# Determination of $|V_{ub}/V_{cb}|$ using baryonic decays

Stefan Meinel

THE UNIVERSITY OF ARIZONA

RIKEN BNL Research Center

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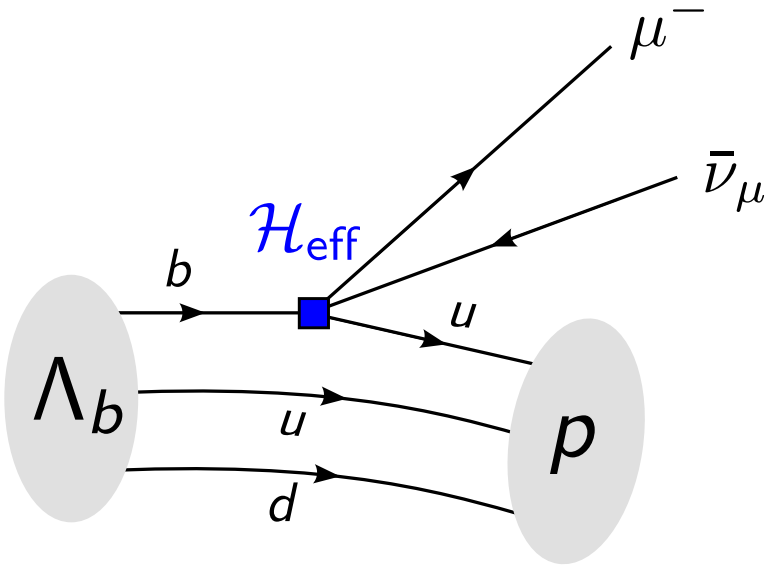
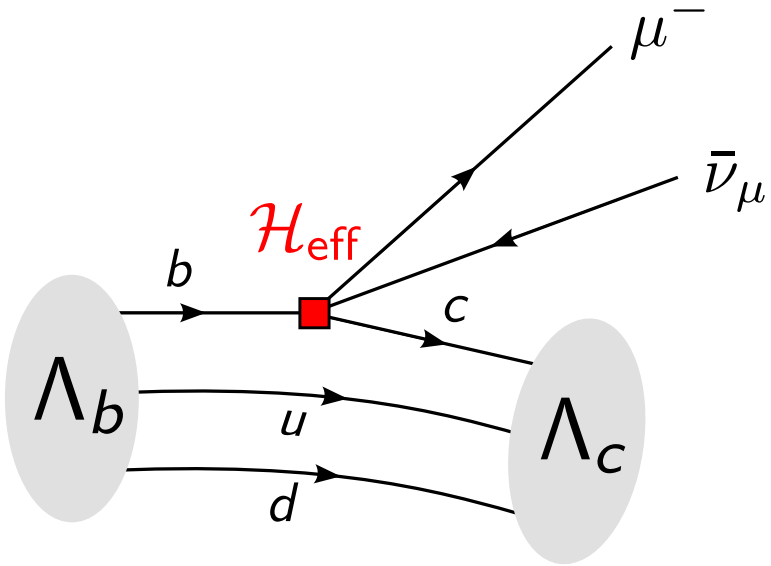
LHCb has measured the ratio of decay rates of



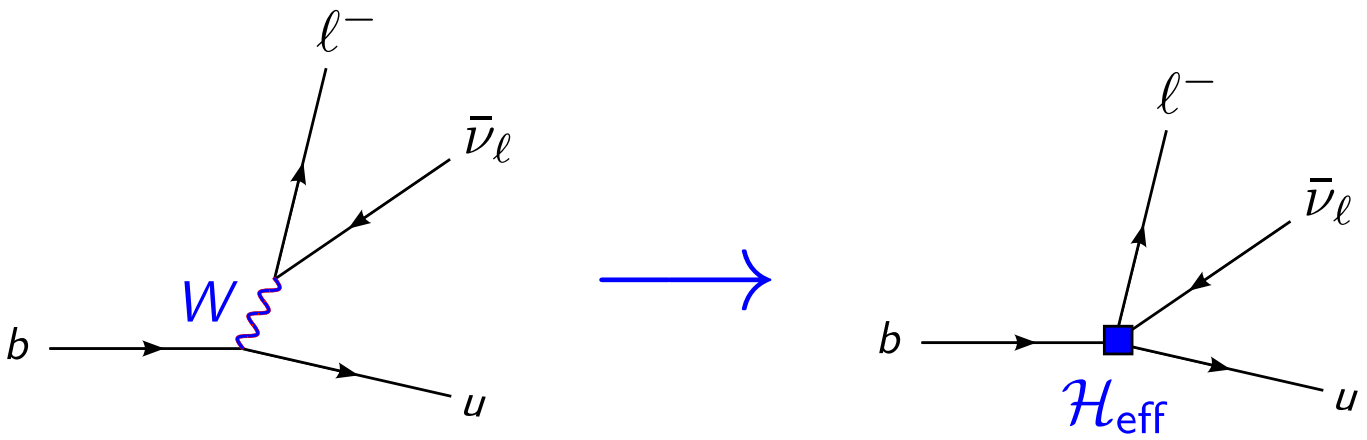

[William Sutcliffe, previous talk; Nature Physics **11**, 743-747 (2015)]

The LHCb result is

$$\frac{\displaystyle\int_{15\ \mathrm{GeV}^2}^{q^2_{\mathrm{max}}} \frac{\mathrm{d}\Gamma(\Lambda_b \to p\,\mu^-\bar{\nu}_\mu)}{\mathrm{d}q^2}\mathrm{d}q^2}{\displaystyle\int_{7\ \mathrm{GeV}^2}^{q^2_{\mathrm{max}}} \frac{\mathrm{d}\Gamma(\Lambda_b \to \Lambda_c\,\mu^-\bar{\nu}_\mu)}{\mathrm{d}q^2}\mathrm{d}q^2} = (1.00 \pm 0.04 \pm 0.08) \times 10^{-2}$$

How can we extract $|V_{ub}/V_{cb}|$ from this?

In the Standard Model,




$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{ub} \; \bar{u}\gamma^\mu(1-\gamma_5)b \; \bar{\ell}\gamma_\mu(1-\gamma_5)\nu$$

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2}(\Lambda_b \to p\mu\bar{\nu}) = |V_{ub}|^2 \times \text{function}\Big[\, \langle p \,|\, \bar{u}\gamma^\mu b \,|\Lambda_b\rangle,\ \langle p \,|\, \bar{u}\gamma^\mu\gamma_5 b \,|\Lambda_b\rangle \,\Big],$$

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2}(\Lambda_b \to \Lambda_c\mu\bar{\nu}) = |V_{cb}|^2 \times \text{function}\Big[\, \langle \Lambda_c \,|\, \bar{c}\gamma^\mu b \,|\Lambda_b\rangle,\ \langle \Lambda_c \,|\, \bar{c}\gamma^\mu\gamma_5 b \,|\Lambda_b\rangle \,\Big]$$

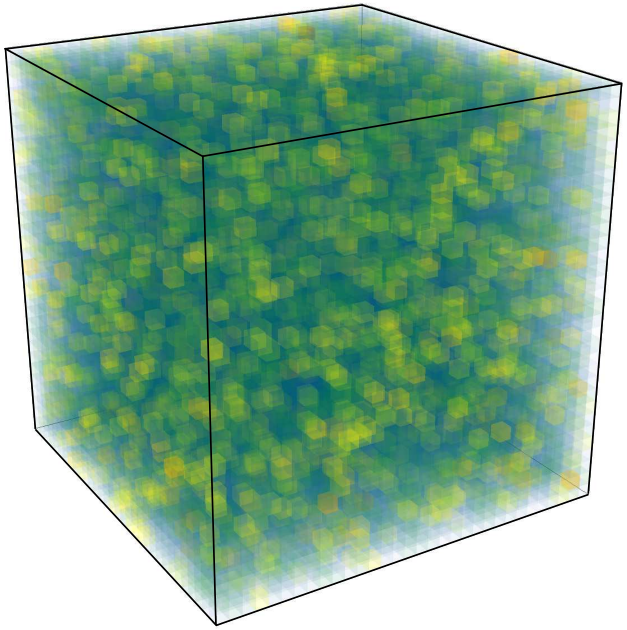
Helicity form factors [T. Feldmann and M. Yip, PRD **85**, 014035 (2012)]:

$$
\begin{aligned}
\langle F|\overline{q}\gamma^\mu b|\Lambda_b\rangle &= \overline{u}_F\Bigg[(m_{\Lambda_b}-m_F)\frac{q^\mu}{q^2}\, f_0 \\
&\quad + \frac{m_{\Lambda_b}+m_F}{s_+}\left(p^\mu + p'^\mu - (m_{\Lambda_b}^2 - m_F^2)\frac{q^\mu}{q^2}\right) f_+ \\
&\quad + \left(\gamma^\mu - \frac{2m_F}{s_+}p^\mu - \frac{2m_{\Lambda_b}}{s_+}p'^\mu\right) f_\perp\Bigg] u_{\Lambda_b}, \\
\langle F|\overline{q}\gamma^\mu\gamma_5 b|\Lambda_b\rangle &= -\overline{u}_F\,\gamma_5\Bigg[(m_{\Lambda_b}+m_F)\frac{q^\mu}{q^2}\, g_0 \\
&\quad + \frac{m_{\Lambda_b}-m_F}{s_-}\left(p^\mu + p'^\mu - (m_{\Lambda_b}^2 - m_F^2)\frac{q^\mu}{q^2}\right) g_+ \\
&\quad + \left(\gamma^\mu + \frac{2m_F}{s_-}p^\mu - \frac{2m_{\Lambda_b}}{s_-}p'^\mu\right) g_\perp\Bigg] u_{\Lambda_b}.
\end{aligned}
$$

$$
F = p, \Lambda_c, \quad \bar{q} = \bar{u}, \bar{c}, \quad s_\pm = (m_{\Lambda_b} \pm m_X)^2 - q^2
$$

"$\Lambda_b \to p\,\ell^-\,\bar{\nu}_\ell$ and $\Lambda_b \to \Lambda_c\,\ell^-\,\bar{\nu}_\ell$ form factors from lattice QCD with relativistic heavy quarks"

[W. Detmold, C. Lehner, S. Meinel, PRD **92**, 034503 (2015)]



- Gauge field configurations generated by the RBC and UKQCD collaborations

  [Y. Aoki *et al.*, PRD **83**, 074508 (2011)]

