Slow modes in rapidly-expanding quark–gluon plasma

Jasmine Brewer

Based on [1910.00021] and ongoing work
with Weiyao Ke, Bruno Scheihing-Hitschfeld, Li Yan, and Yi Yin

What happens before hydrodynamics?

Near-equilibrium

Microscopic theories

Kinetic theory

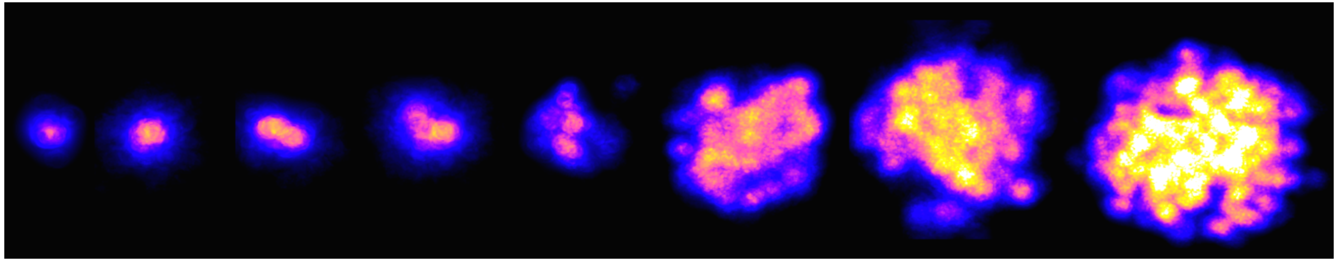
AdS/CFT

…

QGP

Hydrodynamics

$\epsilon(\tau),\ u^\mu(\tau)$



Chun Shen QM'19

Further from equilibrium

Hydrodynamization may be incomplete in small systems

Hydrodynamization near equilibrium

Near-equilibrium

Microscopic theories

Kinetic theory

AdS/CFT

...

QGP

Hydrodynamics

$\epsilon(\tau),\ u^{\mu}(\tau)$

Perturbation in microscopic theory

many degrees of freedom are important

dynamics governed by conservation laws

τ_{Hydro}

τ

Decay of fast "non-hydrodynamic" modes

Slow "hydrodynamic modes"

Hydrodynamics far-from-equilibrium?

Near equilibrium → hydrodynamics → reduction in degrees of freedom

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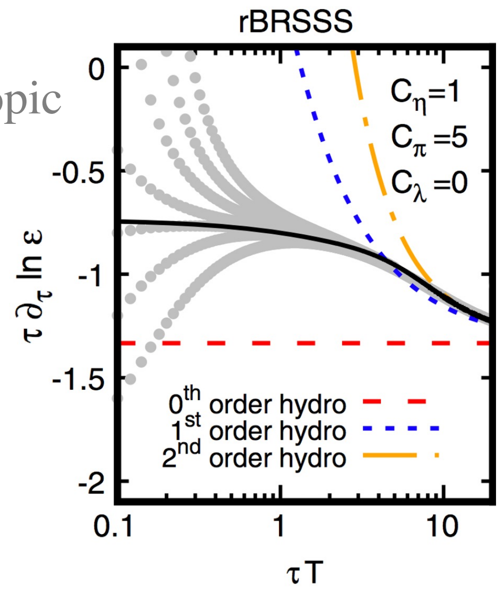
Near equilibrium → hydrodynamics → reduction in degrees of freedom

full microscopic evolution



equilibrium

Romatschke [1704.08699]

Bjorken flow in Israel-Stewart, DNMR, RTA, AdS/CFT [1503.07514, 1609.04803, 1704.08699, 1709.06644, 1712.03865, 1907.08101]

Gubser flow in aHydro, Israel-Stewart, DNMR [1711.01745]

Hydrodynamics far-from-equilibrium?

Near equilibrium → hydrodynamics → reduction in degrees of freedom

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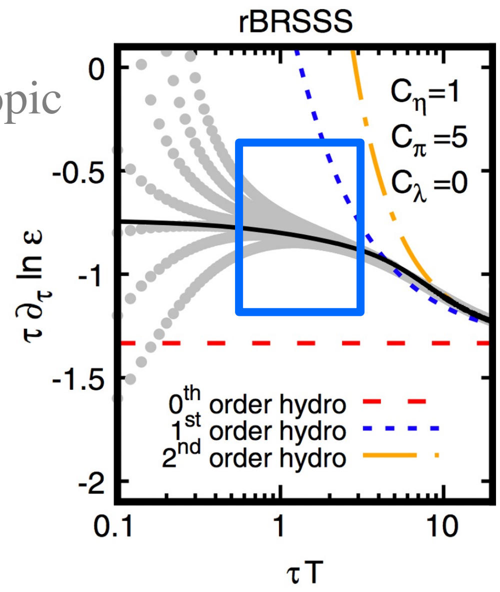


equilibrium

Romatschke [1704.08699]

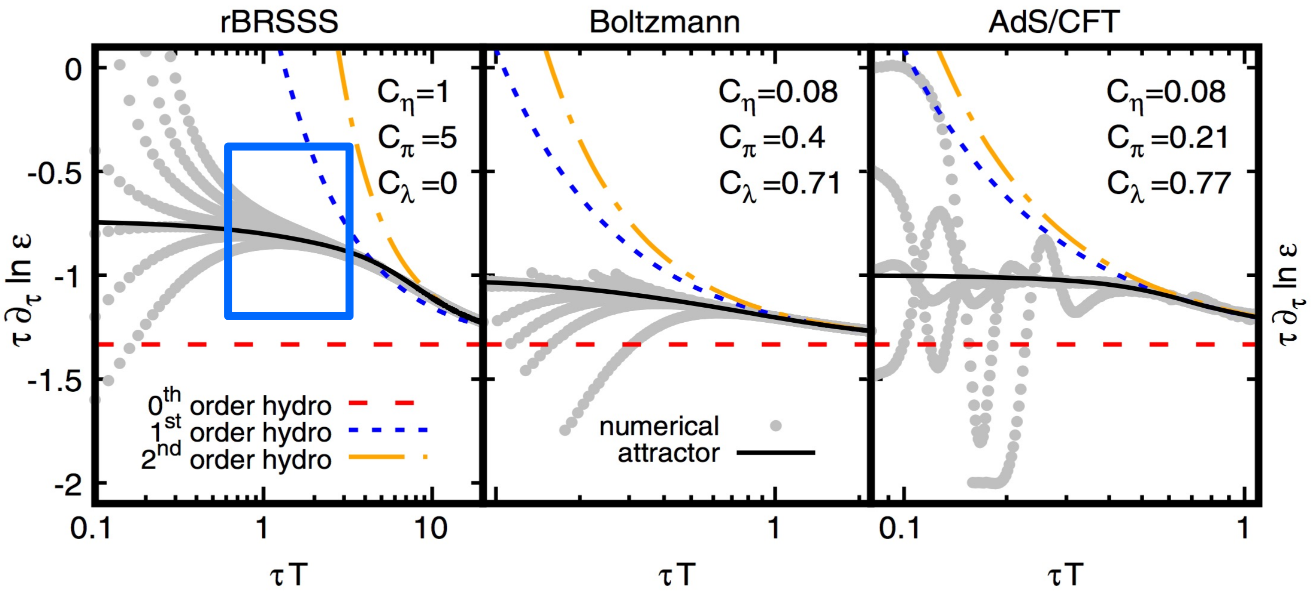
Universality far from equilibrium

Bjorken flow in Israel-Stewart, DNMR, RTA, AdS/CFT [1503.07514, 1609.04803, 1704.08699, 1709.06644, 1712.03865, 1907.08101]

Gubser flow in aHydro, Israel-Stewart, DNMR [1711.01745]

Hydrodynamics far-from-equilibrium?

