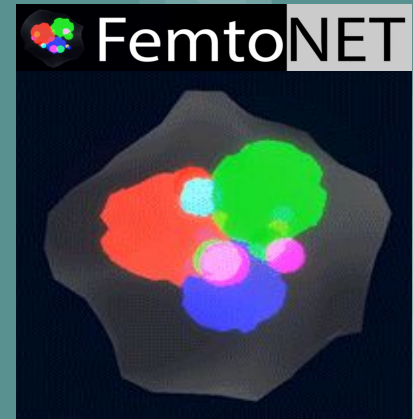


# 3D Tomography of the Nucleon Using GPDs

B. Kriesten



IR2@EIC Joint Argonne and CFNS Workshop  
March 18, 2021



# Outline



- **What is Femtography?**
- **Theoretical Motivation**
- **3 Dimensional Imaging**
- **Experiment and Femtography**
- **Additional Tools for Imaging**

# What is Femtography?

**Femtography** - is **data driven visualizations** of the phase space distribution of the quarks and gluons inside of the proton using a variety of **deeply virtual exclusive processes**.

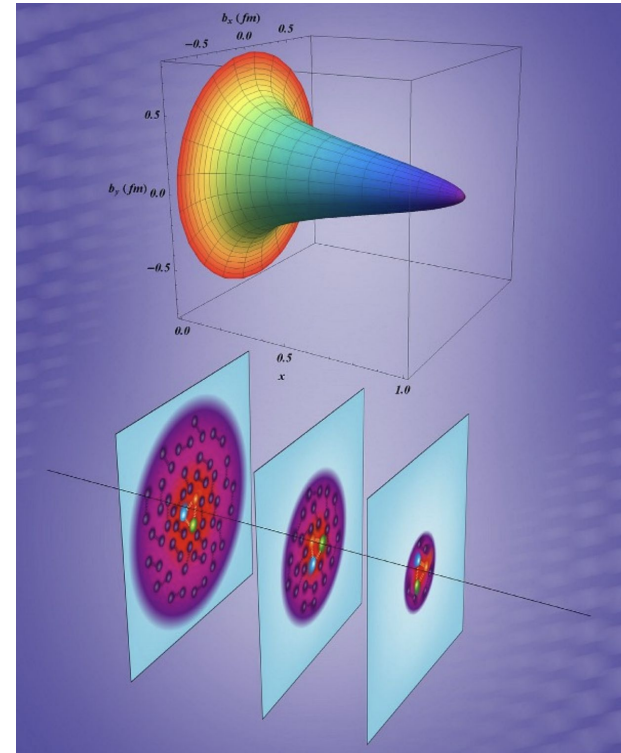


Image credit: **Rafael Dupre**

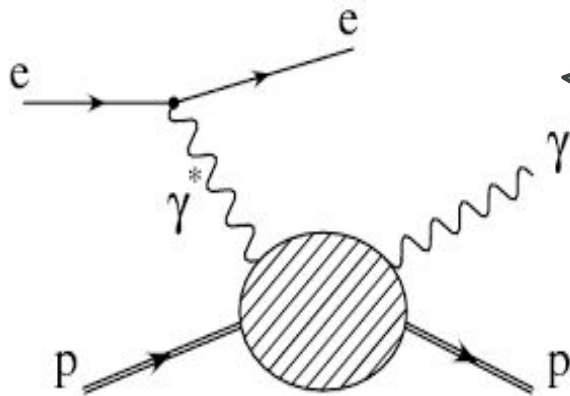
## Nature of Deeply Virtual Exclusive Processes



A **new paradigm** for measuring the fundamental properties of nuclei by probing the quantum mechanical **phase space distributions** of the quarks and gluons.

## Where does the proton spin come from?

$$J^q = \int dx x [H^q(x, 0, 0) + E^q(x, 0, 0)]$$



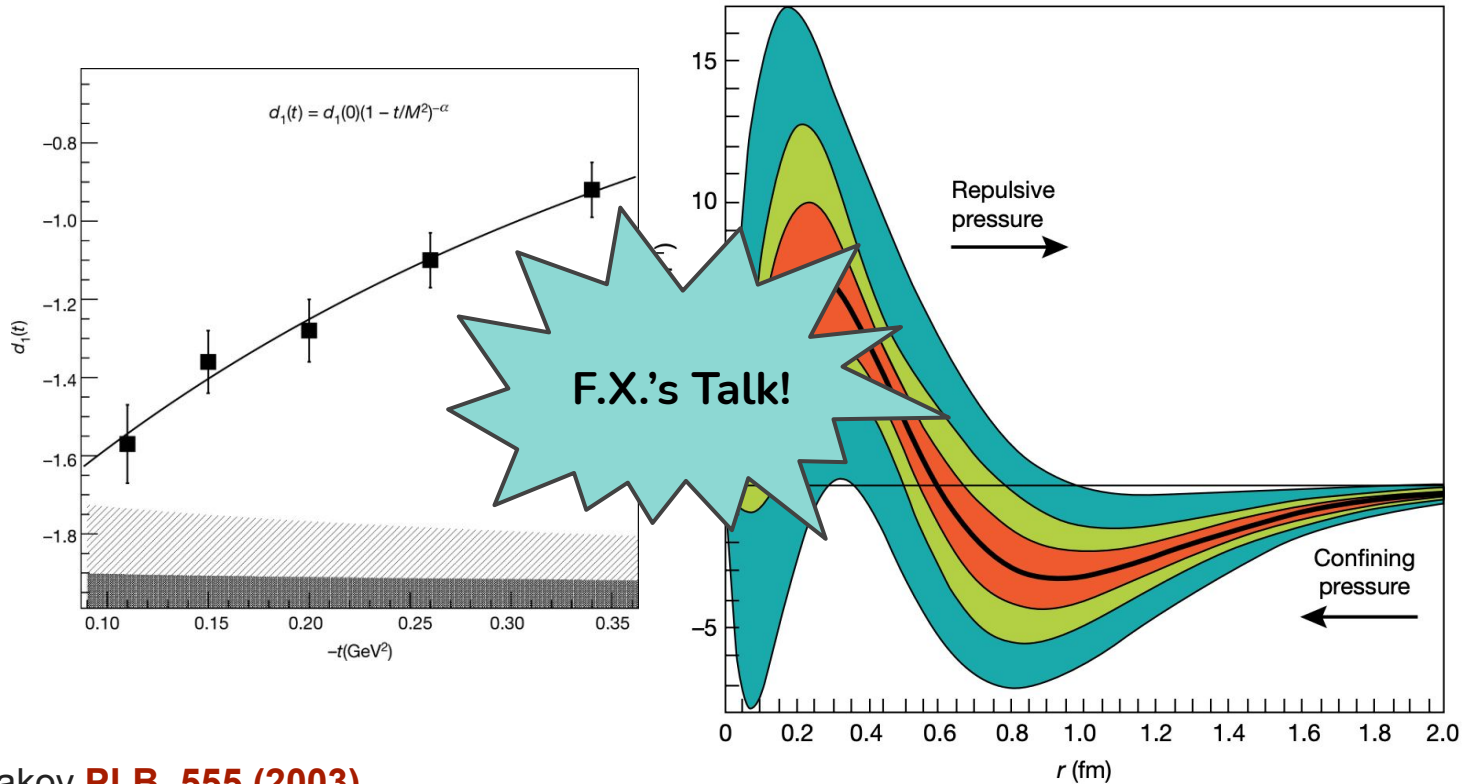
Kyle's Talk

$$J_{q,g}^i = \frac{1}{2} \epsilon^{ijk} \int d^3x M^{0jk}$$

$$J_{q,g}^i = \frac{1}{2} \epsilon^{ijk} \int d^3x (T_{q,g}^{0k} x^j - T_{q,g}^{0j} x^k)$$

