

Improved Estimate of Proton Lifetime in lattice QCD

Eigo Shintani (RIKEN-AICS)

with Y. Aoki (Nagoya), T. Izubuchi (BNL), A. Soni(BNL)

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OUTLINE

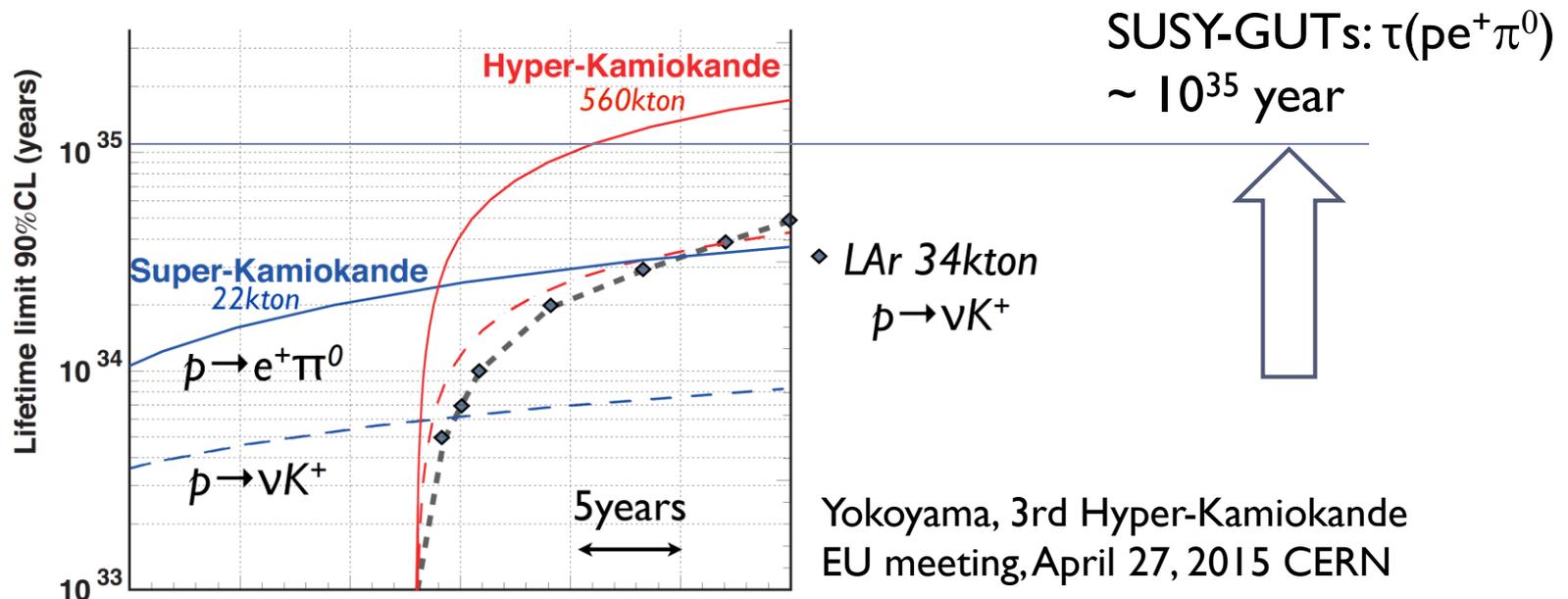
1. Introduction
2. Matrix element of proton decay on the lattice
3. Updated result
4. Summary

1. Introduction

Proton decay

- ▶ SK: $\tau(p \rightarrow e^+ \pi^0) > 1.3 \times 10^{34}$ years, $\tau(p \rightarrow \nu K) > 5.9 \times 10^{33}$ years SK, PRD90(2014)
- ⇒ Hyper-K aims to take $\tau(p \rightarrow e^+ \pi^0) > 1.3 \times 10^{35}$ years, $\tau(p \rightarrow \nu K) > 3.2 \times 10^{34}$ years
HyperK-proto, PTEP 2015

Selected as one of 27 top priority projects in “Japan Master Plan of Large Research Project”, ~\$650M+\$25M/y for 15y. (also J-PARC, Muon g-2/EDM experiment was)
<http://www.scj.go.jp/ja/info/kohyo/pdf/kohyo-22-t188-1.pdf> (in Japanese)



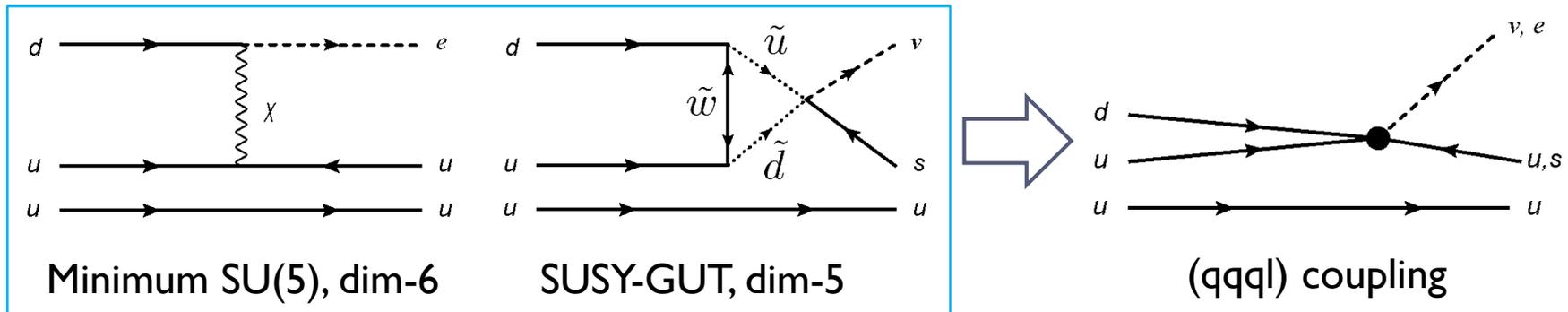
1. Introduction

Decay rate

$$\Gamma_{p \rightarrow \pi^0 e^+} = \frac{m_p}{32\pi} \left[1 - \left(\frac{m_e}{m_p} \right)^2 \right]^2 \left| \sum_i C_i(p \rightarrow P + l) \right|^2 \sim \frac{\alpha_5^2 m_p^5}{M_X^4}$$

