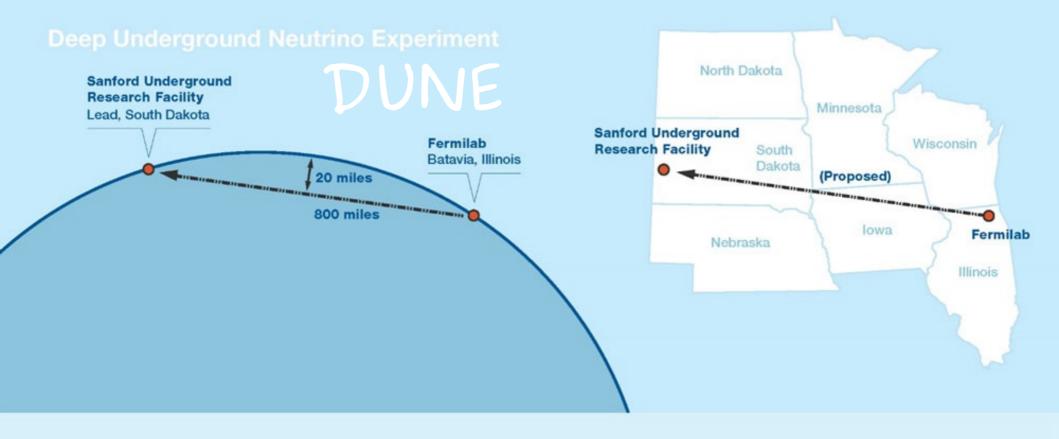
Present Bounds

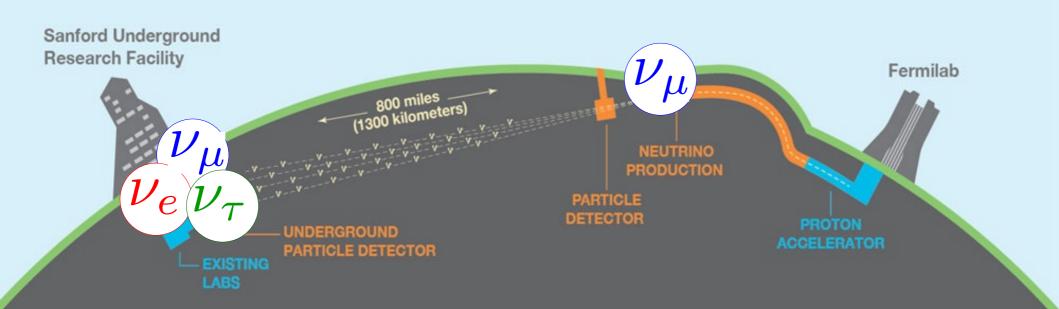
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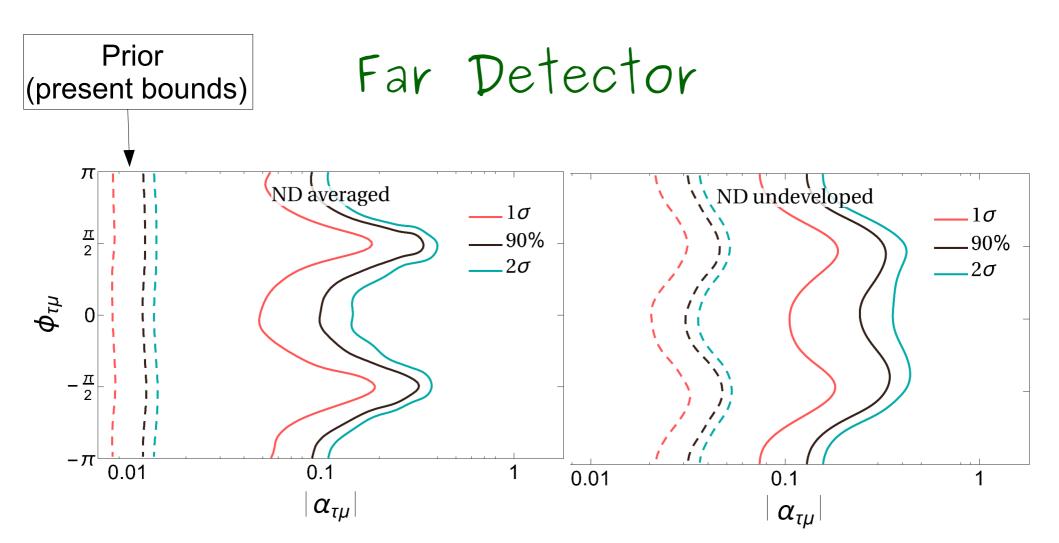
Fernandez-Martinez, Hernandez-Garcia, JLP 1605.08774 Blennow, Coloma, Fernandez-Martinez, Hernandez-Garcia, JLP 1609.08637

AGNOSTIC

See also talk by Julia Gehrlein and Park, Ross-Lonergan 1508.05095 Ellis, Kelly, Weishi Li 2004.13719 Ellis, Kelly, Weishi Li 2008.01088



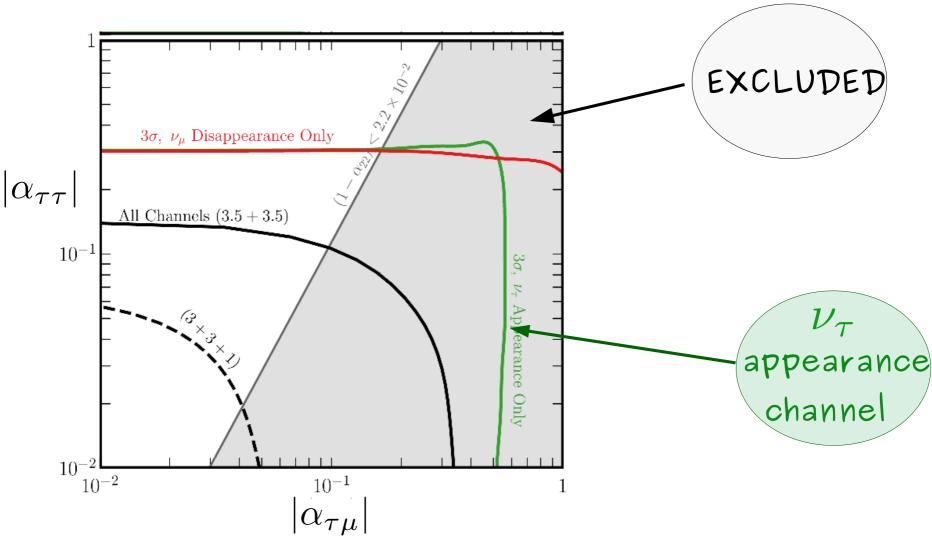




$$\mathcal{P}_{\alpha\beta} = \frac{\left| (N \exp(-iHL)N^{\dagger})_{\beta\alpha} \right|^{2}}{\left[(NN^{\dagger})_{\alpha\alpha} \right]^{2}} \qquad \mathcal{P}_{\alpha\beta} = \left| (N \exp(-iHL)N^{\dagger})_{\beta\alpha} \right|^{2}$$

Blennow, Coloma, Fernandez-Martinez, Hernandez-Garcia, JLP 1609.08637. DUNE CDR configuration 1606.09550

Far Detector



• Including ν_{τ} appearance channel does not change the picture.

Near Detector

Coloma, JLP, Rosauro-Alcaraz, Urrea 2105.11466.

See also Escribuela, Forero, Miranda, Tortola, Valle arXiv:1503.08879 for other Near Detector configurations (without including tau detection).