- $u$, $d$, $s$ quarks: domain-wall action

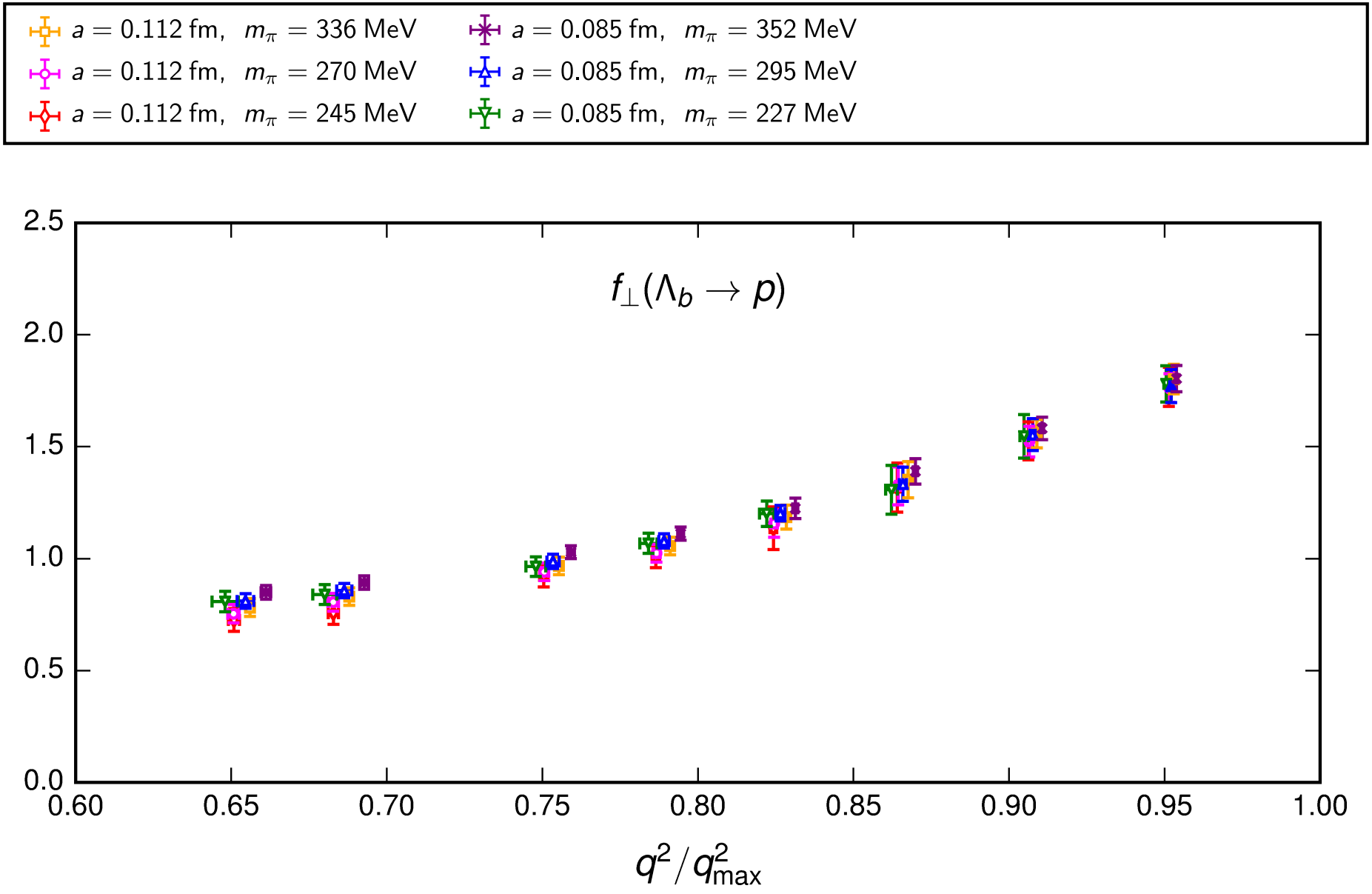
  [D. Kaplan, PLB **288**, 342 (1992); V. Furman and Y. Shamir, NPB **439**, 54 (1995)]

- $c$, $b$ quarks: "relativistic heavy-quark action"

  [A. El-Khadra, A. Kronfeld, P. Mackenzie, PRD **55**, 3933 (1997); Y. Aoki *et al.*, PRD **86**, 116003]

- "Mostly nonperturbative" renormalization

  [A. El-Khadra *et al.*, PRD **64**, 014502 (2001)]


$a = 0.112$ fm, $m_\pi = 336$ MeV
$a = 0.112$ fm, $m_\pi = 270$ MeV
$a = 0.112$ fm, $m_\pi = 245$ MeV
$a = 0.085$ fm, $m_\pi = 352$ MeV
$a = 0.085$ fm, $m_\pi = 295$ MeV
$a = 0.085$ fm, $m_\pi = 227$ MeV
$f_\perp(\Lambda_b \to p)$
2.5
2.0
1.5
1.0
0.5
0.0
0.60
0.65
0.70
0.75
0.80
0.85
0.90
0.95
1.00
$q^2 / q^2_{\mathrm{max}}$

## Combined chiral/continuum/kinematic extrapolation using modified $z$-expansion

[C. Bourrely, I. Caprini, L. Lellouch, PRD **79**, 013008 (2009)]

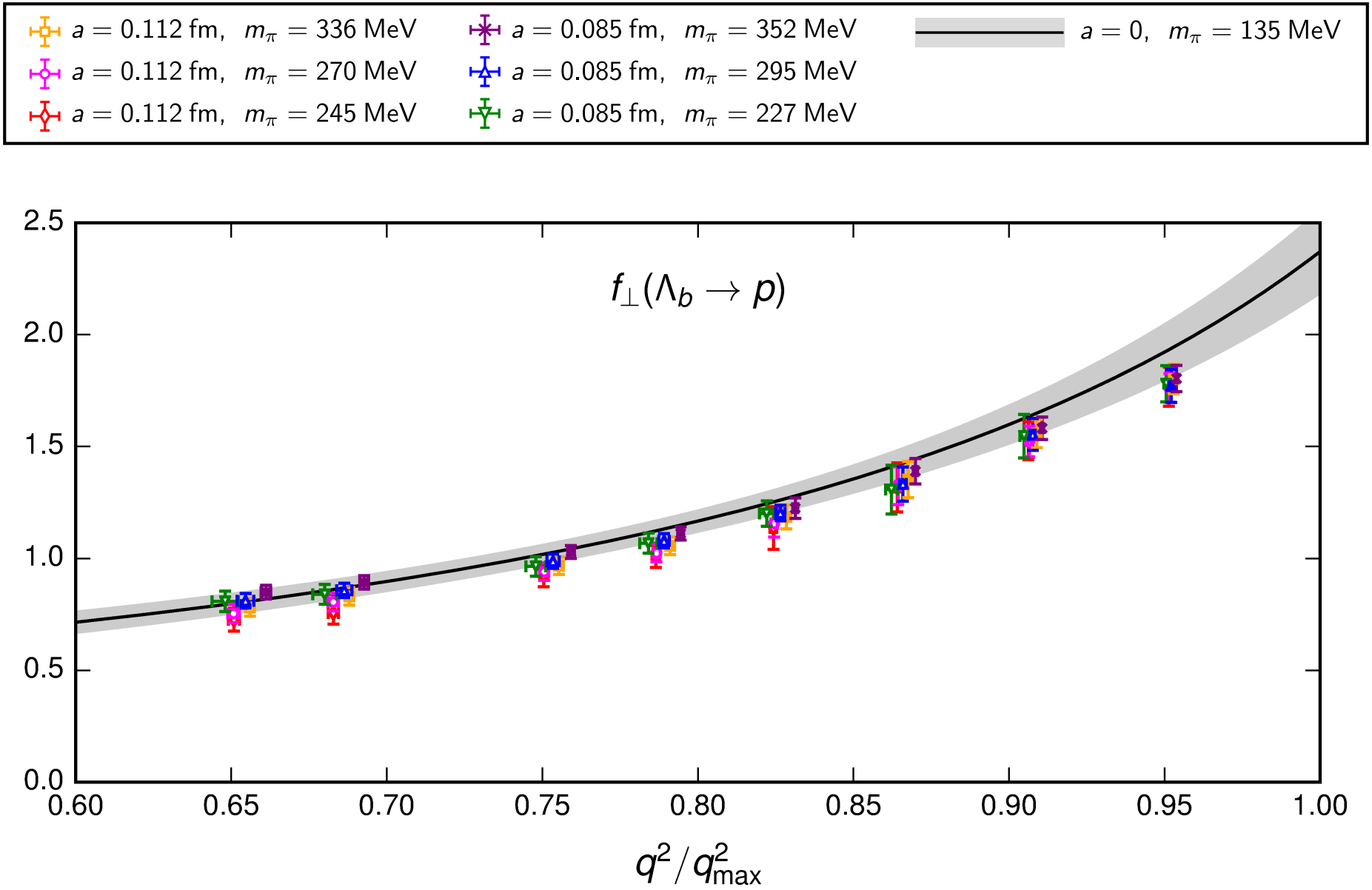
$$z^f(q^2) = \frac{\sqrt{t_+^f - q^2} - \sqrt{t_+^f - t_0}}{\sqrt{t_+^f - q^2} + \sqrt{t_+^f - t_0}},$$

"Nominal fit"

$$f(q^2) = \frac{1}{1 - q^2/(m_{\text{pole}}^f)^2} \left[ a_0^f \left( 1 + c_0^f \frac{m_\pi^2 - m_{\pi,\text{phys}}^2}{\Lambda_\chi^2} \right) + a_1^f \, z^f(q^2) \right]$$
$$\times \left[ 1 + b^f \, \frac{|\mathbf{p}'|^2}{(\pi/a)^2} + d^f \, \frac{\Lambda_{\text{QCD}}^2}{(\pi/a)^2} \right],$$

"Nominal fit" in physical limit $a = 0$, $m_\pi = m_{\pi,\mathrm{phys}}$:

$$f(q^2) = \frac{1}{1 - q^2/(m^f_{\mathrm{pole}})^2} \left[ a^f_0 + a^f_1 \, z^f(q^2) \right]$$




Gray band = statistical uncertainty.

