Mixed QCD-electroweak corrections to vector boson production and their impact on the W-mass measurement

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Outline

- W mass measurements at the LHC:
  - Motivation
  - Methods
- Challenges in calculating QCD-EW corrections
  - Onshell production – infrared subtractions
- IR subtractions in NNLO QCD
- Mixed QCD-EW corrections to Z boson production
- Mixed QCD-EW corrections to W boson production
- Impact on measurements of W-mass

Motivation

- W mass is a fundamental property of an elementary particle.
- Linked to EWSB:

$\sin^2\theta_W = 1 - m_W^2/m_Z^2 = e^2/g^2 \longrightarrow$ Connection between masses and couplings.

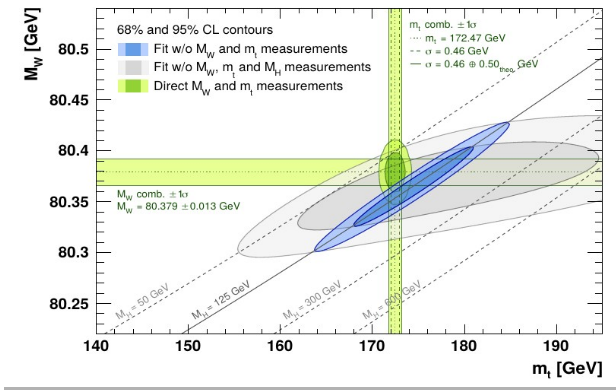
- Radiative corrections: $m_W^2\left(1-\frac{m_W^2}{m_Z^2}\right) = \frac{\pi\alpha}{\sqrt{2}G_F}(1+\Delta r), \quad \Delta r = \Delta r(m_t, m_H, m_Z, ...)$

[Awramik, Czakon, Freitas, Weiglein (2003)]

- Used in global EW fits, test self-consistency of SM.

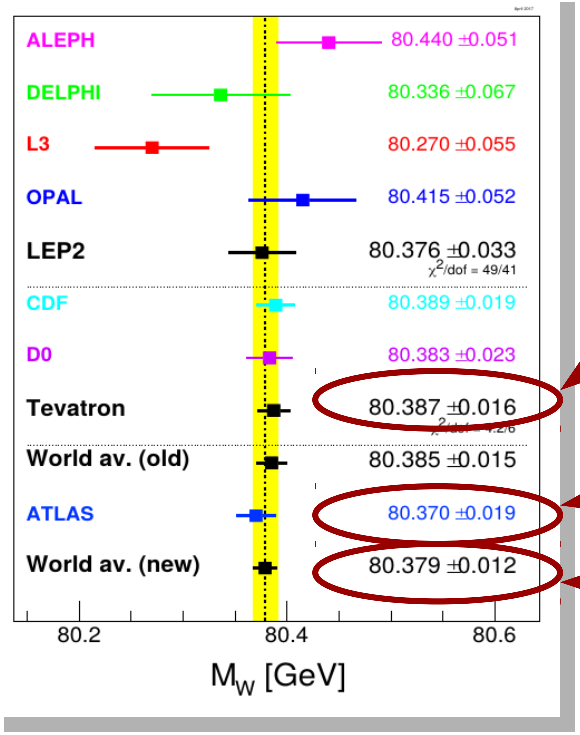
$\longrightarrow m_W = 80.354 \pm 0.007 \text{ GeV}$ [Gfitter Group: Haller *et al.* (2018)]

- Possible probe of BSM physics
  - ➢ Using SMEFT [Bjørn, Trott (2016)]



Experimental measurements

- Theory prediction $m_W = 80.354 \pm 0.007\ \mathrm{GeV}$ sets target precision.



PDG (2019):

Tevatron average

ATLAS (2017)

World average

- Consistent with theory prediction, but higher precision desirable.
- Uncertainty dominated by physics modelling.

Experimental measurements

- W mass directly measured in $pp \to W \to \ell\nu$
- Template fit: simulate data for different values of W-mass and fit to data.
- Three observables: $p_{T,\ell},\ p_{T,\mathrm{miss}},\ m_{T,W}$
- Strongest pull from $p_{T,\ell}$, also most sensitive to higher order corrections. [Carloni Calame *et al.* (2016)].

- Uncontrolled non-perturbative effects enter at the level of $\Lambda_{\mathrm{QCD}}/Q \sim 0.01$

  → theoretical predictions not reliable at the desired precision of 0.1 per mille.
- Use control of process $pp \to Z \to \ell\bar{\ell}$ to calibrate detector response, tune generators, and verify results.

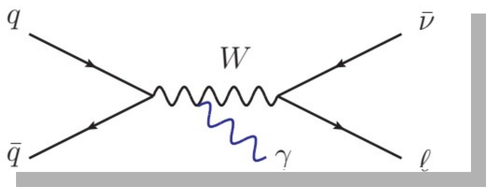
➡ Higher order corrections that decorrelate *W* and *Z* need to be taken into account.