High Scale Non-Unitarity

· What is measured in neutrino oscillation experiments

$$\mathcal{P}_{\alpha\beta} = \left|(NN^\dagger)_{\beta\alpha}\right|^2 = |lpha_{lphaeta}|^2 \quad {
m zero} \ {
m distance}$$

effect!

$$\mathcal{P}_{\alpha\alpha} = \left| (NN^{\dagger})_{\alpha\alpha} \right|^2 = 1 - 4 \,\alpha_{\alpha\alpha}$$

Sterile Neutrinos: 3+1

· What is measured in neutrino oscillation experiments

$$\mathcal{P}_{\alpha\beta} = 4|U_{\alpha4}||U_{\beta4}|\sin^2\frac{\Delta m_{41}^2 L}{4E}$$

$$\mathcal{P}_{\alpha\alpha} = 1 - 4|U_{\alpha 4}|^2 \sin^2 \frac{\Delta m_{41}^2 L}{4E}$$

Averaged-out regime

· What is measured in neutrino oscillation experiments

 $\Delta m_{41}^2 \gtrsim 100 \,\mathrm{eV}^2$

$$\mathcal{P}_{\alpha\beta} = 2|U_{\alpha4}||U_{\beta4}|$$

zero distance effect!

$$\mathcal{P}_{\alpha\alpha} = 1 - 2|U_{\alpha 4}|^2$$

Averaged-out regime

· What is measured in neutrino oscillation experiments

 $\Delta m_{41}^2 \gtrsim 100 \,\mathrm{eV}^2$

$$\mathcal{P}_{\alpha\beta} = 2|\alpha_{\alpha\beta}|^2$$

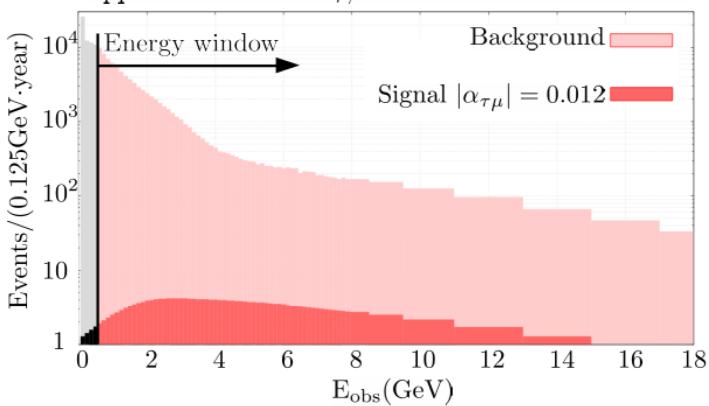
$$\mathcal{P}_{\alpha\alpha} = 1 - 4|\alpha_{\alpha\alpha}|^2$$

zero distance effect!

> Low Scale Non-Unitarity

Role of shape uncertainty

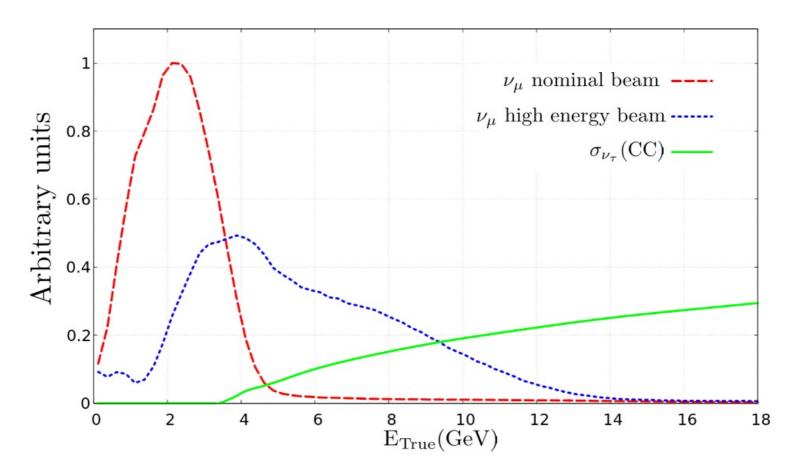
Appearance channel ν_{τ} , nominal beam neutrino mode



- · Sensitivity driven by spectral information.
- Marginal impact of global normalization error.

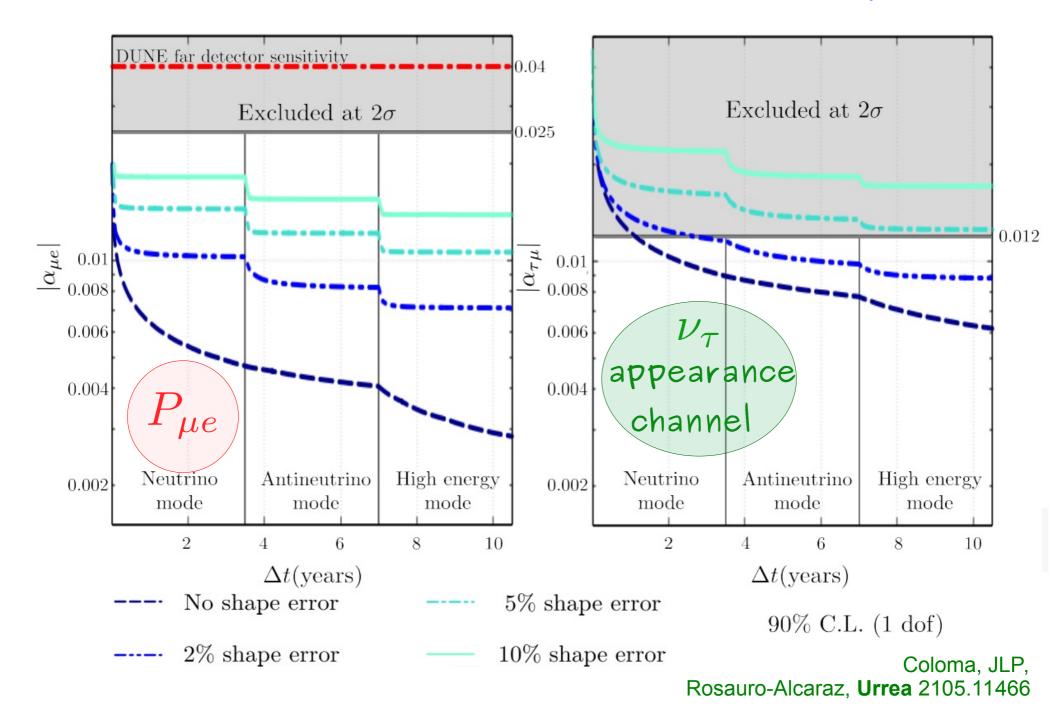
u_{τ} appearance channel

• Energy threshold of τ production 3.2 GeV.



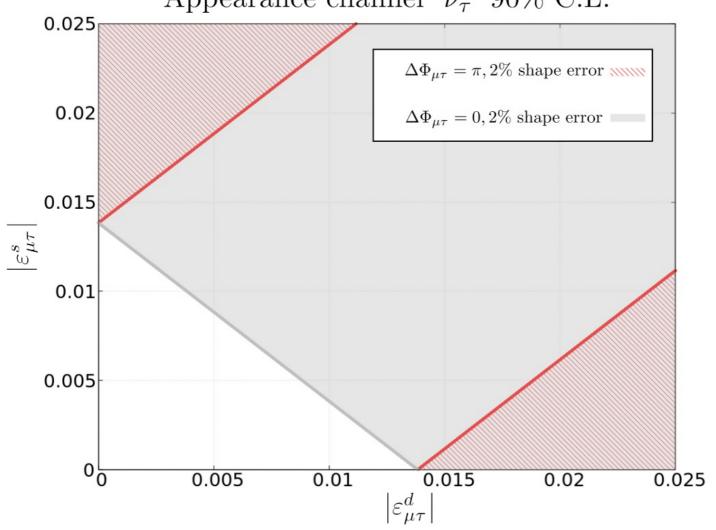
• ν_{τ} detection: we follow de Gouvêa, Kelly, Stenico, Pasquini 1904.07265 See talks by Pedro Machado, Adam Aurisano, Roger Wendell, Thomas Kosc, Dawn Williams, Dario Autiero, Juliana Stachurska

Low Scale Non-Unitarity



NSI in production/detection

Appearance channel ν_{τ} 90% C.L.