Wilson coefficient, relying on GUTs, + QCD matrix element, summing over quantum number i

$$H_{\text{BV}} = \sum_i C_i(\mu) O_i(\mu) / \Lambda_{\text{GUTs}}, \quad O_i = (qqql)_i$$



➤ Wilson coefficient ($p \rightarrow \nu K$) on SUSY-SU(5)

$$C_\mu(\mu) = \frac{\alpha_2^4}{M_{H_C}^2 m_W^4 \sin^2 2\beta} F(M_2, M_S^2) \frac{\beta_p(\mu)}{f_\pi^2} \left\{ 1 + (D + F) \frac{m_p}{m_B} \right\} \bar{m}_s V_{us}^* \sum_{i=2,3} m_{u_i} V_{u_i d} V_{u_i s} e^{i\phi_i} A_R^{(i,2)}(\mu),$$

$$F(M, M_S) = M \left[\frac{1}{M_S^2 - M^2} - \frac{M^2}{(M_S^2 - M^2)^2} \ln \left(\frac{M_S^2}{M^2} \right) \right]$$

M_2 : wino, M_S : sfermion,
 M_{H_C} : color-triplet Higgs

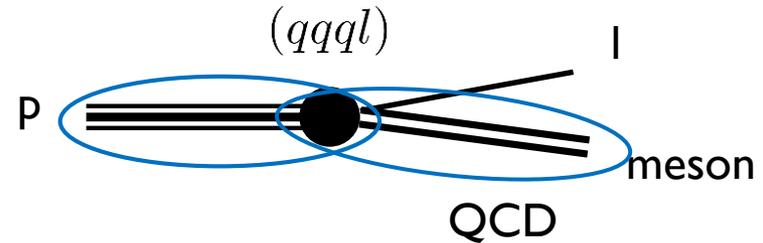
Matrix element described by ChPT

1. Introduction

Matrix element with BV operator

► Transition amplitude

$$\begin{aligned}
 \langle \pi^0 e^+ | p \rangle_{\text{GUT}} &= \sum_{i=\Gamma, \Gamma'} C_i \langle \pi^0 e^+ | (ud)_\Gamma (ul)_{\Gamma'} | p \rangle_{\text{SM}} \\
 &= \sum_{i=\Gamma, \Gamma'} C_i \langle \pi^0 | (ud)_\Gamma u_{\Gamma'} | p \rangle \bar{v}_{e^+}^c \\
 &\quad \Gamma, \Gamma' = \text{L, R}
 \end{aligned}$$



► QCD matrix element appears in $p \rightarrow P(\pi, K, \eta)$

$$\begin{aligned}
 \langle \pi^0(\vec{p}) | (ud)_\Gamma (u)_{\Gamma'} | p(\vec{k}, s) \rangle &= P_{\Gamma'} \left[W_0^{\Gamma\Gamma'}(q^2) - \frac{i\not{q}}{m_p} W_1^{\Gamma\Gamma'}(q^2) \right] u_p(\vec{k}, s) \\
 &= P_{\Gamma'} u_p(\vec{k}, s) W_0^{\Gamma\Gamma'}(0) + \mathcal{O}(m_l/m_N)
 \end{aligned}$$

Aoki et al. (JLQCD), PRD62 (2000); Aoki et al. (RBC), PRD75 (2007)

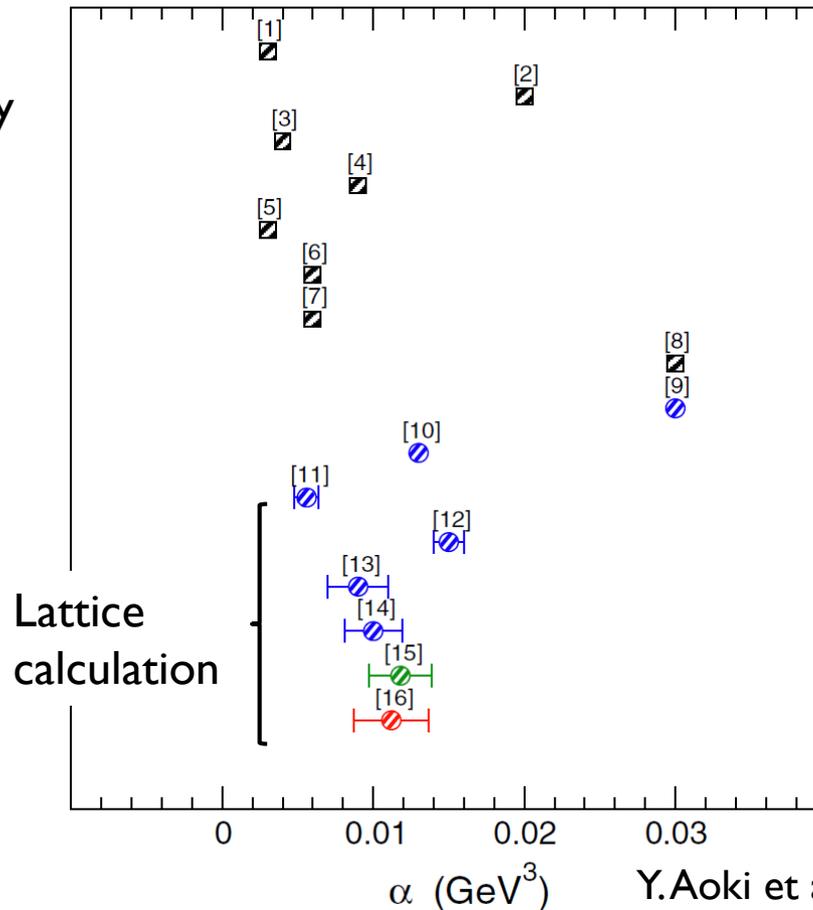
- W_0 contains the matrix element with effective vertex of BSM operator.
- 3-quark operator (qqq) forms L, R combination.
- $m_l/m_N \ll 1 \Rightarrow q^2 = m_l^2 \simeq 0$ is physical kinematics. W_0 is only relevant form factor.

1. Introduction

Model calculations

► Estimate of LECs from effective models and lattice

- Strong model dependence
- Systematic uncertainty may have $O(10)$ for α
 $\Rightarrow O(100)$ in τ_p
- Lattice calculation can determine LEC is around 0.01 GeV^3 .
- More importantly, lattice QCD is able to determine matrix element itself.



