Double spin asymmetries of ion electron collisions

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What spin correlations for elastic EI collisions can act as polarimeters?

Which ion and lepton energies are suitable for using such observables?

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- QED predicts the polarization of light ions scattering on electrons to 1st order
- Measure asymmetry for elastic collisions on electrons polarized L or Normally
- The polarization direction of the light ion can be Longitudinal, N or Sideways
- A_{SL}(41 GeV p) $\approx 22\%$ when static electrons are scattered at 2.5 ± 1 mrad
- Similar analyzing power for 67 GeV/N He-3 when e^- measured near 5 mrad
- Scattering angle of electrons $\approx (1 + m_e E/M)/E$, where E is energy/mass
- Absolute polarimeter of a p or ^{3}He beam if inelastic events can be controlled
- Study breakup of ^{3}He to a proton and deuteron at 5.5 MeV (no excited state)

Proton and Helion Magnetic Moments

Double spin asymmetries for polarimetry have been studied by Nurushev et al [1]. Also, Rekalo & Tomasi-Gustafsson [2], Sofiatti & Donnelly [3], and Gakh et al [4].

- Asymmetries involve $G_M(t)/Z$ and the anomalous Pauli form factor $F_2(t)/Z$

$$\frac{G_M^h(0)/Z_h}{G_M^p(0)/Z_p} = \frac{\mu_h}{\mu_p}\frac{m_h}{m_p}\bigg/\frac{Z_h}{Z_p} = -1.1401 \quad \text{(14\% greater \& opposite in sign)}$$

- Including both charges and moments in the form factors simplifies expressions

$$\frac{F_2^h(0)/Z_h}{F_2^p(0)/Z_p} = \left(\frac{\mu_h\, m_h}{Z_h\, m_p} - 1\right)\bigg/(\mu_p - 1) = -2.334 \quad \text{(factor } -7/3 \text{ greater)}$$

The sizes of asymmetries A_{SL}, A_{LL}, A_{NN} have values appropriate for polarimetry

SPIN CORRELATION ASYMMETRY

The spin observables for polarized proton or ^{3}He ions scattering off polarized electrons may be written in terms of helicity amplitudes which are real for 1γ QED

$$
\begin{aligned}
\mathcal{D}A_{t\,\ell} &= \mathcal{D}A_{\mathrm{SL}} = (\phi_2 + \phi_4)\,\phi_5 - (\phi_1 - \phi_3)\,\phi_6 \\
\mathcal{D}A_{\ell\,\ell} &= \mathcal{D}A_{\mathrm{LL}} = \big(\phi_3^2 - \phi_1^2 + \phi_4^2 - \phi_2^2\big)/\,2 \\
\mathcal{D}A_{nn} &= \mathcal{D}A_{\mathrm{NN}} = \phi_1\,\phi_2 - \phi_3\,\phi_4 - 2\,\phi_5\,\phi_6
\end{aligned}
$$

where the unpolarized differential cross section (upon summing spins) is given by

$$
\mathcal{D} = \big(|\phi_1 + \phi_3|^2 + |\phi_1 - \phi_3|^2 + |\phi_2 + \phi_4|^2 + |\phi_2 - \phi_4|^2\big)/4 + |\phi_5|^2 + |\phi_6|^2
$$

with the real one photon exchange helicity amplitudes ϕ_i given below for ions of mass M and charge Ze elastically scattering off electrons (or muons) of mass m.

TRANSVERSE ION & LONGITUDINAL LEPTON SPIN

The analyzing power for light ions of mass M, charge Z, magnetic moment μ, and energy/M, E, polarized transversely in the scattering plane, scattering off static longitudinally polarized electrons of mass m that recoil at an angle θ_e, is

$$\mathcal{D} A_{\mathrm{SL}} \approx \frac{\theta_e}{2}\left[\frac{M}{m} - \left(\frac{\mu\, M}{Z\, m_p} - 1\right)\frac{M\, E}{m_p}\right]\frac{\mu\, M}{Z\, m_p}$$

Introducing a change of variable prompted by the appearance of a factor in ϕ_6, $x = t_0/t - 1$, enables the analyzing power A_{SL} to be written in the form

$$\mathcal{D} A_{\mathrm{SL}} = G_M \left[2mM(M\, E + m) s^{-1/2} F_1/t_0 + s^{1/2} F_2/2M\right] x^{1/2}$$

where the denominator $\mathcal{D}$ appears as a quadratic in x and $a = \mu t_0/(\sqrt{8}\, Z m_p m E)$

$$\mathcal{D} \propto x^2 + \left(E^{-2} + 1\right) x + E^{-2} + a^2$$

The maximum of A_{SL}, or equivalently the minimum of $\mathcal{D}/(\mathcal{D}A_{\mathrm{SL}})$, again involves a quadratic in x with solution

$$6\,x \;=\; \sqrt{12\,a^2+1}\;-\;1$$

indicating that A_{SL} peaks at 22% (41 GeV), 12% (100 GeV), and 5% (200 GeV), in agreement with Gakh et al (2011)[4] (helions factor -1.14) at electron angles

$$(1+2\,m\,E/M)^{1/2}\left[1+(mM\mu E/Zm_p\,s)^2\right]/2E$$

The squared momentum transfer $-t$ corresponding to the peak in (GeV$/c$)^2 is: 0.001 (41 GeV), 0.005 (100 GeV), 0.008 (250 GeV) for protons (helions similar). The form factors were assumed to have static values in evaluating the peak angles. For other observables and kinematic conditions greater values of $-t$ may obtain, providing a way of measuring the form factors, useful for subsequent polarimetry.

