Heavy Quark Jet Production near Threshold

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[JHEP09(2021)148, PhysRevD.95.074003(2017)]

2nd Workshop on Jets for 3D Imaging at the EIC

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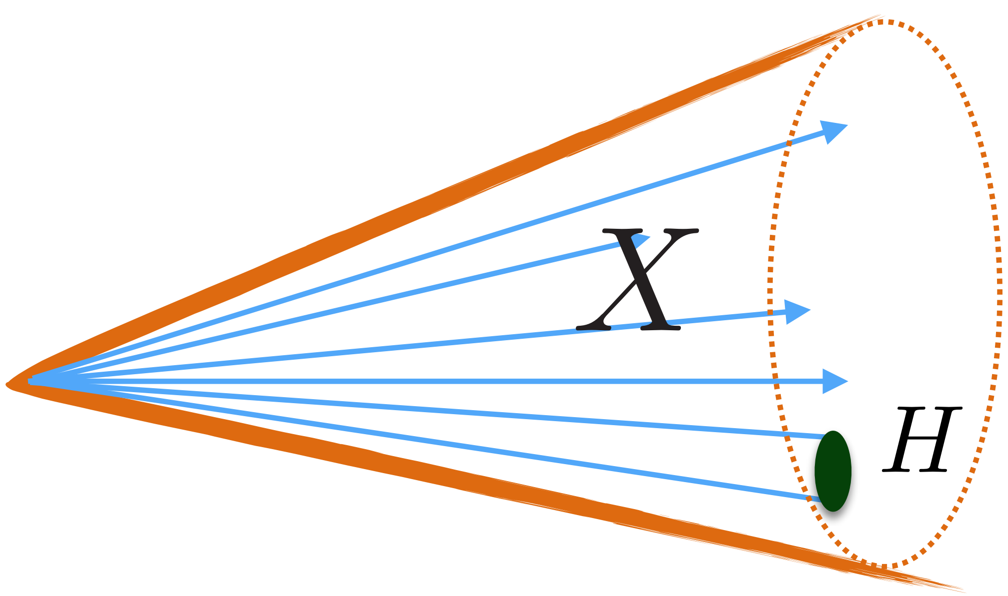
Outline

- Introduction (FJF near threshold)
- Fragmentation to a Jet (Ungroomed)
- Fragmentation to a Jet (Groomed)
- e^+e^- => inclusive heavy jets production
- Summary

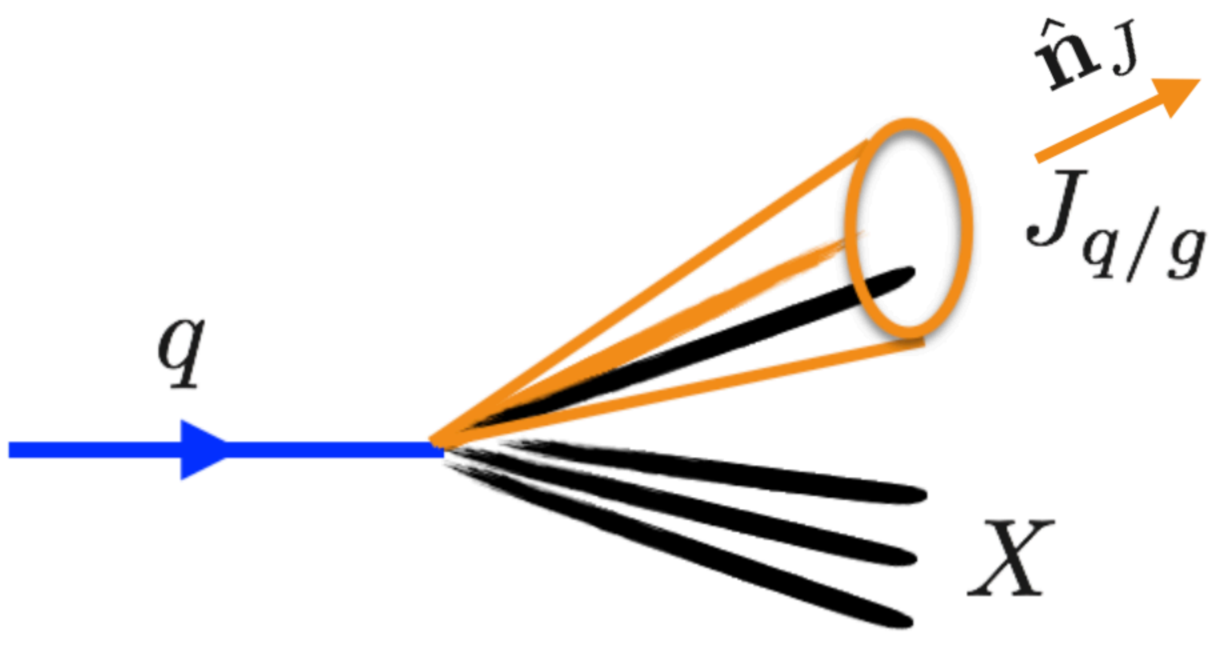
Fragmentation Function to a Jet (FFJ)

JFF, FJF



[Y. Chien, Z. Kang, F. Ringer, I. Vitev, and H. Xing, *JHEP05(2016)125*]
[M. Procura, I. Stewart, *PRD, 81, 074009 (2010)*]

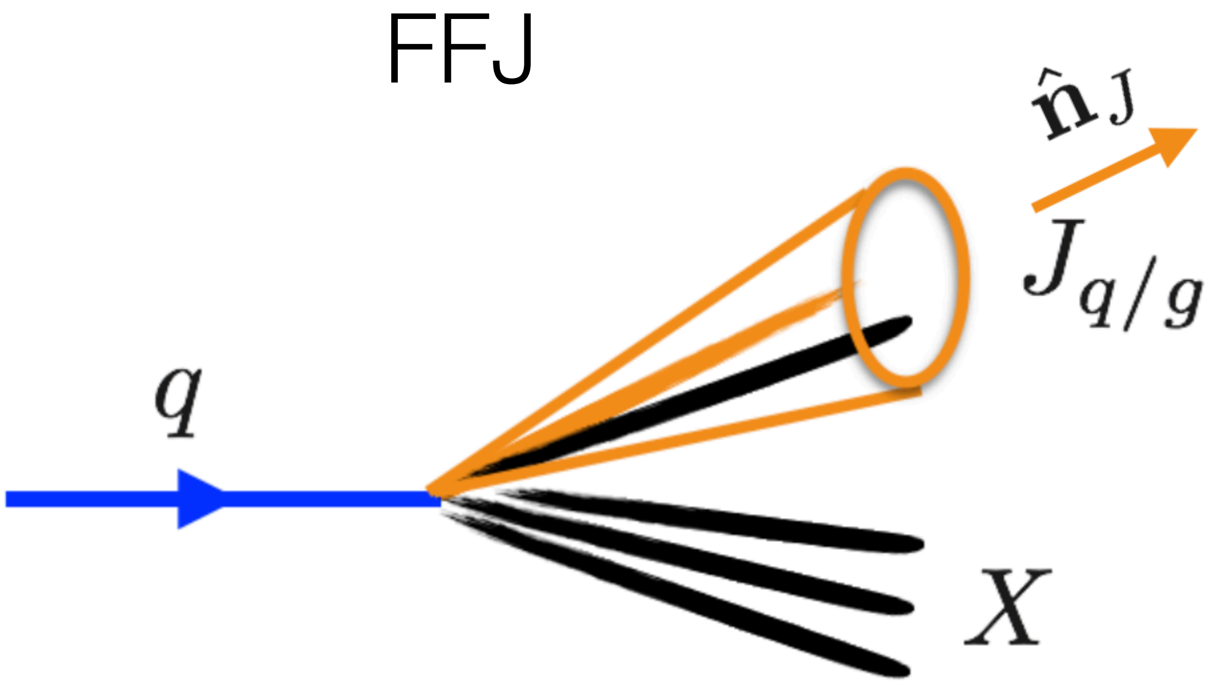
FFJ



$$z = \frac{E_J}{E_q}$$

[LD, C. Kim, and A. Leibovich, *PhysRevD.94.114023(2016)*]

