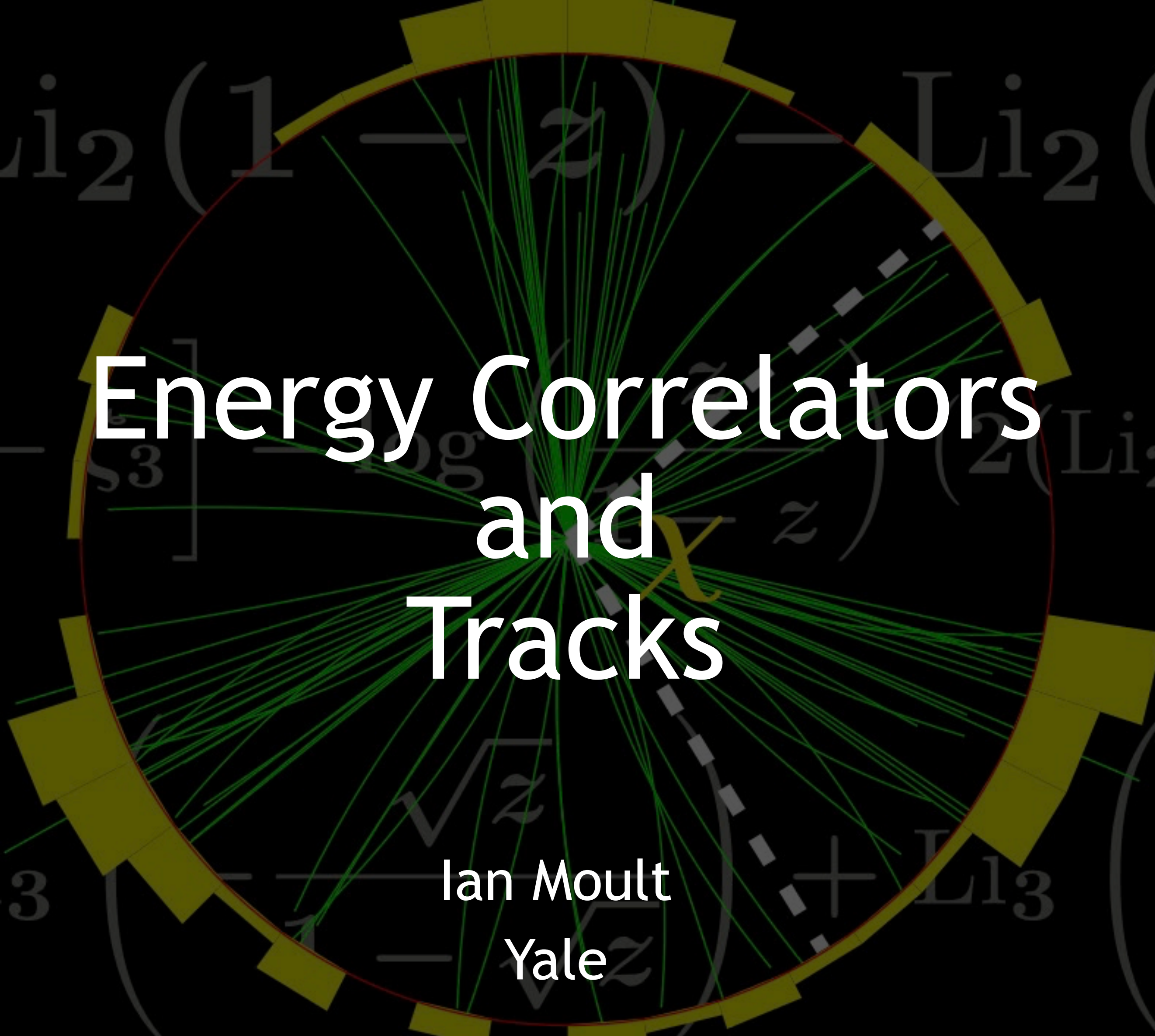
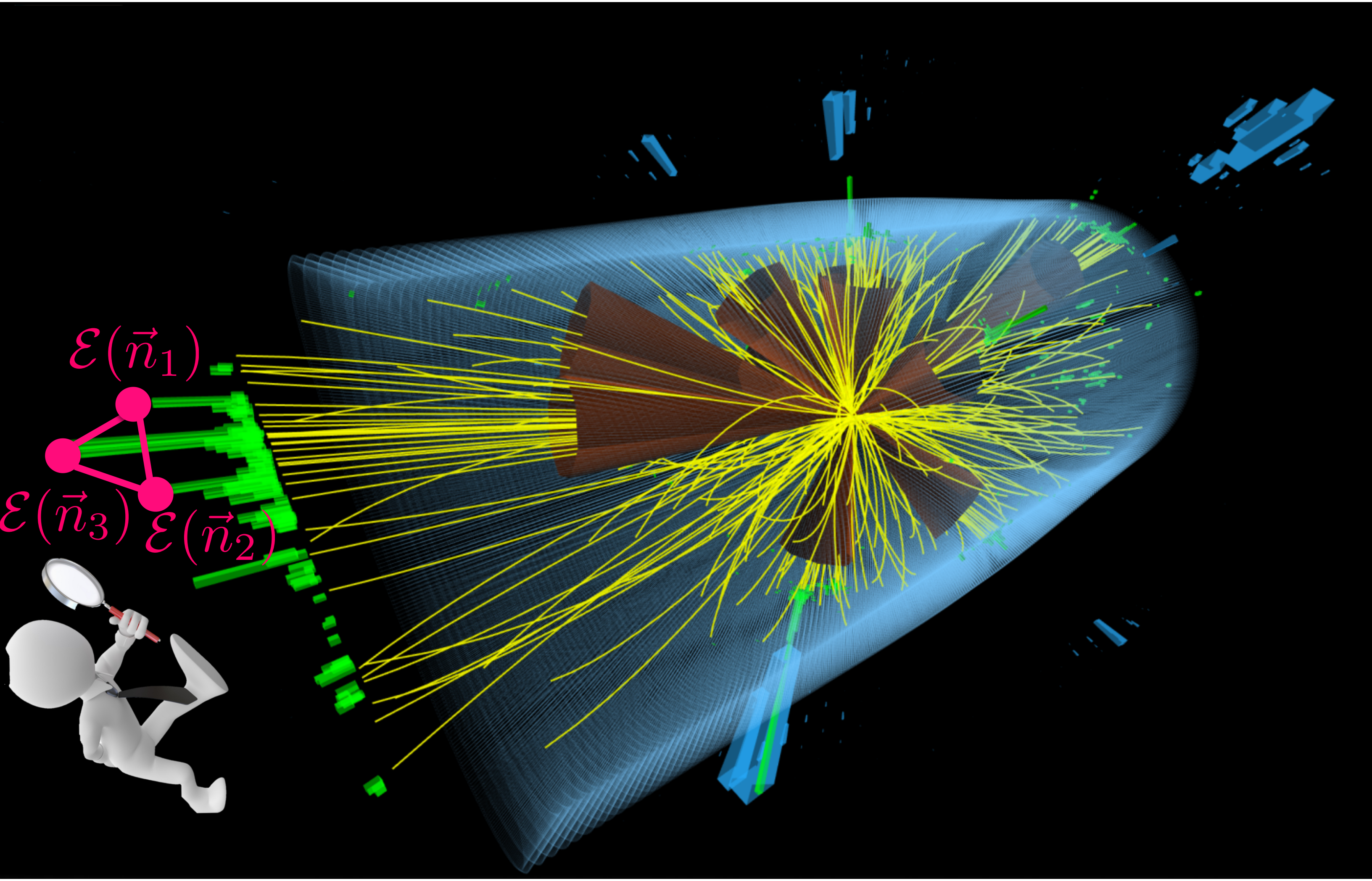


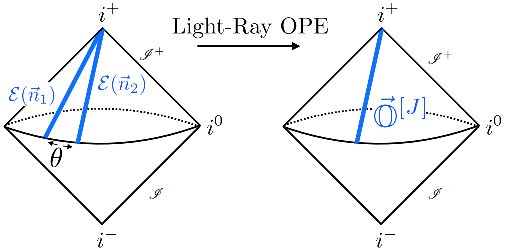
Energy Correlators and Tracks

Ian Moult
Yale

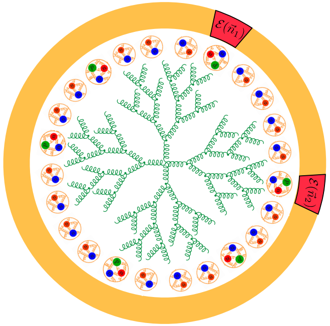

$\mathcal{E}(\vec{n}_1)$
$\mathcal{E}(\vec{n}_3)$
$\mathcal{E}(\vec{n}_2)$

Outline

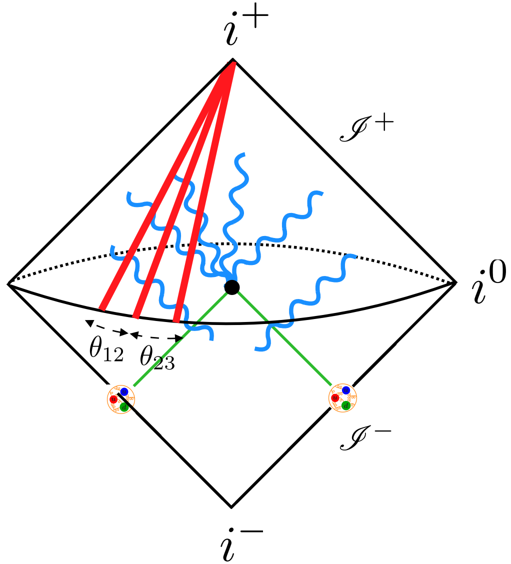
- Rethinking Jets with Energy Correlators



