# Standard-model prediction for direct CP-violation in kaon decays

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# Introduction

# Motivation for studying K→ππ Decays

- Direct CPV first observed in late 90s at CERN and Fermilab in $K_0 \to \pi\pi$:

$$\eta_{00} = \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)}, \qquad \eta_{+-} = \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)}.$$

$$\mathrm{Re}(\epsilon'/\epsilon) \approx \frac{1}{6}\left(1 - \left|\frac{\eta_{00}}{\eta_{\pm}}\right|^2\right) = 16.6(2.3) \times 10^{-4} \qquad \text{(experiment)}$$

measure of direct CPV

measure of indirect CPV

- In terms of isospin states:  ΔI=3/2 decay to I=2 final state, amplitude $A_2$
  ΔI=1/2 decay to I=0 final state, amplitude $A_0$

$$\omega = \mathrm{Re}A_2/\mathrm{Re}A_0$$

$$A(K^0 \to \pi^+\pi^-) = \sqrt{\frac{2}{3}}A_0 e^{i\delta_0} + \sqrt{\frac{1}{3}}A_2 e^{i\delta_2}\ ,$$
$$A(K^0 \to \pi^0\pi^0) = \sqrt{\frac{2}{3}}A_0 e^{i\delta_0} - 2\sqrt{\frac{1}{3}}A_2 e^{i\delta_2}\ .$$

$$\Longrightarrow \quad \epsilon' = \frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}}\left(\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right)$$

($\delta_I$ are strong scattering phase shifts.)

- Small size of ε' makes it particularly sensitive to new direct-CPV introduced by most BSM models.

# Overview of calculation

- Low-energy QCD interactions play an important role in kaon decays.
- Lattice QCD only *ab initio*, systematically improvable technique.
- At energy scales $\mu \ll M_{W,}$ $K \to \pi\pi$ decays use weak EFT:

10 effective four-quark operators

$$H_W^{\Delta S=1} = \frac{G_F}{\sqrt{2}} V_{ud}^* V_{us} \sum_{j=1}^{10} [z_j(\mu) + \tau y_j(\mu)] Q_j$$

perturbative Wilson coeffs.

$$\tau = -\frac{V_{ts}^* V_{td}}{V_{us}^* V_{ud}} = 0.0014606 + 0.00060408i$$

Imaginary part solely responsible for CPV
(everything else is pure-real)

LL finite-volume correction

renormalization
matrix (mixing)

$$A_{2/0} = F \frac{G_F}{\sqrt{2}} V_{ud} V_{us} \sum_{i=1}^{10} \sum_{j=1}^{7} \left[ \Big( z_i(\mu) + \tau y_i(\mu) \Big) Z_{ij}^{\mathrm{lat} \to \overline{\mathrm{MS}}} M_j^{\frac{3}{2}/\frac{1}{2},\mathrm{lat}} \right],$$

$M_j = \langle (\pi\pi)_I | Q_j | K \rangle$ (lattice)

- Operators must be renormalized into same scheme as Wilson coeffs: Use RI-(S)MOM NPR and perturbatively match to MSbar at high scale.

# Lattice Determination of $A_2$

[Phys.Rev. D91 (2015) 7, 074502]

- Separate lattice calculations for $A_2$ and $A_0$.
- RBC & UKQCD have been computing $A_2$ for a number of years.
- Most recently with 2+1f physical quark masses, physical kinematics and in the continuum limit.
- ~3% statistical error!
- 15% sys. error completely dominated by perturbative truncation of RI-SMOM → MSbar matching.
- Can be addressed straightforwardly by step-scaling to a higher μ or computing higher-order PT contributions.

