Associated production of a top pair and a Higgs boson beyond NLO by means of SCET methods

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In collaboration with...

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We thank N. Greiner, T.Hahn, V. Hirshi, P. Maierhofer, G. Ossola, and A. Papanastasiou for their assistance with the use and/or customization of GoSam, Openloops, Collier, MadLoop, MG5 and Cuba

Outline

- Top quark pairs + Higgs boson at the LHC

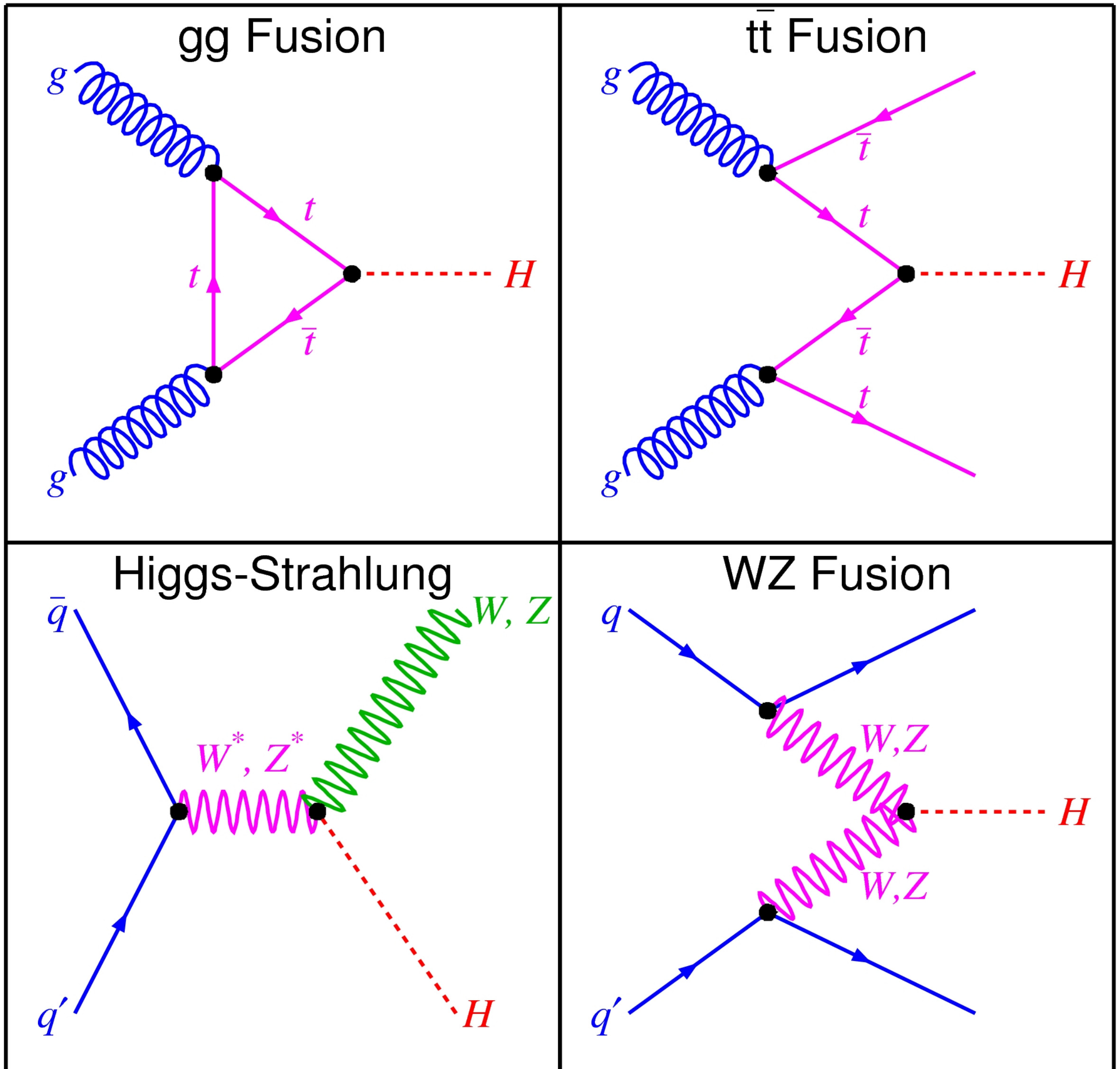
$$p + p \longrightarrow t + \bar{t} + H$$

- Production of heavy particles in the soft emission limit, resummation and approximate formulas
- Approximate NNLO results for total cross section and differential distributions

Higgs boson production channels

$\sigma \sim 49\,\mathrm{pb}$

$\sigma \sim 0.6\,\mathrm{pb}$



$\sigma_W \sim 1.5\,\mathrm{pb}$

$\sigma_Z \sim 0.97\,\mathrm{pb}$

$\sigma \sim 4.2\,\mathrm{pb}$

LHC @ 14 TeV

A brief history of top pair + Higgs calculations

- Cross section and some distributions evaluated to NLO QCD

  Beenakker, Dittmaier, Kraemer, Pluember, Spira, Zerwas ('01-'02)

  Dawson, Reina, Wackeroth, Orr,Jackson ('01,'03)

- In 2 → 3 processes (“multileg processes”), analytic NLO calculations become cumbersome: top pair + Higgs production was one of the first processes to be used to test automated tools

  Frixione et al ('11), Hirshi et al.('11)

  Garzelli et al.('11), Bevilacqua et al.('11)

- EW corrections to the parton level cross section are known

  Frixione, Hirshi,Pagani, Shao, Zaro ('14)

  Zhang, Ma, Chen, Guo ('14)

  Frixione, Hirshi, Pagani, Shao, Zaro ('15)

- NLO QCD corrections were interfaced with `SHERPA` and `POWHEG BOX`

  Gleisberg, Hoeche, Krauss, Schonherr, Schaumann ('09)

  Hartanto, Jaeger, Reina, Wackeroth ('15)

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- EW

  aro ('14)

  Zhang, Ma, Chen, Guo ('14)

  aro ('15)

- NLO

  POW

  ann ('09)

  Hartanto, Jaeger, Reina, Wackeroth ('15)

NLO+NLL resummation of soft gluon emission (production threshold) for the total cross section

Kulesza, Motyka,Stebel,Theeuwes ('15)

NLO corrections with top decays:

$$pp \to e^+\nu_e\mu^-\bar{\nu}_\mu b\bar{b}H$$ @ NLO QCD

Denner, Feger ('15)

Large logarithmic corrections

- The partonic cross section for top pair (+Higgs) production receives potentially large corrections from soft emission diagrams
- Schematically, the partonic cross section depends on logarithms of the ratio of two different scales:

$$L \equiv \ln\left(\frac{\text{“hard” scale}}{\text{“soft” scale}}\right)$$

- It can be that $\alpha_s L \sim 1$
- One needs to reorganize the perturbative series: Resummation
- The resummation of soft emission corrections can be carried out by means of effective field theory methods

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- The re ... ried out by

The resummation framework can also be used in order to reproduce the fixed order partonic cross section in the soft emission limit: approximate formulas

Goal

We want to analyze the factorization properties of

$$p + p \longrightarrow t + \bar{t} + H + X$$

in the soft emission limit in order to

Additional final state radiation

i. Obtain approximate NNLO formulas for the partonic cross section

ii. Evaluate the total cross section and differential distributions depending on the 4-momenta of the final state particles

Soft limit, factorization, and approximate NNLO formulas

Pair Invariant Mass kinematics

- For both top pair and top pair + Higgs production, we have two tree-level partonic processes

$$q(p_1) + \bar{q}(p_2) \to t(p_3) + \bar{t}(p_4) + H(p_5)$$
$$g(p_1) + g(p_2) \to t(p_3) + \bar{t}(p_4) + H(p_5)$$

- For <u>top pair + Higgs production</u>, define the invariants

$$\hat{s} = (p_1 + p_2)^2 \qquad M^2 = (p_3 + p_4 + p_5)^2$$

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Partonic center of mass energy (squared)

Invariant mass of the heavy particles in the final state

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$$\hat{s} = (p_1 + p_2)^2 \qquad M^2 = (p_3 + p_4 + p_5)^2$$

If real radiation in the final state is present, $\hat{s} \neq M^2$

$$z = \frac{M^2}{\hat{s}}$$

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- For top pair + Higgs production, define the invariants

$$\hat{s} = (p_1 + p_2)^2 \qquad M^2 = (p + p + p)^2$$

Partonic threshold limit

$$z \longrightarrow 1$$

If real radiation in the final state is

$$z = \frac{M^2}{\hat{s}}$$

Factorization in a nutshell

Factorization of the hadronic cross section:

$$\frac{d^2\sigma}{dM^2 d\cos\theta} = ff(z) \otimes C(z)$$

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Partonic luminosity

$$ff = \int_y^1 \frac{dx}{x} f_{i/N_1}(x) f_{j/N_2}\left(\frac{y}{x}\right)$$

Factorization in a nutshell

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Partonic luminosity

Hard scattering kernel
(partonic cross section)

Factorization in a nutshell

Factorization of the hadronic cross section:

$$\frac{d^2\sigma}{dM^2 d\cos\theta} = ff(z) \otimes C(z)$$

In the soft emission limit a clear scale hierarchy emerges:

$$\hat{s}, M^2, m_t^2, m_H^2 \gg \hat{s}(1-z)^2 \gg \Lambda^2_{\mathrm{QCD}}$$

Factorization in a nutshell

Differential cross section:

$$\frac{d^2\sigma}{dM^2 d\cos\theta} = ff(z) \otimes C(z)$$

In the soft emission limit a clear scale hierarchy emerges:

$$\hat{s}, M^2, m_t^2, m_H^2 \gg \hat{s}(1-z)^2 \gg \Lambda_{\rm QCD}^2$$

Hard scales

Soft scale

Factorization in a nutshell

Factorization of the hadronic cross section:

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In this limit, the partonic cross section factors into two parts:

$$C_{ij} = \mathrm{Tr}\left[\mathbf{H}_{ij}\left(M, m_t, \cdots, \mu\right)\mathbf{S}_{ij}\left(\sqrt{\hat{s}}(1-z), \cdots, \mu\right)\right]$$

Hard function
(virtual corrections)

Soft function
(real soft emission)

Factorization in a nutshell

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Hard function
(virtual corrections)

both are matrices
(in color space)

Soft function
(real soft emission)

Resummation vs approximate formulas

- The hard and soft functions are free from large logarithms and can be evaluated in fixed order perturbation theory
- We have all of the elements to implement NNLL resummation (in particular the NLO hard function, see later)
- The numerical evaluation of NLO+NNLL resummed formulas is a computationally expensive task
- One can re-expand NNLL cross sections to obtain approximate NNLO predictions (requires a lot of running time, but less than what is needed for NNLL)

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If soft emission accounts for a large fraction of the CS, but effects beyond NNLO are not numerically important, approximate NNLO formulas provide valuable information

Approximate NNLO calculations

- Structure of the NNLO partonic cross section

$$
\begin{aligned}
d\hat{\sigma}_{\text{NNLO}} =& D_3 P_3(z) + D_2 P_2(z) + D_1 P_1(z) \\
&+ D_0 P_0(z) + C_0 \delta(1-z) + R(z)
\end{aligned}
$$

$$
P_n(z) = \left[\frac{\ln^n(1-z)}{1-z} \right]_+
$$

Plus distributions

Terms which are non singular in the z →1 limit

Approximate NNLO formulas include the full set of coefficients Ds, which are functions of the hard scales in the problem

Top pair + Higgs: Hard function

- The calculation of the NLO hard function requires the evaluation of one loop amplitudes for a 2 $\longrightarrow$ 3 process (separating out the various color components)
- The evaluation of the NLO QCD corrections to tT+H corrections was carried out with “traditional” (Passarino-Veltman like) reduction methods
- In order to calculate the NLO hard function, it is convenient to <u>take advantage of the automated tools</u> available on the market. However, to date, none provides hard functions out of the box and all require <u>some level of customization</u>

Solution:

Modified version of GoSam and Openloops (+Collier)

Cullen, Greiner, Heinrich, Luisoni, Mastrolia, Ossola,et al. ('12-'14)

Cascioli, Maierhofer, Pozzorini ('12)

Snippets from the modified code

```
GoSam
An Automated One-Loop
Matrix Element Generator
Version 2.0.1 Rev: 735

(c) The GoSam Collaboration 2011-2014
```

(code modified with the help of N. Greiner and G. Ossola)

Fix the phase space point:

```
Phase space point
vec_1   250.00000000000000      0.0000000000000000      0.0000000000000000     250.00000000000000
vec_2   250.00000000000000      0.0000000000000000      0.0000000000000000    -250.00000000000000
vec_3   187.73850389979739    -58.983229676638324     -41.627869607566652     -11.808257084878599
vec_4   184.32670746268576     39.113315116582932      50.299577830700244     -4.6932008990400051
vec_5   127.93478863751679     19.869914560055363     -8.6717082231335887      16.501457983918591
```

Output LO hard function and (UV renormalized) NLO hard function in the quark annihilation channel, after rotation to the desired color basis:

```
Print Tree Level Hard Function:
(-1.1117307432712692OE-021,  0.0000000000000000     ) (-3.8116482626443515E-021,  0.0000000000000000     )
(-3.3881317890172013E-021,  0.0000000000000000     ) ( 9.1403180646541744E-005,  0.0000000000000000     )

Print One Loop Hard Function:
( 1.4772254600114996E-018, 3.3881317890172013E-021) (-2.3089383098535146E-004, 1.4722285402959297E-004)
(-2.3089383098535146E-004,-1.4722285402959297E-004) (-2.4868451361348187E-004,  0.0000000000000000     )
```

Snippets from the modified code

The IR poles of the HF are can be subtracted by using the Becher-Neubert formula for the IR poles in QCD amplitudes

$$\mathbf{H}^{(1)} = \mathbf{H}^{(1)\mathrm{IR}} - \left(\mathbf{Z}^{(1)}\mathbf{H}^{(0)} - \mathbf{H}^{(0)}\mathbf{Z}^{(1)\dagger}\right)$$

The calculation of the hard function was also implemented by modifying MadLoop. The GoSam, Openloops and MadLoop implementations are in agreement.