"Higher-order fit":

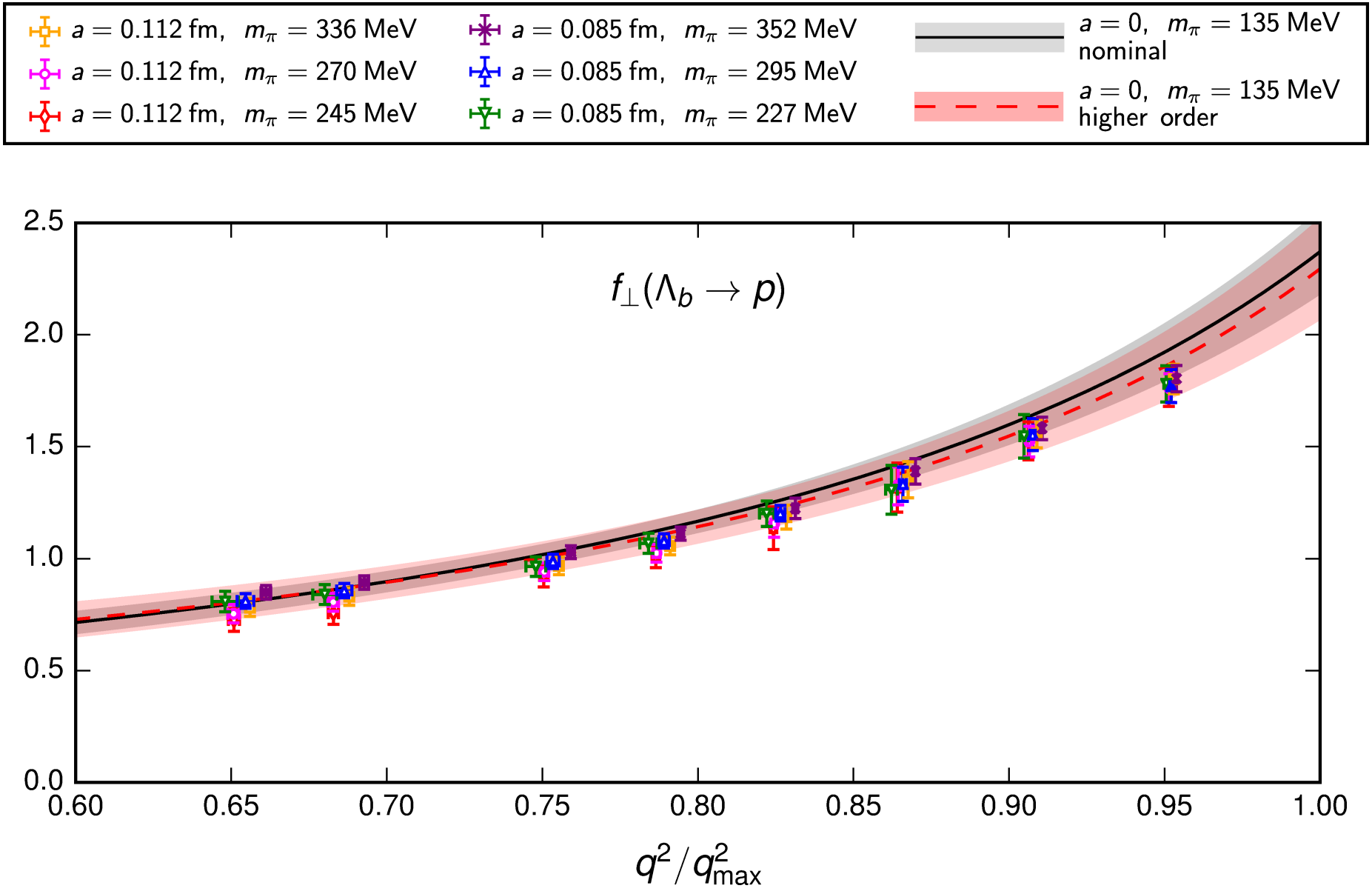
$$f_{\mathrm{HO}}(q^2) = \frac{1}{1 - q^2/(m^f_{\mathrm{pole}})^2} \left[ a_0^f \left( 1 + c_0^f \frac{m_\pi^2 - m_{\pi,\mathrm{phys}}^2}{\Lambda_\chi^2} + \tilde{c}_0^f \frac{m_\pi^3 - m_{\pi,\mathrm{phys}}^3}{\Lambda_\chi^3} \right) \right.$$
$$\left. + a_1^f \left( 1 + c_1^f \frac{m_\pi^2 - m_{\pi,\mathrm{phys}}^2}{\Lambda_\chi^2} \right) z^f(q^2) + a_2^f \left[ z^f(q^2) \right]^2 \right]$$
$$\times \left[ 1 + b^f \frac{|\mathbf{p}'|^2}{(\pi/a)^2} + d^f \frac{\Lambda_{\mathrm{QCD}}^2}{(\pi/a)^2} + \tilde{b}^f \frac{|\mathbf{p}'|^3}{(\pi/a)^3} + \tilde{d}^f \frac{\Lambda_{\mathrm{QCD}}^3}{(\pi/a)^3} \right.$$
$$\left. + j^f \frac{|\mathbf{p}'|^2 \Lambda_{\mathrm{QCD}}}{(\pi/a)^3} + k^f \frac{|\mathbf{p}'| \Lambda_{\mathrm{QCD}}^2}{(\pi/a)^3} \right]$$

Higher-order fit parameters constrained with Gaussian priors to be natural-sized.

Modified data correlation matrix to include other sources of uncertainty.

"Higher-order fit" in physical limit $a = 0$, $m_\pi = m_{\pi,\text{phys}}$:

$$f_{\text{HO}}(q^2) = \frac{1}{1 - q^2/(m^f_{\text{pole}})^2} \left[ a^f_0 + a^f_1 \, z^f(q^2) + a^f_2 \, [z^f(q^2)]^2 \right]$$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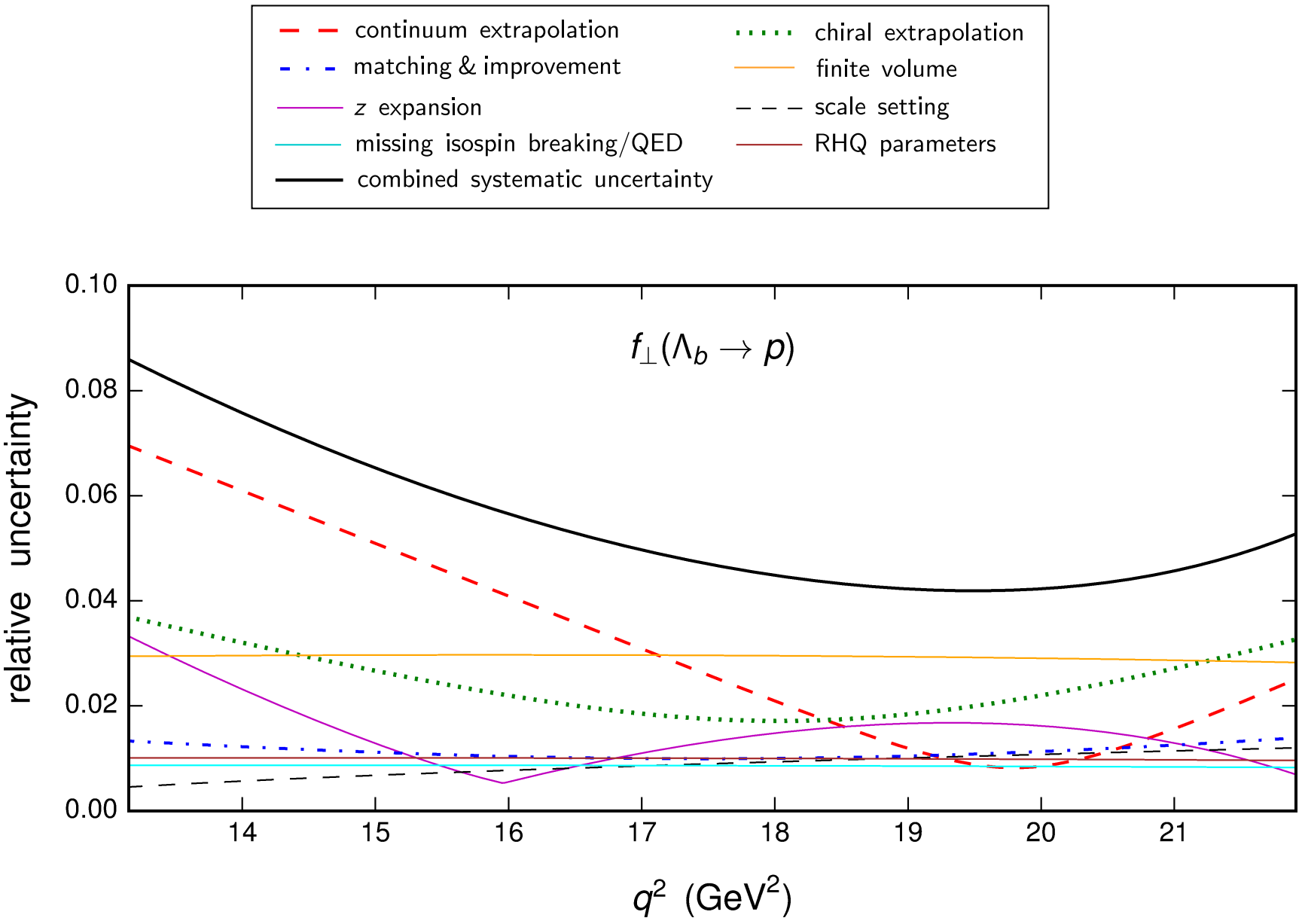
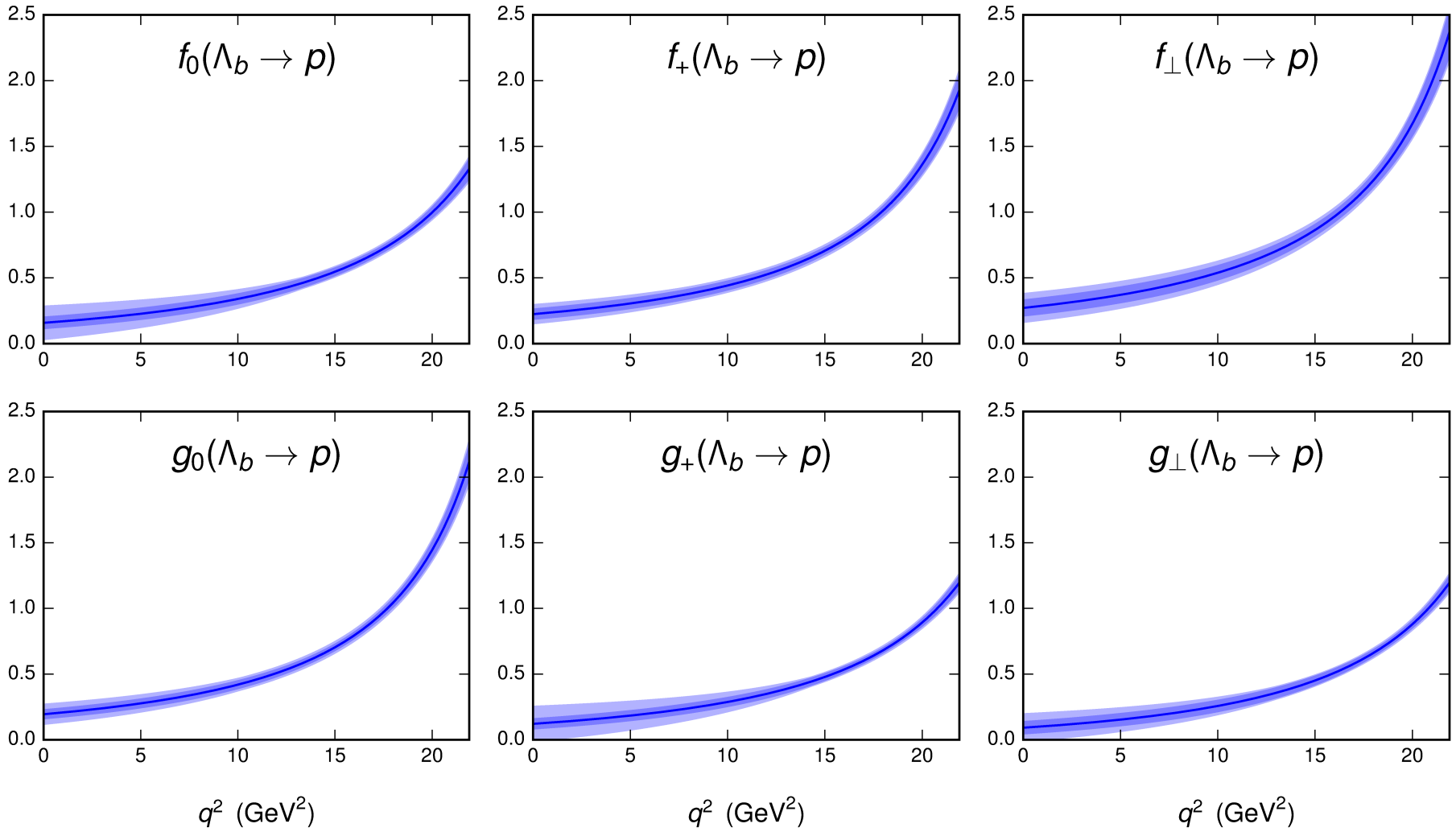
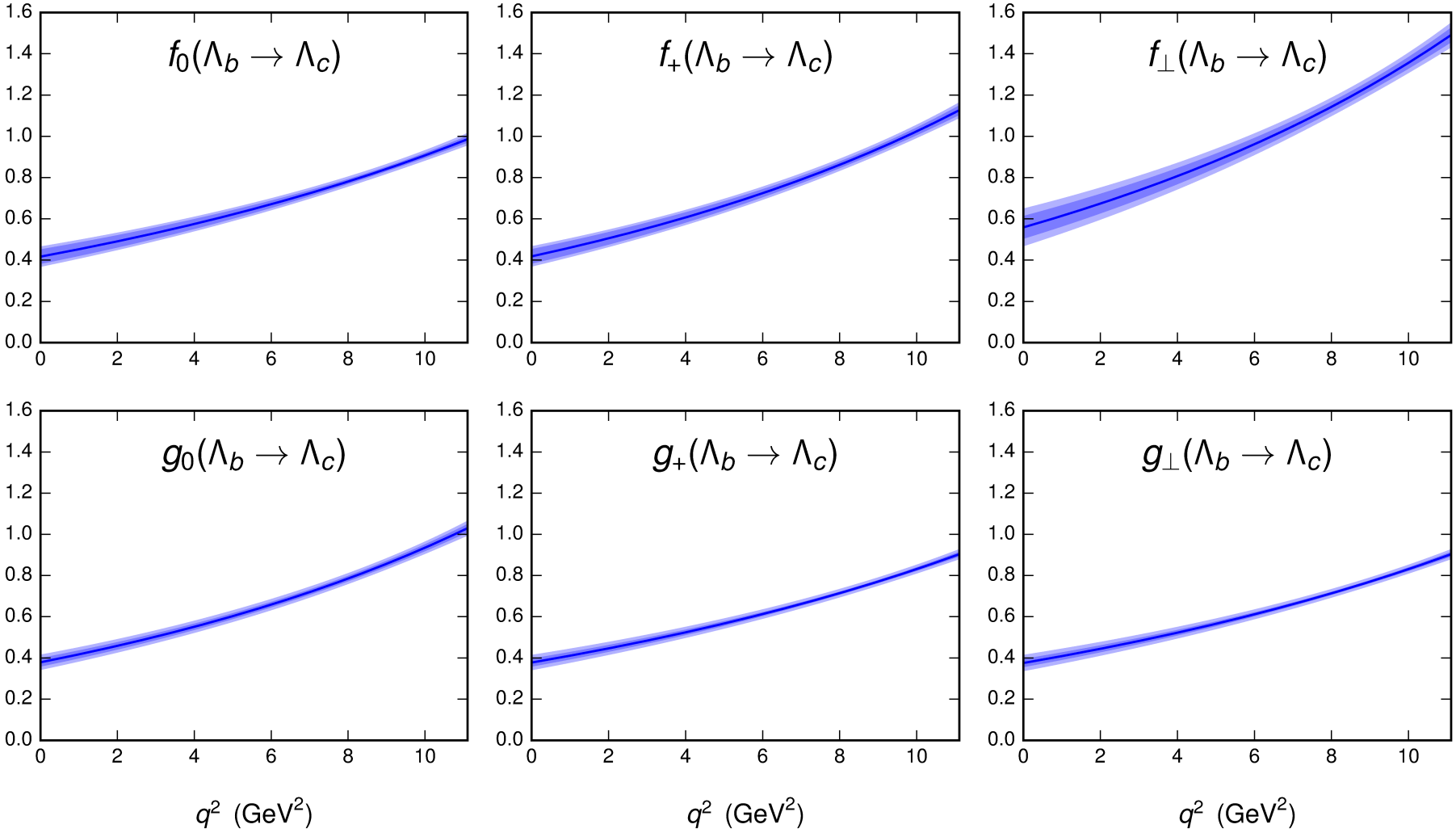
$a = 0.112$ fm, $m_\pi = 270$ MeV
$a = 0.112$ fm, $m_\pi = 245$ MeV
$a = 0.085$ fm, $m_\pi = 352$ MeV
$a = 0.085$ fm, $m_\pi = 295$ MeV
$a = 0.085$ fm, $m_\pi = 227$ MeV
$a = 0$, $m_\pi = 135$ MeV nominal
$a = 0$, $m_\pi = 135$ MeV higher order
$f_\perp(\Lambda_b \to p)$
2.5
2.0
1.5
1.0
0.5
0.0
0.60
0.65
0.70
0.75
0.80
0.85
0.90
0.95
1.00
$q^2 / q^2_{\rm max}$

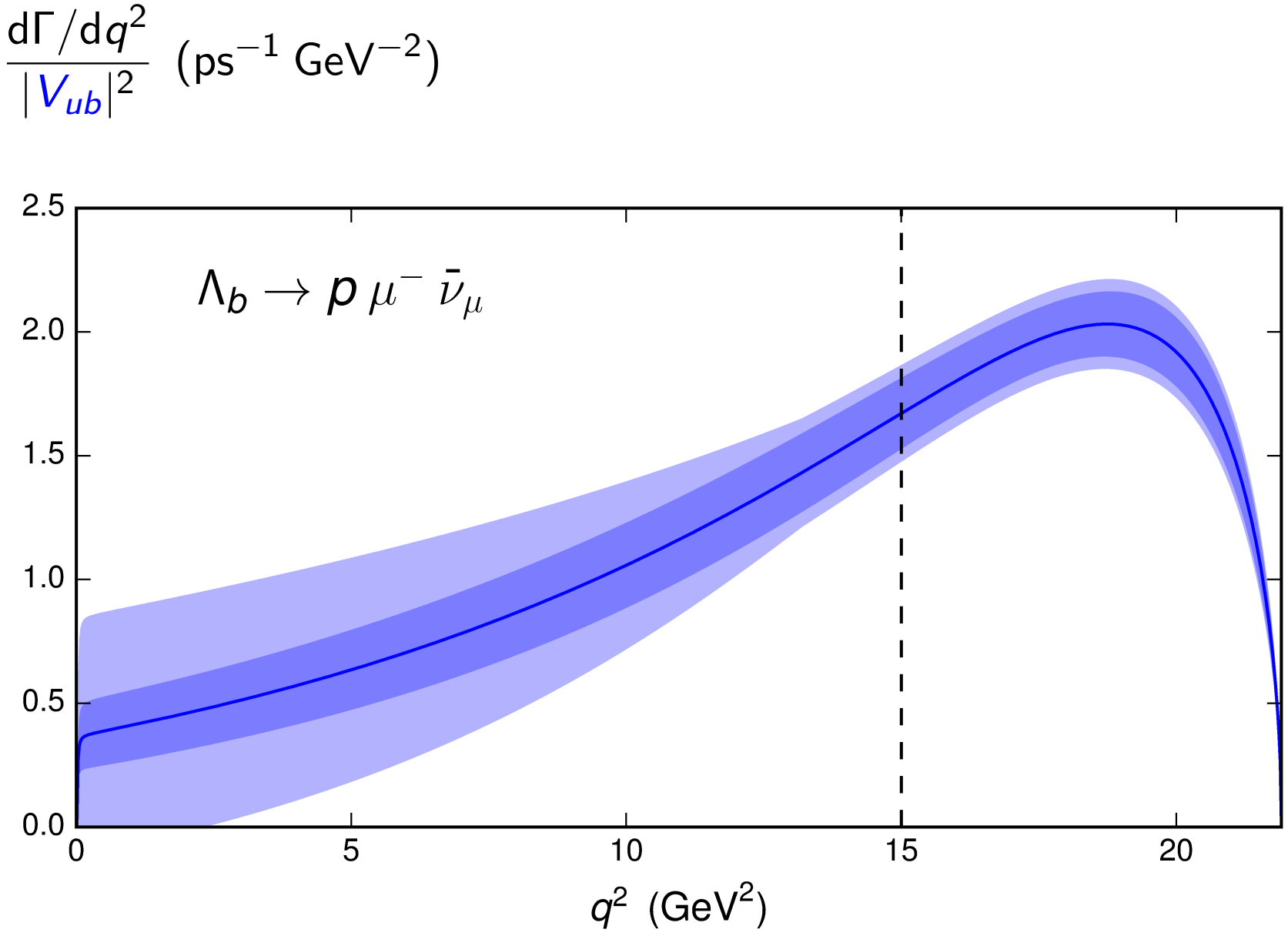
Compute systematic uncertainty of an observable $O$ using