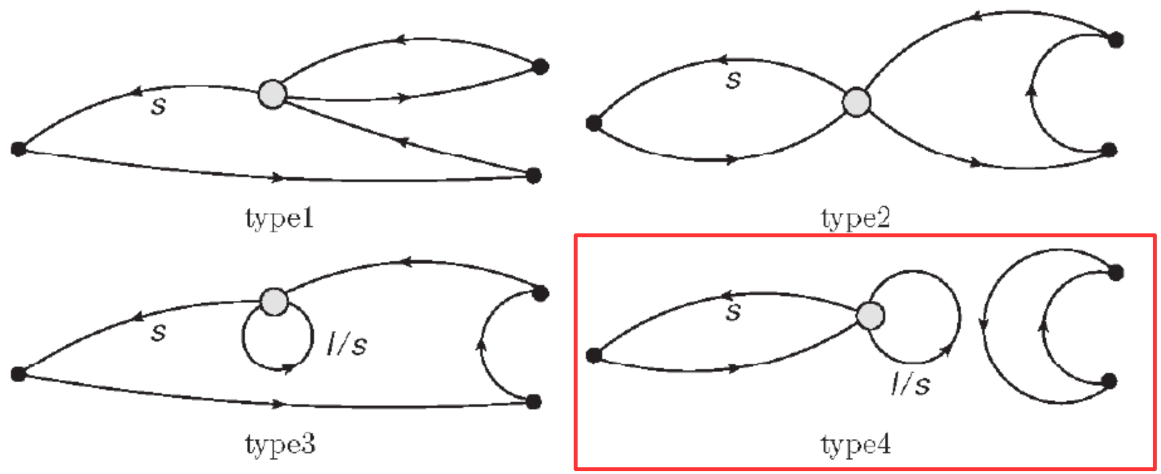
- Lattice calculation of $A_0$ considerably more challenging – topic for most of remainder of this talk.

# Determination of $A_0$

arXiv:1505.07863 [hep-lat]

# Matrix element calculation

- $A_0$ obtained via neutral kaon decays $K^0 \to \pi^+\pi^-$ and $K^0 \to \pi^0\pi^0$
- 4 classes of diagram:




disconnected

- Type 4 disconn. diagrams dominate noise.
- Use Trinity-style all-to-all (A2A) propagators:
  - 900 exact low-eigenmodes computed using Lanczos algorithm
  - Stochastic high-modes with full dilution of indices
- Allows us to perform all spatial and temporal translations to boost statistics.

# Physical Kinematics

- Important to calculate with physical (energy-conserving) kinematics.
- With physical masses: $2 \times m_\pi \sim 270 \text{ MeV} \ll m_K \sim 500 \text{ MeV}$
- Requires moving pions!
- This is excited state of the ππ-system. Possibilities:
  - try to perform multi-state fits to very noisy data (esp. $A_0$ where there are disconn. diagrams)
  - modify boundary conditions to remove the ground-state
- Second approach optimal but technically challenging: must conserve isospin and apply momentum to both charged and neutral pions.
- Solution: Use G-parity BCs:

$$\hat{G} = \hat{C} e^{i\pi \hat{I}_y} \quad : \quad \hat{G}|\pi^\pm\rangle = -|\pi^\pm\rangle \qquad \hat{G}|\pi^0\rangle = -|\pi^0\rangle$$

- As a boundary condition: (i=+, -, 0)

$$\pi^i(x+L) = \hat{G}\pi^i(x) = -\pi^i(x) \longrightarrow |p| \in (\pi/L,\, 3\pi/L,\, 5\pi/L \ldots)$$

(moving ground state)

# Ensemble and state energies

- $32^3$x64 Mobius DWF ensemble with IDSDR gauge action at $\beta$=1.75. Coarse lattice spacing ($a^{-1}$=1.378(7) GeV) but large, (4.6 fm)$^3$ box.
- G-parity BCs in 3 directions.
- Performed 216 independent measurements (4 MDTU sep.).
- Utilized:
  - USQCD 512-node BG/Q machine at BNL
  - DOE "Mira" BG/Q machines at ANL
  - STFC BG/Q "DiRAC" machines at Edinburgh, UK.

- Obtain close matching of kaon and $\pi\pi$ energies:

$$m_K=490.6(2.4)\ \text{MeV}$$

$$E_{\pi\pi}(I=0) = 498(11)\ \text{MeV}$$

$$E_{\pi\pi}(I=2) = 573.0(2.9)\ \text{MeV}$$

$$E_{\pi}=274.6(1.4)\ \text{MeV} \quad (m_{\pi} = 143.1(2.0)\ \text{MeV})$$

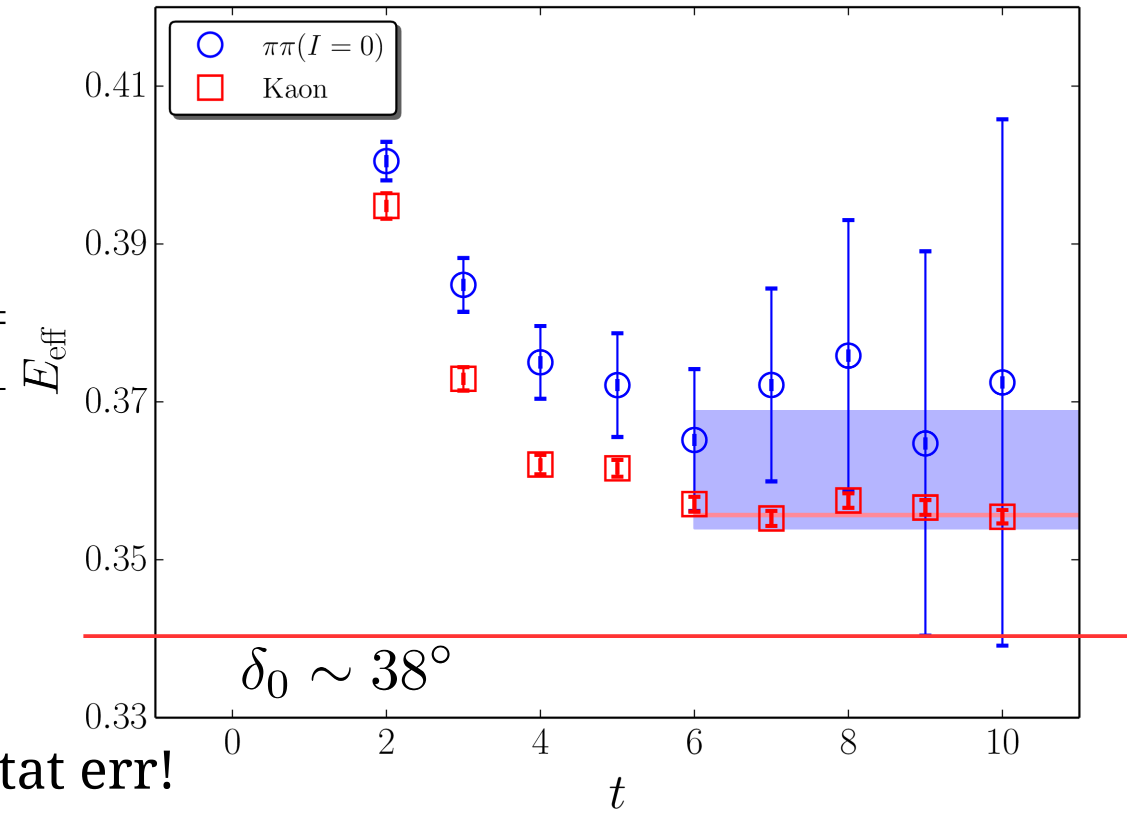
# I=0 ππ energy

- Signal/noise deteriorates quickly due to vacuum contrib.
- Difficult to determine plateau start. Performed both 1- and 2-state fits.

| $t_{\min}$ | $E_{\pi\pi}$ | $E_{\text{exc}}$ | $\chi^2/\text{dof}$ |
|---|---|---|---|
| 2 | 0.363(9) | 1.04(17) | 1.7(7) |
| 3 | 0.367(11) | 1.27(73) | 1.8(8) |
| 4 | 0.364(12) | 0.86(39) | 1.9(8) |

| $t_{\min}$ | $E_{\pi\pi}$ | $\chi^2/\text{dof}$ |
|---|---|---|
| 5 | 0.375(6) | 2.2(9) |
| 6 | 0.361(7) | 1.6(7) |
| 7 | 0.380(11) | 0.9(7) |

