GTMD model predictions for diffractive dijet production at the EIC

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university of groningen

Outline

- GTMDs - introduction:
  - Gluon GTMDs for unpolarized hadrons
  - Gauge link dependence
  - Small-x limit - Wilson loop matrix elements
  - Impact parameter dependence - Frames
- Small-x model for gluon GTMDs
- Diffractive dijet production process
- Phenomenology

GTMDs - introduction

GTMDs - 5D imaging

TMDs: transverse momentum dependent PDFs

GPDs: generalized parton densities = off-forward PDFs

GTMDs: generalized TMDs = off-forward TMDs

GTMDs can also be seen as the transverse momentum dependent GPDs or as Fourier transform of a Wigner distribution

$$G(x, \boldsymbol{k}_T, \boldsymbol{\Delta}_T) \stackrel{FT}{\longleftrightarrow} W(x, \boldsymbol{k}_T, \boldsymbol{b}_T)$$

Meißner, Metz, Schlegel, 2009 — Ji, 2003; Belitsky, Ji & Yuan, 2004

GTMDs inherit and combine all properties of TMDs and GPDs, such as the process dependence & translation non-invariance

But they also display new features, such as elliptic distributions

Elliptic Wigner distributions

$$xW(x, \boldsymbol{b}, \boldsymbol{k}) = x\mathcal{W}_0(x, \boldsymbol{b}^2, \boldsymbol{k}^2) + 2\cos(\phi_b - \phi_k)\, x\mathcal{W}_1(x, \boldsymbol{b}^2, \boldsymbol{k}^2)$$
$$+ 2\cos 2(\phi_b - \phi_k)\, x\mathcal{W}_2(x, \boldsymbol{b}^2, \boldsymbol{k}^2) + \ldots$$

The cos $2(\phi_b$-$\phi_k)$ part is called the elliptic Wigner distribution

[Hatta, Xiao, Yuan, 2016; J. Zhou, 2016; Mäntysaari, Mueller, Schenke, 2019; Salazar, Schenke, 2019]

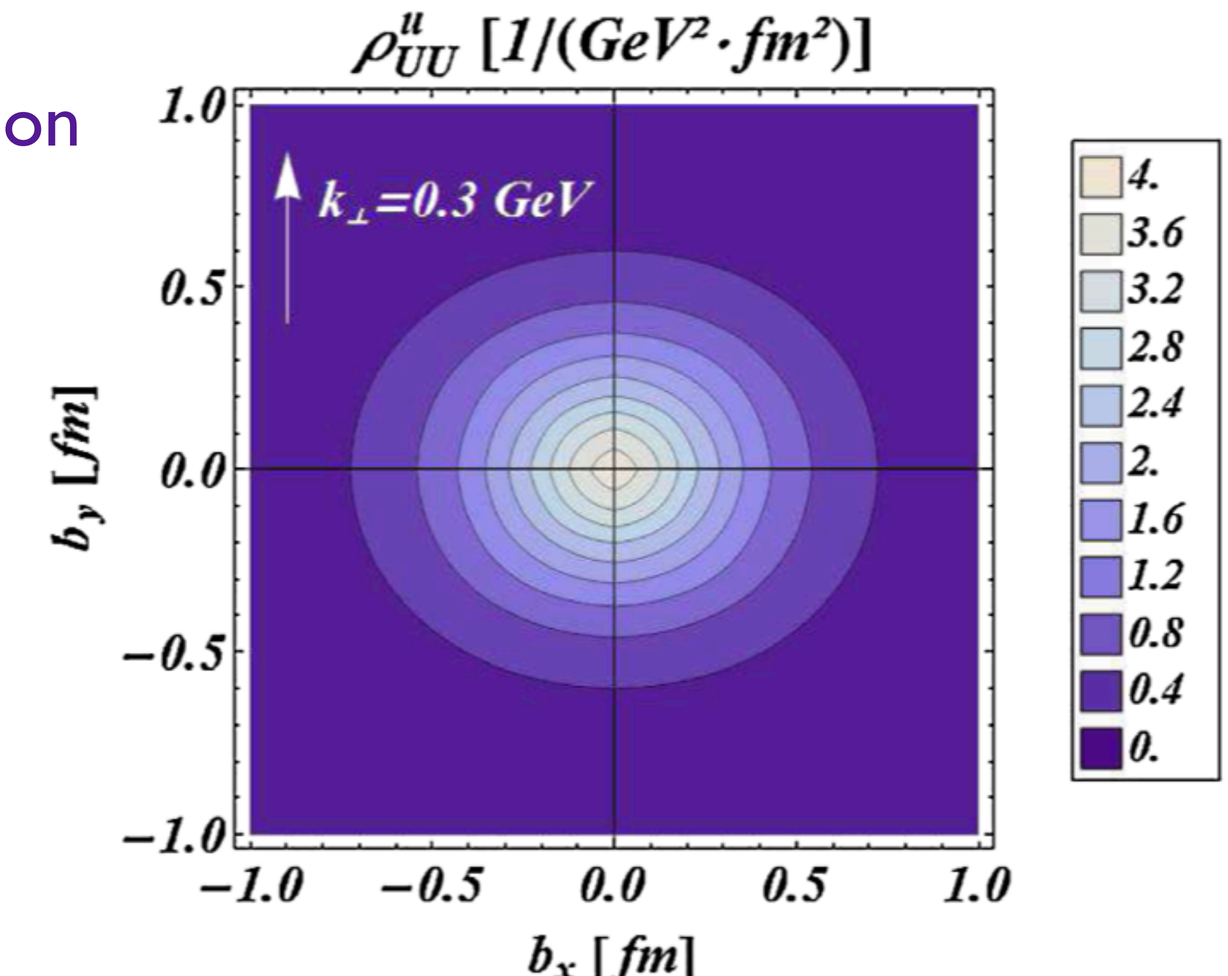
There can be such an elliptic piece in any of the Wigner distributions

A nonzero elliptic quark Wigner distribution in the lightcone constituent quark model:

Lorce & Pasquini, 2011

Due to quark orbital angular momentum

Requires a distribution around a center



Gluon GTMDs for unpolarized protons

For unpolarized protons there are 4 independent (complex valued) gluon GTMDs

$$G^{[U,U']\,ij}(x,\boldsymbol{k},\xi,\boldsymbol{\Delta}) = x\left(\delta_T^{ij}\,\mathcal{F}_1 + \frac{k_T^{ij}}{M^2}\,\mathcal{F}_2 + \frac{\Delta_T^{ij}}{M^2}\,\mathcal{F}_3 + \frac{k_T^{[i}\Delta_T^{j]}}{M^2}\,\mathcal{F}_4\right)$$

[Boer, van Daal, Mulders, Petreska, 2018]

[Lorce, Pasquini, 2013; More, Mukherjee, Nair, 2018]

Likewise there are 4 independent (complex valued) Wigner distributions

In this talk only the unpolarized gluon GTMD $x\mathcal{F}_1$ will be considered and only its isotropic part in the $\xi\to 0$ and $x\to 0$ limit, so 4D imaging

In diffractive dijet production in electron-proton collisions the gluon GTMD correlator with [+,−] link appears:

$$G^{[+,-]}(\boldsymbol{k}_\perp,\boldsymbol{\Delta}_\perp) \equiv \lim_{x,\xi\to 0} G^{[+,-]ij}(x,\boldsymbol{k}_\perp,\xi,\boldsymbol{\Delta}_\perp)\delta_T^{ij}/2$$

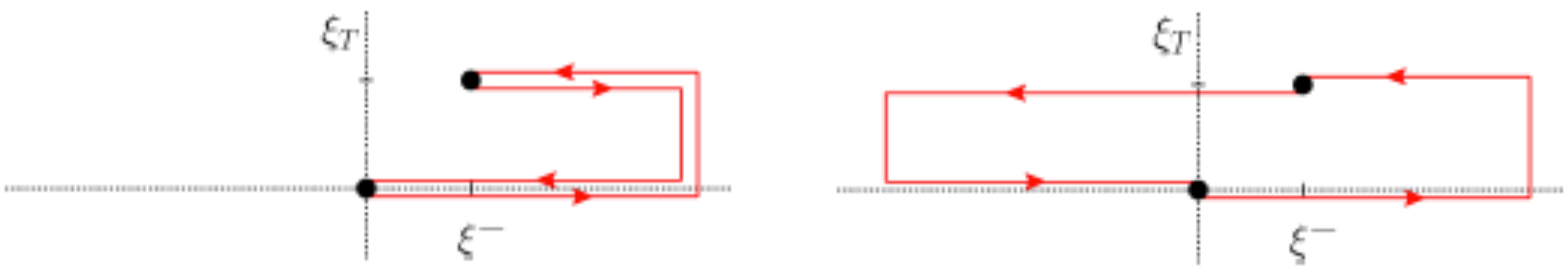
Process dependence of gluon TMDs

The color flow in a process determines the TMD correlator probed

$$\Gamma_g^{\mu\nu[\mathcal{U},\mathcal{U}']}(x,k_T) \equiv \mathrm{F.T.}\langle P|\mathrm{Tr}_c\left[F^{+\nu}(0)\,\mathcal{U}_{[0,\xi]}\,F^{+\mu}(\xi)\,\mathcal{U}'_{[\xi,0]}\right]|P\rangle$$

$$\mathcal{U}_{\mathcal{C}}[0,\xi] = \mathcal{P}\exp\left(-ig\int_{\mathcal{C}[0,\xi]} ds_\mu\, A^\mu(s)\right) \qquad \xi = [0^+,\xi^-,\boldsymbol{\xi}_T]$$