Higher order corrections

- Fixed-order perturbative calculations: expand partonic cross section in $\alpha_s \sim 0.1$ and $\alpha \sim 0.01$

$$\hat{\sigma}ij = \hat{\sigma}_{ij}^{(0,0)} + \alpha_s \hat{\sigma}_{ij}^{(1,0)} + \alpha_s^2 \hat{\sigma}_{ij}^{(2,0)} + \alpha_s^3 \hat{\sigma}_{ij}^{(3,0)} + \ldots + \alpha\, \hat{\sigma}_{ij}^{(0,1)} + \alpha_s \alpha\, \hat{\sigma}_{ij}^{(1,1)} + \ldots$$

- QCD corrections: largely similar for *W* and *Z* production
  - Minor differences: different pdfs, different masses, helicity structures, …
- EW corrections: qualitatively different – *W* charged, can radiate:
- Impact of NLO EW corrections on W-mass measurement studied. [Carloni Calame *et al.* (2016)].
- Investigate impact of mixed QCD-EW corrections.

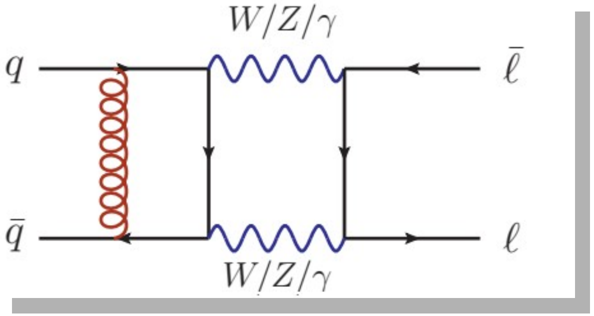


Mixed QCD-EW corrections

Two challenges in computing mixed QCD-EW corrections to $pp \to \ell\bar{\ell}$

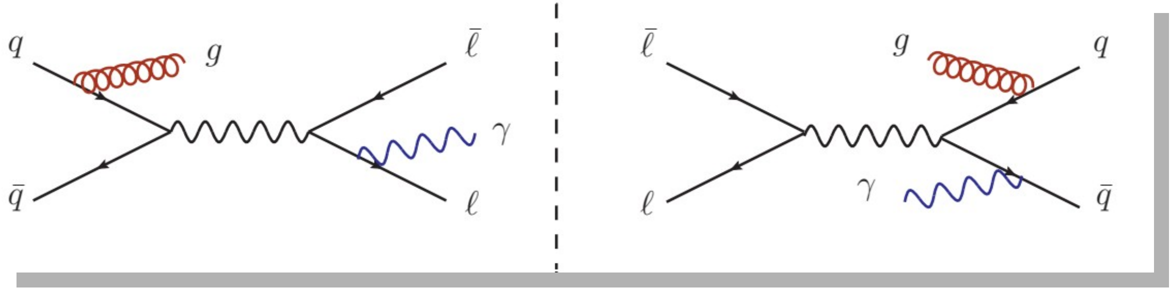
1. Two-loop amplitudes:

- Several energy scales – very demanding!
- Recent computation: [Heller, von Manteuffel, Schabinger, Spiesberger (2020)]



2. QCD and EW singularities:

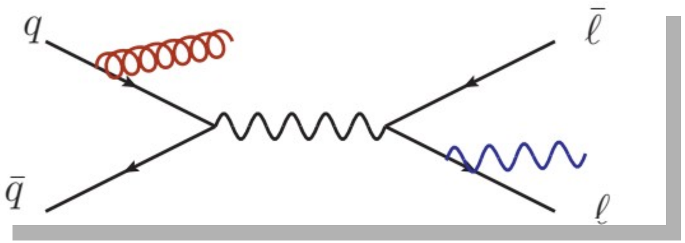
- Infrared singularities arising from radiated and virtual partons and photons.



QCD-EW corrections to onshell vector boson production

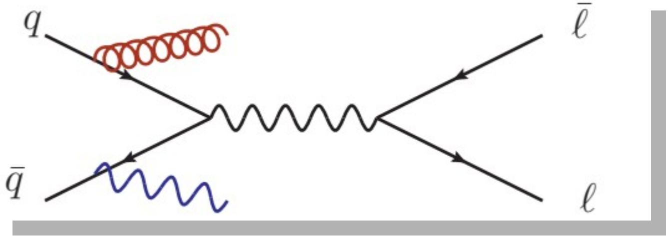
Simplification: consider onshell vector bosons $pp \to V \to \ell\bar{\ell}$

- QCD (production) x EW (decay)



[Dittmaier, Huss, Schwinn, (2014, 2015)]

- QCD x EW (production)

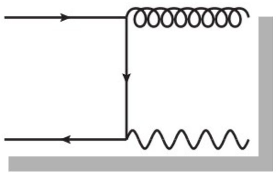


- Two-loop amplitudes → much simpler form factors.
- Major challenge: treating simultaneous QCD and EW IR singularities.
- Insight from NNLO QCD: treatment of IR singularities from double emissions.

Infrared singularities in QCD

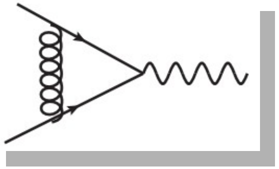
Higher order corrections contain IR singularities from soft and/or collinear radiation.