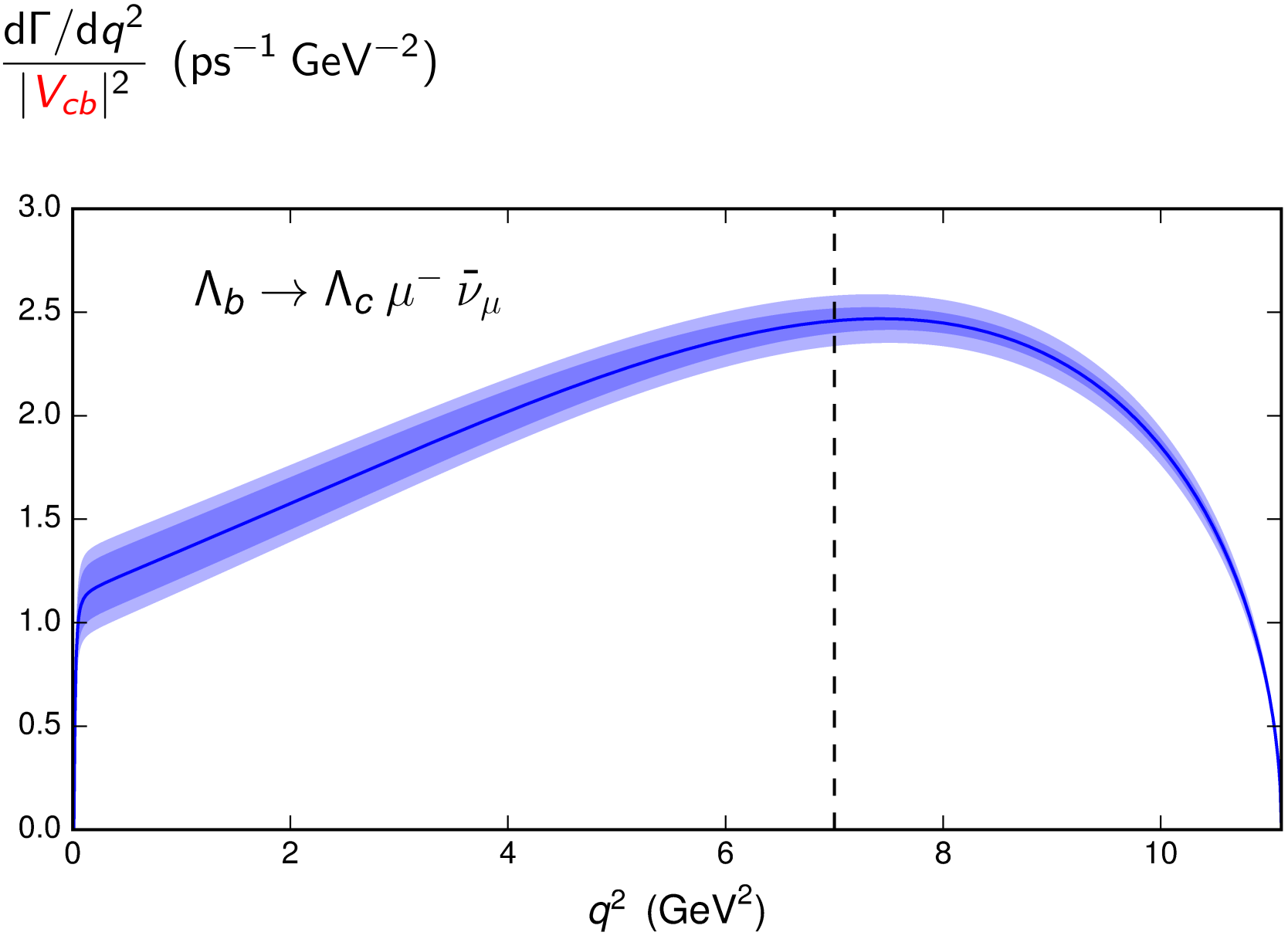
$$\sigma_{O,\text{syst.}} = \max\left(|O_{\text{HO}} - O|,\ \sqrt{|\sigma^2_{\text{HO}} - \sigma^2_O|}\right)$$


continuum extrapolation
matching & improvement
z expansion
missing isospin breaking/QED
combined systematic uncertainty
chiral extrapolation
finite volume
scale setting
RHQ parameters
$f_\perp(\Lambda_b \to p)$
relative uncertainty
0.10
0.08
0.06
0.04
0.02
0.00
14
15
16
17
18
19
20
21
$q^2$ (GeV$^2$)


$f_0(\Lambda_b \to p)$
$f_+(\Lambda_b \to p)$
$f_\perp(\Lambda_b \to p)$
$g_0(\Lambda_b \to p)$
$g_+(\Lambda_b \to p)$
$g_\perp(\Lambda_b \to p)$
0.0
0.5
1.0
1.5
2.0
2.5
0
5
10
15
20
$q^2$ (GeV$^2$)


$f_0(\Lambda_b \to \Lambda_c)$
$f_+(\Lambda_b \to \Lambda_c)$
$f_\perp(\Lambda_b \to \Lambda_c)$
$g_0(\Lambda_b \to \Lambda_c)$
$g_+(\Lambda_b \to \Lambda_c)$
$g_\perp(\Lambda_b \to \Lambda_c)$
$q^2$ (GeV$^2$)
$q^2$ (GeV$^2$)
$q^2$ (GeV$^2$)


$\frac{d\Gamma/dq^2}{|V_{ub}|^2}$ (ps$^{-1}$ GeV$^{-2}$)
$\Lambda_b \to p\,\mu^-\,\bar{\nu}_\mu$
2.5
2.0
1.5
1.0
0.5
0.0
0
5
10
15
20
$q^2$ (GeV$^2$)


$\frac{d\Gamma/dq^2}{|V_{cb}|^2}$ (ps$^{-1}$ GeV$^{-2}$)
$\Lambda_b \to \Lambda_c \, \mu^- \, \bar{\nu}_\mu$
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0
2
4
6
8
10
$q^2$ (GeV$^2$)

$$\zeta_{p\mu\bar{\nu}}(15\ \mathrm{GeV}^2) \equiv \frac{1}{|V_{ub}|^2}\int_{15\ \mathrm{GeV}^2}^{q^2_{\mathrm{max}}} \frac{\mathrm{d}\Gamma(\Lambda_b \to p\ \mu^-\bar{\nu}_\mu)}{\mathrm{d}q^2}\mathrm{d}q^2 = (12.31 \pm 0.76 \pm 0.77)\ \mathrm{ps}^{-1},$$

$$\zeta_{\Lambda_c\mu\bar{\nu}}(7\ \mathrm{GeV}^2) \equiv \frac{1}{|V_{cb}|^2}\int_{7\ \mathrm{GeV}^2}^{q^2_{\mathrm{max}}} \frac{\mathrm{d}\Gamma(\Lambda_b \to \Lambda_c\ \mu^-\bar{\nu}_\mu)}{\mathrm{d}q^2}\mathrm{d}q^2 = (8.37 \pm 0.16 \pm 0.34)\ \mathrm{ps}^{-1},$$

$$\frac{\zeta_{p\mu\bar{\nu}}(15\ \mathrm{GeV}^2)}{\zeta_{\Lambda_c\mu\bar{\nu}}(7\ \mathrm{GeV}^2)} = 1.471 \pm 0.095 \pm 0.109$$

Combine with LHCb measurement:

$$\frac{|V_{ub}|}{|V_{cb}|} = 0.083 \pm 0.004\,(\text{expt}) \pm 0.004\,(\text{lattice})$$

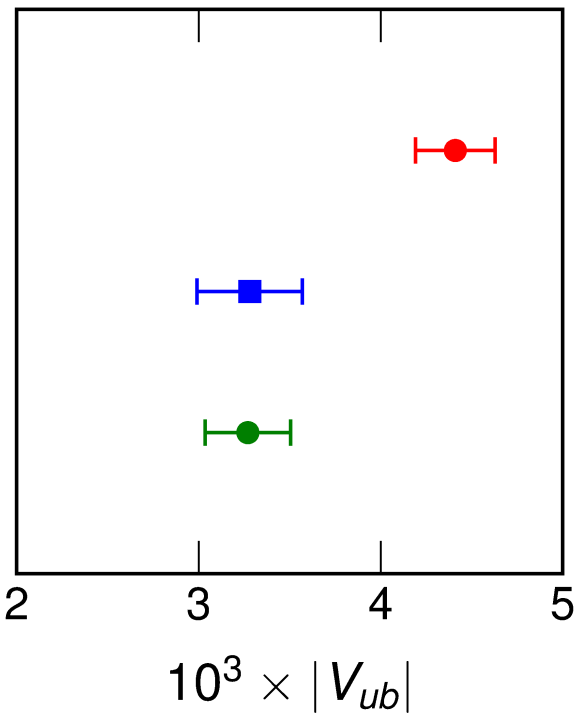
Taking $|V_{cb}|$ from exclusive $B \to D^* \ell \bar{\nu}$
(2014 PDG value, using FNAL/MILC lattice calculation):

$$|V_{ub}| \times 10^3 = 3.27 \pm 0.15\,(\text{expt}) \pm 0.16\,(\text{lattice}) \pm 0.06\,(|V_{cb}|)$$

$B \to X_u \, \ell \, \bar{\nu}_\ell$ (PDG 2014)

$B \to \pi \, \ell \, \bar{\nu}_\ell$ (PDG 2014)

$\Lambda_b \to p \, \ell \, \bar{\nu}_\ell$ (LHCb + this work)

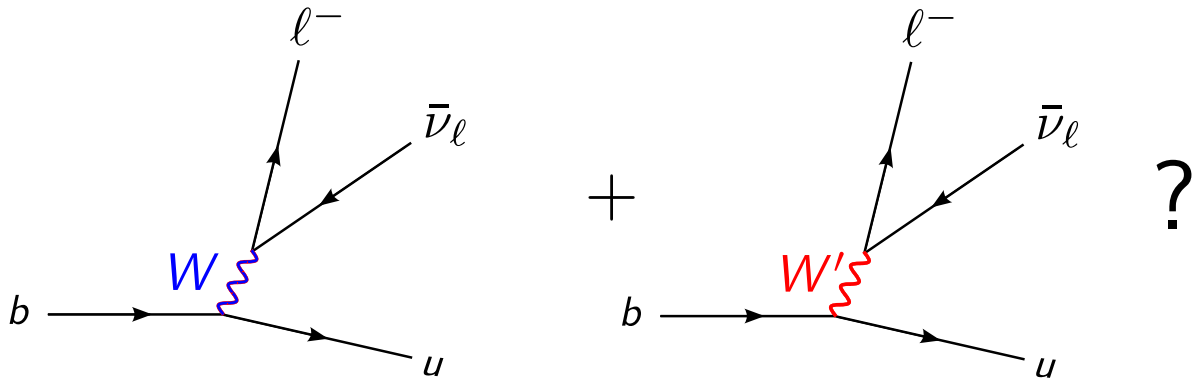


For exclusive $B \to \pi \ell \bar{\nu}$,

$$\langle \pi | \, \bar{u} \gamma^\mu b \, | B \rangle \neq 0,$$
$$\langle \pi | \, \bar{u} \gamma^\mu \gamma_5 b \, | B \rangle = 0.$$

For inclusive $B \to X_s \ell \bar{\nu}$, both currents contribute.

