
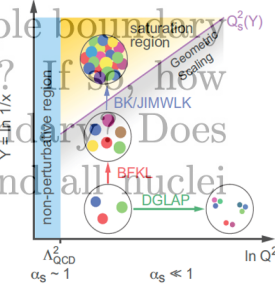
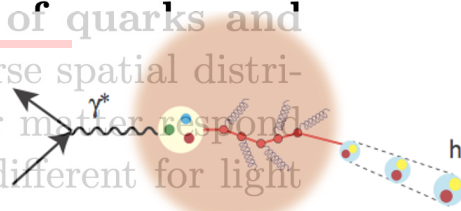


Introduction of Jet for 3D Imaging & Anisotropy in Jet Production

Feng Yuan

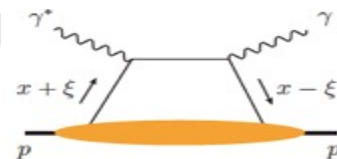
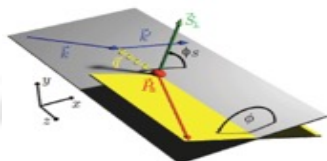
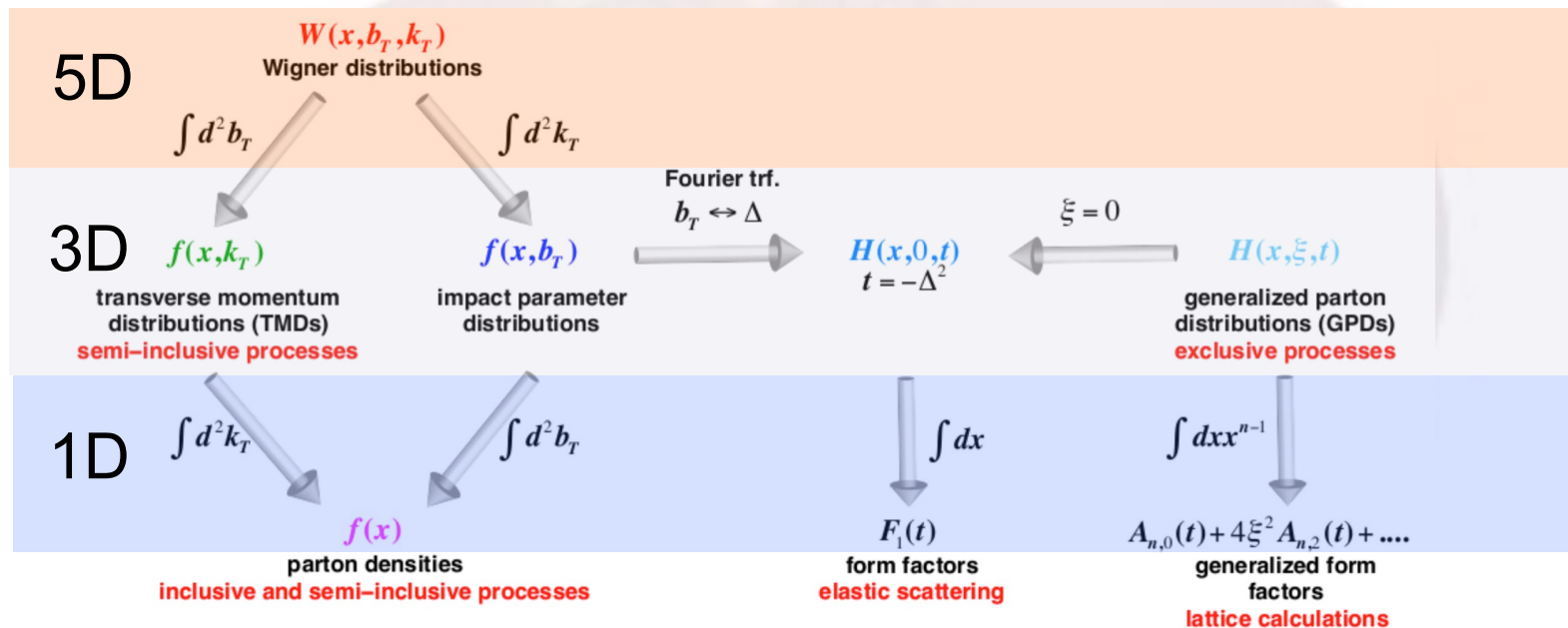
Lawrence Berkeley National Laboratory

Big questions for EIC

- How are the sea quarks and gluons, and their spins, distributed in space and momentum inside the nucleon? How are these quark and gluon distributions correlated with overall nucleon properties, such as spin direction? What is the role of the orbital motion of sea quarks and gluons in building the nucleon spin? 
- Where does the saturation of gluon densities set in? Is there a simple boundary that separates this region from that of more dilute quark-gluon matter? If so, how do the distributions of quarks and gluons change as one crosses the boundary? Does this saturation produce matter of universal properties in the nucleon and all nuclei viewed at nearly the speed of light? 
- How does the nuclear environment affect the distribution of quarks and gluons and their interactions in nuclei? How does the transverse spatial distribution of gluons compare to that in the nucleon? How does nuclear matter respond to a fast moving color charge passing through it? Is this response different for light and heavy quarks? 

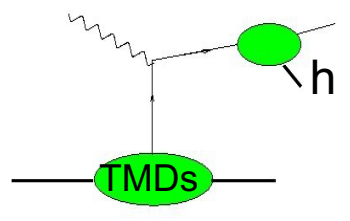
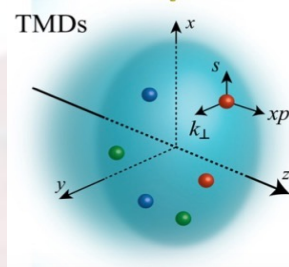
Unified view of the Nucleon

□ Wigner distributions (Belitsky, Ji, Yuan)

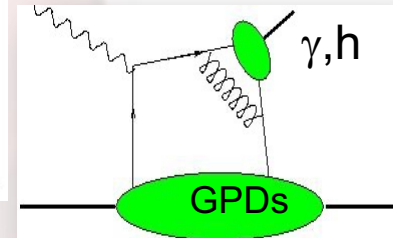
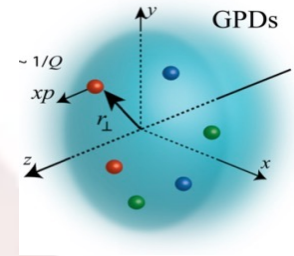


Zoo of TMDs & GPDs

	U	L	T
U	f_1		h_1^\perp
L		g_{1L}	h_{1L}^\perp
T	f_{1T}^\perp	g_{1T}	h_1, h_{1T}^\perp



	U	L	T
U	H		\mathcal{E}_T
L		\tilde{H}	
T	E		H_T, \tilde{H}_T



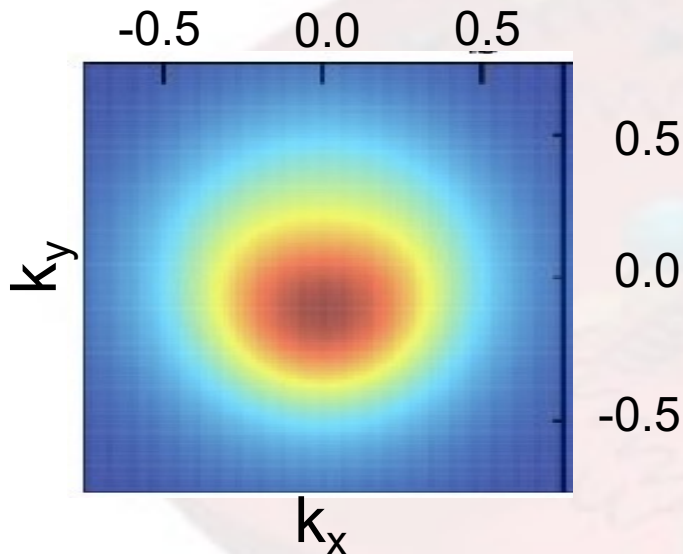
- NOT directly accessible
- Their extractions require measurements of x-sections and asymmetries in a large kinematic domain of x_B , t , Q^2 (GPD) and x_B , P_T , Q^2 , z (TMD)

What can we learn

- 3D Imaging of partons inside the nucleon (non-trivial correlations)
 - Try to answer more detailed questions as Rutherford was doing for atomic matter more than 100 years ago
- QCD dynamics involved in these processes
 - Transverse momentum distributions: universality, factorization, evolutions,...
 - Small-x resummation: BFKL and Sudakov

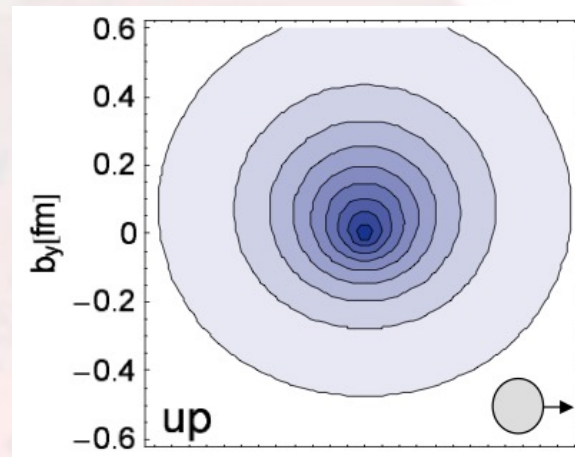
Deformation when nucleon is transversely polarized

TMDs



Quark Sivers function fit to the SIDIS Data, Anselmino, et al. 2009

GPDs



Lattice Calculation of the transverse density of Up quark, QCDSF/UKQCD Coll., 2006