- [1] Donoghue 82
- [2] Thomas 83
- [3] Meljanac 82
- [4] Ioffe 81
- [5] Krasnikov 82
- [6] Ioffe 84
- [7] Tomozawa 81
- [8] Brodsky 84
- [9] Hara 86
- [10] Bowler 88
- [11] Gavela 89
- [12] JLQCD 00
- [13] CP-PACS & JLQCD 04
- [14] RBC 07
- [15] RBC 07
- [16] This work

Y.Aoki et al. (RBC-UKQCD),
PRD78 (2008)

2. Matrix element of proton decay on the lattice

Method in lattice QCD

- ▶ There are two ways on the lattice.
- ▶ Called as “indirect” method
 - ▶ Measurements of LECs in BChPT.
 -  Computation is less cost, and obtain precise value.
 -  Neglecting higher order mass correction → other systematic error
- ▶ Called as “direct” method
 - ▶ Measurement extracted from 3-pt function.
 -  There is no uncertainty depending on models.
 -  Calculates matrix element in each decay mode
 -  Expensive calculation, and large statistical noise appears.

S.Aoki et al. (JLQCD), PRD62 (2000), Y.Aoki et al.(RBC), PRD75,, Y.Aoki et al. (RBC-UKQCD), PRD78 (2008)

2. Matrix element of proton decay on the lattice Indirect method

- ▶ LECs are defined by amplitude of $p \rightarrow \text{vac}$ with BV op.

$$\langle 0 | (ud_R)u_L | p \rangle = \alpha_p P_L u_N, \quad \langle 0 | (ud_L)u_L | p \rangle = \beta_p P_L u_N$$

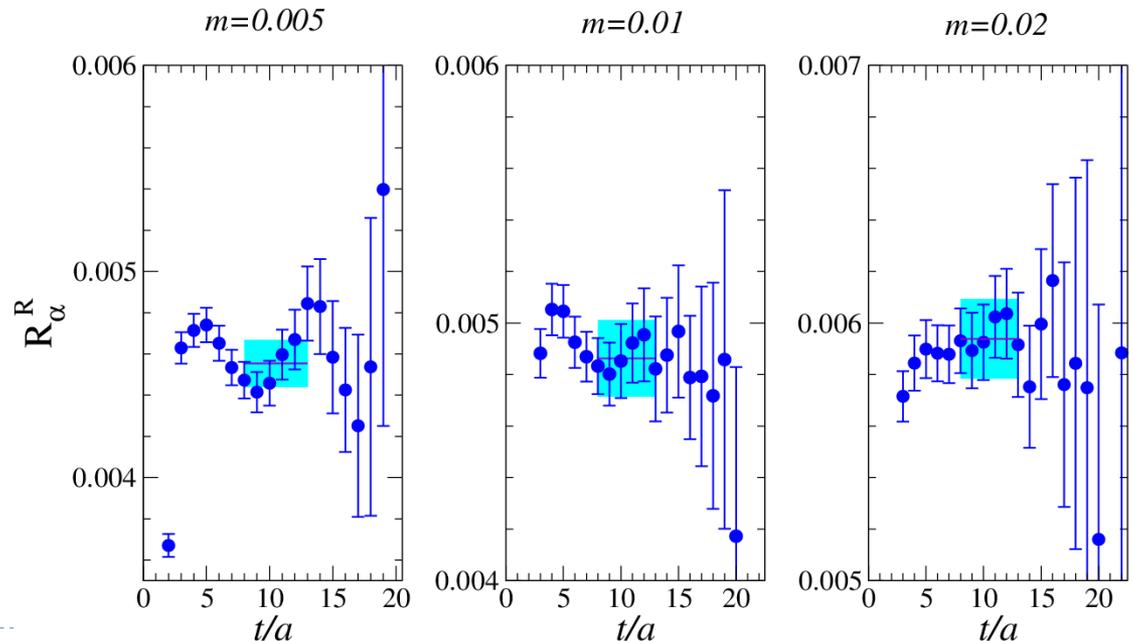
- ▶ Two-point function of BV operator and N operator

$$C_{NN}(t) = \langle 0 | \eta_N(t) \bar{\eta}_N(0) | 0 \rangle \simeq |Z_N|^2 e^{-m_N t}$$

$$C_N^R(t) = \langle 0 | (ud_R)u_L(t) \bar{\eta}_N(0) | 0 \rangle \simeq \langle 0 | (ud_R)u_L | N \rangle Z_N^* e^{-m_N t}$$

$$R_\alpha^R = Z_N \frac{C_N^R(t)}{C_{NN}(t)}$$

$$\simeq \alpha_p$$



2. Matrix element of proton decay on the lattice

Direct method

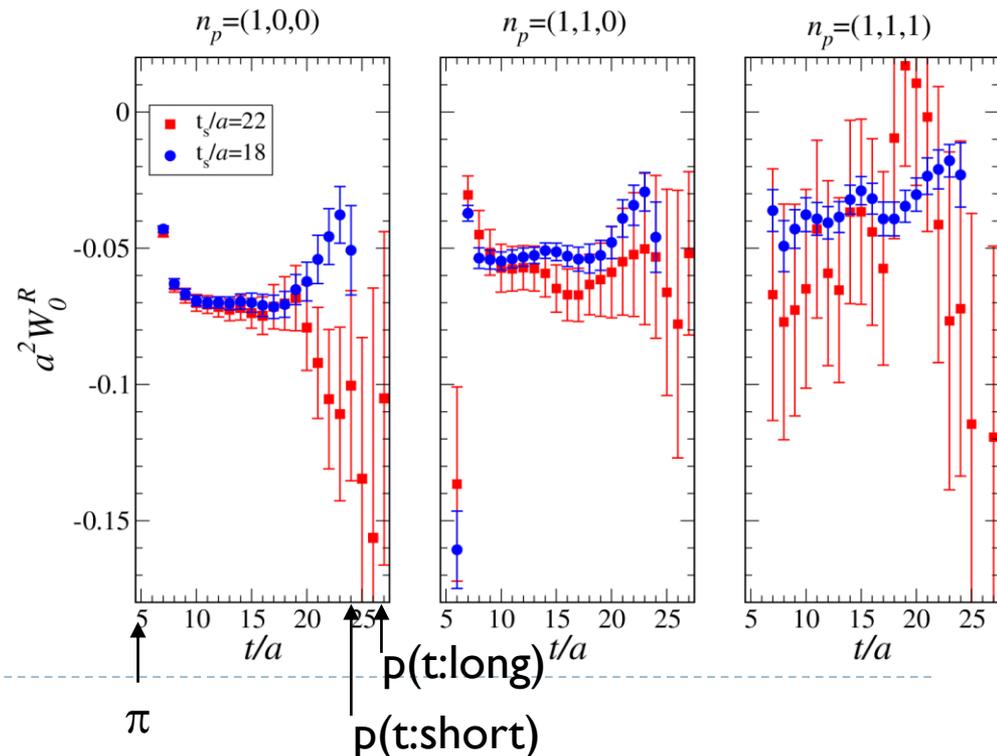
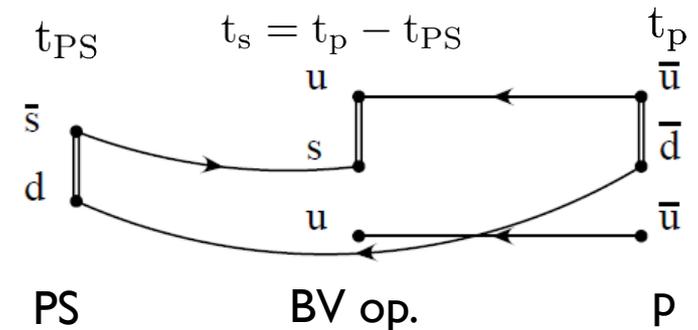
- ▶ **3-pt function**
(PS meson)-(BV operator)-(Nucleon)
- ▶ **Ratio of 3-pt and 2-pt**