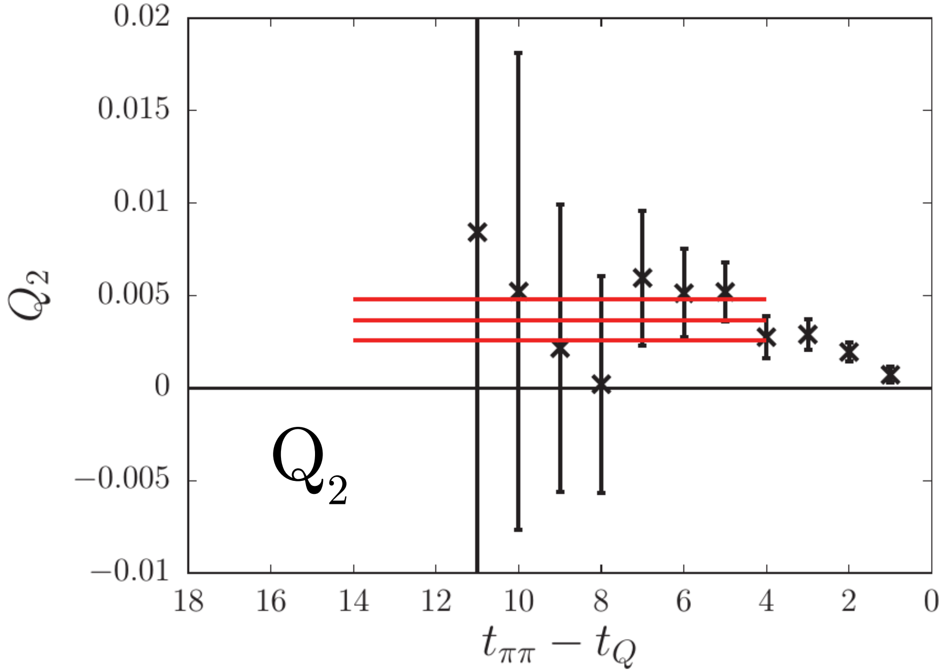
2% stat err!



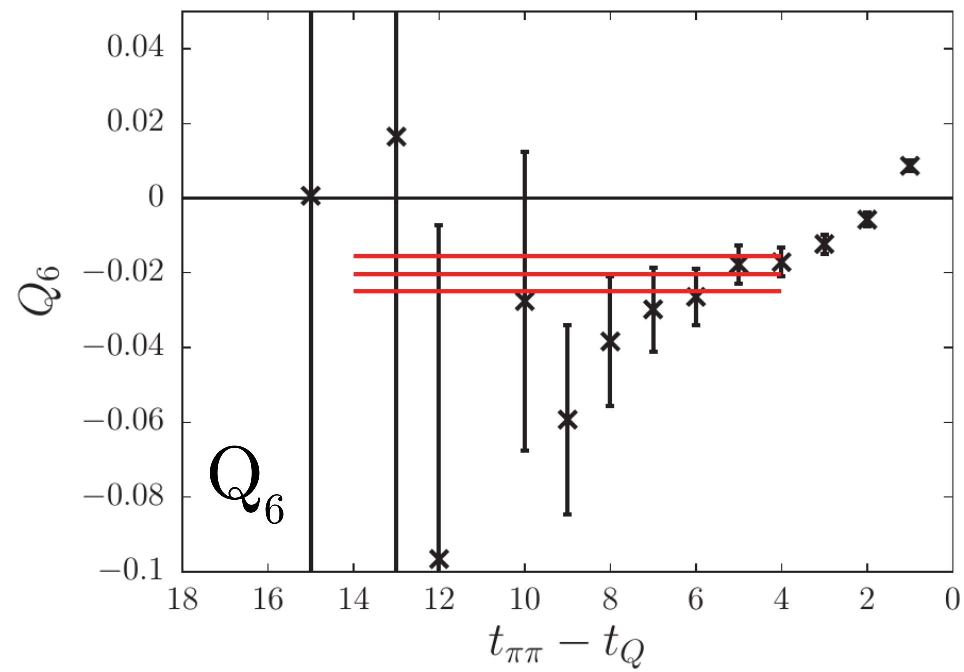

- Our phase shift $\delta_0 = 23.8(4.9)(1.2)^\circ$ ~2.7σ below conventional Roy equation determination of $\delta_0 = 38.0(1.3)^\circ$ [G.Colangelo, private communication]
- Possibly low statistics concealing delayed plateau start?
- Using 38° → ~3% change in $A_0$ : much smaller than other errs.
- For consistency we choose to use our lattice value.