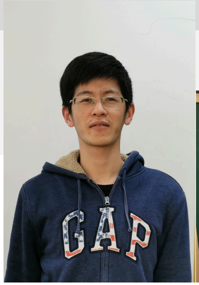
- Extending Precision Perturbative QCD with Track Functions

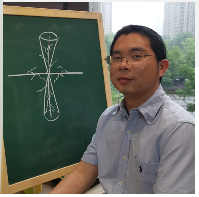


Rethinking Jets with Energy Correlators

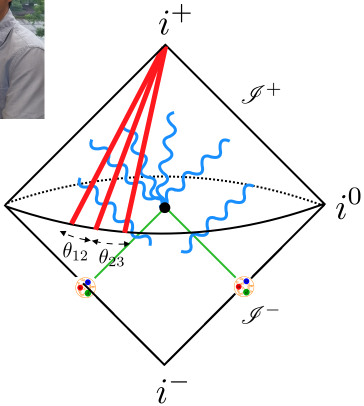


References

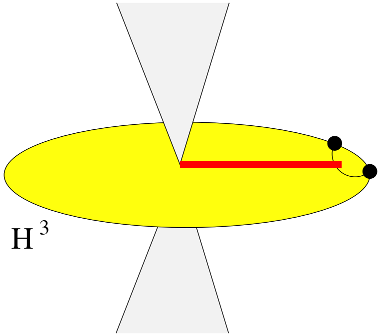




- Directly based on:
  - Dixon, Moult, Zhu, arXiv:1905.01310
  - Chen, Luo, Moult, Yang, Zhang, Zhu, arXiv:1912.11050
  - Chen, Moult, Zhang, Zhu, arXiv:2004.11381
  - Komiske, Moult, Thaler, Zhu, Forthcoming
  - Chen, Moult, Zhu, arXiv:2011.02492
  - Chen, Moult, Sandor, Zhu, Forthcoming



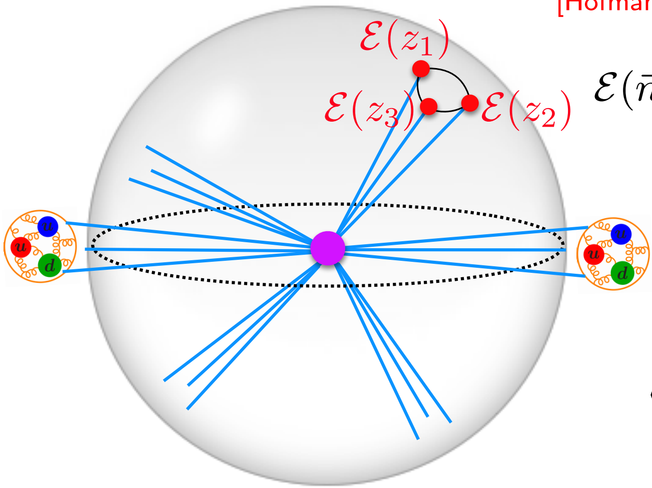
- See also/ Thanks to perspectives from:
  - Hofman, Maldacena
  - Belitsky, Hohenegger, Korchemsky, Sokatchev, Zhiboedov
  - Korchemsky
  - Kravchuk, Simmons Duffin
  - Chang, Kologlu, Simmons Duffin, Kravchuk, Zhiboedov
  - Henn, Sokatchev, Yan, Zhiboedov
  - Chicherin, Henn, Sokatchev, Yan
  - Dixon, Luo, Shtabovenko,Yang, Zhu
  - . . .



Energy Flow Operators

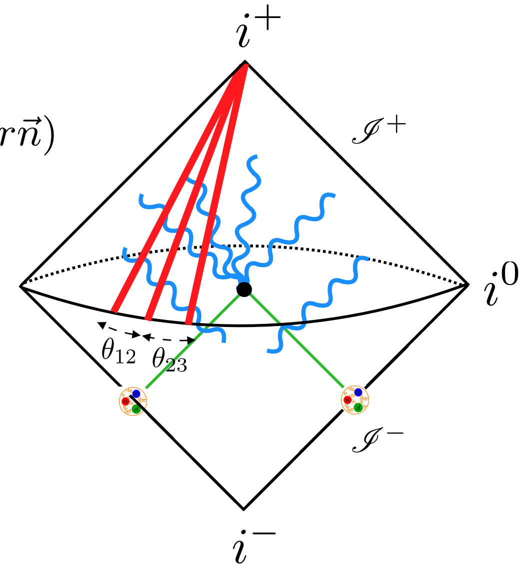
- From a field theory perspective, this is the study of matrix elements of Energy Flow/ ANEC/ Lightray operators

[Sveshnikov, Tkachov; Korchemsky, Sterman]
[Hofman, Maldacena]



$$\mathcal{E}(\vec{n}) = \int_0^\infty dt \lim_{r\to\infty} r^2 n^i T_{0i}(t, r\vec{n})$$

$$\langle \Psi | \mathcal{E}(\hat{n}_1) \cdots \mathcal{E}(\hat{n}_k) | \Psi \rangle$$



- These correlation functions completely characterize the flow of energy (or other charges) at infinity. Have a direct correspondence with "calorimeter cells" in real experiments.

Two Descriptions:

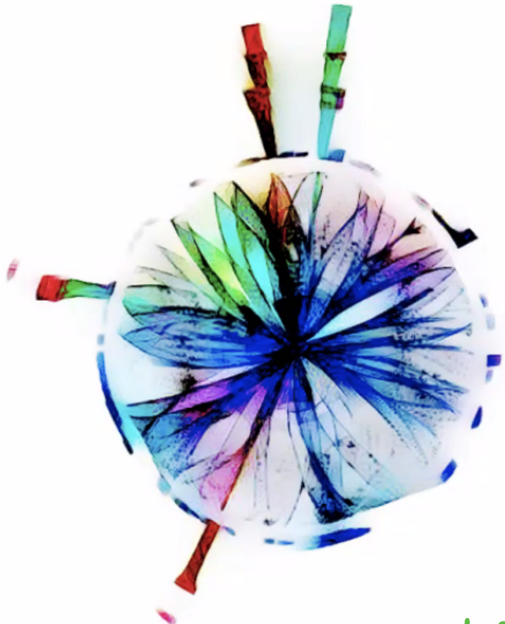
Correlation functions

$\langle \mathcal{E}(n_1) \rangle$

$\langle \mathcal{E}(n_1)\, \mathcal{E}(n_2) \rangle$

$\langle \mathcal{E}(n_1)\, \mathcal{E}(n_2)\, \mathcal{E}(n_3) \rangle$

$\vdots$



Jet Shapes

- Jet mass
- angularities

$\vdots$

all standard substructure observables.

Related by expansion in moments:

$$m = \langle \delta(m - m[\mathcal{E}(n_1), \mathcal{E}(n_2)]) \rangle = \sum_{n=0}^{\infty} \delta^{(n)}(m) \langle (m[\mathcal{E}(n_1), \mathcal{E}(n_2)])^n \rangle$$

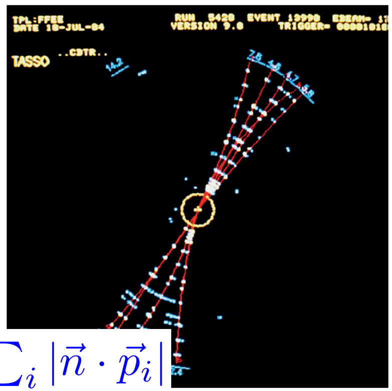
→ destroys symmetries.

In principle equivalent, but probe physics in very different ways

Jet Shapes

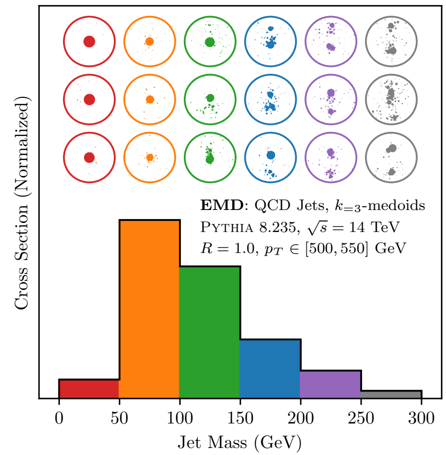
- The classic approach to studying jets (dating back to Farhi's "thrust" in 1977) is to use "jet shapes", which measure the spread of radiation.