GoSam or OpenLoops without Collier takes about 100ms to calculate the HF in a phase space point. OpenLoops + Collier takes about 10ms pr phase point.

```
Print Tree Level Hard Function:
(-1.1117307432712692OE-021,  0.0000000000000000     ) (-3.8116482626443515 3E-021,  0.0000000000000000     )
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```

Numbers at approx NNLO + NLO (nNLO)

Implementation of the differential cross section in the code

- Running time was not an issue in the calculation of the top pair production cross section (the hard function was calculated analytically). However, here GoSam or OpenLoops performs a numerical calculation in each phase-space point: the management of the running time is crucial
- We set up things in such a way that one can easily calculate differential distributions depending on the momenta of the final state particles (invariant mass dist, pT of top-quark and Higgs, rapidities, etc...)
- We developed an in-house Monte Carlo to evaluate

$$\int d\sigma = \frac{1}{2s}\int_{\tau_{\min}}^{1}\frac{d\tau}{\tau}\int_{\tau}^{1}\frac{dz}{\sqrt{z}} ff\left(\frac{\tau}{z}\right)\int d\phi_{t\bar{t}H}\,\mathrm{Tr}\left[\mathbf{HS}(z,\cdots)\right]$$

$$\tau_{\min} = \frac{(2m_t + m_H)^2}{s}$$

Complete NLO vs approx. NLO

We need complete NLO results for the total cross section and the differential distributions we are interested in, both to validate the approximate formulas and to match results to the full NLO:

MadGraph5_aMC@NLO

Complete NLO vs approx. NLO: total cross section

Focus on the LHC at 13 TeV:

$$\mathrm{m}_t = 172.5 \text{ GeV} \qquad m_H = 125 \text{ GeV} \qquad \mu_0 = \frac{2m_t + m_H}{2} = 235 \text{ GeV}$$

NLO MG5 [fb]	NLO no qg channel MG5 [fb]	NLO approx. [fb]
$515.5^{+30.6}_{-49.4}$	$499.5^{+0.0}_{-30.1}$	$486.5^{+0.0}_{-47.6}$

(MSTW 2008 NLO PDFs)

Approximate NLO:
Predictions obtained by re-expanding the resummation formulas to NLO. They include all of the terms singular in the soft limit $z \to 1$ in the NLO partonic cross section

Complete NLO vs approx. NLO: total cross section