X. Ji **PRL. 78 (1997)**

# The mechanical properties of nuclei: pressure in the proton?



M. V. Polyakov **PLB. 555 (2003)**

M. V. Polyakov, P. Schweitzer **Int.J.Mod.Phys. A33 (2018)**

V. Burkert, L. Elouadhiri, F.X. Girod **Nature v. 557 (2018)**

# Phase Space (Wigner) Distributions in the Proton



Wigner Distribution  $W(x, \vec{k}_T, \vec{b}_T)$

$k_{\perp} = 0.4 \text{ GeV}$

$b_T \rightarrow \Delta_T$

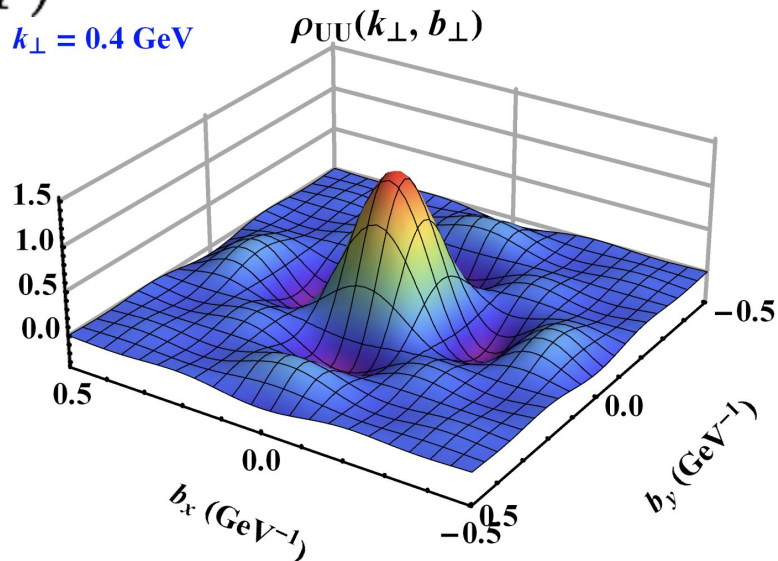
Generalized Transverse  
Momentum Distribution  
Functions

$W(x, \vec{k}_T, \vec{\Delta}_T)$

$$\int d^2 k_T$$

Generalized Parton  
Distribution Functions

$H(x, \vec{\Delta}_T)$

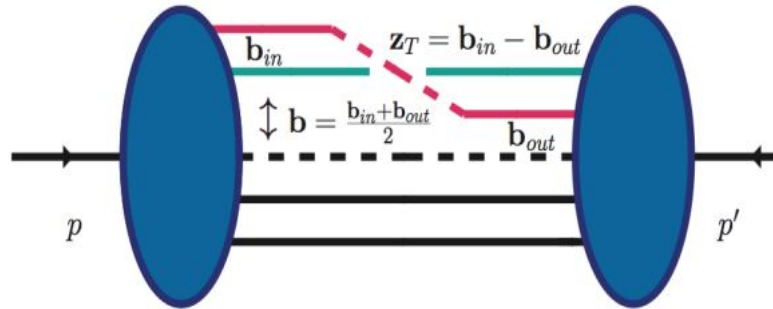


A.V. Belitsky, X. Ji, F. Yuan **PRD. 69 (2003)**  
 Ji, Xiong, Yuan **PRD 88 (2013) (gluon)**

Lorce and Pasquini **PRD.84 (2011)**  
 Lorce and Pasquini **JHEP 1309 (2013) (gluon)**  
 More, Mukherjee, Nair, **PRD. 95 (2017)**

# Generalized parton distributions

$$F_{\Lambda, \Lambda'}^{[\Gamma]}(x, \xi, t) = \frac{1}{2} \int \frac{dz^-}{2\pi} e^{ikz} \langle p', \Lambda' | \bar{\psi}(-\frac{z}{2}) \Gamma \mathcal{W}(-\frac{z}{2}, \frac{z}{2} | n) \psi(\frac{z}{2}) | p, \Lambda \rangle \Big|_{z^+ = z_T = 0}$$



$b_T$  Relative average transverse position from the center of momentum of the system  $\longleftrightarrow \Delta_T$

$k_T$  Relative average transverse momentum (integrated)  $\longleftrightarrow z_T$

