Energy-energy correlators in DIS

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Based on the works arXiv:2006.02437, arXiv:2102.05669
In collaboration with Yiannis Makris, Ivan Vitev and YuJiao Zhu

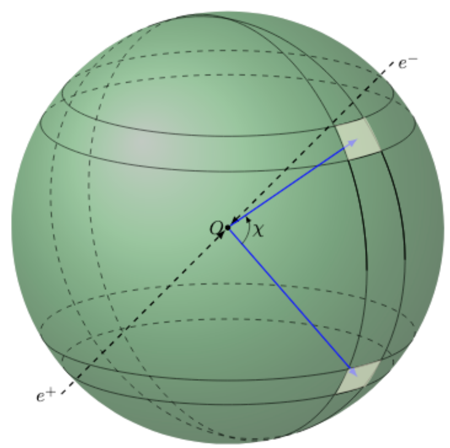
2nd Workshop on Jets for 3D Imaging at the EIC

09-29-2021

Introduction to EEC/TEEC

Basham et al 1978

e^+e^- Collisions

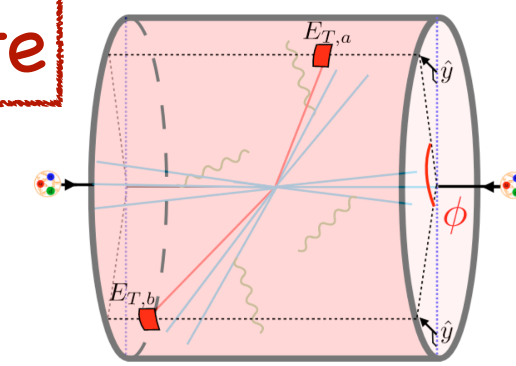


$$\mathrm{EEC} = \sum_{a,b} \int d\sigma_{V\to a+b+X} \frac{2E_a E_b}{Q^2 \sigma_{\mathrm{tot}}} \delta(\cos(\theta_{ab}) - \cos(\chi))$$

- sum over all the jets for each event
- sum over all the particles for each event

Ali et al 1984

Hadronic initial state



observable

$$\mathrm{TEEC} = \sum_{a,b} \int d\sigma_{pp\to a+b+X} \frac{2E_{T,a} E_{T,b}}{|\sum_i E_{T,i}|^2} \delta(\cos\phi_{ab} - \cos\phi)$$

- weighted cross section
- the soft radiation does not contribute directly to the observable at leading power
- soft gluon contributes only via recoil

Introduction to EEC/TEEC

EEC/TEEC is a class of observables which can be studied for various processes

In DIS

$$\text{TEEC} = \sum_a \int d\sigma_{lp\to l+a+X} \frac{E_{T,l}E_{T,a}}{E_{T,l}\sum_i E_{T,i}} \delta(\cos\phi_{la} - \cos\phi)$$

For Drell-Yan

$$\frac{d\sigma}{d\cos\phi} = \int d\sigma_{pp\to l^+ + l^- + X} \delta\left(\cos\phi_{l^+l^-} - \cos\phi\right)$$

For V+Jets

$$\frac{d\sigma}{d\cos\phi} = \sum_a \int d\sigma_{pp\to V+a+X} \frac{E_V E_{T,a}}{E_V \sum_i E_{T,i}} \delta\left(\cos\phi_{Va} - \cos\phi\right)$$

- From the definition, for (T)EEC contribution from soft radiations is suppressed by construction
- (T)EEC is simply defined in comparison with other event shapes
- It is calculable at high orders

Introduction to EEC/TEEC

EEC predictions

Analytical NLO *Dixon et al arXiv:1801.03219*

NNLO *Vittorio Del Duca et al arXiv:1606.03453*

NNLL+NLO *Florian, Grazzini arXiv:hep-ph/0407241*

NNLL+NNLO *Tulipánt, et al arXiv:1708.04093*

NNNLL$'$+NNLO *Elbert, et al, arXiv:2012.07859*

3-loop soft function *Moult, Zhu arXiv:1801.02627*

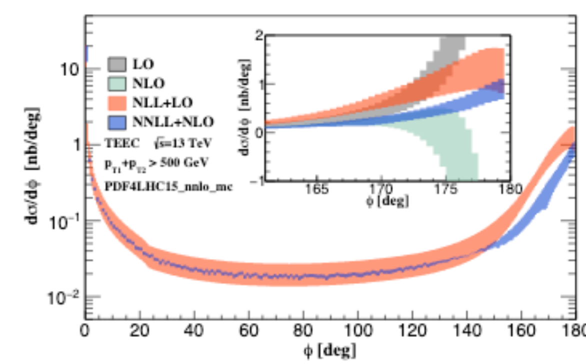
Analytical NNLO *Henn et al arXiv:1903.05314*

For Higgs decay *Luo et al arXiv:1903.07277*
Gao et al arXiv:2012.14188

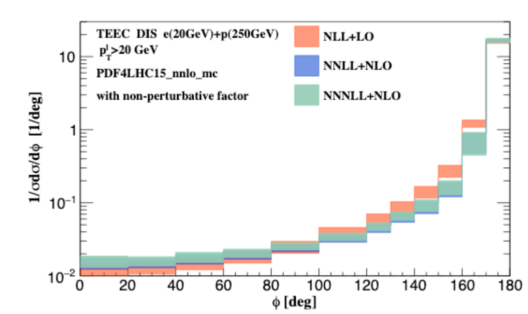
Collinear Limit
Dixon, Moult, Zhu, arXiv:1905.01310

Three Point Energy Correlators
Chen et al, arXiv:1912.11050
See Ian's talk on Monday

