Calculating Neutron Electric Dipole Moments using Lattice Quantum Chromodynamics

Tanmoy Bhattacharya

Los Alamos National Laboratory

Santa Fe Institute

Oct 8, 2015

Introduction
CP violation and nEDM

CP violation needed in the universe.
Observed baryon asymmetry: $n_B/n_\gamma = 6.1^{+0.3}_{-0.2} \times 10^{-10}$.

WMAP + COBE 2003

Without CP violation, freezeout ratio: $n_B/n_\gamma \approx 10^{-20}$.

Kolb and Turner, *Front. Phys.* **69** (1990) 1.

Either asymmetric initial conditions or baryogenesis!
Sufficiently asymmetric initial conditions kills inflation.

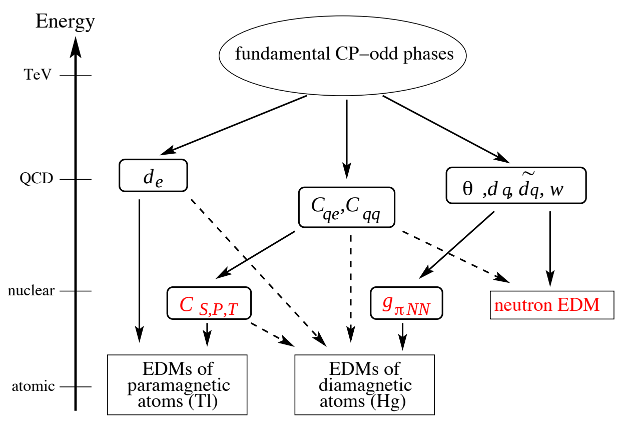
Sakharov Conditions

Sakharov, *Pisma Zh. Eksp. Teor. Fiz.* **5** (1967) 32.

- Baryon Number violation
- C, CP and T violation
- Out of equilibrium evolution

Introduction

Effective Field Theory



Pospelov and Ritz, *Ann. Phys.* **318** (2005) 119.

Introduction
BSM Operators

Standard model CP violation in the weak sector.
Anomalously small strong CP violation from dim 3 and 4.

- Dimension 3 and 4:
  - CP violating mass $\bar{\psi}\gamma_5\psi$.
  - Toplogical charge $G_{\mu\nu}\tilde{G}^{\mu\nu}$.
- Suppressed by $v_{\rm EW}/M^2_{\rm BSM}$:
  - Electric Dipole Moment $\bar{\psi}\Sigma_{\mu\nu}\tilde{F}^{\mu\nu}\psi$.
  - Chromo-electric Dipole Moment $\bar{\psi}\Sigma_{\mu\nu}\tilde{G}^{\mu\nu}\psi$.
- Suppressed by $1/M^2_{\rm BSM}$:
  - Weinberg operator (Gluon chromo-electric dipole moment): $G_{\mu\nu}G_{\lambda\nu}\tilde{G}_{\mu\lambda}$.
  - Various four-fermi operators.

Introduction
Phase choice

Consider the chiral and CP violating parts of the action $\mathcal{L} \supset d_i^\alpha O_i^\alpha$, where i is flavor and α is operator index.
Consider only one chiral symmetric CP violating term: $\Theta G\tilde{G}$
Convert to polar basis

$$d_i \equiv |d_i| e^{i\phi_i} \equiv \frac{\sum_\alpha d_i^\alpha \langle\Omega|\,\mathcal{I}m\, O_i^\alpha|\pi\rangle}{\sum_\alpha \langle\Omega|\,\mathcal{I}m\, O_i^\alpha|\pi\rangle}$$

Then CP violation is proportional to:

$$\bar{d}\bar{\Theta}\,\mathcal{R}e\,\frac{d_i^\alpha}{d_i} - |d_i|\,\mathcal{I}m\,\frac{d_i^\alpha}{d_i} \quad \text{with} \quad \frac{1}{\bar{d}} \equiv \sum_i \frac{1}{d_i} \quad \bar{\Theta} \equiv \Theta - \sum_i \phi_i$$

CP violation depends on $\bar{\Theta}$ and on a *mismatch* of phases between d_i^α and d_i.

Introduction
Lattice QCD

Ultraviolet divergence regulated by the periodicity:

$$\int_{-\infty}^{\infty} dp = \sum_{m=-\infty}^{\infty} \int_{\pi(m-1)/a}^{\pi(m+1)/a} dp \to \int_{-\pi/a}^{\pi/a} dp$$

Infrared controlled by calculating in a finite universe.

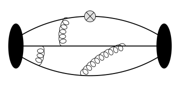
$$\int dp\, f(p) \to \sum_n (\frac{2\pi}{L}) f(\frac{2\pi n}{L} + p_0)$$

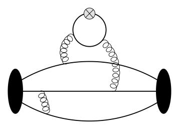
Integration by time average over a stochastic ergodic process.
Real world reached by

$$\lim_{\substack{L\to\infty \\ a\to 0}} .$$

Current calculations $a \sim 0.05$–$0.15\,\mathrm{fm}$ and $L \sim 3$–$5\,\mathrm{fm}$.