Image credit: A. Rajan, M. Engelhardt, S. Liuti **PRD 98 (2018)**

X. Ji **PRL. 78 (1997)**

A. Radyushkin **PRD. 56 (1997)**

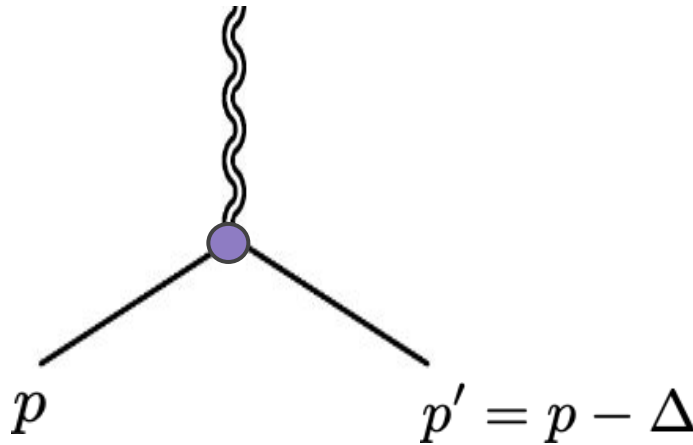
D. Muller, et. al. **(1994)**

M. Diehl **Phys.Rep. (2003)**



## Energy Momentum Tensor Form Factors

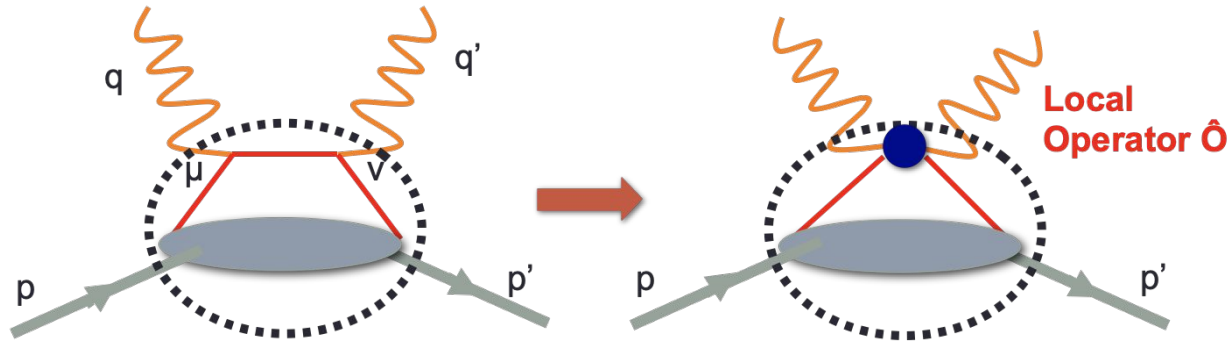
$$\langle P' | T_{q,g}^{\mu\nu} | P \rangle = \bar{U}(P') [A_{q,g}(\Delta^2) \gamma^{(\mu} \bar{P}^{\nu)} + B_{q,g}(\Delta^2) \bar{P}^{(\mu} i\sigma^{\nu)\alpha} \Delta_\alpha / 2M + C_{q,g}(\Delta^2) (\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2) / M + \bar{C}_{q,g}(\Delta^2) g^{\mu\nu} M] U(P)$$



X. Ji **PRL. 78 (1997)**

The matrix elements of the energy momentum tensor can be parameterized by **form factors** describing elastic scattering of a graviton off a proton.

## Connection between Local Operators and GPDs



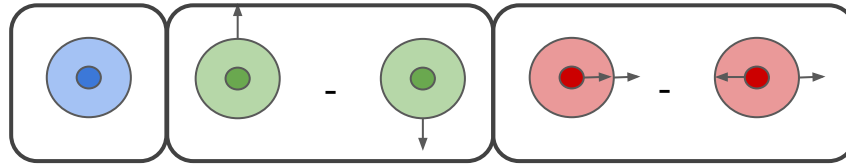
$$\int dx x H(x, \xi, t) = A(t) + \xi^2 C(t)$$

$$\int dx x E(x, \xi, t) = B(t) - \xi^2 C(t)$$

## Momentum Space to Transverse Position Space

Probability density of finding a quark at transverse position  $\mathbf{b}$  from the center of momentum as a function of quark and proton polarization.

$$\rho_{\Lambda\lambda}^q(\mathbf{b}) = H_q(\mathbf{b}^2) + \frac{b^i}{M} \epsilon_{ij} S_T^j \frac{\partial}{\partial b} E_q(\mathbf{b}^2) + \Lambda\lambda \tilde{H}_q(\mathbf{b}^2)$$



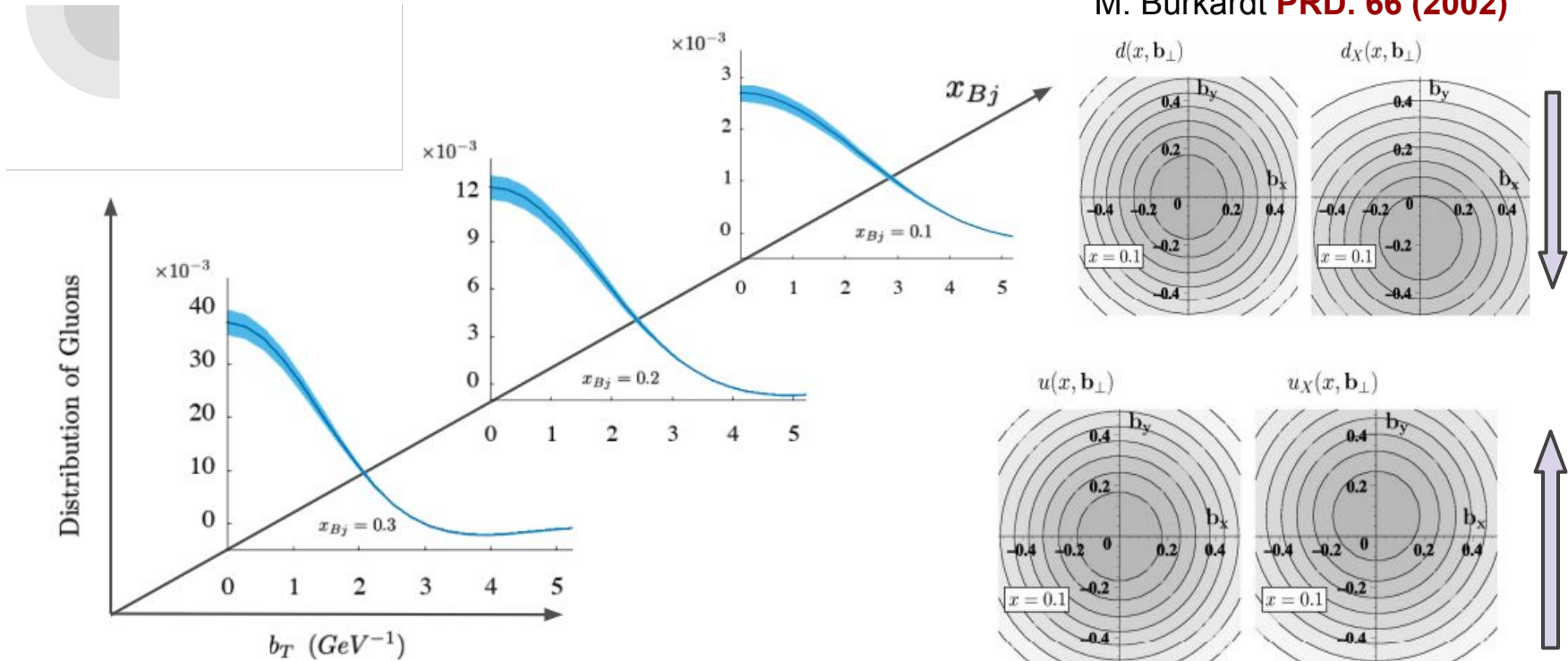
$$\rho_{\uparrow}^q(x, b_T) = \int \frac{d^2 \Delta_T}{(2\pi)^2} \left[ H(x, 0, t) - \frac{1}{2M} E(x, 0, t) \frac{\partial}{\partial b_y} \right] e^{-i\Delta_T \cdot b_T}$$

M. Burkardt **PRD. 62 (2000)**  
 M. Burkardt **Int.J.Mod.Phys.A 18 (2003)**

Transverse polarization shifts the unpolarized distribution proportional to the GPD E.