$$R_3(t, t_s, p) = \frac{\langle 0 | \{ \eta_\pi(t_{PS}, p) [(ud_R)u_L](t_s, -p) \bar{\eta}_N(0) \}_T | 0 \rangle}{C_\pi(t_{PS} - t_s, p) C_{NN}(t_s, 0)}$$

$$\text{Tr} [R_3(t, t_s, p) P_L P_4] \simeq W_0^L - \frac{iq_4}{m_N} W_1^L,$$

$$\text{Tr} [R_3(t, t_s, p) P_L i P_4 \gamma_j] \simeq \frac{q_j}{m_N} W_1^L$$

- Pion and BV op have momentum, however N does not.
- To suppress the excited state, t_s and $t_s - t_{PS}$ should be large.
- Because excited state effect is unknown, we investigate by varying t_s with several values.



3. Updated results

Lattice setting

- ▶ Domain-wall fermion (DWFs) $N_f=2+1$ RBC-UKQCD, 2011
 - ▶ $24^3 \times 64$ size at $a^{-1} = 1.73$ GeV (2.5 fm³ box size)
 - ▶ Light quark mass $m=0.005, 0.01, 0.02, 0.03$ ($m_\pi = 0.3$ -- 0.6 GeV)
 - ▶ Strange quark mass $m_s = 0.04$ ($m_K = 0.5$ GeV)
 - ▶ 5th dimension, $L_s = 16$ in which $am_{res} = 0.003 \rightarrow$ small chiral sym. breaking
 - ▶ AMA with low-mode deflation Blum, Izubuchi, ES (2013--2014)
- ▶ Renormalization constant
 - ▶ The mixing with different chirality due to $am_{res} \neq 0$ is negligible.

$$U_L^{\overline{MS}}(\mu = 2\text{GeV}) = 0.705(10), \quad U_R^{\overline{MS}}(\mu = 2\text{GeV}) = 0.706(11)$$

Error is statistical one. In addition, truncation error in one-loop matching is around 8% (roughly), so this will be not negligible.

3. Updated results

LECs in BChPT

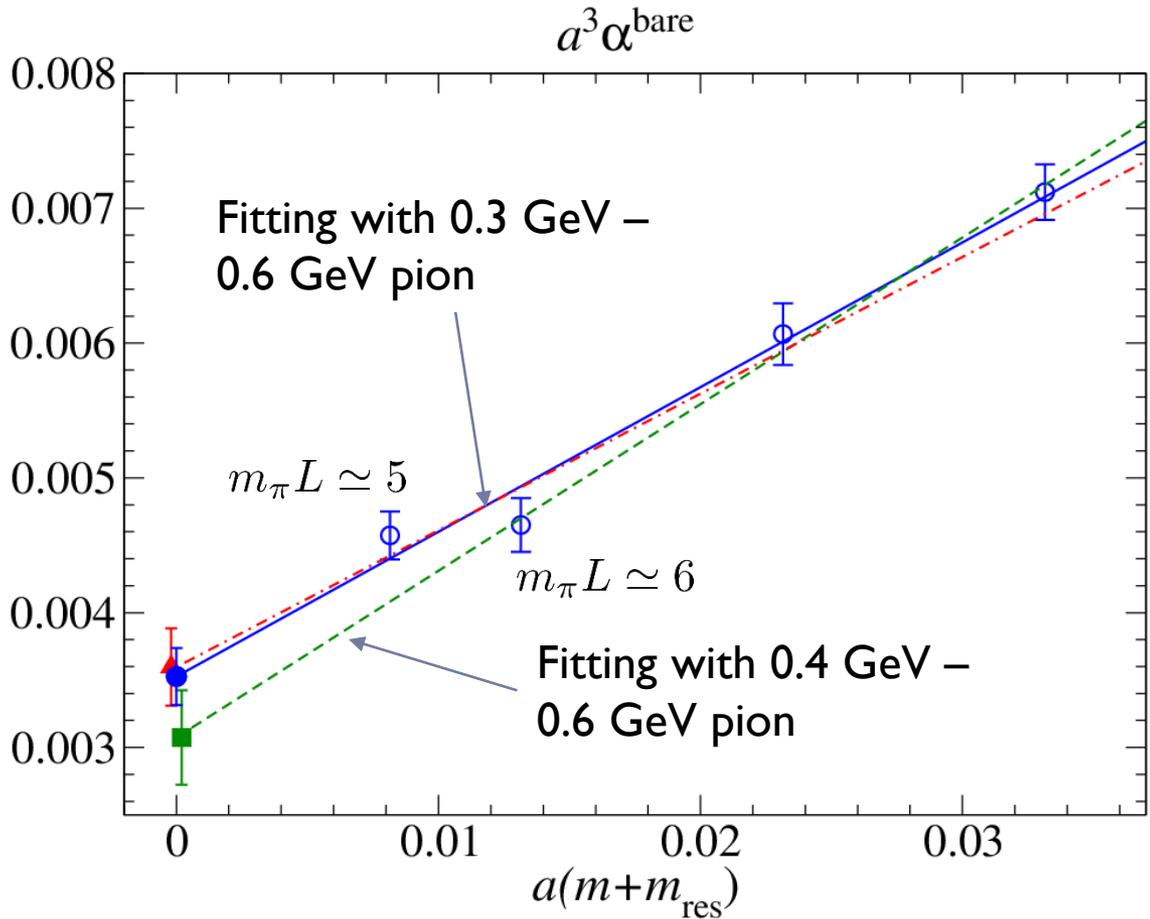
- Finite volume correction
 $\sim \exp[-m_\pi L]$
- Higher order of $O(m_\pi^2)$,
 \Rightarrow discrepancy from linearity.