NNLL+NLO in pp
Gao, HTL, Moult, Zhu, arXiv:1901.04497

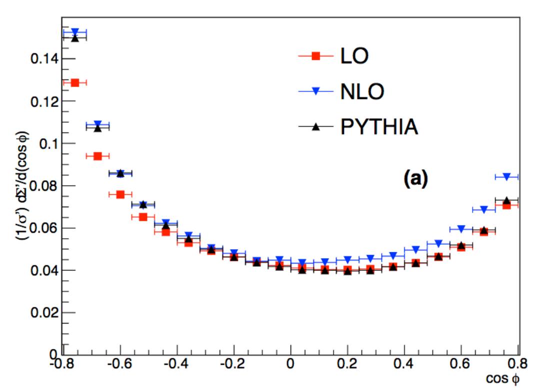


NNNLL+NLO in DIS
HTL, Vitev, Zhu, arXiv:2006.02437



TEEC predictions

NLO QCD corrections
Ali, Barreiro, Llorente, Wang arXiv:1205.1689

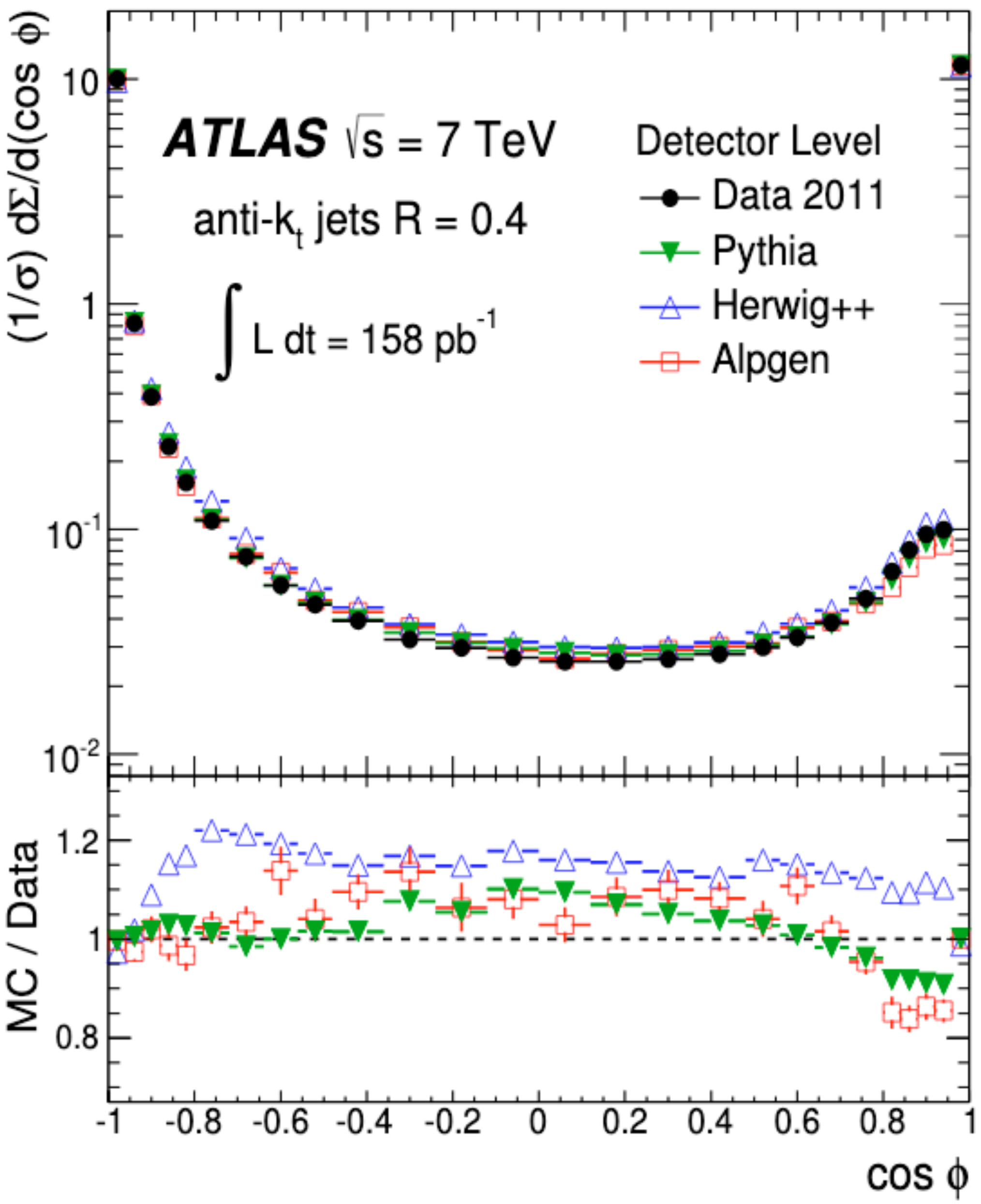


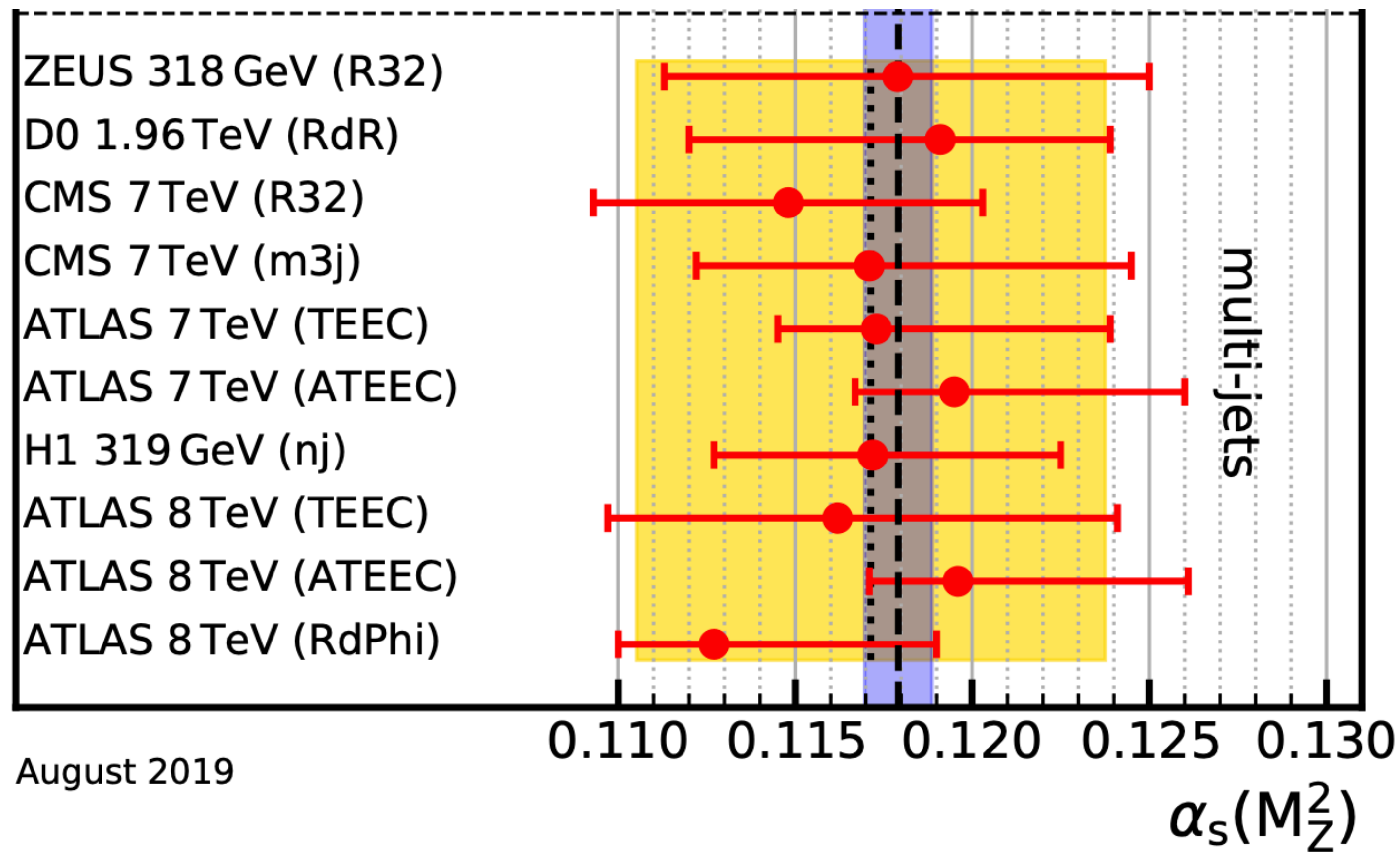
HTL, Markis, Vitev, arXiv:2102.05669
Ali, Li, Wang, Xing, arXiv:2008.00271

And many other works…..

Measurements at the LHC

Measurements at the LHC



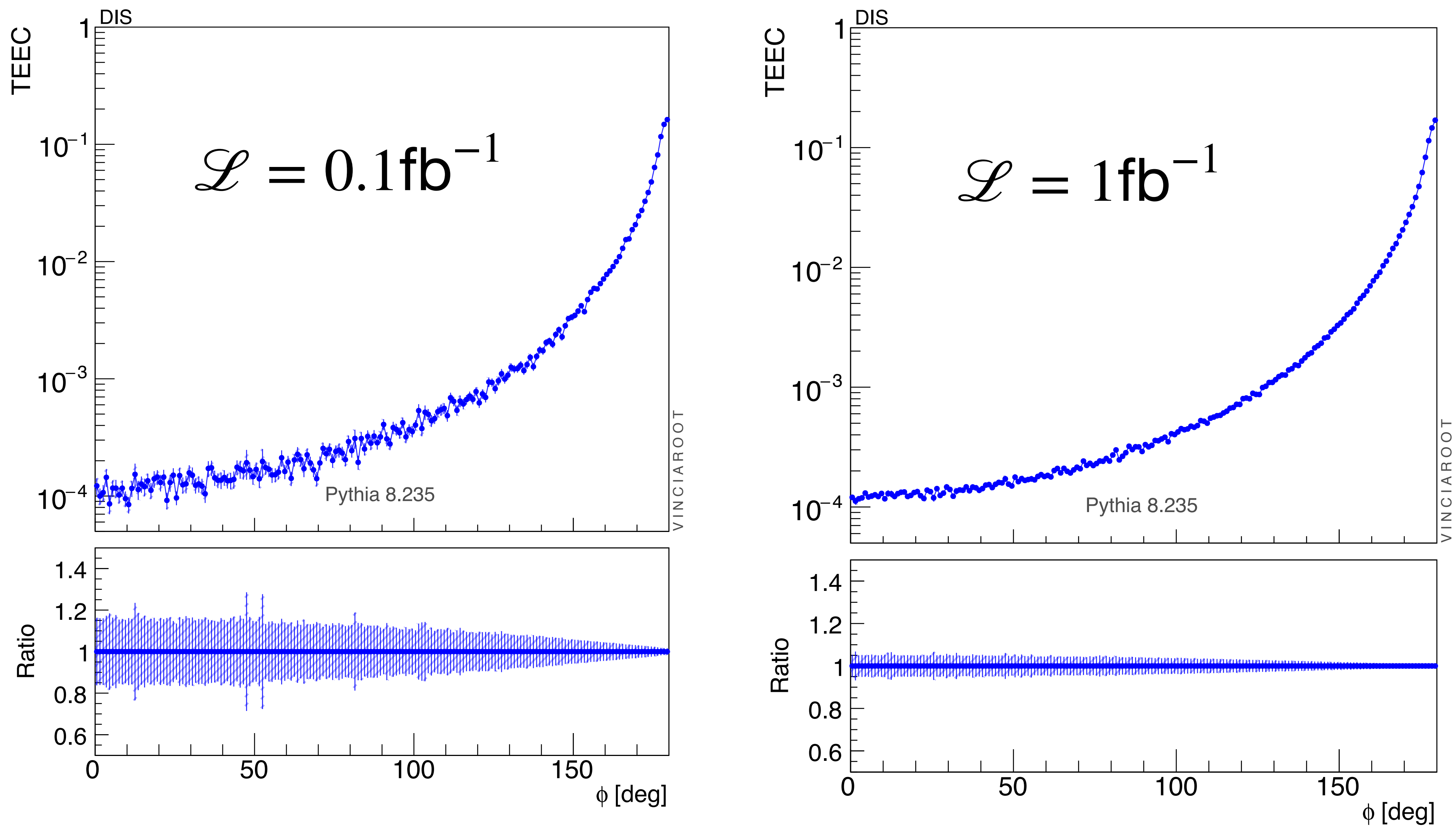


$$\alpha_s(m_Z) = 0.1162 \pm 0.0011\,(\text{exp.})\,{}^{+0.0076}_{-0.0061}\,(\text{scale}) \pm 0.0018\,(\text{PDF}) \pm 0.0003\,(\text{NP}),$$

in good agreement with the determinations in other experiments and with the world average

Simulations at the EIC

$e(18\text{GeV}) + p(275\text{GeV})$ Select events with $p_{T,l} > 20\text{GeV}, \quad -1 < \eta_h < 3$



It is possible to study this observable in percent level

TEEC in DIS

HTL, Vitev, Zhu, 2020

In Lab Frame at the EIC

Definition

$$\mathrm{TEEC} = \boxed{\sum_a} \int d\sigma_{lp\to l+a+X} \boxed{\frac{E_{T,l}E_{T,a}}{E_{T,l}\sum_i E_{T,i}}} \boxed{\delta(\cos\phi_{la} - \cos\phi)}$$

sum over all hadrons

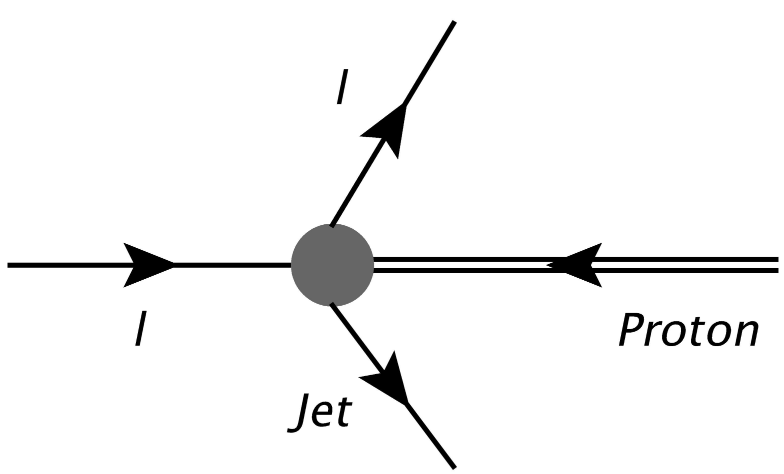
energy weighted

measure azimuthal angle correlations

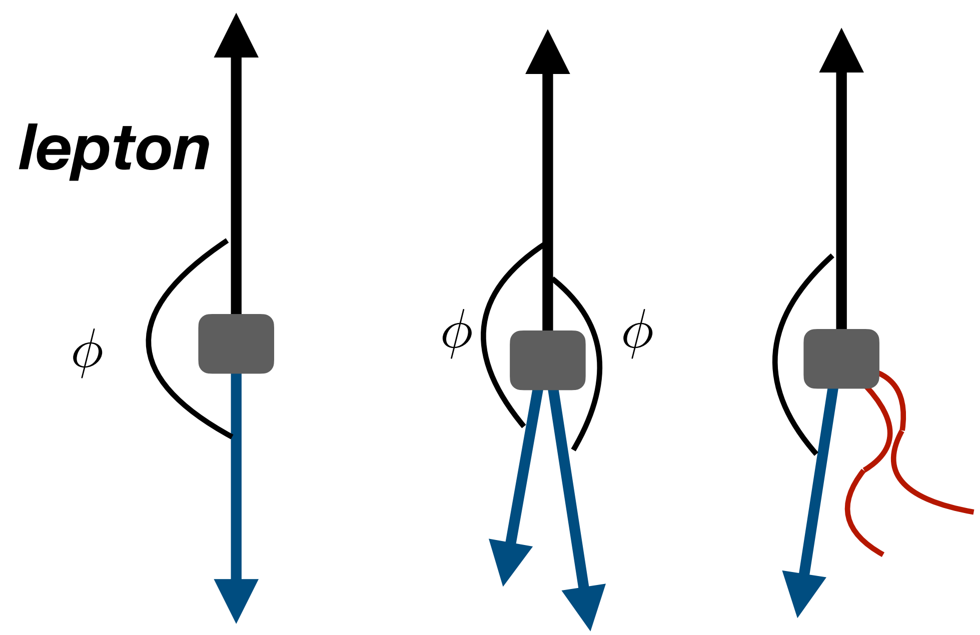
azimuthal angle correction between particles and lepton $\tau = \dfrac{1+\cos\phi}{2}$