FFJ near Threshold



Threshold:

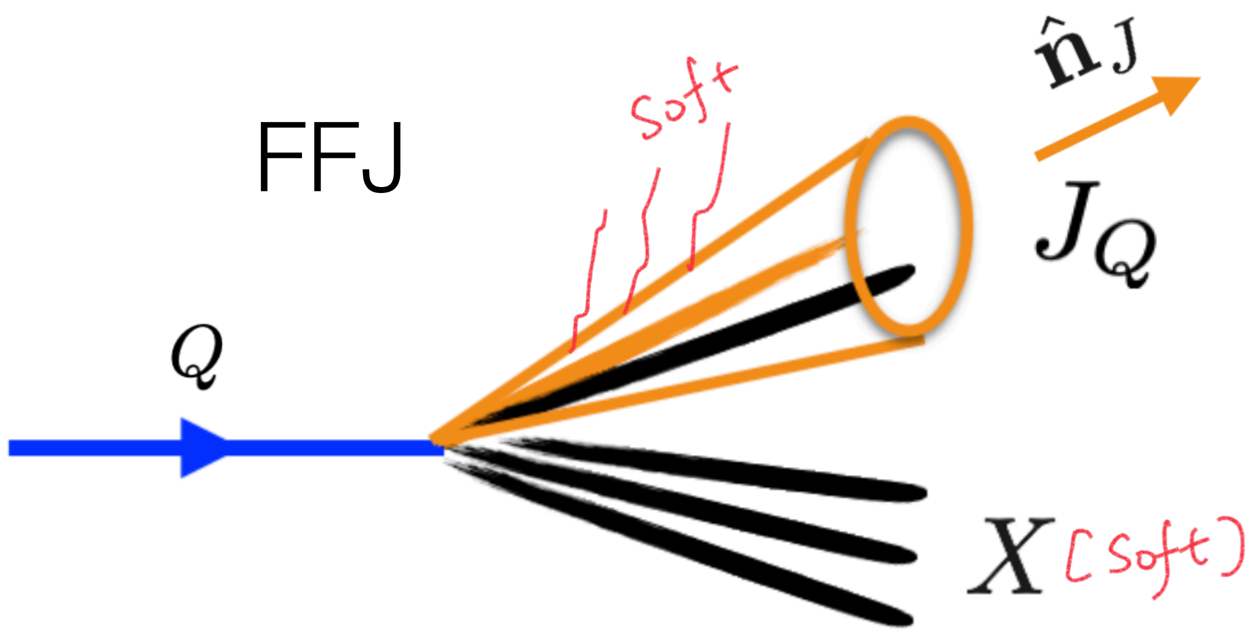
$$z = \frac{E_J}{E_q} \to 1$$

$q \to J_g + q$ Supressed by (1-z), $\boxed{q \to J_q + g}$

ln(1-z) large

[LD, C. Kim, and A. Leibovich, *PhysRevD.95.074003(2017)*]

FFJ near Threshold



$$z = \frac{E_J}{E_q} \to 1$$

$$E_{soft} \lesssim (1-z)E_J$$

- Work in the framework of SCET

- In this talk, focus on heavy quark

- Adding a csoft mode to resume ln(1-z). Should scale as:

$$p_{cs} = \left(p_{cs}^{+}, p_{cs}^{\perp}, p_{cs}^{-}\right) \sim (1-z)E_J\left(1, R, R^2\right)$$

[LD, C. Kim, and A. Leibovich, *JHEP09(2021)148*]
[M. Procura W. Waalewijn, and L. Zeune, JHEP 02(2015) 117]

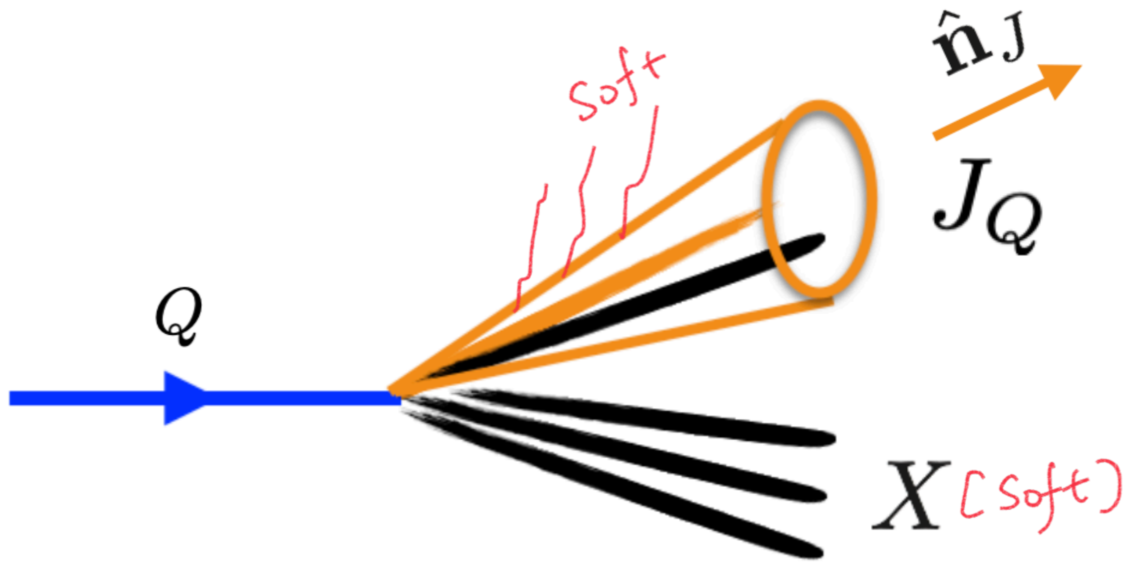
FFJ Definition

$$D_{J/Q}(z; E_J R', m, \mu) = \sum_X \frac{1}{2N_c z} \int d^{D-2}\mathbf{p}_J^{\perp} \mathrm{Tr}\langle 0|\delta\Big(\frac{p_J^+}{z} - \mathcal{P}_+\Big)\delta^{(D-2)}(\boldsymbol{\mathcal{P}}_\perp)\frac{\bar{n}\!\!\!/}{2}\Psi_n^Q$$

$$\times |J(p_J^+, \mathbf{p}_J^{\perp})X_{\notin J}\rangle\langle J(p_J^+, \mathbf{p}_J^{\perp})X_{\notin J}|\bar{\Psi}_n^Q|0\rangle,$$

- LO: $\delta(1-z)$

- Generalization of FF



[LD, C. Kim, and A. Leibovich, *PhysRevD.94.114023(2016)*]