See also, Guo-Ji-Shiells, 2101.05243

Parton's orbital motion through the Wigner Distributions

Phase space distribution:

Projection onto $p(x)$ to get the momentum (probability) density

Quark orbital angular momentum

$$L(x) = \int (\vec{b}_\perp \times \vec{k}_\perp) W(x, \vec{b}_\perp, \vec{k}_\perp) d^2\vec{b}_\perp d^2\vec{k}_\perp$$

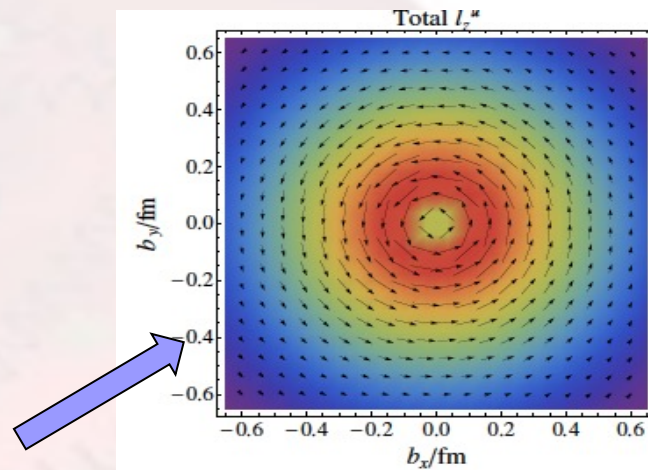
Well defined in QCD:

Ji, Xiong, Yuan, PRL, 2012; PRD, 2013

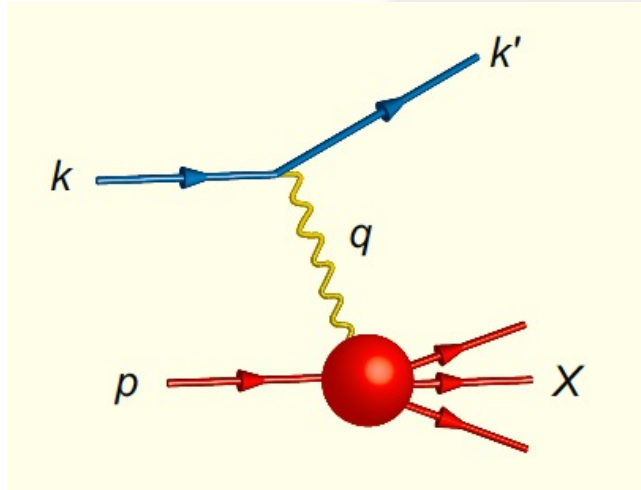
Lorce, Pasquini, Xiong, Yuan, PRD, 2012

Lorce-Pasquini 2011

Hatta 2011



Basics



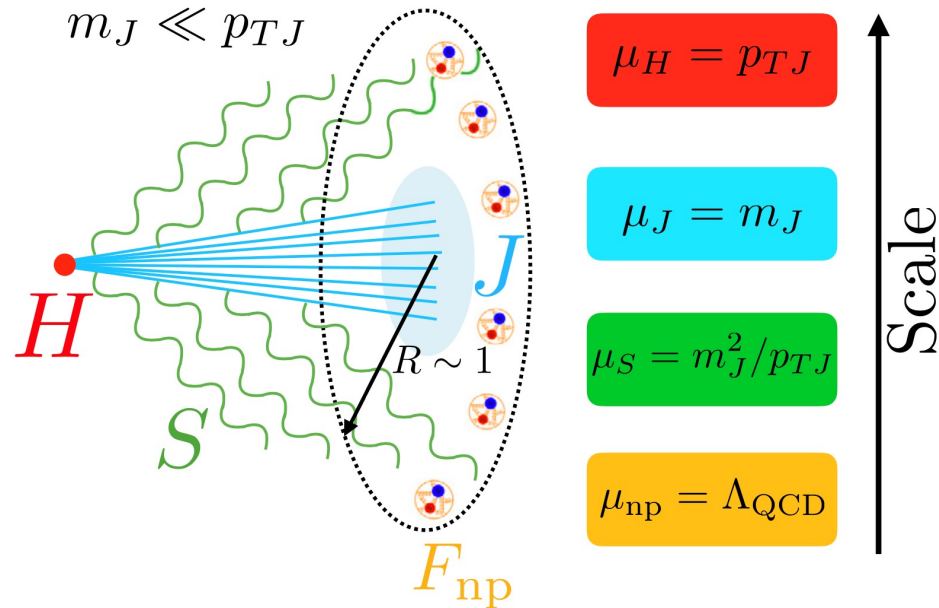
- Inclusive DIS
 - Parton distributions
- Semi-inclusive DIS, measure additional hadron in final state
 - K_t -dependence
- Exclusive Processes, measure recoiled nucleon
 - Nucleon tomography
- Parity violating process

Luminosity requirement

Tremendous theory advance of jet physics at colliders in recent years

A.J. Larkoski, I. Moult and B. Nachman / Physics Reports 841 (2020) 1–63

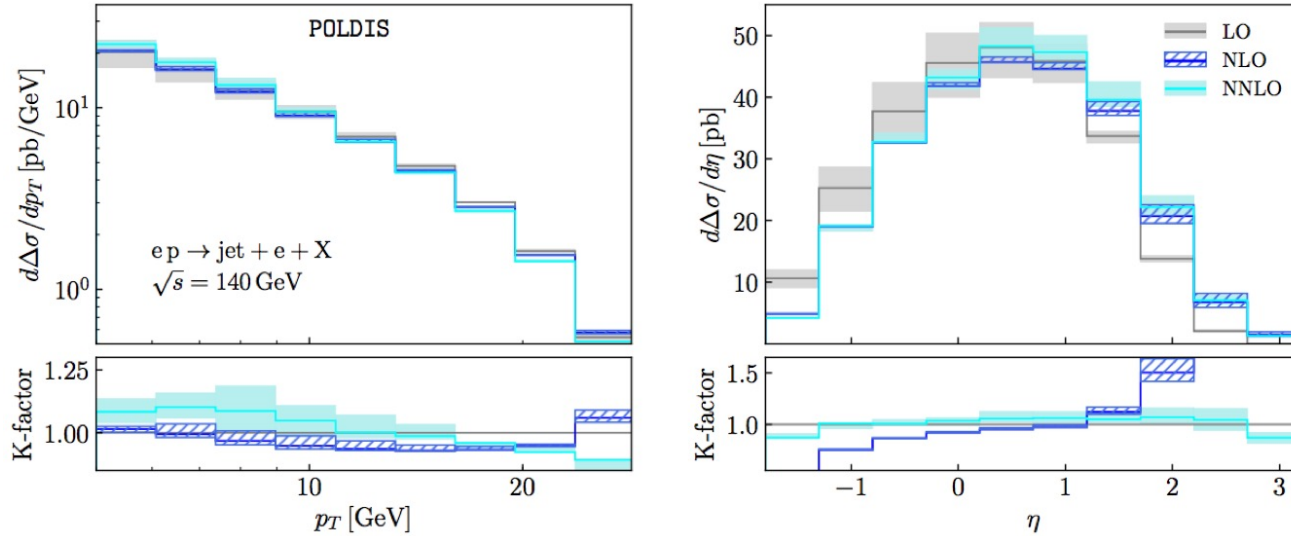
e.g.



Jet @EIC has been very active in recent years

- Contribute to explore
 - Spin/tomography of nucleon
 - Small-x gluon saturation
 - Hard probe interaction with cold nuclei matter
- QCD dynamics in precision study
 - E.g., jet substructure to measure α_s , jet algorithms, jet angularity, hadronization, etc.
- Observables:
 - Leading jet/hadron, dijet/dihadron, jet substructure, ...

Inclusive jet: state of art



Will contribute
to a global fit of
parton helicity
distributions in
the EIC-era

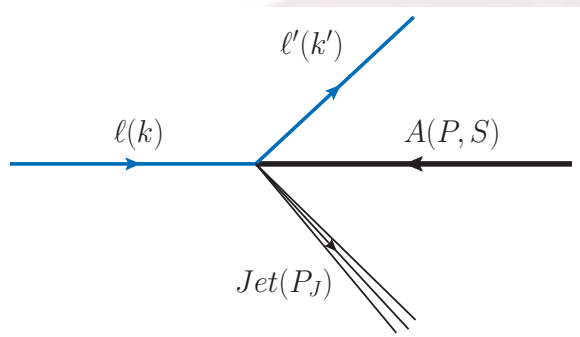
Benchmark measurements:

Borsa-de Florian-Pedron, PRL 2020, 2005.10705, 2010.07354

See also: Hinderer, Schlegel, Vogelsang, 1703.10872; Boughezal, Petriello,

Xing, 1704.05457, 1806.07311; Page, Chu, Aschenauer, 1911.00657

Semi-inclusive processes: lepton-jet correlation



Quark distribution \otimes soft factor

$$\frac{d^5 \sigma(\ell p \rightarrow \ell' J)}{dy_\ell d^2 k_{\ell\perp} d^2 q_\perp} = \sigma_0 \int d^2 k_\perp d^2 \lambda_\perp x f_q(x, k_\perp, \zeta_c, \mu_F) \\ \times H_{\text{TMD}}(Q, \mu_F) S_J(\lambda_\perp, \mu_F) \delta^{(2)}(q_\perp - k_\perp - \lambda_\perp) .$$