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Approximate NLO results for the total cross section are reasonably close to the complete NLO results, especially when one excludes the quark-gluon channel from the comparison

However the scale uncertainty looks weird, why?

Complete NLO vs approx. NLO: total cross section

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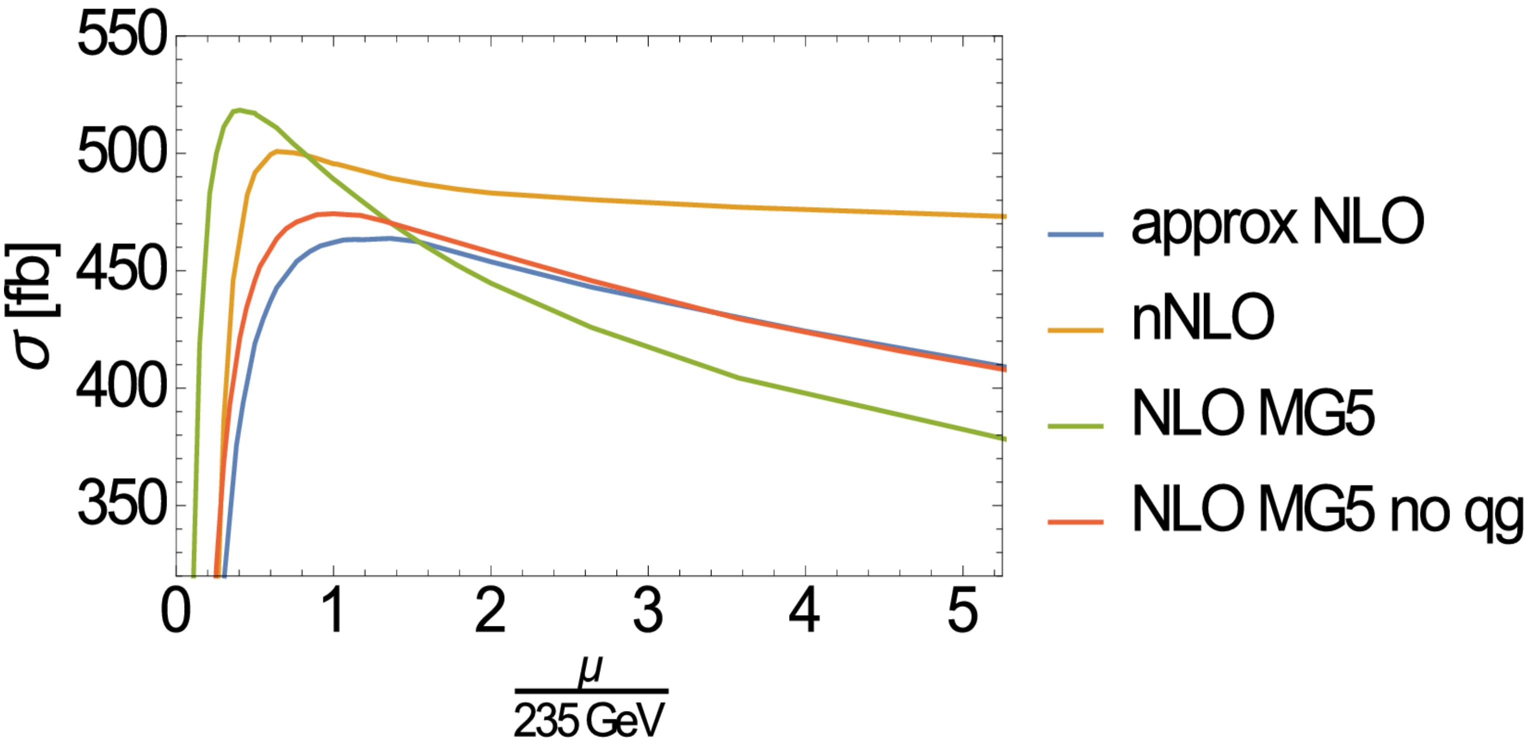
Scale variation: the cross section is near a maximum at μ = 235 GeV

The quark gluon channel has a small impact on the cross section, but a large effect on the scale variation

Scale uncertainty obtained by evaluating the cross section at $\mu_0, \mu_0/2, 2\mu_0$ and by then looking at the maximum and minimum value obtained

NLO and approx NNLO: Scale dependence

Let's use MSTW 2008 NNLO PDFs, and plot also nNLO



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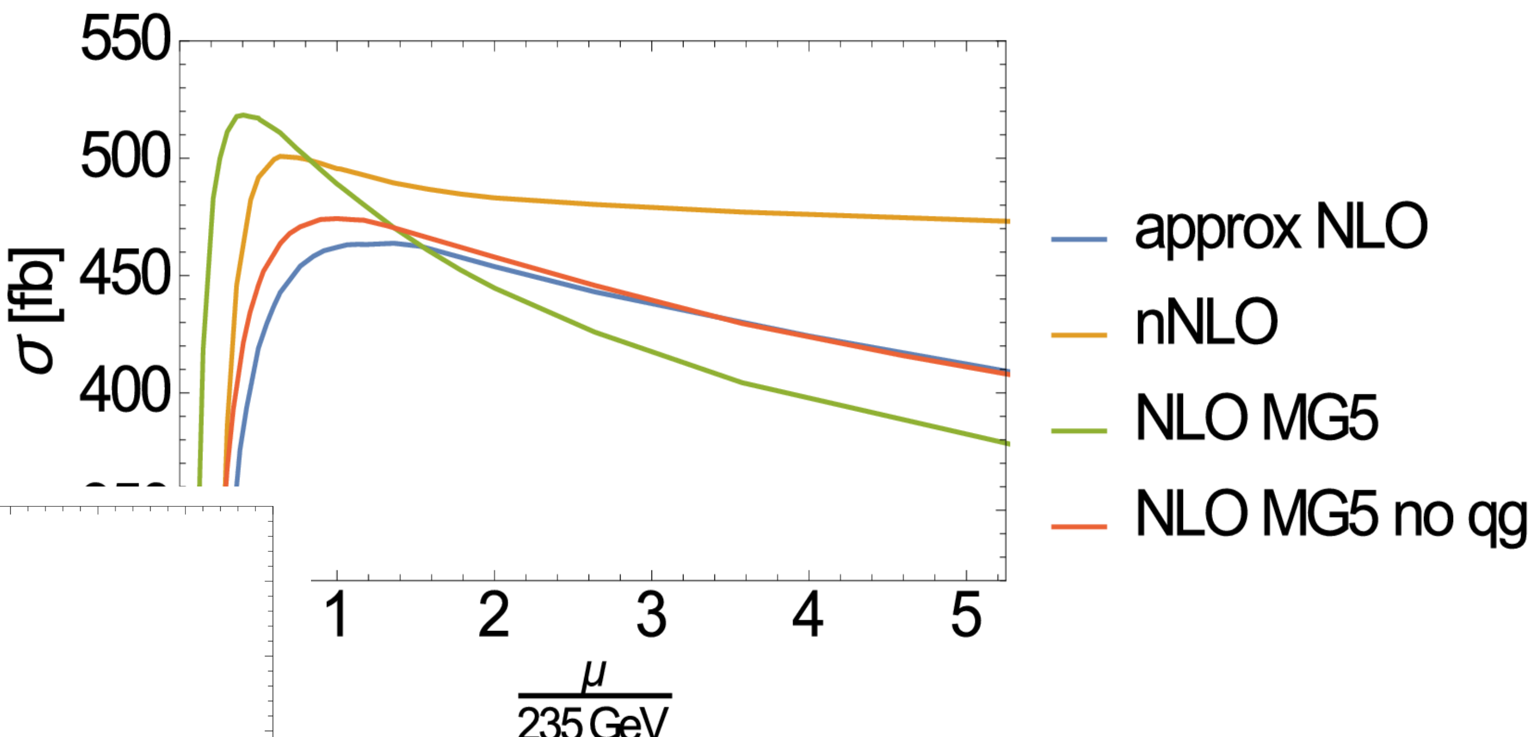

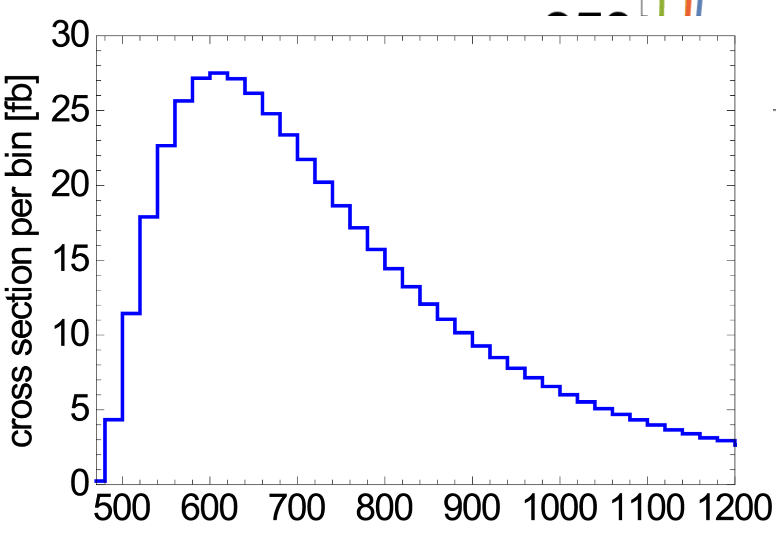

The peak of the invariant mass distribution (M~ 620 GeV) is not very sensitive to the collider energy. It corresponds to μ/235~ 2.6. What happens if we chose this as the renormalization/factorization scale?

NLO vs approx NLO vs nNLO @ 620 GeV

μ_0 [GeV]	NLO MG5 [fb]	NLO no qg chan MG5 [fb]	NLO app. [fb]
620	$445.7^{+51.4}_{-51.4}$	$467.1^{+28.1}_{-41.0}$	$464.5^{+22.2}_{-38.1}$

Uncertainty brackets obtained by scale variation in the range

$$\mu_0/2 \leq \mu \leq 2\mu_0$$

The approximate NLO reproduces to a very good extent the NLO without the qg channel contribution.

NLO vs approx NLO vs nNLO @ 620 GeV

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Since the soft limit approximation works well at NLO, the NNLO could be well approximated by soft emission at NNLO + complete NLO (nNLO)

NLO vs approx NLO vs nNLO @ 620 GeV

μ_0 [GeV]	NLO MG5 [fb]	NLO no qg chan MG5 [fb]	NLO app. [fb]
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μ_0 [GeV]	LO [fb]	NLO MG5 [fb]	nNLO [fb]
620	$317.2^{+97.4}_{-69.2}$	$445.7^{+51.4}_{-51.4}$	$479.8^{+10.3}_{-7.1}$