- Real corrections
  - Integrate over phase space of radiated parton:



$\longrightarrow \int |\mathcal{M}|^2 F_J d\phi_g$ diverges

- Virtual corrections
  - Explicit IR singularities from loop integration



$$\longrightarrow \mathcal{M}_{1-\mathrm{loop}} = \frac{c_{-2}}{\epsilon^2} + \frac{c_{-1}}{\epsilon} + c_0$$

- Singularities unphysical, guaranteed to cancel in sum (KLN theorem).
- Cancellation only manifest after integrating over full phase space of emitted parton:

  → lose kinematic information.

Subtracting IR singularities in QCD

Subtraction scheme:

Extract singularities without integrating over full phase space of radiated parton:

- Singularities manifest as poles in $1/\epsilon$ cancel against poles in virtual correction $\rightarrow$ finite fully differential result.

$$\int |\mathcal{M}|^2 F_J d\phi_d = \int \left(|\mathcal{M}_J|^2 F_J - S\right) d\phi_4 + \int S d\phi_d$$

Finite;

integrate in 4-dim.

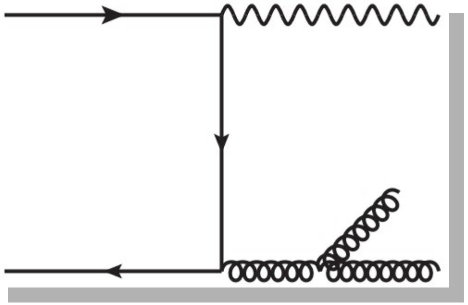
Counterterm;

Explicit singularities

- Subtractions at NLO fully solved. [Catani, Seymour (1996); Frixione, Kunszt, Signer (1996, 1997)]
- Constructing NNLO subtraction schemes is an active area of research.

IR singularities at NNLO in QCD

Complicated singularity structure at NNLO:



Singularities arise when:

- *Either* gluon or *both* gluons → soft.
- *Either* gluon or *both* gluons → collinear to either initial state quark.
- Gluons → collinear to each other.

- Any combination of above – overlapping singularities.
  - Can approach each limit in different ways.
- Need to separate the singularities.
- Multiple approaches: Antennas, STRIPPER, CoLoRFulNNLO, Projection-to-Born, **nested soft-collinear subtraction,** geometric subtraction, local analytic subtraction, ...

Nested soft-collinear subtractions

[Caola, Melnikov, R.R. (2017)]

- Extension of FKS subtraction to NNLO.
- Colour coherence → independent subtraction of soft and collinear divergences.
- Overlapping soft singularities separated by energy ordering.
- Overlapping collinear singularities separated using partitions and sectors.
  - Natural splitting by rapidity.

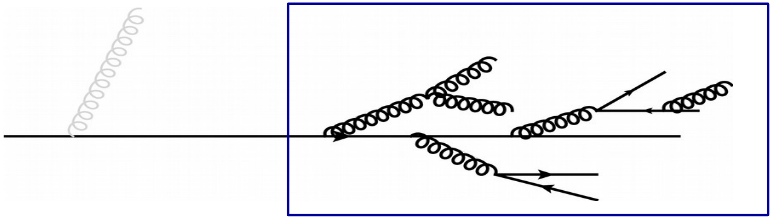
- Fully local and fully analytic.

  [Caola, Melnikov, R.R. (2019); Asteriadis, Caola, Melnikov, R.R. (2019)]

  [Delto, Frellesvig, Caola, Melnikov (2018); Delto, Melnikov (2019)]
- Clear physical origin of singularities (soft & collinear).
- Flexible → use for mixed QCD-EW singularities.

Color coherence

- On-shell, gauge-invariant QCD scattering amplitudes : color coherence.
- Used in resummation & parton showers; not manifest in subtractions.



- Soft gluon cannot resolve details of collinear splittings; only sensitive to total color charge.

➡ No overlap between soft and collinear limits – treated independently:

- Regularize soft singularities first, then collinear singularities.
- Energies and angles decouple.

Soft singularities

- Consider partonic process $q(p_1)\bar{q}(p_2) \to V(p_3)g(p_4)g(p_5)$
- Define $F_{LM}(1,2,4,5) = \mathrm{dLips}_V \, |\mathcal{M}(1,2,4,5,V)|^2 \, \mathcal{F}_{\mathrm{kin}}(1,2,4,5,V)$

- Overlapping double-soft and single-soft limits: order energies:

$$2s \cdot \mathrm{d}\sigma^{\mathrm{RR}} = \int [\mathrm{d}g_4][\mathrm{d}g_5]\theta(E_4 - E_5)F_{LM}(1,2,4,5) \equiv \langle F_{LM}(1,2,4,5)\rangle.$$

→ soft singularities: either double soft or *g*$_5$ soft.