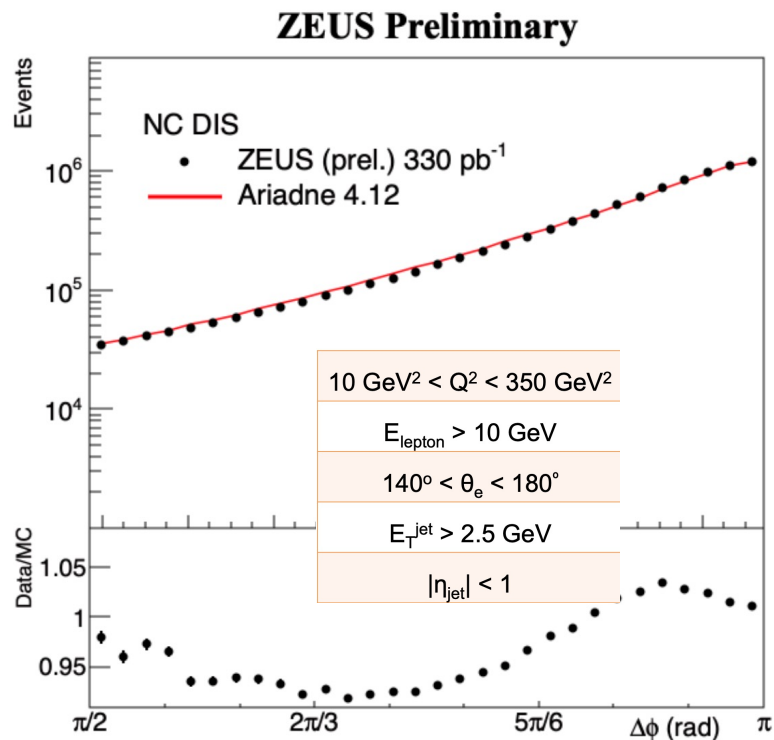
Liu-Ringer-Vogelsang-Yuan 1812.08077, 2007.12866

(Lab frame)

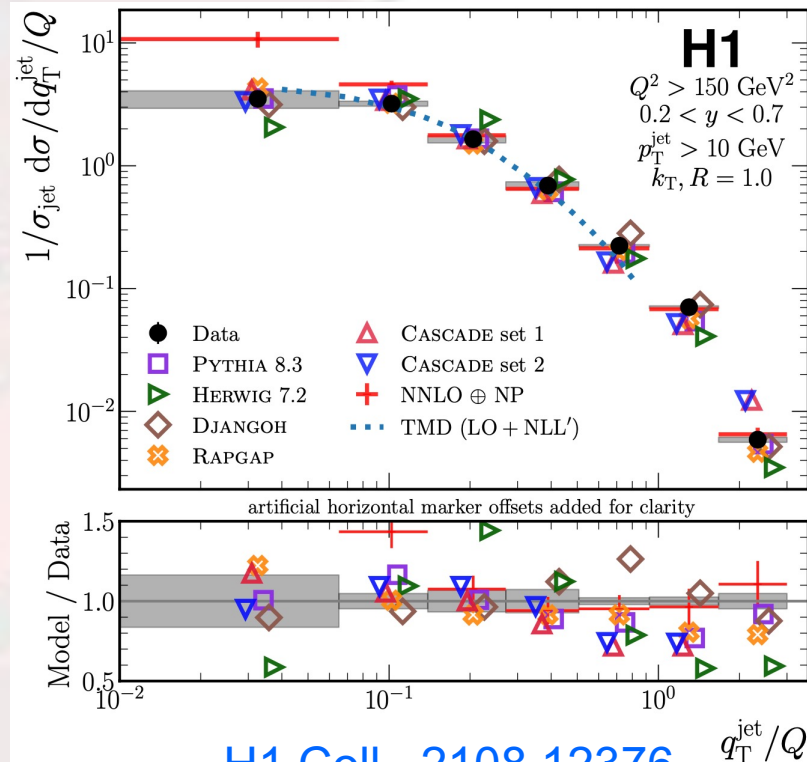
Total transverse momentum of the lepton+jet probes
the TMD quark distribution