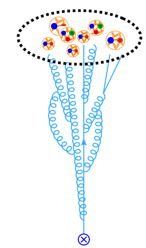
LEP



$$T = \max_{\vec{n}} \frac{\sum_i |\vec{n} \cdot \vec{p}_i|}{Q}$$

LHC

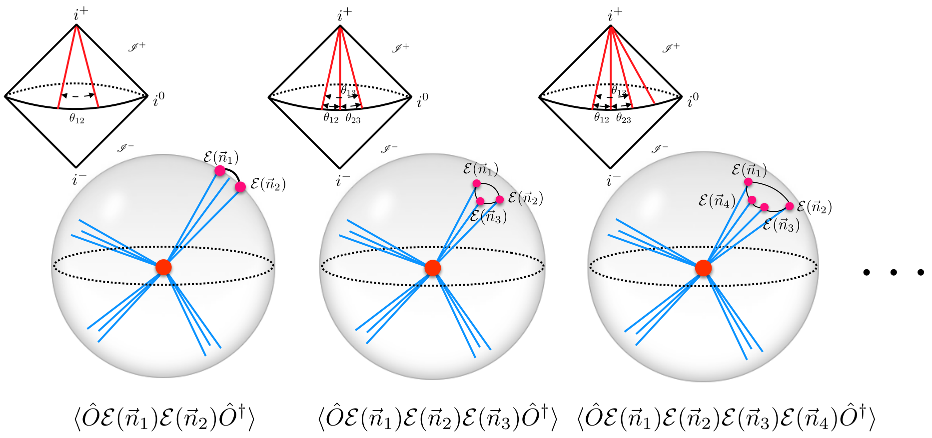




- Organize jets by phase space regions $\implies$ useful for tagging.

Correlators

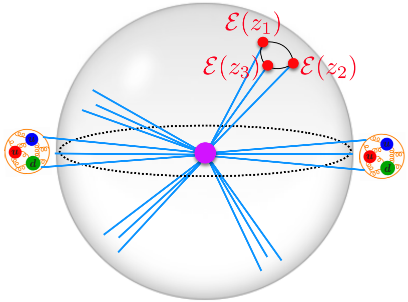
- A second approach is to directly measure the fundamental correlators:

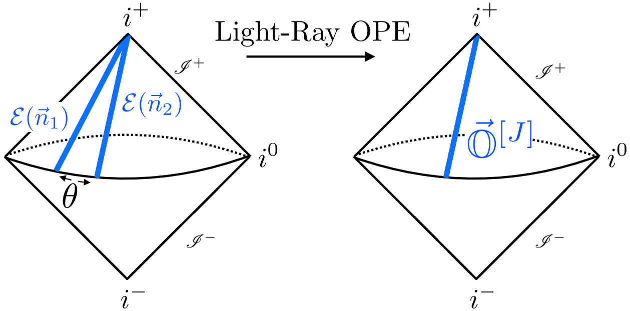


- Not defined event by event. Only defined as ensemble average.
- These objects are much simpler to understand theoretically and offer new ways to study jets.

Why Correlators?

- Reason 1: These are genuine correlation functions living on the celestial sphere. This enables the use of a variety of sophisticated techniques (OPE, conformal blocks, symmetry, ...) that can't be used for jet shapes.

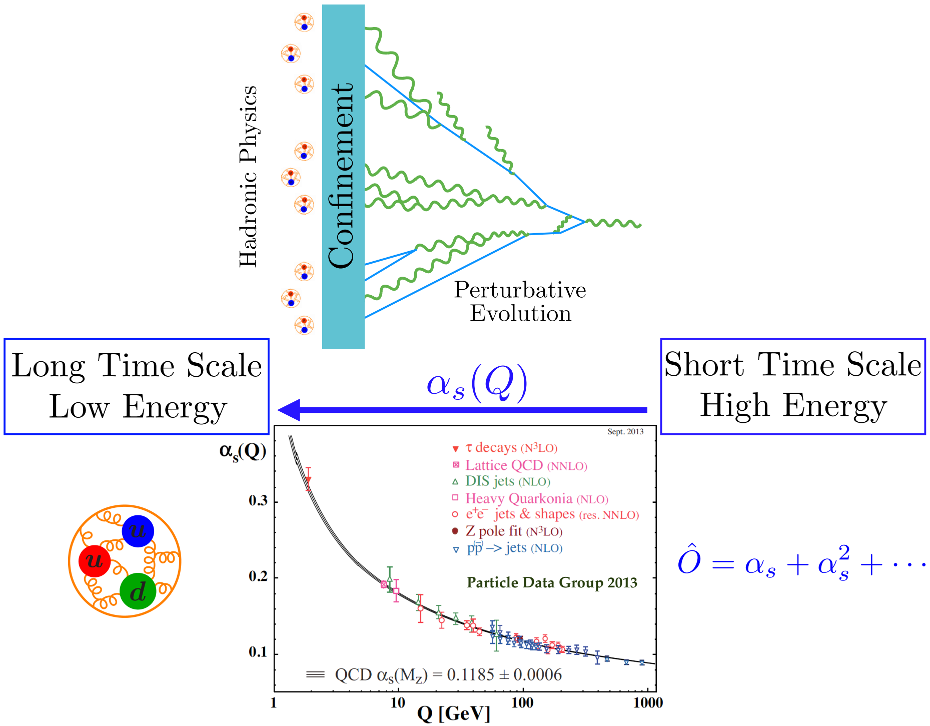




- Reason 2: Correlators probe dynamics at a fixed scale. While I will use an angle, this can easily be converted to "formation time", $\tau_f \sim \frac{1}{E\ \theta^2}$.

Imaging Jets with Correlators

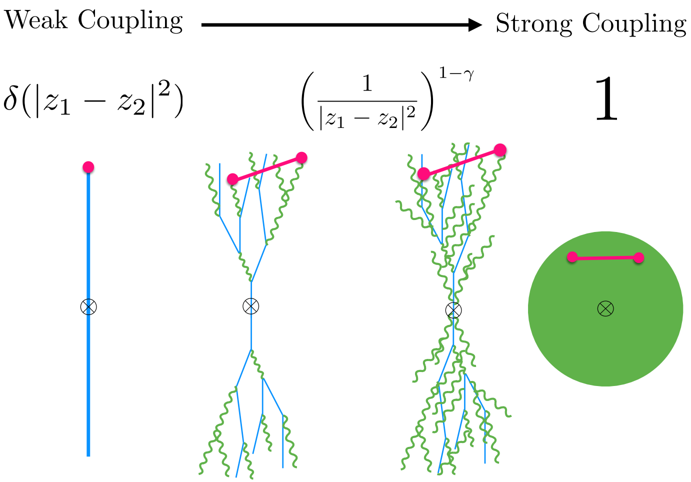
- Real world QCD is complicated: Jets exhibit a transition from weakly coupled quarks and gluons to freely propagating hadrons.

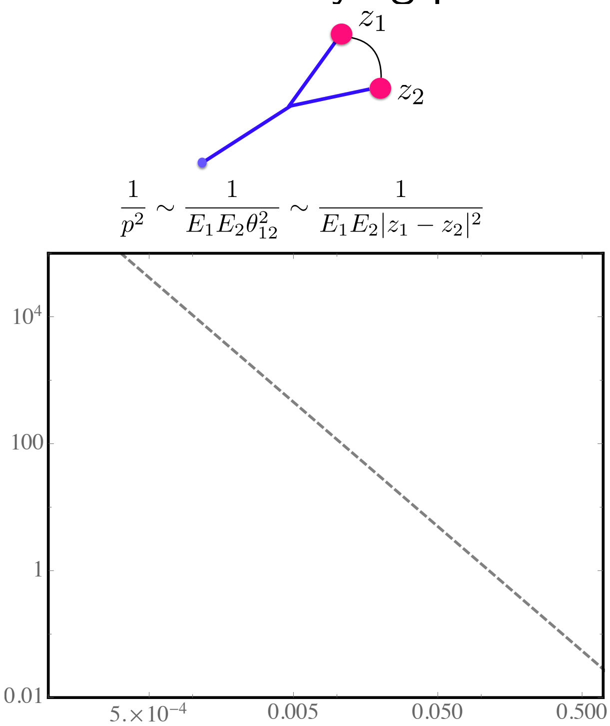


- Expect two qualitatively different regimes in a correlation function.

Limiting Behavior

- In a weakly coupled conformal theory, have a power law scaling.
- Scaling $(\theta^2)^{1-\gamma}$ associated with the pole of the underlying partons.