Observed in several microscopic theories…



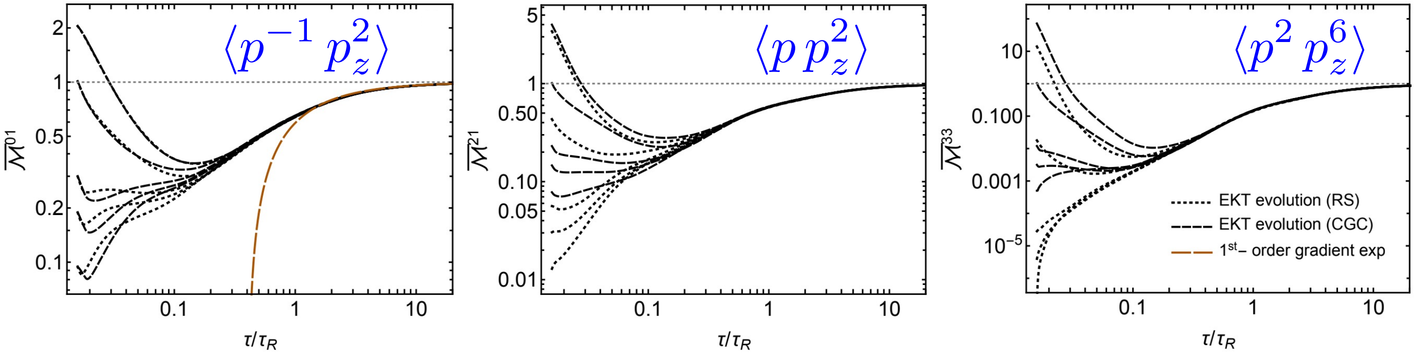
Romatschke [1704.08699]

Relevance for phenomenology
Giacalone, Mazeliauskas, Schlichting [1908.02866]

Hydrodynamics far-from-equilibrium?

And for higher moments of the distribution function… [1809.01200, 2004.05195]

QCD kinetic theory



Almaalol, Kurkela, Strickland [2004.05195]

Hydrodynamics far-from-equilibrium?

And for higher moments of the distribution function… [1809.01200, 2004.05195]

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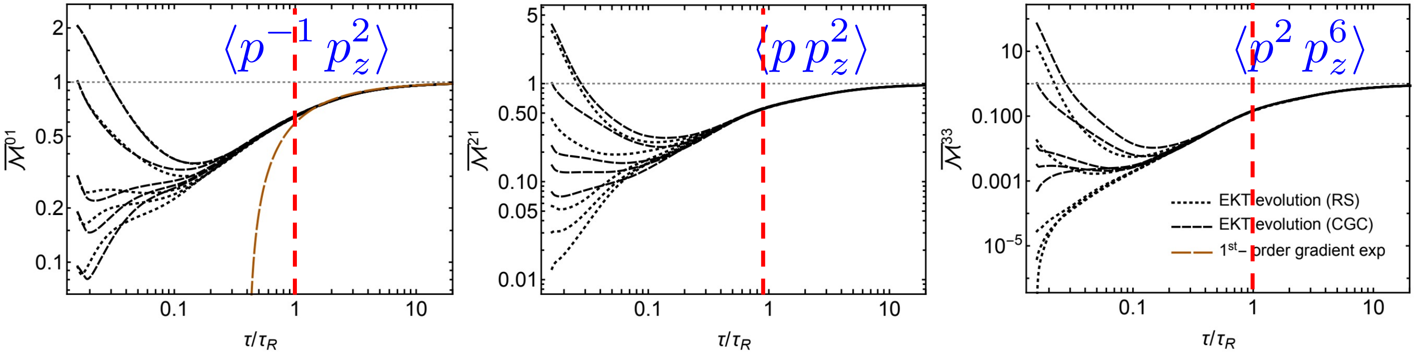


Almaalol, Kurkela, Strickland [2004.05195]

Apparent reduction in degrees of freedom before relaxation time

What is the origin of reduced degrees of freedom before collisions?

What is the origin of reduced degrees of freedom before collisions?

Hydrodynamics is one way to cause a reduction in the effective degrees of freedom

Emerging picture: rapid expansion can also cause a reduction in degrees of freedom

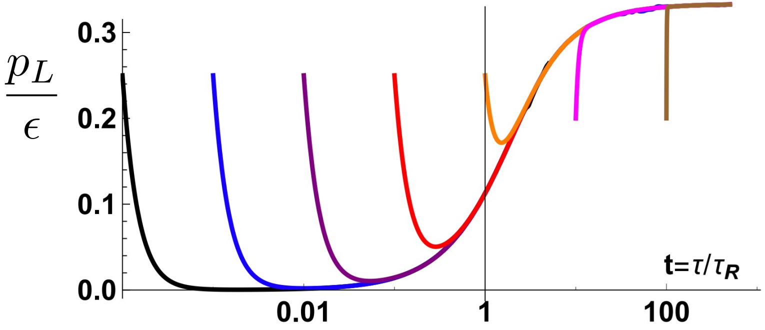
Kurkela, van der Schee, Wiedemann, Wu [1907.08101]
JB, Yan, Yin [1910.00021]

Motivates understanding the attractor in terms of far-from-equilibrium slow degrees of freedom

Berges, Mazeliauskas [1810.10554]
JB, Yan, Yin [1910.00021]
JB, Ke, Yan, Yi (in preparation)
JB, Scheihing-Hitschfeld, Yin (in preparation)

Hint: different physical origin of early- and late-time attractors

relaxation time approximation



(qualitatively similar results for Israel-Stewart)

τ_R: relaxation timescale

Kurkela, van der Schee, Wiedemann, Wu [1907.08101]

Reduction in degrees of freedom driven by…

Rapid expansion without collisions

Collisions

hydrodynamic modes

Suggests "slow mode" describing rapid expansion without collisions

Hydrodynamization in kinetic theory

Boost-invariant longitudinal expansion:

$$\partial_\tau f - \underbrace{\frac{p_z}{\tau}\partial_{p_z} f}_{\text{longitudinal expansion}} = \underbrace{-C[f]}_{\text{collisions}}$$