Comparing two fitting,
 excluding smallest pion or
 excluding largest pion mass,
 seemingly finite size is large.

Preliminary:

$$\alpha_p^{\mu=2\text{GeV}} = -0.0142(9)(19)\text{GeV}^3,$$

$$\beta_p^{\mu=2\text{GeV}} = 0.0126(8)(13)\text{GeV}^3$$

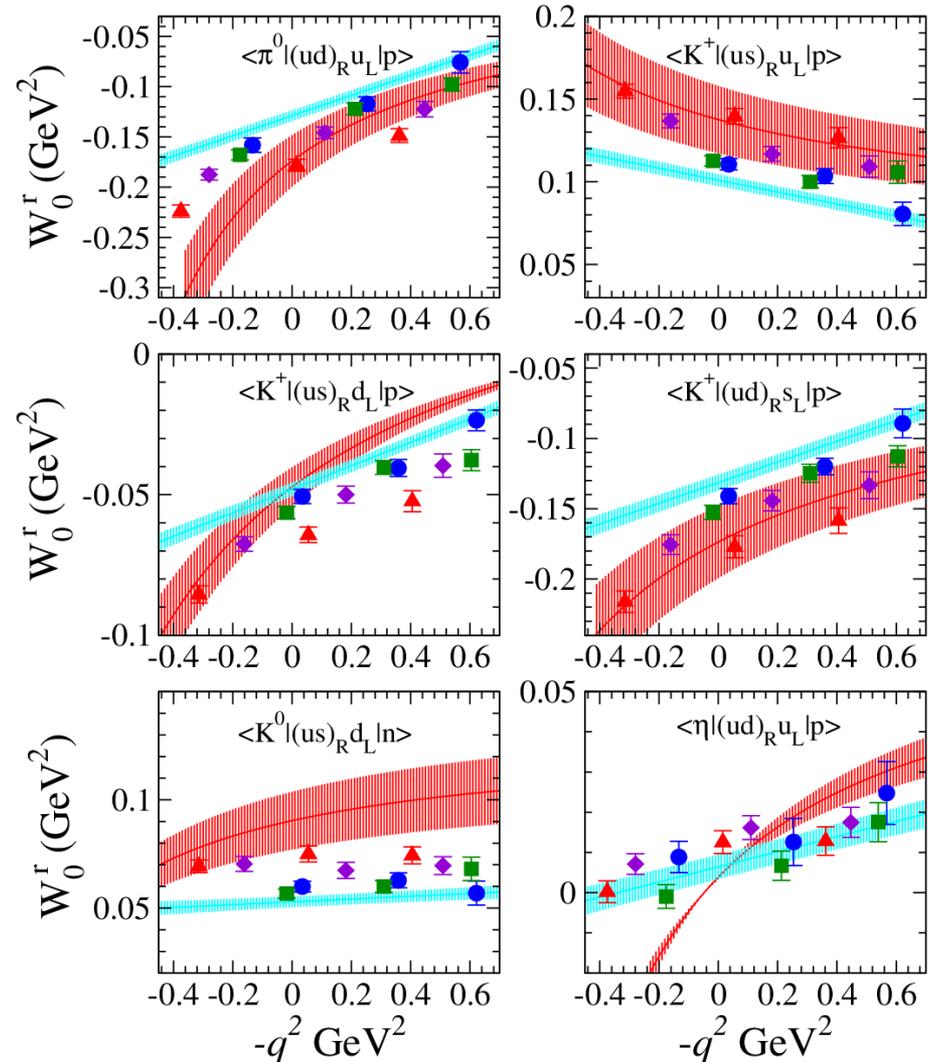


Statistical precision is good, but systematic error of finite size is predominant.
 \Rightarrow need to use larger volume than $L=3.5\text{fm}$.

3. Updated results

Behavior on and off physical kinematics

- **Straight line:** linear extrapolation function in physical pion
- **Red line:** BChPT using lattice value of α_p .
- Lattice result is not much different from BChPT lines, roughly 10 -- 20%.



3. Updated results

W_0 in physical point

Matrix element	W_0^r GeV ²	fit	artifact	Δ_Z	Δ_a
$-\langle \pi^0 (ud)_{RuL} p \rangle$	0.124(6)(14)	0.008	0.006	0.010	0.001
$\langle \pi^0 (ud)_{LuL} p \rangle$	0.127(7)(19)	0.016	0.006	0.010	0.001
$\langle K^0 (us)_{RuL} p \rangle$	0.099(4)(13)	0.010	0.005	0.008	0.001
$\langle K^0 (us)_{LuL} p \rangle$	0.057(3)(6)	0.003	0.003	0.004	0.000
$-\langle K^+ (us)_{RdL} p \rangle$	0.046(2)(5)	0.003	0.002	0.004	0.000
$\langle K^+ (us)_{LdL} p \rangle$	0.038(2)(8)	0.008	0.002	0.003	0.000
$-\langle K^+ (ud)_{RsL} p \rangle$	0.128(6)(14)	0.009	0.006	0.010	0.001
$\langle K^+ (ud)_{LsL} p \rangle$	0.136(7)(21)	0.018	0.007	0.010	0.001
$-\langle K^+ (ds)_{RuL} p \rangle$	0.052(2)(7)	0.006	0.003	0.004	0.000
$-\langle K^+ (ds)_{LuL} p \rangle$	0.097(4)(13)	0.010	0.005	0.007	0.001
$\langle \eta (ud)_{RuL} p \rangle$	0.006(3)(5)	0.005	0.000	0.000	0.000
$\langle \eta (ud)_{LuL} p \rangle$	0.109(6)(14)	0.011	0.005	0.008	0.001