The NNLO soft emission corrections increase the value of the cross section and reduce significantly the scale uncertainty.

NLO vs approx NLO vs nNLO @ 620 GeV

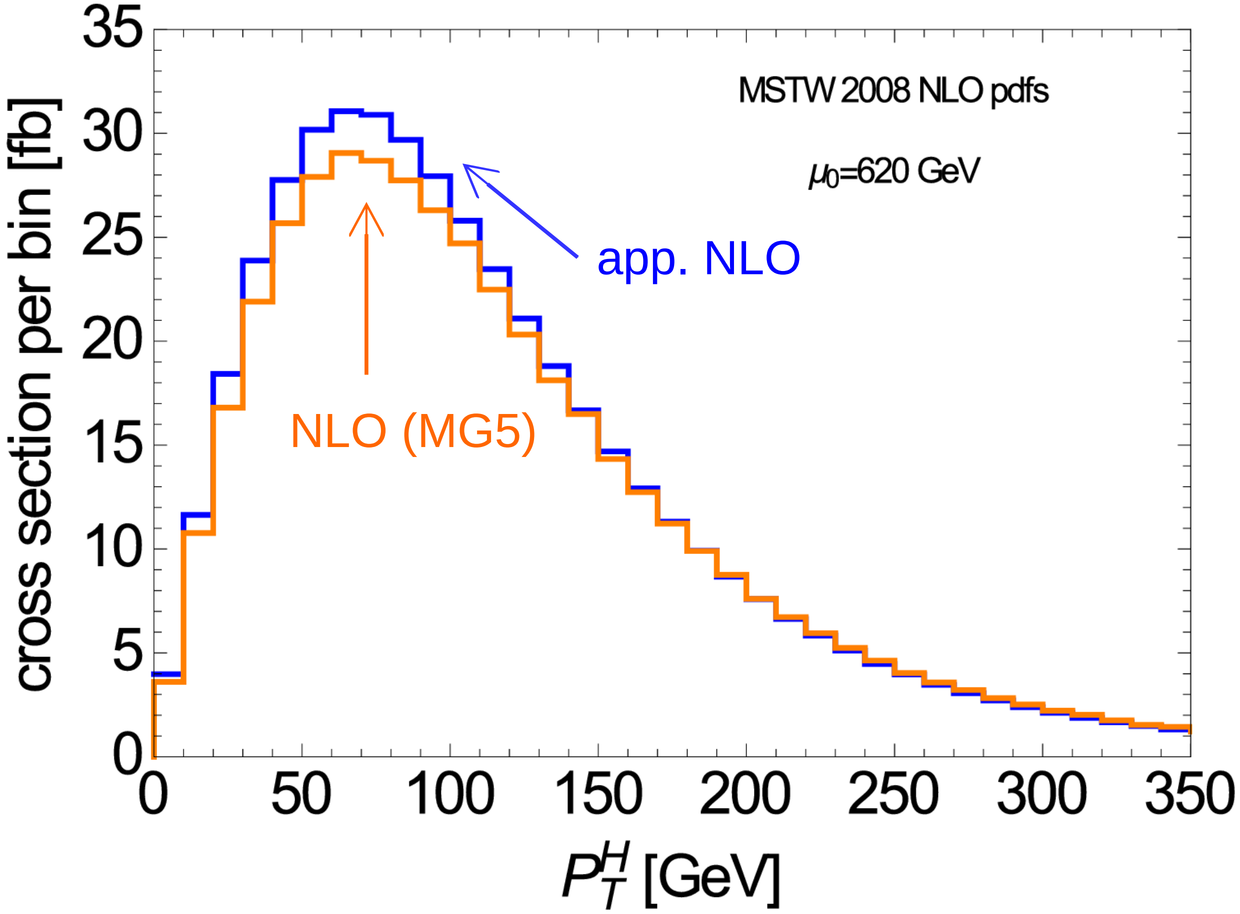
μ_0 [GeV]	NLO MG5 [fb]	NLO no qg chan MG5 [fb]	NLO app. [fb]
620	$445.7^{+51.4}_{-51.4}$	$467.1^{+28.1}_{-41.0}$	$464.5^{+22.2}_{-38.1}$

We can also estimate the theoretical uncertainty in a more conservative way, by implementing terms subleading in the soft limit in two different ways: very good agreement between complete NLO and approximate NLO

MG5 [fb]	nNLO [fb]
$.7^{+51.4}_{-51.4}$	$479.8^{+10.3}_{-7.1}$

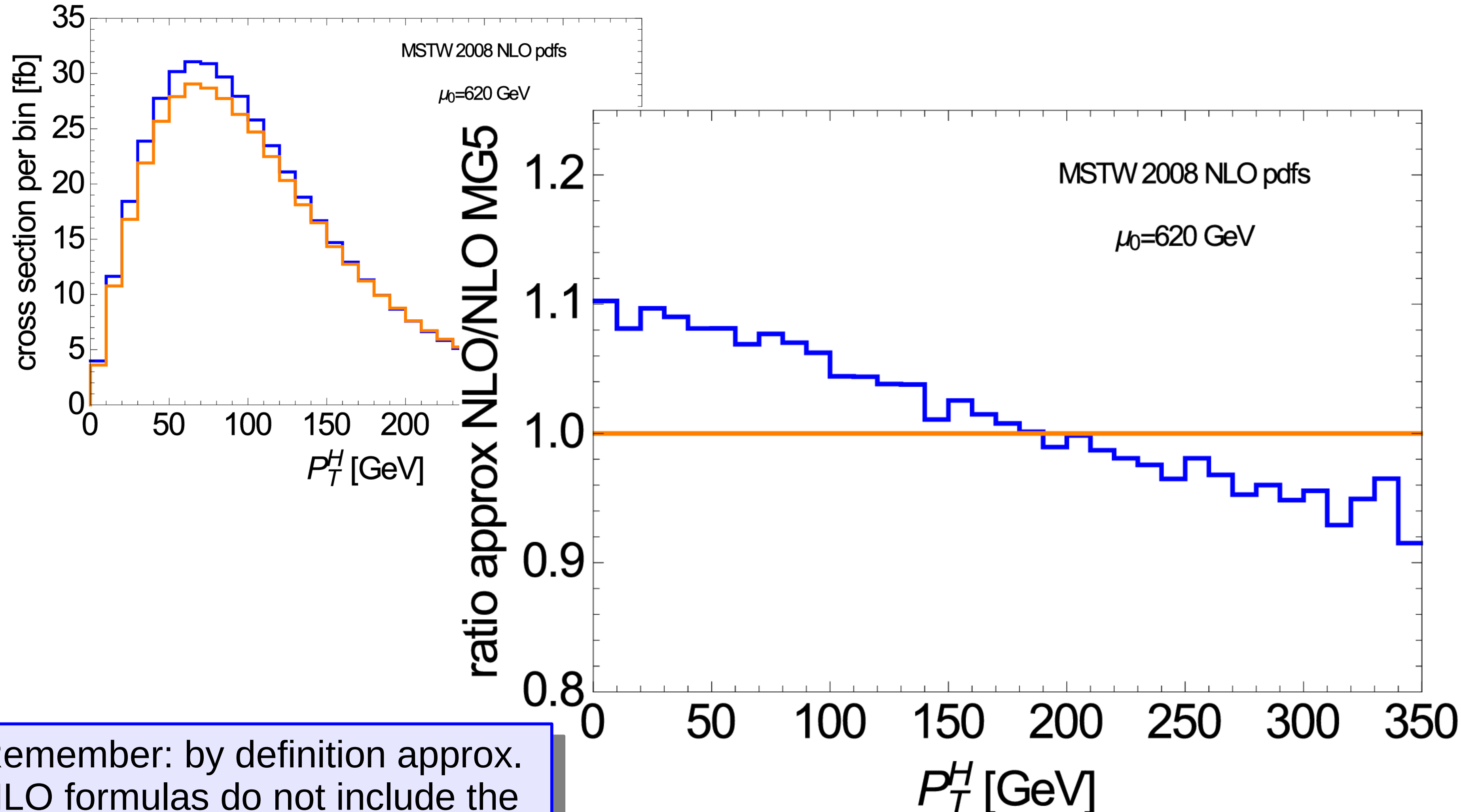
μ_0 [GeV]	NLO MG5 [fb]	approx. NLO [fb]	nNLO [fb]
620	$445.7^{+51.4}_{-51.4}$	$442.4^{+44.3}_{-44.3}$	$467.2^{+22.9}_{-22.9}$

Distributions: NLO vs approx NLO



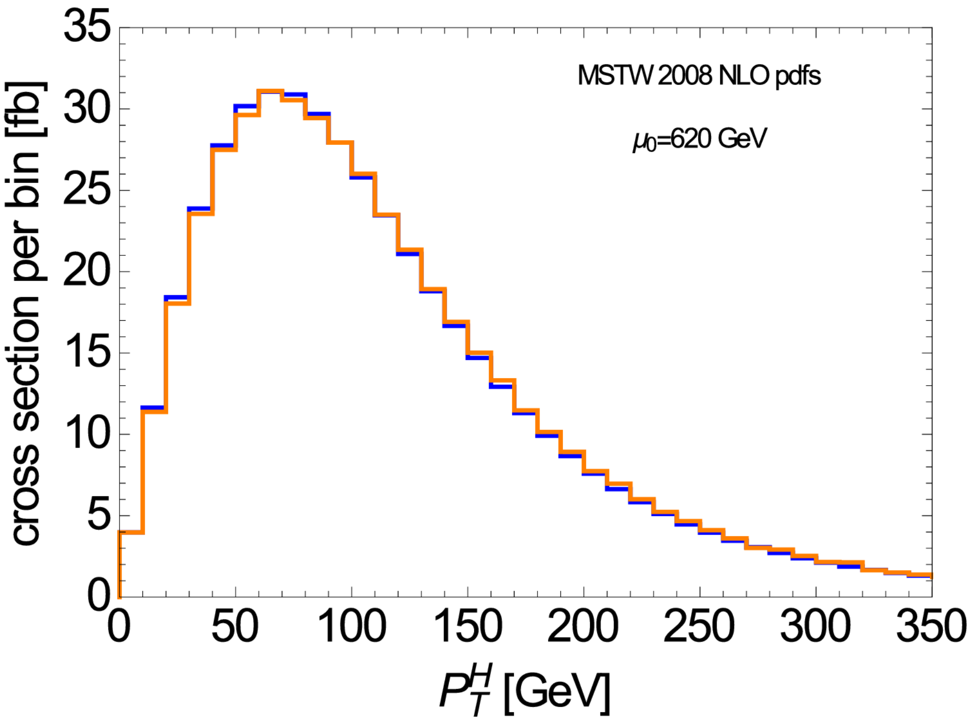
Transverse momentum of the Higgs boson (LHC at 13 TeV)

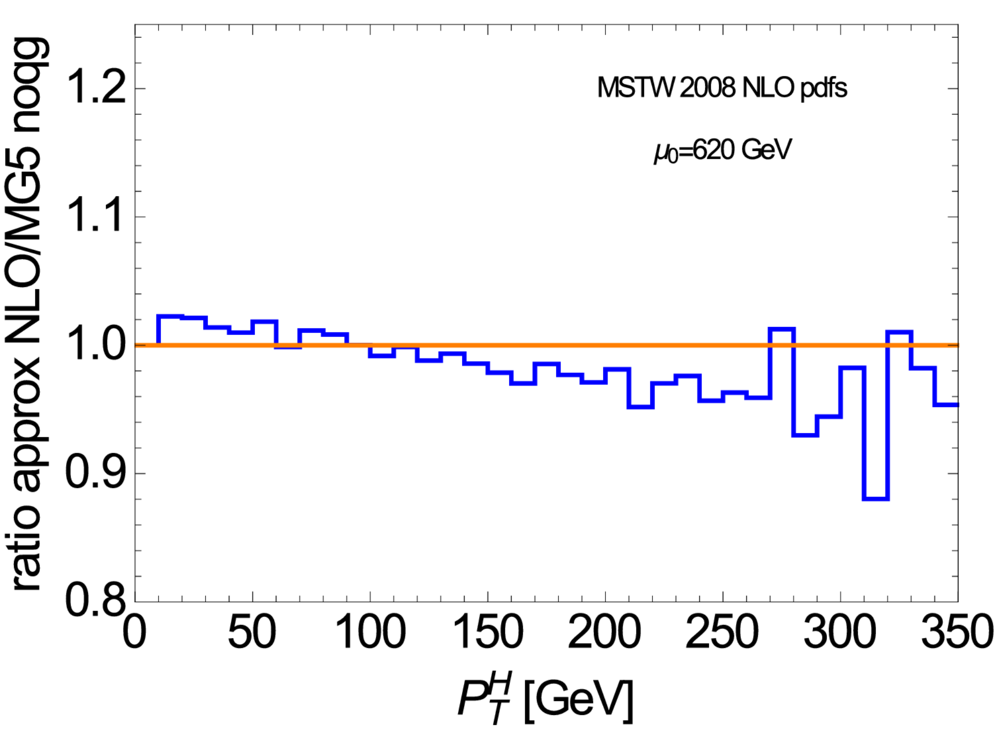
Distributions: NLO vs approx NLO



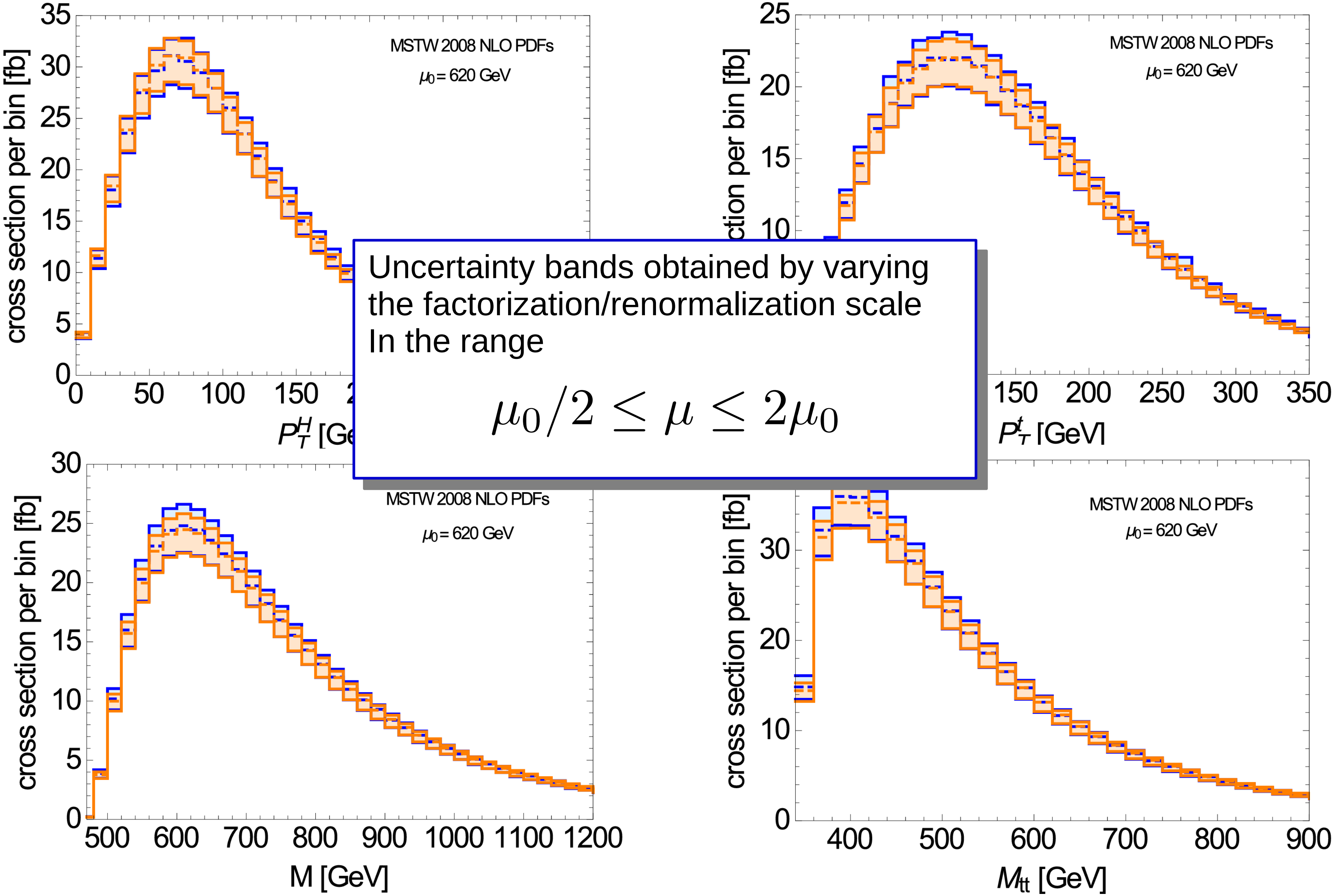
Remember: by definition approx. NLO formulas do not include the qg channel (which will be accounted for by matching when working at approx. NNLO)

NLO without qg vs approx NLO





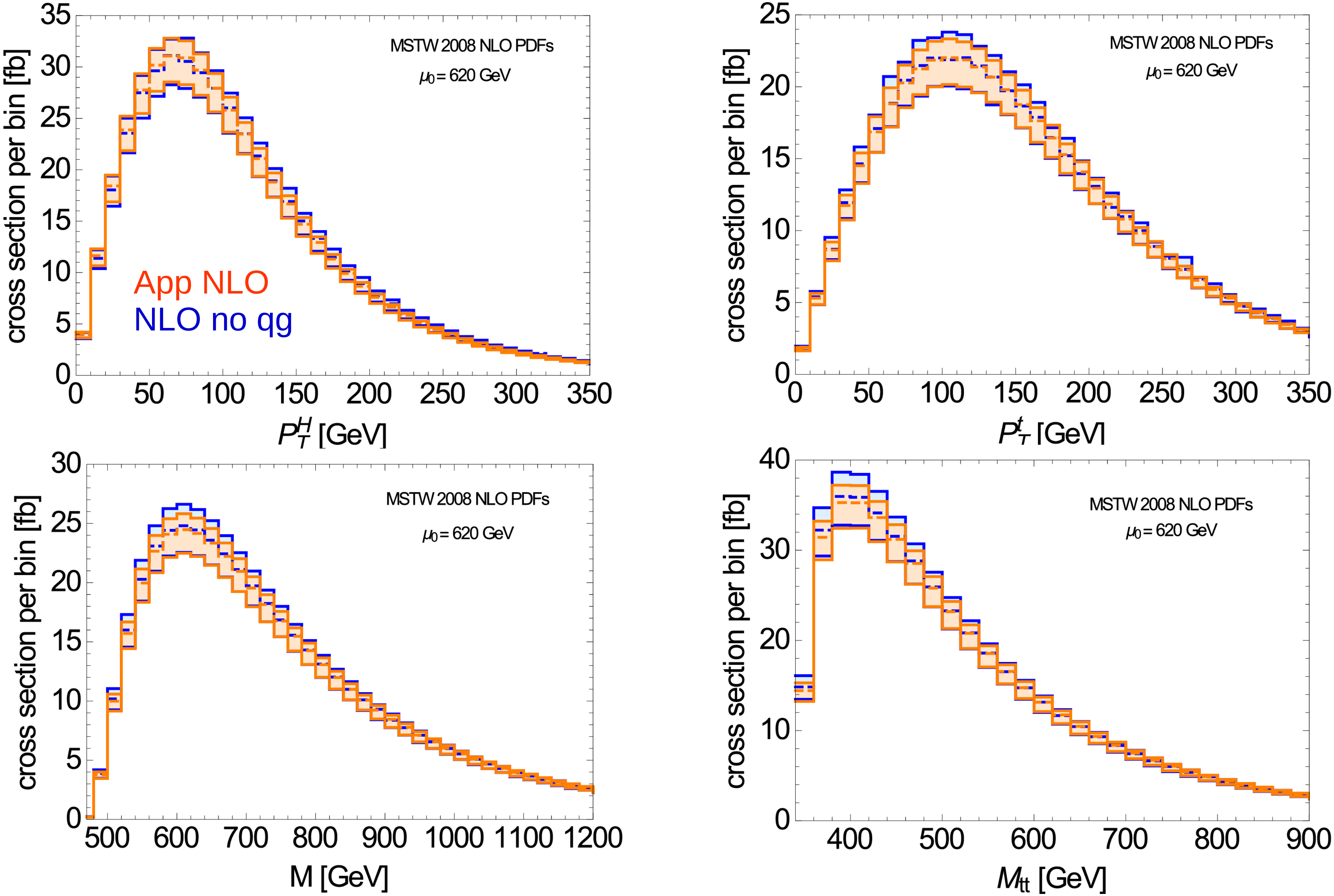
NLO without qg vs approx NLO



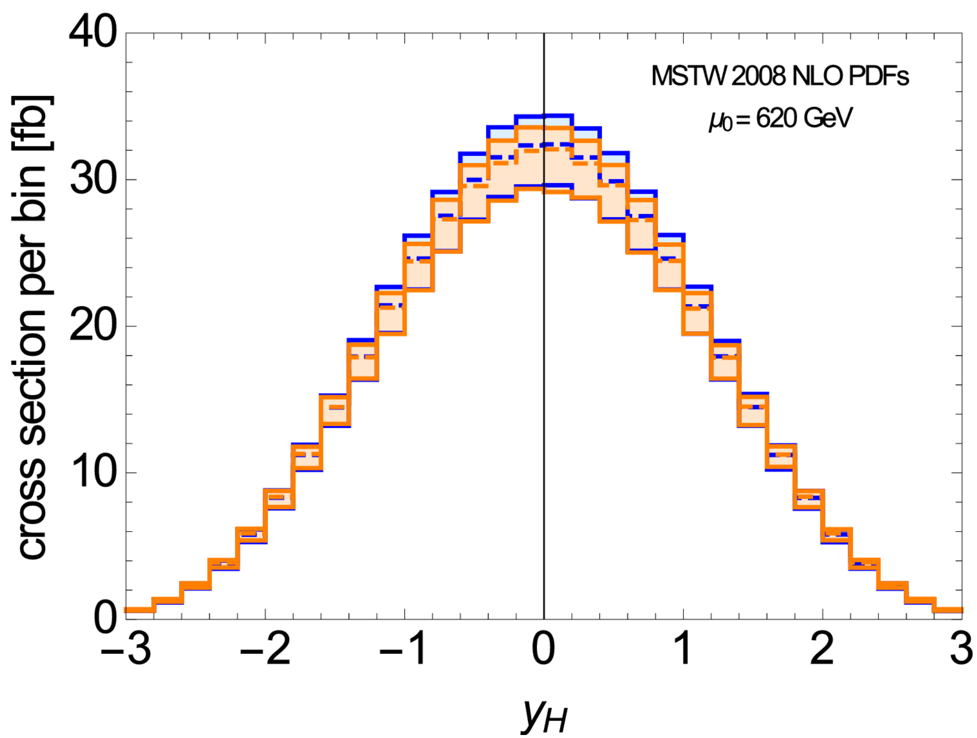
Uncertainty bands obtained by varying
the factorization/renormalization scale
In the range

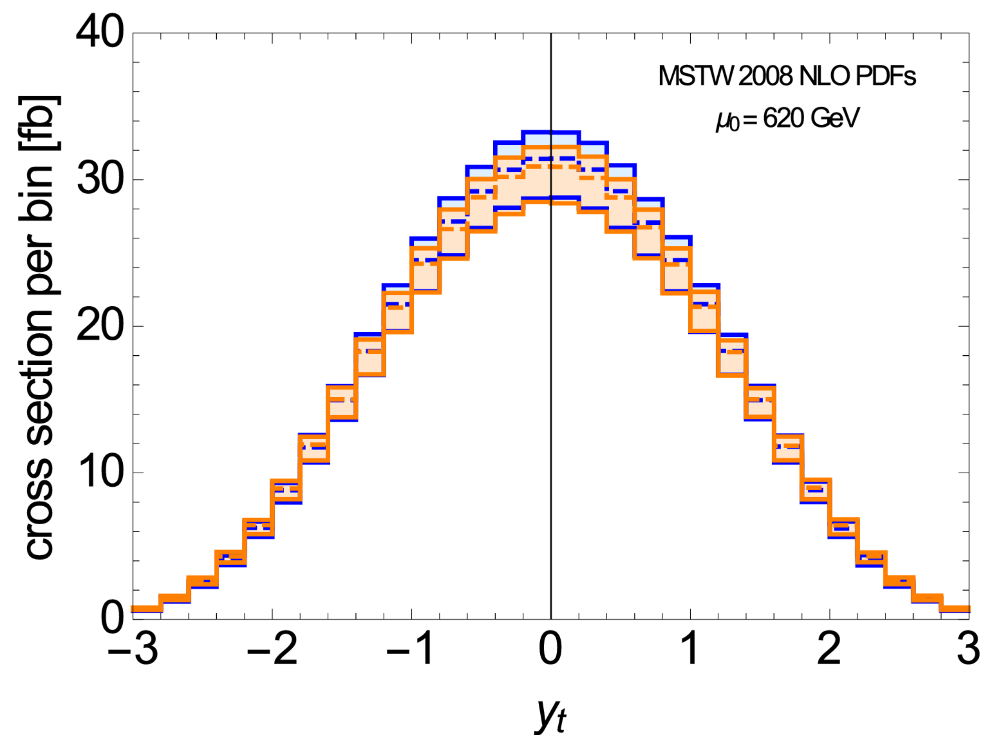
$$\mu_0/2 \leq \mu \leq 2\mu_0$$

NLO without qg vs approx NLO



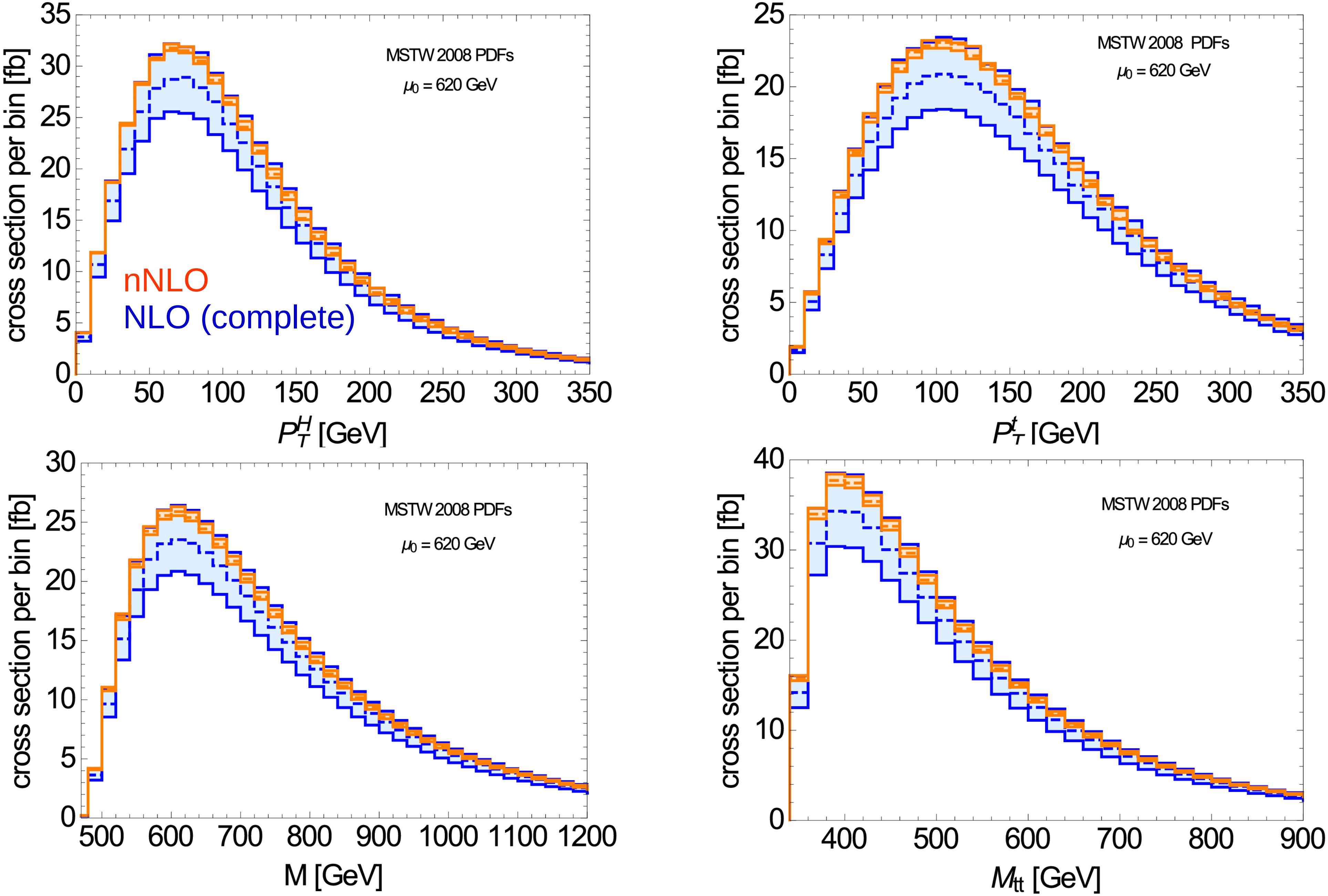
NLO without qg vs approx NLO: Rapidities



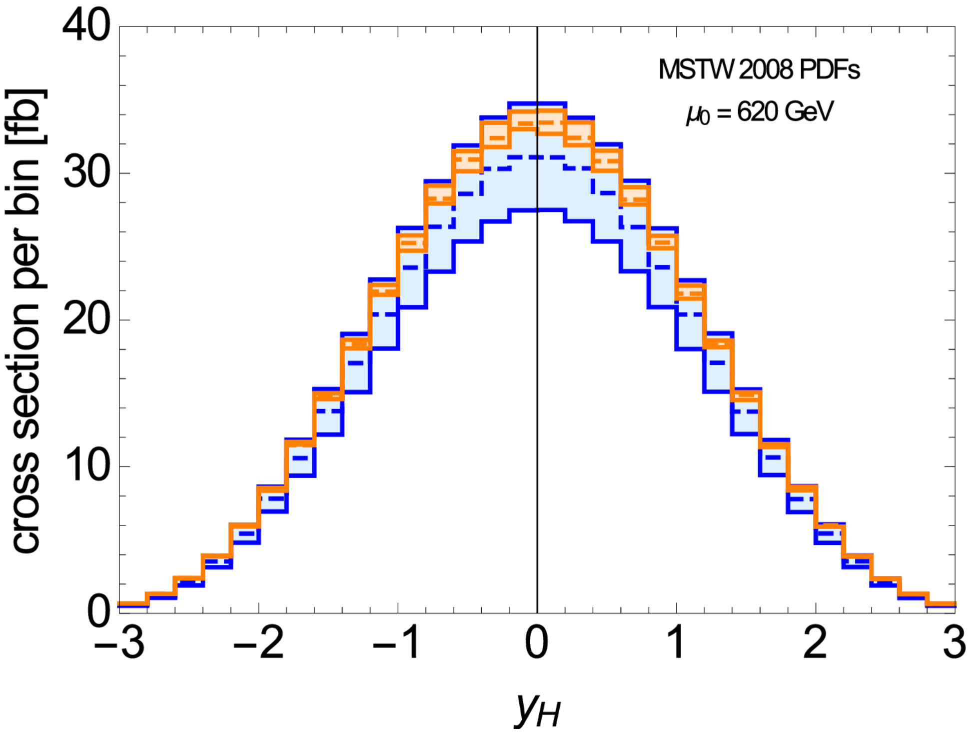


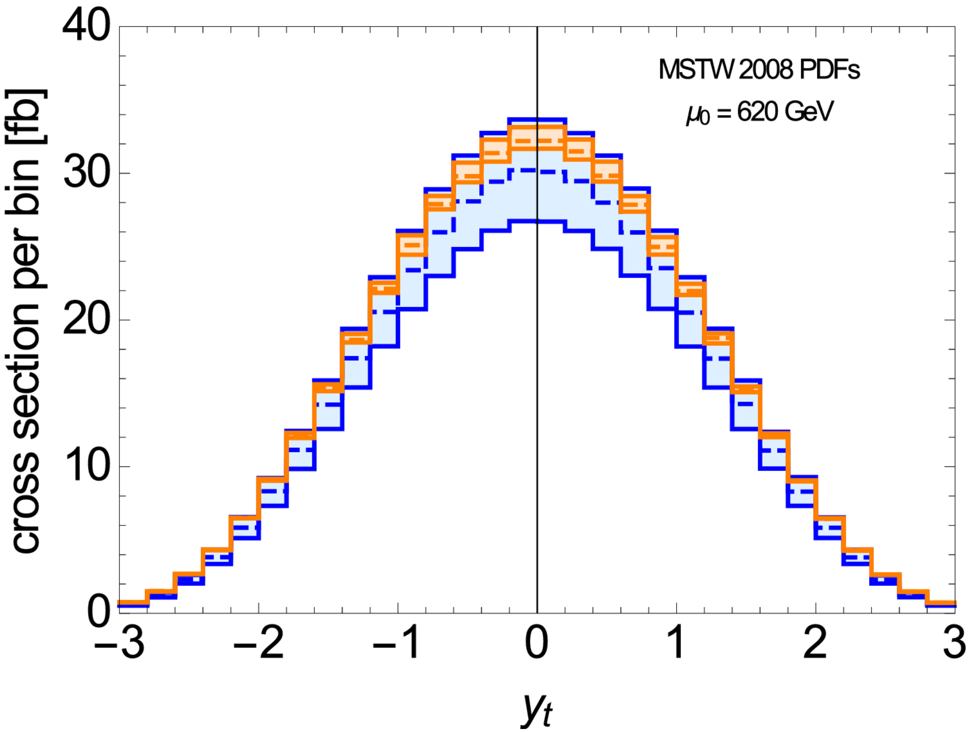
Excellent agreement between NLO no qg and approx. NLO
Let's see what happens if we calculate distributions at NLO+approx. NNLO = nNLO

nNLO distributions vs complete NLO



nNLO distributions vs NLO: rapidities





Conclusions and Outlook

What was done so far:

We built an in-house parton level Monte Carlo for the process

$$p + p \to t + \bar{t} + H$$