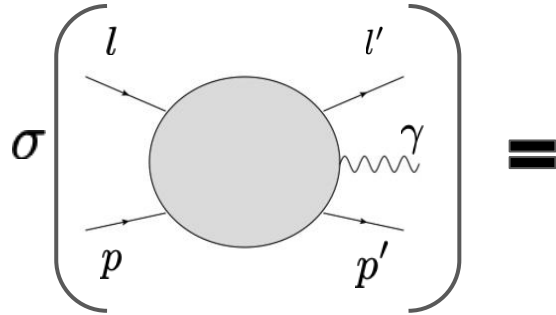
# 2+1 Dimensional Imaging in Impact Parameter Space

M. Burkardt **PRD. 66 (2002)**



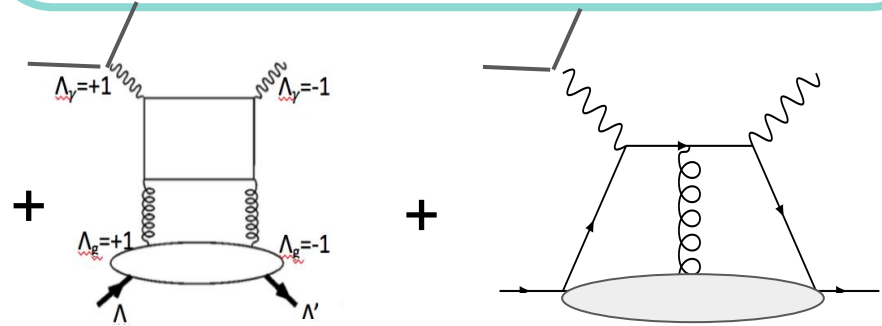
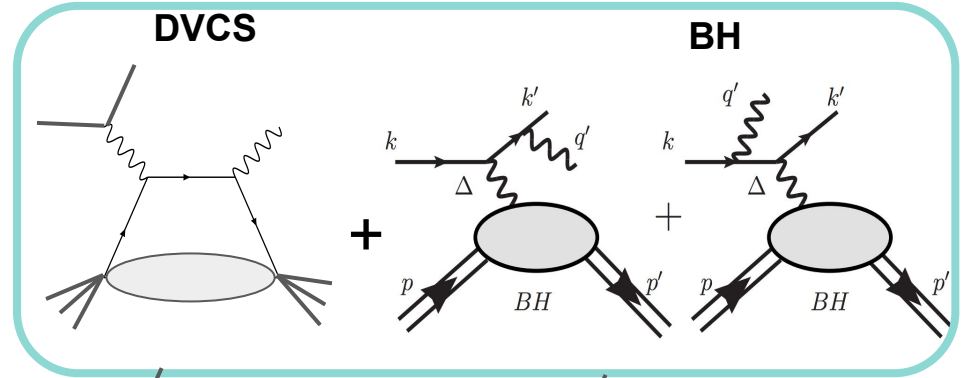
**Gluon GPDs:** B. Kriesten, P. Velie, E. Yeats, F.Y. Lopez, S. Liuti [arXiv:2101.01826](https://arxiv.org/abs/2101.01826)  
**Fourier Transforms:** A. Rajan, B. Kriesten, S. Liuti **(in progress)**

# Deeply Virtual Compton Scattering



DVCS is known to probe **generalized parton distributions** and is accompanied by various background processes.

X. Ji, **PRD. 55 (1997)**



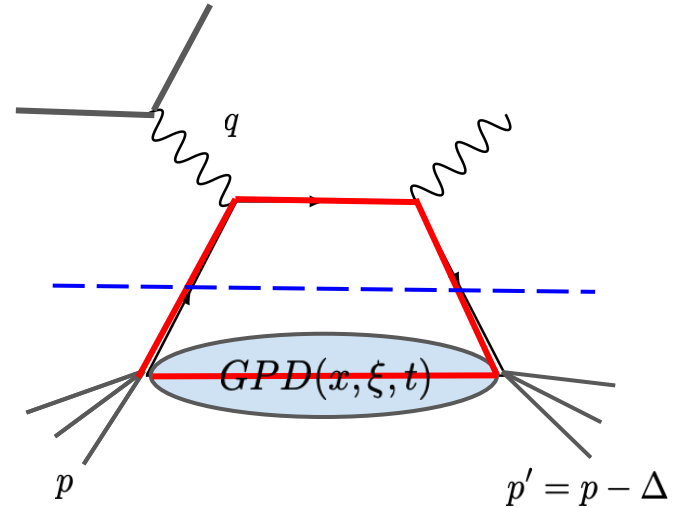
**Gluon Transversity**

**Higher Twist**

B.Kriesten, S.Liuti, et. al. **PRD. 101 (2020)**

## Inverse Problem for Extracting GPDs?

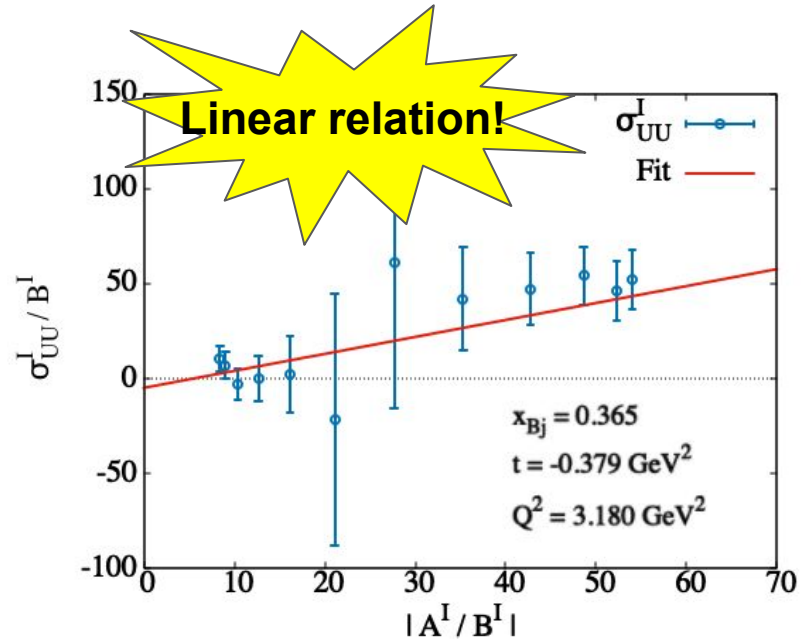
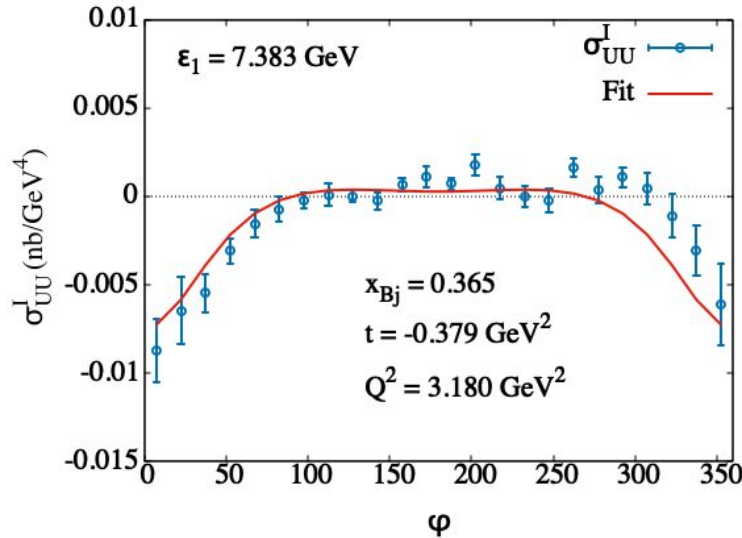
**Issues with GPDs:** they come convoluted with Wilson coefficient functions (Compton Form Factors) meaning we only have experimental access to integrals (ReCFF) or specific points in  $x$  (ImCFF) of these distributions.