$\longrightarrow$ New physics with right-handed coupling?




$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{ub}^L \, [(1+\epsilon_R)\bar{u}\gamma^\mu b - (1-\epsilon_R)\, \bar{u}\gamma^\mu\gamma_5 b] \; \bar{\ell}\gamma_\mu(1-\gamma_5)\nu$$

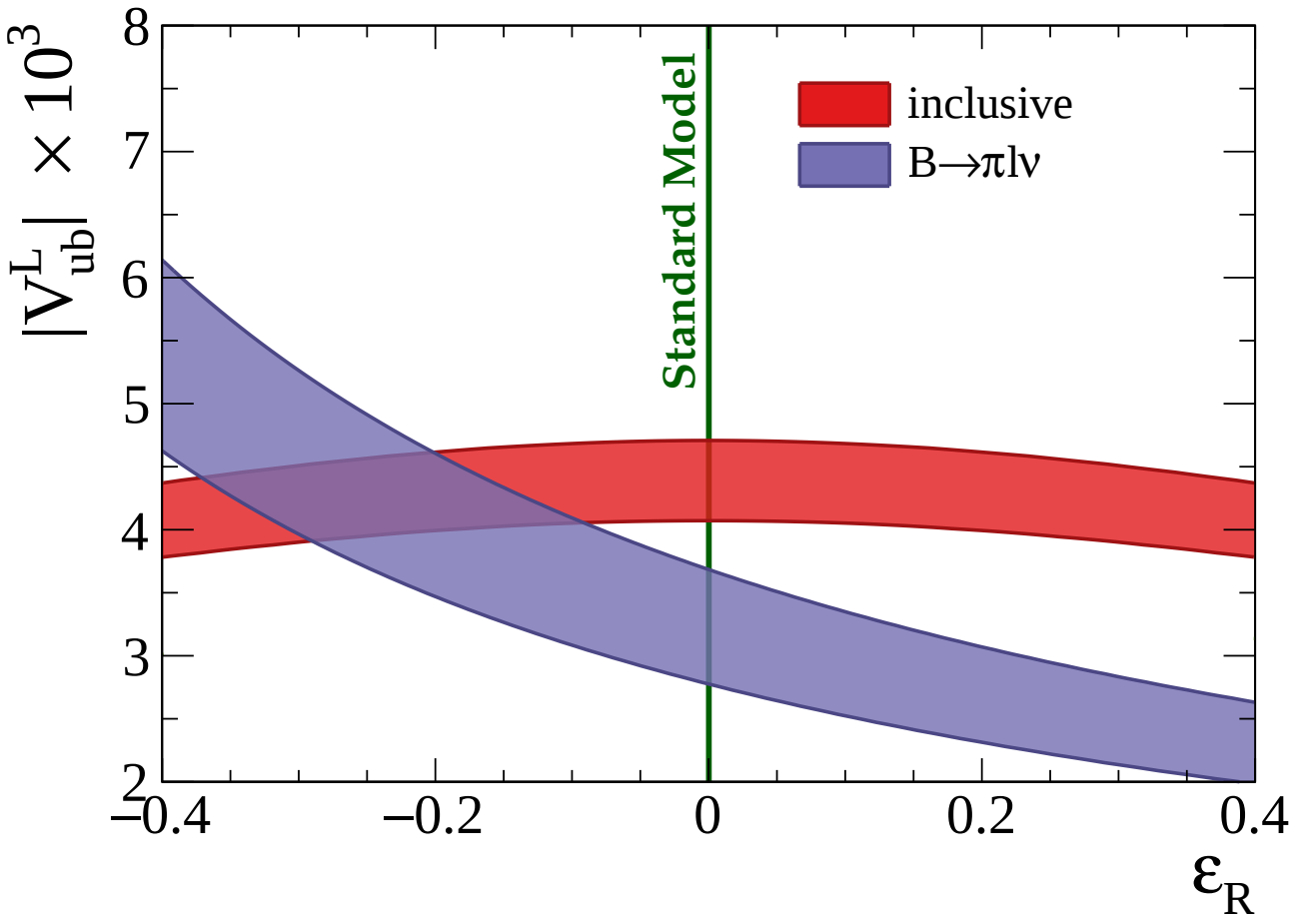



Figure (modified) from Nature Physics **11**, 743-747 (2015)

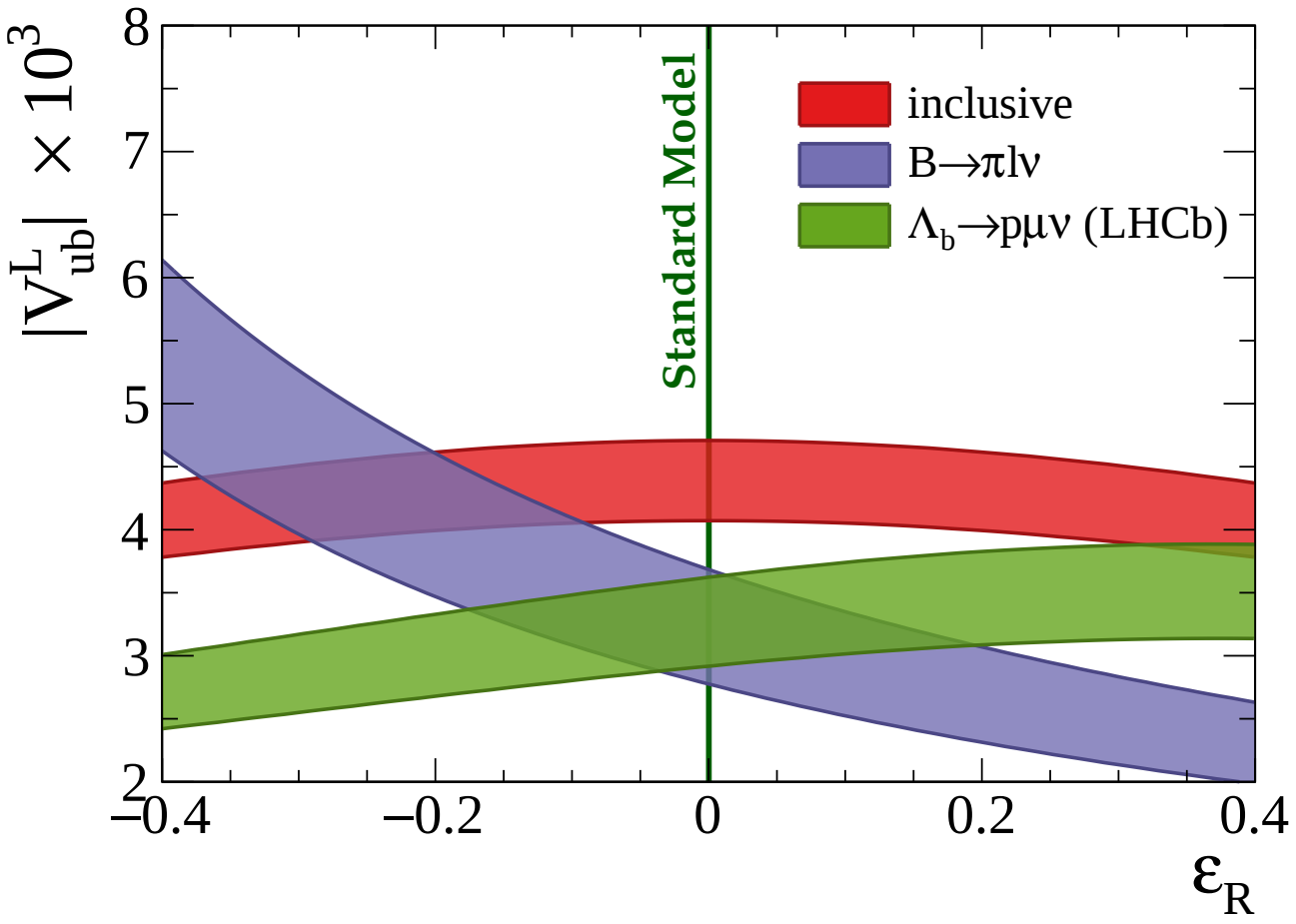



Figure (modified) from Nature Physics **11**, 743-747 (2015)

Conclusion:

$\Lambda_b$ baryons provide exciting new opportunities for flavor physics.