(homogenous in transverse plane)

Can connect to hydrodynamics by considering the distribution contributing to the stress tensor

$$F = \int_p p\, f$$

(e.g. Kurkela, Wiedemann, Wu [1905.05139])

Evolution of F can be described by effective “Hamiltonian”

$$\int_p p\left(\underbrace{\partial_\tau f}_{\partial_\tau F} - \underbrace{\frac{p_z}{\tau}\partial_{p_z} f}_{-\frac{1}{\tau}(\dots)F} = \underbrace{-C[f]}_{\substack{\text{(sometimes)}\\ (\dots)F}}\right)$$

Evolution of F $\longleftrightarrow$ $\partial_y \psi = -\mathcal{H}(y)\psi$

Eigenstates give effective degrees of freedom

Slow modes: ground states

$$y = \log\left(\frac{\tau}{\tau_I}\right)$$

JB, Yan, Yin [1910.00021]

Evolution of F can be described by effective "Hamiltonian"

$$\int_p p\left(\underbrace{\partial_\tau f}_{\partial_\tau F} - \underbrace{\frac{p_z}{\tau}\partial_{p_z} f}_{-\frac{1}{\tau}(\dots)F} = -\underbrace{C[f]}_{\substack{\text{(sometimes)}\\ (\dots)F}}\right)$$

$$C[f] = \frac{1}{\tau_R}(f - f_{\mathrm{eq}})$$

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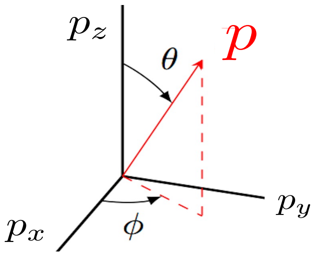
Bjorken expansion $\quad F = F(\theta;\tau)$

JB, Yan, Yin [1910.00021]



Transverse momentum anisotropy $\quad F = F(\theta,\phi;\tau)$

JB, Ke, Yan, Yin (in progress)

Fokker-Planck collision kernel: JB, Scheihing-Hitschfeld, Yin (in progress)

Truncate $\mathcal{H}$ using moment expansion

Bjorken expansion

$$F\left(\cos\theta;\tau\right) = \epsilon(\tau) + \sum_{n=1} \frac{4n+1}{2}\mathcal{L}_n(\tau)P_{2n}(\cos\theta) \quad \longleftrightarrow \quad \psi = (\epsilon, \mathcal{L}_1, \mathcal{L}_2, ...)$$

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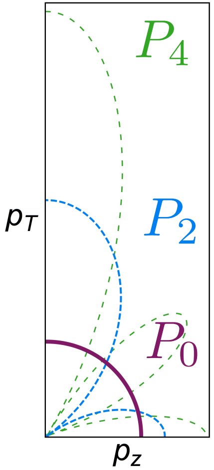


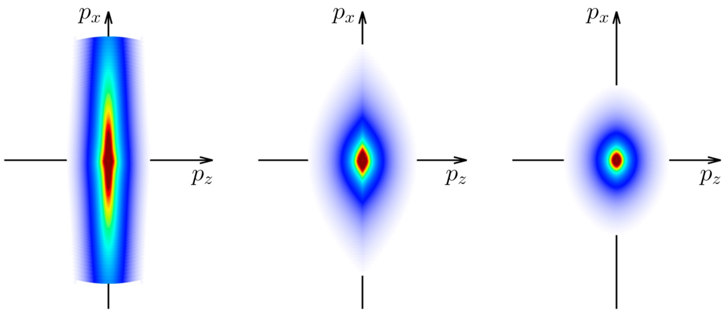
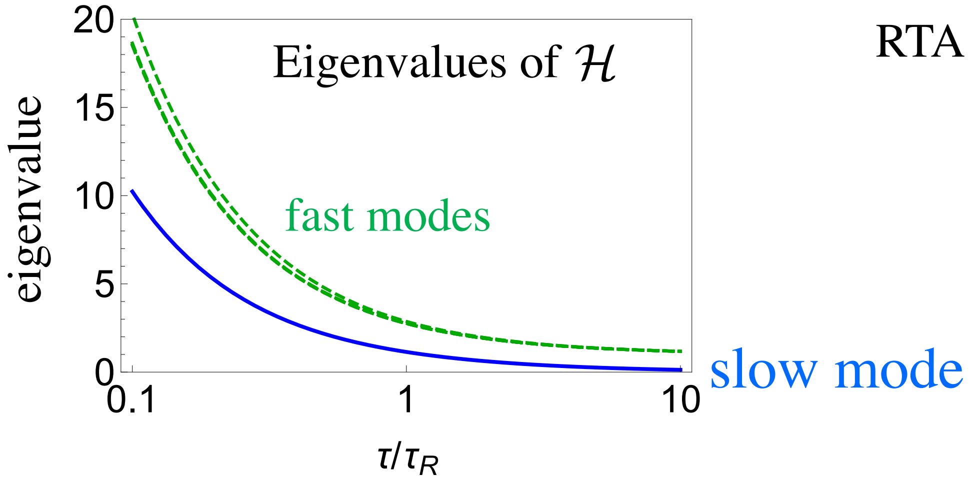


Fig adapted from KoMPoST [1805.00961]

$$(\epsilon, \mathcal{L}_1, \mathcal{L}_2, \ldots) \longrightarrow (\epsilon, 0, 0, \ldots)$$