# Matrix element fits

[Dominant contribution to Re($A_0$)]




[Dominant contribution to Im($A_0$)]




- No statistically resolvable excited state dependence with $t_{min}(\pi \to Q) > 3$.
- Signal quickly decays: +40% stat. error between $t_{min}(\pi \to Q)=4$ and 5!
- Use $t_{min}(\pi \to Q) = 4$ .
- Estimate 5% excited state systematic by comparing ππ(I=0) amplitude computed using one- and two-state fits.

# Systematic errors

- Errors for each separate operator matrix element:

| Description | Error | Description | Error |
|---|---|---|---|
| Finite lattice spacing | 12% | Finite volume | 7% |
| Wilson coefficients | 12% | Excited states | $\leq$ 5% |
| Parametric errors | 5% | Operator renormalization | 15% |
| Unphysical kinematics | $\leq$ 3% | Lellouch-Lüscher factor | 11% |
| Total (added in quadrature) | | | 27% |

- 15% ren. error due to one-loop PT truncation and low, 1.53 GeV matching scale. (Est. by comparing two different RI/SMOM intermediate schemes.)
- 12% Wilson coefficient error large for same reason. (Est. from difference between LO and NLO.)
- 12% discretization error due to coarse lattice spacing. (Est. from $A_2$ calculations.)

# Results for $A_0$

$$\mathrm{Re}(A_0) = 4.66(1.00)_{\mathrm{stat}}(1.21)_{\mathrm{sys}} \times 10^{-7}\ \mathrm{GeV} \qquad \text{(This work)}$$

$$\mathrm{Re}(A_0) = 3.3201(18) \times 10^{-7}\ \mathrm{GeV} \qquad \text{(Experiment)}$$

- Good agreement for Re($A_0$) serves as test for method.
- Expt far more precise. Physics dominated by tree-level current-current diagrams hence unlikely to receive large BSM contributions.
- Use expt. for computing ε'.

$$\mathrm{Im}(A_0) = -1.90(1.23)_{\mathrm{stat}}(1.04)_{\mathrm{sys}} \times 10^{-11}\mathrm{GeV} \qquad \text{(This work)}$$

- ~85% total error on the predicted Im($A_0$) due to strong cancellation between dominant $Q_4$ and $Q_6$ contributions:

$$\begin{aligned}\Delta[\mathrm{Im}(A_0), Q_4] &= \quad 1.82(0.62)(0.32) \times 10^{-11} \\ \Delta[\mathrm{Im}(A_0), Q_6] &= -3.57(0.91)(0.24) \times 10^{-11}\end{aligned}$$

despite only 40% and 25% respective errors for the matrix elements.

# Results for ε' and concluding remarks

# Results for ε'

- Re($A_0$) and Re($A_2$) from expt.
- Lattice values for Im($A_0$), Im($A_2$) and the phase shifts,

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = \mathrm{Re}\left\{\frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}\varepsilon}\left[\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right]\right\}$$

$$\boxed{\begin{array}{ll} = \quad 1.38(5.15)(4.43)\times 10^{-4}, & \text{(this work)} \\ \quad 16.6(2.3)\times 10^{-4} & \text{(experiment)} \end{array}}$$

- Find discrepancy between lattice and experiment at the 2.1σ level.

# Conclusions and Outlook

- First direct computation of $A_0$ with controllable errors performed.
- Measured $\mathrm{Re}(A_0)$ in good agreement with experiment.
- 85% total error on $\mathrm{Im}(A_0)$ despite 25% and 40% errors on dominant $Q_6$ and $Q_4$ contributions resp., due to strong mutual cancellation.
- On final result, stat. error currently dominant.
- Sys. errors dominated by perturbative truncation errors on the renormalization and Wilson coeffs due to low, 1.53 GeV scale.
- Currently computing NPR running to higher energies in order to reduce this systematic.
- Total error on $\mathrm{Re}(\varepsilon'/\varepsilon)$ is ~3x the experimental error, and we observe a $2.1\sigma$ discrepancy. Strong motivation for continued study!
- Hope to achieve O(10%) errors on $\mathrm{Re}(\varepsilon'/\varepsilon)$ on a timescale of ~5 years.
- We hope these results with spur new efforts in the experimental community to reduce the current 15% error on the experimental number.

Thank you!