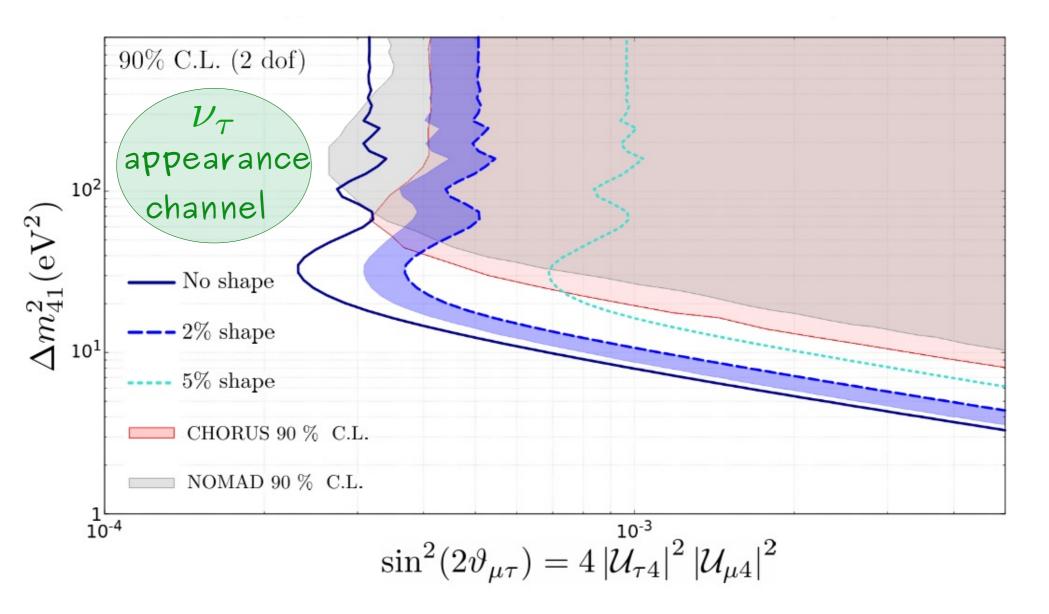


 u_{τ} appearance channel

Coloma, JLP, Rosauro-Alcaraz, **Urrea** 2105.11466

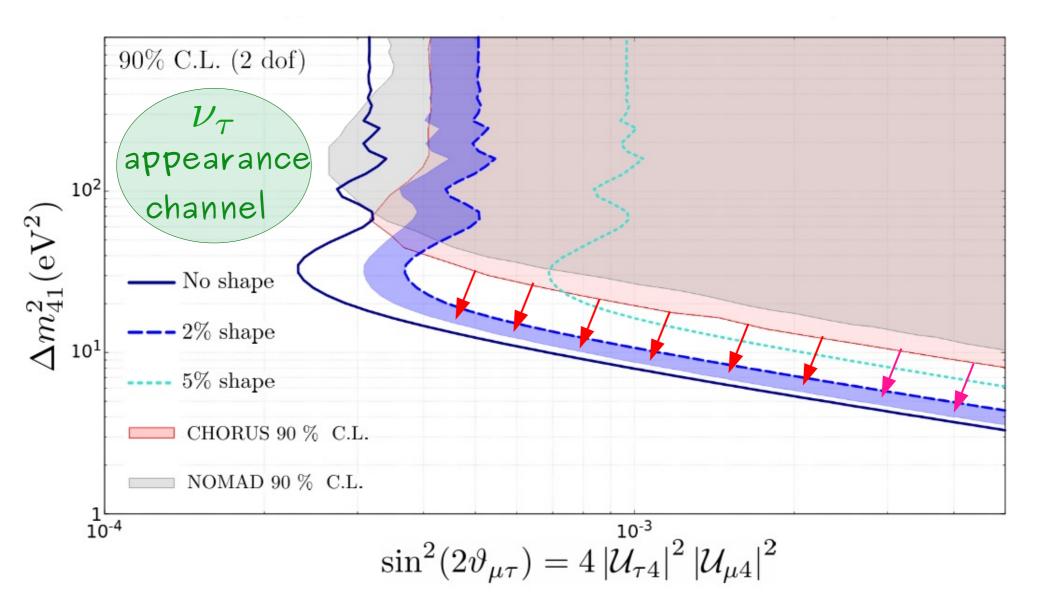
• Mapping: $2|\alpha_{\beta\gamma}|^2 = |\epsilon_{\beta\gamma}^d|^2 + |\epsilon_{\beta\gamma}^s|^2 + 2|\epsilon_{\beta\gamma}^d||\epsilon_{\beta\gamma}^s|\cos(\Phi_{\beta\gamma}^s - \Phi_{\beta\gamma}^d)$