This talk:

- $|V_{ub}/V_{cb}|$ from $\Lambda_b \to p\,\mu^-\bar{\nu}_\mu$ and $\Lambda_b \to \Lambda_c\,\mu^-\bar{\nu}_\mu$

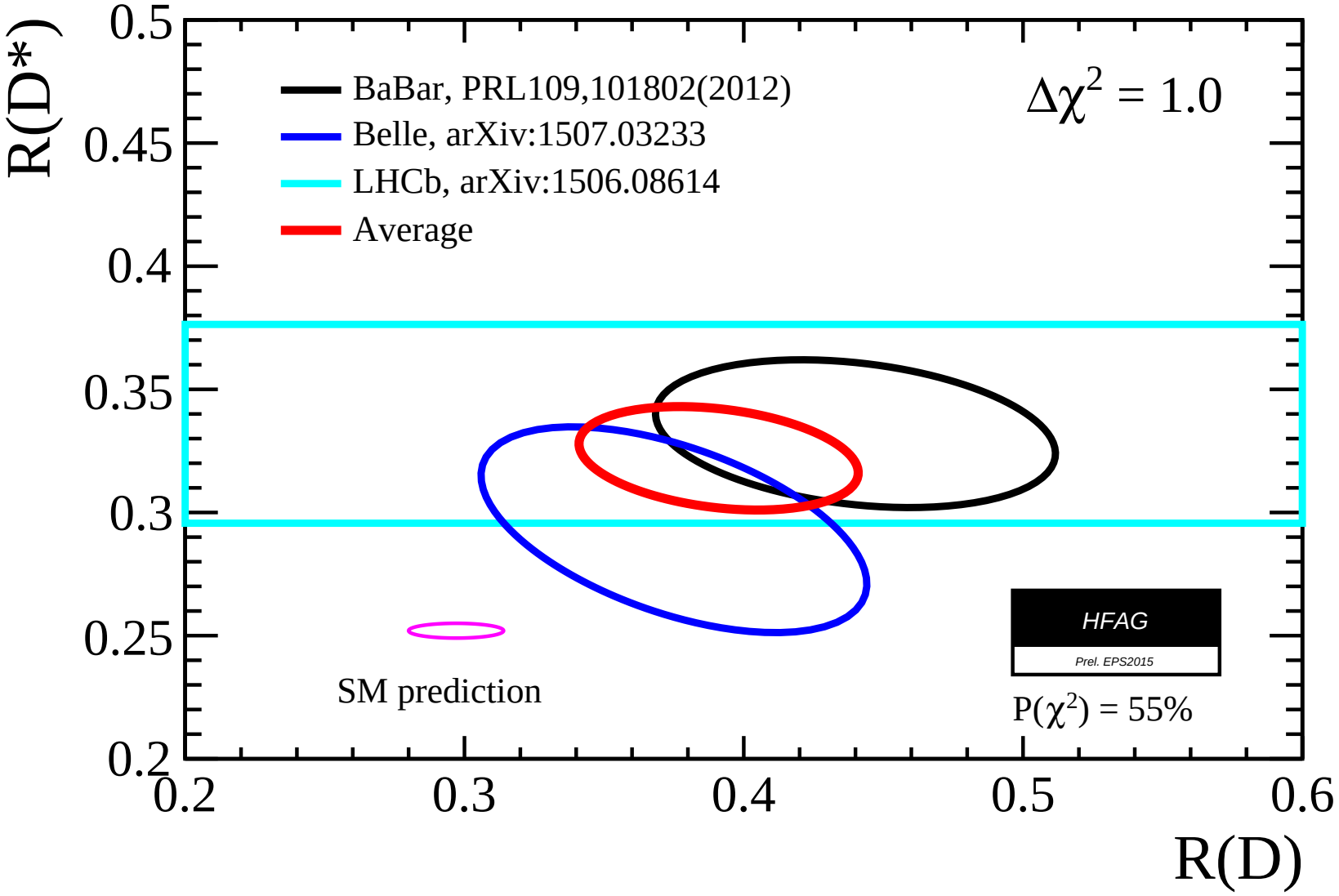
Future work:

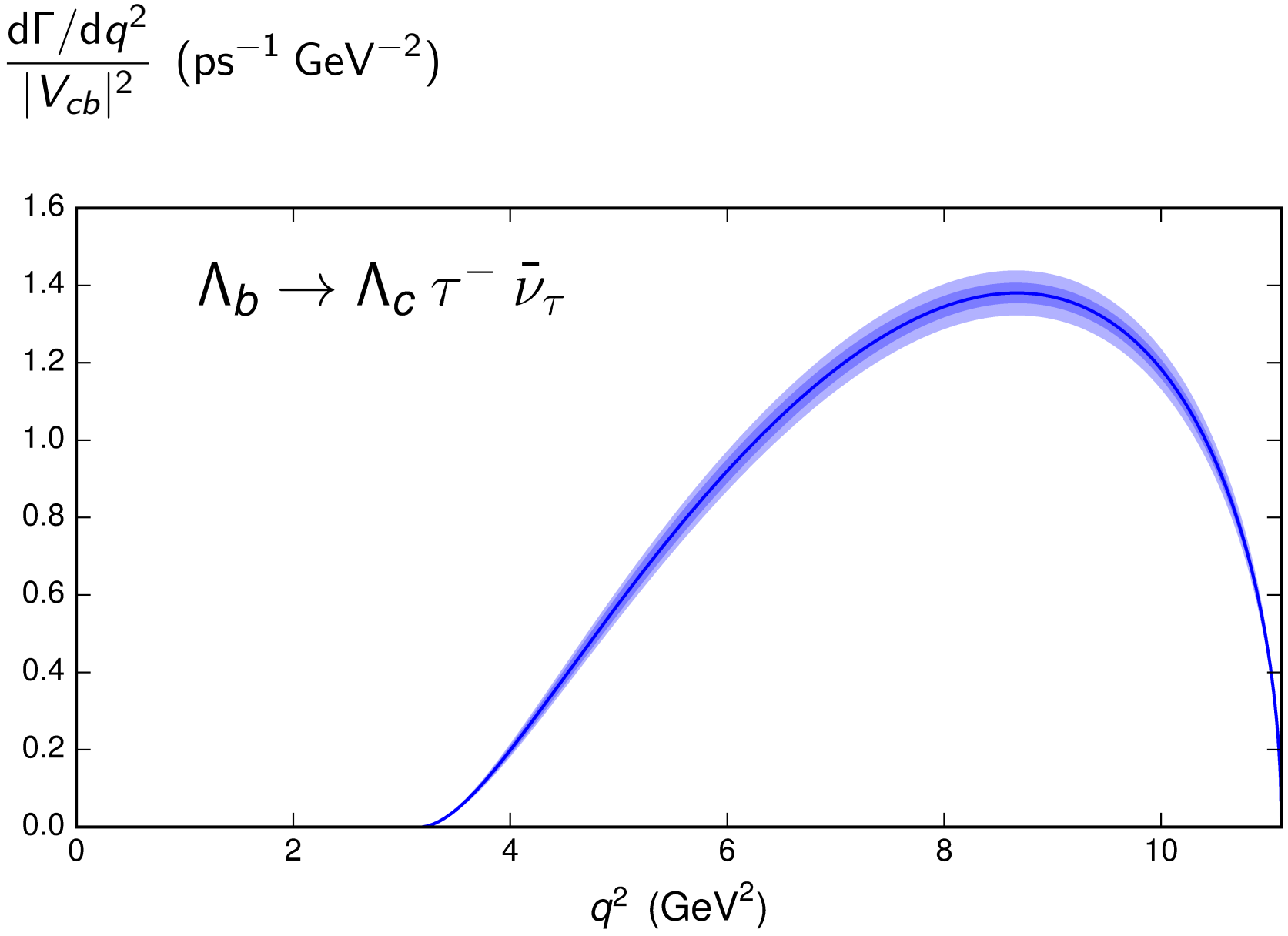
- New insights on $b \to s\mu^+\mu^-$ anomalies from $\Lambda_b \to \Lambda\,\mu^+\mu^-$
- New insights on $R[D^{(*)}]$ puzzle from $\Lambda_b \to \Lambda_c\,\tau^-\bar{\nu}_\tau$

# Extra slides

| | $\zeta_{p\mu\bar{\nu}}(15\ \mathrm{GeV}^2)$ | $\zeta_{\Lambda_c\mu\bar{\nu}}(7\ \mathrm{GeV}^2)$ | $\frac{\zeta_{p\mu\bar{\nu}}(15\ \mathrm{GeV}^2)}{\zeta_{\Lambda_c\mu\bar{\nu}}(7\ \mathrm{GeV}^2)}$ |
|---|---|---|---|
| Statistics | 6.2 | 1.9 | 6.5 |
| Finite volume | 5.0 | 2.5 | 4.9 |
| Continuum extrapolation | 3.0 | 1.4 | 2.8 |
| Chiral extrapolation | 2.6 | 1.8 | 2.6 |
| RHQ parameters | 1.4 | 1.7 | 2.3 |
| Matching & improvement | 1.7 | 0.9 | 2.1 |
| Isospin breaking/QED | 1.2 | 1.4 | 2.0 |
| Scale setting | 1.7 | 0.3 | 1.8 |
| $z$ expansion | 1.2 | 0.2 | 1.3 |
| Total | 8.8 | 4.5 | 9.8 |

$$R[D^{(*)}] = \frac{\Gamma[\, B \to D^{(*)}\, \tau\, \bar{\nu}_\tau \,]}{\Gamma[\, B \to D^{(*)}\, \ell\, \bar{\nu}_\ell \,]_{\ell=e,\mu}}$$


R(D*)
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.2
0.3
0.4
0.5
0.6
R(D)
BaBar, PRL109,101802(2012)
Belle, arXiv:1507.03233
LHCb, arXiv:1506.08614
Average
Δχ² = 1.0
HFAG
Prel. EPS2015
P(χ²) = 55%
SM prediction


$\frac{d\Gamma/dq^2}{|V_{cb}|^2}$ (ps$^{-1}$ GeV$^{-2}$)
$\Lambda_b \to \Lambda_c \tau^- \bar{\nu}_\tau$
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10
$q^2$ (GeV$^2$)