Equal-time vacuum matrix elements of Weyl-ordered operators. To extract $\langle n|O|n\rangle$:





$$
\begin{aligned}
&\mathrm{Tr}\, e^{-\beta H}\hat{n}e^{-HT_f}Oe^{-HT_i}\hat{n}^\dagger \\
&= \quad e^{-\beta E_s}\langle s|\hat{n}e^{-HT_f}Oe^{-HT_i}\hat{n}^\dagger|s\rangle \\
&\underset{\beta\to\infty}{\longrightarrow} \quad \langle\Omega|\hat{n}|n_f\rangle e^{-M_fT_f}\langle n_j|O|n_i\rangle e^{-M_iT_i}\langle n_i|\hat{n}^\dagger|\Omega\rangle \\
&\underset{T_i,T_f\to\infty}{\longrightarrow} \quad \langle n|O|n\rangle e^{-M_0(T_i+T_f)}
\end{aligned}
$$

Introduction

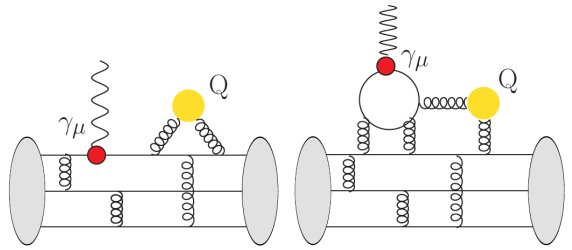
Systematics summary

- *Number of quarks* 2+1
  Isospin breaking beyond current calculations.
  Charm is sometimes included.
- *Quark mass* $M_{\pi,\mathrm{min}} < 200$ MeV.
  May be possible to work at the physical point.
  At least, χPT from $M_\pi < 400$ MeV.
- *Discretization* $a < 0.1$ fm (2/3 points)
  Discretization errors differ in different schemes.
  May be problematic if all points $a > 0.1$ fm.
- *Volume* $M_\pi L > 4$
  At least $M_\pi L > 3$. OK if $\exp(-M_\pi L)$ at 3 masses.
- *Renormalization* Nonpeturbative matching
  At least improved 1-loop perturbation theory.
- *Excited states* $t_{\mathrm{sep,max}} > 1.5$ fm
  At least $t_{\mathrm{sep}} > 1.2$ fm. Extrap from 3 t_{sep} OK.
- *Disconnected diagrams*

Theta term

Dimension 3 and 4 operators

Axial anomaly links $\Theta G\tilde{G}$ and $m\bar{\psi}\gamma_5\psi$.
No connected diagrams.



Typical results: $d_n = -3.8(2)(9) \times 10^{-16}\Theta$ e cm. arXiv:1502.02295 [hep-lat]

Quark EDM
Quark EDM

$$\mathcal{L} \supset -\frac{i}{2} \sum d_q \bar{q} \sigma_{\mu\nu} \gamma_5 q F^{\mu\nu}$$

Note that $\sigma_{\mu\nu}\gamma_5 \propto \epsilon_{\mu\nu\alpha\beta}\sigma^{\alpha\beta}$.

$$d_N = \sum d_q \langle N | \bar{q} \sigma_{\mu\nu} q | N \rangle \equiv d_q g_T^q$$

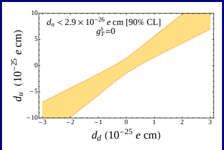
g_T calculated on the lattice using MILC lattices:

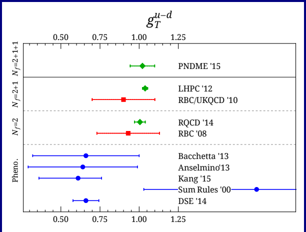
Bhattacharya, Cirigliano, Gupta, Lin, Yoon, **arXiv:1506.04196 [hep-lat]**

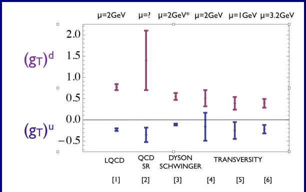
Bhattacharya, Cirigliano, Cohen, Gupta, Joseph, Lin, Yoon **arXiv:1506.06411 [hep-lat]**

$$a \in [0.06, 0.12] \text{ fm}, \quad m_\pi \in [130, 310] \text{ MeV}, \quad m_\pi L \in [3.3, 5.5]$$

Quark EDM
Results







[1] Bhattacharya *et al.* 2015
[2] Pospelov-Ritz 2000
[3] Pitschmann *et al.* 2014
[4] Bacchetta *et al.* 2013
[5] Anselmino *et al.* 2013
[6] Kang *et al.* 2015

Quark CEDM

Renormalization and Mixing

RI-$\tilde{\mathrm{S}}$Mom Conditions:

$$\begin{pmatrix} O \\ N \end{pmatrix}_{\mathrm{ren}} = \begin{pmatrix} Z_O & Z_{ON} \\ 0 & Z_N \end{pmatrix} \begin{pmatrix} O \\ N \end{pmatrix}_{\mathrm{bare}}$$

O: Gauge-invariant operators, does not vanish by equation of motion.