For $\tau \to 1$, **large angle radiation**

For $\tau \to 0$, **small angle radiation**



In transverse plane



$\delta(\tau)$

$A\delta(\tau) + B\frac{1}{\tau} + C\frac{\ln\tau}{\tau}\cdots$

$$\tau \approx \frac{\left|k_{2,y} - k_{s,y} + \frac{k_{4,y}}{\xi_4}\right|^2}{4p_T^2}$$

EEC in DIS

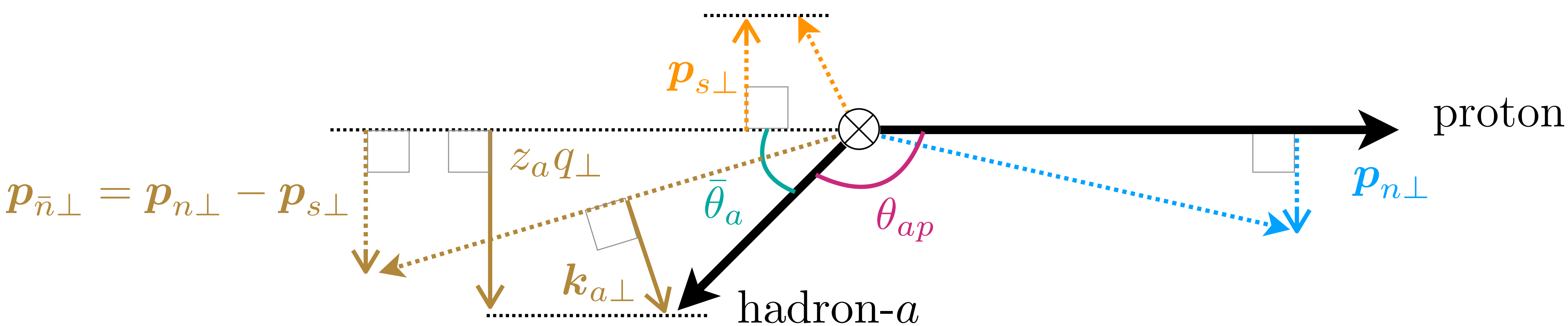
In Breit Frame at the EIC

HTL, Makris, Vitev 2021

From Lab frame to Breit Frame

- **boost the system to proton rest frame**
- **rotate the system: virtual photon has zero $\vec{q}_T$**
- **boost along z direction: virtual photon has zero energy**

$$\gamma^* + \textbf{proton} \to \textbf{jet/hadron} + X$$



We proposed a new definition of EEC in DIS:

correlation between initial proton and final state hadron

$$\mathrm{EEC} = \sum_a \int d\sigma_{lp\to l+a+X} \left(\frac{p\cdot p_a}{\sum_i p\cdot p_i}\right) \delta(\cos\chi - \cos\theta_{ap})$$

(T)EEC in DIS

No singularities in the forward region

the backward region is most important: TMD physics

TEEC: Similar to 1-dimensional TMD factorization
EEC: Similar to usual TMD factorization

hadron with small p_T

TMD PDF

$$\frac{d\sigma_h}{d^2p_\perp} = \sum_f \int \frac{d\xi dQ^2}{\xi Q^2} Q_f^2 H(Q,\mu) \int \frac{db}{2\pi} e^{ib_\perp \cdot p_\perp} \boxed{f_{f/N}\left(b,\xi,\mu,\nu\right)} \boxed{S\left(b,\frac{n_2\cdot n_4}{2},\mu,\nu\right)} \boxed{\int \frac{dz}{z^2} F_{h/f}\left(z,b/z,E_4,\mu,\nu\right)}$$

TMD soft TMDFF

sum over all hadrons in the final state

$$\frac{d\sigma_h}{d\tau} = \sum_f \int \frac{d\xi dQ^2}{\xi Q^2} Q_f^2 H(Q,\mu) \int dk_y \int \frac{db}{2\pi} e^{-ib_y\cdot k_y} f_{f/N}\left(b,\xi,\mu,\nu\right) S\left(b,\frac{n_2\cdot n_4}{2},\mu,\nu\right) \sum_h \int zdz\ F_{h/f}\left(z,b/z,E_4,\mu,\nu\right) \delta(\tau-\tau(k_y))$$

Jet function **the second Mellin-Moment of the TMDFFs**

$$\sum_N \int_0^1 dzz F_{N/q}\left(z,b_\perp/z,\nu\right) = \sum_{i,N} \int_0^1 dzz \int_z^1 \frac{d\xi}{\xi}\ d_{N/i}(z/\xi)\ \ \mathscr{C}_{iq}\left(\xi,b_\perp/\xi,\nu\right) + \mathcal{O}\left(b_T^2\Lambda_{\rm QCD}^2\right)$$

$$= \sum_{i,N} \int_0^1 dx x \mathscr{C}_{iq}\left(x,b_\perp/\xi,\nu\right) \int_0^1 d\xi \xi d_{N/i}(\xi) + \mathcal{O}\left(b_T^2\Lambda_{\rm QCD}^2\right)$$

Factorizations for difference processes

(T)EEC is a class of observale which can be studies in various processes

$ep \to e + \text{jet}$

$$\frac{d\sigma^{(0)}}{d\tau} = \sum_f \int \frac{d\xi dQ^2}{\xi Q^2} Q_f^2 \sigma_0 \int \frac{db}{2\pi} e^{-2ib\sqrt{\tau}p_T} H(p_T, Q, \mu) S(b, Q, \mu, \nu) B_{f/N}(b, \xi, \mu, \nu) J_f(b, \mu, \nu)$$

$e^+e^- \to \text{dijet}$

$$\frac{d\sigma}{d\tau} = \frac{1}{2}\int d^2\vec{k}_\perp \int \frac{d^2\vec{b}_\perp}{(2\pi)^2} e^{-i\vec{b}_\perp\cdot\vec{k}_\perp} H(Q,\mu) J^q_{\text{EEC}}\left(\vec{b}_\perp, \mu, \nu\right) J^{\bar{q}}_{\text{EEC}}\left(\vec{b}_\perp, \mu, \nu\right) S_{\text{EEC}}\left(\vec{b}_\perp, \mu, \nu\right) \delta\left(1 - \tau - \frac{\vec{k}_\perp^2}{Q^2}\right)$$

$pp \to \text{dijet}$

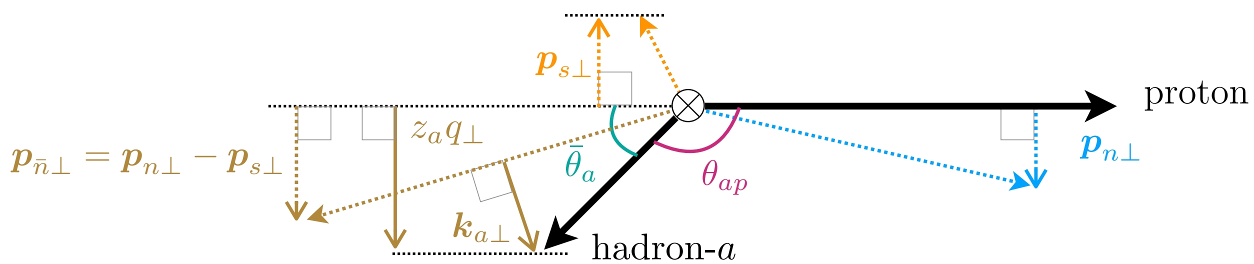
$$\frac{d\sigma^{(0)}}{d\tau} = \frac{1}{16\pi s^2 \left(1 + \delta_{f_5 f_4}\right)\sqrt{\tau}} \sum_{\text{channels}} \frac{1}{N_{\text{init}}} \int \frac{dy_3 dy_4 p_T dp_T^2}{\xi_1 \xi_2} \int_{-\infty}^{\infty} \frac{db}{2\pi} e^{-2ib\sqrt{\tau}p_T}$$
$$\times \operatorname{tr}\left(\mathbf{H}^{f_1 f_2 \to f_3 f_4}\left(p_T, y^*, \mu\right) \mathbf{S}\left(b, y^*, \mu, \nu\right)\right] B_{f_1/N_1}\left(b, \xi_1, \mu, \nu\right) B_{f_2/N_2}\left(b, \xi_2, \mu, \nu\right) J_{f_3}(b, \mu, \nu) J_{f_4}(b, \mu, \nu)$$