- Regulate the soft singularities:

$$\mathrm{F}_{LM}(1,2,4,5)\rangle = \langle \mathbb{S} F_{LM}(1,2,4,5)\rangle + \langle S_5(I - \mathbb{S})F_{LM}(1,2,4,5)\rangle + \langle (I - S_5)(I - \mathbb{S})F_{LM}(1,2,4,5)\rangle.$$

Double- and single-soft counterterms

Soft-subtracted term – still has (overlapping) collinear singularities

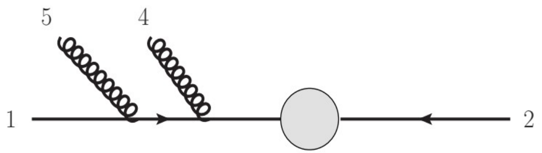
Phase-space partitioning

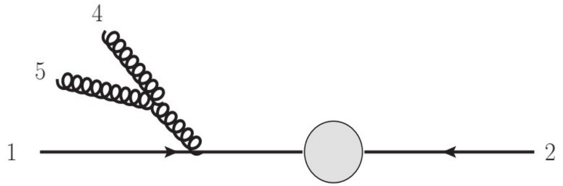
Separate overlapping collinear limits in two stages:

1. Introduce phase-space partitions $1 = w^{14,15} + w^{24,25} + w^{14,25} + w^{15,24}.$

$$\mathrm{C}_{42} w^{14,15} = C_{52} w^{14,15} = 0 \;\Longrightarrow\; w^{14,15} \text{ contains } C_{41},\ C_{51},\ C_{45}$$

Triple collinear partition

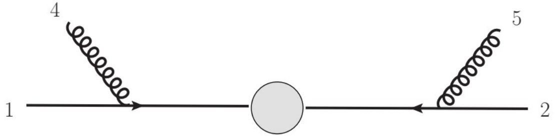




and

$$\mathrm{C}_{42} w^{14,25} = C_{51} w^{14,25} = C_{45} w^{14,25} = 0 \;\Longrightarrow\; w^{14,25} \text{ contains } C_{41},\ C_{52}$$

Double collinear partition



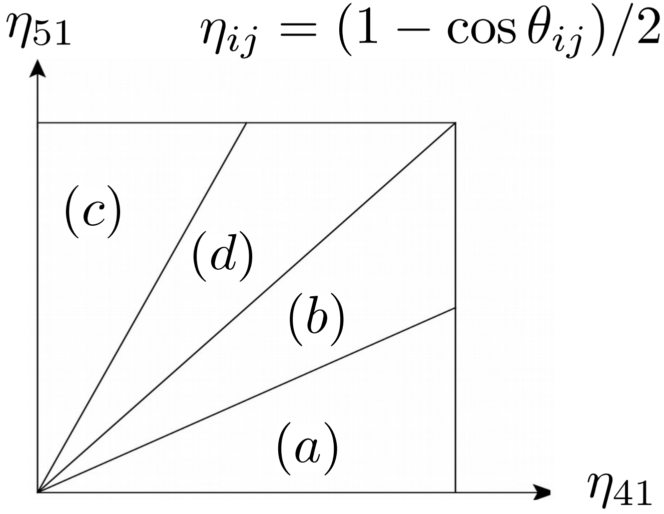
Sector Decomposition

2. Sector decomposition to remove remaining overlapping singularities in triple collinear partitions.

- Define angular ordering to separate singularities.

$$\eta_{ij} = (1-\cos\theta_{ij})/2$$

$$1 = \theta\left(\eta_{51} < \frac{\eta_{41}}{2}\right) + \theta\left(\frac{\eta_{41}}{2} < \eta_{51} < \eta_{41}\right) + \theta\left(\eta_{41} < \frac{\eta_{51}}{2}\right) + \theta\left(\frac{\eta_{51}}{2} < \eta_{41} < \eta_{51}\right) \equiv \theta^{(a)} + \theta^{(b)} + \theta^{(c)} + \theta^{(d)}.$$



- Thus the limits are

$$\theta^{(a)} : C_{51} \qquad \theta^{(b)} : C_{45}$$
$$\theta^{(c)} : C_{41} \qquad \theta^{(d)} : C_{45}$$

- Achieved using angular phase-space parametrization [Czakon (2010, 2011)].

Removing collinear singularities

Separates collinear limits – subtract iteratively from soft-regulated term

$$\left\langle (I - S_5)(I - \mathbb{S}) F_{LM}(1,2,4,5) \right\rangle =$$

$$\left\langle F_{LM}^{s_r c_s}(1,2,4,5) \right\rangle + \left\langle F_{LM}^{s_r c_t}(1,2,4,5) \right\rangle + \left\langle F_{LM}^{s_r c_r}(1,2,4,5) \right\rangle$$

(Soft-regulated) single and triple collinear counterterms.

Fully subtracted term – finite

Integrate four singular counterterms

$$\left\langle \mathbb{S} F_{LM}(1,2,4,5) \right\rangle \quad \left\langle S_5 (I - \mathbb{S}) F_{LM}(1,2,4,5) \right\rangle \quad \left\langle F_{LM}^{s_r c_s}(1,2,4,5) \right\rangle \quad \left\langle F_{LM}^{s_r c_t}(1,2,4,5) \right\rangle$$

over unresolved phase space :

- cancel IR poles against loop amplitudes;
- Finite remainder: subtraction counterterm.