$$\int_{-1}^{+1} dx \frac{F^q(x, \xi, t)}{x - \xi + i\epsilon} = \mathcal{P} \int_{-1}^{+1} dx \frac{F^q(x, \xi, t)}{x - \xi} - i\pi F(\xi, \xi, t)$$

## Rosenbluth Separation using DVCS Data

$$\frac{F_{UU}^{\mathcal{I},tw2}}{B_{UU}^{\mathcal{I}}} = \frac{A_{UU}^{\mathcal{I}}}{B_{UU}^{\mathcal{I}}} \Re(F_1 \mathcal{H} + \tau F_2 \mathcal{E}) + G_M \Re(\mathcal{H} + \mathcal{E}) + \frac{C_{UU}^{\mathcal{I}}}{B_{UU}^{\mathcal{I}}} \Re \tilde{\mathcal{H}}$$

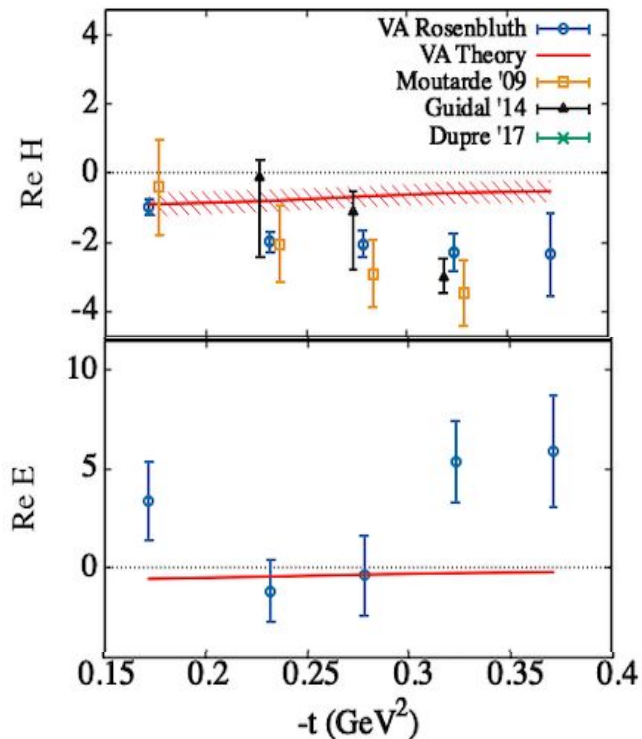


B. Kriesten, S. Liuti, A. Meyer [arXiv: 2011.04484](#)

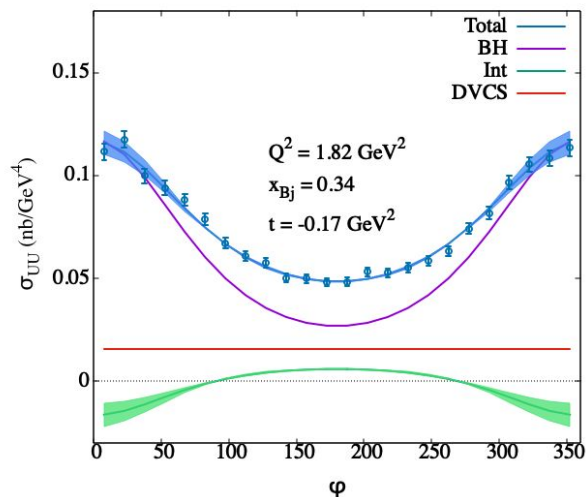
B.Kriesten, S. Liuti [arXiv: 2004.08890](#)

Data: F. Georges et. al. [Ph.D. Thesis \(2018\)](#)

## CFF Extractions: A comparison of methods

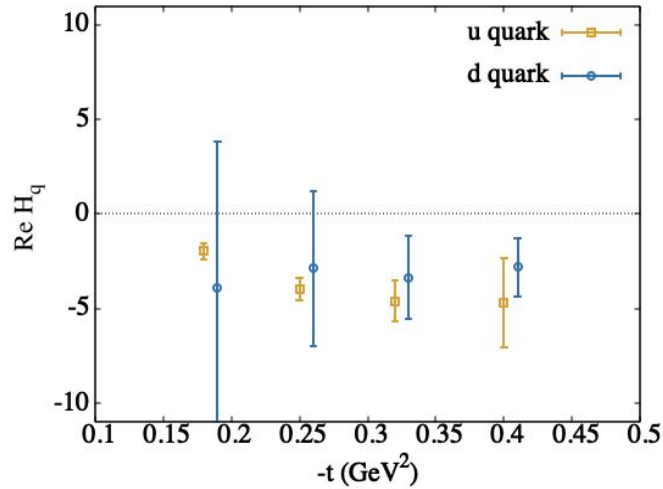


All CFF extractions use the same data, but with different extraction methods and cross section formula you extract different values and different size error bars.

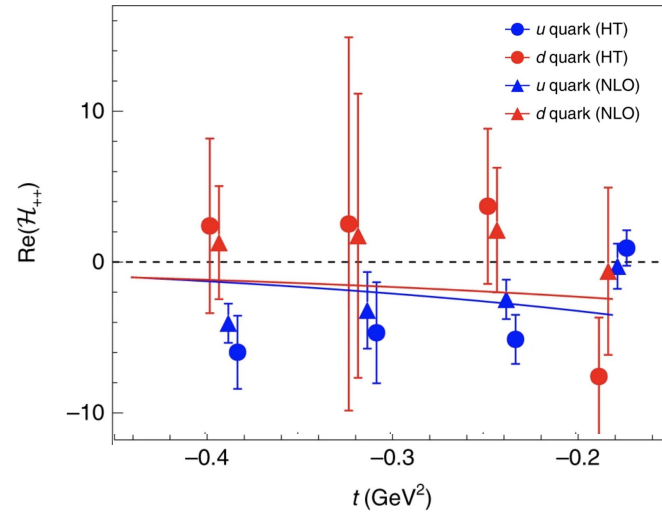




# Flavor Separation using Neutron DVCS Data



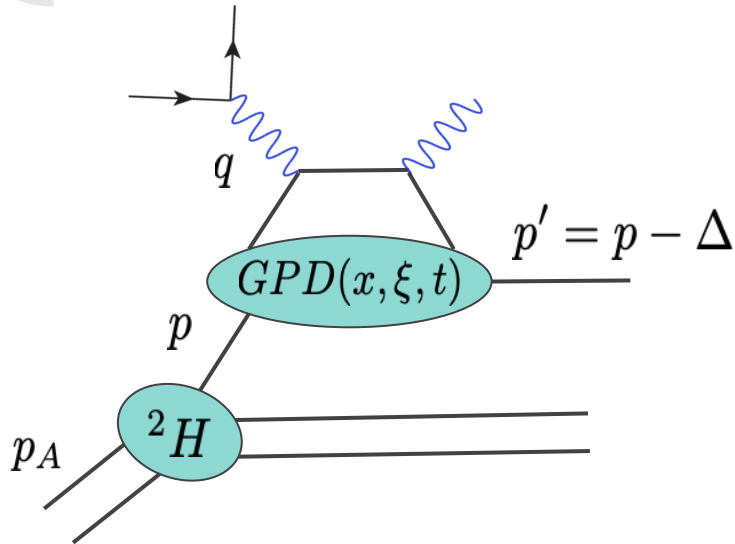
B. Kriesten, S. Liuti (in progress)