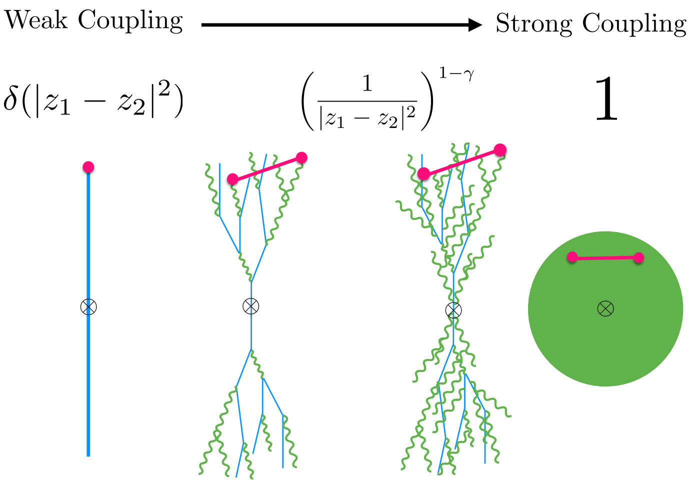


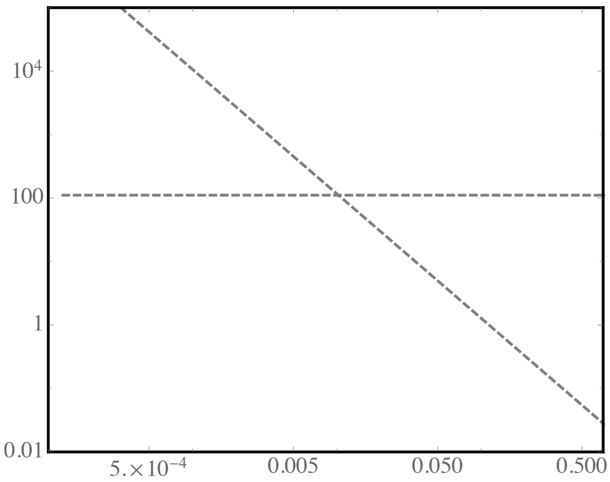


- Small angle singularity associated with the existence of jets in QCD.

Limiting Behavior

- A second limiting behavior is a uniform distribution of energy.
- This occurs in strongly coupled $\mathcal{N} = 4$ where you produce "mush" instead of jets. [Hofman, Maldacena]

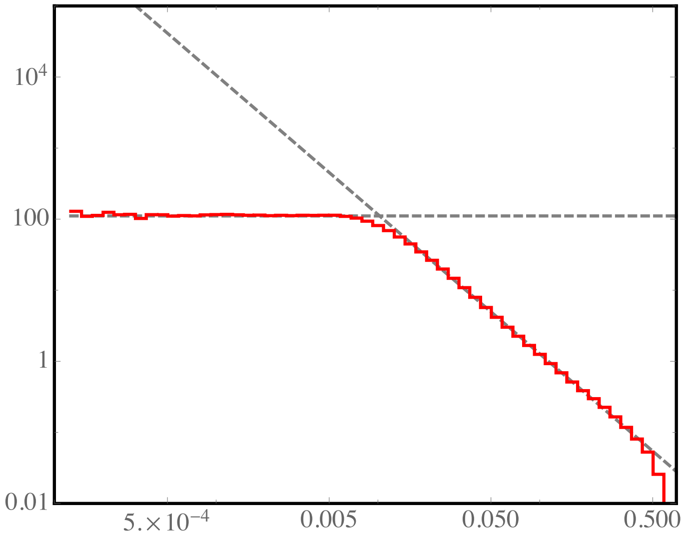


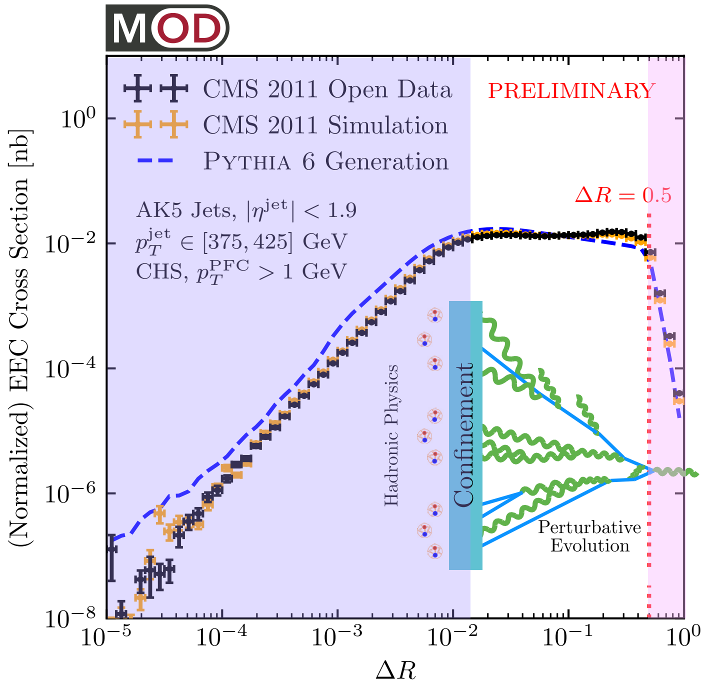


- What do we see in QCD?

Two-point Correlator with Data

- Allow one to “image” a jet at a given scale.

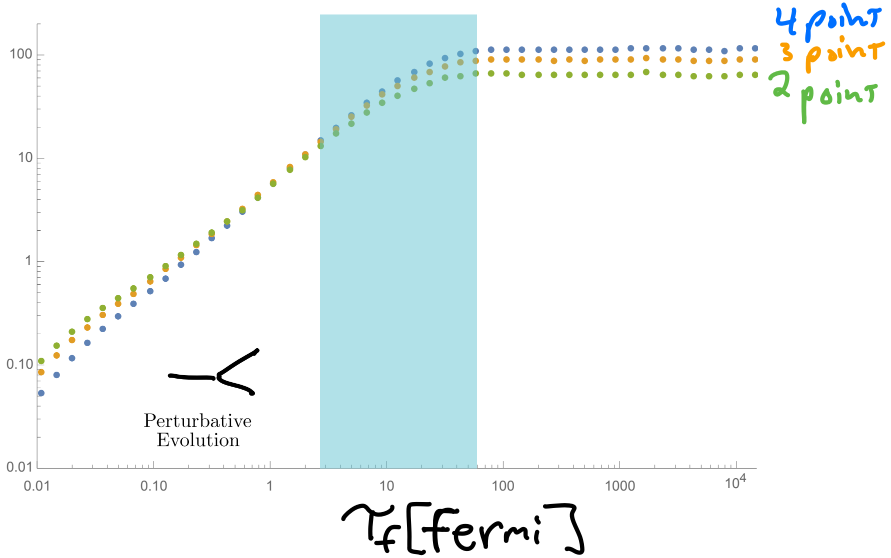




- Distinct phases of QCD clearly visible by eye. Clean scaling behavior in each regime.

Relation with Formation Time

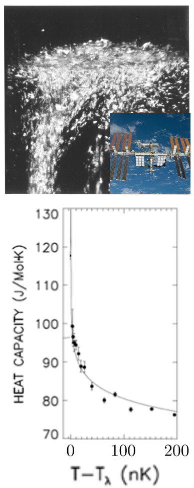
- Note: These correlators have a close relationship with "formation time", where one has the approximate relation $\tau_f \sim \frac{1}{E\ \theta^2}$.
- All our calculations can therefore be mapped to τ_f using a simple Jacobian.
- Useful to "reintroduce scales" for interpretation in Heavy Ion/ Nuclear applications.
- See Raghav's Talk!



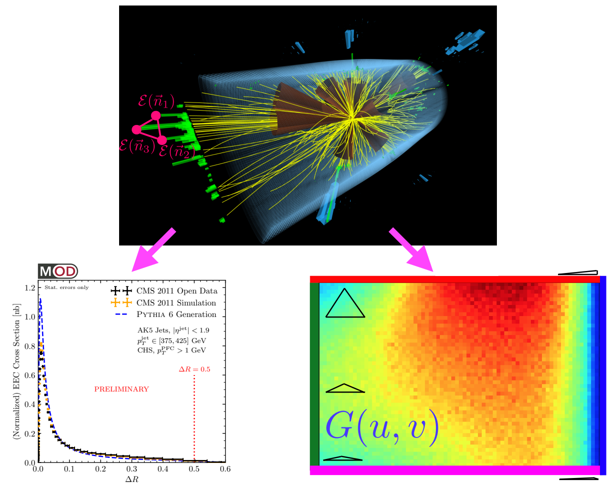
Conformal Colliders Meet the LHC

- Progress in jet substructure allows for the direct measurement of these correlators, and their associated lightray OPE scaling, inside high energy jets at the LHC.

Local OPE



Lightray OPE



Scaling

Shape

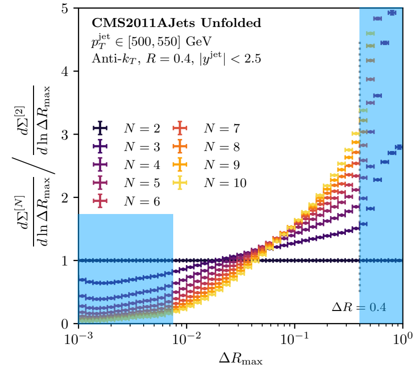
- Formulating the study of jet substructure in terms of correlation functions opens door to use of new techniques and opportunity for interplay with CFT developments.