See also, Gutierrez-Reyes, Scimemi, Waalewijn, Zoppi,
1807.07573, 1904.04259

Re-analysis of HERA Data

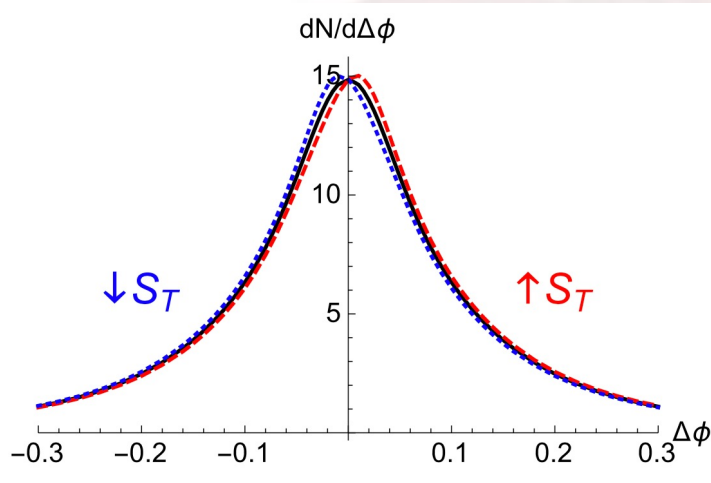


Amilkar Quintero, 2019
EIC-meeting, Paris 9/26/21

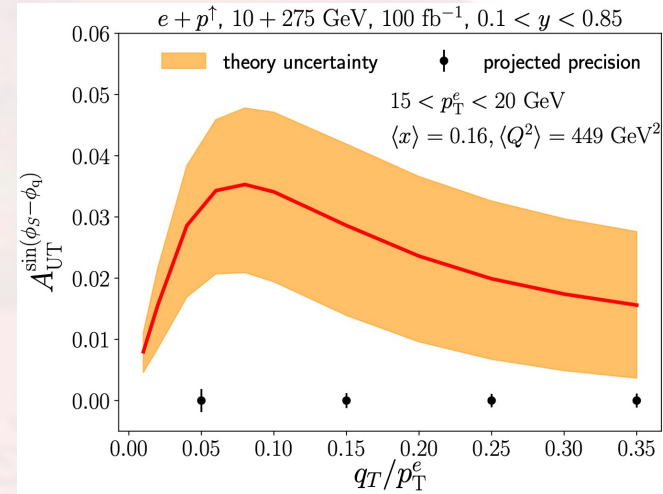


H1 Coll., 2108.12376
M. Arratia, DIS 2021

Complementary and unique perspective for Sivers asymmetries

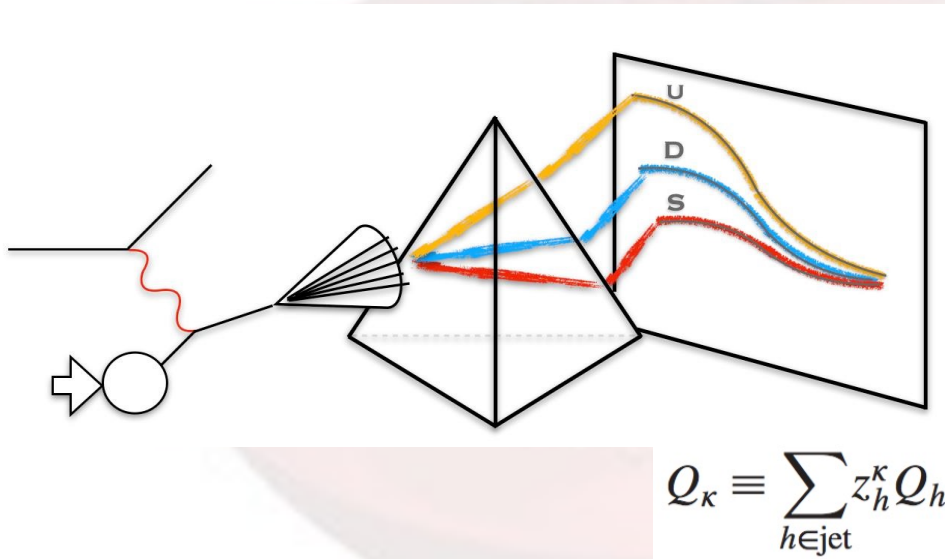


Originally proposed by Boer, Voegelsang,
for pp collisions in 2003

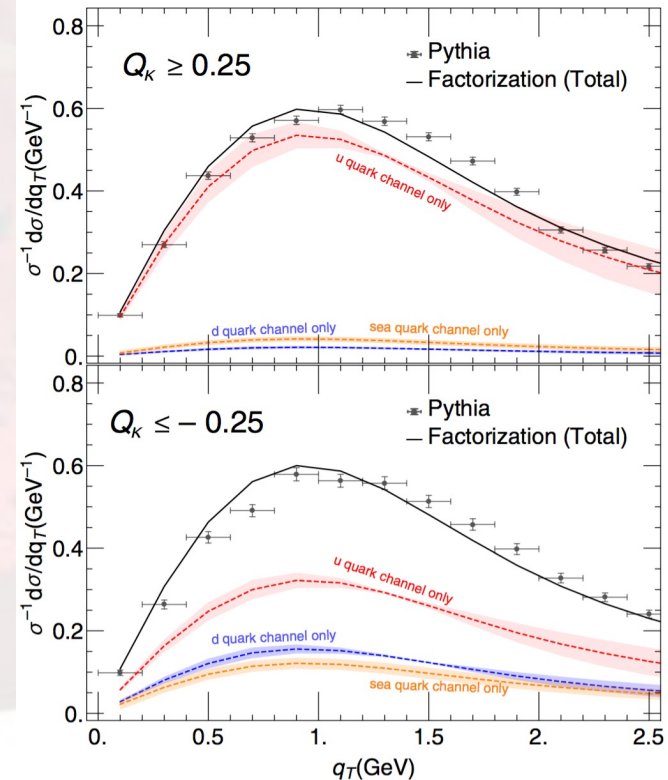


Arratia, Kang, Prokudin, Ringer, 2007.07281

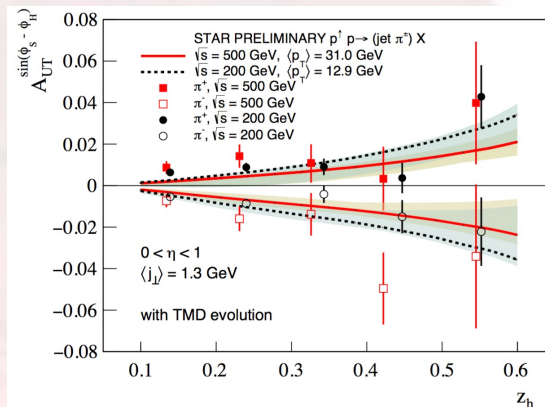
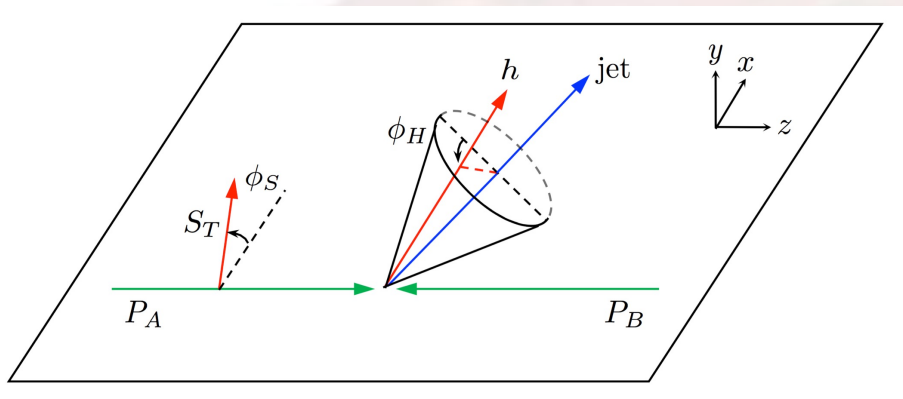
Jet Charge: A Flavor Prism for Spin Asymmetries at the Electron-Ion Collider



Kang, Liu, Mantry, Shao, PRL 2020, 2008.00655



Jet can be utilized to measure the hadron fragmentation as well

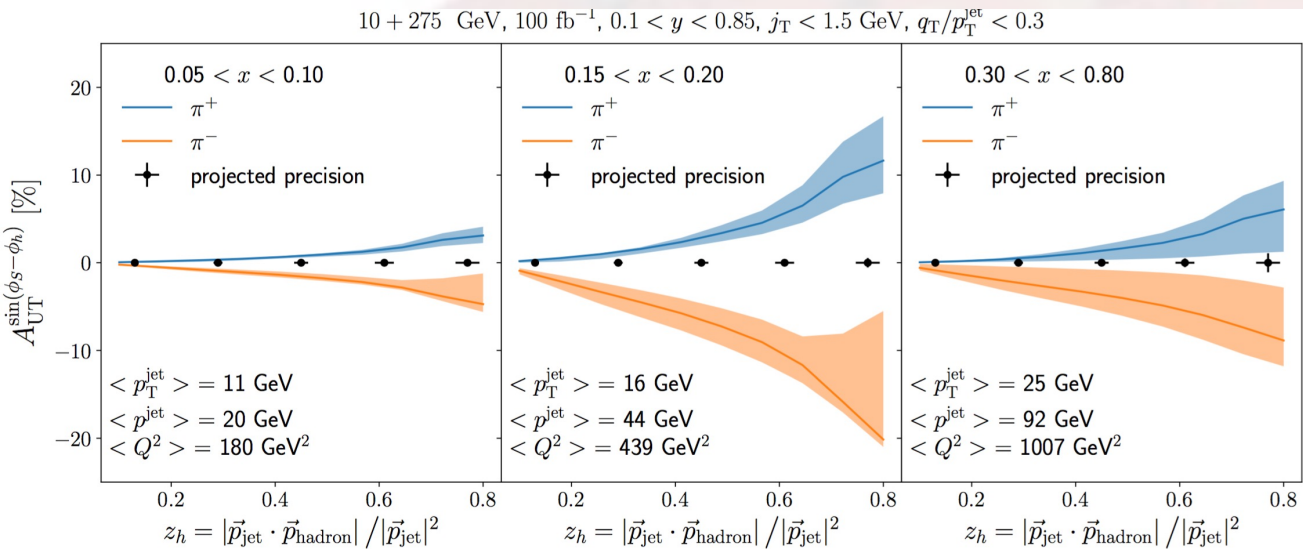


STAR coll.:
1708.07080

Kang, Prokudin, Ringer, Yuan, 1707.00913

earlier: Yuan, 0709.3272; D'Alesio, Murgia, Pisano, 1011.2692

TMD fragmentation in jet: Collins asymmetries at EIC



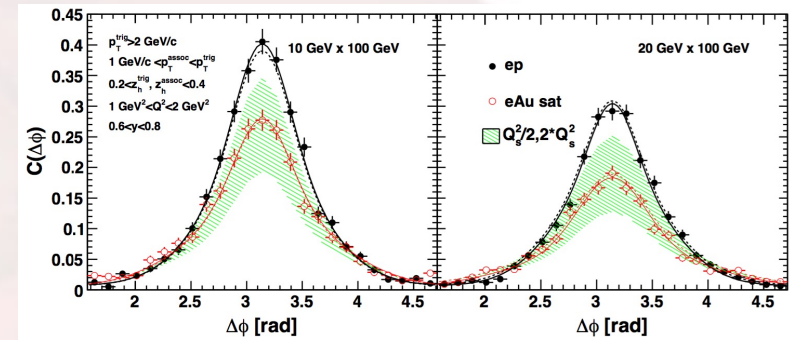
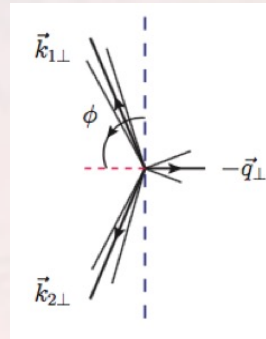
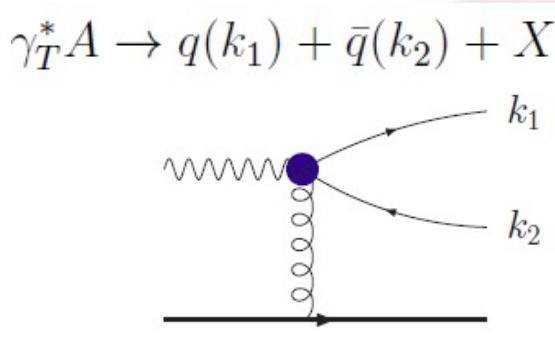
Factorization involves jet axis definition, applying jet thrust as an alternate approach:

- Kang, Shao, Zhao, 2007.14425
- Boglione, Simonelli, 2011.07366
- Will be studied by Belle II experiments

Arratia, Kang, Prokudin, Ringer, 2007.07281

See also, polarized jet fragmentation functions: Kang, Lee, Zhao, 2005.02398

Semi-inclusive process: DIS dijet probes gluon TMDs

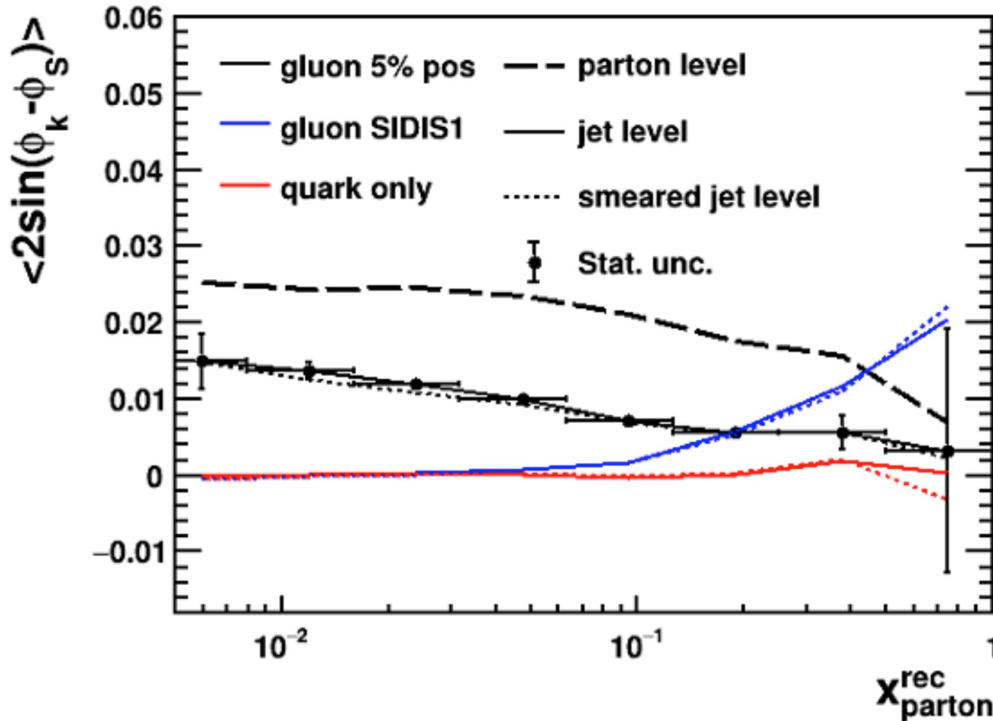


Zheng, et al., 1403.2413

- q_t -dependence measure the gluon distribution
 - Weizsacker-Williams gluon distribution in nucleus (CGC predictions)
- Various channels at the EIC: heavy flavor production, real and virtual photon