ANOMALOUS MAGNETIC MOMENT

Study of the electromagnetic current matrix element leads to an anomaly factor

$$\mu\, M/(m_{\mathrm{p}}\, Z) \;-\; 1$$

for a light ion of mass M, charge Ze, with initial and final 4-momenta p_ν and p'_ν

$$\bar{u}' \left\{ \,(\, p' + p\,)^\mu\, F_1 \;-\; \tfrac{1}{2}\, [\, \gamma^\mu ,\, \gamma^\nu\,]\; (\, p' - p\,)_\nu \; G_{\mathrm{M}} \right\} u \,/\, 2\, M$$

where the electromagnetic form factors $F_1(t)$ & $F_2(t)$, and $G_{\mathrm{E}}(t)$ & $G_{\mathrm{M}}(t)$ with

$$t \;=\; (p' - p)_\nu\, (p' - p)^\nu, \quad \text{(for which, in this metric, } p_\nu\, p^\nu \;=\; M^2\text{)}$$

have static values equal to the charge and magnetic moment of the light fermion

$$F_1(0) \;=\; Z\, e\,, \qquad \frac{G_{\mathrm{M}}(0)}{2\, M} \;=\; \mu' = \mu\, \frac{e}{2\, m_{\mathrm{p}}}$$

noting that the magnetic moment μ' is normally given as μ in nuclear magnetons.

Alternative expressions of the electromagnetic current use form factors $F_1(t)$, $F_2(t)$

$$\bar{u}' \, \{ \, \gamma^\mu \, F_1 \, - \, [\, \gamma^\mu , \, \gamma^\nu \,] \, (p' - p)_\nu \, F_2 \, / 4 \, M \, \} \; u$$

in a normalisation where the various electromagnetic form factors are related by

$$G_{\mathrm{E}}(t) \, = \, F_1(t) \, + \, t \, F_2(t)/4 \, M^2 \, , \qquad G_{\mathrm{M}}(t) \, = \, F_1(t) \, + \, F_2(t)$$

so that a fermionic light ion with charge Ze has anomalous magnetic moment

$$\frac{F_2(0)}{2\,M} = \mu' - \frac{Z\,e}{2\,M} = \frac{Z\,e}{2\,M} \left(\frac{\mu\,M}{m_p\,Z} - 1 \right) .$$

The Dirac magnetic moment here is $Ze/2M$, or Zm_p/M, in nuclear magnetons

The Rosenbluth formula for leptons of mass m colliding with ions of mass M is

$$\frac{\mathcal{D}}{s} = \frac{-t_0}{4\pi} \frac{d\sigma}{dt} = \frac{t \, - \, t_0}{t^2} \left(F_1^{\,2} - \frac{t}{4M^2} F_2^{\,2} \right) + \frac{4\,m^2 M^2}{t^2\,s} G_{\mathrm{E}}^{\,2} + \frac{1}{2\,s} G_{\mathrm{M}}^{\,2}$$

where s is the relativisticaly invariant square of the total centre-of-mass energy and t_0 is the algebraic minimum of the squared momentum transfer variable t

$$-t_0 \;=\; s \,-\, 2\,M^2 \,-\, 2\,m^2 \,+\, \left(M^2 - m^2\right)^2 / \, s$$

Note that $t = t_0$ corresponds to backward scattering and that both the ion's charge and magnetic moment have been incorporated in the electromagnetic form factors. Other asymmetries A_{LL} and A_{NN} are only suitable for polarimetry when the ion energy per mass E is close to 20 GeV in the case of static electrons, in which case the analyzing power is around 70%. Choosing another reference frame, electrons with rapidity lower or above the rapidity of the ion would need to be considered. The CEC, for example, has variable lower electron energies but recoil angles may well be beyond the reach of detection. And there is the difficulty of polarizing the electrons though some polarization would be generated naturally in the returns.

Ion-lepton one photon exchange helicity amplitudes for a proton or helion of mass M with given form factors scattering on an electron or muon of mass m are:

$$
\begin{aligned}
\phi_1 + \phi_3 &= G_{\mathrm{M}} + \left(\frac{2}{t_0} - \frac{2}{t}\right)\left(s - M^2 - m^2\right) F_1 \\
\phi_1 - \phi_3 &= G_{\mathrm{M}} \\
\phi_2 - \phi_4 &= \frac{4mM}{t_0} F_1 + \frac{m}{M} F_2 \\
\phi_2 + \phi_4 &= 0 \\
\phi_5 &= \left(\frac{t_0}{t} - 1\right)^{1/2} \left(\frac{m^2 - M^2 - s}{t_0 \sqrt{s}\,/M} F_1\right) \\
\phi_6 &= \left(\frac{t_0}{t} - 1\right)^{1/2} \left(\frac{m^2 - M^2 + s}{t_0 \sqrt{s}\,/m} F_1 + \frac{\sqrt{s}}{2M} F_2\right)
\end{aligned}
$$

References

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