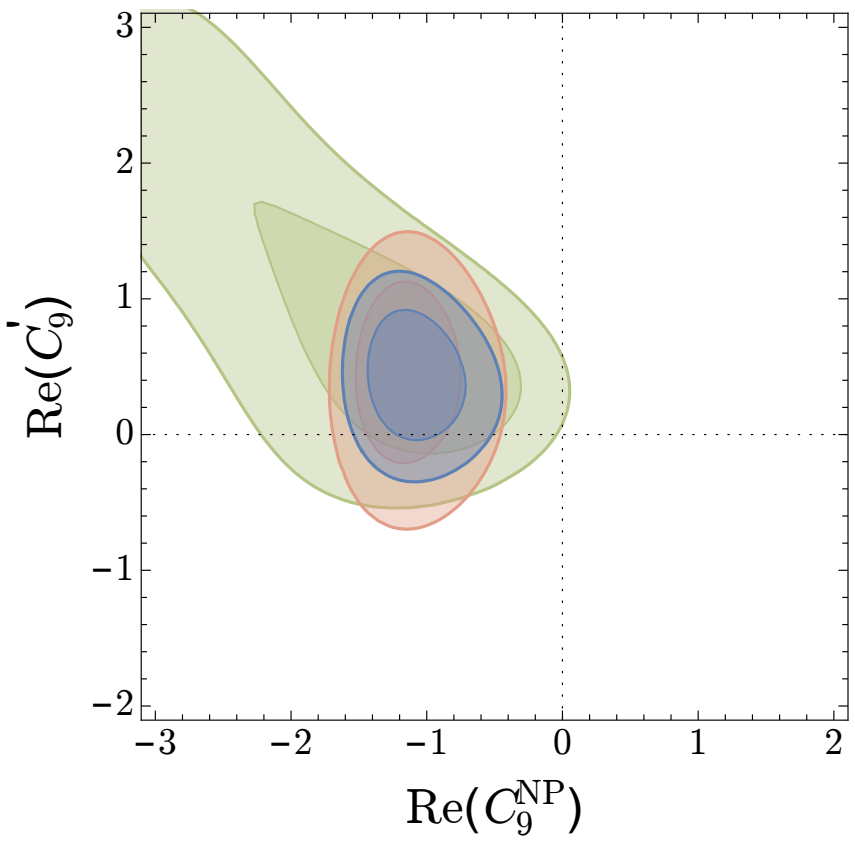
SM Prediction:

$$R[\Lambda_c] = \frac{\Gamma(\Lambda_b \to \Lambda_c \, \tau^- \bar{\nu}_\tau)}{\Gamma(\Lambda_b \to \Lambda_c \, \mu^- \bar{\nu}_\mu)} = 0.3328 \pm 0.0074 \pm 0.0070$$

LHCb measurement?

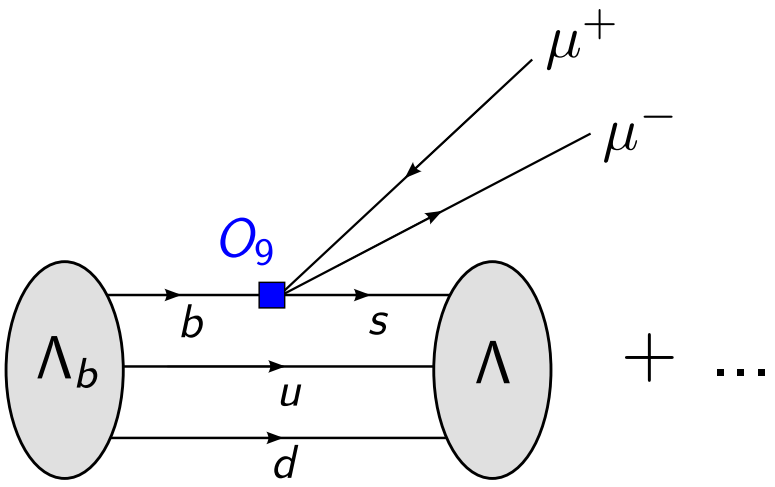
New physics in $b \to s\mu^+\mu^-$?

Analysis of mesonic decays gives:




[W. Altmannshofer, D. Straub, arXiv:1503.06199]

Complementary information can be obtained from $\Lambda_b \to \Lambda \mu^+ \mu^-$



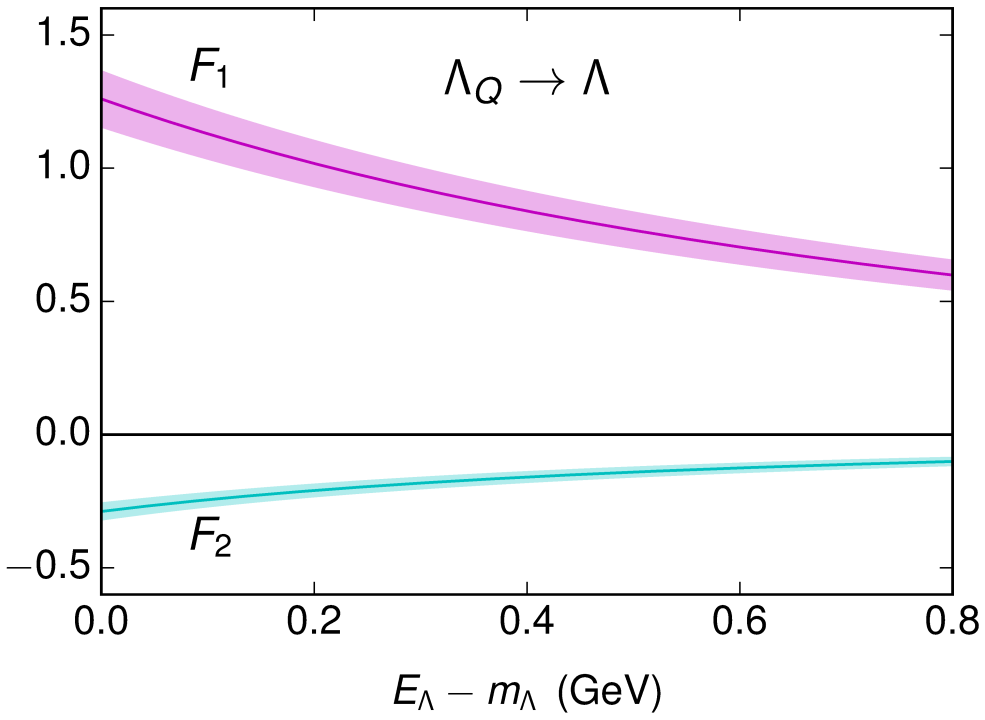

Combines the best aspects of $B \to K^* \mu^+ \mu^-$ and $B \to K \mu^+ \mu^-$:

$\Lambda$ has nonzero spin and is stable under strong interactions.

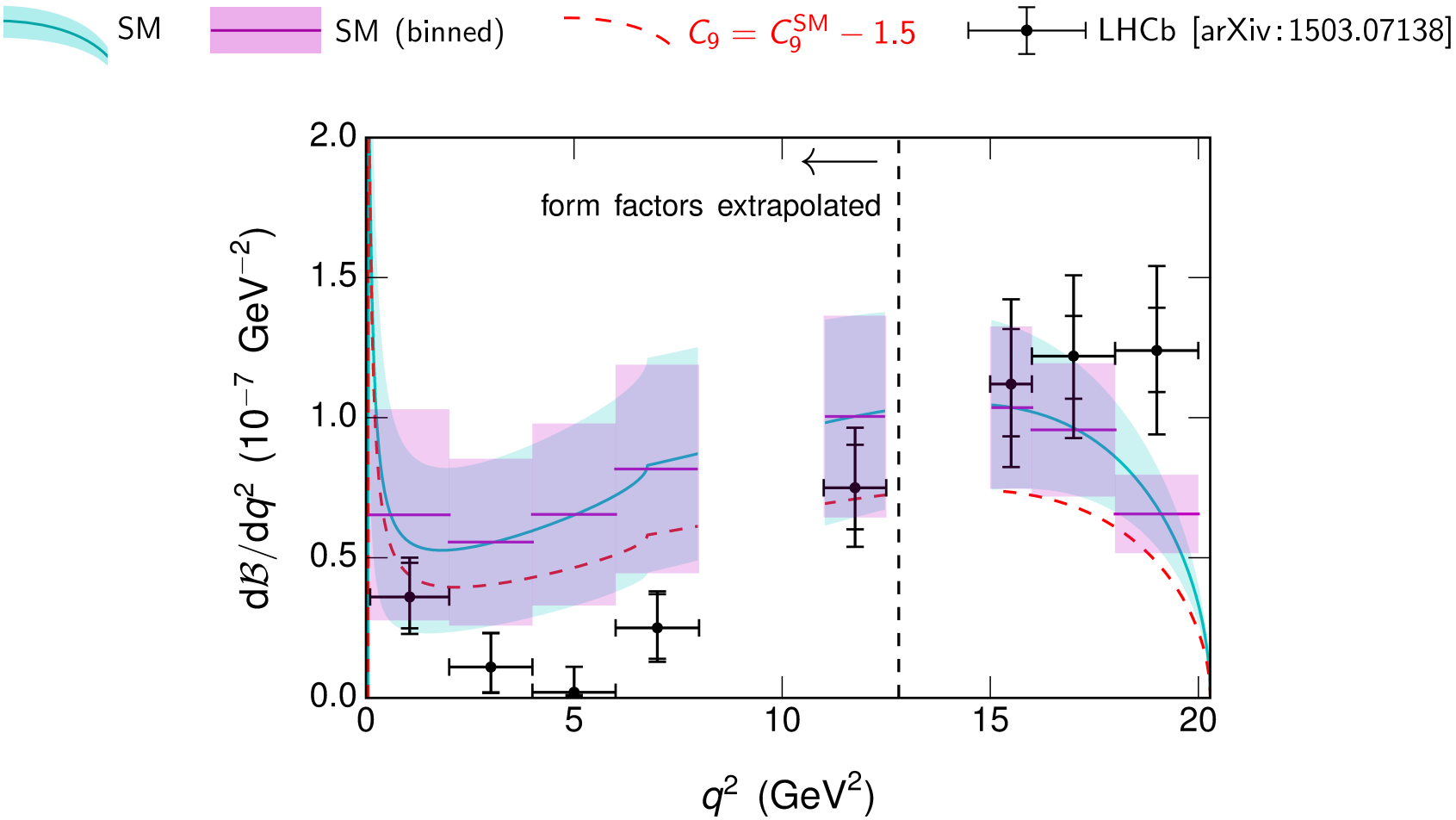
$\Lambda_b \to \Lambda$ form factors so far only with static $b$ quarks

[W. Detmold, C.-J. D. Lin, S. Meinel, M. Wingate, PRD **87**, 074502 (2013)]

$$\langle \Lambda | \, \bar{s} \, \Gamma \, Q \, | \Lambda_Q \rangle \;=\; \bar{u}_\Lambda \, [F_1 + \not{v} \, F_2] \, \Gamma \, u_{\Lambda_Q}$$



# $\Lambda_b \to \Lambda \mu^+ \mu^-$ differential branching fraction



# $\Lambda_b \to \Lambda\mu^+\mu^-$ lepton-side forward-backward asymmetry

$$\frac{d^2\Gamma}{dq^2\, d\cos\theta_\ell} = \frac{d\Gamma}{dq^2}\left[\frac{3}{8}\left(1+\cos^2\theta_\ell\right)(1-f_L) + A_{FB}^{(\ell)}\cos\theta_\ell + \frac{3}{4}f_L\sin^2\theta_\ell\right]$$