Dominguez-Marquet-Xiao-Yuan 2011

Gluon Sivers function at EIC

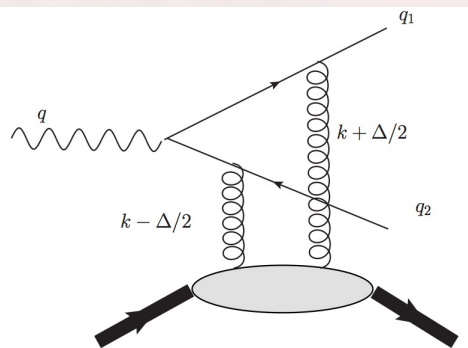
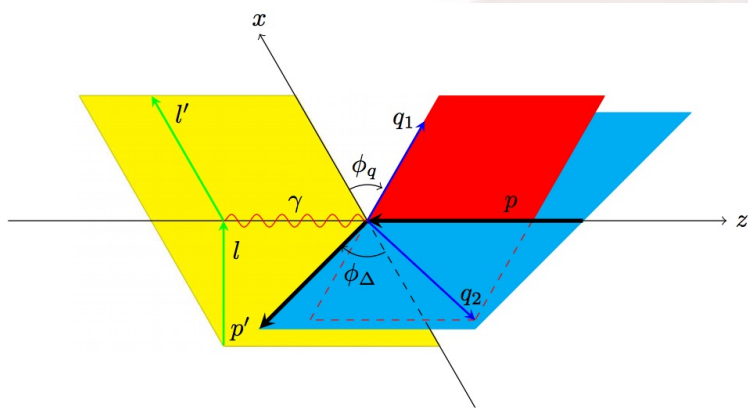


Kinematics:

- 18X275GeV, $Q > 1\text{ GeV}$
Leading jet $> 4.5\text{ GeV}$,
subleading jet $> 4\text{ GeV}$,
 $R = 0.5$, $|\eta| < 2.5$

L. Zheng's talk at the jet
workshop, see also, 1805.05290
See also, Charm jet as a probe:
Arrington et al, 2102.08337

Diffraction Dijet: Probe the Gluon Orbital Angular Momentum



- The longitudinal spin asymmetry depends on the gluon OAM distribution
- More quantitative studies needed to show the impact from EIC measurements

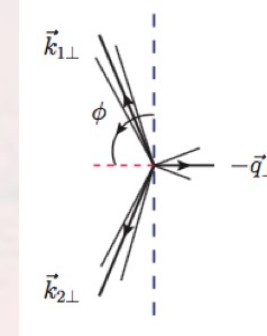
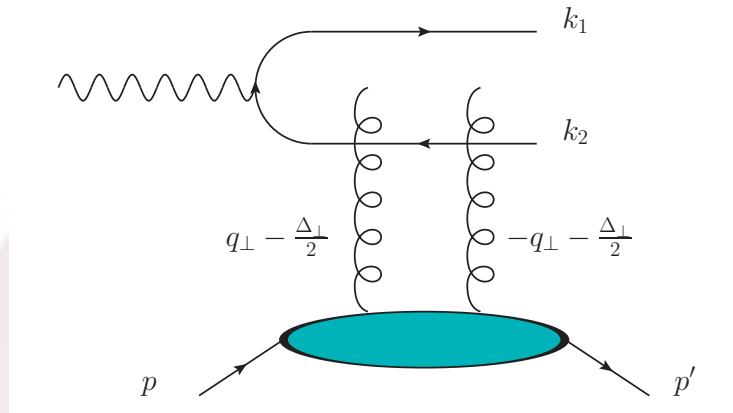
$$A_{\sin(\phi_q - \phi_\Delta)} \propto \frac{(\bar{z} - z) |\vec{q}_\perp| |\vec{\Delta}_\perp|}{\vec{q}_\perp^2 + \mu^2} \mathcal{L}_g(\xi, t)$$

Ji, Yuan, Zhao, arXiv:1612.02438

Hatta, Nakagawa, Yuan, Zhao, arXiv:1612.02445

Nucleon/Nucleus Tomography at Small-x

Hatta-Xiao-Yuan, 1601.01585



$\cos(2\phi)$
anisotropy

- In the Breit frame, by measuring the recoil of final state proton, one can access Δ_T . By measuring jets momenta, one can approximately access q_T .
- The diffractive dijet cross section is proportional to the square of the Wigner distribution \rightarrow nucleon/nucleus tomography

$$x\mathcal{W}_g^T(x, |\vec{q}_\perp|, |\vec{b}_\perp|) + 2 \cos(2\phi) x\mathcal{W}_g^\epsilon(x, |\vec{q}_\perp|, |\vec{b}_\perp|)$$

\rightarrow **Anisotropy \sim few %**

Azimuthal Angular Asymmetries from the Soft Gluon Radiation Associated with Jet

Reference: Hatta, Xiao, Yuan, Zhou, arXiv: 2010.10774; arXiv: 2106.05307

Soft gluon radiation leads to Sudakov Logarithms

Sudakov, 1956; Collins-Soper-Sterman 1985

- Differential cross section depends on $Q_1=q_T$, where $Q^2 \gg Q_1^2 \gg \Lambda_{\text{QCD}}^2$

$$\frac{d\sigma}{dQ_1^2} = \frac{1}{Q_1^2} f_1 \otimes f_2 \otimes \sum_i \alpha_s^i \ln^{2i-1} \frac{Q^2}{Q_1^2} + \dots$$

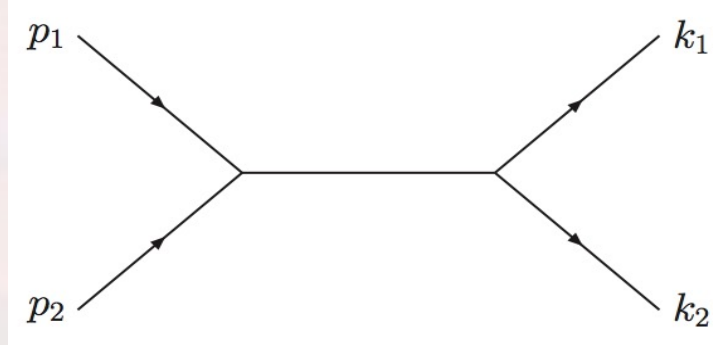
- Resummation of these large logs
 - In terms of transverse momentum dependent parton distributions and fragmentation functions and apply to
 - Semi-inclusive hadron production in DIS, Drell-Yan type of hard processes in pp collisions, e.g., Higgs, Z/W boson, ...

Hard process with jet is different

- Final states carry color
 - Soft gluon radiation associated with the jet will contribute
- Jet algorithm will enter into the calculations as well
 - Only out of cone radiation contributes to the imbalance between the two jets

Leading double logs in dijet case

- Power counting: each **incoming** parton contributes to a half of the associated color factor



DL coefficient:
 $A^{(1)} = (C_{p_1} + C_{p_2})/2$

Banfi-Dasgupta-Delenda, PLB 2008
Mueller-Xiao-Yuan, PRD 2013

Beyond the leading double logs

- Jet size-dependence is computed by averaging the azimuthal angle between the soft gluon and leading jet
- Matrix form due to colored final state [Kidonakis-Sterman 1997](#)

$$x_1 f_a(x_1, \mu = b_0/b_\perp) x_2 f_b(x_2, \mu = b_0/b_\perp) e^{-S_{\text{Sud}}(Q^2, b_\perp)} \\ \text{Tr} \left[\mathbf{H}_{ab \rightarrow cd} \exp \left[- \int_{b_0/b_\perp}^Q \frac{d\mu}{\mu} \gamma^{s\dagger} \right] \mathbf{S}_{ab \rightarrow cd} \exp \left[- \int_{b_0/b_\perp}^Q \frac{d\mu}{\mu} \gamma^s \right] \right]$$

([Sun, C.-P. Yuan, F. Yuan, PRL 2014](#))

$$S_{\text{Sud}}(Q^2, b_\perp) = \int_{b_0^2/b_\perp^2}^{Q^2} \frac{d\mu^2}{\mu^2} \left[\ln \left(\frac{Q^2}{\mu^2} \right) A + B + D_1 \ln \frac{Q^2}{P_T^2 R_1^2} + D_2 \ln \frac{Q^2}{P_T^2 R_2^2} \right]$$

D: color-factor for the jet

R: jet size

see also, heavy quark pair resummation:
[Zhu-Li-Li-Shao-Yang 2012](#)
[Catani-Grazzini-Torre 2014](#)

Azimuthal angular asymmetries

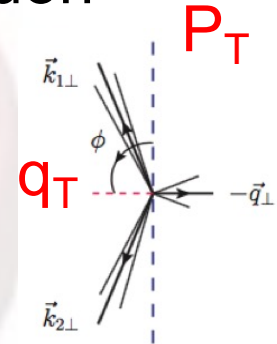
Catani-Grazzini-Sargsyan 2017

- Azimuthal angular asymmetries arise from soft gluon radiations

- ϕ is defined as angle between total and different transverse momenta of the two final state particles