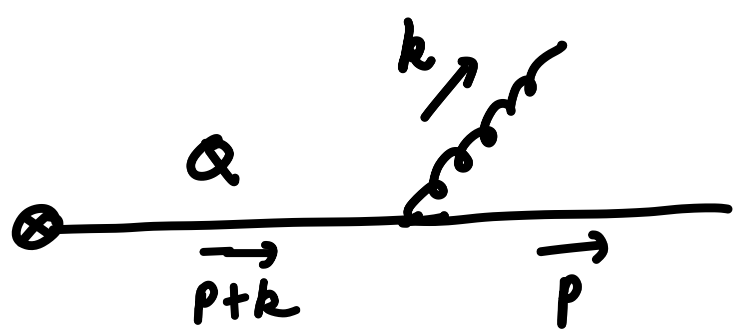
Integrating out Collinear Integration

- Integrate out collinear int.

$$p_c^2 \sim E^2R^2, p_{cs}^2 \sim E^2R^2(1-z)^2$$

- Denominator of heavy Quark Propagator:

$$(p+k)^2 - m^2 = k^2 + 2p\cdot k \approx 2p\cdot k \sim 2mv$$



  - $v \sim p/m_Q$
  - bHQET: $\xi_n(x) = \sqrt{\frac{v_+}{2}} e^{-imv\cdot x} h_n(x)$
  - $\mathcal{L}^{(0)}_{\mathrm{bHQET}} = \bar{h}_n v\cdot iD \frac{\bar{\not{n}}}{2} h_n$

Integrating out Collinear Interaction

$$D_{J/Q}(z;E_JR',m,\mu) = \sum_X \frac{1}{2N_c z}\int d^{D-2}\mathbf{p}_J^{\perp}\,\mathrm{Tr}\langle 0|\delta\Big(\frac{p_J^+}{z}-\mathcal{P}_+\Big)\delta^{(D-2)}(\boldsymbol{\mathcal{P}}_\perp)\frac{\bar{n}\!\!\!/}{2}\Psi_n^Q$$
$$\times\,|J(p_J^+,\mathbf{p}_J^\perp)X_{\notin J}\rangle\langle J(p_J^+,\mathbf{p}_J^\perp)X_{\notin J}|\bar{\Psi}_n^Q|0\rangle,$$

(Integrating out collinear interaction)

$$D_{J/Q}(z\to 1;E_JR',m,\mu) = \mathcal{J}_Q(E_JR',m,\mu)S_{J/Q}(z;E_JR',m,\mu).$$

Csoft-Function

where,

$$\mathcal{J}_Q(E_JR',m,\mu) = 1+\frac{\alpha_s C_F}{2\pi}\Bigg[\frac{1}{\epsilon^2}+\frac{1}{\epsilon}\Big(\ln\frac{\mu^2}{E_J^2R'^2+m^2}+\frac{3+b}{2(1+b)}\Big)+\frac{1}{2}\ln\frac{\mu^2}{E_J^2R'^2+m^2}$$
$$+\frac{1}{1+b}\Big(\ln\frac{\mu^2}{E_J^2R'^2}+2\Big)+\frac{1}{2}\ln^2\frac{\mu^2}{E_J^2R'^2+m^2}-\frac{1}{2}\ln^2(1+b)$$
$$+f(b)+g(b)-\mathrm{Li}_2(-b)+2-\frac{\pi^2}{12}\Bigg].$$

$b \equiv m^2/(E_J^2R'^2)$

UV & Check massless limit

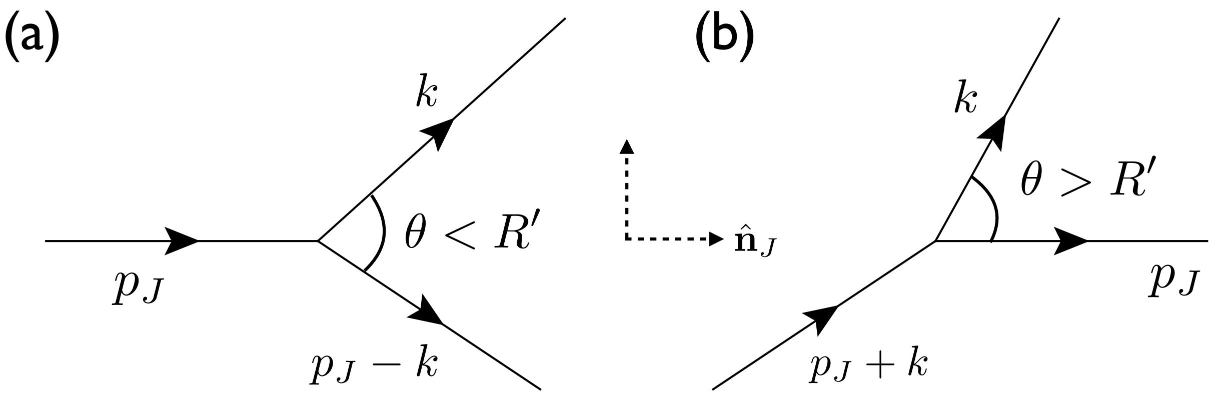
$$S_{J/Q}(z;E_JR',m,\mu) = \sum_{X_{cs}}\frac{1}{2N_c}\mathrm{Tr}\frac{v_+}{2}\langle 0|\,\delta\big((1-z)p_J^+-i\partial_+\big)Y_{\bar{n}}^{cs\dagger}h_n\,|J(p_J^+,\mathbf{p}_J^\perp=0)X_{\notin J}^{cs}\rangle$$
$$\times\langle J(p_J^+,\mathbf{p}_J^\perp=0)X_{\notin J}^{cs}|\,\bar{h}_nY_{\bar{n}}^{cs}\frac{\bar{n}\!\!\!/}{2}\,|0\rangle,$$

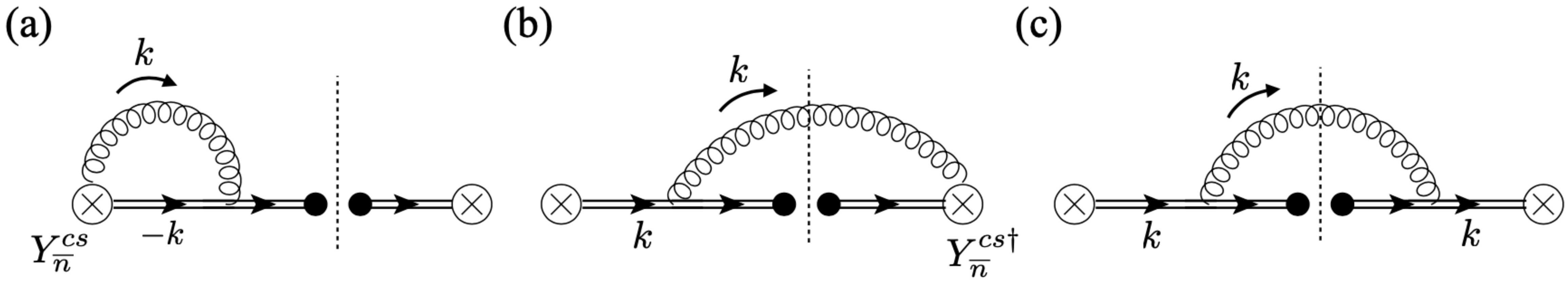
[LD, C. Kim, and A. Leibovich, *JHEP09(2018)109*]