NNLO corrections to Drell-Yan production

Separates collinear limits – subtract iteratively from soft-regulated term

$$\left\langle (I - S_5)(I - \not{S}) F_{LM}(1,2,4,5) \right\rangle =$$

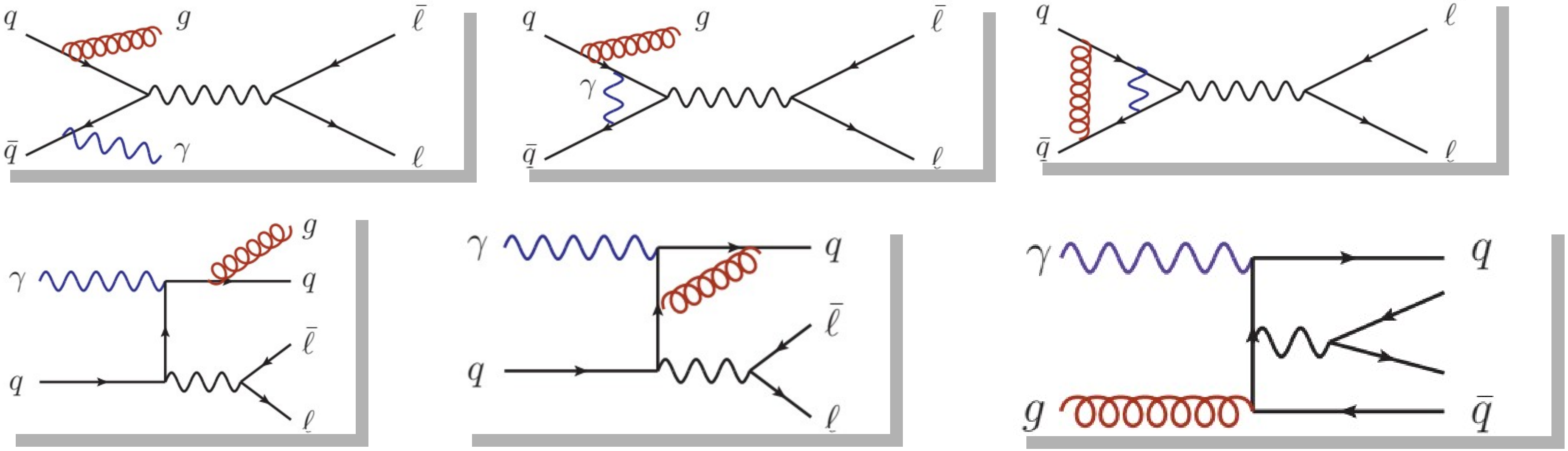
$$\left\langle F_{LM}^{s_r c_s}(1,2,4,5) \right\rangle + \left\langle F_{LM}^{s_r c_t}(1,2,4,5) \right\rangle + \left\langle F_{LM}^{s_r c_r}(1,2,4,5) \right\rangle$$

- Developed fully differential parton-level code for Drell-Yan production at NNLO in QCD.
- Isolate individual colour factors – detailed checks against analytic results of [Hamberg, van Neerven, Matsuura (1990)].

- cancel IR poles against loop amplitudes;
- Finite remainder: subtraction counterterm.

Return to QCD-EW corrections

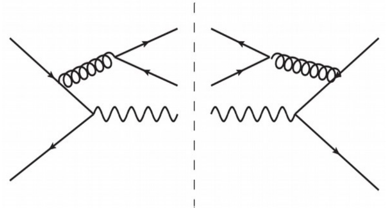
Consider QCD-QED corrections to $pp \to Z \to \ell^- \ell^+$



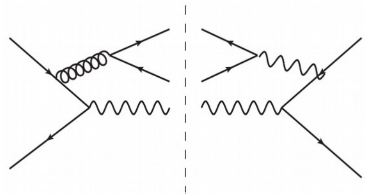
➡ NNLO QCD corrections with one gluon replaced by a photon.

QCD-QED corrections to Z production

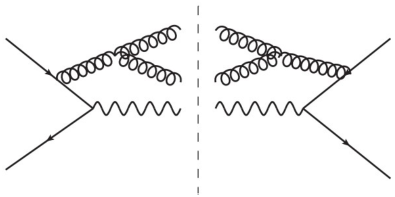
Abelianization achieved by modifying colour factors. [De Florian, Der, Fabre (2018)]



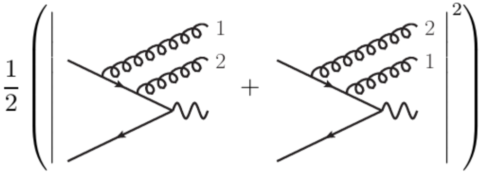
$\sim C_F T_R \longrightarrow$



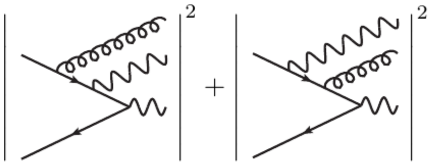
$= 0$



$\sim C_F C_A \longrightarrow 0$



$\sim C_F^2 \longrightarrow$



$\sim 2e_q^2 C_F$

$T_R \to 0$

$C_A \to 0$

$C_F^2 \to 2e_q^2 C_F$

(quark-antiquark channel)

$C_F^2 \to e_q^2 C_F$

(other channels)

QCD-QED results for Z production

- Procedure applied by [De Florian, Der, Fabre (2018)] to analytic formulas for inclusive Drell-Yan production at NNLO in QCD [Hamberg, van Neerven, Matsuura (1991)].