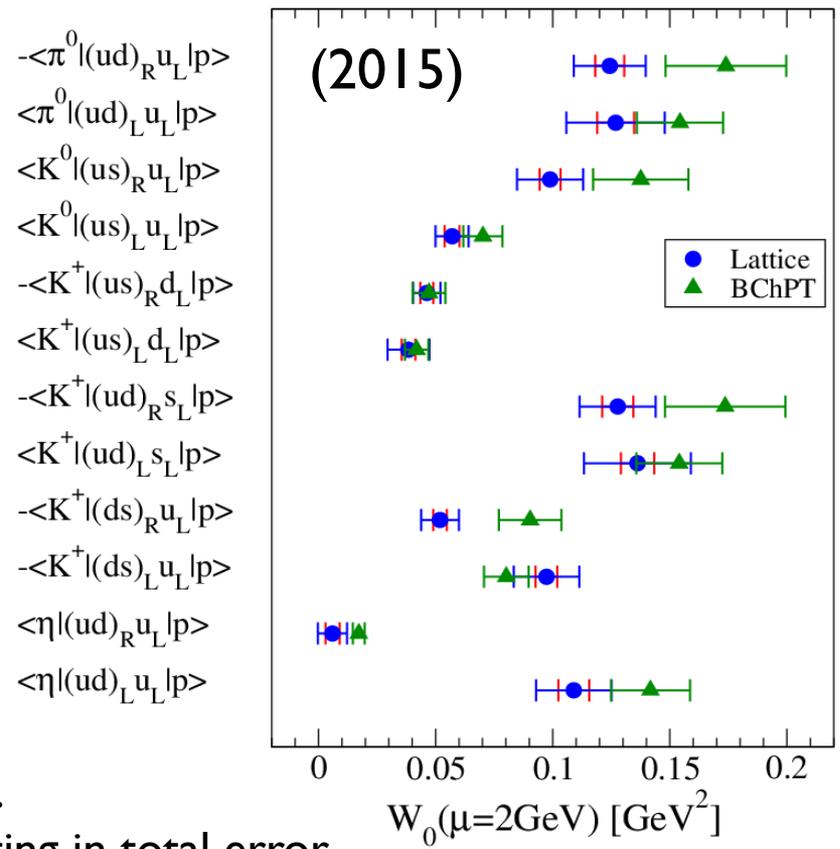
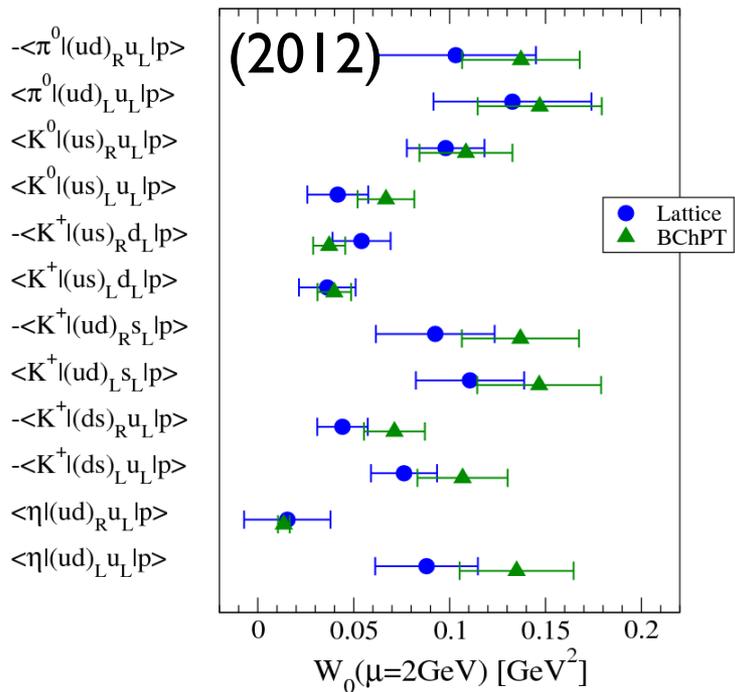
Extrapolation error,
finite size correction
is rather large.

Truncation error of
renormalization
constant.



3. Updated results

Comparison with BChPT



- Precision of W_0 is much improved.
- Systematic error are now dominating in total error.
- Decay width ($\text{p} \rightarrow \pi$) : factor 1.3 difference,
 ⇒ impact to p lifetime is about factor 2.

3. Updated results

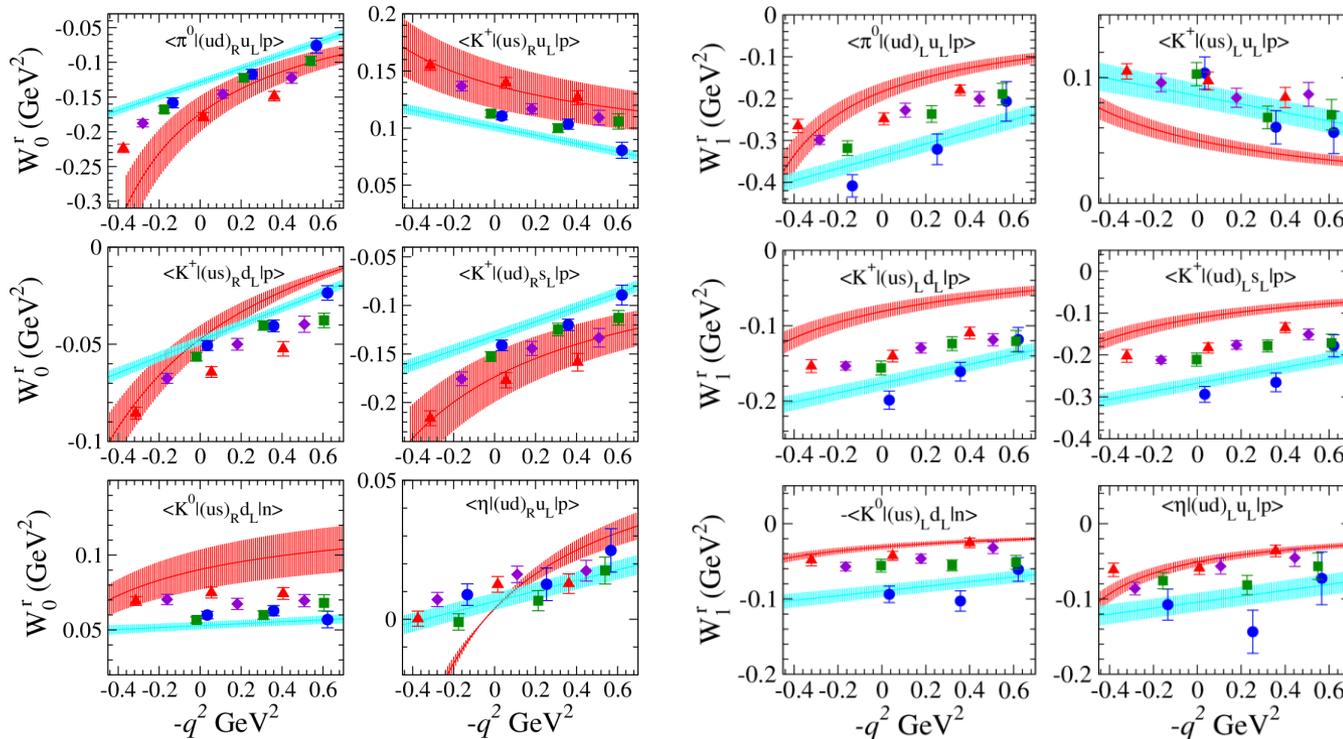
Matrix elements of $(p, DM) \rightarrow (\text{meson}, DM)$