hydrodynamization

Ground state: far-from-equilibrium slow mode

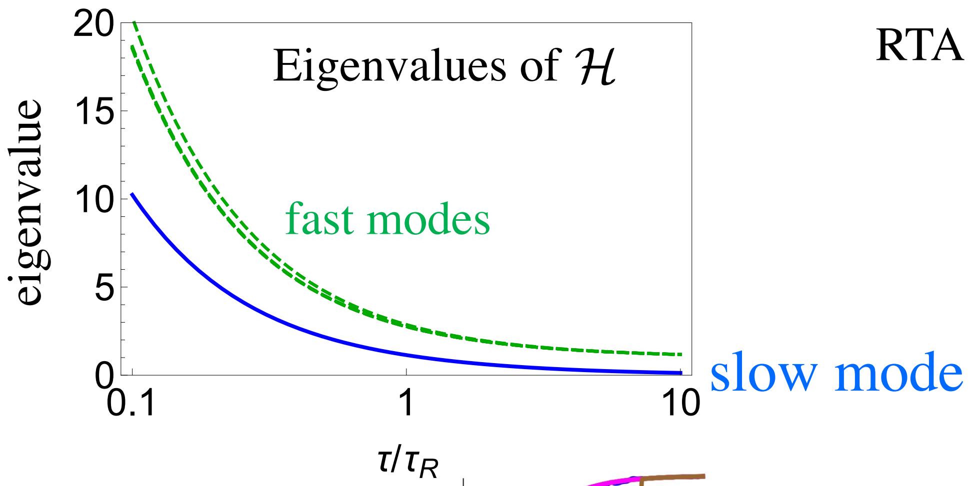


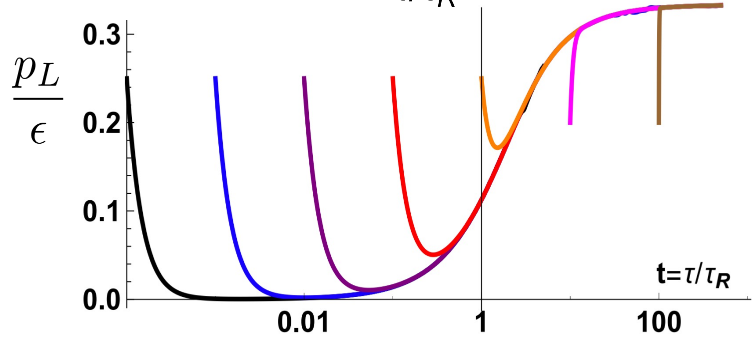
RTA: $\mathcal{H} = \mathcal{H}_F + \frac{\tau}{\tau_R}\mathcal{H}_H$

Early times: $\Delta E \sim \frac{1}{\tau}$

Late times: $\Delta E \sim \frac{1}{\tau_R}$

Ground state: far-from-equilibrium slow mode





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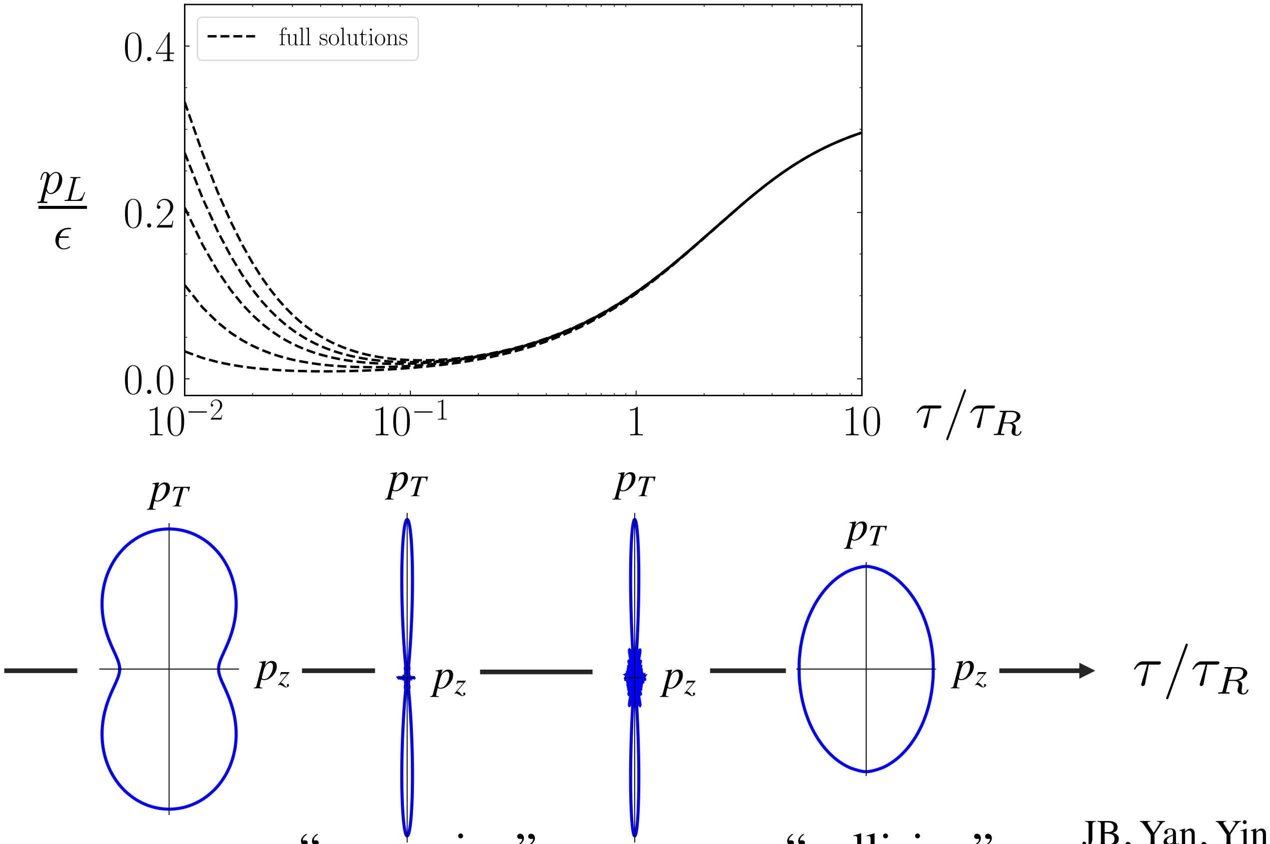
Late times: $\Delta E \sim \frac{1}{\tau_R}$

Initial conditions decay to ground state on time scale set by energy gap

Kurkela, van der Schee, Wiedemann, Wu [1907.08101]