The 2 link combinations of most interest: [+,+] & [+,−]



[−,−] & [−,+] are related to them by parity and time reversal

The same links can appear in gluon GTMDs

Dipole gluon GTMD

In diffractive dijet production in ep the GTMD correlator with the [+,−] link appears

In the x→0 this becomes a correlator of a single Wilson loop:

$$U^{[\square]}(y,x) = U^{[+]}_{[x,y]} U^{[-]}_{[y,x]} \qquad S^{[\square]}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp) \equiv \frac{1}{N_c} \mathrm{Tr}\left[U^{[\square]}(\boldsymbol{y}_\perp, \boldsymbol{x}_\perp)\right]$$

$$G^{[+,-]}(\boldsymbol{k}_\perp, \boldsymbol{\Delta}_\perp) \equiv \frac{1}{2\pi g^2}\left[\boldsymbol{k}_\perp^2 - \frac{\boldsymbol{\Delta}_\perp^2}{4}\right] G^{[\square]}(\boldsymbol{k}_\perp, \boldsymbol{\Delta}_\perp)$$

$$G^{[\square]}(\boldsymbol{k}, \boldsymbol{\Delta}) \equiv \int \frac{d^2\boldsymbol{x}\, d^2\boldsymbol{y}}{(2\pi)^4}\, e^{-i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{y}) + i\boldsymbol{\Delta}\cdot\frac{\boldsymbol{x}+\boldsymbol{y}}{2}}\, \frac{\langle p'|\, S^{[\square]}(\boldsymbol{x}, \boldsymbol{y})\, |p\rangle\big|_{\mathrm{LF}}}{\langle P|P\rangle},$$

[Boer, van Daal, Mulders, Petreska, 2018]

This is the off-forward generalization of the dipole (DP) gluon TMD distribution:

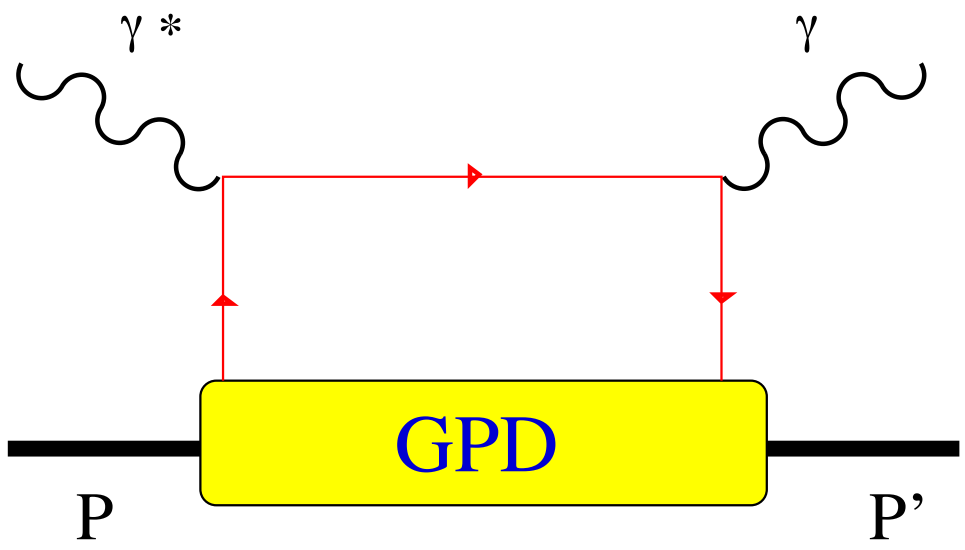
$$xG^{(2)}(x, q_\perp) = \frac{q_\perp^2 N_c}{2\pi^2 \alpha_s} S_\perp \int \frac{d^2 r_\perp}{(2\pi)^2} e^{-iq_\perp \cdot r_\perp} \frac{1}{N_c} \left\langle \mathrm{Tr} U(0) U^\dagger(r_\perp)\right\rangle_{x_g}$$

[Dominguez, Marquet, Xiao, Yuan, 2011]

GPDs

Deeply Virtual Compton Scattering (DVCS):

$$\gamma^* + p \rightarrow \gamma + p'$$



Theoretical description involves Generalized Parton Distributions (GPDs)

[Müller *et al.*, 1994; Radyushkin, 1996; Ji, 1997; ...]

GPDs are off-forward matrix elements (P’ ≠ P)

GPDs provide information about the spatial distribution of quarks inside nucleons

It describes the transverse spatial distance of quarks w.r.t. the “center” of the proton

The transverse center of longitudinal momentum:
[Burkardt, 2000; Soper, 1977]

$$\mathbf{R}_\perp^{CM} \equiv \sum_i x_i \mathbf{r}_{\perp i}$$

Impact parameter dependent GPDs

Impact parameter dependent quark GPD [Burkardt, 2000]

$$q(x,\boldsymbol{b}_\perp) = \int \frac{d\lambda}{2\pi P^+}\, e^{i\lambda x}\, \langle P^+, \boldsymbol{R}_\perp = 0|\, \overline{\psi}(-\frac{\lambda}{2}n + \boldsymbol{b}_\perp)\, \gamma^+\, \mathcal{L}\, \psi(\frac{\lambda}{2}n + \boldsymbol{b}_\perp)\, |P^+, \boldsymbol{R}_\perp = 0\rangle$$

Normalized proton state localized in the spatial ⊥ direction for some wave packet Φ

$$|P^+, \boldsymbol{R}_\perp = 0\rangle = \mathcal{N} \int \frac{d^2 \boldsymbol{P}_\perp}{(2\pi)^2} \Phi(\boldsymbol{P}_\perp) |P^+, \boldsymbol{P}_\perp\rangle$$

The wave packet must be sufficiently localized in transverse position space, in order to be viewed as the FT of a GPD:

$$q(x,\boldsymbol{b}_\perp) \approx \int \frac{d^2 \boldsymbol{\Delta}_\perp}{(2\pi)^2}\, e^{-i\boldsymbol{b}_\perp\cdot\boldsymbol{\Delta}_\perp}\, H(x, 0, -\boldsymbol{\Delta}_\perp^2)$$

$\Phi(\boldsymbol{P}_\perp + \boldsymbol{\Delta}_\perp) \approx \Phi(\boldsymbol{P}_\perp)$

[Burkardt, 2000; Diehl, 2002]

For the 3D spatial distribution of the proton: inevitable wave packet dependence

See e.g. Jaffe, 2021

For 2D spatial distributions one can take P^+ large, in order to have a sufficiently broad distribution in $P_\perp$ while $P^- \ll P^+$ (ΔP_z small, Δz large, while Δx and Δy small)

Wigner distributions

This applies to the relation between the Wigner distribution and the GTMD as well:

$$W(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp) \equiv \int \frac{d\lambda}{2\pi P^+} d^2\boldsymbol{r}_\perp e^{i\lambda x} e^{i\boldsymbol{k}_\perp \cdot \boldsymbol{r}_\perp}$$

$$\times \langle P^+, \boldsymbol{R}_\perp = 0 | \, \overline{\psi}(-\frac{\lambda}{2} n + \boldsymbol{b}_\perp - \frac{\boldsymbol{r}_\perp}{2}) \gamma^+ \mathcal{L} \psi(\frac{\lambda}{2} n + \boldsymbol{b}_\perp + \frac{\boldsymbol{r}_\perp}{2}) \, | P^+, \boldsymbol{R}_\perp = 0 \rangle$$

$$q_W(x, \boldsymbol{k}_\perp, \boldsymbol{\Delta}_\perp) \equiv \int \frac{d^2\boldsymbol{b}_\perp}{(2\pi)^2} \, e^{i\boldsymbol{b}_\perp \cdot \boldsymbol{\Delta}_\perp} W(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp)$$

The GTMD definition as an off-forward TMD coincides with the definition as a Fourier transform of Wigner distribution in the limit $P^+ \to \infty$ ($P^- \to 0$)

At small x ($s \gg Q^2$) one considers this for gluons and in the dipole frame ($q^- \gg q^+$), which allows to include also photo-production

Note that the Wigner distribution involves a diagonal matrix element, but its Fourier transform corresponds to an off-forward matrix element