3+1 Sterile Neutrinos

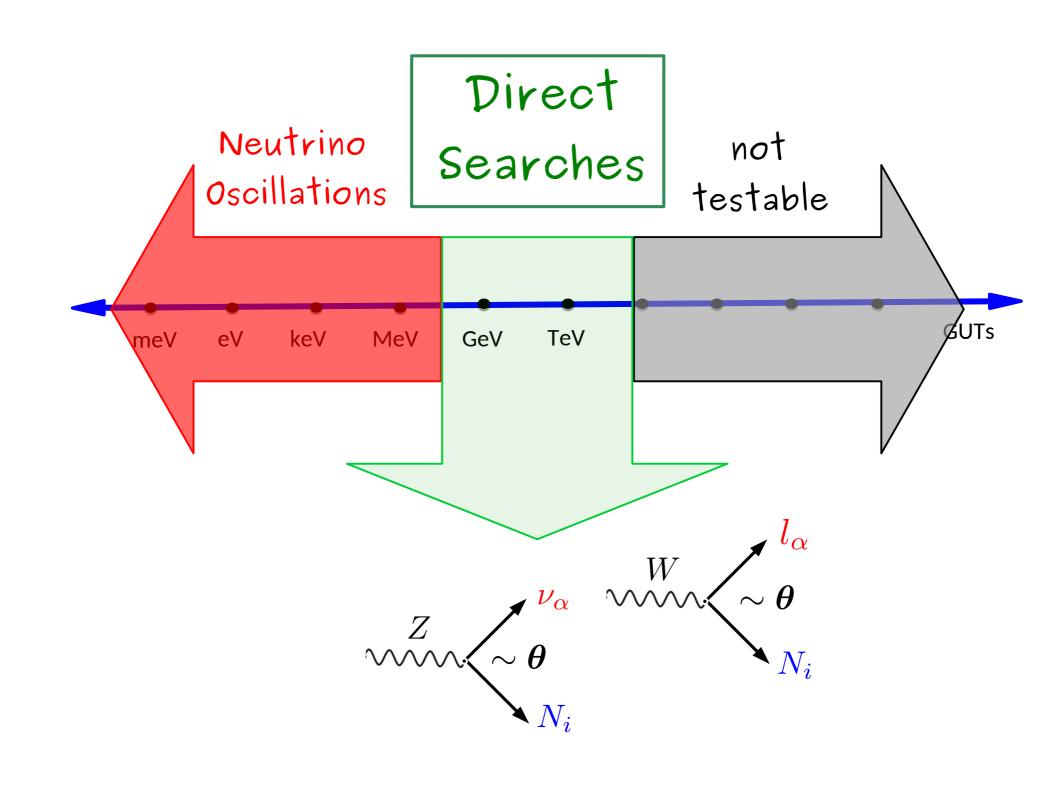


Coloma, JLP, Rosauro-Alcaraz, Urrea 2105.11466

3+1 Sterile Neutrinos

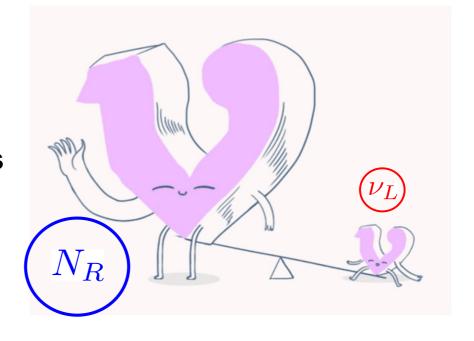


Coloma, JLP, Rosauro-Alcaraz, Urrea 2105.11466



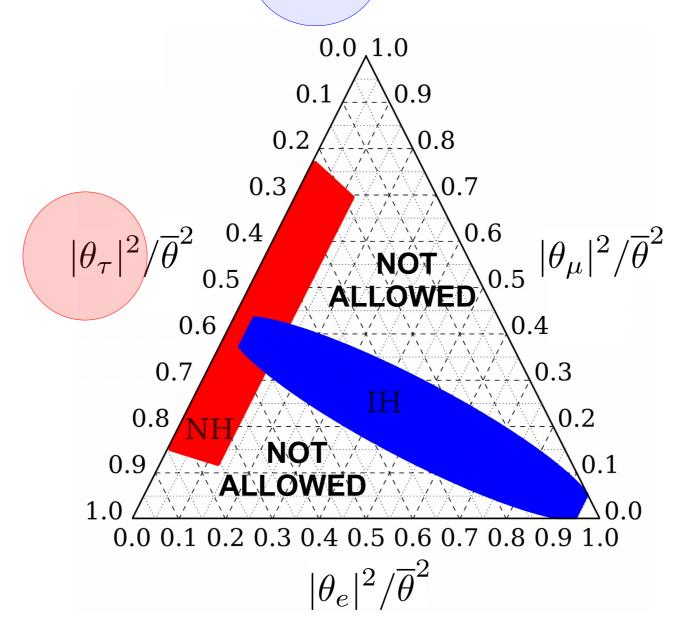
Light Neutrino mass generation

 Generation of light neutrino masses imposes constraints on mixing between HNLs and active neutrinos from light neutrino sector

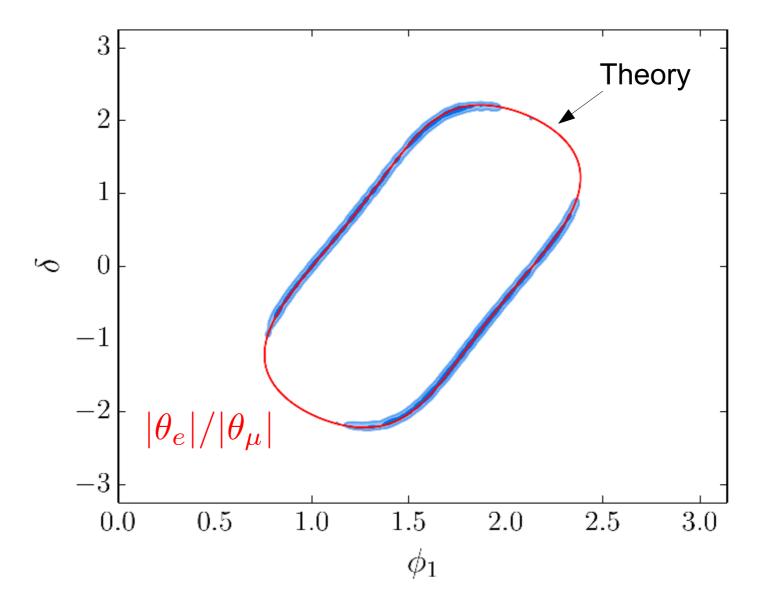


$$m_{\nu} = \frac{v^2}{2} Y^T M^{-1} Y = \underbrace{\theta \, M \, \theta^T}_{\text{HNL sector}} = \underbrace{U \, m \, U^T}_{\text{Light-active neutrino sector}}$$

Minimal model $N_R=2$: Flavor Structure



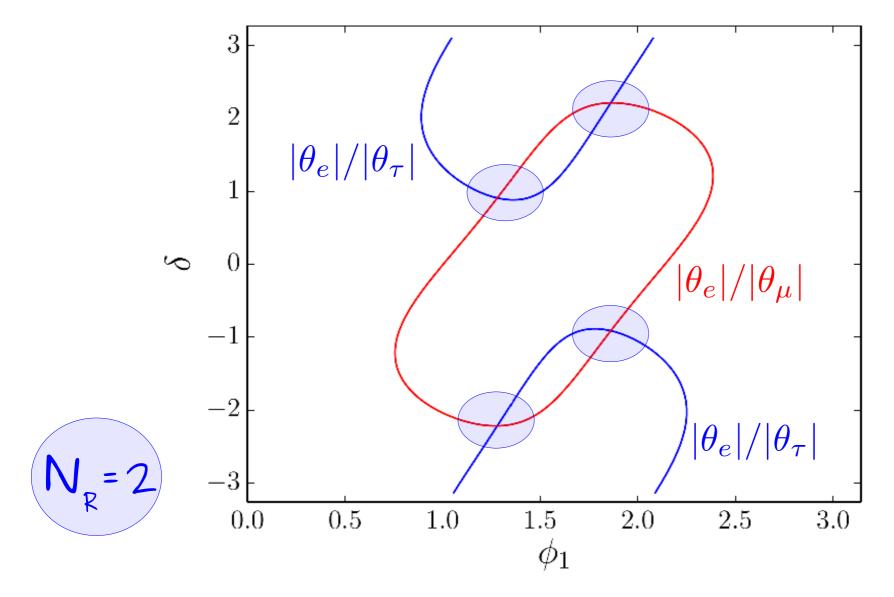
PMNS CP-phases from HNLs searches





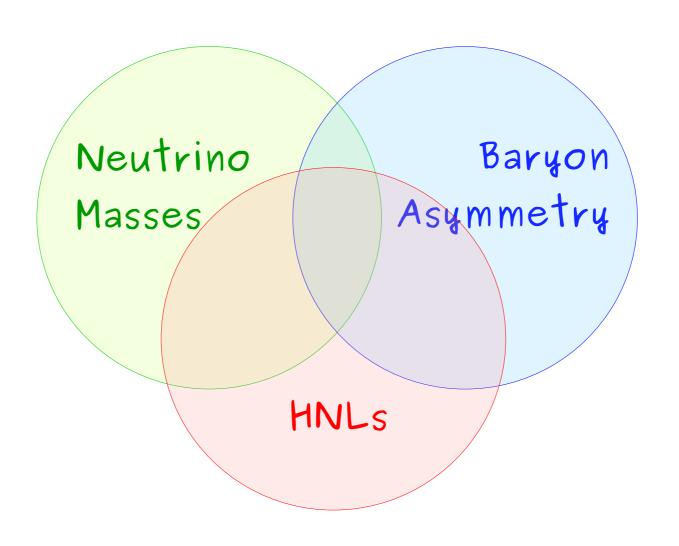
Hernandez, Kekic, JLP, Racker, Salvado 1606.06719 Caputo, Hernandez, Kekic, JLP, Salvado 1611.05000

PMNS CP-phases from HNLs searches

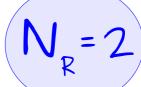


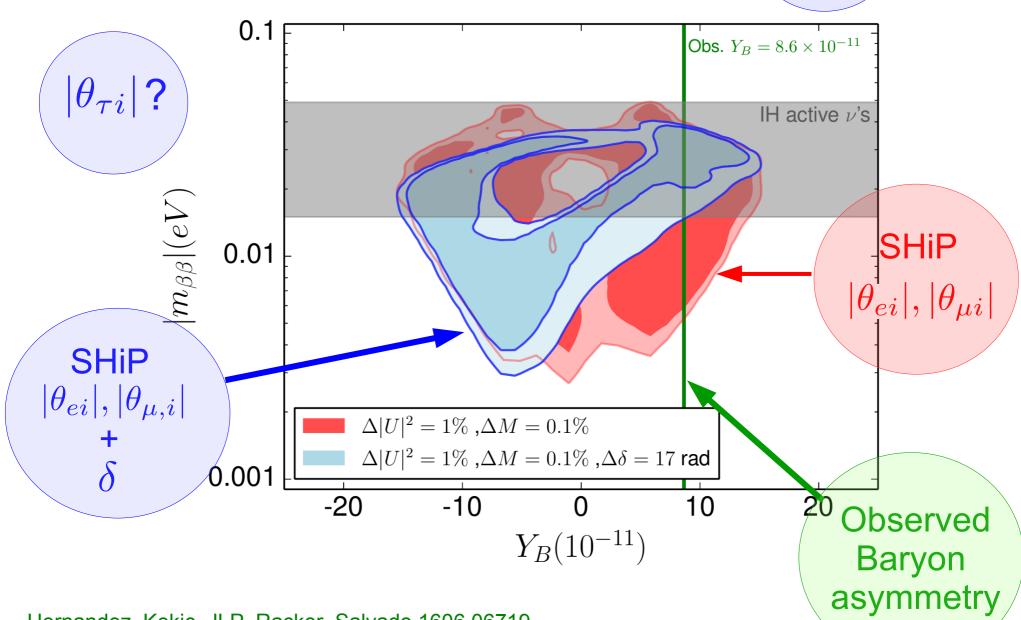
 Measurement of mixing with tau neutrinos would allow to break degeneracies!
 Hernandez, Kekic, JLP, Racker, Salvado 1606.06719