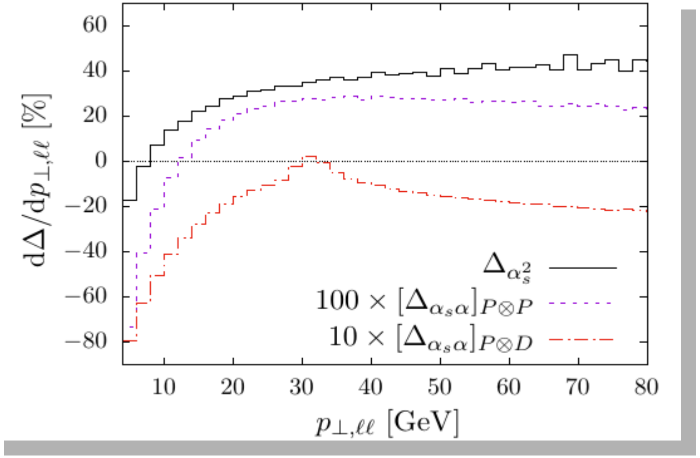
  → QCD-QED corrections to cross section.

- Procedure applied to fully differential corrections using nested soft-collinear subtraction scheme.

  [Delto, Jaquier, Melnikov, R.R. (2019)]

  → fully differential QCD-QED corrections.

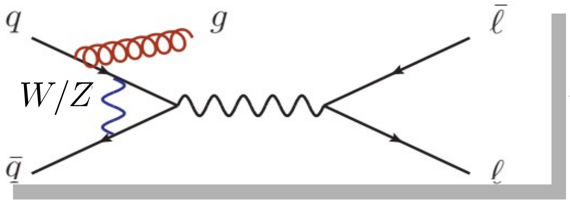
- Corrections generally below per mille level.



- IR singularities arise only from QCD-QED corrections – abelianization solves this problem for QCD-EW corrections to Z production.

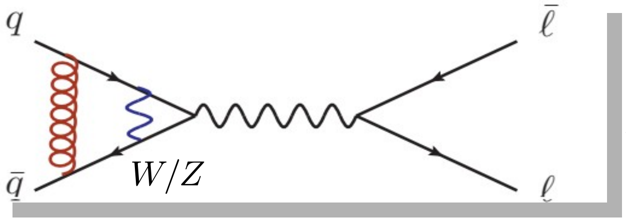
QCD-EW corrections for Z production

- QCD-EW corrections to Z boson production: include QCD-weak corrections.
- Contain virtual weak bosons



→ OpenLoops

[Cascioli, Maierhöfer, Pozzorini (2012); Buccioni, Pozzorini, Zoller (2018); Buccioni *et al.* (2019)]



→ [Kotikov, Kühn, Veretin (2008)]

+ renormalization terms

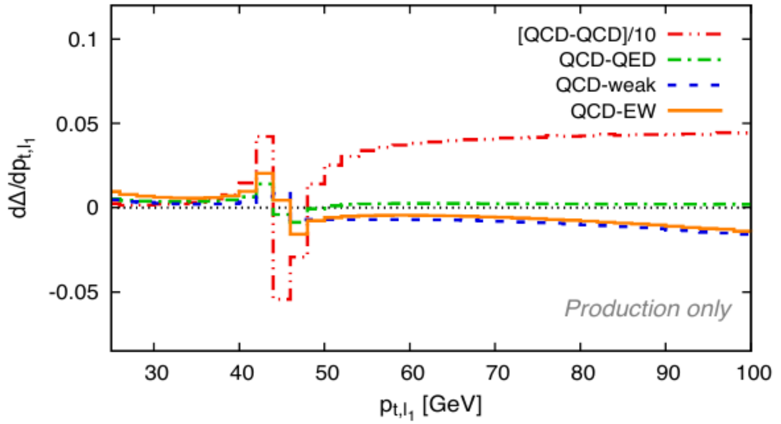
QCD-EW results for Z production

➡ Mixed QCD-EW corrections to $pp \to Z \to \ell^- \ell^+$

[Buccioni, Caola, Delto, Jaquier, Melnikov, R.R. (2020)]

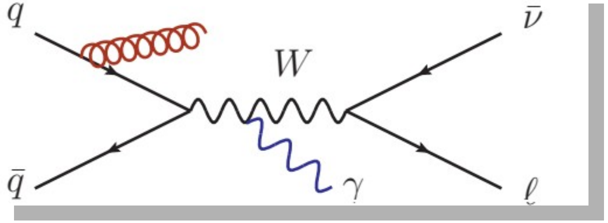
- Fully differential.
- Good agreement with inclusive calculation [Bonciani, Buccioni, Rana, Vicini (2020)]
- QCD-weak effects dominate

  → QCD-EW corrections ~ 0.1%.
- Corrections strongly cut-dependent.
- No clear hierarchy with NNLO QCD.



QCD-EW corrections to *W* boson production

Can we do the same for $pp \to W \to \ell\nu$?