- DM model of proton annihilation with B number DM (Ψ, Φ) scattering.

$$\mathcal{L}_{\text{eff}} = M^{-2} u_R u_R d_R \Psi_R \Phi + h.c.$$

- Matrix element involves **two relevant form factor W_0 and W_1**

$$p \rightarrow (\text{meson}, DM) \quad \langle \pi^0(\vec{p}) | (ud)_{\Gamma}(u)_{\Gamma'} | p(\vec{k}, s) \rangle_{\text{IND}} = P_{\Gamma'} \left[W_0^{\Gamma\Gamma'}(q^2) + \frac{m_{\Phi, \Psi}}{m_p} W_1^{\Gamma\Gamma'}(q^2) \right] u_p(\vec{k}, s)$$



Lattice W_1 is largely distinct from BChPT.

Large uncertainty may appear into IND model estimate.

5. Summary

Summary

- ▶ Improved the precision of matrix element for p decay.
- ▶ Limitation of the BChPT became apparent.
- ▶ It may increase p lifetime to factor 2--3.
- ▶ Lattice QCD allows to provide more reliable estimate in other kinematics, e.g. induced DM model
- ▶ Approaching to physical pion is more important for p decay amplitude.
Martin and Stavenga, PRD85(2012)
 - ▶ There may appear significant effect of pion mass dependence.
An order of magnitude may be different.
 - ▶ Computation close to physical pion (~ 170 MeV) is underway.

Backup

BV effective operators at low-energy

▶ 4-fermi BV operators in the SM

$$\begin{aligned}
 \mathcal{O}_{abcd}^1 &= (D_a^i, U_b^j)_R (q_c^k, l_d^\beta)_L \varepsilon^{ijk} \varepsilon^{\alpha\beta}, & : (\mathbf{q}, \mathbf{q})_R (\mathbf{q}, \mathbf{l})_L \\
 \mathcal{O}_{abcd}^2 &= (q_a^{i\alpha}, q_b^{j\beta})_L (U_c^k, l_d)_R \varepsilon^{ijk} \varepsilon^{\alpha\beta}, & : (\mathbf{q}, \mathbf{q})_L (\mathbf{q}, \mathbf{l})_R \\
 \tilde{\mathcal{O}}_{abcd}^4 &= (q_a^{i\alpha}, q_b^{j\beta})_L (q_c^k, l_d^\delta)_L \varepsilon^{ijk} \varepsilon^{\alpha\delta} \varepsilon^{\beta\gamma}, & : (\mathbf{q}, \mathbf{q})_L (\mathbf{q}, \mathbf{l})_L \\
 \mathcal{O}_{abcd}^5 &= (D_a^i, U_b^j)_R (U_c^k, l_d)_R \varepsilon^{ijk} & : (\mathbf{q}, \mathbf{q})_R (\mathbf{q}, \mathbf{l})_R
 \end{aligned}$$

a, b, c, d : generation,
 $\alpha, \beta, \gamma, \delta$: SU(2) indices,
 i, j, k : color indices

Weinberg, PRL43, 1566(1979), Wilczek and Zee, PRL43, 1571(1979)

➤ 3-quark operator

$$\mathcal{O}_{abc}^{\Gamma\Gamma'} = (q_a q_b)_\Gamma q_{c\Gamma'} = (q_a^{Ti} C P_\Gamma q_b^j) P_{\Gamma'} q_c^k \varepsilon^{ijk}$$

➤ Reduced number of matrix element with Parity and flavor SU(2) symmetry

$$\langle PS; \vec{p} | \mathcal{O}^{LL} | N; \vec{k}, s \rangle = \gamma_4 \langle PS; -\vec{p} | \mathcal{O}^{RR} | N; -\vec{k}, s \rangle,$$

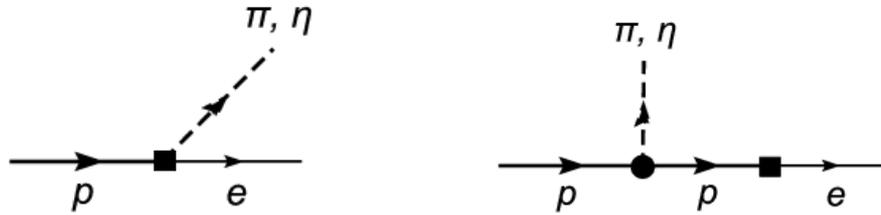
$$\langle PS; \vec{p} | \mathcal{O}^{LR} | N; \vec{k}, s \rangle = \gamma_4 \langle PS; -\vec{p} | \mathcal{O}^{RL} | N; -\vec{k}, s \rangle$$

$$\langle \pi^+ | \mathcal{O}_{udu}^{L/R} | p \rangle = \sqrt{2} \langle \pi^0 | \mathcal{O}_{udu}^{L/R} | p \rangle$$

⇒ Total matrix element is 12.