Very relevant input to leptogenesis



GeV Scale Leptogeneis





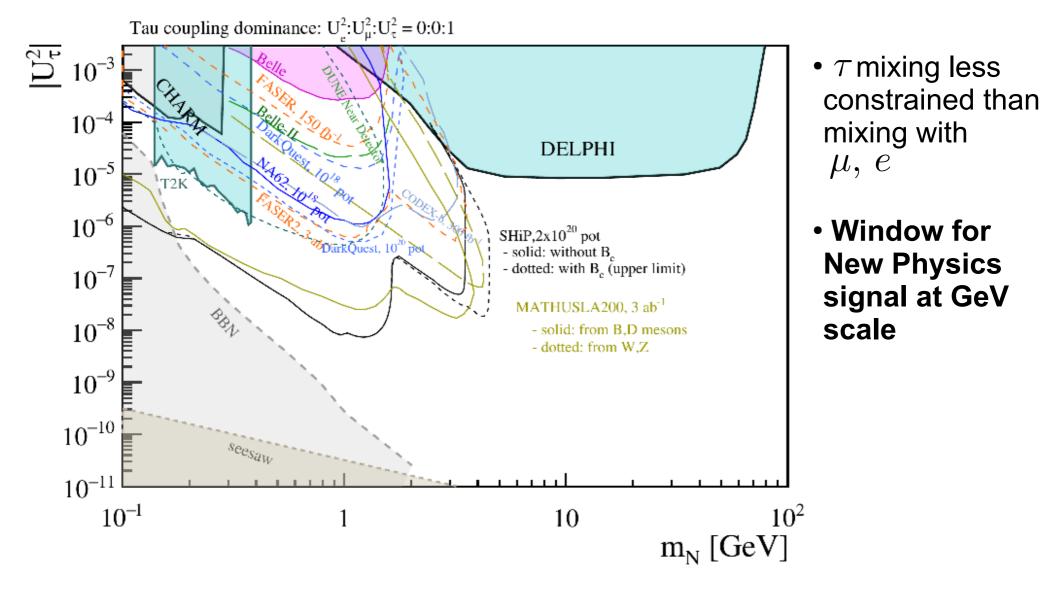
Hernandez, Kekic, JLP, Racker, Salvado 1606.06719

Conclusions

- ν_{τ} detection can play a relevant role in testing the robustness of the 3-neutrino picture (low scale Non-Unitarity, sterile neutrino oscillations, NSI...)
- Measurement of HNL tau mixing would allow to test low scale minimal neutrino mass model and to indirectly measure the PMNS phases (including Majorana phase!!) in this framework: very relevant input for testing Leptogenesis!
 - More opportunities expected in non minimal scenarios, experiments and observables not considered in this talk.
 Stay tuned for the rest of the workshop!

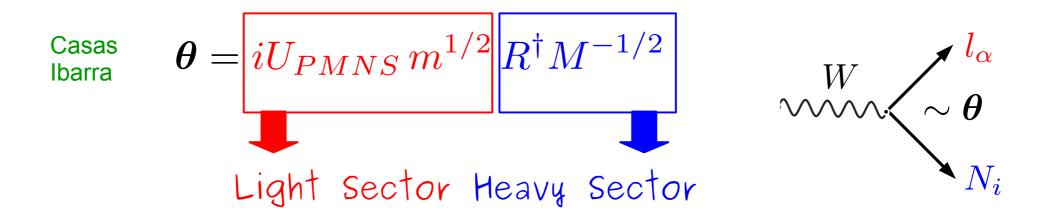
Thank you!

Direct Searches of HNLs



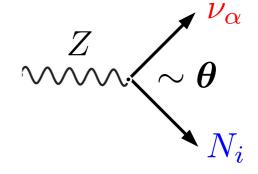
Feebly-Interacting Particles:FIPs 2020 Workshop 2102.12143

Direct searches of HNLs



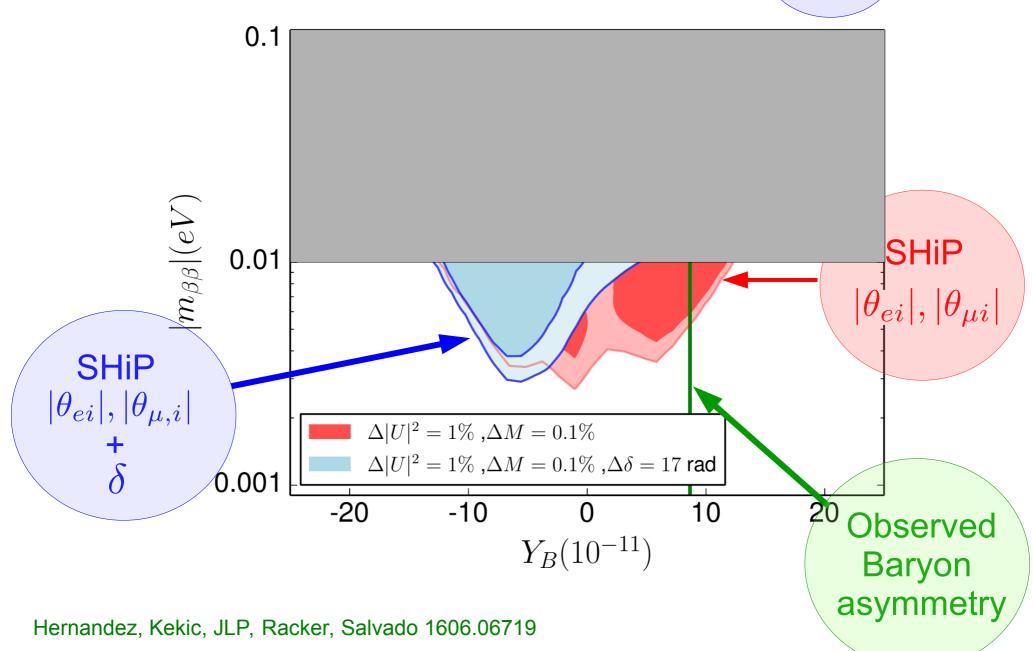
· Direct detection requires:

$$\theta \gg \sqrt{m/M} \iff R_{ij} \gg 1$$

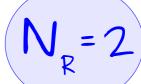


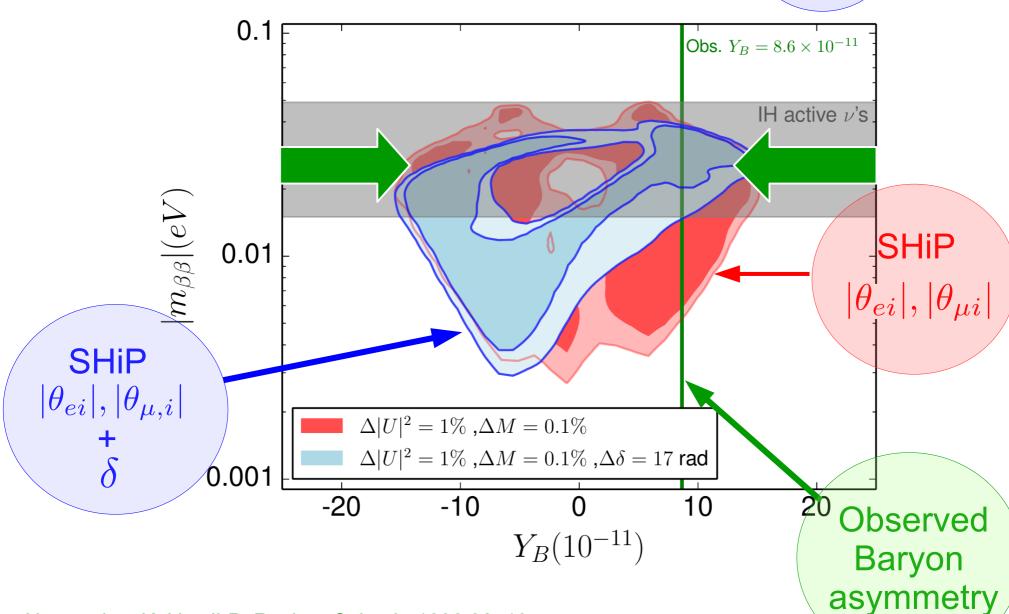
• Phenomenological constraint automatically satisfied in inverse and direct seesaw realizations based on a symmetry protected scenario.