But this slow mode is not a hydrodynamic mode

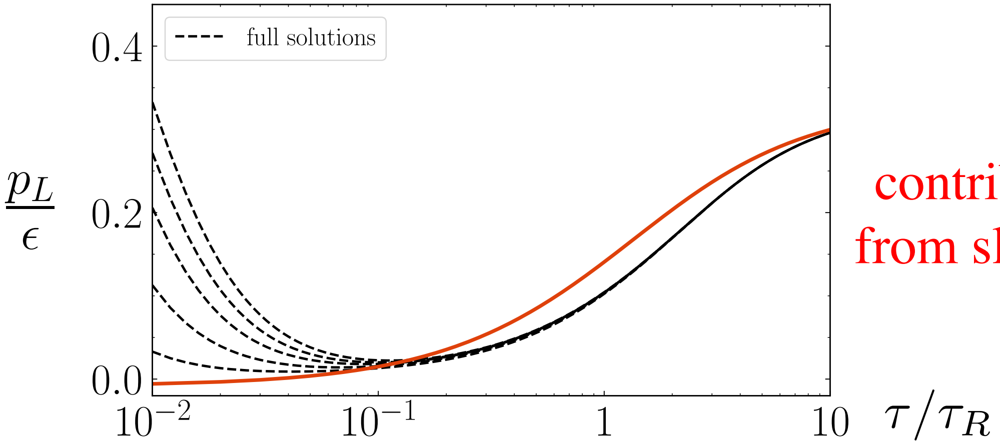


“expansion” slow mode

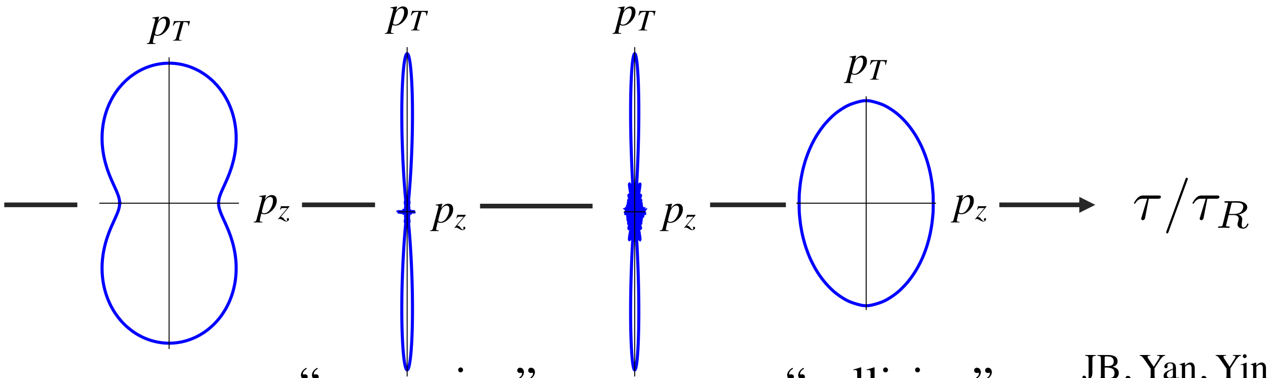
“collision” slow mode

JB, Yan, Yin [1910.00021]

But this slow mode is not a hydrodynamic mode



contribution only from slowest mode



"expansion" slow mode

"collision" slow mode

JB, Yan, Yin [1910.00021]

A system prepared in its (instantaneous) ground state remains in its (instantaneous) ground state if transitions are suppressed

$$\text{Transition rate } \sim \frac{\partial_\tau \log \lambda}{\Delta E_n} \langle 0(\tau)|H|n(\tau)\rangle \qquad \lambda = \tau/\tau_R$$

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“Slow quench” adiabaticity

- Hamiltonian evolution slow compared to energy gap
- Small close to hydro limit

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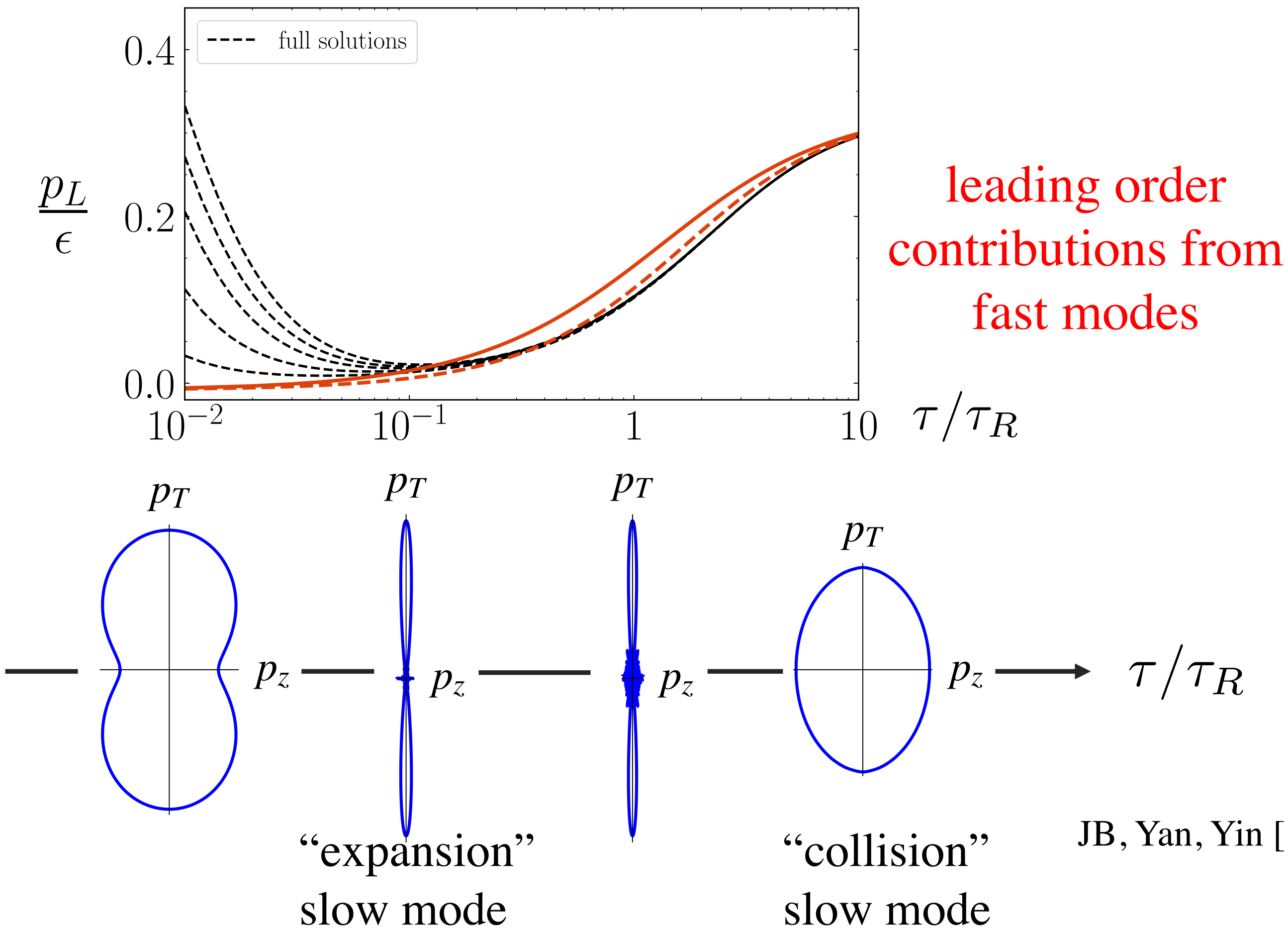
"Slow quench" adiabaticity
- Hamiltonian evolution slow compared to energy gap
- Small close to hydro limit

"Fast quench" adiabaticity
- Matrix element suppressed
- Small at early times because H suppressed by τ

But this slow mode is not a hydrodynamic mode

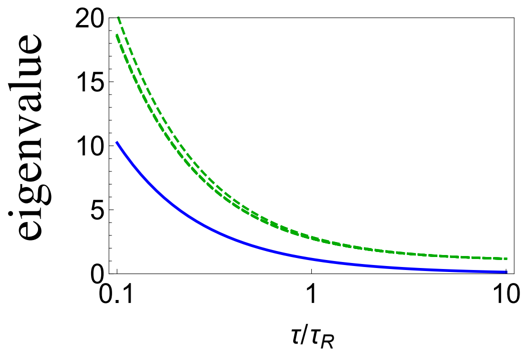


JB, Yan, Yin [1910.00021]