Resummation can be achieved through solving the RG equations

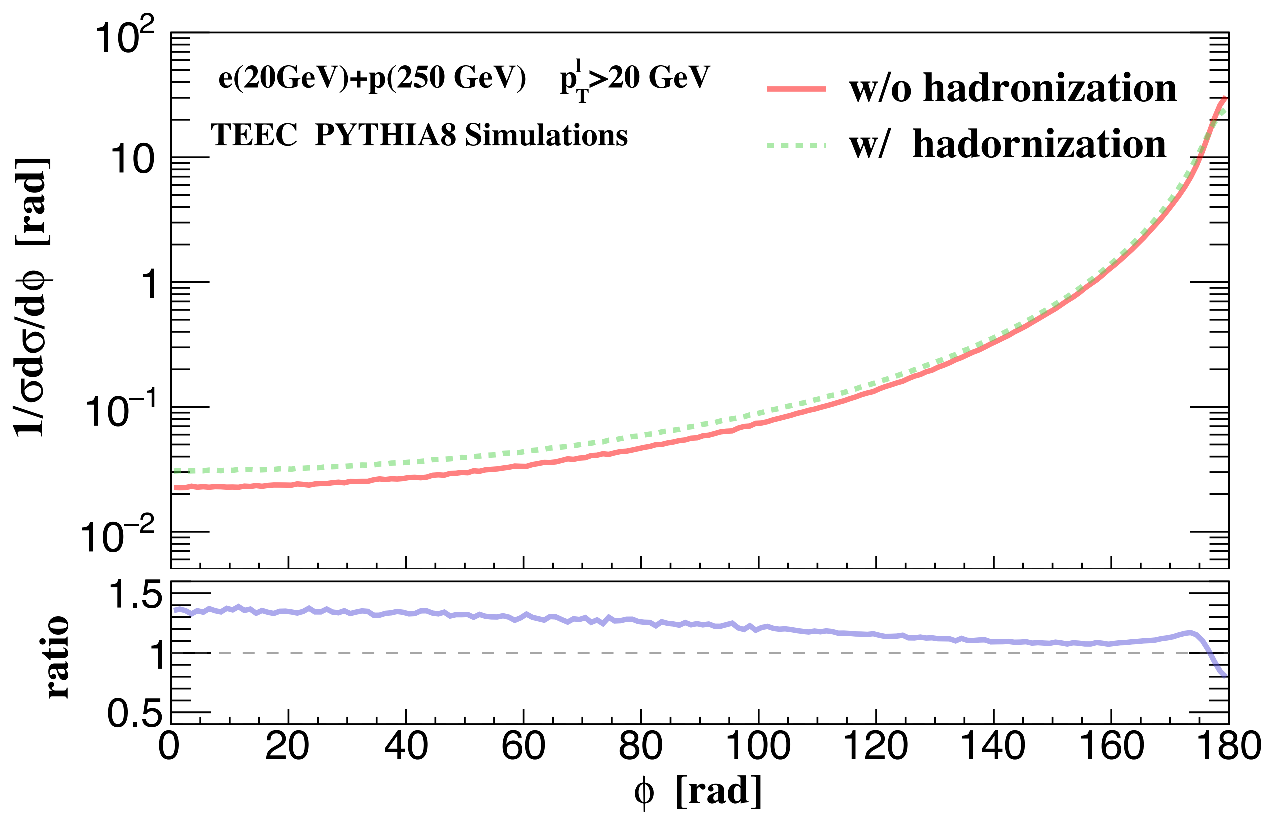
Universality of QCD in the infrared regime

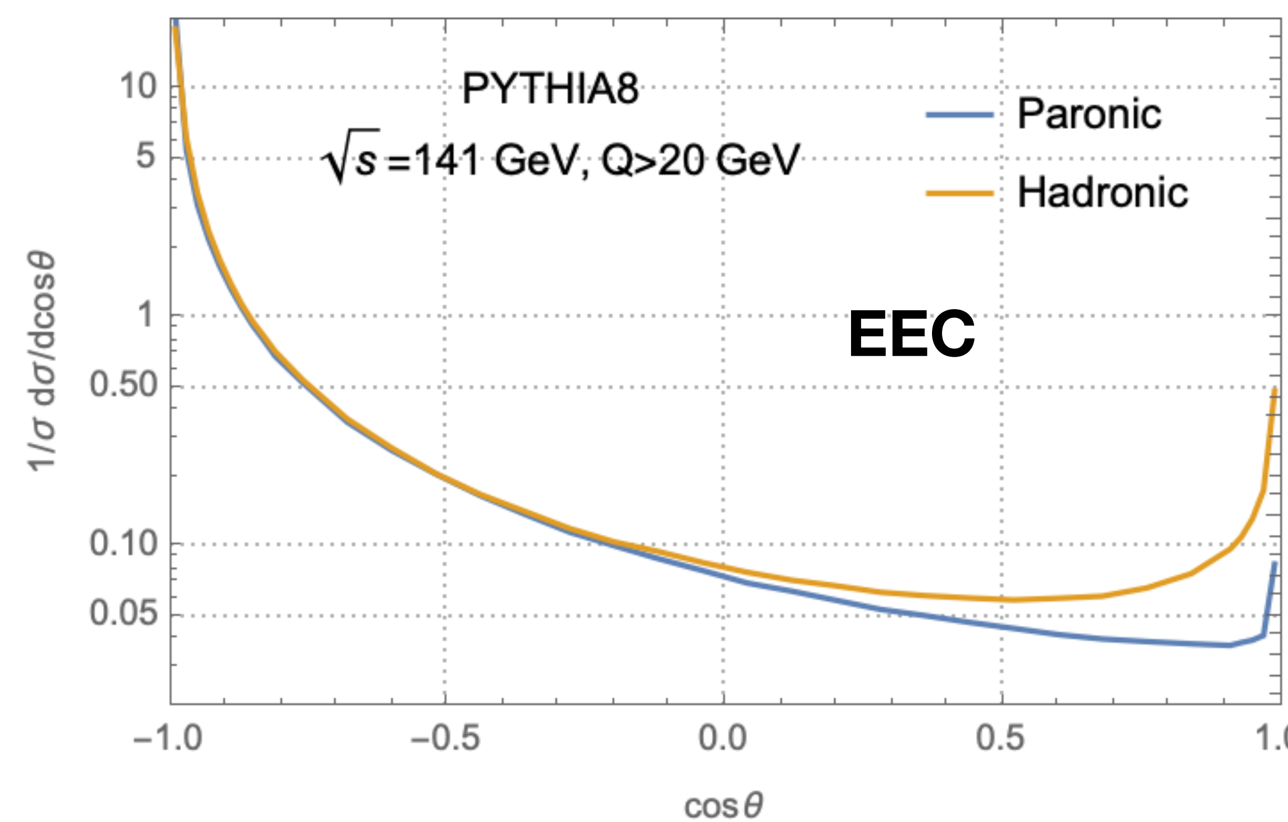
EEC in DIS

$$\mathrm{EEC} = \sum_a \int d\sigma_{lp\to l+a+X} \left(\frac{p\cdot p_a}{\sum_i p\cdot p_i}\right) \delta(\cos\chi - \cos\theta_{ap})$$



- ❏ weight function is Lorentz Invariant
- ❏ radiation close to the beam direction is suppressed
- ❏ soft radiation/hadronization effect is suppressed





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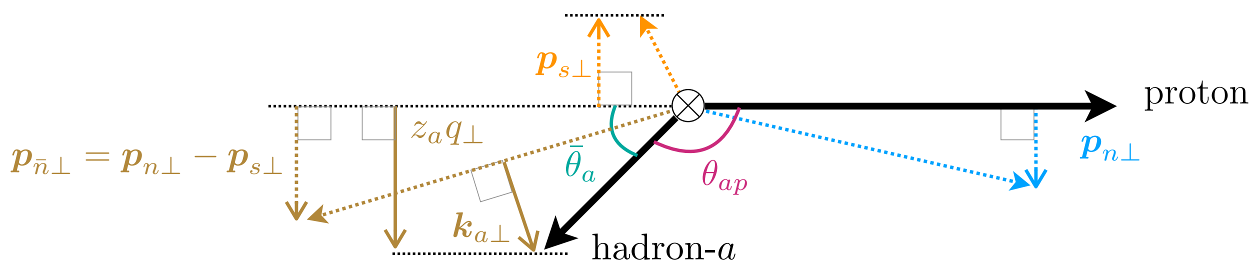

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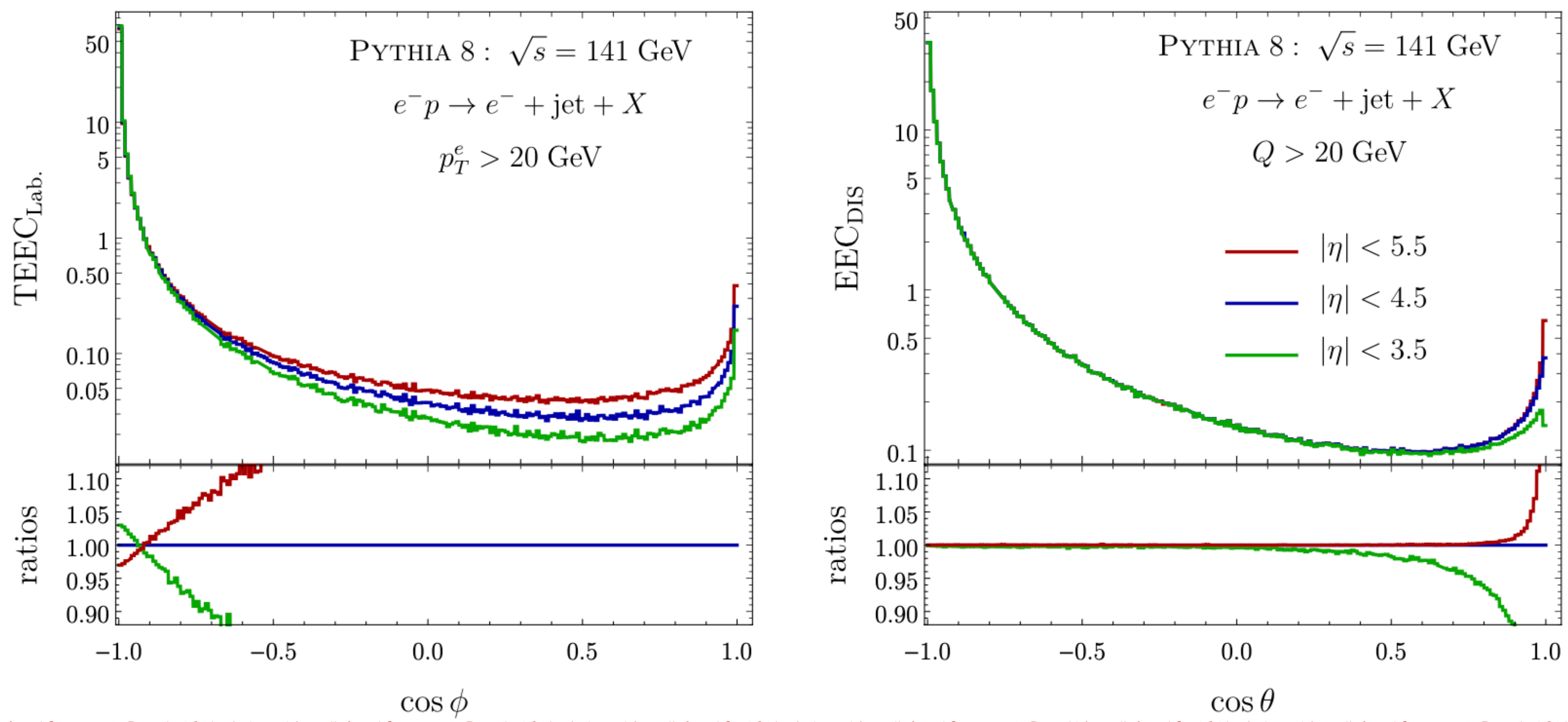
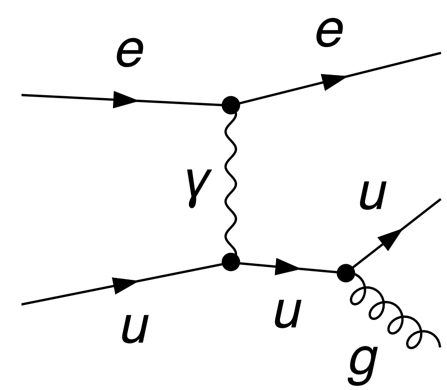