M. Benali, et. al. **Nature Physics 16, (2020)**

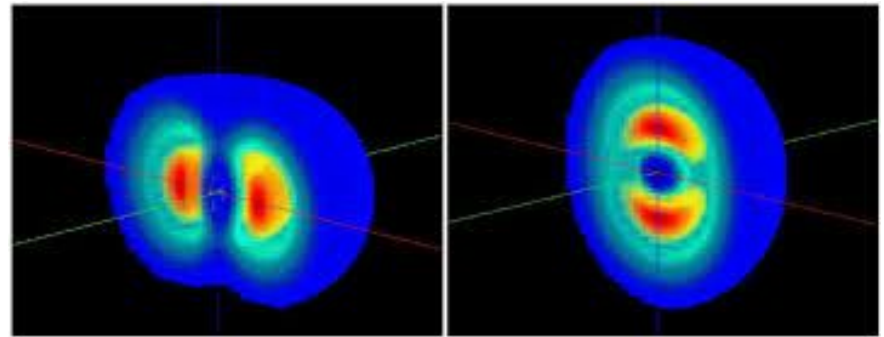
Using a combination of proton and neutron data we can flavor separate CFFs where an EIC will help constrain the neutron DVCS errors.

# DVCS on Light Nuclei



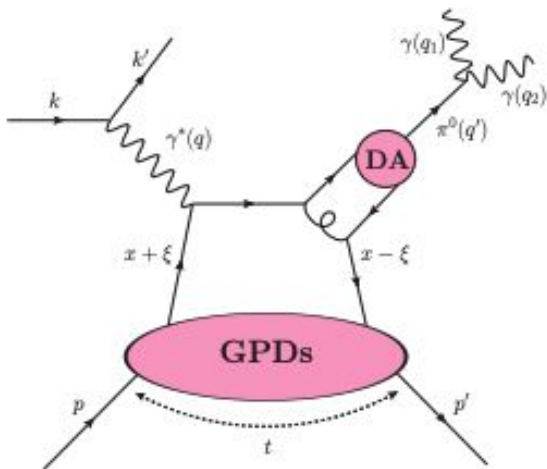
Using the Spin-1 Energy Momentum Tensor, a similar angular momentum sum rule has been developed for the Spin-1 deuteron

$$J_q = \frac{1}{2} \int dx x H_2^q(x, 0, 0)$$



S. Taneja, K. Kathuria, S Liuti, and  
G.R. Goldstein **PRD. 86 (2012)**

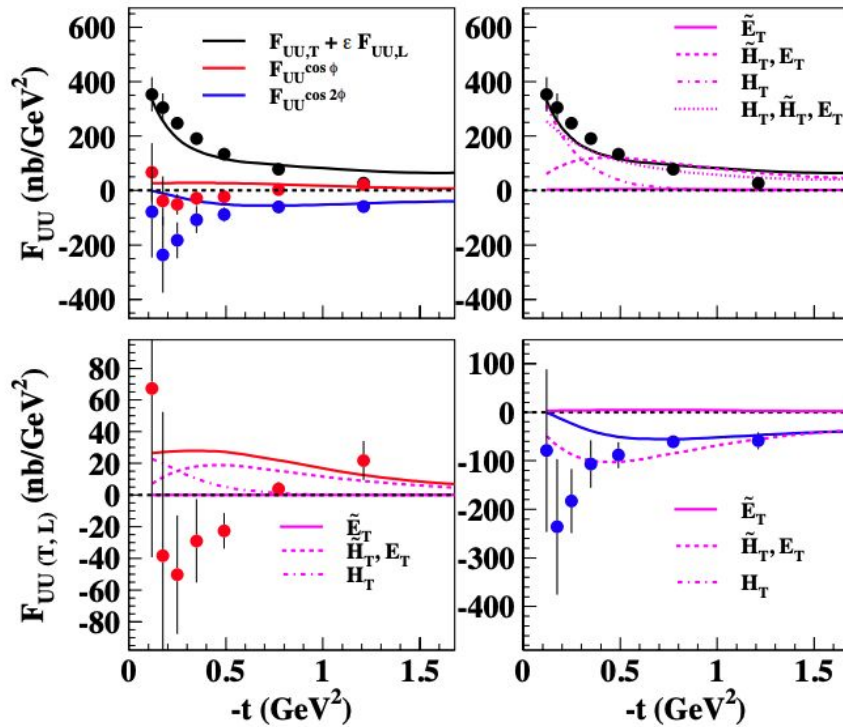
# pi0 electroproduction and Chiral Odd GPDs



Ahmad, Goldstein, Liuti **PRD.79 (2009)**

Goloskokov, Kroll **EPJA (2011)**

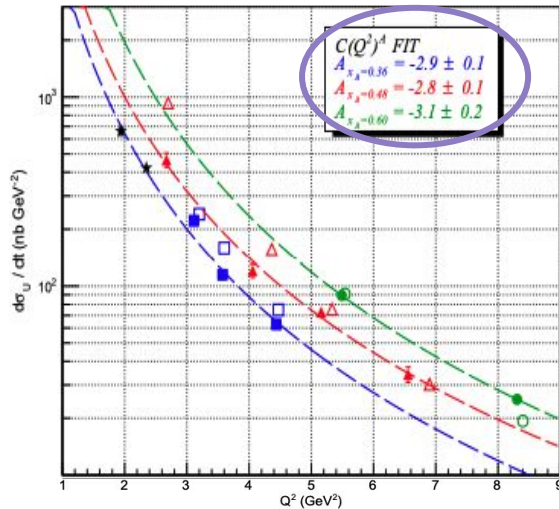
**Data:** Bedlinsky et. al. **PRL. 109, (2012)**



# Deeply Virtual pi0 Production Q2 dependence

Chance to study scaling effects and the scattering mechanism of pi0 electroproduction.