- Qualitatively new feature: photon radiated off W.
- Collinear limits regulated by W-mass, but soft limit is singular.
- Changes form of eikonal function in soft limit:

Soft gluon → $\mathrm{Eik}_g(p_1, p_2; p_g) = \dfrac{2C_F(p_1 \cdot p_2)}{(p_1 \cdot p_g)(p_2 \cdot p_g)}$

Soft photon → $$\mathrm{Eik}_\gamma(p_1, p_2, p_W; p_\gamma) = Q_u Q_d \frac{2(p_1 \cdot p_2)}{(p_1 \cdot p_\gamma)(p_2 \cdot p_\gamma)} - Q_W^2 \frac{p_W^2}{(p_W \cdot p_\gamma)^2} + Q_W \left(Q_u \frac{2(p_W \cdot p_1)}{(p_W \cdot p_\gamma)(p_1 \cdot p_\gamma)} - Q_d \frac{2(p_W \cdot p_2)}{(p_W \cdot p_\gamma)(p_2 \cdot p_\gamma)} \right)$$

➡ Cannot just abelianize as for *Z* production – more substantial changes to subtraction scheme needed.

QCD-EW corrections to *W* boson production

Can make subtraction scheme simpler:

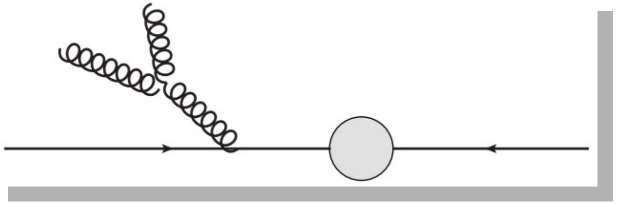
- Recall NNLO QCD: soft limits of gluons overlap → introduced energy ordering.
- Mixed QCD-EW: soft limits of gluons and photons are independent → no energy ordering needed.

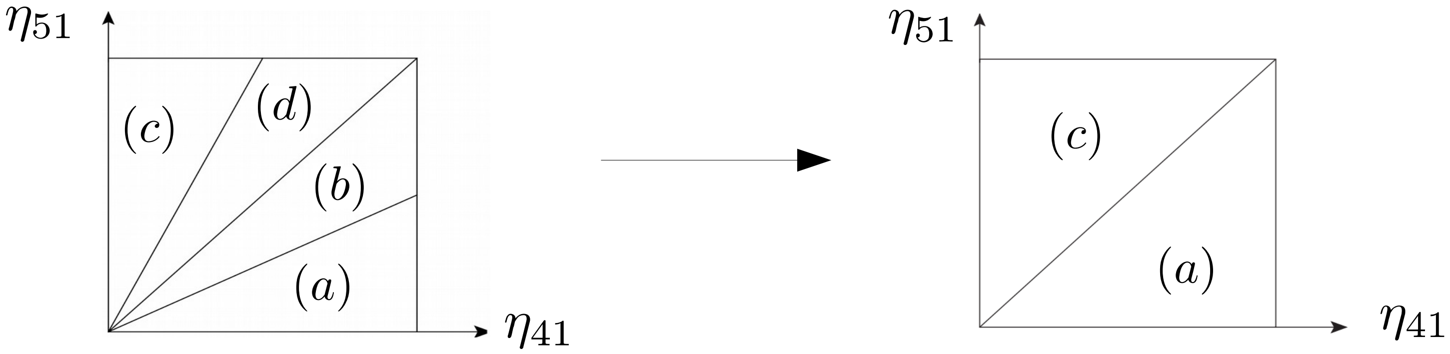
$$\mathrm{F}_{LM}(1,2,4,5)\rangle = \langle \not{S} F_{LM}(1,2,4,5)\rangle + \langle S_5(I-\not{S})F_{LM}(1,2,4,5)\rangle + \langle (I-S_5)(I-\not{S})F_{LM}(1,2,4,5)\rangle .$$

$$\Rightarrow \langle F_{LM}(1,2,4,5)\rangle = \langle S_g S_\gamma F_{LM}(1,2,4,5)\rangle + \langle S_\gamma(I-S_g)F_{LM}(1,2,4,5)\rangle + S_g(I-S_\gamma)F_{LM}(1,2,4,5)\rangle + \langle (I-S_5)(I-S_\gamma)F_{LM}(1,2,4,5)\rangle .$$

- Soft subtraction: iterated NLO-like soft limits.

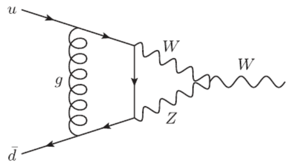
QCD-EW corrections to W boson production

- Genuine NNLO-like singularities in collinear limits → require phase-space partitioning and sectoring.
- Fewer collinear singularities:  disappears.
- Fewer sectors required:.

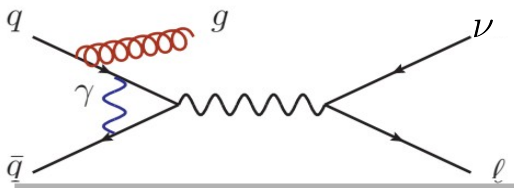


QCD-EW corrections to *W* boson production

- First full computation of two-loop form factor:



- Real-virtual amplitudes → OpenLoops



➡ Mixed QCD-EW corrections to $pp \to W \to \ell\nu$

[Behring, Buccioni, Caola, Delto, Jaquier, Melnikov, R.R. (2020)]

We now have all ingredients to calculate impact of QCD-EW corrections on W mass determination.