N: Gauge-dependent operators, restricted by BRST, vanish by equation of motion.

Impose conditions on matrix elements of quarks and gluons:

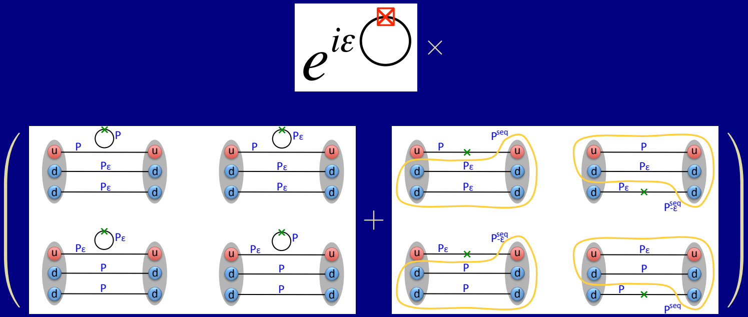
- Use $\overline{\mathrm{MS}}$ quark masses in the expansion.
- Three point functions at $p^2 = p'^2 = q^2 = -\Lambda^2 \ll 0$ (RI-SMOM).
- Four point functions at $p^2 = p'^2 = k^2 = q^2 = s = u = t/2 = -\Lambda^2$.

This choice eliminates most non-1PI contributions.

(See arXiv:1502.07325 [hep-ph]).

Quark CEDM

Schwinger source method

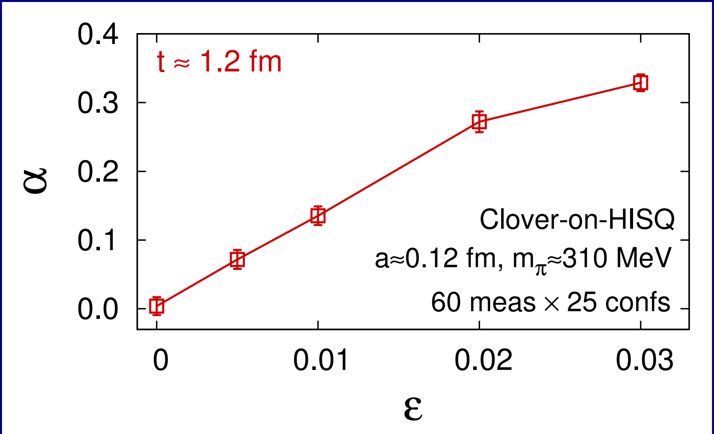


The chromoEDM operator is dimension 5.

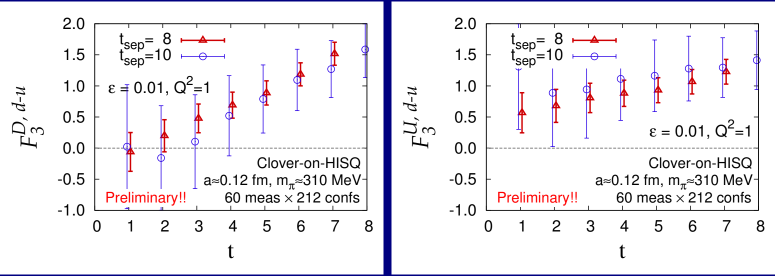
Uncontrolled divergences unless $\epsilon \lesssim 4\pi a\Lambda_{\mathrm{QCD}} \sim 1$.

Need to check linearity.

Quark CEDM
Numerical tests



Propagator $= \exp(-i\alpha\gamma_5)(\not{p}+m)exp(-i\alpha\gamma_5)$; Preliminary; Connected Diagrams Only



Preliminary; Connected Diagrams Only

- Connected F_3 does not get contribution from $\dim \mathcal{O} < 5$.
- Observed F_3 from CQEDM, QEDM, or will vanish on extrapolation.
- Perturbative subtraction of QEDM contribution possible, and determination of proportionality possible.

Conclusions

Summary

- QEDM contributions from u, d, and s quarks under control.
- Methods developed for QCEDM.
- Study of systematics for QCEDM needed.
- Most divergent mixing with $\frac{\alpha_s}{a^2}\bar{\psi}\gamma_5\psi$.
  nEDM due to this same as due to $\frac{\alpha_s}{ma^2} G \cdot \tilde{G}$.
  Current estimates of nEDM due to
  - $\mathrm{CEDM}^{\overline{\mathrm{MS}}} \Rightarrow O(1)$
  - $\frac{\alpha_s}{ma^2}\Theta G \cdot \tilde{G} \Rightarrow \frac{O(0.1)}{5\mathrm{MeV}a^2} O(10^{-3})$e-fm $= O(1)$
    at $a \approx 0.1\mathrm{fm}$.
  Expect O(1–10) cancellation. Important for disconnected diagrams.
- Chiral symmetry does not remove this mixing.