- Infrared safe but divergent

- $\langle \cos(\phi) \rangle$, $\langle \cos(2\phi) \rangle$, ... divergent, $\sim 1/q_T^2$
 - Integral is finite for small q_T -cutoff, resummation can be carried out for the harmonics, $\langle \cos(n\phi) \rangle \sim q_T^n$
 - Examples discussed include Vj, top quark pair production



Azimuthal angular correlations in jet production processes

Hatta, Xiao, Yuan, Zhou, arXiv: 2010.10774; 2106.05307

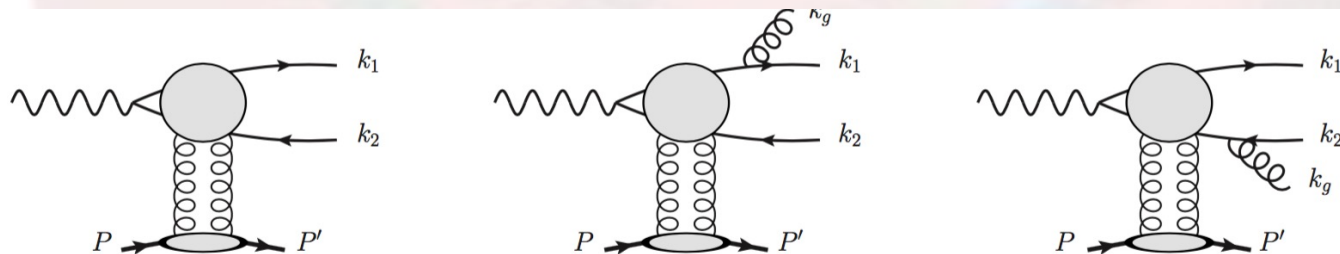
- Diffractive photoproduction of dijet
- Lepton plus jet production at the EIC
- Inclusive dijet in DIS

1. Diffractive dijet production

- Gluon radiation tends to be aligned with the jet direction

$$S_J(q_\perp) = \delta(q_\perp) + \frac{\alpha_s}{2\pi^2} \int dy_g \left(\frac{k_1 \cdot k_2}{k_1 \cdot k_g k_2 \cdot k_g} \right)_{\vec{q}_\perp = -\vec{k}_{g\perp}}$$

$$S_{J0}(|q_\perp|) + 2 \cos(2\phi) S_{J2}(|q_\perp|) + \dots$$



[Hatta-Xiao-Yuan-Zhou, 2010.10774](#)

anisotropy was neglected in an earlier paper:
[Hatta-Mueller-Ueda-Yuan, 1907.09491](#)

Leading power contributions, explicit result at α_s

$$S_J(q_\perp) = S_{J0}(|q_\perp|) + 2 \cos(2\phi) S_{J2}(|q_\perp|)$$

$$S_{J0}(q_\perp) = \delta(q_\perp) + \frac{\alpha_0}{\pi} \frac{1}{q_\perp^2}, \quad S_{J2}(q_\perp) = \frac{\alpha_2}{\pi} \frac{1}{q_\perp^2},$$

where

$$\alpha_0 = \frac{\alpha_s C_F}{2\pi} 2 \ln \frac{a_0}{R^2}, \quad \alpha_2 = \frac{\alpha_s C_F}{2\pi} 2 \ln \frac{a_2}{R^2}.$$

a_0, a_2 are order 1 constants, so,

in the small- R limit, $\langle \cos(2\phi) \rangle$ goes to 1

Additional gluon radiation contributions,

- In the momentum space, it will be a convolution
 - $q_T = k_{g1} + k_{g2} + \dots$
 - Dominant contributions will be ϕ -independent
- It is convenient to perform resummation in Fourier-b space

$$\begin{aligned}\tilde{S}_J(b_\perp) &= \int d^2 q_\perp e^{i q_\perp \cdot b_\perp} S_J(q_\perp) \\ &= \tilde{S}_{J0}(|b_\perp|) - 2 \cos(2\phi_b) \tilde{S}_{J2}(|b_\perp|) + \dots\end{aligned}$$

$$\tilde{S}_{J0}(b_\perp) = 1 + \alpha_0 \ln(\mu_b^2 / P_\perp^2) , \quad \tilde{S}_{J2}(b_\perp) = \alpha_2$$

All order resummation, in Fourier-b space

$$\tilde{S}_{J0}(b_{\perp}) = e^{-\Gamma_0(b_{\perp})} , \quad \tilde{S}_{J2}(b_{\perp}) = \alpha_2 e^{-\Gamma_0(b_{\perp})} \quad \Gamma_0(b_{\perp}) = \int_{\mu_b^2}^{P_{\perp}^2} \frac{d\mu^2}{\mu^2} \alpha_0$$

EIC

Kinematics:

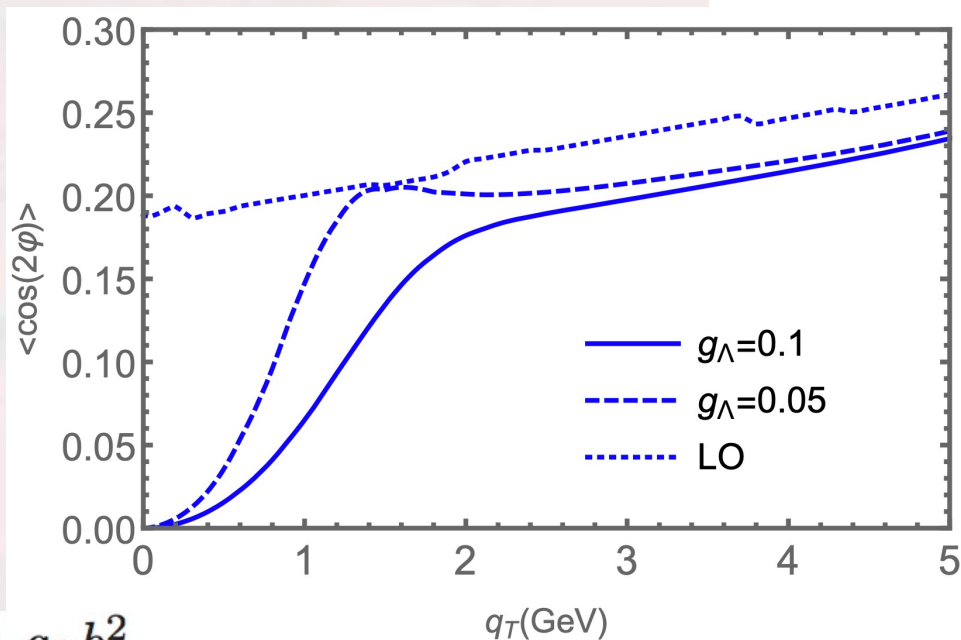
$P_T \sim 15 \text{ GeV}$

$R=0.4$

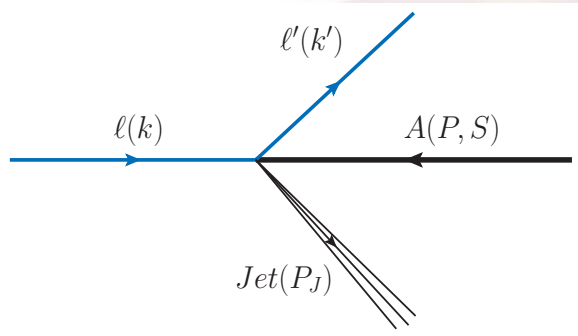
$y_1=y_2$

Non-pert. input:

$$\Gamma_0(b_{\perp}) \Rightarrow \Gamma_0(b_*) + g_{\Lambda} b_{\perp}^2$$



2. Lepton-jet correlation in DIS



Quark distribution \otimes soft factor

$$\frac{d^5 \sigma(\ell p \rightarrow \ell' J)}{dy_\ell d^2 k_{\ell\perp} d^2 q_\perp} = \sigma_0 \int d^2 k_\perp d^2 \lambda_\perp x f_q(x, k_\perp, \zeta_c, \mu_F) \\ \times H_{\text{TMD}}(Q, \mu_F) S_J(\lambda_\perp, \mu_F) \delta^{(2)}(q_\perp - k_\perp - \lambda_\perp) .$$

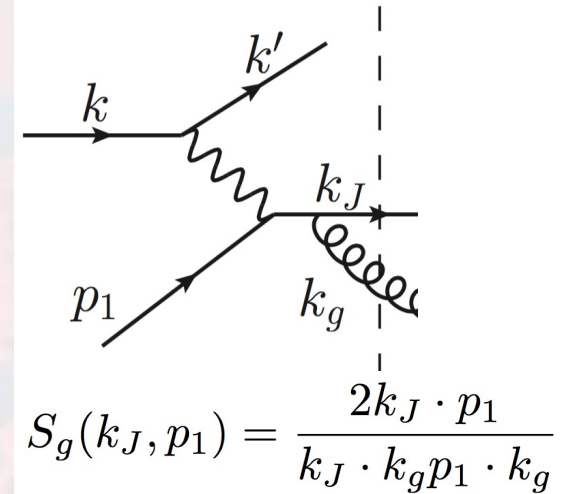
Liu-Ringer-Vogelsang-Yuan 1812.08077, 2007.12866

(Lab frame)