Small-x model for the unpolarized gluon GTMD

Impact parameter dependent MV model

In the saturation regime at small x one often employs the McLerran-Venugopalan (MV) model for the dipole scattering amplitude:

$$\left\langle \frac{S^{[\Box]}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp) + S^{[\Box]\dagger}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp)}{2} \right\rangle_C = \exp\left(-\frac{1}{4} r_\perp^2 Q_s^2 \ln\left[\frac{1}{r_\perp^2 \Lambda^2} + e \right] \right)$$

where an average over the color configurations of the target is taken

$$\boldsymbol{r}_\perp = \boldsymbol{y}_\perp - \boldsymbol{x}_\perp \qquad \boldsymbol{b}_\perp = (\boldsymbol{x}_\perp + \boldsymbol{y}_\perp)/2$$

Often Q_s is given a $b_\perp$ dependence (position w.r.t. the center of the target):

$$Q_s^2 \to Q_s^2(b_\perp) = \frac{4\pi\alpha_s^2 C_F}{N_c} T_p(b_\perp)$$

If Q_s is $b_\perp$ dependent, then the Fourier transform corresponds to an off-forward matrix element and hence to a GTMD:

$$\langle S^{[\Box]}(\boldsymbol{b}_\perp, \boldsymbol{r}_\perp) \rangle_C = \langle P^+, \boldsymbol{R}_\perp = 0 | S^{[\Box]}(\boldsymbol{b}_\perp, \boldsymbol{r}_\perp) | P^+, \boldsymbol{R}_\perp = 0 \rangle$$

MV-like model

This leads to a small-x MV-like model for the gluon GTMD:

$$G^{[\Box]}(\boldsymbol{k}_\perp, \boldsymbol{\Delta}_\perp) = 4N_c \int \frac{d^2\boldsymbol{r}_\perp d^2\boldsymbol{b}_\perp}{(2\pi)^2}\, e^{-i\boldsymbol{k}_\perp\cdot\boldsymbol{r}_\perp} e^{i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp} \exp\left(-\frac{1}{4}r_\perp^2 Q_s^2(b_\perp)\ln\left[\frac{1}{r_\perp^2\Lambda^2}+e\right]\right)$$

Similar expressions were considered by Hagiwara, Hatta, Pasechnik, Tasevsky & Teryaev, 2017; Salazar, Schenke, 2019

Consider a Gaussian profile for the proton

$$T_p(b_\perp) = \exp\left(-b_\perp^2/(2R_p^2)\right)$$

R_p = gluonic radius of the target

When $R_p \to \infty$, then one retrieves the forward case: $\Delta_\perp = 0$

The MV model expression for the gluon TMD is retrieved

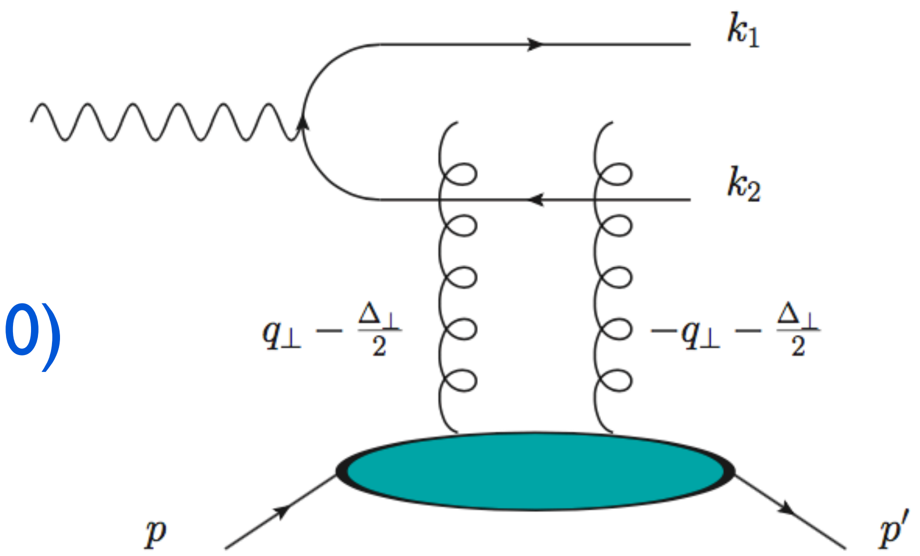
Side remark: the wave packet of the state |P+, $R_\perp$ = 0> describes how well the state is localized, which should be much smaller than the width of the $b_\perp$ profile

Diffractive dijet production

Diffractive dijet production

First suggestion to measure gluon GTMDs: hard diffractive dijet production in eA

Altinoluk, Armesto, Beuf, Rezaeian, 2016; Hatta, Xiao, Yuan, 2016



Earlier suggested to probe gluon GPDs ($\Delta_\perp = 0$, $\xi \neq 0$)

Braun, Ivanov, 2005

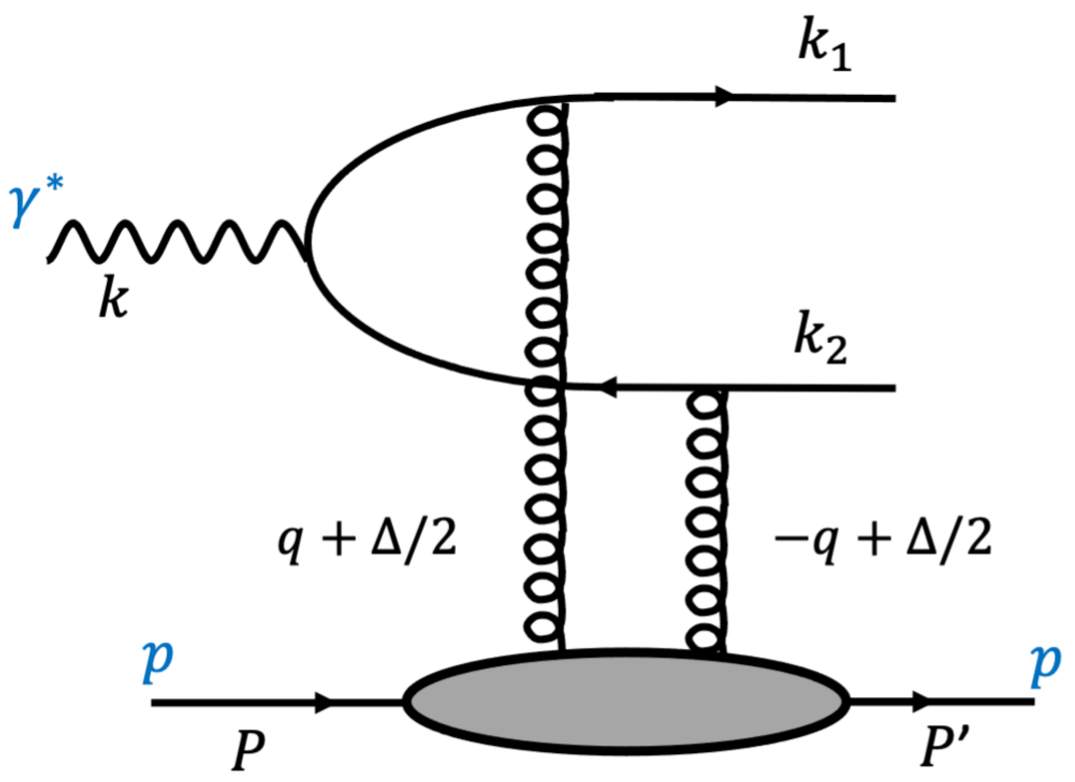
This is considered in the dipole frame at small x (large s), where the photon splits into a quark-antiquark dipole before interacting with the target (ISI & FSI)

Both jets have large transverse momenta to guarantee small (perturbative) dipoles

But the transverse momentum of the jet pair is taken small ($\Delta_\perp \neq 0$, $\xi=0$)

The transverse momentum dependence of the GTMD is probed indirectly

Diffractive dijet production



$$K = (k_1 - k_2)/2$$
$$\Delta = k_1 + k_2$$

Back-to-back correlation limit:

$$\Delta_\perp \ll K_\perp$$

$$\frac{d\sigma}{dy_1 dy_2 d^2\boldsymbol{k}_{1\perp} d^2\boldsymbol{k}_{2\perp}} \propto \int d^2\boldsymbol{q}_\perp d^2\boldsymbol{q}'_\perp \mathcal{F}^{[\Box]}(\boldsymbol{q}_\perp, \boldsymbol{\Delta}_\perp)\mathcal{F}^{[\Box]}(\boldsymbol{q}'_\perp, \boldsymbol{\Delta}_\perp)\mathcal{A}(\boldsymbol{K}_\perp, \boldsymbol{q}_\perp, \boldsymbol{q}'_\perp, \epsilon_f^2)$$

Altinoluk, Armesto, Beuf, Rezaeian, 2016; Hatta, Xiao, Yuan, 2016

$$\epsilon_f^2 = z(1-z)Q^2$$

$$G^{[\Box]} \to \mathcal{F}^{[\Box]} \qquad S^{[\Box]}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp) \to 1 - S^{[\Box]}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp)$$