GeV Scale Leptogeneis N_R=2



GeV Scale Leptogeneis





Hernandez, Kekic, JLP, Racker, Salvado 1606.06719

u_{τ} appearance channel

$\nu_{ au}$ detection:

- Energy threshold of τ production 3.2 GeV.
- Short lifetime of τ , indirect measurement via hadronic decays (~ 65% branching ratio).
- NC background. We have considered a sample in which 30% of the hadronic events are identified keeping 0.5% of NC background.

de Gouvêa, Kelly, Stenico, Pasquini 1904.07265

See talks by Pedro Machado & Adam Aurisano

DUNE set up

Globes files

DUNE Collaboration, arXiv:2103.04797 [hep] 8 Mar 2021.

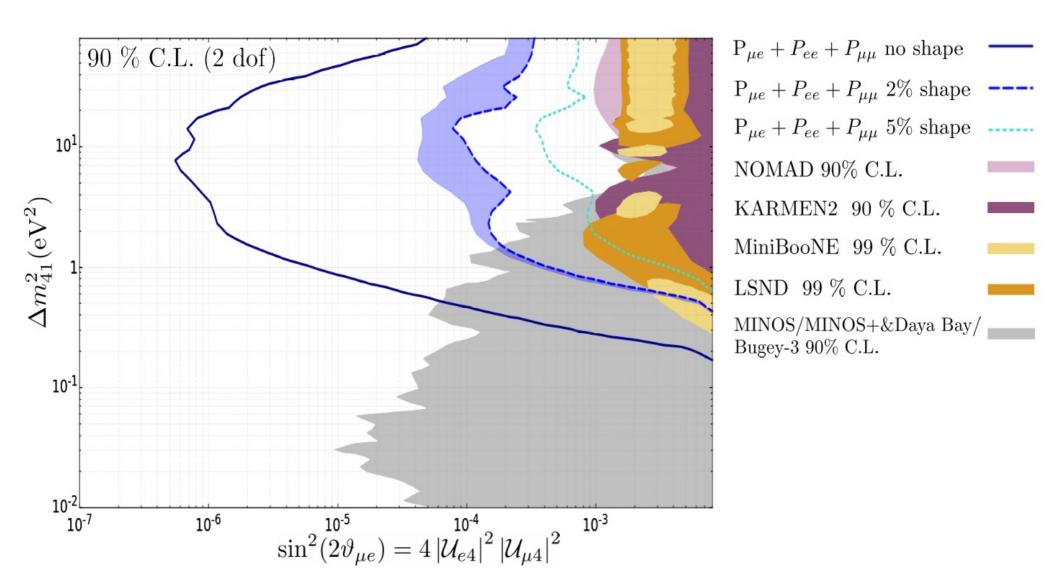
Flux configuration

Beam configuration	Power	E_p	PoT/yr	$t_{\nu} ({\rm yr})$	$t_{\bar{\nu}} \; (\mathrm{yr})$	$M_{ m det}$
Nominal	1.2 MW	120 GeV	1.1×10^{21}	3.5	3.5	67.2 tons
High-Energy	$1.2~\mathrm{MW}$	$120~{\rm GeV}$	1.1×10^{21}	3.5	_	67.2 tons

Running mode	Sample	Contribution	Event rates $(\times 10^5)$	$E_{ m obs}^{ m max} \ ({ m GeV})$	
u mode (nominal)	ν_e -like	Intrinsic cont.	20.18		
		Flavor mis-ID	4.61	7.125	
		NC	6.77		
	$ u_{\mu}$ -like	$\nu_{\mu}, \bar{\nu}_{\mu} \text{ CC } (P_{\mu\mu} = 1)$	2,235.72	7 195	
		NC	17.35	7.125	
	$ u_{ au}$ -like	$\nu_{\tau}, \bar{\nu}_{\tau} \text{ CC } (P_{\mu\tau} = 1)$	39.33	18	
		NC	3.23	10	
$ar{ u}$ mode (nominal)	$\bar{\nu}_e$ -like	Intrinsic cont.	11.18		
		Flavor mis-ID	1.07	7.125	
		NC	3.89		
	$ar{ u}_{\mu}$ -like	$\nu_{\mu}, \bar{\nu}_{\mu} \text{ CC } (P_{\mu\mu} = 1)$	1,013.42	7.125	
		NC 9.76		1.120	
	$\bar{\nu}_{\tau}$ -like	$\nu_{\tau}, \bar{\nu}_{\tau} \text{ CC } (P_{\mu\tau} = 1)$	27.75	18	
		NC	1.80	10	
$ u \mod (HE) $	$ u_e$ -like	Intrinsic cont. 38.10			
		Flavor mis-ID	12.98	18	
		NC	30.51		
	ν_{μ} -like	$\nu_{\mu}, \bar{\nu}_{\mu} \text{ CC } (P_{\mu\mu} = 1)$	5,784.30	10	
		NC 72.15		18	
	ν_{τ} -like	$\nu_{\tau}, \bar{\nu}_{\tau} \text{ CC } (P_{\mu\tau} = 1)$	259.67	1.0	
		NC	9.42	18	

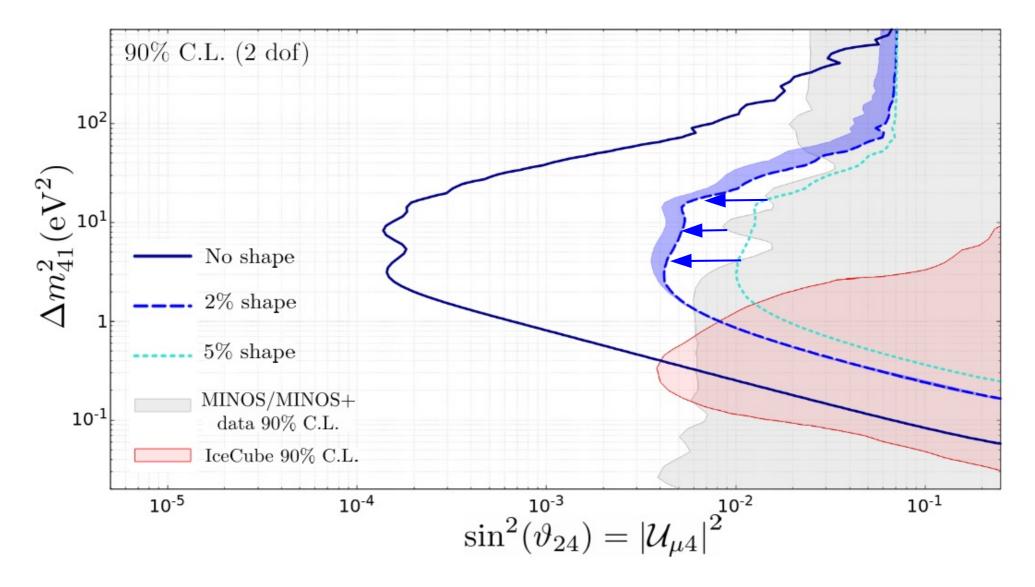
Event sample	Contribution	Benchmark 1		Benchmark 2		Benchmark 3	
Event sample	Contribution	σ_{norm}	σ_{shape}	σ_{norm}	σ_{shape}	σ_{norm}	σ_{shape}
$ u_e$ -like	Signal	5%	_	5%	_	5%	_
	Intrinsic cont.	10%	_	10%	2%	10%	5%
	Flavor mis-ID	5%		5%	2%	5%	5%
	NC	10%		10%	2%	10%	5%
$ u_{\mu}$ -like	$\nu_{\mu}, \bar{\nu}_{\mu}$ CC (signal)	10%		10%	2%	10%	5%
	NC	10%		10%	2%	10%	5%
$\nu_{ au}$ -like	Signal	20%	_	20%	_	20%	_
	NC	10%	_	10%	2%	10%	5%