CSoft Function Calculation

$$S_{J/Q}(z;E_JR',m,\mu)=\sum_{X_{cs}}\frac{1}{2N_c}\mathrm{Tr}\frac{\not{v}_+}{2}\langle 0|\,\delta\big((1-z)p_J^+-i\partial_+\big)Y_{\overline{n}}^{cs\dagger}h_n\,|J(p_J^+,\mathbf{p}_J^\perp=0)X_{\notin J}^{cs}\rangle$$
$$\times\langle J(p_J^+,\mathbf{p}_J^\perp=0)X_{\notin J}^{cs}|\,\bar{h}_nY_{\overline{n}}^{cs}\frac{\not{\overline{n}}}{2}\,|0\rangle,$$

NLO: In, out Jet





CSoft Function Calculation

$$S^{(1)}_{J/Q}(z) = \frac{\alpha_s C_F}{2\pi}\Big\{\delta(1-z)\Big[-\frac{1}{\epsilon^2} - \frac{1}{\epsilon}\ln\frac{\mu^2}{E_J^2R'^2+m^2} + \frac{b}{1+b}\big(\frac{1}{\epsilon} + \ln\frac{\mu^2}{E_J^2R'^2+m^2}\big)$$
$$-\frac{1}{2}\ln^2\frac{\mu^2}{E_J^2R'^2+m^2} - \frac{1}{1+b}\ln(1+b) + \frac{1}{2}\ln^2(1+b) + \frac{\pi^2}{12} + \mathrm{Li}_2(-b)\Big]$$
$$\Big[\frac{2}{1-z}\big(\frac{1}{\epsilon} + \ln\frac{\mu^2}{(E_J^2R'^2+m^2)(1-z)^2} - \frac{b}{1+b}\big)\Big]_+\Big\},$$

$$b \equiv m^2/(E_J^2R'^2)$$

- Poles are UV

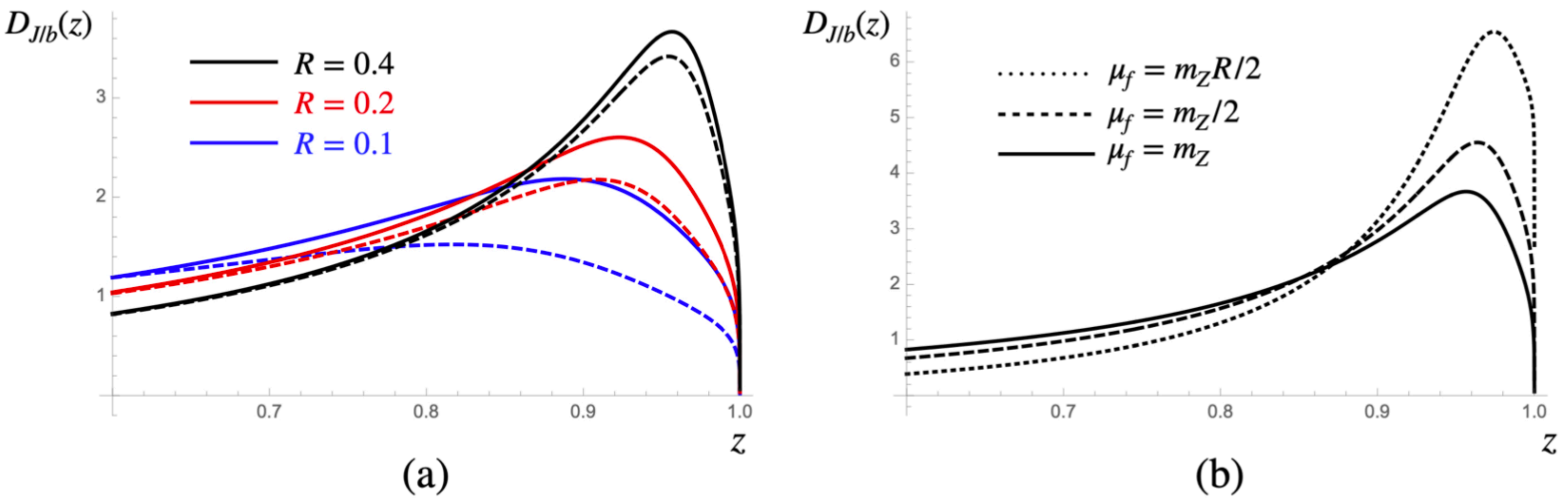
- Natural scale: $\mu_{cs} \sim (1-z)\sqrt{(E_JR')^2+m^2}$

- Resum ln(1-z):

$$\begin{aligned} D_{J/Q}(z\to 1; E_JR', m, \mu_f) &= \mathcal{J}_Q(E_JR', m, \mu_f)S_{J/Q}(z; E_JR', m, \mu_f) \\ &= \exp[\mathcal{M}(\mu_f, \mu_c, \mu_{cs})]\mathcal{J}_Q(E_JR', m, \mu_c)(1-z)^{-1+\eta} \\ &\quad \times \tilde{S}_{J/Q}\Big[\ln\frac{\mu_{cs}^2}{B^2(1-z)^2} - 2\partial_\eta\Big]\frac{e^{-\gamma_E\eta}}{\Gamma(\eta)}. \end{aligned}$$

FJF Near Threshold

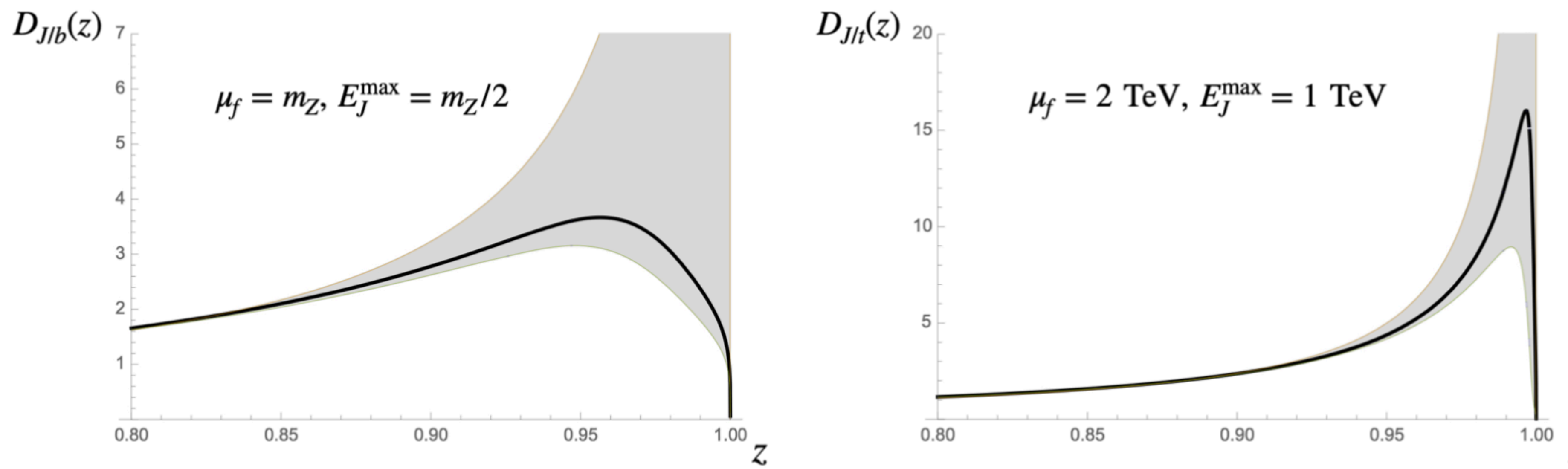
$$
\begin{aligned}
D_{J/Q}(z \to 1; E_J R', m, \mu_f) &= \mathcal{J}_Q(E_J R', m, \mu_f) S_{J/Q}(z; E_J R', m, \mu_f) \\
&= \exp[\mathcal{M}(\mu_f, \mu_c, \mu_{cs})] \mathcal{J}_Q(E_J R', m, \mu_c)(1-z)^{-1+\eta} \\
&\quad \times \tilde{S}_{J/Q}\Big[\ln \frac{\mu_{cs}^2}{B^2(1-z)^2} - 2\partial_\eta\Big] \frac{e^{-\gamma_E \eta}}{\Gamma(\eta)}.
\end{aligned}
$$