One can access the off-forwardness through $\Delta_\perp$: $t = -\Delta_\perp^2$

MV-like model

Apply an MV-like model that is similar to the model by Hagiwara et al, 2017:

$$\mathcal{F}^{[\Box]}(\boldsymbol{k}_\perp, \boldsymbol{\Delta}_\perp) = 4N_c \int \frac{d^2\boldsymbol{r}_\perp d^2\boldsymbol{b}_\perp}{(2\pi)^2} e^{-i\boldsymbol{k}_\perp\cdot\boldsymbol{r}_\perp} e^{i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp} e^{-\epsilon_r r_\perp^2} \left[1 - \exp\left(-\frac{1}{4} r_\perp^2 \chi Q_s^2(b_\perp) \ln\left[\frac{1}{r_\perp^2\Lambda^2} + e\right]\right)\right]$$

Two free parameters are to be fitted to the data: ϵ_r and χ [Boer, Setyadi, 2021]

ϵ_r cuts out the region where the $q\bar{q}$ dipole size becomes large compared to the target size, where the model should not be applicable (cut-off in b not needed)

χ is viewed as determining the average small-x value

The inclusive DIS HERA data (GBW model) is described well by

$$Q_s^2(x) = A^{1/3}(3 \cdot 10^{-4}/x)^{0.3} \ [\mathrm{GeV}^2]$$

So we expect

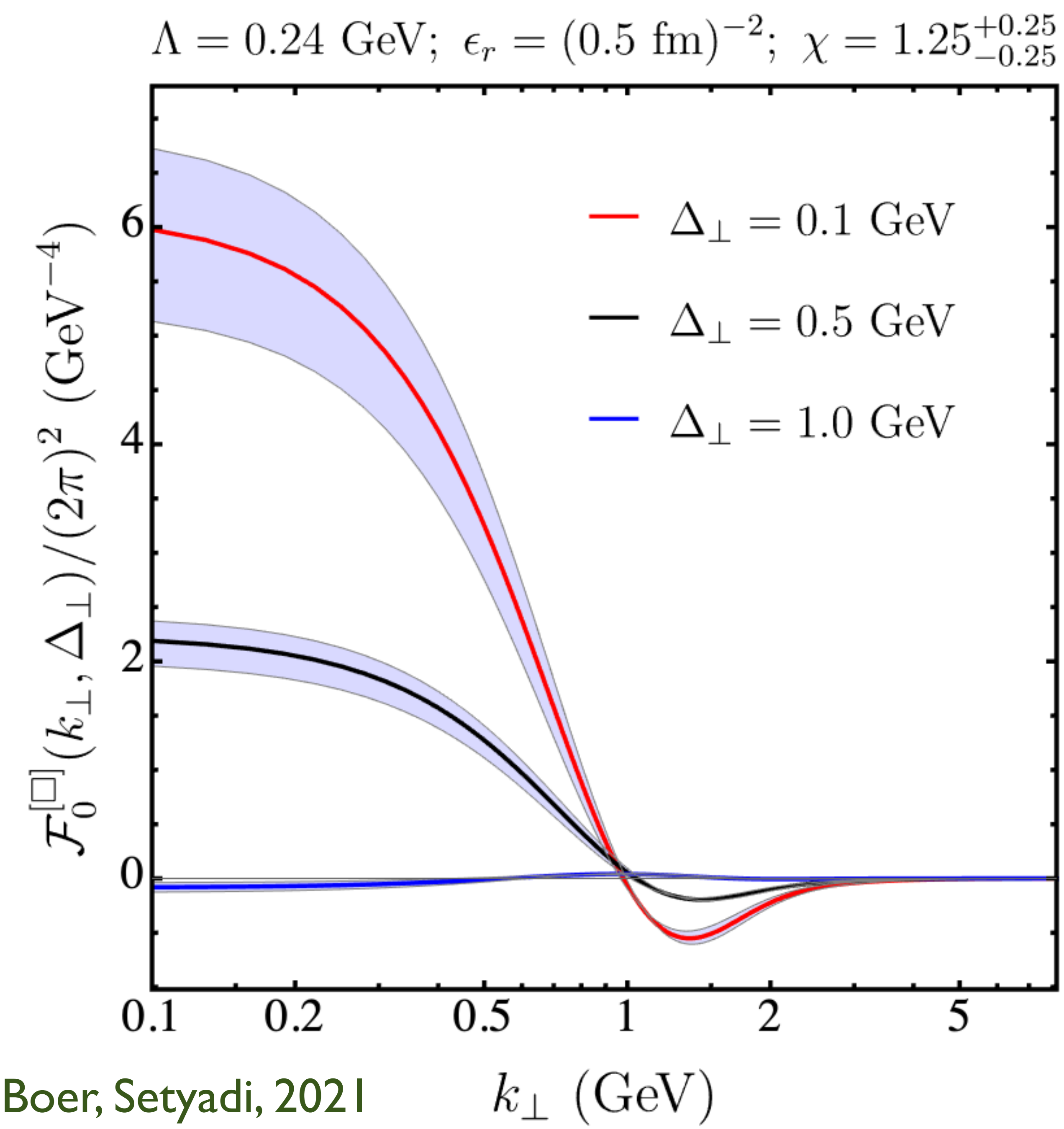
$$\chi \sim x^{-0.3}$$

Smaller x data will require larger χ

Model

$$\mathcal{F}^{[\Box]}(\boldsymbol{k}_\perp, \boldsymbol{\Delta}_\perp) = 4N_c \int \frac{d^2\boldsymbol{r}_\perp d^2\boldsymbol{b}_\perp}{(2\pi)^2}\, e^{-i\boldsymbol{k}_\perp\cdot\boldsymbol{r}_\perp} e^{i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp}\, e^{-\epsilon_r r_\perp^2} \left[1 - \exp\left(-\frac{1}{4} r_\perp^2 \chi Q_s^2(b_\perp) \ln\left[\frac{1}{r_\perp^2\Lambda^2} + e\right]\right)\right]$$

$$\mathcal{F}^{[\Box]}(\boldsymbol{k}_\perp, \boldsymbol{\Delta}_\perp) = \mathcal{F}_0^{[\Box]}(k_\perp, \Delta_\perp) + 2\mathcal{F}_2^{[\Box]}(k_\perp, \Delta_\perp)\cos 2\theta_{k\Delta} + \ldots$$



Boer, Setyadi, 2021

The data is ϕ integrated, therefore we restrict to $\mathcal{F}_0$

ϵ_r is chosen to correspond to the gluonic radius

Data will be for K⊥ > 5 GeV
Dominant contribution from:

$$\Delta_\perp \ll K_\perp \approx k_{1\perp}$$

$$K_\perp = k_{1\perp} - \Delta_\perp/2$$

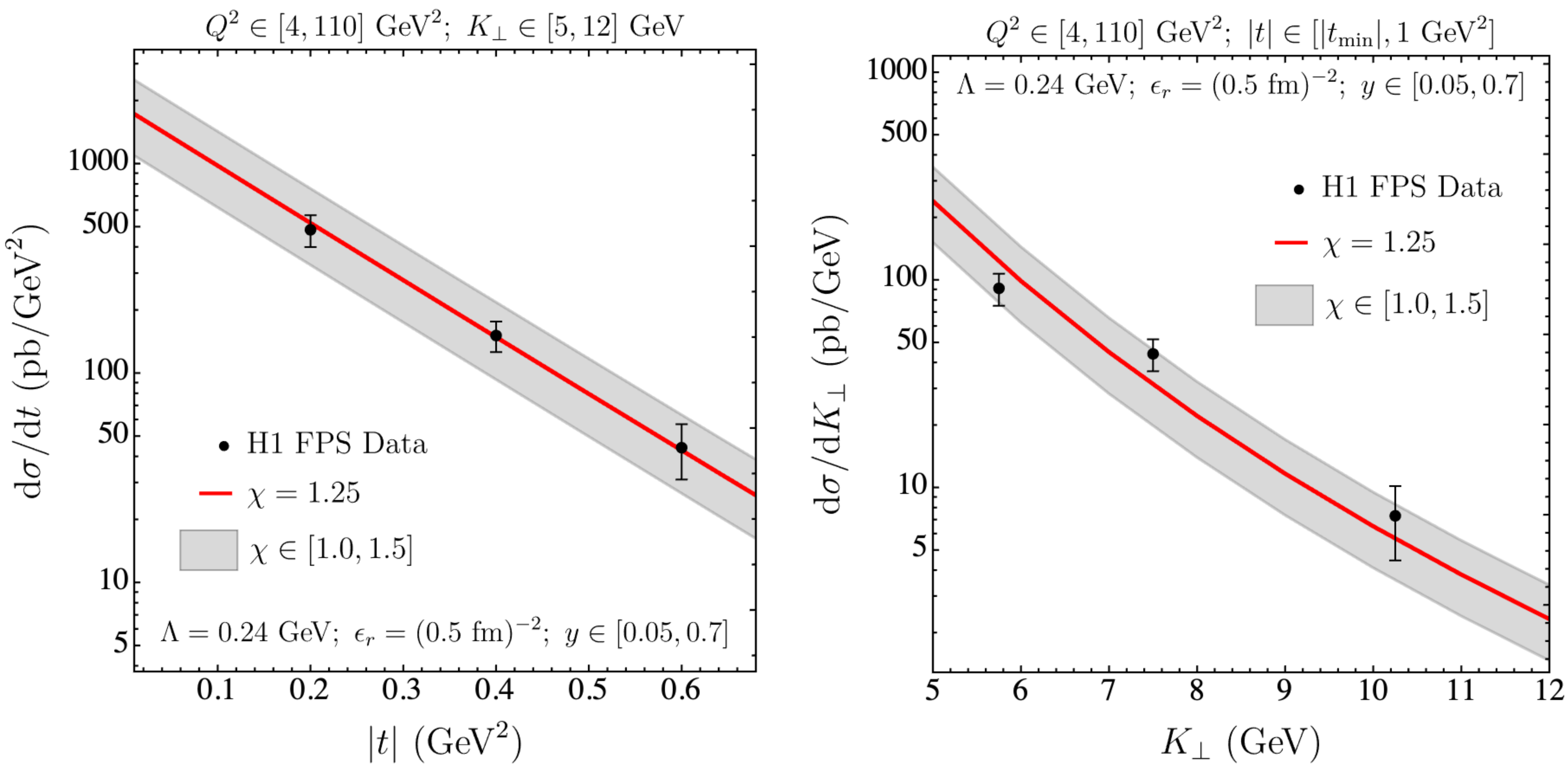
H1 data

The H1 data considered is on ep → e' j j X' p, so not strictly exclusive dijets, but it will be dominated by dijets (the LO contribution)

EPJC 72 (2012) 1970

The ZEUS data on exclusive diffractive dijets are not differential enough

EPJC 76 (2016) 16



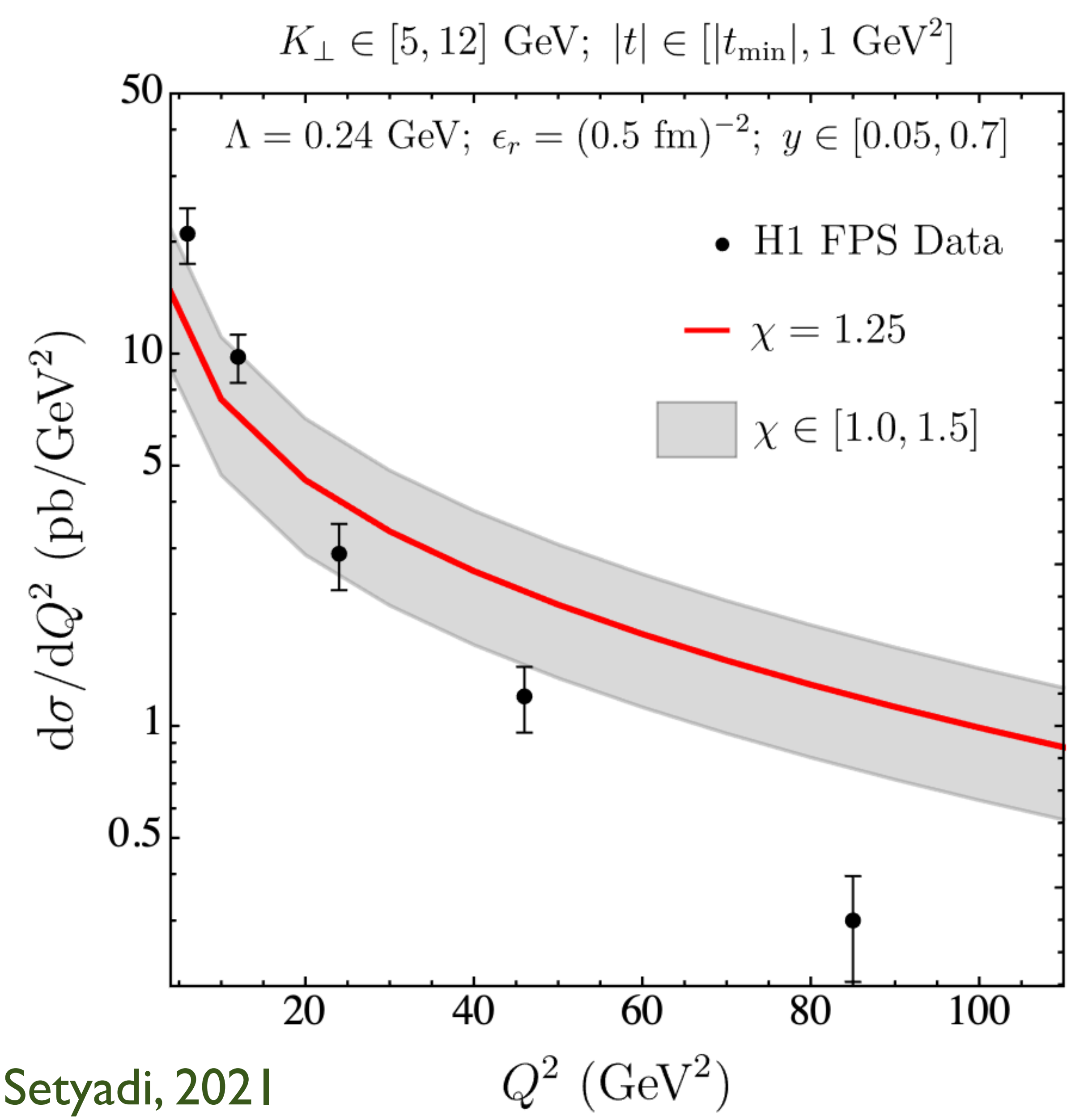
$K_\perp \approx k_{1\perp}$

Boer, Setyadi, 2021

H1 data

The data as function of t and $K_\perp$ is integrated over Q and y ($\sqrt{s}$ = 319 GeV)
This corresponds to an average small-x value on the order of 10^{-3}

The data as a function of Q is only integrated over y, hence all data points correspond to a different average x value, not fully captured by this x→0 model