Total transverse momentum of the lepton+jet probes the TMD quark distribution

Soft gluon radiation

$$\begin{aligned}
 & g^2 \int \frac{d^3 k_g}{(2\pi)^3 2E_{k_g}} \delta^{(2)}(q_\perp + k_{g\perp}) C_F S_g(k_J, p_1) \\
 &= \frac{\alpha_s}{2\pi^2} \frac{1}{q_\perp^2} \left[\ln \frac{Q^2}{q_\perp^2} + \ln \frac{Q^2}{k_{\ell\perp}^2} \right. \\
 &\quad \left. + c_0 + 2c_1 \cos(\phi) + 2c_2 \cos(2\phi) + \dots \right],
 \end{aligned}$$



■ Small-R limit,

$$\ln \frac{1}{R^2} + 2 \cos(\phi) \left(\ln \frac{1}{R^2} + 2 \ln(4) - 2 \right) + 2 \cos(2\phi) \left(\ln \frac{1}{R^2} - 1 \right)$$

Final result depends on the quark TMD

$$\begin{aligned} \frac{d^5\sigma}{dy_\ell d^2k_{\ell\perp} d^2q_\perp} &= \sum_{n=1} 2 \cos(n\phi) \int \frac{b_\perp db_\perp}{(2\pi)} J_n(|q_\perp||b_\perp|) \\ &\times e^{-\text{Sud}} \sum_q \sigma_0 x_q f_q(x_q, \mu_b) \quad (15) \\ &\times \int d|q'_\perp| J_n(|b_\perp||q'_\perp|) \frac{C_F \alpha_s c_n(q'^2_\perp)}{|q'_\perp| \pi}. \end{aligned}$$

$$\begin{aligned} \text{Sud}(\mu_b^2, P_\perp^2, R) &= \int_{\mu_b}^Q \frac{d\mu}{\mu} \left\{ \frac{\alpha_s(\mu) C_F}{\pi} \left[\ln \frac{Q^2}{\mu^2} + \ln \frac{Q^2}{P_\perp^2} \right. \right. \\ &\quad \left. \left. - \frac{3}{2} + c_0(R) \right] \right\}, \quad (14) \end{aligned}$$

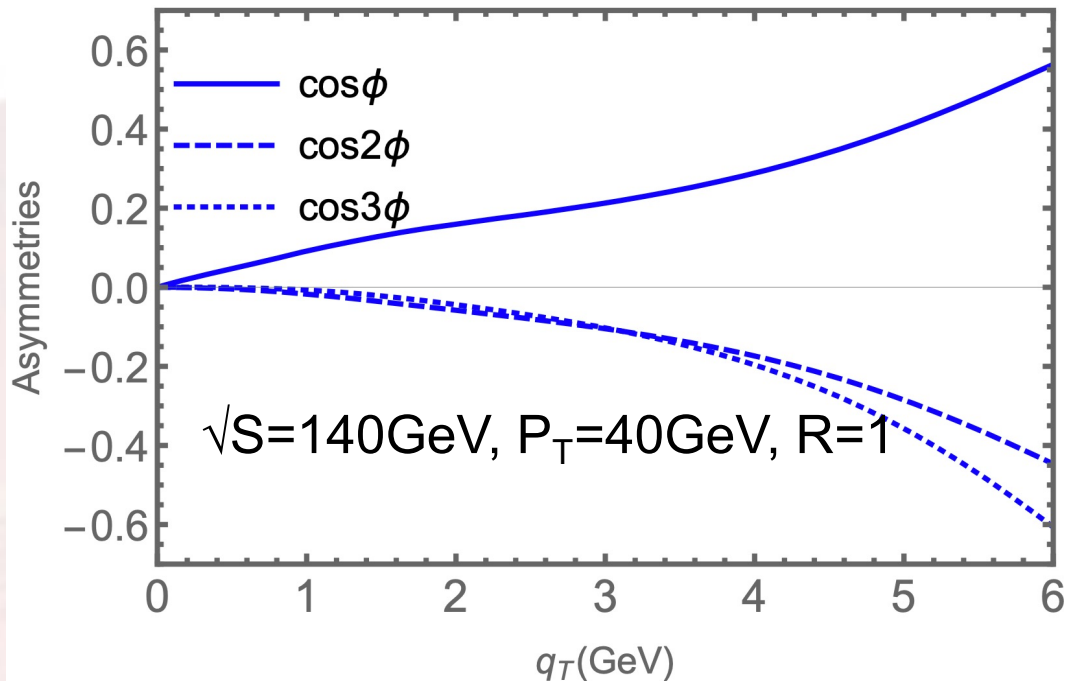
Estimate for EIC kinematics

- TMD quark follows SIYY parameterization

□ 1406.3073

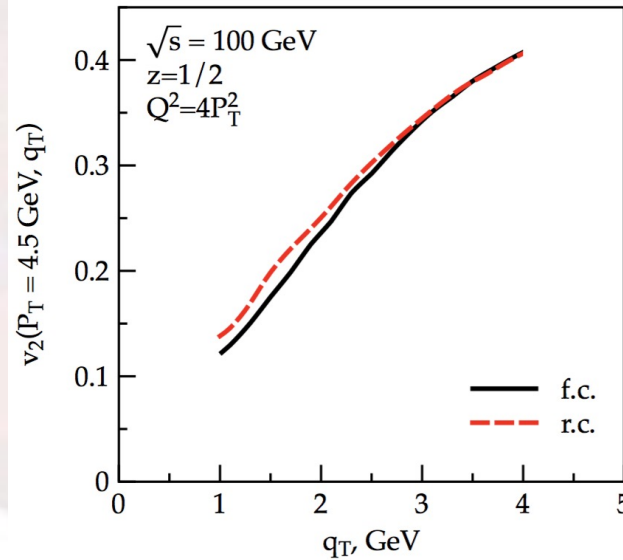
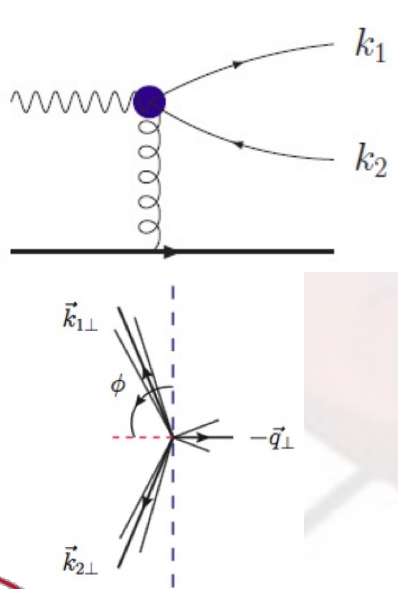
$$\text{Sud}(b_{\perp}) \rightarrow \text{Sud}(b_{*}) + \text{Sud}_{\text{NP}}^q(b_{\perp}) + \text{Sud}_{\text{NP}}^{\text{jet}}(b_{\perp})$$

$$\text{Sud}_{\text{NP}}^q(b_{\perp}) = 0.106 b_{\perp}^2 + 0.42 \ln(Q/Q_0) \ln(b_{\perp}/b_{*}) \quad \text{Sud}_{\text{NP}}^{\text{jet}}(b_{\perp}) = g_{\Lambda} b_{\perp}^2$$



3. Inclusive Dijet in DIS

- $\cos(2\phi)$ anisotropy was proposed to study the linearly polarized gluon distribution



CGC calculation:
Dumitru-Lappi-Skokov,
1508.04438

see also,
Boer-Brodsky-Mulders-Pisano
1011.4225

Metz-Zhou, 1105.1991
Boer et al., 1702.08195,
1605.07934

Mantysaari et al.,
1902.05087, 1912.05586

Three contributions to $\cos(2\phi)$ asymmetry

$$\frac{d^4\sigma}{d\Omega} = \sigma_0 \int \frac{d^2\vec{b}_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp \cdot \vec{b}_\perp} \left[\widetilde{W}_0^{\gamma^*p}(|b_\perp|) - 2 \cos(2\phi_b) \widetilde{W}_2^{\gamma^*p}(|b_\perp|) \right]$$

$$\widetilde{W}_0^{\gamma^*p}(b_\perp) = x_g f_g(x_g, \mu_b) e^{-\text{Sud}_{\text{pert}}^{\gamma^*p}(b_*) - \text{Sud}_{\text{NP}}^{\gamma^*p}(b_\perp)}$$

$$\widetilde{W}_2^{\gamma^*p}(b_\perp) = e^{-\text{Sud}_{\text{pert}}^{\gamma^*p}(b_*) - \text{Sud}_{\text{NP}}^{\gamma^*p}(b_\perp)}$$

$$\times \left[x_g f_g(x_g, \mu_b) \left(\alpha_2^{\gamma g} + \frac{\sigma_2}{\sigma_0} g_h(b_\perp) \right) \right]$$

$$+ \frac{\sigma_2}{\sigma_0} \int \frac{dx'}{x'} x_g f_i(x', \mu) C_{h/i}^{(1)} \left(\frac{x_g}{x'} \right)$$

Soft gluon from jet

Intrinsic linearly
polarized gluon

Collinear splitting
contribution

- Two loop calculation:
Gutierrez-Reyes, et al.,
1907.03780

- Numerically, contribution from soft gluon with jet is sizable
 - This can also be studied in real photon scattering process, where there is no linearly polarized gluon contribution
- The difference between the transverse and longitudinal photons purely comes from the linearly polarized gluon distribution