3+1 Sterile Neutrinos: $P_{\mu\mu} + P_{\mu e} + P_{ee}$



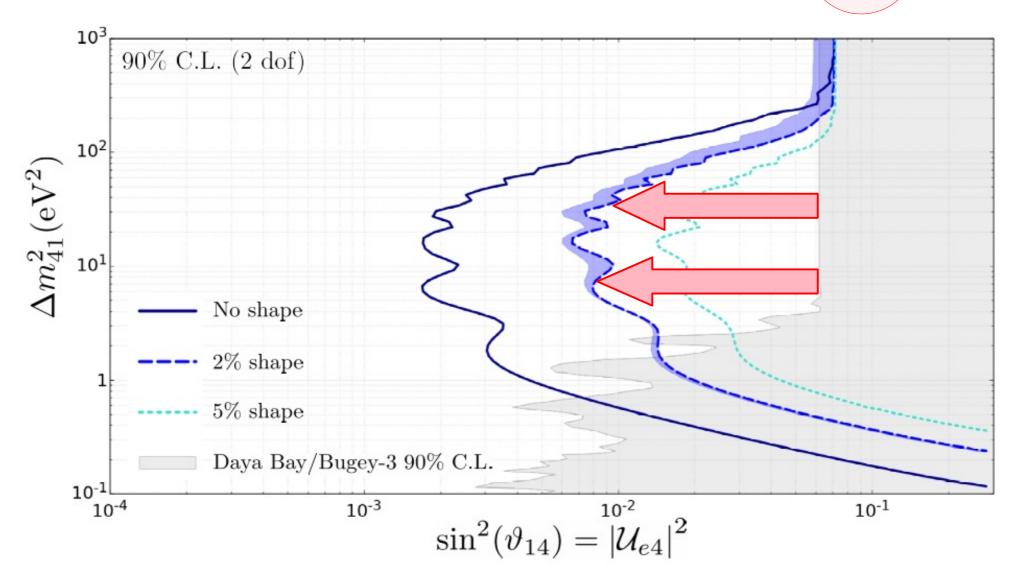
3+1 Sterile Neutrinos: $P_{\mu\mu}$



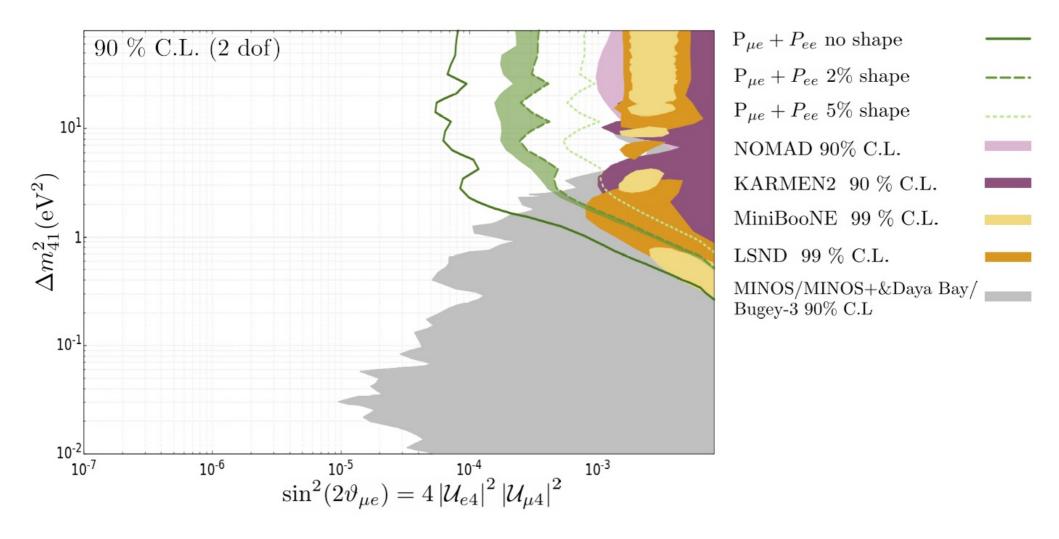


3+1 Sterile Neutrinos: Pee





3+1 Sterile Neutrinos: $P_{\mu e} + P_{ee}$



PMNS CP-phases from HNLs searches

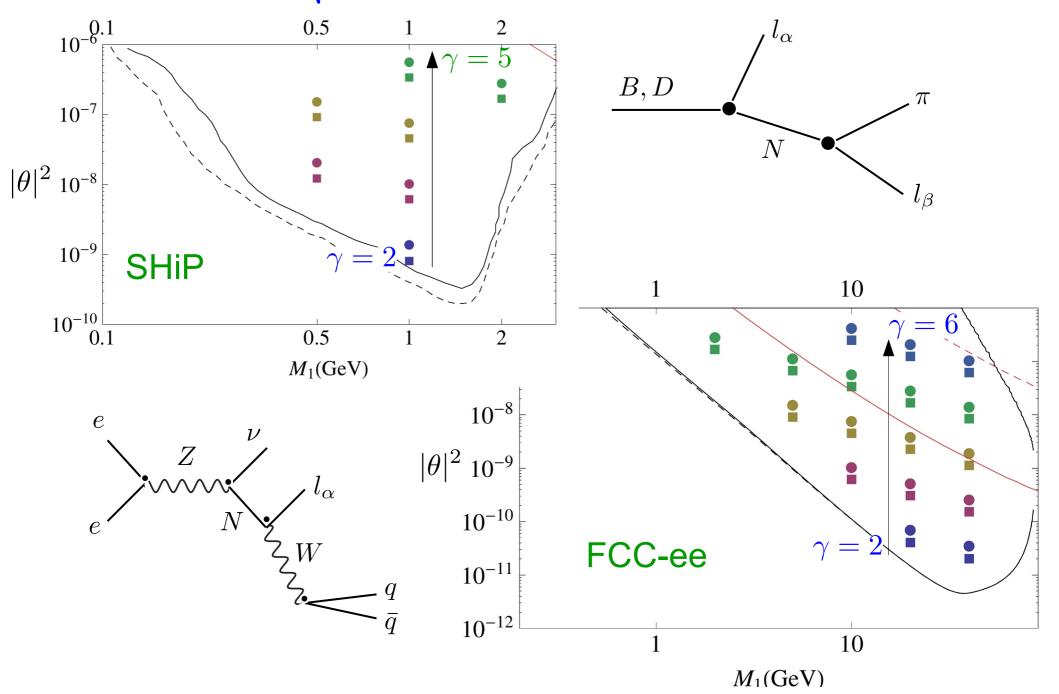
• For instance, SHiP and FCC-ee can measure HNLs parameters:

$$M_1, M_2, |\theta_{ei}|, |\theta_{\mu i}| \qquad \text{Sensitivity to} \\ \text{PMNS CP-phases!} \\ \bullet |\theta_{e1}|^2/|\theta_{\mu 1}|^2 \simeq |\theta_{e2}|^2/|\theta_{\mu 2}|^2 \simeq \\ \hline (1+s_{\phi_1}\sin 2\theta_{12})(1-\theta_{13}^2) + \frac{1}{2}r^2s_{12}(c_{12}s_{\phi_1}+s_{12}) \\ \hline (1-\sin 2\theta_{12}s_{\phi_1}\left(1+\frac{r^2}{4}\right) + \frac{r^2c_{12}^2}{2}\right)c_{23}^2 + \theta_{13}(c_{\phi_1}s_{\delta} - \cos 2\theta_{12}s_{\phi_1}c_{\delta})\sin 2\theta_{23} + \theta_{13}^2(1+\sin 2\theta_{12})s_{23}^2s_{\phi_1}}$$