It can evaluate the NNLO corrections in the soft limit for the total cross section and differential distribution. The results are matched to the full NLO $\longrightarrow$ nNLO predictions

Conclusions and Outlook

- Implement NNLL resummation and study the dependence on hard and soft scales

- Carry out a phenomenological study (PDFs and α_s errors, several collider energies, etc...)

- Our program can be easily adapted to study processes such as top pair + Z boson

Backup Slides

Large logarithmic corrections

- The partonic cross section for top pair (+Higgs) production receives potentially large corrections from soft gluon emission dia
- So log

Renormalization group improved perturbation theory schematically:

➔ Separation of scales ↔ factorization

➔ Evaluate each (single-scale) factor in fixed order perturbation theory at a scale for which it is free of large logs

➔ Use Renormalization Group Equations to evolve the factors to a common scale

- It c
- On tion
- Th d out by means of effective field theory methods

Hard function at NLO

- Perturbative expansion of the hard function

$$\boldsymbol{H}_{ij} = \alpha_s^2 \frac{1}{d_R} \left(\boldsymbol{H}_{ij}^{(0)} + \frac{\alpha_s}{4\pi} \boldsymbol{H}_{ij}^{(1)} + \dots \right)$$

- The matrix elements can be written in terms of UV finite, IR subtracted QCD amplitudes, projected on a channel dependent color basis

$$H_{IJ}^{(0)} = \frac{1}{4} \frac{1}{\langle c_I | c_I \rangle \langle c_J | c_J \rangle} \left\langle c_I \middle| \mathcal{M}_{\text{ren}}^{(0)} \right\rangle \left\langle \mathcal{M}_{\text{ren}}^{(0)} \middle| c_J \right\rangle ,$$

$$H_{IJ}^{(1)} = \frac{1}{4} \frac{1}{\langle c_I | c_I \rangle \langle c_J | c_J \rangle} \left[\left\langle c_I \middle| \mathcal{M}_{\text{ren}}^{(0)} \right\rangle \left\langle \mathcal{M}_{\text{ren}}^{(1)} \middle| c_J \right\rangle + \left\langle c_I \middle| \mathcal{M}_{\text{ren}}^{(1)} \right\rangle \left\langle \mathcal{M}_{\text{ren}}^{(0)} \middle| c_J \right\rangle \right]$$

- For top-quark pair production, the NLO matrix elements can still be calculated analytically → Things are more complicated for top-pair+Higgs