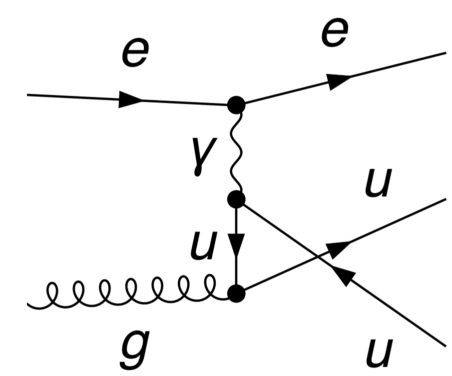
normalized to the cross section with cut $|\eta| < 5.5$

In Breit frame, rapidity cut only changes the cross section tail region

In back-to-back limit, TMD factorization can be used which has a better connection to the usual TMD physics

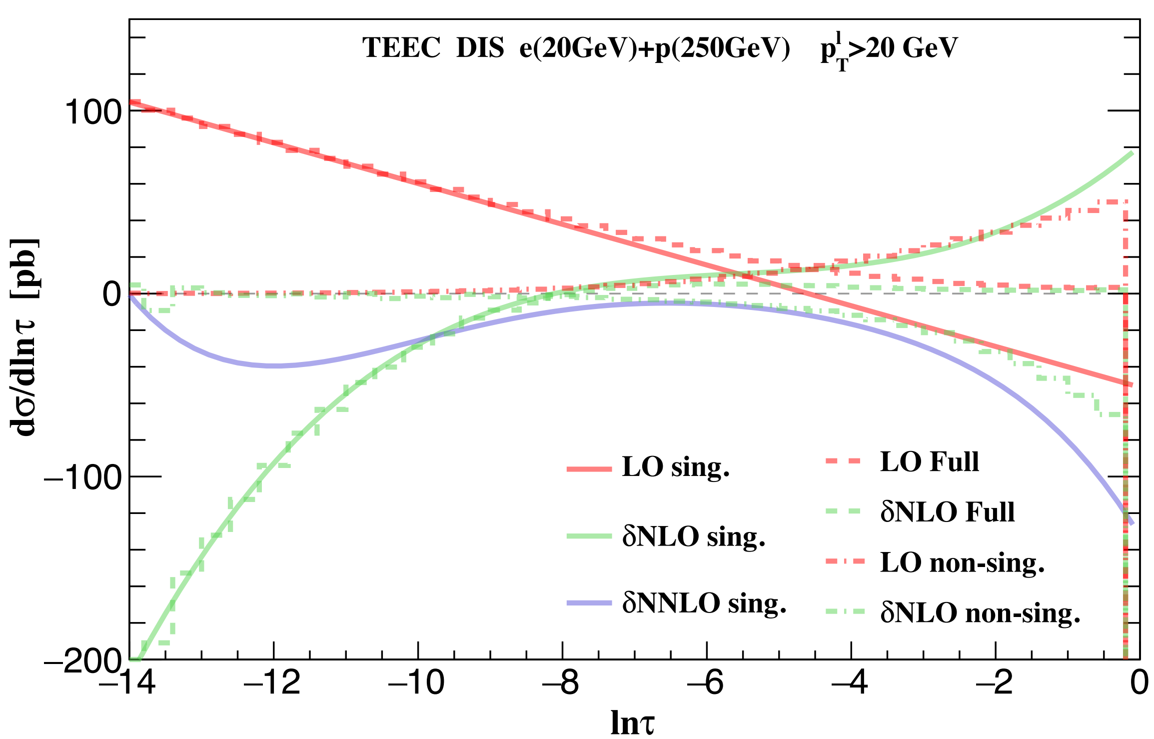
Fixed order results

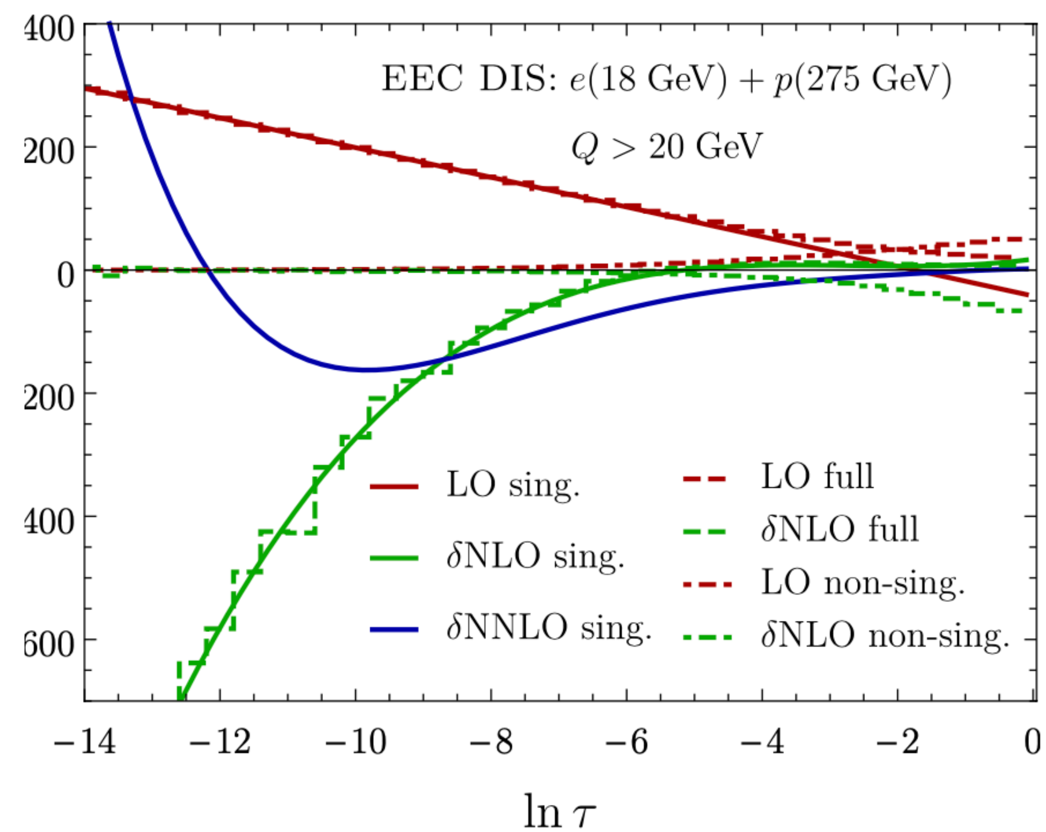




The leading order process is

calculated using NLOJET++





- Reproduced the singular behaviors
- Full control of the distributions in the back-back limit at LO and NLO.
- We obtained singular distribution up to NNLO (three loop anomalous dimensions)