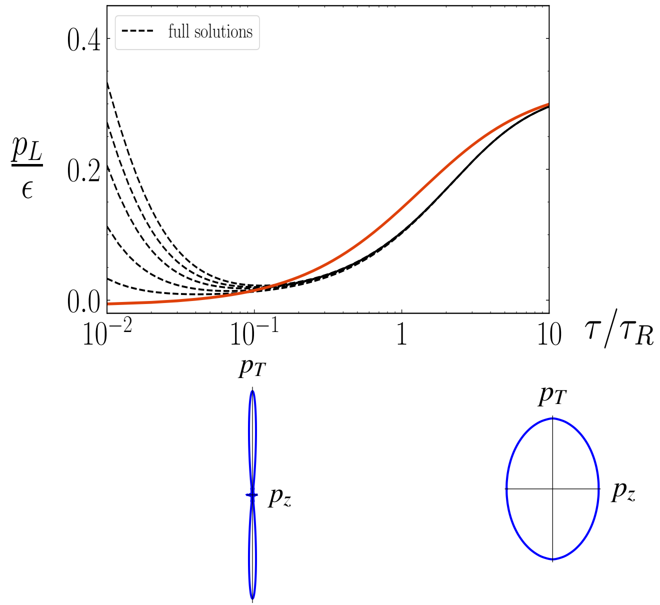
Bjorken expansion:

far-from-equilibrium evolution described by ground state

ground state "slow mode" gapped from excited states



slow mode



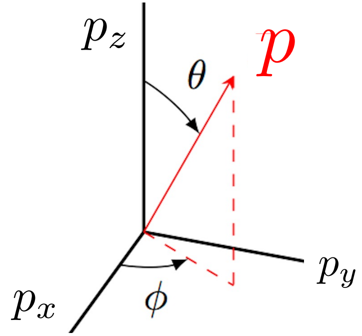
Bjorken is special because there is only one hydrodynamic mode.

Beyond Bjorken expansion

Bjorken expansion

$$F\left(\cos\theta;\tau\right) = \epsilon(\tau) + \sum_{n=1} \frac{4n+1}{2}\mathcal{L}_n(\tau)P_{2n}(\cos\theta) \quad \longleftrightarrow \quad \psi = (\epsilon, \mathcal{L}_1, \mathcal{L}_2, \ldots)$$

Transverse momentum anisotropy $\quad Y_l^m(\theta,\phi) \sim \cos(m\phi)P_l^m(\cos\theta)$



Matveenko, Gryaznykh, Kondratev, Litvinenko
JB, Weiyao Ke, Li Yan, Yi Yin (in progress)

Beyond Bjorken expansion

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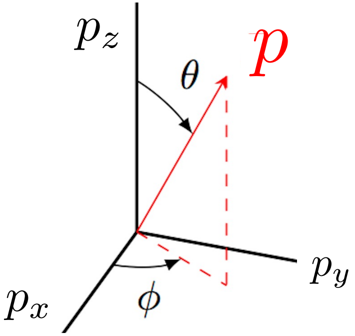


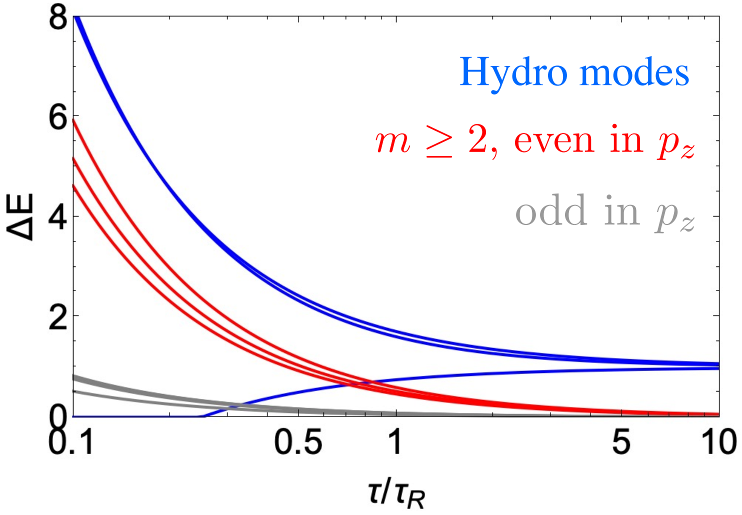
Boost-invariant expansion, spatially homogeneous in transverse plane:

Different m, even/odd $t = l + m$ do not mix $\qquad \mathcal{H}^{m,t} = \mathcal{H}_F^{m,t} + \frac{\tau}{\tau_R}\mathcal{H}_H^{m,t}$

Matveenko, Gryaznykh, Kondratev, Litvinenko
JB, Weiyao Ke, Li Yan, Yi Yin (in progress)

Beyond Bjorken expansion

Energy gap of $\mathcal{H}^{m,t} = \mathcal{H}_F^{m,t} + \frac{\tau}{\tau_R}\mathcal{H}_H^{m,t}$



$Y_l^m(\theta,\phi) \sim \cos(m\phi) P_l^m(\cos\theta)$

JB, Weiyao Ke, Li Yan, Yi Yin (in progress)

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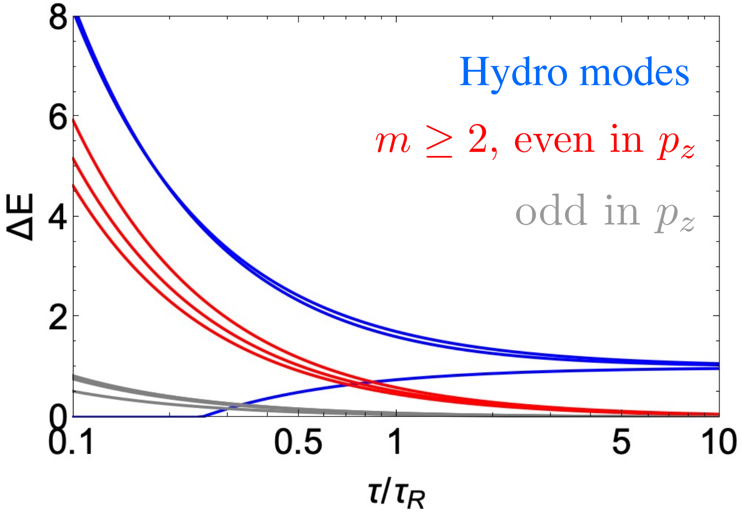

$Y_l^m(\theta,\phi) \sim \cos(m\phi)P_l^m(\cos\theta)$

Hydrodynamic modes are gapped at late times.
Some but not all are also gapped at early times.