Energy Correlators in Open Data

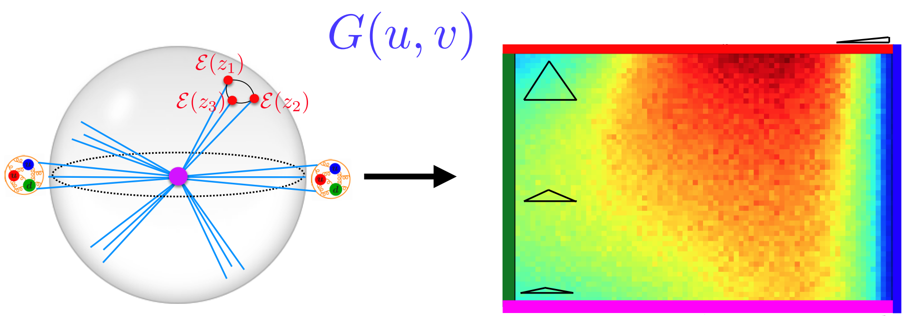
- Many interesting features of the energy correlators can be directly measured in data.
- To illustrate, I will focus on two of the most important features

1. Scaling behavior:

$$\mathcal{E}(\hat{n}_1)\mathcal{E}(\hat{n}_2) = \theta^{\gamma_i} \sum \mathbb{O}_i(\hat{n}_1)$$

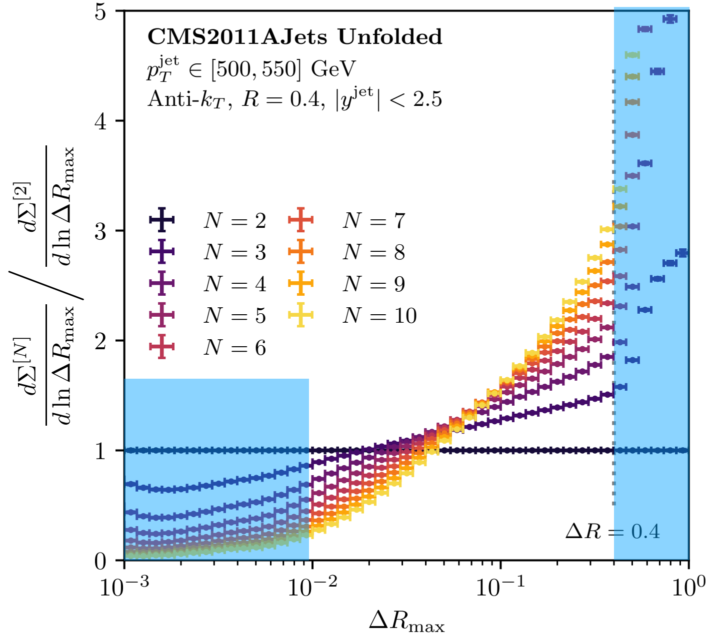


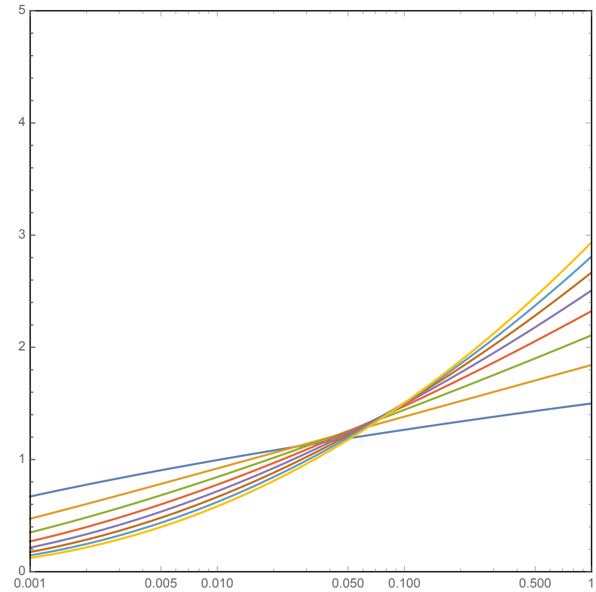
2. Shape dependence of the Three-Point Correlator



Anomalous Scaling in Data

- Anomalous scaling of energy correlators up to 10 points:

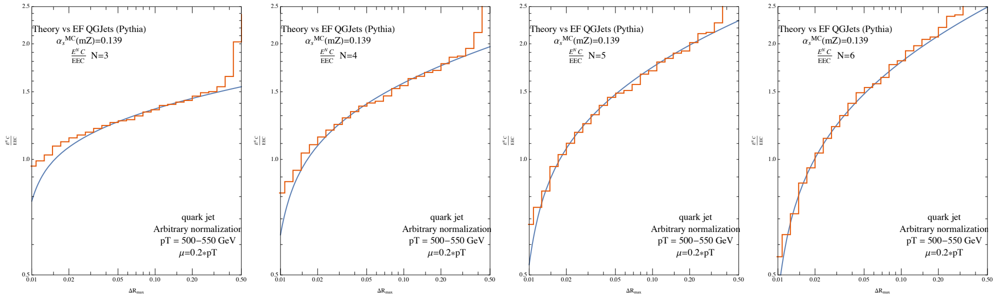




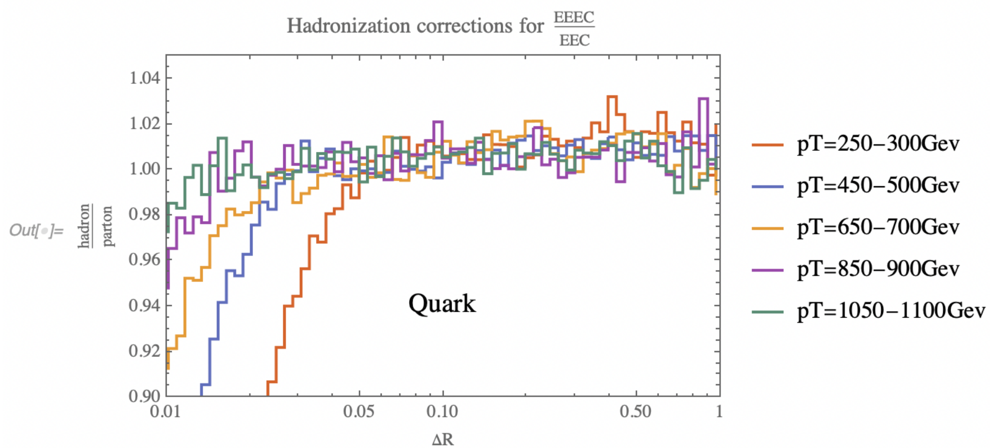
- Lorentzian scaling of lightray operators at the quantum level!!
- Proper treatment of exp/theory uncertainties, etc. ongoing.

Anomalous Scaling

- Quantitative Comparison

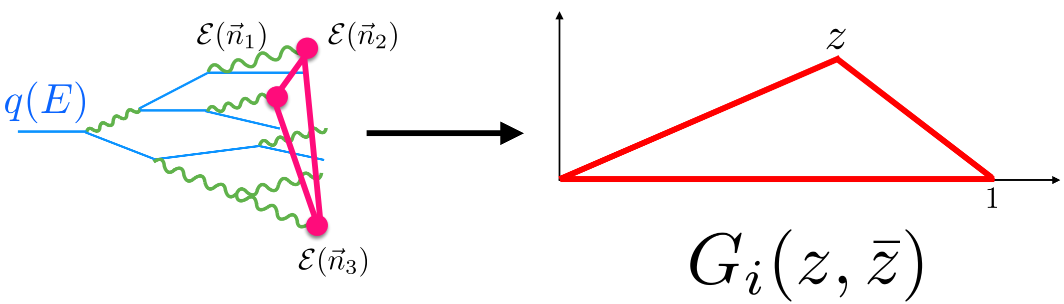


- Ratios have remarkably small non-perturbative corrections!



The Three-Point Energy Correlator

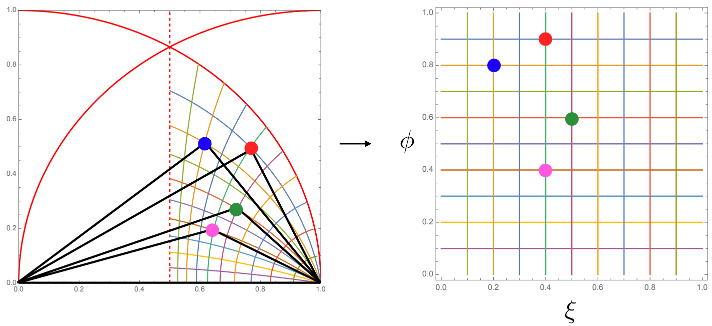
- The first correlator with non-trivial shape dependence in the collinear limit is the three-point correlator.
- Depends on a scaling variable x, and cross ratio variables $(z, \bar{z})$ or (u, v).