[LD, C. Kim, and A. Leibovich, *JHEP09(2021)148*]

FJF Near Threshold

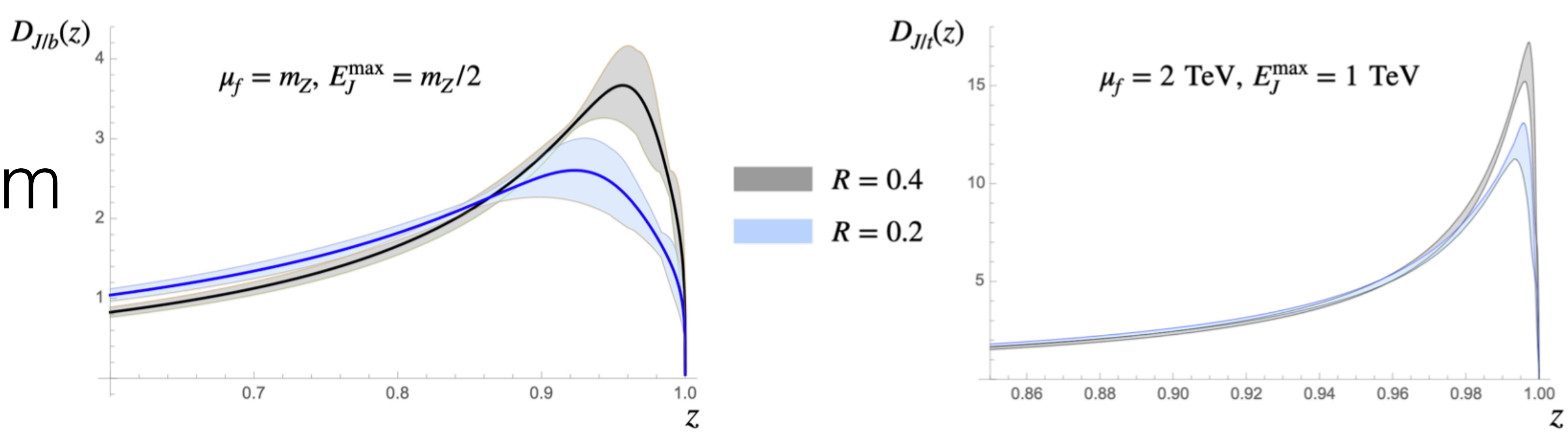
DGLAP



(a) (b)

$$D_{J/Q}(z \to 1; E_J R', m, \mu_f) = \exp[\tilde{\mathcal{M}}(\mu_f, \mu_J)] D_{J/Q}(z \to 1; E_J R', m, \mu_J).$$

ln(1-z) Resum



(a) (b)

[LD, C. Kim, and A. Leibovich, *JHEP09(2021)148*]

Groomed FFJ

- Soft drop: $\frac{\min(E_{j_1}, E_{j_2})}{E_{j_1} + E_{j_2}} > z_{\rm cut} \left(\frac{\theta_{12}}{R}\right)^{\beta}$

[A. Larkoski, S. Marzani, G. Soyez, and J. Thaler, *JHEP05(2014)146*]

- A new scale z_{cut} (small like, 0.1) complicates factorization

$$\frac{d\sigma}{dE_J dz_G} = \int_{x=2E_J/Q}^{1} \frac{dz}{z} \frac{d\sigma(x/z;\mu)}{dE_i} D_{J_Q/i}(z; E_J R, \mu) \Phi_G(z_G)$$

$$z_G = p^+_{J_G}/p^+_J$$

$$\Phi_G(z_G; z_{\rm cut}, E_J R, m) = \sum_{X_{cs}} \frac{1}{2N_c} \mathrm{Tr} \frac{\not{v}_+}{2} \langle 0|\, \delta\big((1-z_G)p^+_J - \Theta_{\rm in} \cdot i\partial_+\big) Y^{cs\dagger}_{\bar{n}} h_n \,|J_G X^{cs}_{\notin J_G}\rangle$$
$$\times \langle J_G(p^+_{J_G}, \mathbf{p}^\perp_{J_G} = 0) X^{cs}_{\notin J_G}|\, \bar{h}_n Y^{cs}_{\bar{n}} \frac{\not{\bar{n}}}{2}\, |0\rangle,$$

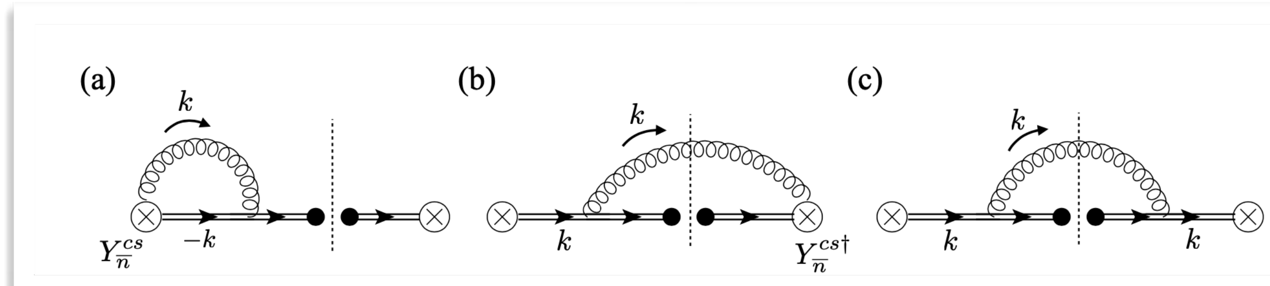
- Near the threshold (1-z) < z_{cut}, only in-jet frag. contribute
- Grooming Function scale indep.

Groomed FFJ

$$\Phi_G(z_G; z_{\rm cut}, E_J R, m) = \sum_{X_{cs}} \frac{1}{2N_c} \mathrm{Tr}\frac{\not{v}_+}{2} \langle 0|\, \delta\big((1-z_G)p_J^+ - \Theta_{\rm in}\cdot i\partial_+\big) Y_{\bar{n}}^{cs\dagger} h_n \,|J_G X^{cs}_{\notin J_G}\rangle$$
$$\times \langle J_G(p^+_{J_G}, \mathbf{p}^\perp_{J_G}=0) X^{cs}_{\notin J_G}|\, \bar{h}_n Y_{\bar{n}}^{cs} \frac{\not{\bar{n}}}{2}\,|0\rangle,$$

(NLO)

$$\Phi_G^{(1)}(z_G) = \Big[\int_{gr} d\mathbf{k}_\perp^2 M_R(1-z_G, \mathbf{k}_\perp^2)\Big]_+$$
$$= \frac{\alpha_s C_F}{\pi}\left[\frac{\Theta(z_G+z_{\rm cut}-1)}{1-z_G}\left(\ln\frac{1+b}{\left(\frac{1-z_G}{z_{\rm cut}}\right)^{2/\beta}+b} + \frac{b}{1+b} - \frac{b}{\left(\frac{1-z_G}{z_{\rm cut}}\right)^{2/\beta}+b}\right)\right]_+$$



$b \equiv m^2/(E_J^2 R'^2)$

- Only plus distribution is left.
- Massless limit: $\Phi_G^{(1)}(z_G; m=0) = \frac{\alpha_s C_F}{\pi}\frac{2}{\beta}\left[\frac{\Theta(z_G+z_{\rm cut}-1)}{1-z_G}\ln\frac{z_{\rm cut}}{1-z_G}\right]_+$
- Heavy Quark, Infrared safe:

$$\Phi_G^{(1)}(z_G; \beta=0) = \frac{\alpha_s C_F}{\pi}\left[\frac{\Theta(z_G+z_{\rm cut}-1)}{1-z_G}\left(\ln\frac{1+b}{b} - \frac{1}{1+b}\right)\right]_+$$

ln(b)?