Data: M. Defurne et. al. **PRL 117 (2017)**  
 E. Fuchey **PRC 83 (2011)**  
 Bedlinsky et. al. **PRL. 109, (2012)**

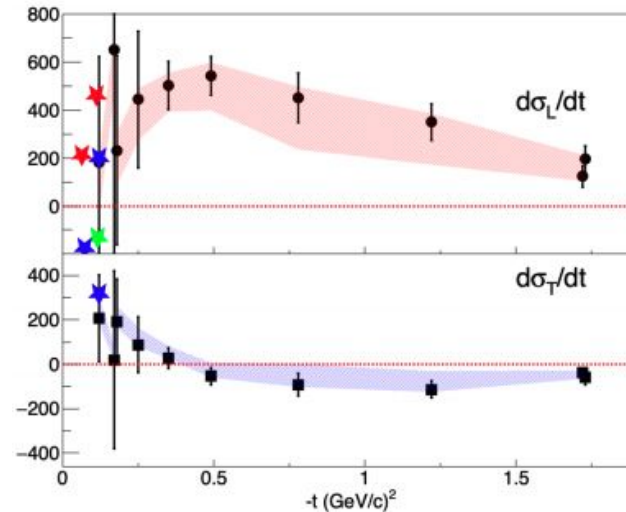


M. Dlamini et. al. **arXiv:2011.11125**

$$\frac{d\sigma_U}{dt} = \frac{d\sigma_T}{dt} + \epsilon \frac{d\sigma_L}{dt}$$

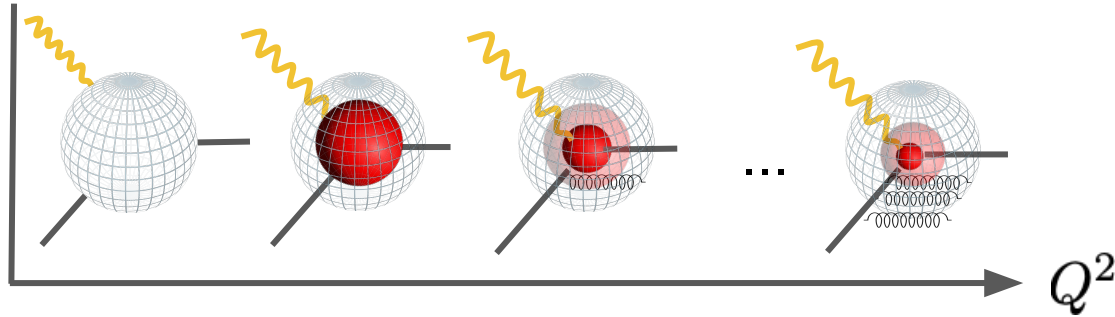
$Q^{-8}$        $Q^{-6}$

J. Collins et. al. **PhysRevD.56.2982**



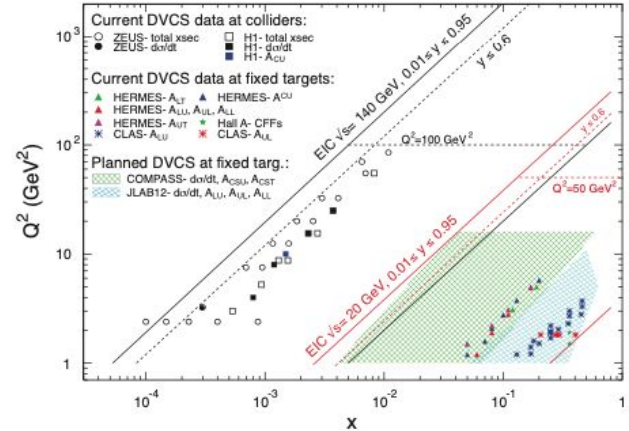
I. Korover, R. Milner **arXiv:2103.00611**

# Where is the glue?



$$\frac{\partial F_2(x, Q^2)}{\partial \ln Q^2} = \frac{\alpha_S(Q^2)}{2\pi} \left[ P_{QQ} \otimes F_2 + 2e^2 P_{QG} \otimes xg(x) \right]$$

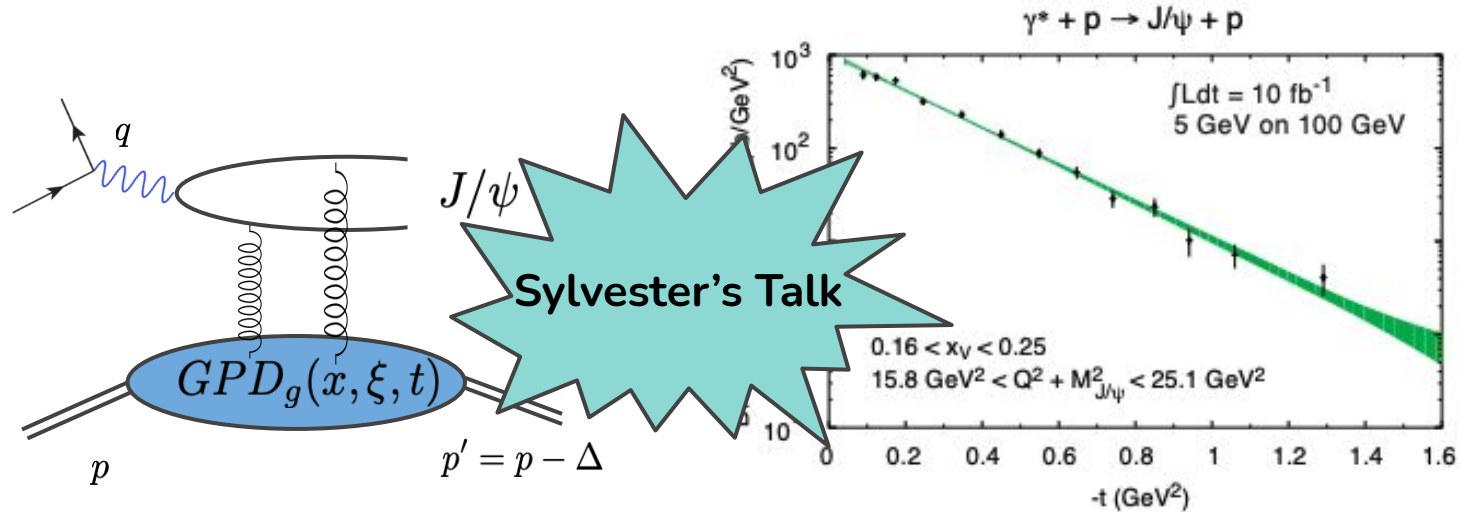
A **lever arm in  $Q^2$**  hopefully allows us to use perturbative evolution to extract the gluon distribution.