Integration over z

- You might have noticed a difference in the cross section formulas I used for top-pair and top-pair + Higgs formulas:

$$\int d\sigma_{t\bar{t}} \propto \int_{\tau_{\min}}^{1} \frac{d\tau}{\tau} \int_{\tau}^{1} \frac{dz}{z} \; ff \int d\phi_{t\bar{t}} \, \mathrm{Tr}\left[\mathbf{HS}\right]$$

$$\int d\sigma_{t\bar{t}H} \propto \int_{\tau_{\min}}^{1} \frac{d\tau}{\tau} \int_{\tau}^{1} \frac{dz}{\sqrt{z}} \; ff \int d\phi_{t\bar{t}H} \, \mathrm{Tr}\left[\mathbf{HS}\right]$$

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Why this difference?

Integration over z

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- It depends on the way one treats cofactors of the soft function in deriving the cross section formula:

$$\text{if} \quad M^2 \simeq \hat{s} \;\longrightarrow\; \frac{1}{z}$$

$$\text{if} \quad M^2 = z\hat{s} \longrightarrow \frac{1}{\sqrt{z}}$$

Integration over z

- You might have noticed a difference in the cross section
  for

- It d … ion
  in

- The two implementations are equivalent, up to power suppressed terms of O(1-z), therefore in our limit they are formally both correct
- However, the choice of the power of z to be used has some numerical impact on the results
- The choice leading to 1/z was adopted in our top pair production calculations, in part becuase it is very close to what one find when studying other processes, e.g. Drell-Yan
- The choice leading to $1/\sqrt{z}$ provides results which are numerically closer to the ones obtained by "direct QCD" methods

$\sqrt{z}$

Integration over z and “direct QCD”

- The choice of the power of z in the integrand can be recast in terms of a redefinition of the soft function

$$\frac{1}{\sqrt{z}} S_{\mathrm{OLD}}(z) = \frac{1}{z} \underbrace{\sqrt{z} S_{\mathrm{OLD}}(z)}_{\equiv S_{\mathrm{NEW}}(z)}$$

- Compare the new S with the one that can be obtain with direct QCD methods:

$$S_{\mathrm{NEW}}(z, M^2, \mu_s) = \tilde{s}\left(\frac{M^2}{\mu_s^2} + \partial_\eta, \mu_s\right) \frac{\sqrt{z}}{1-z} \left(\frac{1-z}{\sqrt{z}}\right)^{2\eta} \frac{e^{-2\gamma_{\mathrm{E}}\eta}}{\Gamma(2\eta)}$$