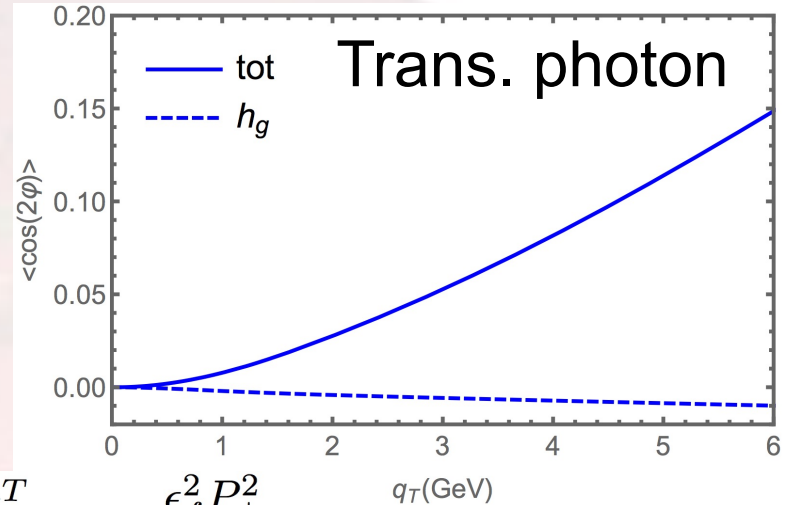
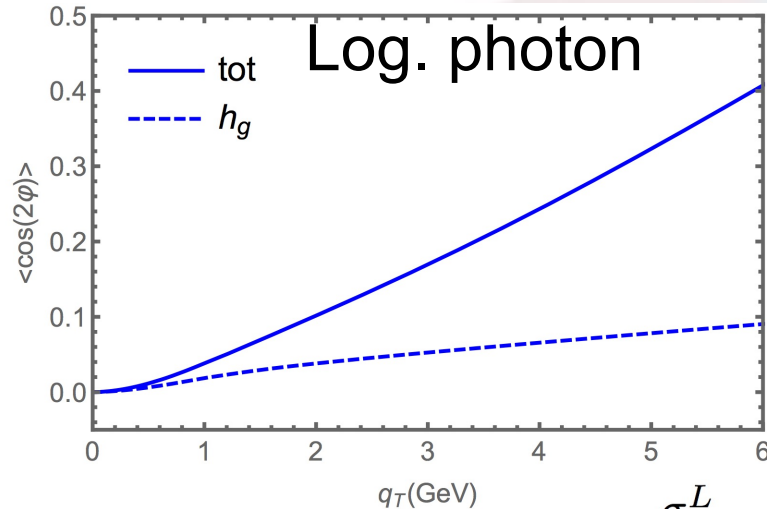
$$\frac{\sigma_2^L}{\sigma_0^L} = \frac{1}{2}$$

$$\frac{\sigma_2^T}{\sigma_0^T} = -\frac{\epsilon_f^2 P_\perp^2}{\epsilon_f^4 + P_\perp^4}$$

Conclusion

- Soft gluon radiation can generate a sizable azimuthal asymmetry between the total and different transverse momenta of two final particles
- It provides an opportunity to explore QCD dynamics in the final state soft gluon radiation
- This physics has to be understood before we can apply the dijet azimuthal correlations to study the nucleon/nucleus tomography

Numerically, contribution from soft gluon with jet is sizable,
 $Q=10\text{GeV}$, $P_T=15\text{GeV}$, $R=0.4$



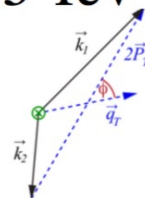
$$\frac{\sigma_2^L}{\sigma_0^L} = \frac{1}{2} \quad \frac{\sigma_2^T}{\sigma_0^T} = -\frac{\epsilon_f^2 P_\perp^2}{\epsilon_f^4 + P_\perp^4}$$

- The difference between the above two purely comes from the linearly polarized gluon distribution

CMS measurement on diffractive dijet

Exclusive dijets in UPC PbPb @5 TeV

(CMS-PAS-HIN-18-011)

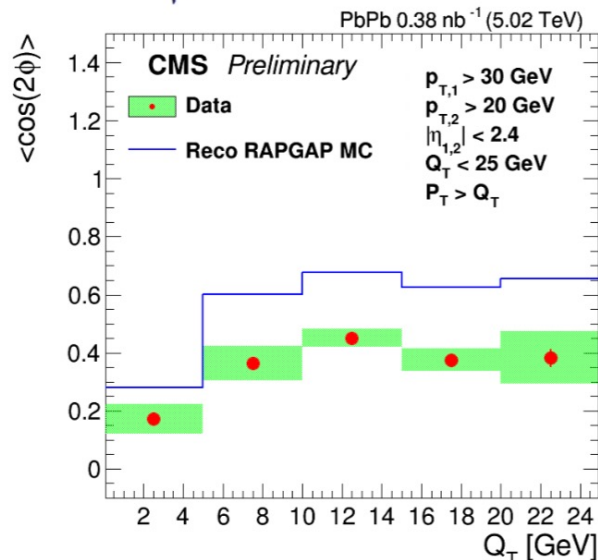
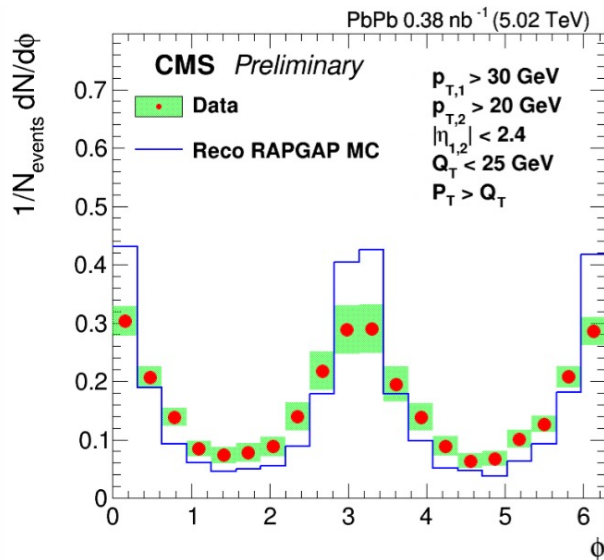


Vector sum of 2 jets:

$$\vec{Q}_T = \vec{k}_1 + \vec{k}_2$$

Vector difference of 2 jets

$$\vec{P}_T = \frac{1}{2}(\vec{k}_1 - \vec{k}_2)$$



Bylinkin@ICHEP 2020, Takaki's group at Kansas U

All order resummation, in Fourier-b space (no power corrections)

$$\tilde{S}_{J0}(b_{\perp}) = e^{-\Gamma_0(b_{\perp})}, \quad \tilde{S}_{J2}(b_{\perp}) = \alpha_2 e^{-\Gamma_0(b_{\perp})} \quad \Gamma_0(b_{\perp}) = \int_{\mu_b^2}^{P_{\perp}^2} \frac{d\mu^2}{\mu^2} \alpha_0$$

CMS

Kinematics:

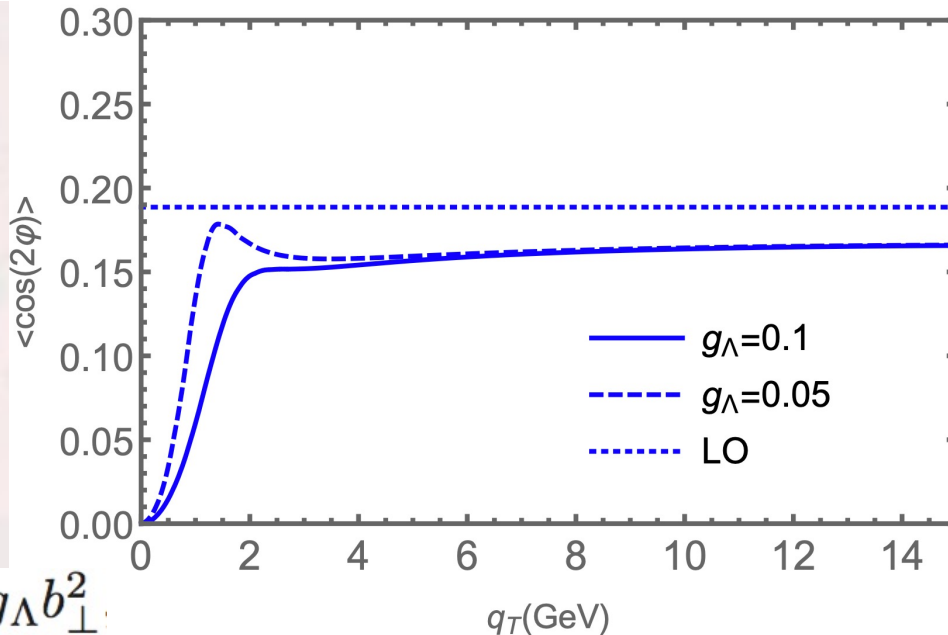
$P_T \sim 35 \text{ GeV}$

$R=0.4$

$y_1=y_2$

Non-pert. input:

$$\Gamma_0(b_{\perp}) \Rightarrow \Gamma_0(b_*) + g_{\Lambda} b_{\perp}^2$$



All order resummation, in Fourier-b space (with power corrections)

$$\tilde{S}_{J0}(b_\perp) = e^{-\Gamma_0(b_\perp)} , \quad \tilde{S}_{J2}(b_\perp) = \alpha_2 e^{-\Gamma_0(b_\perp)} \quad \Gamma_0(b_\perp) = \int_{\mu_b^2}^{P_\perp^2} \frac{d\mu^2}{\mu^2} \alpha_0$$

CMS

Kinematics:

$P_T \sim 35 \text{ GeV}$

$R=0.4$

$y_1=y_2$

Non-pert. input:

$$\Gamma_0(b_\perp) \Rightarrow \Gamma_0(b_*) + g_\Lambda \hat{b}_\perp^2$$