EIC Yellow Report [arXiv: 2103.05419](https://arxiv.org/abs/2103.05419)  
 EIC White Paper [arXiv: 1212.1701](https://arxiv.org/abs/1212.1701)

# Exclusive Measurements of Gluon Distributions

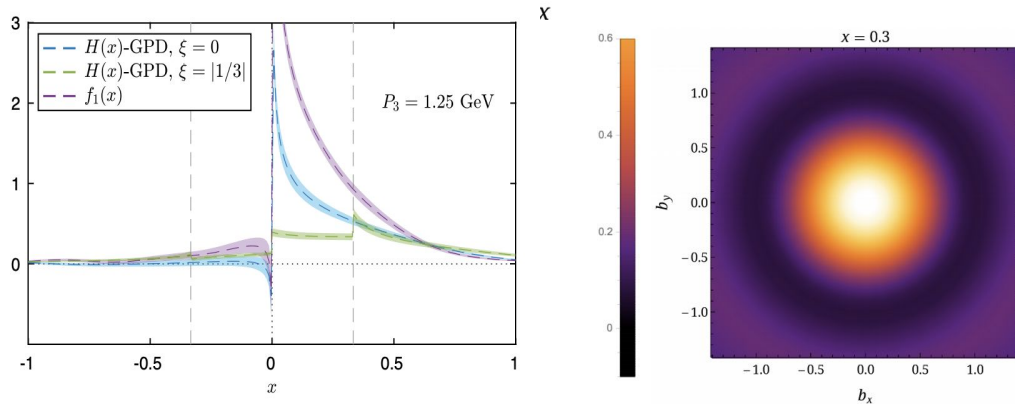
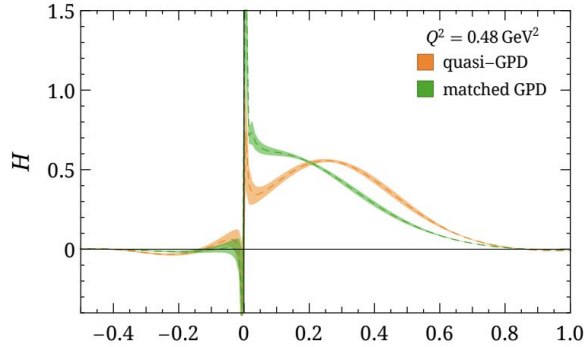
Exclusive electroproduction of vector mesons (such as the  $J/\psi$ ) can also probe the gluon content of nuclei



EIC White Paper [arXiv: 1212.1701](https://arxiv.org/abs/1212.1701)

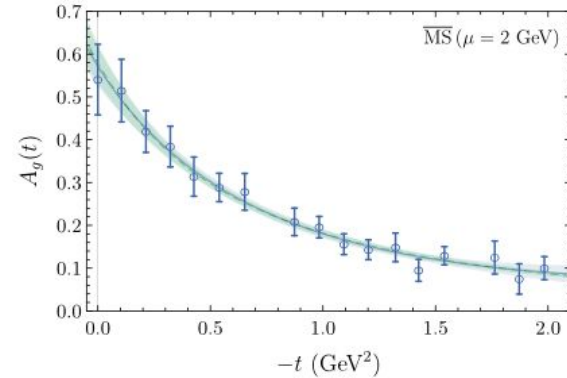
# Additional Tools: Constraints from Lattice QCD

H.W. Lin [arXiv:2008:12474](https://arxiv.org/abs/2008.12474)



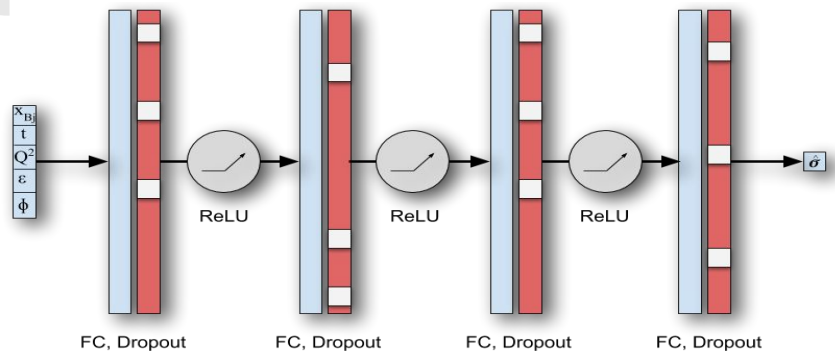
ETMC [PRL 125 \(2020\)](https://arxiv.org/abs/2008.12474)

P.E. Shanahan, W. Detmold [PRD 99 \(2019\)](https://arxiv.org/abs/1901.08211)



Lattice QCD can place **constraints** on extracted gluon observables through calculated moments.

## Additional Tools: Femtography using Artificial Intelligence

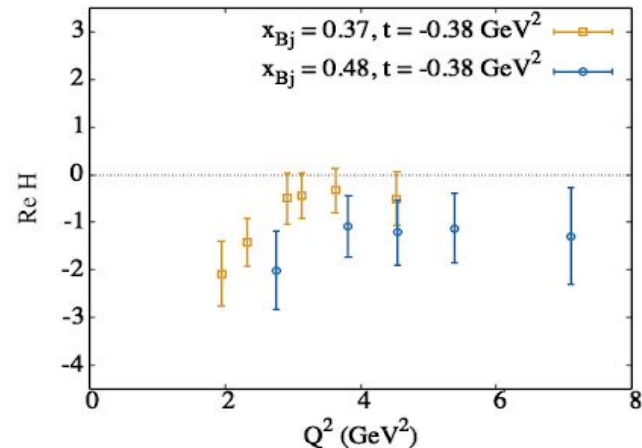
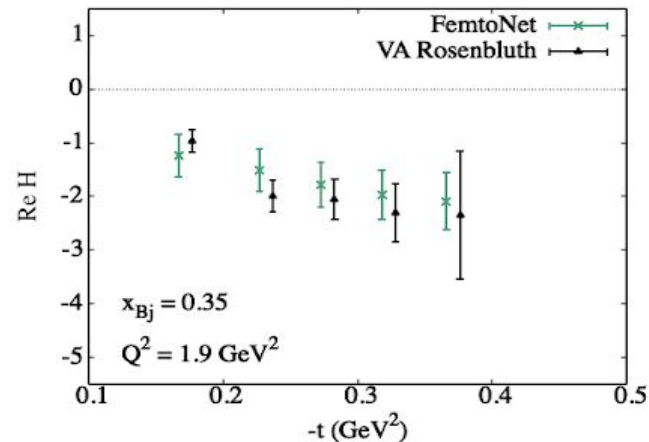


Why do we need a deep neural network?

- DNN provide **efficient** and **accurate** predictions in the range of the data while disentangling **complex final states** with a **large number of observables**.

J. Grigsby, B. Kriesten, S. Liuti, et. al. [arXiv: 2012.04801](#)

J. Grigsby, B. Kriesten, S. Liuti, et. al. [\(in progress\)](#)





# Conclusions



- An EIC will help us explore **new kinematic regimes** for femtography using a variety of exclusive processes to place constraints on the quark and gluon distributions.
- Femtography is **extremely difficult**, there are many steps from cross sections -> images.
  - Some of these connections are well defined but **CFFs -> GPDs** (inverse problem) must be explored using additional data analysis tools.
- Creating femtographic images will require **coalitions of experts** including experimentalists, theorists, data scientists, machine learning experts, lattice QCD specialists, and visualization experts to work together.