Boer, Setyadi, 2021

Smaller x means larger χ

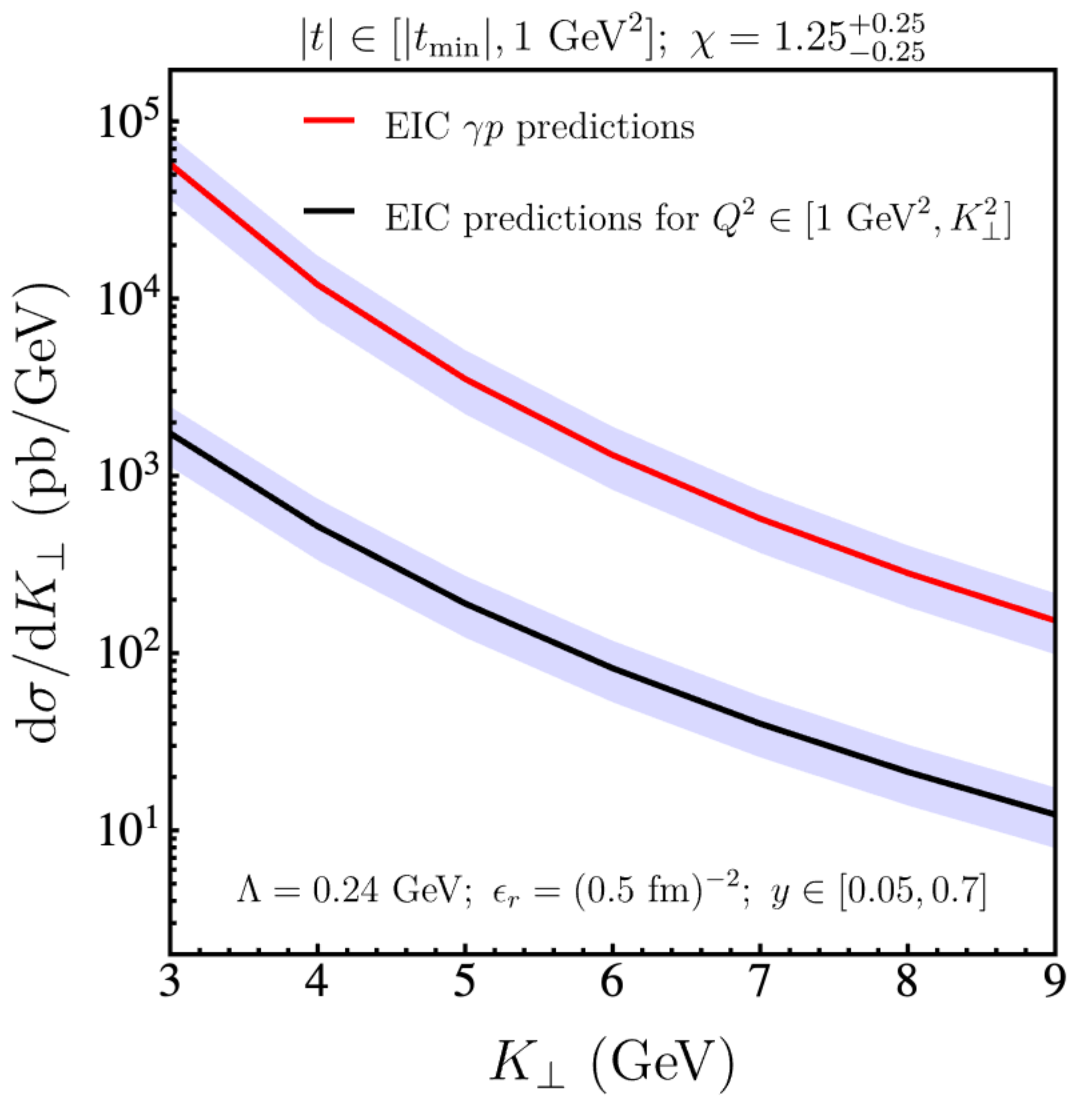
$$\chi \sim x^{-0.3}$$

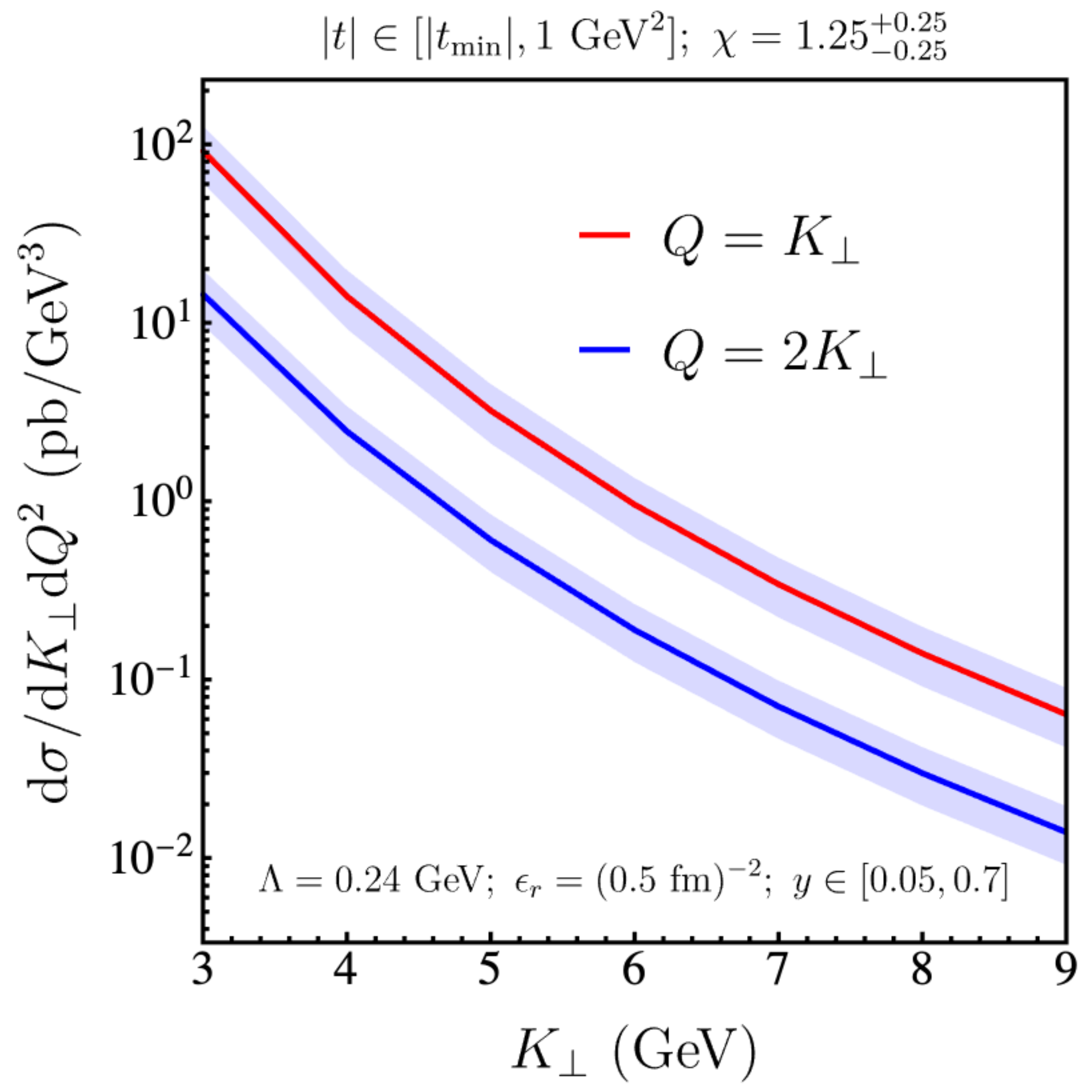
$Q^2 = xys$

This explains why smaller Q^2 data points favor larger χ and vice versa

For effects of soft gluon radiation, see Hatta, Mueller, Ueda, Yuan, 2019; Hatta, Yuan, Xiao, Zhou, 2020

Predictions for EIC





Predictions for photo-production and for electroproduction in a somewhat different regime

For $\sqrt{s}$=140 GeV the average x is between 10^{-3} and 10^{-2} (expected to favor somewhat smaller χ)

For lower $K_\perp$, the data are expected to favor higher χ and overshoot the lines, and vice versa for higher $K_\perp$

Boer, Setyadi, 2021

Conclusions

Conclusions

- Like TMDs the GTMDs are process dependent and like GPDs they require translation non-invariance, such as a $b_\perp$ profile w.r.t. a center
- This $b_\perp$ profile of the proton is the one as seen by a relativistic probe (here γ^* or γ)
- Diffractive dijet production in ep collisions probes the dipole gluon GTMD, which becomes a Wilson loop correlator in the small-x limit
- An MV-like model for the unpolarized dipole gluon GTMD was shown to provide a reasonable description of the H1 data, leading to predictions for EIC
- Comparing H1 and EIC data for the angular-integrated cross sections at small x would offer a first test of the underlying GTMD description

Back-up slides

Small-x limit of GTMDs

For unpolarized hadrons there are 4 independent (complex valued) gluon GTMDs

$$G^{[U,U']\,ij}(x,\boldsymbol{k},\xi,\boldsymbol{\Delta}) = x\left(\delta_T^{ij}\,\mathcal{F}_1 + \frac{k_T^{ij}}{M^2}\,\mathcal{F}_2 + \frac{\Delta_T^{ij}}{M^2}\,\mathcal{F}_3 + \frac{k_T^{[i}\Delta_T^{j]}}{M^2}\,\mathcal{F}_4\right)$$

For [+,-] there is only one gluon GTMD in the limit x→0 (at leading twist)

$$G^{[+,-]\,ij}(\boldsymbol{k},\boldsymbol{\Delta}) = \frac{2N_c}{\alpha_s}\left[\frac{1}{2}\left(\boldsymbol{k}^2 - \frac{\boldsymbol{\Delta}^2}{4}\right)\delta_T^{ij} + k_T^{ij} - \frac{\Delta_T^{ij}}{4} - \frac{k_T^{[i}\Delta_T^{j]}}{2}\right] G^{[\square]}(\boldsymbol{k},\boldsymbol{\Delta})$$

where

$$G^{[\square]}(\boldsymbol{k},\boldsymbol{\Delta}) \equiv \int \frac{d^2\boldsymbol{x}\,d^2\boldsymbol{y}}{(2\pi)^4}\, e^{-i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{y})+i\boldsymbol{\Delta}\cdot\frac{\boldsymbol{x}+\boldsymbol{y}}{2}}\, \frac{\langle p'|\,S^{[\square]}(\boldsymbol{x},\boldsymbol{y})\,|p\rangle\big|_{\rm LF}}{\langle P|P\rangle},$$