JB, Weiyao Ke, Li Yan, Yi Yin (in progress)

Beyond Bjorken expansion

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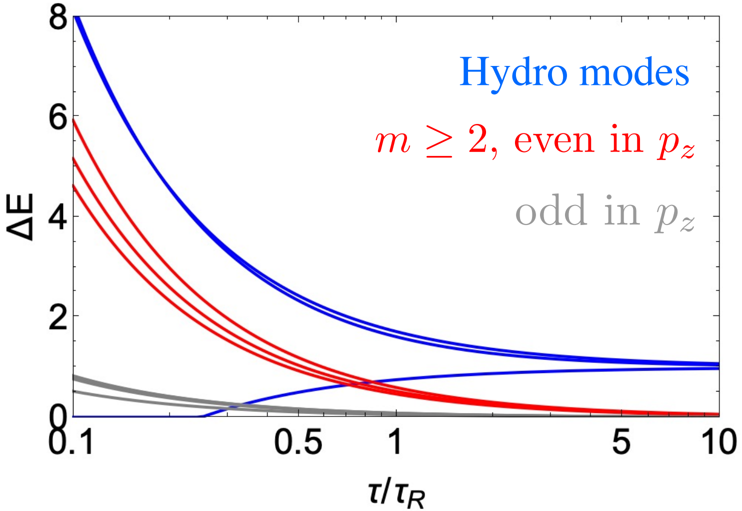

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Some non-hydro modes are gapped at early times

JB, Weiyao Ke, Li Yan, Yi Yin (in progress)

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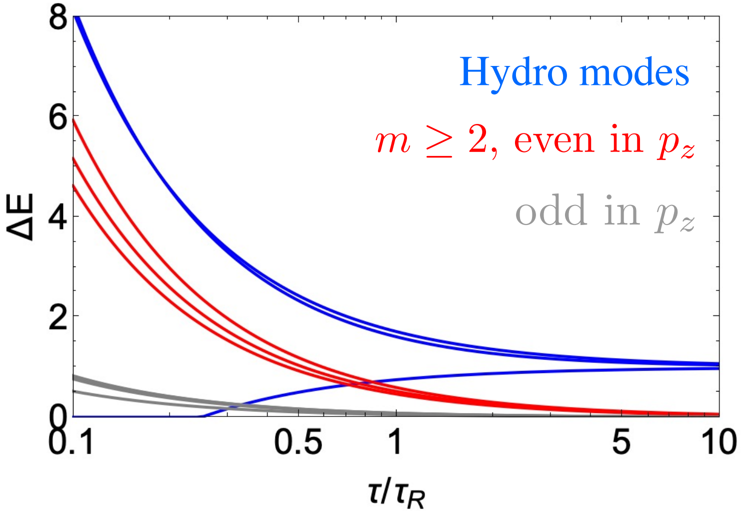

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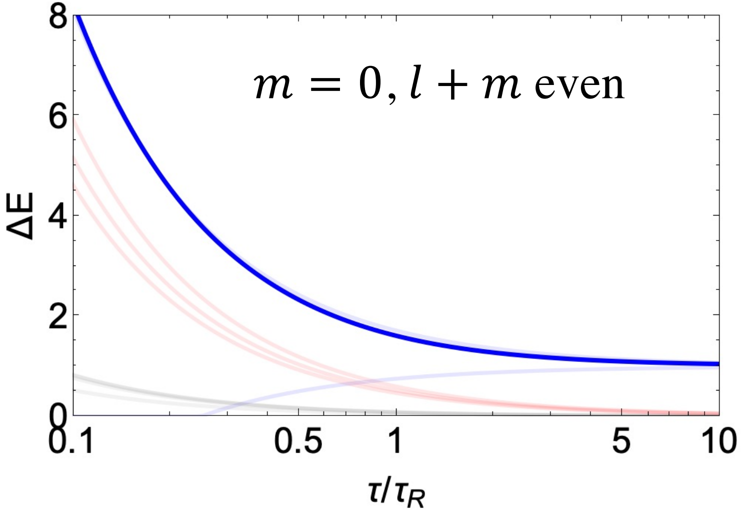
Some but not all are also gapped at early times.

Some non-hydro modes are gapped at early times

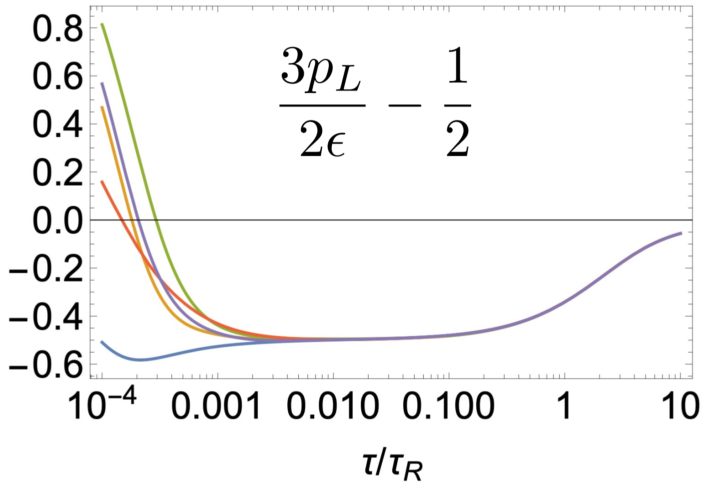
Attractor associated with gapped modes, which are not the same as hydrodynamic modes

JB, Weiyao Ke, Li Yan, Yi Yin (in progress)

Early-time attractor not associated with hydrodynamic modes



$\langle FY_2^0\rangle / \langle FY_0^0\rangle$



JB, Weiyao Ke, Li Yan, Yi Yin (in progress)