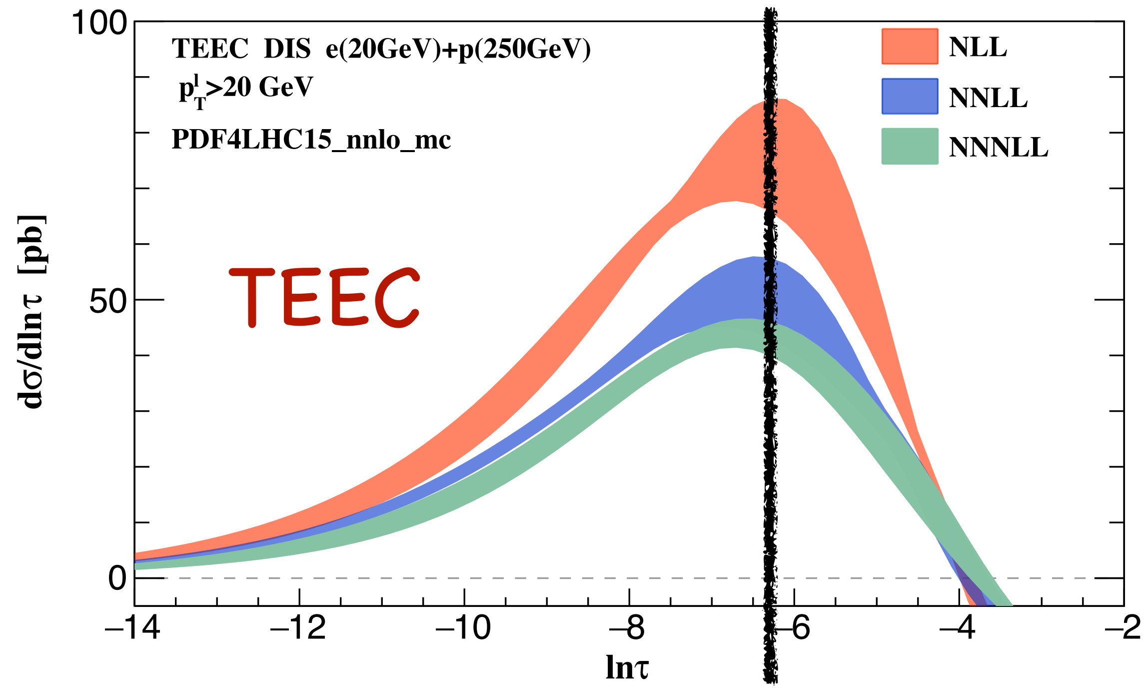
Resummation

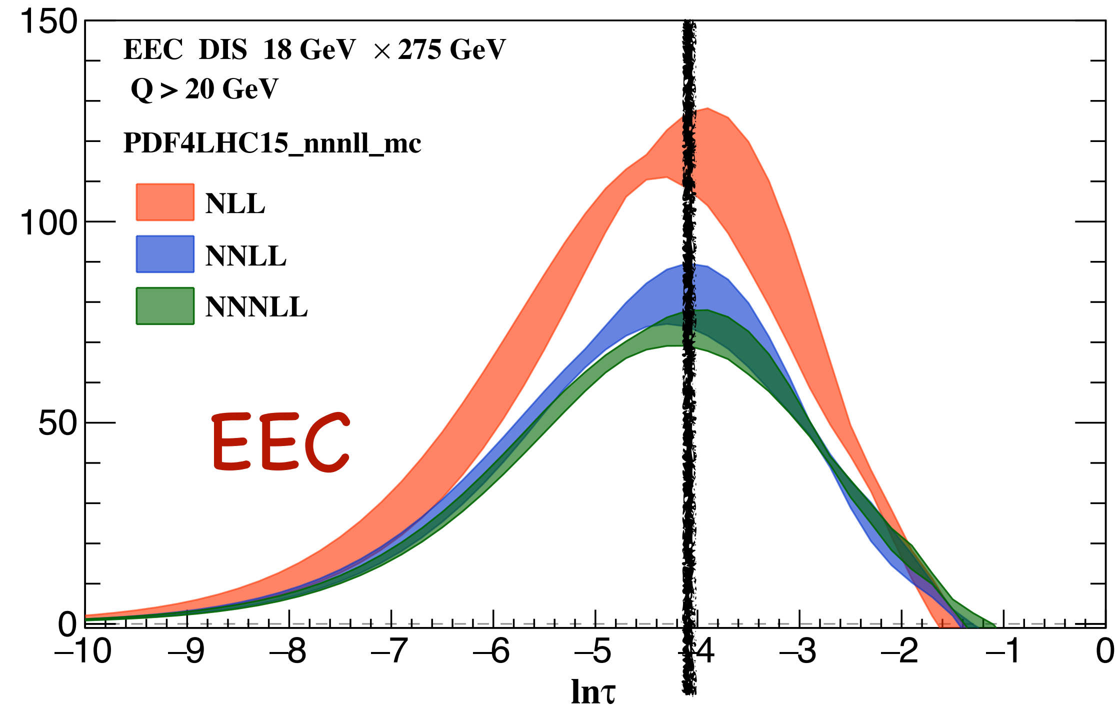
$$\underbrace{\sum_{i=1} \alpha_s^i L^{2i} + \sum_{i=1} \alpha_s^i L^{2i-1}}_{\text{NLL}} + \underbrace{\sum_{i=1} \alpha_s^i L^{2i-2} + \sum_{i=2} \alpha_s^i L^{2i-3}}_{\text{NNLL}} + \underbrace{\sum_{i=2} \alpha_s^i L^{2i-4} + \sum_{i=3} \alpha_s^i L^{2i-5}}_{\text{NNNLL}}$$

$\boxed{N^3LL}$

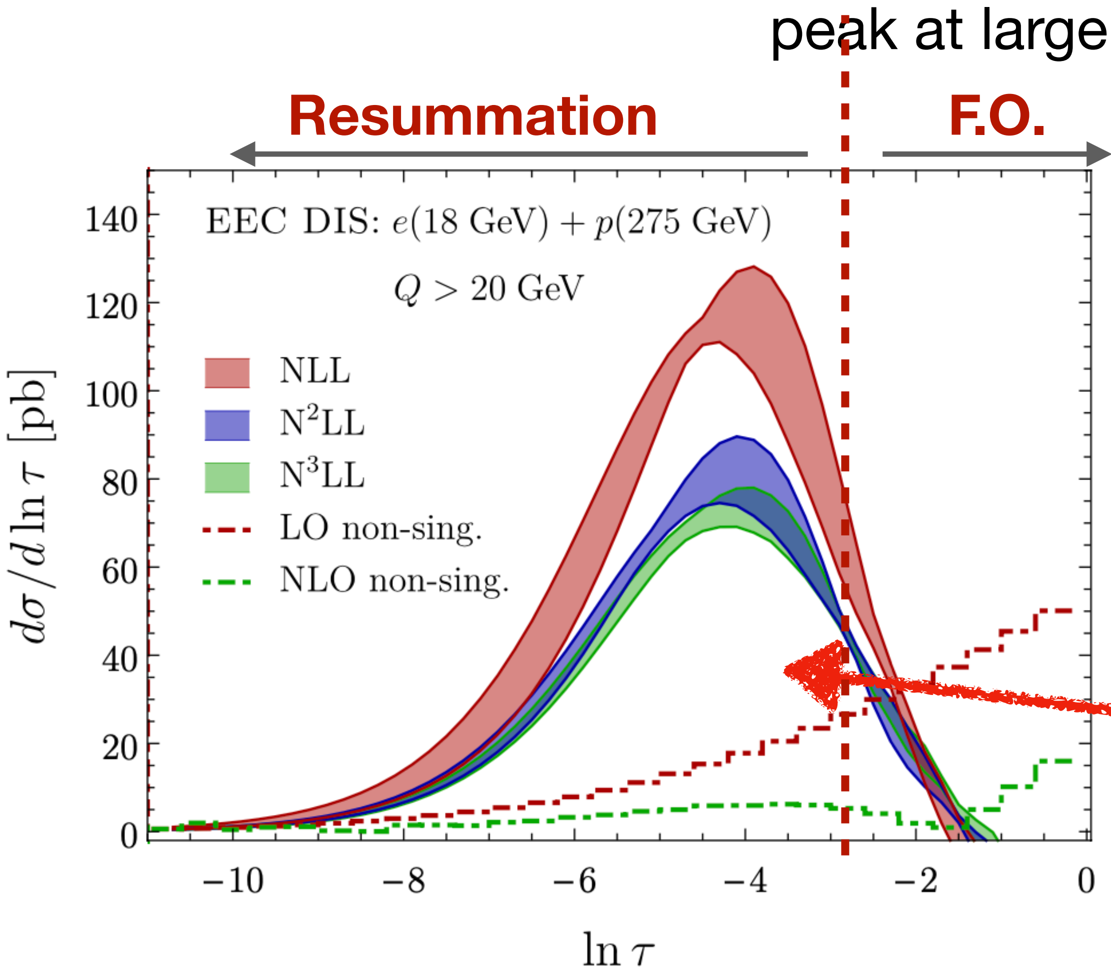
Four-loop Cusp anomalous dimensions
Three-loop anomalous dimensions for Beam, Hard, Soft, Jet functions

Resummation





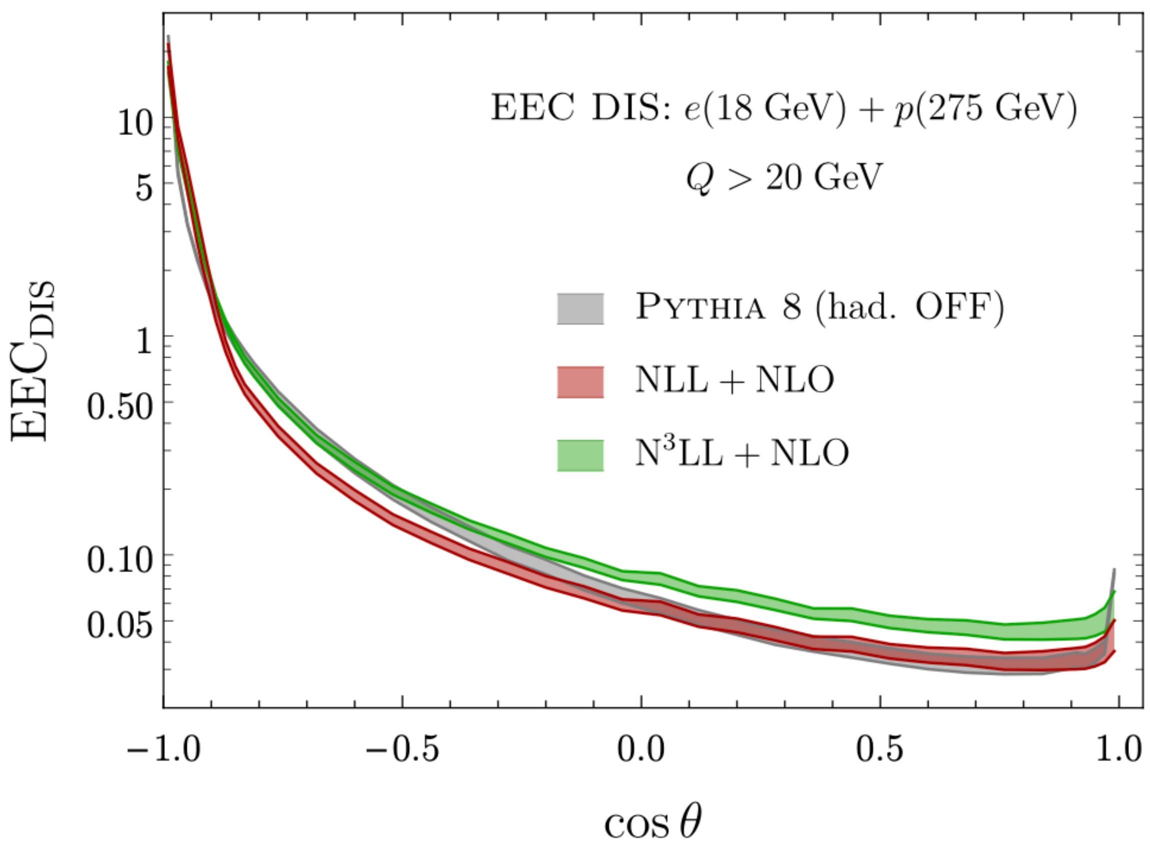
peak at larger tau, means small NP effects



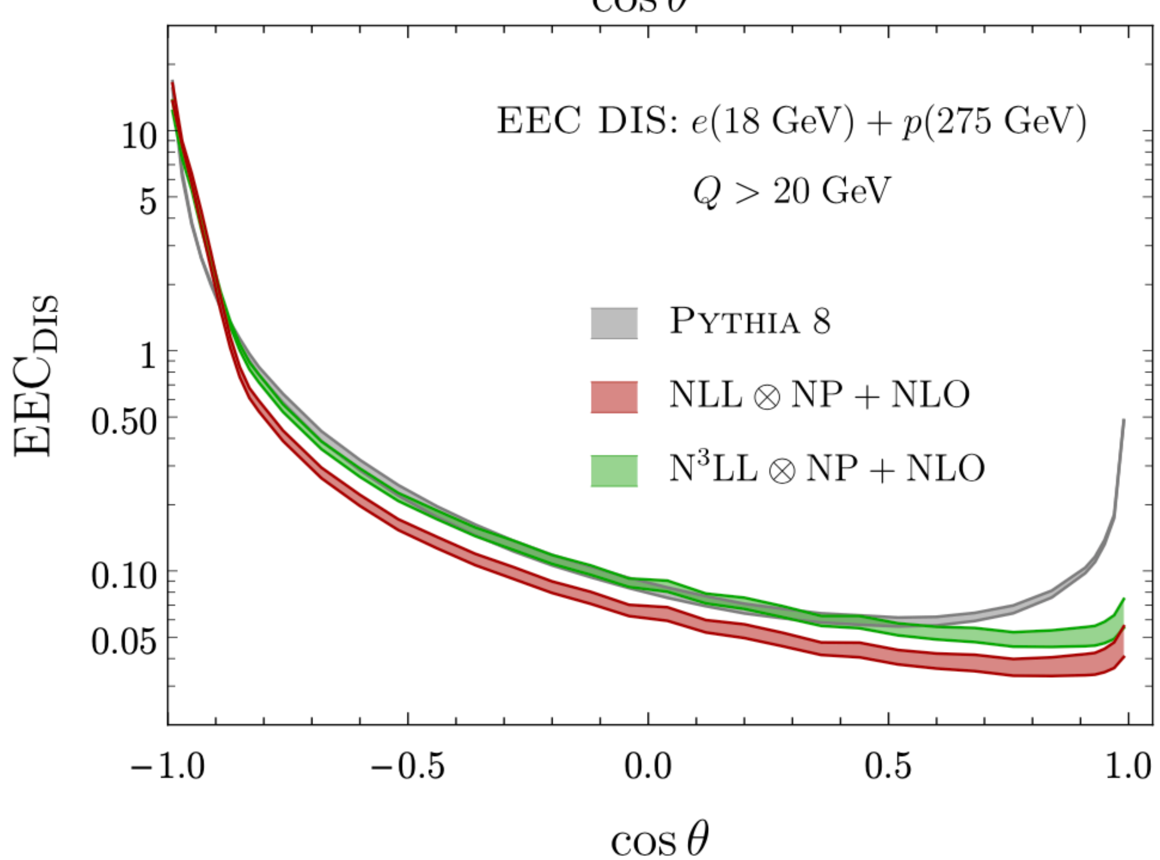
- Convergence in back-to-back limit after resummation
- Huge difference from NLL to NNLL and good perturbative convergence from NNLL to NNNLL
- Reduction of scale uncertainties order by order from NLL to NNNLL