Groomed FFJ near Threshold

- Threshold region: $1 - z \ll z_{\rm cut}$
  - Csoft and CUSoft mode ($E_J R \sim m$):

$$p_{cs} \sim z_{\rm cut} E_J (1, R, R^2) \qquad p_{cus} \sim (1-z) E_J (1, R, R^2)$$

$$S_{J_G/Q}(z \to 1; z_{\rm cut}, E_J R, m, \mu) = S_G(z_{\rm cut}, E_J R, b, \mu) U_{cus}(z; m, \mu)$$

  - If $E_J R \gg m$

$$p_{cs} \sim z_{\rm cut} E_J (1, R, R^2) \qquad p_{ucs} \sim z_{\rm cut} E_J \left(1, \frac{m}{E_J}, \frac{m^2}{E_J^2}\right)$$

$$S_G(z_{\rm cut}, E_J R \gg m, \mu) = S_{G,0}(z_{\rm cut}, E_J R, \mu) U_{G,ucs}(z_{\rm cut}, m, \mu).$$

Groomed FFJ near Threshold

$$S_G(z_{\rm cut}, E_J R, m, \mu) = 1 + \frac{\alpha_s C_F}{2\pi}\Big[\Big(\ln\frac{1+b}{b} - \frac{1}{1+b}\Big)\ln\frac{\mu^2}{z_{\rm cut}^2 E_J^2 R^2} + \frac{\pi^2}{6} + \frac{1}{2}\ln^2 b + \mathrm{Li}_2(-b) - \ln\frac{1+b}{b}\Big].$$

$$E_J R \sim m$$

$$\mu_{cs} = z_{cut} E_J R$$

$$S_{G,0}(z_{\rm cut}, E_J R, \mu) = 1 + \frac{\alpha_s C_F}{2\pi}\Big(-\frac{1}{2}\ln^2\frac{\mu^2}{z_{\rm cut}^2 E_J^2 R^2} + \frac{\pi^2}{12}\Big)$$

$$U_{G,ucs}(z_{\rm cut}, m, \mu) = 1 + \frac{\alpha_s C_F}{2\pi}\Big(-\ln\frac{\mu^2}{z_{\rm cut}^2 m^2} + \frac{1}{2}\ln^2\frac{\mu^2}{z_{\rm cut}^2 m^2} + \frac{\pi^2}{12}\Big)$$

$$E_J R \gg m$$

$$\mu_{cs} = z_{cut} E_J R$$

$$\mu_{ucs} = z_{cut} m$$

$$U_{cus}(z; m, \mu) = \delta(1-z) + \frac{\alpha_s C_F}{2\pi}\Big\{\delta(1-z)\Big[\ln\frac{\mu^2}{m^2} - \frac{1}{2}\ln\frac{\mu^2}{m^2} - \frac{\pi^2}{12}\Big] + \Big[\frac{2}{1-z}\Big(\ln\frac{\mu^2}{m^2(1-z)^2} - 1\Big)\Big]_+\Big\}$$