$$S_{\mathrm{QCD}}(z, M^2, \mu) = \tilde{s}\left(\frac{M^2}{\mu_s^2} + \partial_\eta, \mu_s\right) (-\ln z)^{-1+2\eta} \frac{e^{-2\gamma_{\mathrm{E}}\eta}}{\Gamma(2\eta)}$$

Integration over z and “direct QCD”

- The choice of the power of z in the integrand can be recast in terms o
- Compa … th direct QCD m

$S_{\rm NEW}(z$ … $\dfrac{e^{-2\gamma_{\rm E}\eta}}{\Gamma(2\eta)}$

$$S_{\rm QCD}(z,M^2,\mu)=\tilde{s}\left(\frac{M^2}{\mu_s^2}+\partial_\eta,\mu_s\right)(-\ln z)^{-1+2\eta}\,\frac{e^{-2\gamma_{\rm E}\eta}}{\Gamma(2\eta)}$$

The NEW and QCD soft functions differ only quadratic power suppressed terms

$$\frac{S_{\rm NEW}}{S_{\rm QCD}}=1+\mathcal{O}\left((1-z)^2\right)$$

Calculations carried out by using the factor $1/\sqrt{z}$ in the integration formula give results which are expected to be close to the ones obtained by means of “direct QCD” methods

Final state phase space

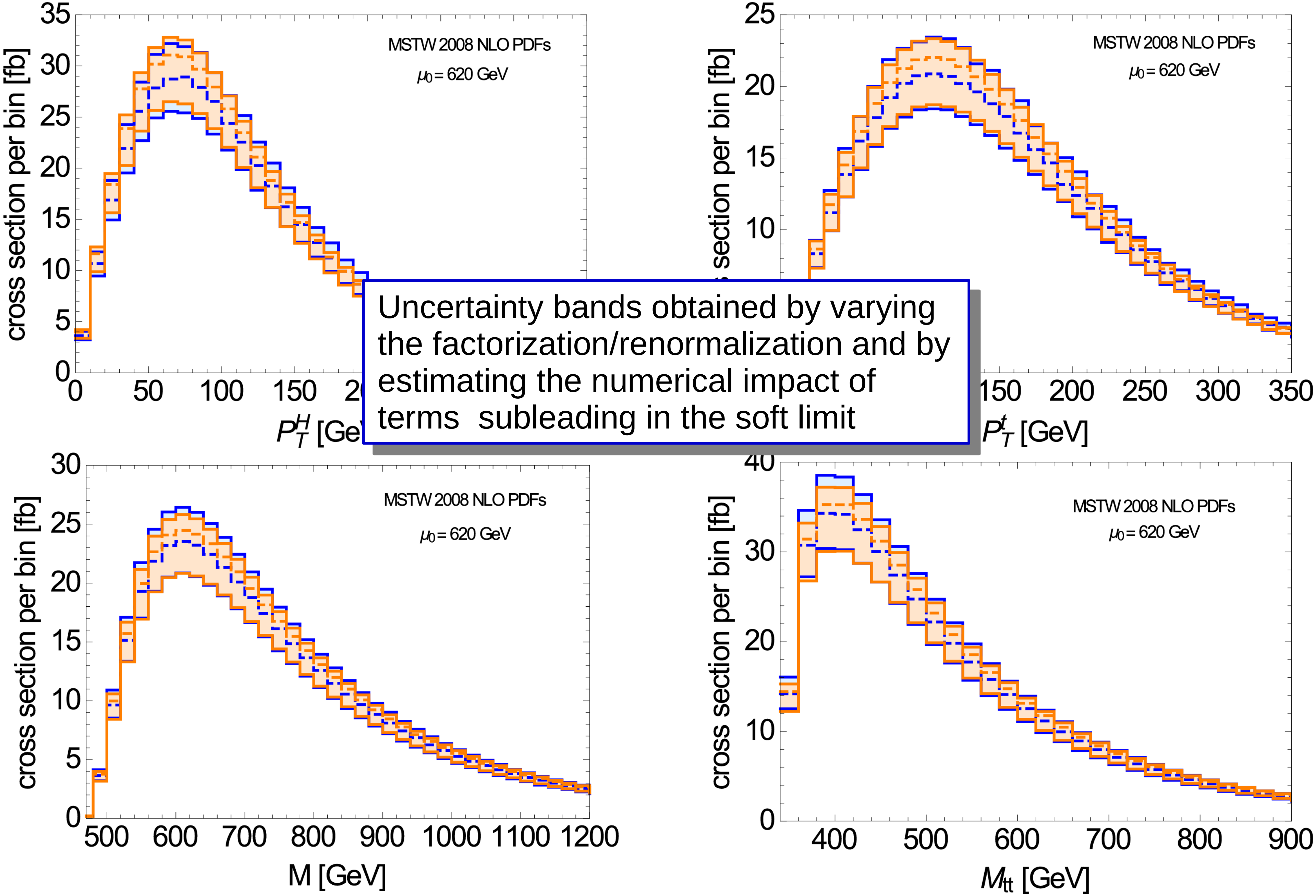
- The final state phase space is written as the convolution of two two-particle phase spaces:

$$\int d\Phi_{t\bar{t}H} = \int \frac{ds_{t\bar{t}}}{2\pi} \frac{1}{2\hat{s}} \frac{d\Omega}{16\pi^2} K\left(\hat{s}, s_{t\bar{t}}, m_H^2\right) \frac{1}{2s_{t\bar{t}}} \frac{d\Omega^*}{16\pi^2} K\left(s_{t\bar{t}}, m_t^2, m_t^2\right)$$

$$K(a,b,c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$$

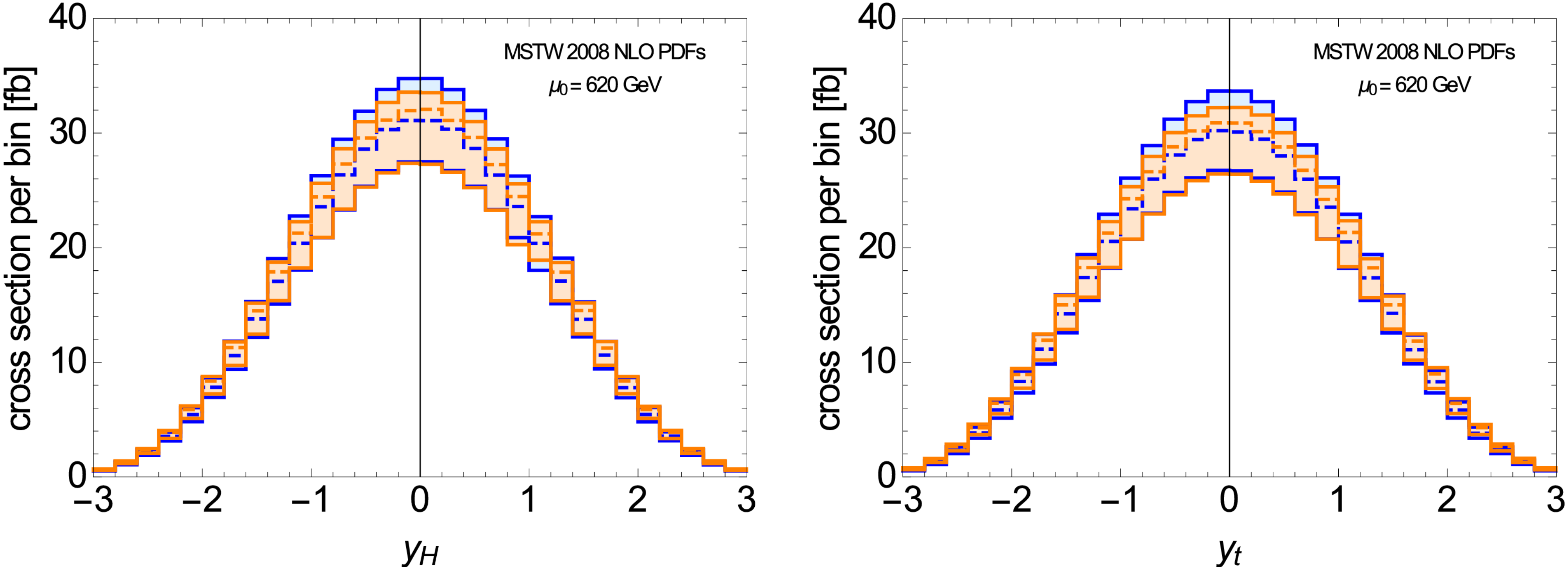
- Five integrations left in the final state phase space
- Three integrations for the initial state (τ, z, and the luminosity variable x)
- One needs to build a Monte Carlo integration over 8 variables
- The 8 integration variables determine the top, antitop, Higgs and incoming parton momenta: one can bin events and plot distributions

NLO vs approx NLO large bands

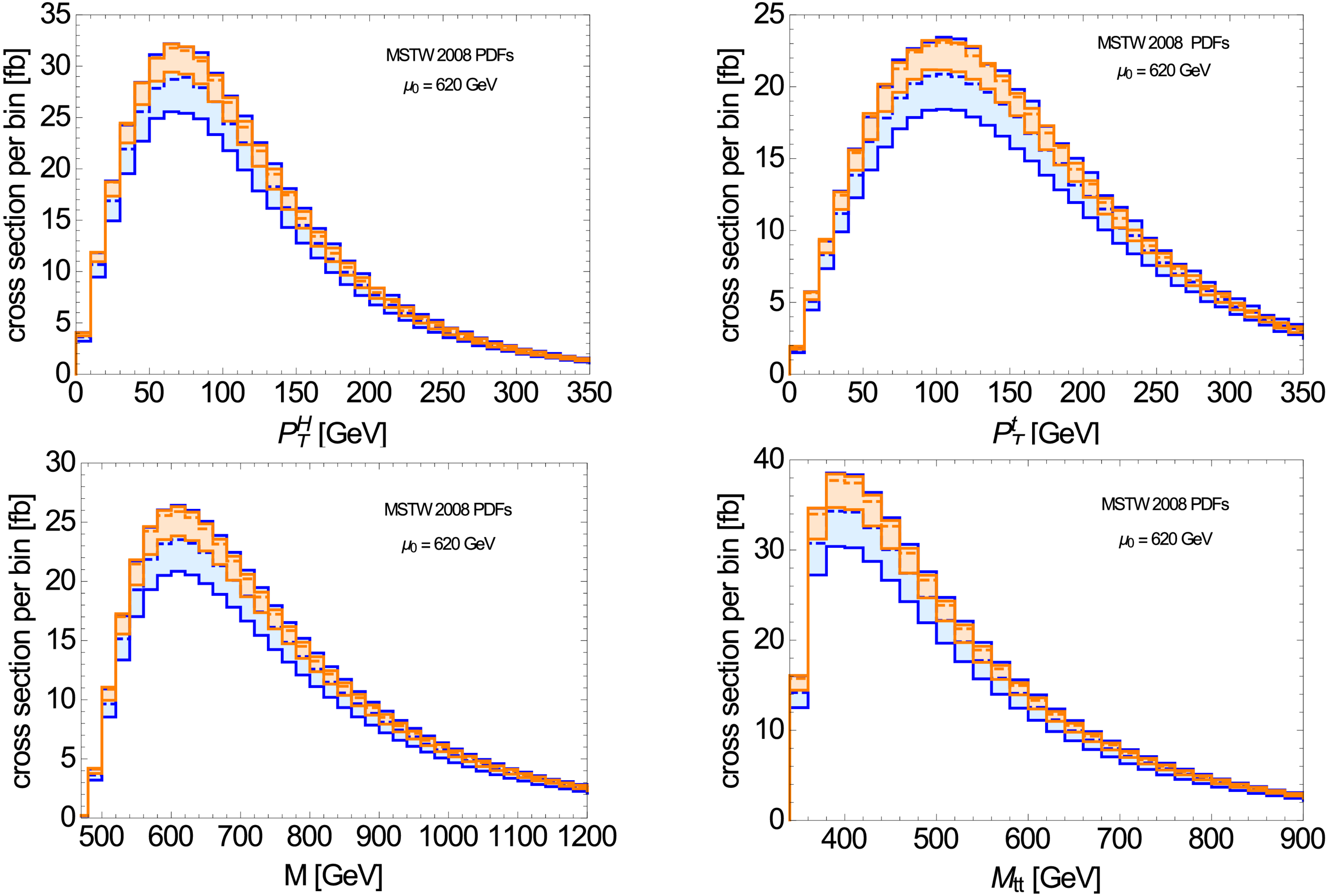


Uncertainty bands obtained by varying the factorization/renormalization and by estimating the numerical impact of terms subleading in the soft limit

NLO vs approx NLO large bands: Rapidities



nNLO large bands vs complete NLO



nNLO large bands vs NLO: rapidities