Non-singular terms start to contribute which is less important for EEC

EEC in DIS



EEC DIS: e(18 GeV) + p(275 GeV)
$Q > 20$ GeV
Pythia 8 (had. OFF)
NLL + NLO
N^{3}LL + NLO
EEC$_{\mathrm{DIS}}$
10
5
1
0.50
0.10
0.05
−1.0
−0.5
0.0
0.5
1.0
$\cos\theta$



EEC DIS: e(18 GeV) + p(275 GeV)
$Q > 20$ GeV
Pythia 8
NLL ⊗ NP + NLO
N^{3}LL ⊗ NP + NLO
EEC$_{\mathrm{DIS}}$
10
5
1
0.50
0.10
0.05
−1.0
−0.5
0.0
0.5
1.0
$\cos\theta$

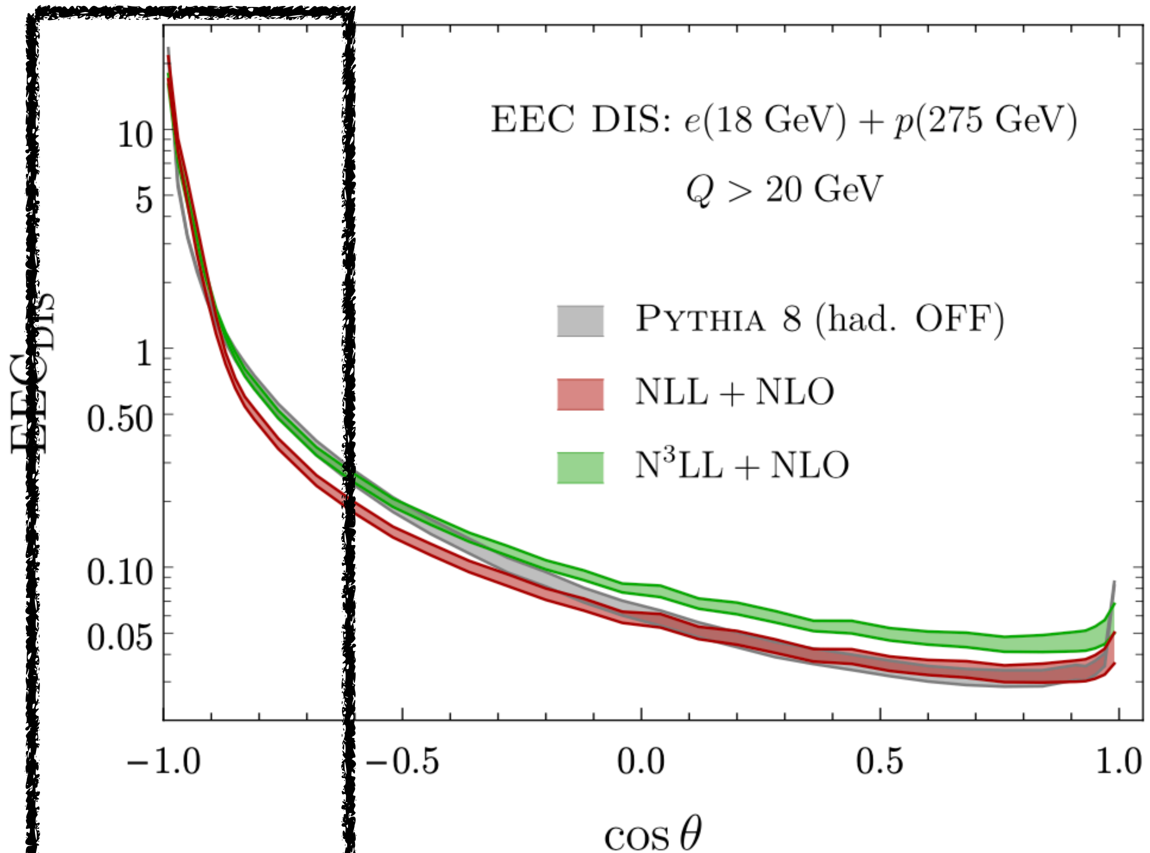
EEC in DIS

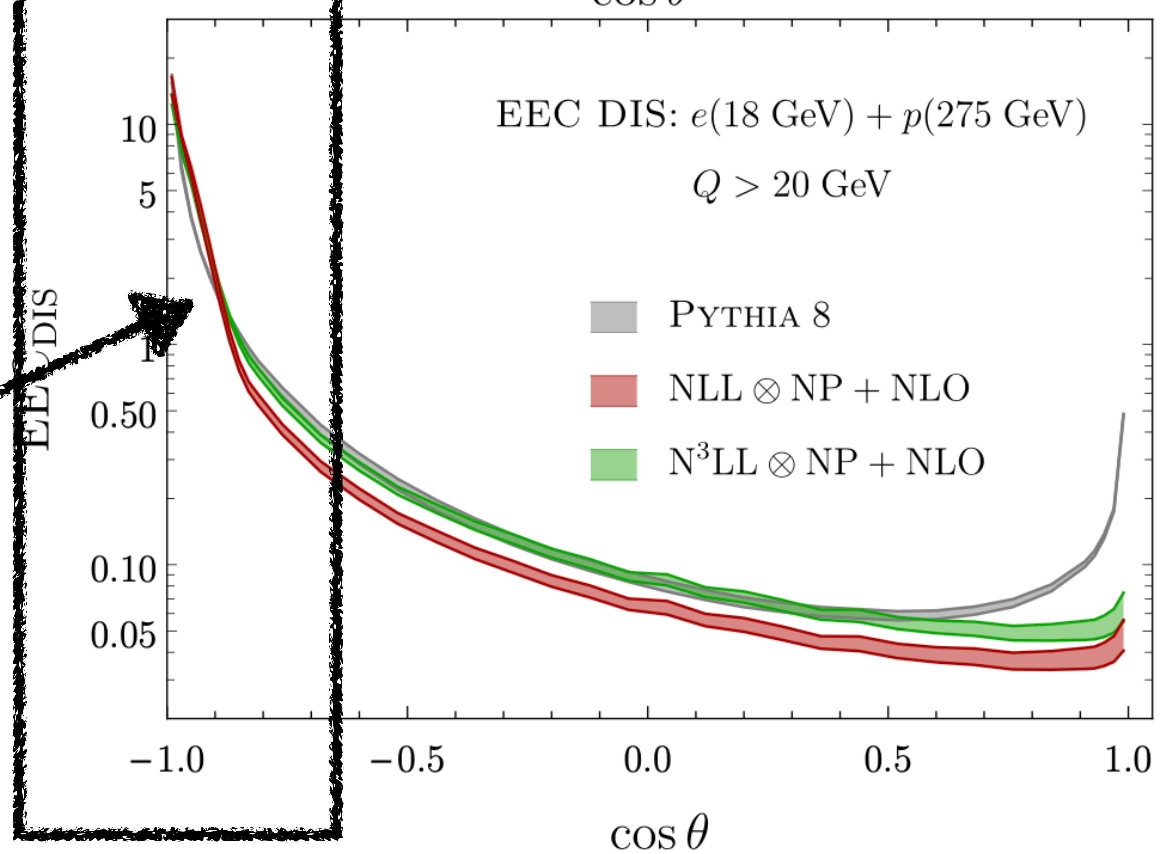
Resummation Dominant region

Scale uncertainties are reduced

Bands almost are on top of each other for NNLL +NLO and NNNLL+NLO

Slightly different with PYTHIA simulation with hadronization





EEC in DIS

Resummation
Dominant region

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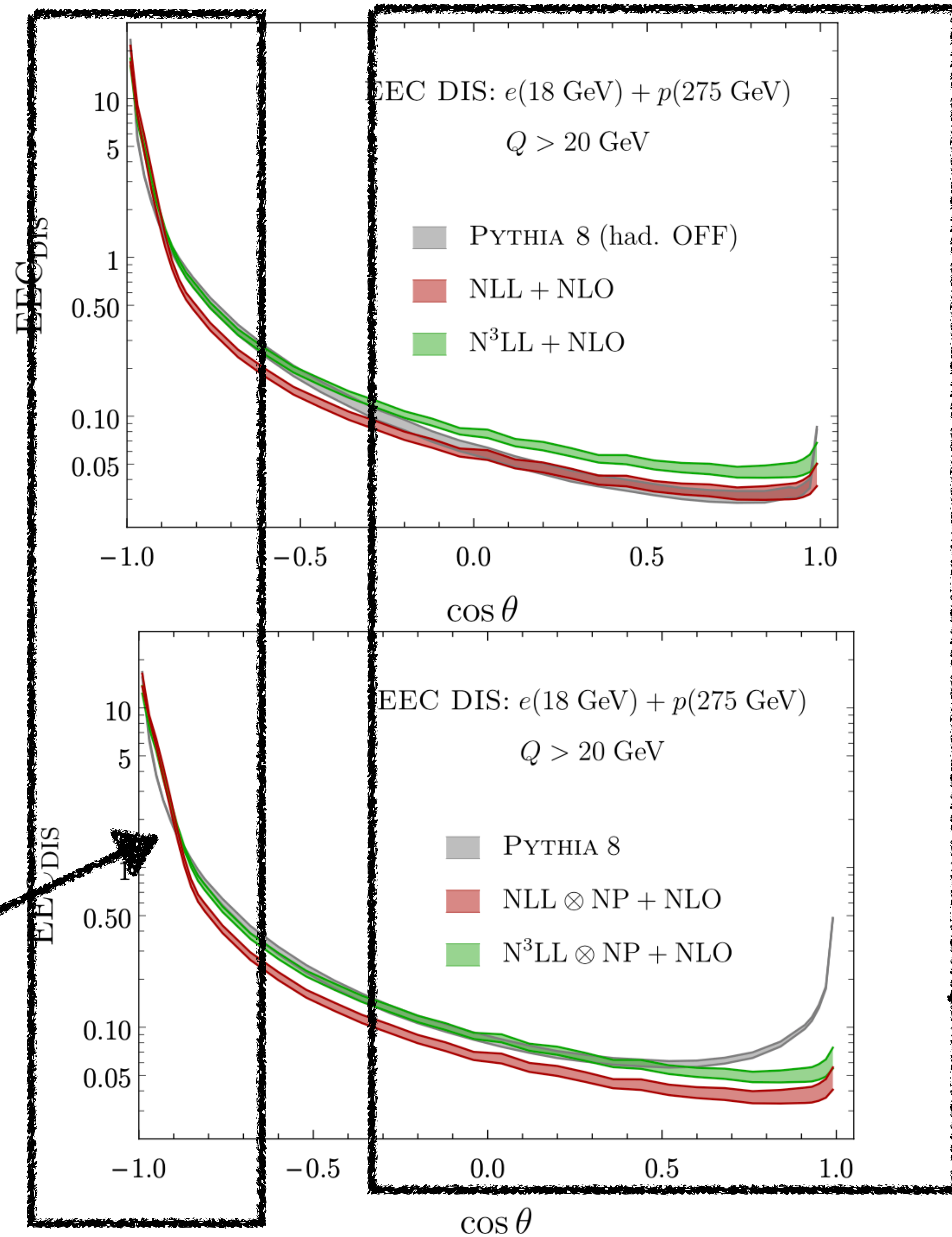
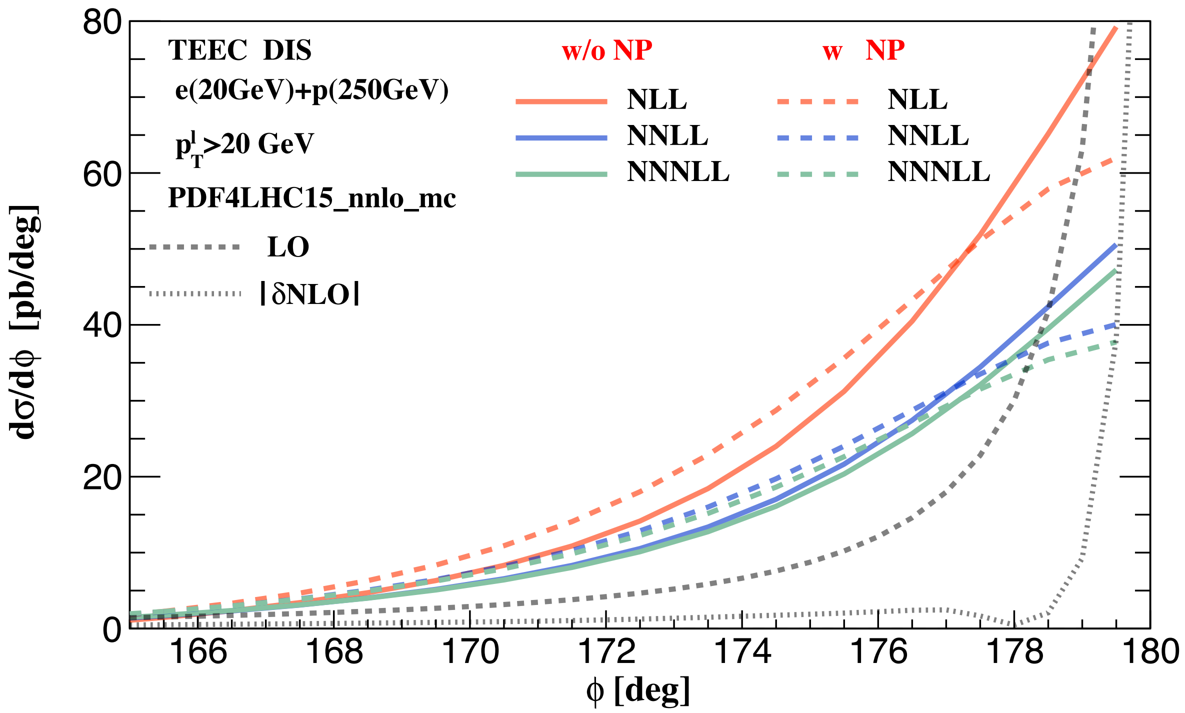


Scale uncertainties
dominated by fixed
order

NNLO matching
would improve the
predictions

Hadronization
effects are only
important in tail
region

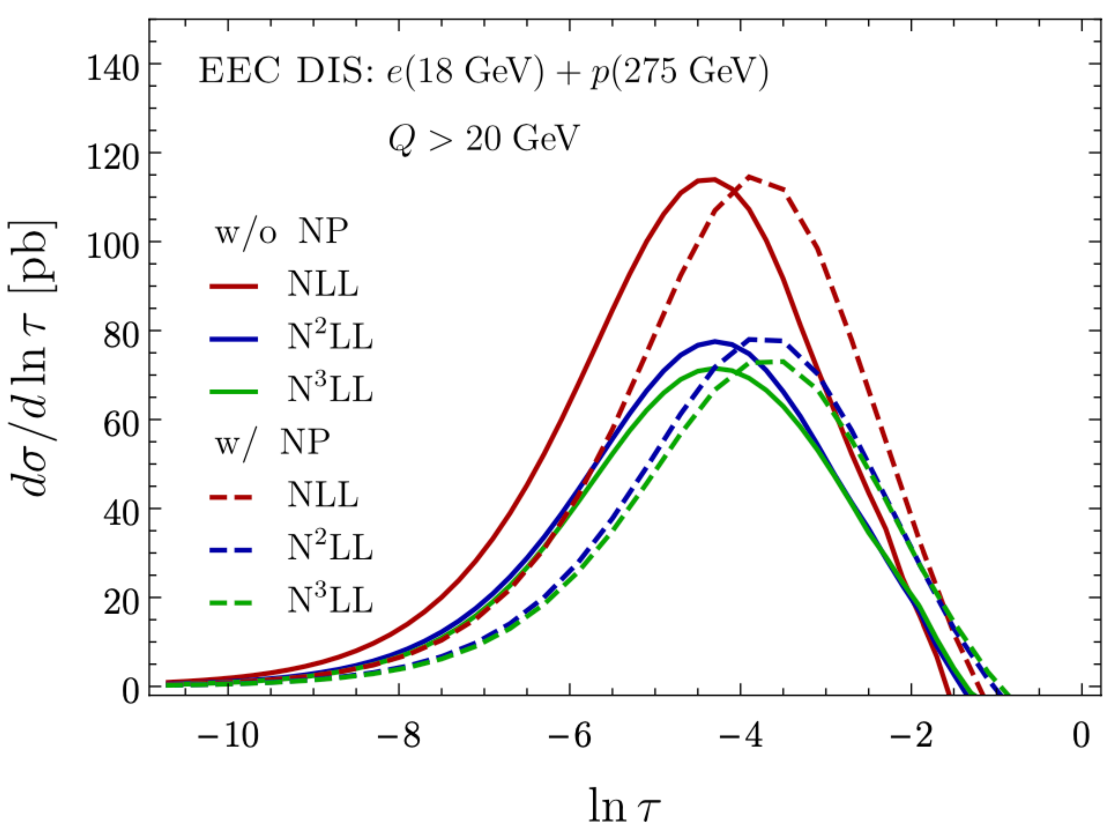
Non-perturbative effects



- ☑ corrections to rapidity evolution
- ☑ corrections to the TMD matrix element

Non-perturbative form factors, which extracted from the semi-inclusive hadron production in DIS.

$$S_{\rm NP} = \exp\left[-0.106\ b^2 - 0.84 \ln Q/Q_0 \ln b/b^*\right]$$



from TMD FFs

$$D_{i/a}^{\rm NP}(y,b) = \exp\left(-0.042\frac{b^2}{y^2}\right) \Rightarrow$$

$$j_i(b) = \exp\left(-0.59b - 0.03b^2\right)$$

NP shifts the cross section

Sizable NP effects in back-to-back limit