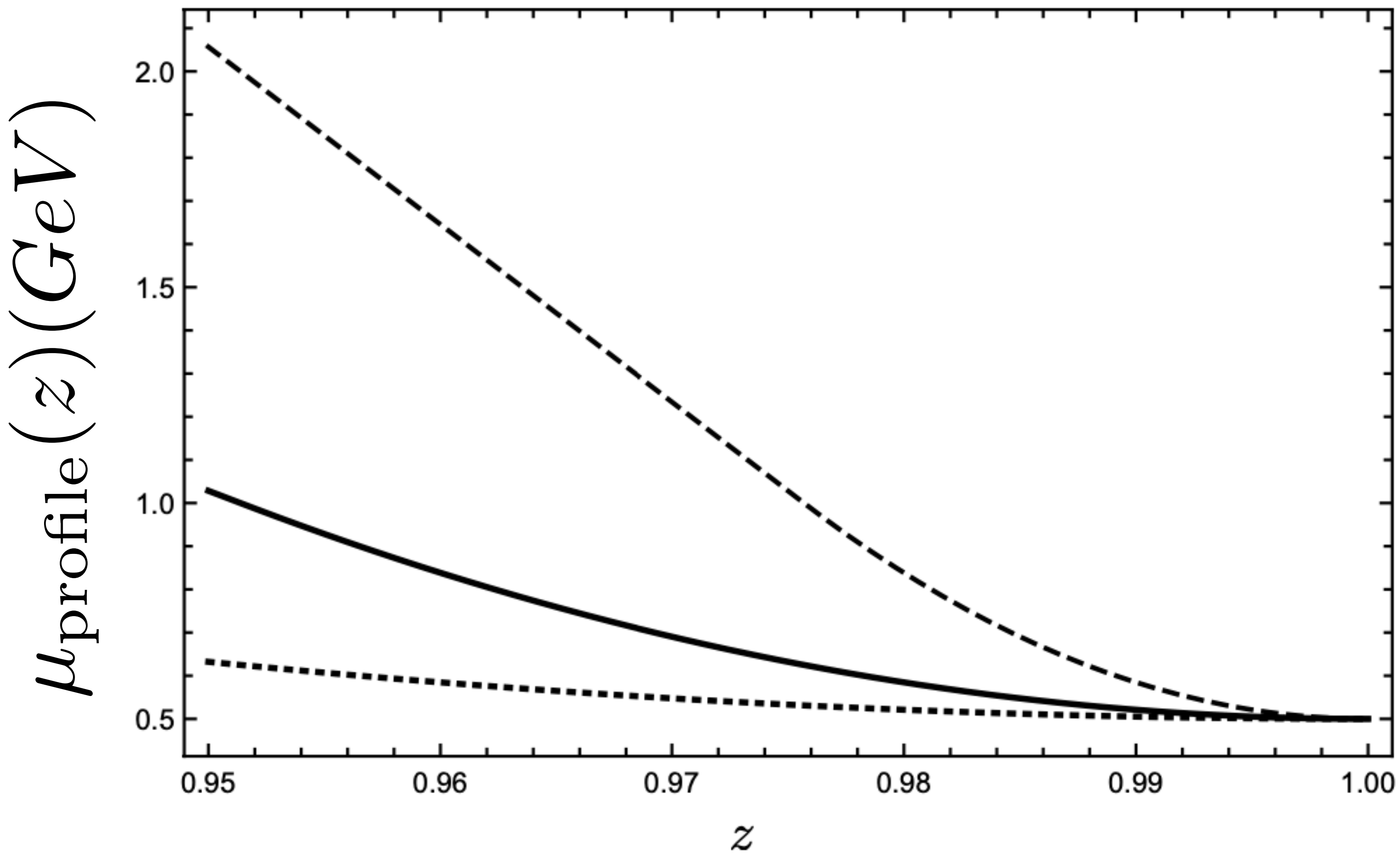
$E_J R \sim m \quad E_J R \gg m$

$$\mu_{cus} = (1-z)m$$

Groomed FFJ near Threshold $E_J R \sim m$

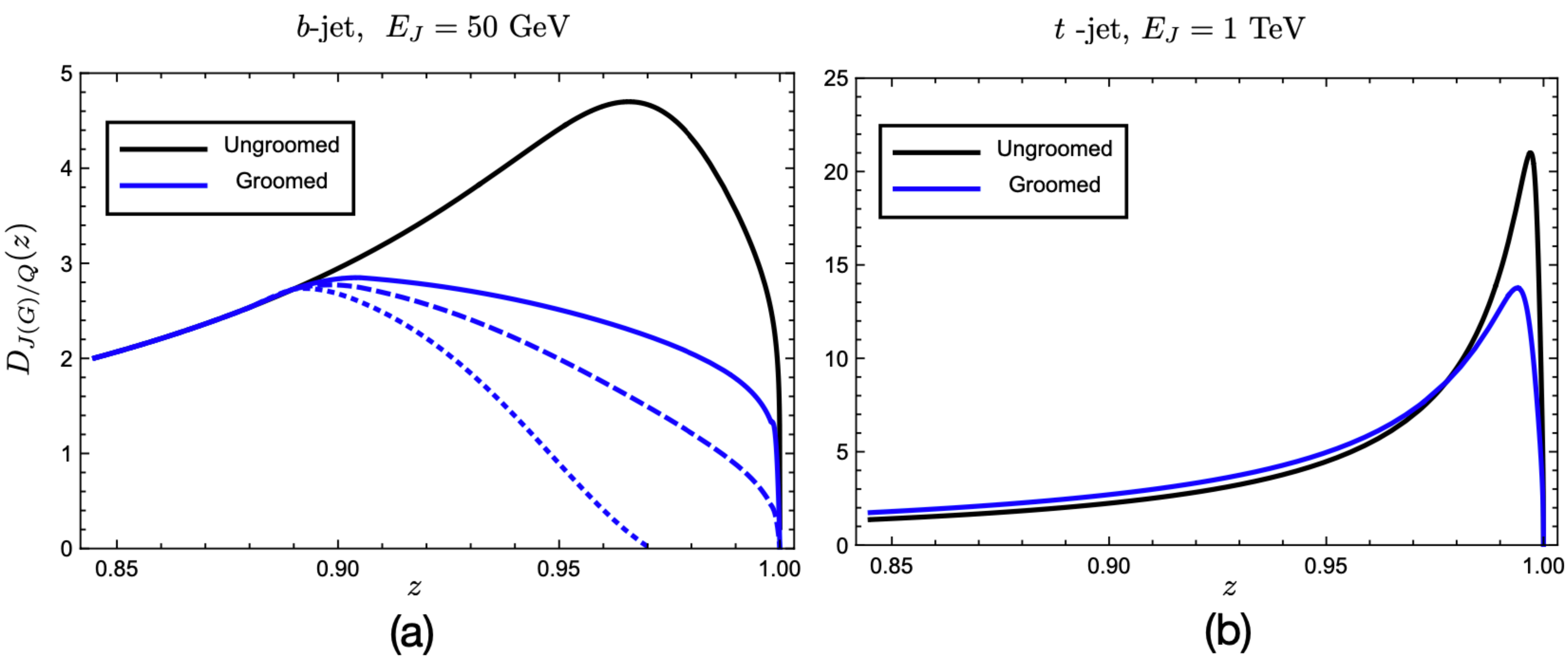
$$D_{J_G/Q}(z \to 1; E_J R, m, \mu) = \mathcal{J}_Q(E_J R, m, \mu) S_{J_G/Q}(z \to 1; E_J R, m, \mu_f)$$
$$= \mathcal{J}_Q(E_J R, m, \mu) S_G(z_{\rm cut}, E_J R, b, \mu) U_{cus}(z \to 1; m, \mu)$$

$\mu_{cs} = z_{cut} E_J R$ $\quad$ $\mu_{cus} = (1-z)m$ $\quad$ $\mu_{\rm profile}(z) = \begin{cases} \mu_0(1-z), & \text{for } z < x; \\ \mu_{min} + a\mu_0(1-z)^2, & \text{for } z \geq x. \end{cases}$



Groomed FFJ near Threshold $E_J R \sim m$

$$\mu_{cus} = (1-z)m$$

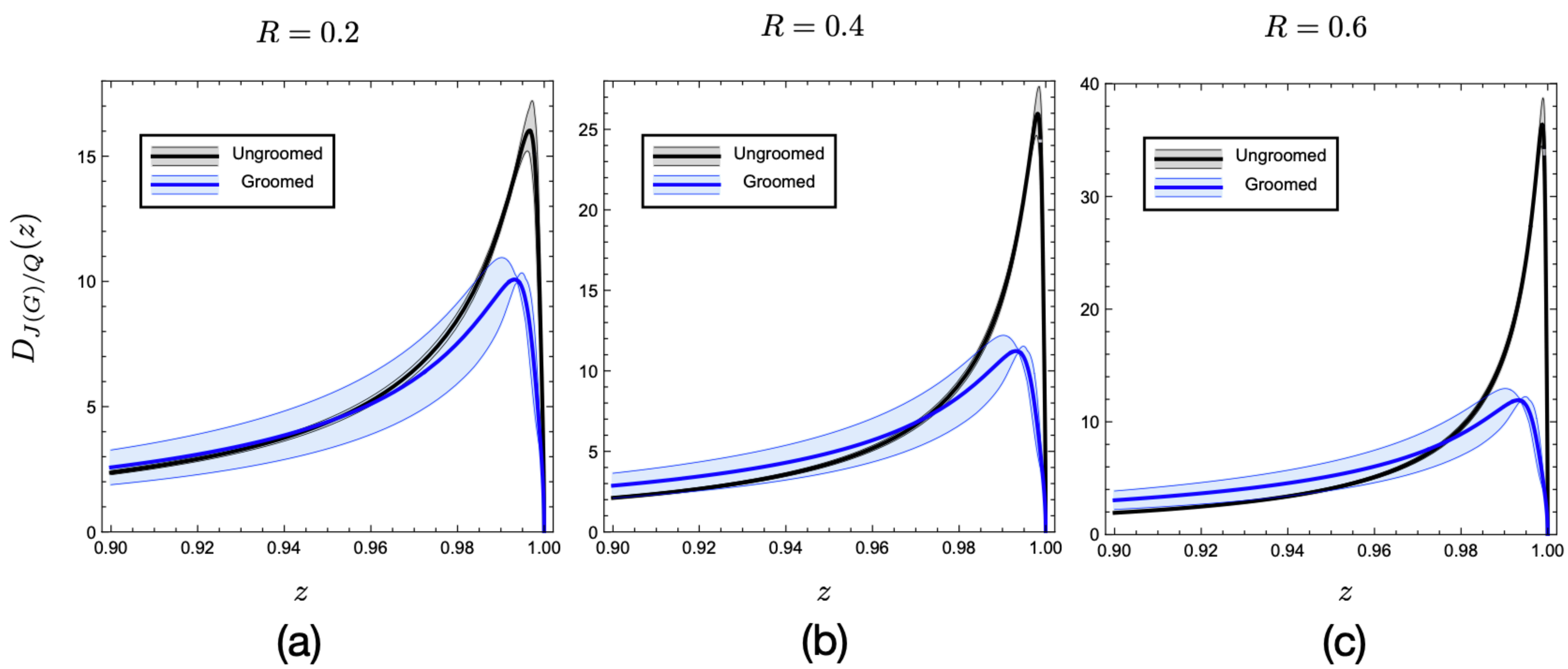


$$z_{cut} = 0.1, R = 0.4, \mu_{min} = 0.3, 0.4, 0.5 \text{ GeV}$$

[LD, C. Kim, and A. Leibovich, *JHEP09(2021)148*]