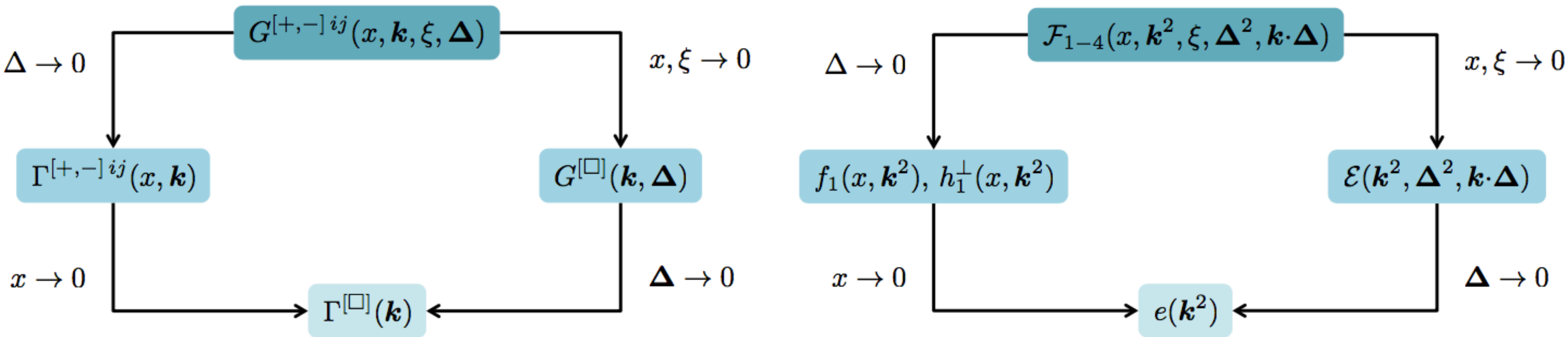
This implies: $$\lim_{x,\xi\to 0} x\mathcal{F}_1 = \lim_{x,\xi\to 0} x\mathcal{F}_2^{(1)} = -4\lim_{x,\xi\to 0} x\mathcal{F}_3^{(1)} = -2\lim_{x,\xi\to 0} x\mathcal{F}_4^{(1)} = \mathcal{E}^{(1)}$$

$$\mathcal{F}_i^{(n)} \equiv [(\boldsymbol{k}^2 - \boldsymbol{\Delta}^2/4)/(2M^2)]^n\,\mathcal{F}_i$$

The imaginary part corresponding to the odderon operator, which for an unpolarized proton vanishes in the forward limit

Boer, van Daal, Mulders, Petreska, 2018

Gluon GTMDs



PhD thesis by Tom van Daal, 2018

$$G^{[U,U']\,ij}(x,\boldsymbol{k},\xi,\boldsymbol{\Delta}) = x\left(\delta_T^{ij}\,\mathcal{F}_1 + \frac{k_T^{ij}}{M^2}\,\mathcal{F}_2 + \frac{\Delta_T^{ij}}{M^2}\,\mathcal{F}_3 + \frac{k_T^{[i}\Delta_T^{j]}}{M^2}\,\mathcal{F}_4\right)$$

1 In principle one could also have a function that comes with the symmetric and traceless Lorentz structure $k_T^{\{i}\Delta_T^{j\}} + (\boldsymbol{k}\cdot\boldsymbol{\Delta})\,g_T^{ij}$. However, this function would not be independent from the other ones; more specifically, it could be eliminated by suitable redefinitions of the functions $\mathcal{F}_2$ and $\mathcal{F}_3$ through the (two-dimensional) relation $\boldsymbol{k}\cdot\boldsymbol{\Delta}\left[k_T^{\{i}\Delta_T^{j\}} + (\boldsymbol{k}\cdot\boldsymbol{\Delta})\,g_T^{ij}\right] = \boldsymbol{\Delta}^2 k_T^{ij} + \boldsymbol{k}^2\Delta_T^{ij}$.

Boer, van Daal, Mulders, Petreska, 2018

MV-like model

Apply an MV-like model that is similar to the model by Hagiwara et al, 2017:

$$\mathcal{F}^{[\Box]}(\boldsymbol{k}_\perp, \boldsymbol{\Delta}_\perp) = 4N_c \int \frac{d^2\boldsymbol{r}_\perp d^2\boldsymbol{b}_\perp}{(2\pi)^2}\, e^{-i\boldsymbol{k}_\perp\cdot\boldsymbol{r}_\perp} e^{i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp}\, e^{-\epsilon_r r_\perp^2} \left[1 - \exp\left(-\frac{1}{4} r_\perp^2 \chi Q_s^2(b_\perp) \ln\left[\frac{1}{r_\perp^2\Lambda^2} + e\right]\right)\right]$$

Two free parameters are to be fitted to the data: ϵ_r and χ

ϵ_r cuts out the region where the $q\bar{q}$ dipole size becomes large compared to the target size, where the model should not be applicable (cut-off in b not needed)

χ is viewed as determining the average small-x value

The inclusive DIS HERA data (GBW model) is described well by

$$Q_s^2(x) = A^{1/3}(3\cdot 10^{-4}/x)^{0.3}\ [\mathrm{GeV}^2]$$

Equating this expression for $\alpha_s \approx 0.3$ with $\chi Q_s^2(b_\perp = 0_\perp) = 0.5\chi A^{1/3}\ [\mathrm{GeV}^2]$ one finds

$$\chi = 2(3\cdot 10^{-4}/x)^{0.3}$$

Smaller x means larger χ

Boer, Setyadi, 2021

GPDs

Impact parameter dependent quark GPD [Burkardt 2000]

$$q(x,\boldsymbol{b}_\perp) = \int \frac{d\lambda}{2\pi P^+}\, e^{i\lambda x}\, \langle P^+, \boldsymbol{R}_\perp = 0|\, \overline{\psi}(-\frac{\lambda}{2}n + \boldsymbol{b}_\perp)\, \gamma^+\, \mathcal{L}\, \psi(\frac{\lambda}{2}n + \boldsymbol{b}_\perp)\, |P^+, \boldsymbol{R}_\perp = 0\rangle$$

Normalized proton state localized in the spatial $\perp$ direction for some wave packet Φ

$$|P^+, \boldsymbol{R}_\perp = 0\rangle = \mathcal{N} \int \frac{d^2\boldsymbol{P}_\perp}{(2\pi)^2}\, \Phi(\boldsymbol{P}_\perp) |P^+, \boldsymbol{P}_\perp\rangle$$

$$\begin{aligned} q(x,\boldsymbol{b}_\perp) &= |\mathcal{N}|^2 \int \frac{d\lambda}{2\pi P^+} e^{i\lambda x} \int \frac{d^2\boldsymbol{P}_\perp}{(2\pi)^2} \frac{d^2\boldsymbol{P}'_\perp}{(2\pi)^2} \Phi^*(\boldsymbol{P}'_\perp)\Phi(\boldsymbol{P}_\perp) \langle P^+, \boldsymbol{P}'_\perp|\, \overline{\psi}(-\frac{\lambda}{2}n + \boldsymbol{b}_\perp)\gamma^+ \mathcal{L}\psi(\frac{\lambda}{2}n + \boldsymbol{b}_\perp)\, |P^+, \boldsymbol{P}_\perp\rangle \\ &= |\mathcal{N}|^2 \int \frac{d\lambda}{2\pi P^+}\, e^{i\lambda x} \int \frac{d^2\boldsymbol{P}_\perp}{(2\pi)^2} \frac{d^2\boldsymbol{P}'_\perp}{(2\pi)^2}\, \Phi^*(\boldsymbol{P}'_\perp)\Phi(\boldsymbol{P}_\perp) e^{-i\boldsymbol{b}_\perp\cdot\boldsymbol{\Delta}_\perp} \langle P^+, \boldsymbol{P}'_\perp|\, \overline{\psi}(-\frac{\lambda}{2}n)\, \gamma^+\, \mathcal{L}\, \psi(\frac{\lambda}{2}n)\, |P^+, \boldsymbol{P}_\perp\rangle \\ &\approx |\mathcal{N}|^2 \int \frac{d^2\boldsymbol{P}_\perp}{(2\pi)^2} |\Phi(\boldsymbol{P}_\perp)|^2 \frac{d^2\boldsymbol{\Delta}_\perp}{(2\pi)^2}\, e^{-i\boldsymbol{b}_\perp\cdot\boldsymbol{\Delta}_\perp} H(x,0,-\boldsymbol{\Delta}_\perp^2) = \int \frac{d^2\boldsymbol{\Delta}_\perp}{(2\pi)^2}\, e^{-i\boldsymbol{b}_\perp\cdot\boldsymbol{\Delta}_\perp} H(x,0,-\boldsymbol{\Delta}_\perp^2) \end{aligned}$$

$\Phi(\boldsymbol{P}_\perp + \boldsymbol{\Delta}_\perp) \approx \Phi(\boldsymbol{P}_\perp)$

Wave packet slowly varying on the scale of the off-forwardness, i.e. sufficiently localized in transverse position space

For 3D spatial distribution of the proton this leads to wave packet dependence: insufficient room between Compton wave length (0.21 fm) and system size (0.85 fm)