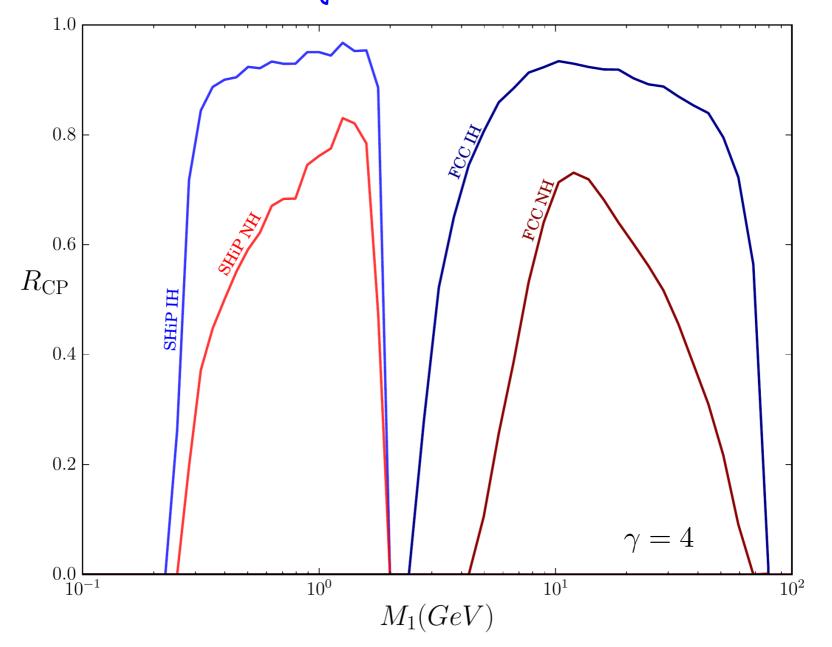
•
$$|\theta_{ei}|^2, |\theta_{\mu i}|^2 \propto e^{2\gamma}$$
 parametrizes size of R_{ij} (and thus heavy mixing)



PMNS CP-phases from HNLs searches



5σ discovery PMNS CP-violation



Caputo, Hernandez, Kekic, JLP, Salvado arXiv:1611.05000

Systematics: Disappearance

$$N_{\nu_{\alpha} \to \nu_{\beta}} \sim \frac{\Phi_{\alpha}(E)}{L^2} P_{\alpha\beta}(L/E) \sigma_{\beta}(E) \epsilon_{\beta}(E)$$

• Using near detectors is a very effective way of reducing systematics in disappearance experiments (K2K, MINOS, reactors...).

$$\frac{N_{\nu_{\alpha}}^{\mathrm{FD}}}{N_{\nu_{\alpha}}^{\mathrm{ND}}} \sim \frac{L_{\mathrm{ND}}^{2}}{L_{\mathrm{FD}}^{2}} \frac{\Phi_{\alpha} \sigma_{\alpha} \epsilon_{\alpha}}{\Phi_{\alpha} \sigma_{\alpha} \epsilon_{\alpha}} P_{\alpha \alpha}$$

Systematics: Appearance (CP violation)

$$N_{\nu_{\alpha} \to \nu_{\beta}} \sim \frac{\Phi_{\alpha}(E)}{L^2} P_{\alpha\beta}(L/E) \sigma_{\beta}(E) \epsilon_{\beta}(E)$$

• For appearance experiments the situation is more complicated

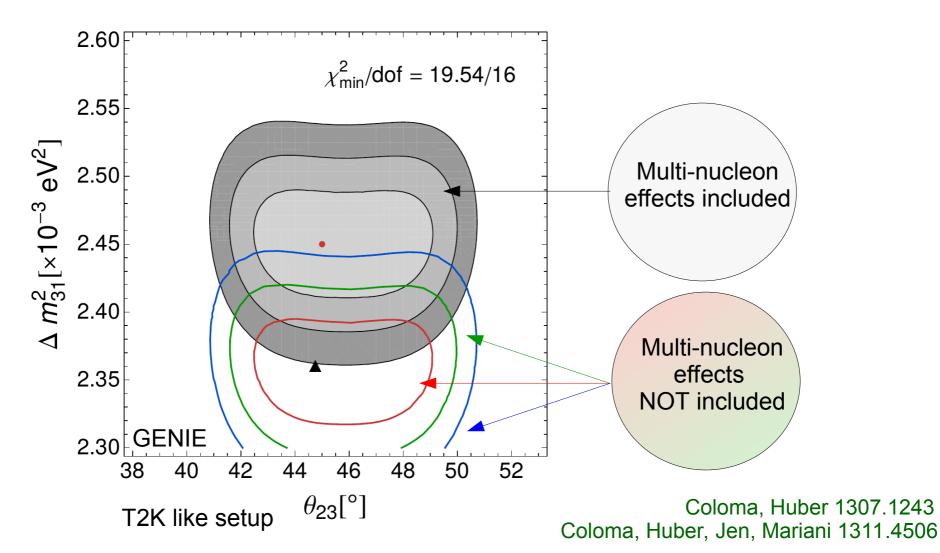
$$rac{N_{
u_e}^{
m FD}}{N_{
u_\mu}^{
m ND}} \sim rac{L_{
m ND}^2}{L_{
m FD}^2} rac{\sigma_e \epsilon_e}{\sigma_\mu \epsilon_\mu} P_{\mu e}$$

• CP violation requires comparison between neutrino and anti-netrino signals.

$$\frac{N_{\nu_e}^{\rm Far}}{N_{\bar{\nu}_e}^{\rm Far}} \sim \frac{N_{\nu_{\mu}}^{\rm ND}}{N_{\bar{\nu}_{\mu}}^{\rm ND}} \frac{\sigma_e \epsilon_e}{\sigma_{\mu} \epsilon_{\mu}} \frac{\sigma_{\bar{\mu}} \epsilon_{\bar{\mu}}}{\sigma_{\bar{e}} \epsilon_{\bar{e}}} \frac{P_{\mu e}}{P_{\bar{\mu}\bar{e}}}$$

Nuclear Cross sections

 Neutrino-nucleus cross section missmodeling could lead to unacceptably large systematic uncertainties or biased measurements, even after the inclusion of a near detector.



. . .

$$\begin{split} \chi^2_{\min}(\{\Theta\}) &= \min_{\{\xi,\zeta\}} \left[\chi^2_{\text{stat}}(\{\Theta,\xi,\zeta\}) + \sum_s \left(\frac{\zeta_s}{\sigma_{\text{norm},s}} \right)^2 + \sum_b \left(\frac{\zeta_b}{\sigma_{\text{norm},b}} \right)^2 \right. \\ &\left. + \sum_i \left(\frac{\xi_i^{\text{sig}}}{\sigma_{\text{shape,sig}}} \right)^2 + \sum_i \left(\frac{\xi_i^{\text{bg}}}{\sigma_{\text{shape,bg}}} \right)^2 \right] \,, \end{split}$$

$$\chi_{\text{stat}}^{2}(\{\Theta, \xi, \zeta\}) = \sum_{i} 2\left(N_{i}(\{\Theta, \xi, \zeta\}) - O_{i} + O_{i} \ln \frac{O_{i}}{N_{i}(\{\Theta, \xi, \zeta\})}\right)$$

$$N_i(\{\Theta, \xi, \zeta\}) = \sum_s (1 + \xi_i^{\text{sig}} + \zeta_s) \, s_i(\{\Theta\}) + \sum_b (1 + \xi_i^{\text{bg}} + \zeta_b) \, b_i(\{\Theta\})$$