Groomed FFJ near Threshold $E_J R \gg m$

$$
\begin{aligned}
D_{J_G/Q}(z \to 1; E_J R, m, \mu_f) &= \mathcal{J}_Q(E_J R, m, \mu_f) S_{J_G/Q}(z; E_J R, m, \mu_f) \\
&= \exp[\mathcal{M}(\mu_f, \mu_c, \mu_{cs}, \mu_{ucs}, \mu_{cus})] \mathcal{J}_Q(E_J R, m, \mu_c) S_{G,0}(z_{\rm cut}, E_J R, \mu_{cs}) \\
&\quad \times (1-z)^{-1+\eta} U_{G,ucs}(z_{\rm cut}, m, \mu_{ucs}) \tilde{U}_{cus}\left[\ln \frac{\mu_{cus}^2}{m^2(1-z)^2} - 2\partial_\eta\right] \frac{e^{-\gamma_E \eta}}{\Gamma(\eta)},
\end{aligned}
$$



- Top jets: $E_J = 2$ TeV and $z_{\rm cut} = 0.1$.

[LD, C. Kim, and A. Leibovich, *JHEP09(2021)148*]

Groomed FFJ near Threshold $E_J R \gg m$

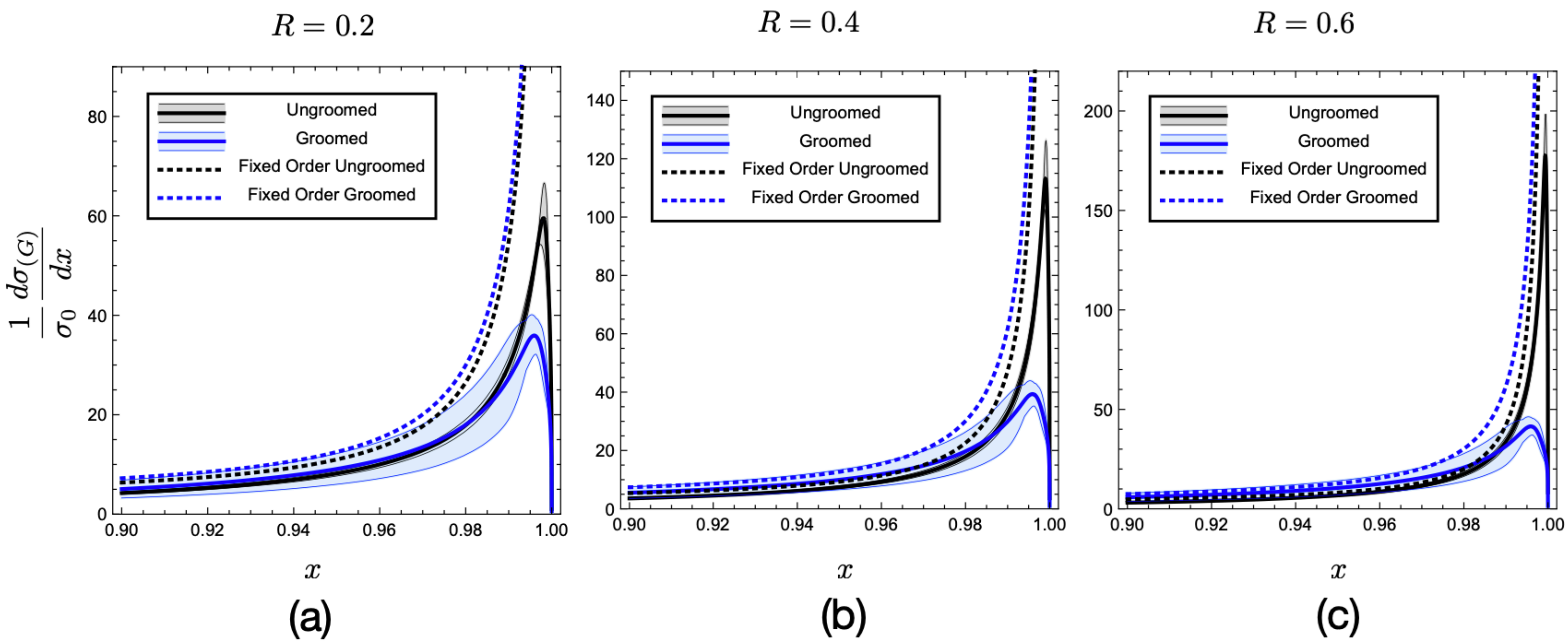
- e+e- => top jet:

$$\frac{1}{\sigma_0}\frac{d\sigma_G}{dx} = 2\exp[\mathcal{M}(\mu_h, \mu_{hc}, \mu_c, \mu_{cs}, \mu_{ucs}, \mu_{cus})]$$
$$\times H(Q, \mu_h)\mathcal{J}_Q(E_J R, m, \mu_c) S_{G,0}\left(z_{\rm cut} E_J R, \mu_{cs}\right) U_{G,ucs}\left(z_{\rm cut} m, \mu_{ucs}\right)$$
$$\times (1-x)^{-1+\eta} \tilde{J}_{\overline{n}}\Big[\ln\frac{\mu_{hc}^2}{Q^2(1-x)} - \partial_\eta\Big] \tilde{U}_{cus}\left[\ln\frac{\mu_{us}^2}{m^2(1-x)^2} - 2\partial_\eta\right]\frac{e^{-\gamma_E \eta}}{\Gamma(\eta)}$$

[LD, C. Kim, and A. Leibovich, *JHEP09(2021)148*]

Groomed FFJ near Threshold $E_J R \gg m$

- e+e- => top jet:



- Resum ln(1-z) is crucial

- Fixed order:

$$\frac{1}{\sigma_0}\left(\frac{d\sigma}{dx}\right)_{\text{fix}} \approx \frac{\alpha_s C_F}{\pi}\frac{1}{1-x}\ln\frac{Q^2}{B^2(1-x)} \sim \frac{\alpha_s C_F}{\pi}\frac{1}{1-x}\ln\frac{Q^2}{(E_J R)^2(1-x)},$$

$$\frac{1}{\sigma_0}\left(\frac{d\sigma_G}{dx}\right)_{\text{fix}} \approx \frac{\alpha_s C_F}{\pi}\frac{1}{1-x}\ln\frac{Q^2}{m^2(1-x)},$$

[LD, C. Kim, and A. Leibovich, *JHEP09(2021)148*]

Summary

- Study FFJ near threshold for heavy quarks (both ungroomed and groomed jets)
- Heavy quark mass as an IR regulator makes grooming IR safer
- Using RG, resum different logs involved: ln(1-z), log($E_J R z_{cut}$)…