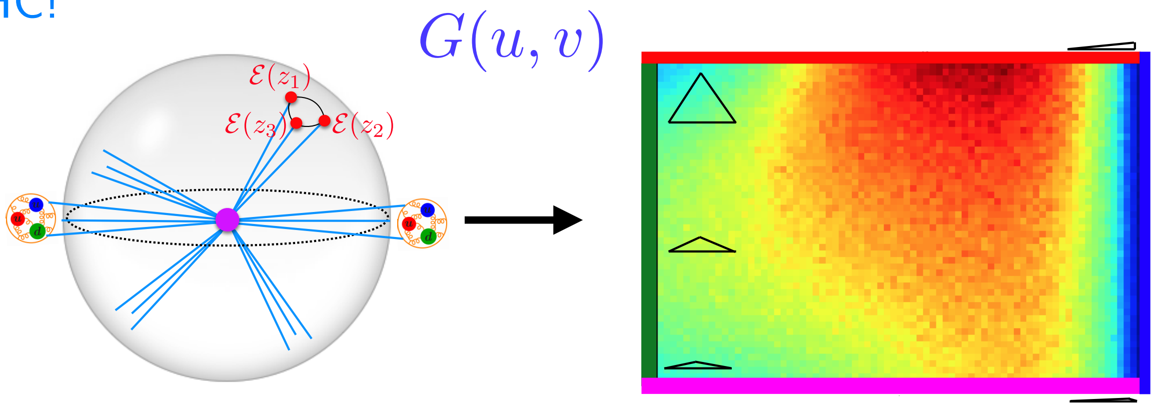
- "Kinematics" equivalent to a CFT 4 point function.

Shape Dependence in Data

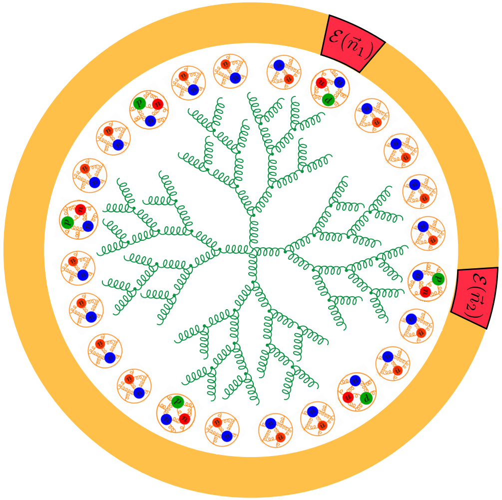
- Standard parametrization is inconvenient for experimental binning: We can map to a square grid by "blowing up" the OPE region.



- Can directly measure correlation functions of lightray operators at the LHC!



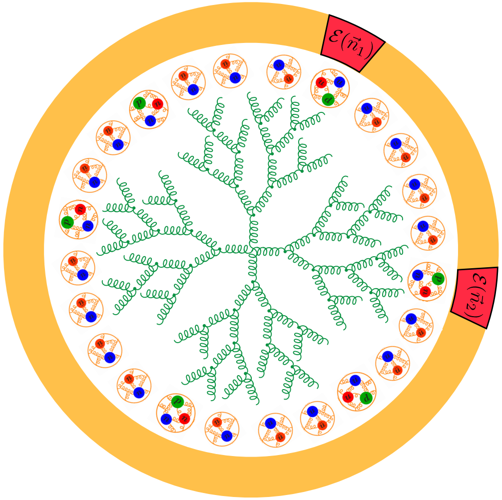
Extending Precision Perturbative QCD with Track Functions



Li, IM, Schrijnder van Velzen, Waalewijn, Zhu

Energy Flow of Hadrons

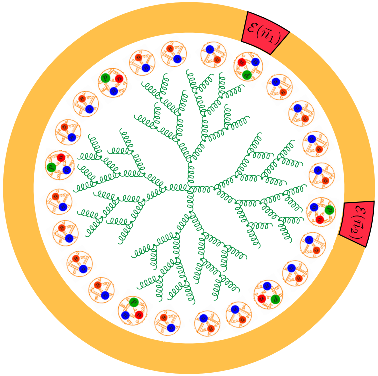
- Improving precision is not just performing higher order calculations, one also has to develop techniques to perform new classes of calculations.
- Many interesting measurements of energy flow on a restricted set of hadronic states, R, e.g. Charged hadrons (tracks). These cannot be computed in perturbation theory.



$$\langle \mathcal{E}_R(\vec{n}_1)\mathcal{E}_R(\vec{n}_2)\cdots\mathcal{E}_R(\vec{n}_k)\rangle$$

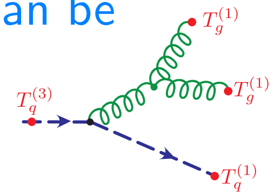
Tracks and Energy Correlators

- Energy correlators are weighted by energy flow through detector cells as a function of angle.
- How to go from full calorimeter to tracks? simply multiply by first moment of a track function!



$$\langle \mathcal{E}_R(\vec{n}_1)\mathcal{E}_R(\vec{n}_2)\cdots\mathcal{E}_R(\vec{n}_k)\rangle = \sum_{i_1,i_2,\cdots,i_k} T_{i_1}(1)\cdots T_{i_k}(1)\langle \mathcal{E}_{i_1}(\vec{n}_1)\mathcal{E}_{i_2}(\vec{n}_2)\cdots\mathcal{E}_{i_k}(\vec{n}_k)\rangle\,.$$

- Upshot: Any analytic calculation of energy correlators can be upgraded to tracks!
- Exhibit interesting evolution.

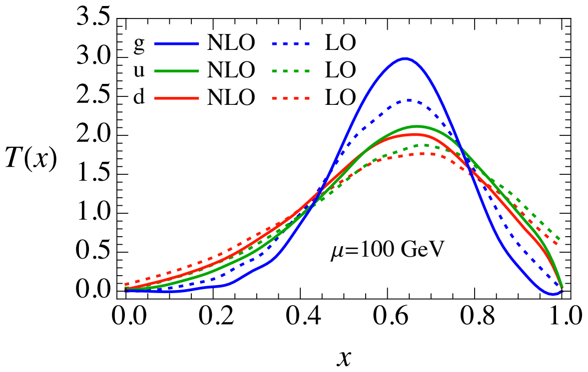


Track Functions

- Track functions are a non-perturbative function describing the total energy fraction going into hadrons with a particular property, R.

$$T_q(x) = \int dy^+ d^2 y_\perp e^{ik^- y^+/2} \frac{1}{2N_c} \sum_X \delta\left(x - \frac{P_R}{k_-}\right) \text{tr}\left[\frac{\gamma^-}{2} \langle 0|\psi(y^+, 0, y_\perp)|R\bar{R}\rangle \langle R\bar{R}|\bar{\psi}(0)|0\rangle\right]$$

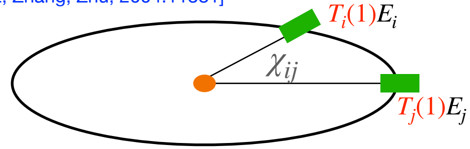
- e.g. for charged particles:



- Note: Fragmentation functions describe energy fraction carried by an individual hadron. Track functions describe total energy fraction carried by all particles of type R. This is necessary for computing energy flow observables.

Tracks

[Chen, Moult, Zhang, Zhu, 2004.11381]



Incorporating Tracks

[1303.6637]

- For a δ-function type observable e measured using partons:

$$\frac{d\sigma}{de} = \sum_N \int d\Pi_N \frac{d\sigma_N}{d\Pi_N} \delta\left[e - \hat{e}(p_i^\mu)\right]$$

↓ tracks

full functional form of T

$$\frac{d\sigma}{d\bar{e}} = \sum_N \int d\Pi_N \frac{d\bar{\sigma}_N}{d\Pi_N} \int \prod_{i=1}^{N} dx_i T_i(x_i) \delta\left[\bar{e} - \hat{e}(x_i p_i^\mu)\right]$$

- For an energy correlator at partonic level: e.g. 2-point correlator (EEC)

$$\frac{d\Sigma}{d\cos\chi} = \sum_{i,j} \int \frac{E_i E_j}{Q^2} \delta\left(\cos\chi - \cos\chi_{ij}\right) d\sigma$$

$$E_i^n \to \int dx_i T_i(x_i) x_i^n E_i^n = T_i(n) E_i^n$$

Mellin moments

only moments of T

$$\left(\frac{d\Sigma}{d\cos\chi}\right)_{\text{tr}} = \sum_{i,j} T_i(1) T_j(1) \int \frac{E_i E_j}{Q^2} \delta\left(\cos\chi - \cos\chi_{ij}\right) d\bar{\sigma}$$

Track EEC

- Energy correlators: tracking easily included and can use modern fixed-order techniques.

- Note: This is simply a factual distinction between amount of non-perturbative input needed for (groomed) mass vs. energy correlators. No amount of thought/ theory work can overcome it.
- My conclusion: Energy correlators are superior for studies of substructure where angular resolution is required!

- Unlike DGLAP, the evolution of track functions is non-linear.