About the definition

A Lorentz Invariant definition

$$\bar{\eta}_a = 2\sqrt{1+\frac{q\cdot p_a}{x_B P\cdot p_a}} \quad \frac{\textbf{Breit}}{\textbf{frame}} \quad \frac{2p_{a\perp}}{p_a^+}$$

$$\mathrm{EEC}_{\mathrm{LI}} = \frac{1}{\sigma}\sum_a \int d\sigma(\ell + h \to \ell + a + X)\, \frac{P\cdot p_a}{P\cdot q}\, \delta(\tanh\bar{\eta}_a - \tanh\bar{\eta})$$

Definitions in DIS

$$\mathrm{TEEC} = \boxed{\frac{1}{\sigma}\sum_a \int} d\sigma(l + h \to l + a + X)\, \frac{E_{T,a}}{\sum_i E_{T,i}}\, \delta(\cos\phi_{al} - \cos\phi)$$

$$\mathrm{BEEC} = \boxed{\frac{1}{\sigma}\sum_a \int} d\sigma(\ell + h \to \ell + a + X)\, \frac{P\cdot p_a}{P\cdot q}\, \delta(\cos\theta_{ap} - \cos\theta)$$

In principle we could sum over all hadrons in final state, or

$$\sum_a \to \sum_{a\in S}$$

In TMD region, the summation only changes the summation over fragmentation functions

Introduce a non-perturbative factor in the formula $\mathcal{F}_{j\to\mathbb{S}}(\mu) = \sum_{a\in\mathbb{S}} \int_0^1 dz\, z\, d_{i\to a}(z,\mu)$

sum over charged particles, hadrons with a certain flavors, or an identified hadron,

which allows up to probe initial/final state flavor information

Conclusion

- Introduce definition of (T)EEC in DIS
- Provide a factorized cross-section with TMD elements
- Provide $\mathrm{N^3LL} + \mathcal{O}(\alpha_s^2)$ predictions
- Evaluate NP component of EEC jet function

- measure QCD coupling
- test TMD factorization and QCD universality
- extract TMD PDFs and TMD FFs

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Thank you!