W mass determination

- *Estimate* effect of QCD-EW corrections on W mass measurement.
- Decorrelated corrections between *Z* and *W* production.
- Correlation between average transverse momentum of leptons and mass of boson:

$$\frac{m_W}{m_Z} = \frac{\langle p_{T,l}^W \rangle}{\langle p_{T,l}^Z \rangle} \Rightarrow m_W^{\text{meas.}} = m_Z \frac{\langle p_{T,l}^{W,\text{meas.}} \rangle}{\langle p_{T,l}^{Z,\text{meas.}} \rangle} C_{\text{th.}}$$

- Theoretical correction: assume input masses, compute W-mass, and compare with input W-mass.

$$\Rightarrow C_{\text{th.}} = \frac{m_W^{\text{in}}}{m_Z^{\text{in}}} \frac{\langle p_{T,l}^{Z,\text{th.}} \rangle}{\langle p_{T,l}^{W,\text{th.}} \rangle}$$

→ estimate impact of decorrelations in *W* and *Z* spectra from higher order corrections:

$$\frac{\delta m_W^{\text{meas.}}}{m_W^{\text{meas.}}} = \frac{\delta C_{\text{th.}}}{C_{\text{th.}}} = \frac{\delta \langle p_{T,l}^{Z,\text{th.}} \rangle}{\langle p_{T,l}^{Z,\text{th.}} \rangle} - \frac{\delta \langle p_{T,l}^{W,\text{th.}} \rangle}{\langle p_{T,l}^{W,\text{th.}} \rangle}$$

Impact on W mass determination

Shifts in W-mass: inclusive setup

- NLO EW: $\Delta m_W = 1 \text{ MeV}$
- QCD-EW: $\Delta m_W = -7 \text{ MeV}$

➔ Impact of QCD-EW corrections larger than NLO EW:

- NLO EW corrections **suppressed** in G_μ scheme.
- NLO EW corrections **more correlated** between *W* and *Z* production.
- Consider QCD-EW corrections to *W* production only:
  - NLO EW: $\Delta m_W = -31 \text{ MeV}$
  - QCD-EW: $\Delta m_W = 54 \text{ MeV}$

G_μ scheme

$m_Z = 91.1876 \text{ GeV}$

$m_W = 80.398 \text{ GeV}$

$m_t = 173.2 \text{ GeV}$

$m_H = 125 \text{ GeV}$

$G_F = 1.16339 \cdot 10^{-5} \text{ GeV}^{-2}$

NNPDF31_luxQED

$\mu_R = \mu_F = m_V/2$

Impact on W mass determination

Shifts in W-mass: fiducial setup

- Inclusive setup: $\Delta m_W = -7 \text{ MeV}$
- "ATLAS" cuts: $\Delta m_W = -17 \text{ MeV}$
- "Tuned" cuts: $\Delta m_W = -1 \text{ MeV}$

➔ Cuts can have dramatic impact: shifts vary by factor of 20.

➢ "ATLAS" cuts have stronger cuts on leptons from (lighter) *W* than from *Z* → decorrelation.

➔ QCD-EW shifts potentially relevant for target precision of 8 MeV.

$p_{T,\ell}^{Z} > 25 \text{ GeV}; |\eta_\ell^{\mathrm{Z}}| < 2.4$

"ATLAS" cuts: $\mathrm{p}_{T,\ell}^{W} > 30 \text{ GeV}; \mathrm{p}_{\mathrm{T,miss}}^{\mathrm{W}} > 30 \text{ GeV}; |\eta_\ell^{\mathrm{W}}| < 2.4.$

"Tuned" cuts: $\mathrm{p}_{T,\ell}^{W} > 25.44 \text{ GeV}; \mathrm{p}_{\mathrm{T,miss}}^{\mathrm{W}} > 25.44 \text{ GeV}; |\eta_\ell^{\mathrm{W}}| < 2.4.$

Interpretation

- These results are **estimates** of impact of QCD-EW corrections on W-mass measurements at the LHC.
- Indicate that QCD-EW corrections could be relevant for 0.1 permille precision on W-mass measurements.
- Further investigations are essential:
  - What is the impact when using the full transverse momentum spectrum?
  - What is the impact on other observables?
  - How well are these captured with standard experimental simulation tools?
  - How reliable are these results – do we need to include parton showers to handle Sudakov shoulder?
  - ...

Conclusions

- Performed first fully differential calculation of mixed QCD-EW corrections to onshell *W* and *Z* boson production.
- IR singularities treated using nested soft-collinear subtractions:
  - *Z* production: abelianization of NNLO QCD subtraction procedure.
  - *W* production: more dramatic changes to subtractions.
- Estimated impact on measurement of W-mass at LHC ~ 10 MeV.
  - Strongly cut-dependent.
  - Potentially relevant for target uncertainty of 0.1 per mille.
  - Further investigations needed.

Thank you for your attention!