Track Function Evolution

- LO evolution [1303.6637]

$$\frac{d}{d\ln\mu^2}T_i(x,\mu) = a_s(\mu)\sum_{j,k}\int dz dx_1 dx_2 P^{(0)}_{i\to jk}(z_1,z_2)\delta(1-z_1-z_2) \times T_j(x_1,\mu)T_k(x_2,\mu)\delta[x-z_1x_1-z_2x_2]\ .$$

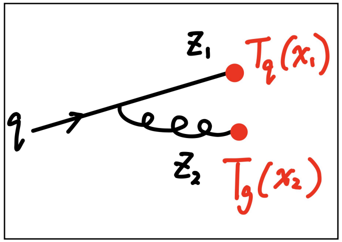
Nonlinear, involving contributions from both branches of the splitting.

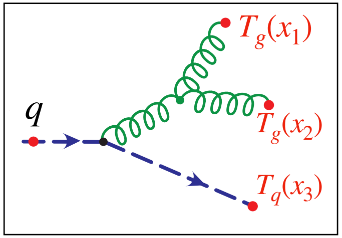
- Beyond leading order: Involves contributions from multiple branchings.

- While for fragmentation functions: Only one branch observed → Linearity

$$\frac{d}{d\ln\mu^2}d_{h/i}(z,\mu) = \sum_j d_{h/j}\otimes P^T_{ji}(z,\mu)$$

10





RG Evolution

- Evolution of low moments largely fixed by symmetries

For fragmentation functions:

$$\frac{d}{d\ln\mu^2} d_{h/i}(z,\mu) = \sum_j d_{h/j} \otimes P_{ji}(z,\mu)$$

- Scale invariant $d(y) \to d(ay)$.

A Surprising Symmetry:

- Energy conservation implies the evolution is **shift-symmetric**: $x \to x + a$

$$\frac{d}{d\ln\mu^2} T_i(x+a) = \sum_X \int \left(\prod_m dx_m dz_m T_{i_m}(x_m + a) \right) P_{i\to i_1\cdots i_m\cdots}(\{z_m\})\, \delta\left(1 - \sum_m z_m\right) \delta\left(x - \sum_m x_m z_m\right)$$

- This uniquely fixes the form of the evolution of the first three moments:

$$\frac{d}{d\ln\mu^2}\Delta = [-\gamma_{qq}(2) - \gamma_{gg}(2)]\,\Delta\ ,$$

$$\frac{d}{d\ln\mu^2}\begin{bmatrix}\sigma_g(2)\\ \sigma_q(2)\end{bmatrix} = \begin{bmatrix}-\gamma_{gg}(3) & -\gamma_{qg}(3)\\ -\gamma_{gq}(3) & -\gamma_{qq}(3)\end{bmatrix}\begin{bmatrix}\sigma_g(2)\\ \sigma_q(2)\end{bmatrix} + \begin{bmatrix}\gamma_g^{\Delta^2}\\ \gamma_q^{\Delta^2}\end{bmatrix}\Delta^2\ ,$$

$$\frac{d}{d\ln\mu^2}\begin{bmatrix}\sigma_g(3)\\ \sigma_q(3)\end{bmatrix} = \begin{bmatrix}-\gamma_{gg}(4) & -\gamma_{qg}(4)\\ -\gamma_{gq}(4) & -\gamma_{qq}(4)\end{bmatrix}\begin{bmatrix}\sigma_g(3)\\ \sigma_q(3)\end{bmatrix} + \begin{bmatrix}\gamma_{gg}^{\sigma\Delta} & \gamma_{qg}^{\sigma\Delta}\\ \gamma_{gq}^{\sigma\Delta} & \gamma_{qq}^{\sigma\Delta}\end{bmatrix}\begin{bmatrix}\sigma_g(2)\\ \sigma_q(2)\end{bmatrix}\Delta + \begin{bmatrix}\gamma_g^{\Delta^3}\\ \gamma_q^{\Delta^3}\end{bmatrix}\Delta^3$$

shift-invariant objects:

$$\Delta := T_q(1) - T_g(1)$$
$$\sigma_i(2) := T_i(2) - T_i(1)^2$$
$$\sigma_i(3) := T_i(3) - 3T_i(2)T_i(1) + 2T_i(1)^3$$

Here $\gamma_{ji}(n) = -\int_0^1 dz\, z^{n-1} P_{ji}(z, a_s)$ where P_{ji} denotes the singlet timelike splitting function.

RG Evolution

- We have computed the NLO evolution of the first three moments of the quark and gluon track functions.
- e.g. for gluons:

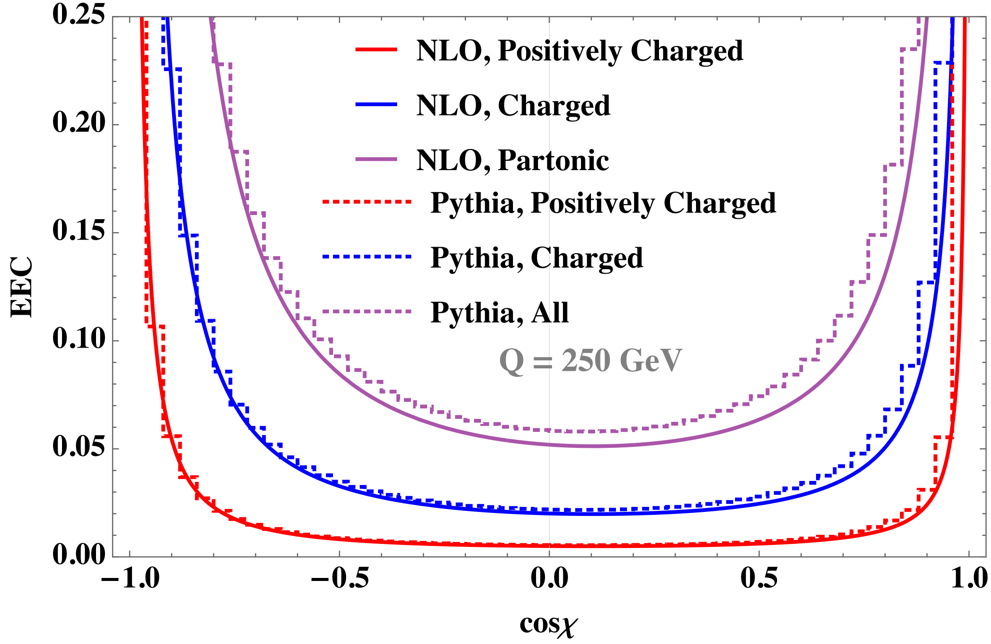
$$\frac{d}{d\ln\mu^2}T_g(2) = -\gamma_{gg}^{(1)}(3)T_g(2) + \sum_q\left(-2\gamma_{qg}^{(1)}(3)\right)T_q(2) + \left[C_A^2\left(-8\zeta_3+\frac{2158}{675}+\frac{26\pi^2}{45}\right)-\frac{4}{9}C_An_fT_F\right]T_g(1)T_g(1)+\cdots\ ,$$

$$\frac{d}{d\ln\mu^2}T_g(3) = -\gamma_{gg}^{(1)}(4)T_g(3) + \sum_q\left(-2\gamma_{qg}^{(1)}(4)\right)T_q(3) + \left[C_A^2\left(24\zeta_3+\frac{767263}{4500}-\frac{278\pi^2}{15}\right)-\frac{2}{3}C_An_fT_F\right]T_g(2)T_g(1)$$
$$+\sum_q\left[C_AT_F\left(\frac{23051}{1125}-\frac{28}{15}\pi^2\right)-C_FT_F\frac{28}{15}\right]T_g(1)T_q(1)T_q(1)+\cdots\ .$$

- Importantly, this allows us to predict and absorb IR poles in energy correlator event shapes to $\mathcal{O}(\alpha_s^2)$

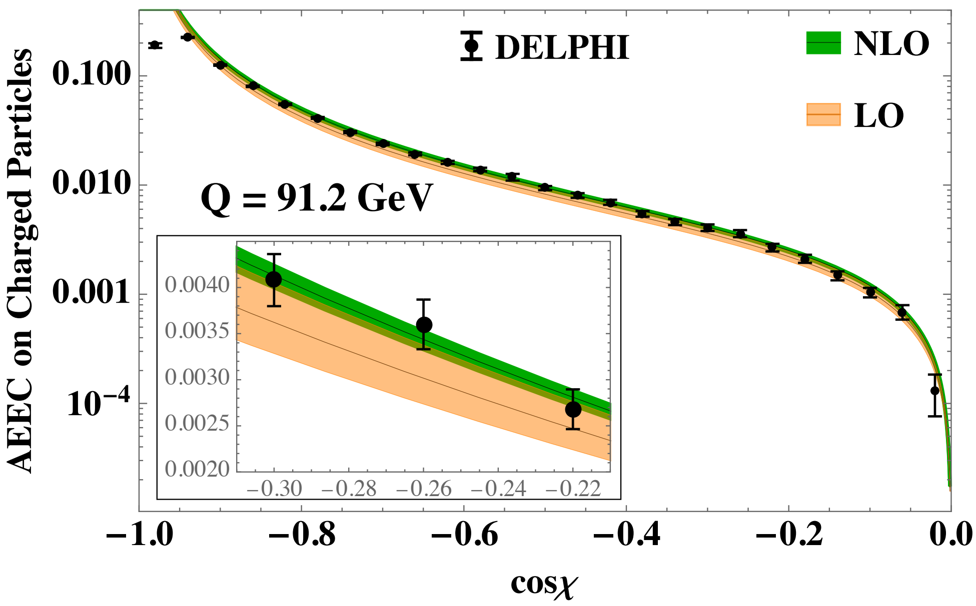
Preliminary Track Phenomenology

- We can now begin to ask more refined questions about energy flow, while maintaining state of the art perturbative precision.