Jaffe, 2021

Wilson loop correlator

$$\Gamma^{[+,-]\,ij}(x,\boldsymbol{k}_T) \;\xrightarrow{x\to 0}\; \frac{k_T^i k_T^j}{2\pi L}\,\Gamma_0^{[\Box]}(\boldsymbol{k}_T)$$

a single Wilson loop matrix element

D.B., Cotogno, van Daal, Mulders, Signori & Ya-Jin Zhou, JHEP 2016

$$U^{[\Box]} = U^{[+]}_{[0,y]} U^{[-]}_{[y,0]}$$

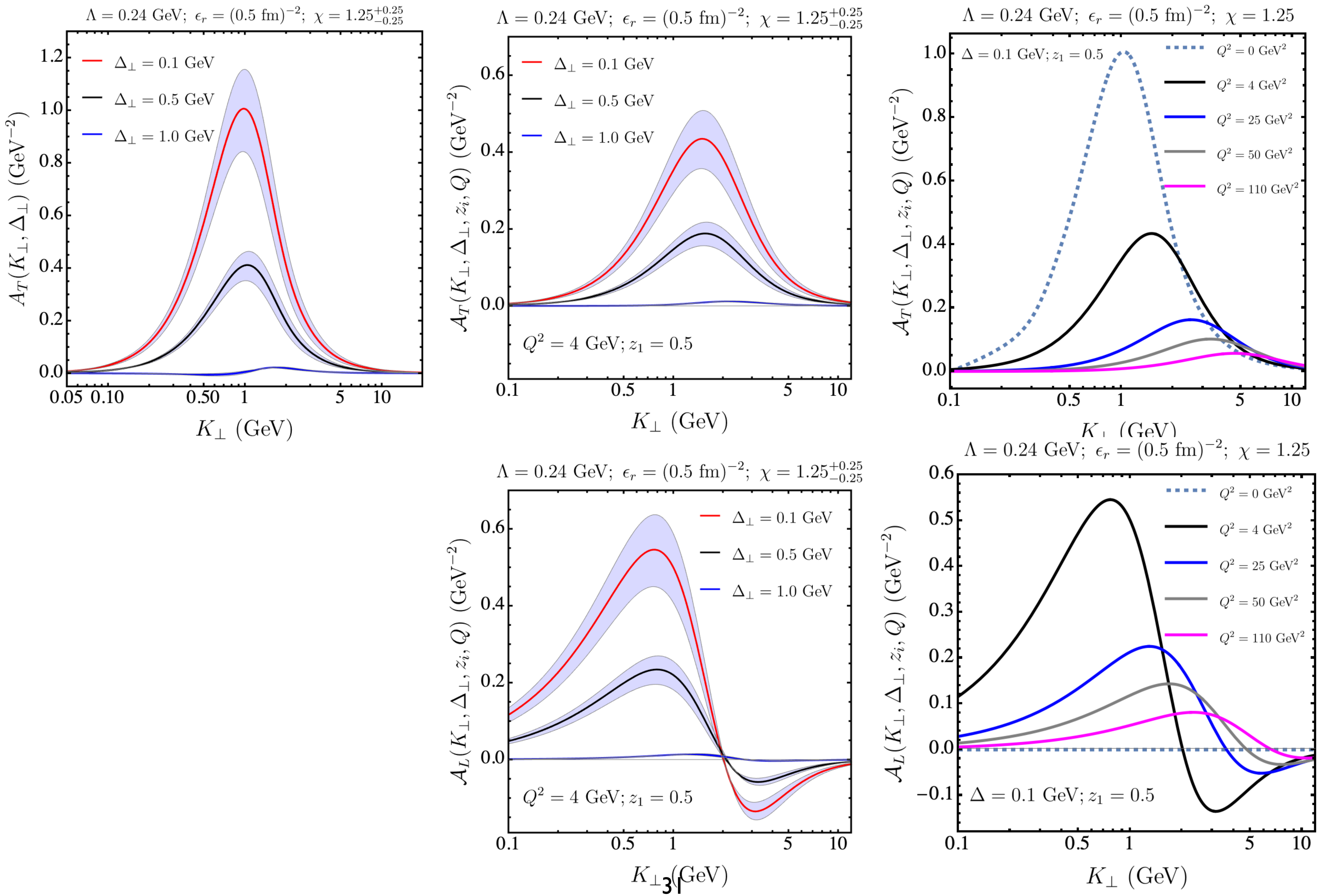
$$\begin{aligned}
k_T^i k_T^j\,\Gamma_0^{[\Box]}(\boldsymbol{k}_T) &= 4\int \frac{d^2\xi_T}{(2\pi)^2}\, e^{ik_T\cdot\xi_T}\,\langle P|\, G_T^i(0)\, U^{[+]}_{[0,\xi]}\, G_T^j(\xi)\, U^{[-]}_{[\xi,0]}\,|P\rangle\Big|_{\xi\cdot n=0} \\
&= \int \frac{d\eta\cdot P\, d\eta'\cdot P\, d^2\xi_T}{(2\pi)^2}\, e^{ik_T\cdot\xi_T}\,\langle P|\, F^{ni}(\eta')\, U^{[+]}_{[\eta',\eta]}\, F^{nj}(\eta)\, U^{[-]}_{[\eta,\eta']}\,|P\rangle\Big|_{\substack{\eta'\cdot n=\eta\cdot n=0,\\ \eta'_T=0_T,\,\eta_T=\xi_T}} \\
&= 2\pi L\int \frac{d\xi\cdot P\, d^2\xi_T}{(2\pi)^3}\, e^{ik\cdot\xi}\,\langle P|\, F^{ni}(0)\, U^{[+]}_{[0,\xi]}\, F^{nj}(\xi)\, U^{[-]}_{[\xi,0]}\,|P\rangle\Big|_{\xi\cdot n=k\cdot n=0} \\
&= 2\pi L\,\Gamma^{[+,-]\,ij}(0,\boldsymbol{k}_T),
\end{aligned} \qquad (3.1)$$

$$\left[i\partial_z^k, U^{[\pm]}_{[a,z]}\right] = \mp g U^{[\pm]}_{[a,z]}\, G_T^k(z),$$

$$G_T^k(z) \equiv \frac{1}{2}\int_{-\infty}^{\infty} d\eta^-\, U^n_{[z^-,\eta^-;z]}\, F^{+k}(z^+,\eta^-,z)\, U^n_{[\eta^-,z^-;z]}\,.$$

$$\begin{aligned}
G^{[+,-]\,ij}(\boldsymbol{k},\boldsymbol{\Delta}) &= \frac{16}{\langle P|P\rangle}\int \frac{d^2\boldsymbol{x}\, d^2\boldsymbol{y}}{(2\pi)^3}\, e^{-i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{y})+i\boldsymbol{\Delta}\cdot\frac{\boldsymbol{x}+\boldsymbol{y}}{2}} \\
&\quad\times \langle p'|\,\mathrm{Tr}\left(G_T^j(x)\, U^{[-]}_{[x,y]}\, G_T^i(y)\, U^{[+]}_{[y,x]}\right)|p\rangle\Big|_{x^+=y^+=0} \\
&= \frac{4}{g^2\,\langle P|P\rangle}\int \frac{d^2\boldsymbol{x}\, d^2\boldsymbol{y}}{(2\pi)^3}\, e^{-i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{y})+i\boldsymbol{\Delta}\cdot\frac{\boldsymbol{x}+\boldsymbol{y}}{2}} \\
&\quad\times \langle p'|\,\partial_x^j\partial_y^i\,\mathrm{Tr}\left(U^{[\Box]}_{[y,x]}\right)|p\rangle\Big|_{x^+=y^+=0},
\end{aligned}$$

Amplitudes



$\Lambda = 0.24$ GeV; $\epsilon_r = (0.5 \text{ fm})^{-2}$; $\chi = 1.25^{+0.25}_{-0.25}$
$\Delta_\perp = 0.1$ GeV
$\Delta_\perp = 0.5$ GeV
$\Delta_\perp = 1.0$ GeV
$A_T(K_\perp, \Delta_\perp)$ (GeV^{-2})
$K_\perp$ (GeV)
$\Lambda = 0.24$ GeV; $\epsilon_r = (0.5 \text{ fm})^{-2}$; $\chi = 1.25^{+0.25}_{-0.25}$
$A_T(K_\perp, \Delta_\perp, z_i, Q)$ (GeV^{-2})
$Q^2 = 4$ GeV; $z_1 = 0.5$
$K_\perp$ (GeV)
$\Lambda = 0.24$ GeV; $\epsilon_r = (0.5 \text{ fm})^{-2}$; $\chi = 1.25$
$\Delta = 0.1$ GeV; $z_1 = 0.5$
$Q^2 = 0$ GeV^2
$Q^2 = 4$ GeV^2
$Q^2 = 25$ GeV^2
$Q^2 = 50$ GeV^2
$Q^2 = 110$ GeV^2
$A_T(K_\perp, \Delta_\perp, z_i, Q)$ (GeV^{-2})
$K_\perp$ (GeV)
$\Lambda = 0.24$ GeV; $\epsilon_r = (0.5 \text{ fm})^{-2}$; $\chi = 1.25^{+0.25}_{-0.25}$
$A_L(K_\perp, \Delta_\perp, z_i, Q)$ (GeV^{-2})
$Q^2 = 4$ GeV; $z_1 = 0.5$
$K_\perp$ (GeV)
$\Lambda = 0.24$ GeV; $\epsilon_r = (0.5 \text{ fm})^{-2}$; $\chi = 1.25$
$A_L(K_\perp, \Delta_\perp, z_i, Q)$ (GeV^{-2})
$\Delta = 0.1$ GeV; $z_1 = 0.5$
$K_\perp$ (GeV)