Matrix element in BChPT

▶ Leading order interaction in $p \rightarrow \pi, K$ transition



Claudson, et al. NPB195(1982),
JLQCD, PRD62(2000)

$$\begin{aligned} \langle \pi^0 | (ud_R)u_L | p \rangle &= \alpha_p P_L u_p \left[1 \frac{\sqrt{2}f_\pi}{\sqrt{2}f_\pi} \frac{D+F}{-q^2 - m_N^2} - \frac{4b_1}{\sqrt{2}f_\pi} \frac{m_u m_N}{-q^2 - m_N^2} \right] \\ &- \alpha_p P_L \frac{i\hat{q}}{m_N} u_p \left[\frac{D+F}{\sqrt{2}f_\pi} \frac{2m_N^2}{-q^2 - m_N^2} + \frac{4b_1}{\sqrt{2}f_\pi} \frac{m_u m_N}{-q^2 - m_N^2} \right] \end{aligned}$$

α_p : LECs

$$D+F = g_A^{(np)} = 1.27$$

$$D-F = g_A^{(n\Sigma)} = 0.33$$

$$\begin{aligned} \langle K^+ | (us_R)d_L | p \rangle &= \alpha_p P_L u_N \left[-\frac{D-F}{2f_\pi} \frac{-q^2 + m_N m_\Sigma}{-q^2 - m_\Sigma^2} - \frac{D+3F}{6f_\pi} \frac{-q^2 + m_N m_\Lambda}{-q^2 - m_\Lambda} \right. \\ &- \left. \frac{b_2}{f_\pi} \frac{(m_u + m_s)m_\Sigma}{-q^2 - m_\Sigma^2} + \frac{b_2 - 2b_1}{3f_\pi} \frac{(m_u + m_s)m_\Lambda}{-q^2 - m_\Lambda^2} \right] \\ &- \alpha_p P_L \frac{i\hat{q}}{m_N} \left[\frac{D-F}{2f_\pi} \frac{m_N(m_N + m_\Sigma)}{-q^2 - m_\Sigma^2} + \frac{D+3F}{6f_\pi} \frac{m_N(m_N + m_\Lambda)}{-q^2 - m_\Lambda^2} \right. \\ &+ \left. \frac{b_2}{f_\pi} \frac{m_N(m_u + m_s)}{-q^2 - m_\Sigma^2} - \frac{b_2 - 2b_1}{3f_\pi} \frac{m_N(m_u + m_s)}{-q^2 - m_\Lambda^2} \right] \end{aligned}$$

Renormalization

- ▶ Renormalization for (qqq) operator (also other kind of high D op.) is also given from matching lattice scheme with MSbar scheme.

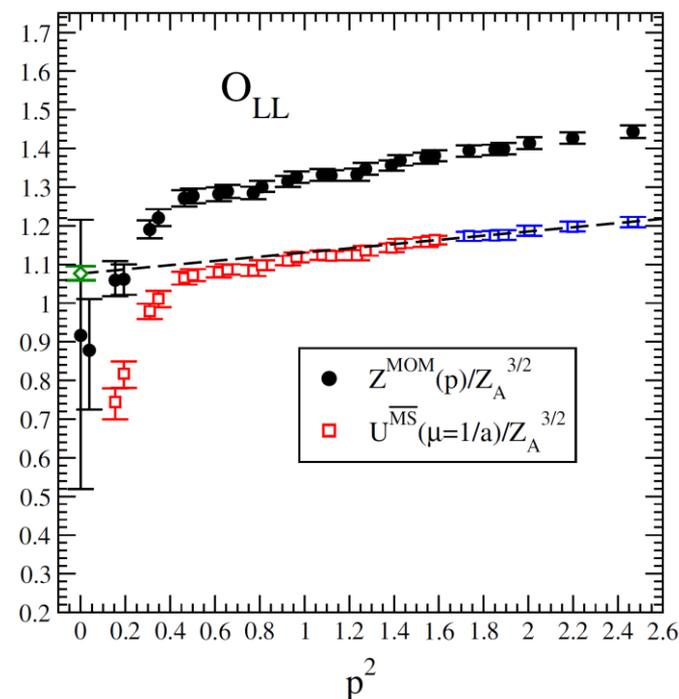
See Buchoff's talk

- ▶ RI-(s)MOM is easy way to evaluate on present gauge configurations.

- Non-perturbatively compute three-quark vertex function amputated by quark propagator in Landau gauge fixing.
- Setting window of p^2 where is small $O((ap)^2)$ at $p \gg \Lambda$, and fitting with function $A + B (ap)^2$
- One-loop matching

$$U^{\overline{MS}}(\mu) = U^{\overline{MS}}(\mu; p) \underbrace{\frac{Z^{\overline{MS}}(p)}{Z^{MOM}(p)}}_{\text{one loop perturbation}} \underbrace{Z_{ND}(p)}_{\text{Lattice}}$$

$$U^{\overline{MS}}(\mu; p) = \left[\frac{\alpha_s(\mu)}{\alpha_s(p)} \right]^{\gamma_0/2\beta_0} \left[1 + \left(\frac{\gamma_1}{2\beta_0} - \frac{\beta_1\gamma_0}{2\beta_0^2} \right) \frac{\alpha_s(\mu) - \alpha_s(p)}{4\pi} \right]$$



Induced n decay model

Davoudiasl et al., PRL105(2010),
PRD84(2011), Davoudiasl PRL114(2015)

- ▶ Assume the dark matter (Φ, Ψ) annihilate nucleon, producing energetic meson,

$$\Phi N \rightarrow \bar{\Psi} P(\pi, K, \eta), \quad \Psi N \rightarrow \Phi^\dagger P(\pi, K, \eta)$$

- ▶ Assume DM has B number, total $B_\Phi + B_\Psi = -1$, then its coupling is

$$\mathcal{L}_{\text{eff}} = M^{-2} u_R u_R d_R \Psi_R \Phi + h.c.$$

- ▶ DM mass is estimated from $\Omega_{\text{DM}}/\Omega_{\text{N}} \simeq 5$, $m_{\Phi, \Psi} = 1.7 - 2.9$ GeV.
- ▶ Proton can decay into meson and DM (induced nucleon decay)

- Proton decay cross section is evaluated as a function of DM mass based on chiral Lagrangian,

$$\mathcal{L}_{\text{IND}} = \beta_p \text{Tr}[c \xi^\dagger (B_R \Psi_R) \Phi \xi]$$

In the IND model, the QCD matrix element is same as ordinal p decay.