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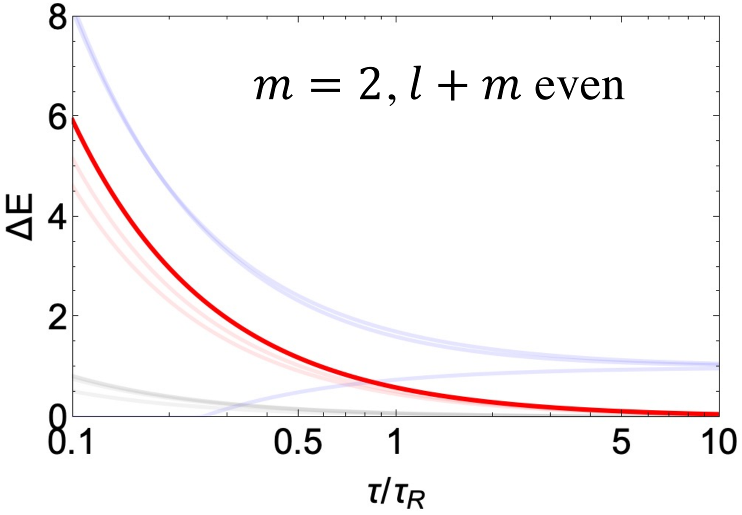
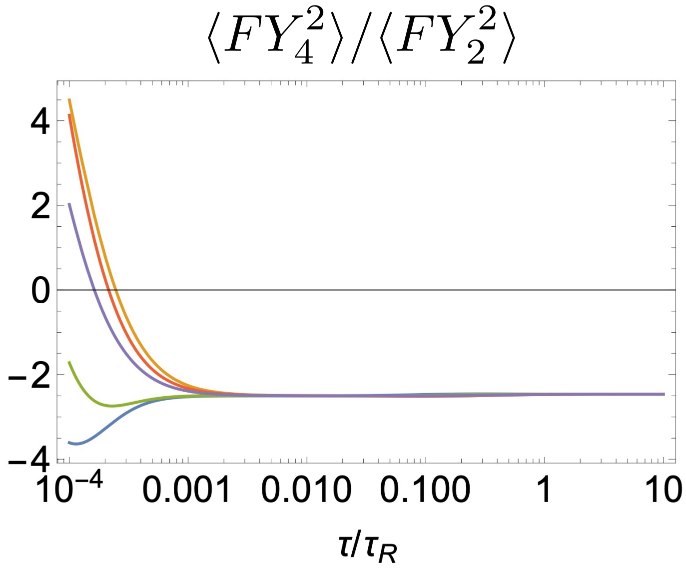
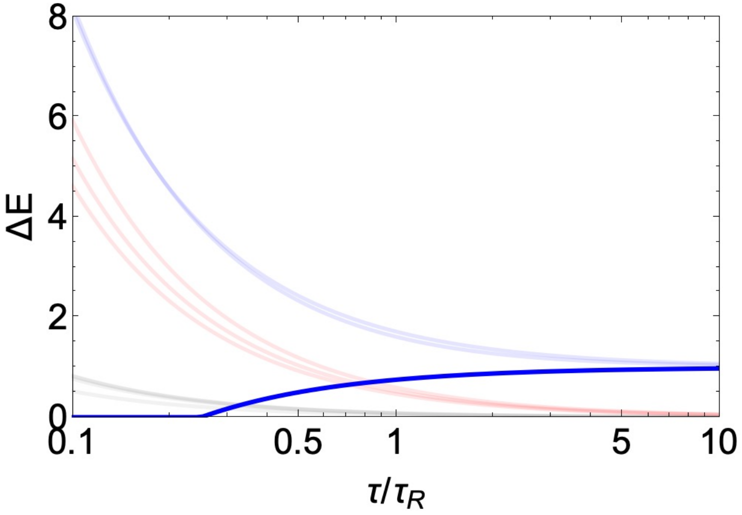


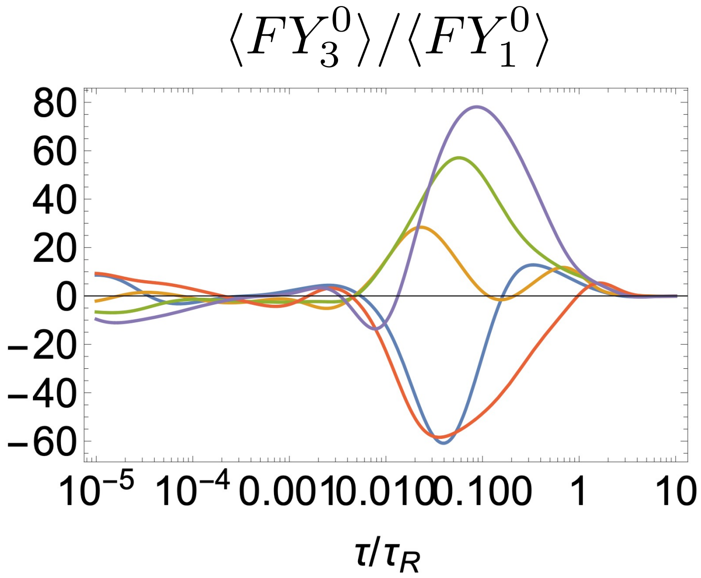


Attractor associated with non-hydrodynamic mode

JB, Weiyao Ke, Li Yan, Yi Yin (in progress)

Early-time attractor not associated with hydrodynamic modes



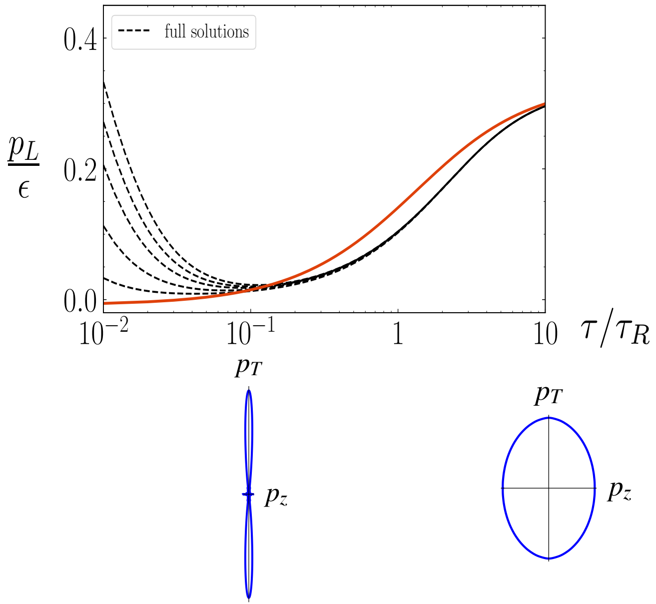


No attractor associated with hydrodynamic mode

JB, Weiyao Ke, Li Yan, Yi Yin (in progress)

Summary and Outlook

Connection between attractor and far-from-equilibrium slow modes

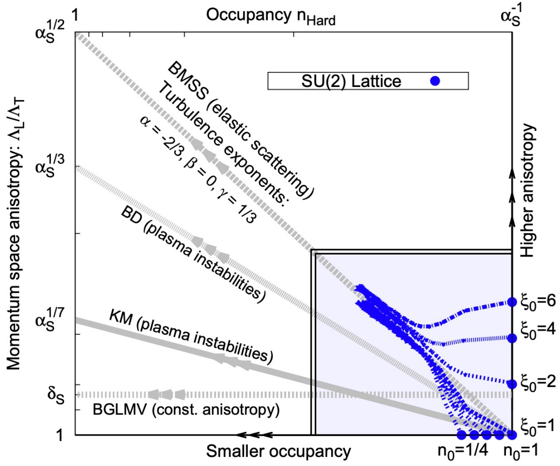


Beyond Bjorken, slow modes at early times are qualitatively different (including in number) than hydrodynamic modes

Scaling solution as ground state of Fokker-Planck

$$f(p_\perp, p_z, \tau) = \tau^{\alpha(\tau)} f_S(\tau^{\beta(\tau)} p_\perp, \tau^{\gamma(\tau)} p_z)$$

$$C[f] = -\hat{q}\partial_{p_z}^2 f \qquad \hat{q} \sim \alpha_s^2 N_c^2 \int_p f^2$$



Berges, Boguslavski, Schlichting, Venugopalan [1311.3005]

Scaling exponents of BMSS achieved as ground state

$$\alpha = -2/3,\ \gamma = 1/3,\ \beta = 0$$

Bruno Scheihing-Hitschfeld
MIT graduate student