Preliminary Track Phenomenology

- Higher order predictions with tracks!



Interplay with Factorization Formulas

- Interfaces nicely with standard factorization formulas, allowing high order resummation on track based observables.

Jet Substructure

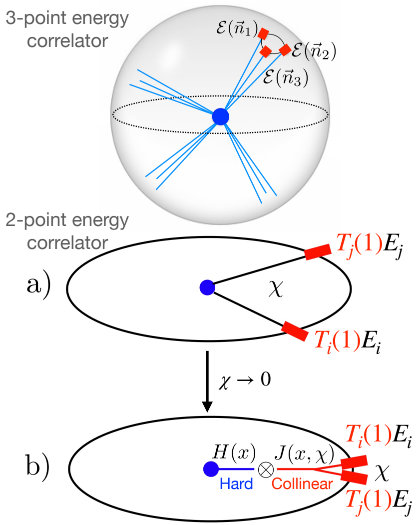
In the collinear limit:

- The energy correlator is a jet substructure observable.
- Jet function constants (jet functions with the logarithmic dependence excluded):
  - The moments $T_i(n)$ appear as the coefficients.
  - e.g. for track EECs, up to $\mathcal{O}(\alpha_s^2)$

$$j^g = \frac{1}{4}T_g(2) + a_s\left\{T_g(1)T_g(1)C_A\left(-\frac{449}{150}\right) + \sum_q T_q(1)T_{\bar{q}}(1)T_F\left(-\frac{7}{25}\right)\right\}$$
$$+a_s^2\left\{T_g(1)T_g(1)\left\{C_A^2\left(-\frac{527\zeta(3)}{10} + \frac{133639871}{3240000} - \frac{2159\pi^2}{1800} + \frac{19\pi^4}{90}\right) + C_A n_f T_F\frac{139}{270}\right\} + \sum_q T_q(1)T_{\bar{q}}(1)\cdots\right\}$$

- Matches the state-of-the-art calculation for jet substructure, but now on tracks!

[Kardos, Larkoski, Trocsanyi, 2002.05730]



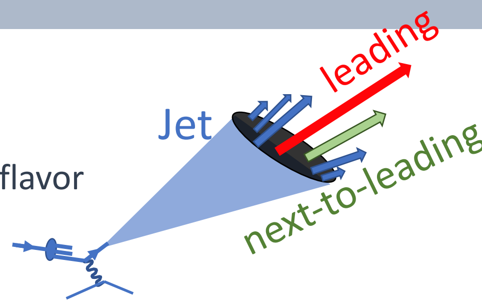
Towards Interesting Questions

- Ability to probe charged energy flow opens the door to an interesting class of questions.

New charge-energy correlation

Observable : charge-energy correlation, r_c

- Correlations in momentum, charge and flavor
- **Leading(L)** and **next-to-leading (NL)** momentum particles in a jet



$$r_c \equiv \frac{N_{CC} - N_{C\bar{C}}}{N_{CC} + N_{C\bar{C}}}$$

N_{CC} : # Jets where L and NL particles with same sign charges

$N_{C\bar{C}}$: # Jets where L and NL particles with opposite sign charges

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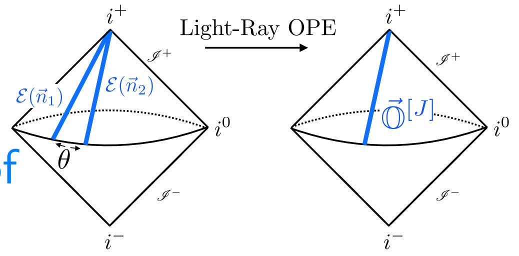
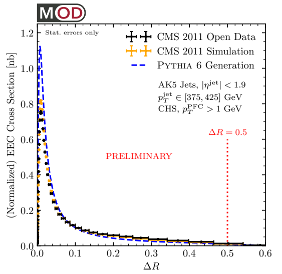
- This can be formulated as the correlator $\langle \mathcal{E}_+(n_1)(\mathcal{E}_-(n_2) - (\mathcal{E}_+(n_2)))\rangle$, and computed to high perturbative accuracy.

Summary

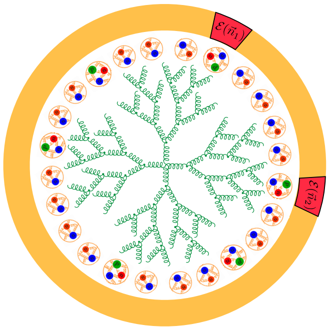
- There has been significant recent progress understanding the theoretical structure of energy correlator observables from a number of directions.

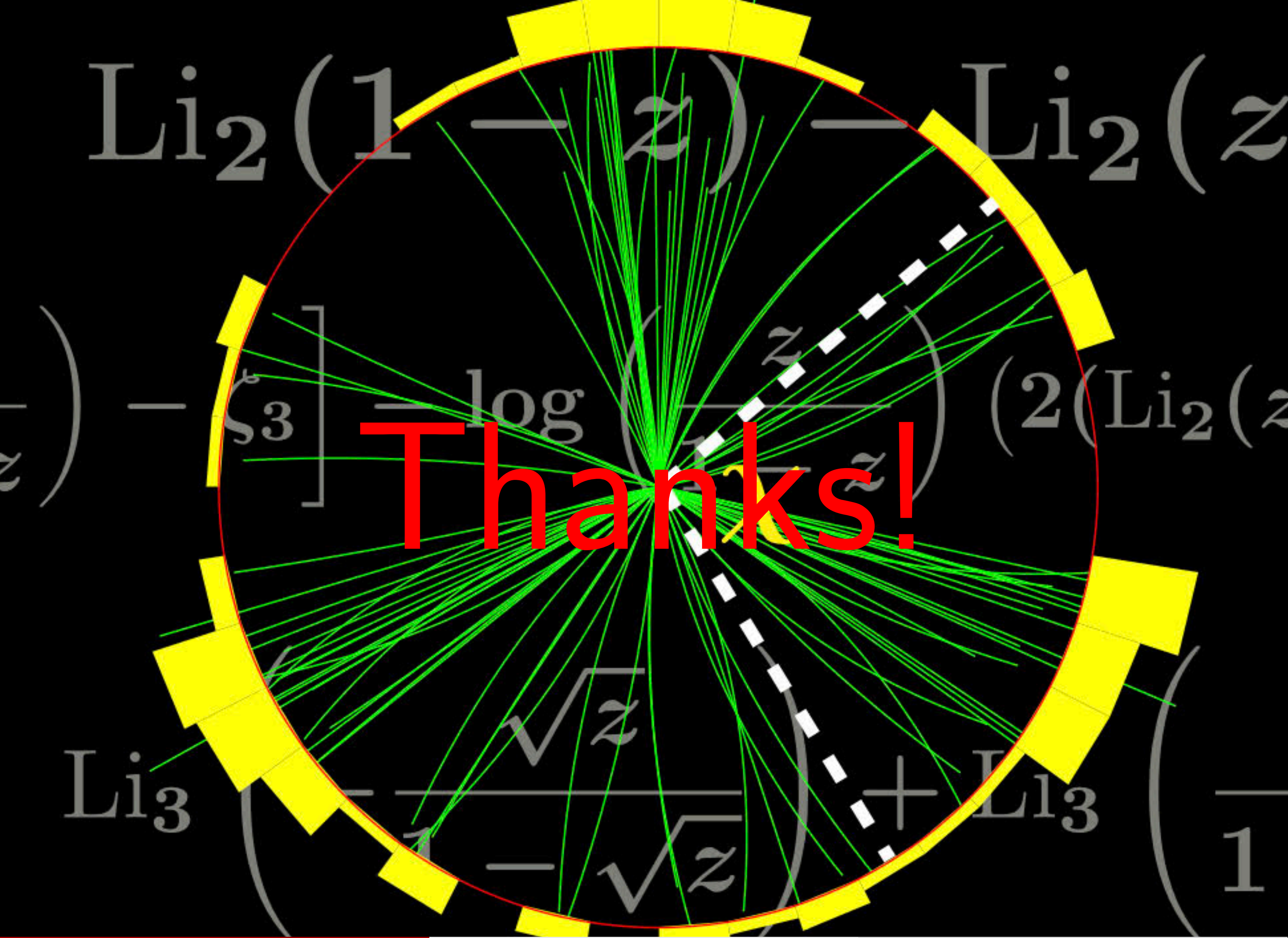


- Energy Correlators can be directly measured, and provide detailed probes of the physics of QCD jets.



- Track functions provide a bridge between perturbative and non-perturbative physics.





Thanks!