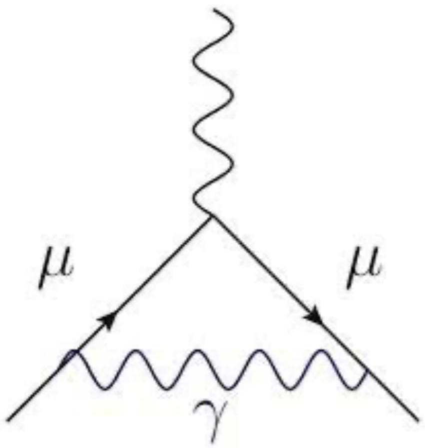
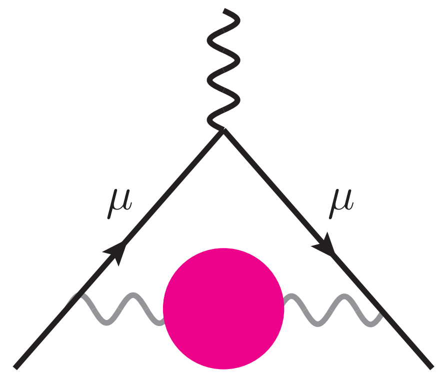
Discovering the new physics of g-2 with a muon collider

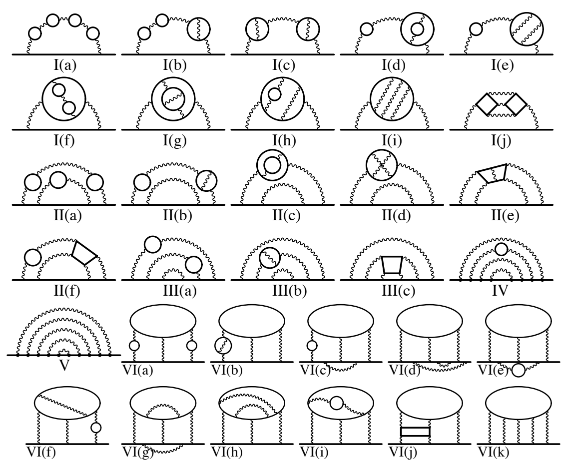
Yoni Kahn
University of Illinois at Urbana-Champaign
BNL HEP theory seminar, 4/1/21

w/R. Capdevilla, D. Curtin, G. Krnjaic
arXiv:2006.16277, arXiv:2101.10334

The anomaly







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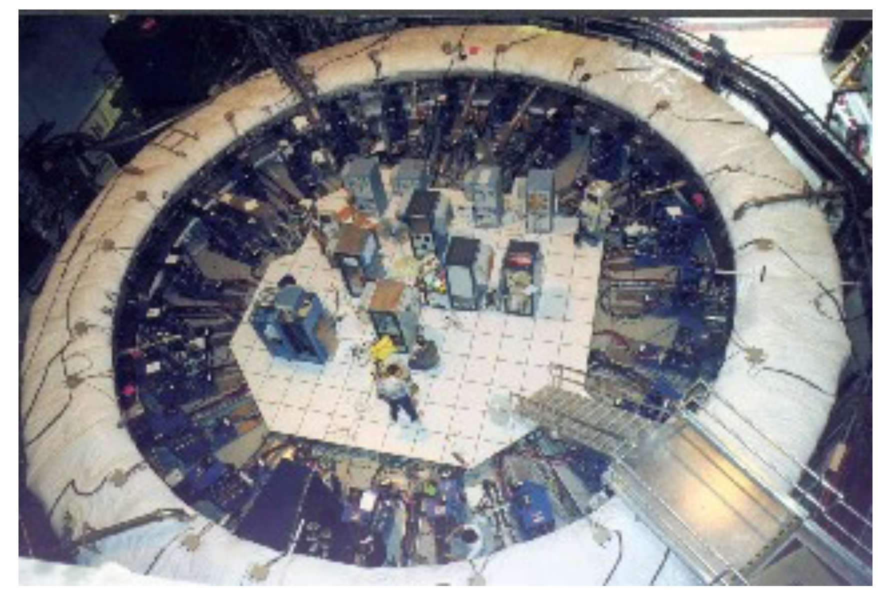


Tour-de-force theory calculation:

$$a_\mu^{\rm SM} = a_\mu^{\rm QED} + a_\mu^{\rm EW} + a_\mu^{\rm HVP,\,LO} + a_\mu^{\rm HVP,\,NLO} + a_\mu^{\rm HVP,\,NNLO} + a_\mu^{\rm HLbL} + a_\mu^{\rm HLbL,\,NLO}$$
$$= 116\,591\,810(43) \times 10^{-11}\,.$$

E821 @ BNL (2004):

$$a_\mu^{\rm exp} = 116\,592\,089(63) \times 10^{-11}$$

$$\Delta a_\mu := a_\mu^{\rm exp} - a_\mu^{\rm SM} = 279(76) \times 10^{-11} \approx 3.7\sigma$$

Next week...

First results from the Muon g-2 experiment at Fermilab

April 7, 2021, 10:00 am US/Central

Kevin Pitts, Fermilab

Aida El-Khadra, UIUC

Chris Polly, Fermilab

The first results from the Muon g-2 experiment at Fermilab will be unveiled and discussed in a special seminar to be held Wednesday, April 7, 2021, at 10:00 AM US Central Time.

The Muon g-2 experiment searches for telltale signs of new particles and forces by examining the muon's interaction with a surrounding magnetic field. By precisely determining the magnetic moment of the muon and comparing with similarly exact theoretical predictions, the experiment is sensitive to new physics lurking in the subatomic quantum fluctuations surrounding the muon. A previous experiment performed two decades ago at Brookhaven National Laboratory revealed an intriguing hint of such physics. The highly anticipated result from Fermilab pushes the precision of the experiment into uncharted territory in the quest to confirm or refute that finding.

The experimental result will be presented by Chris Polly, Fermilab physicist and co-spokesperson for the Muon g-2 scientific collaboration, following a summary of the current theoretical status given by Aida El-Khadra, a UIUC theoretical physicist and co-chair of the Muon g-2 Theory Initiative.

Seminar agenda:
10:00 – 10:05 Introduction — Kevin Pitts, Fermilab chief research officer
10:05 – 10:20 Theory overview — Aida El-Khadra, UIUC theoretical physicist
10:20 – 11:00 Muon g-2 results — Chris Polly, Fermilab experimental physicist
11:00 – 11:20 Question & Answer

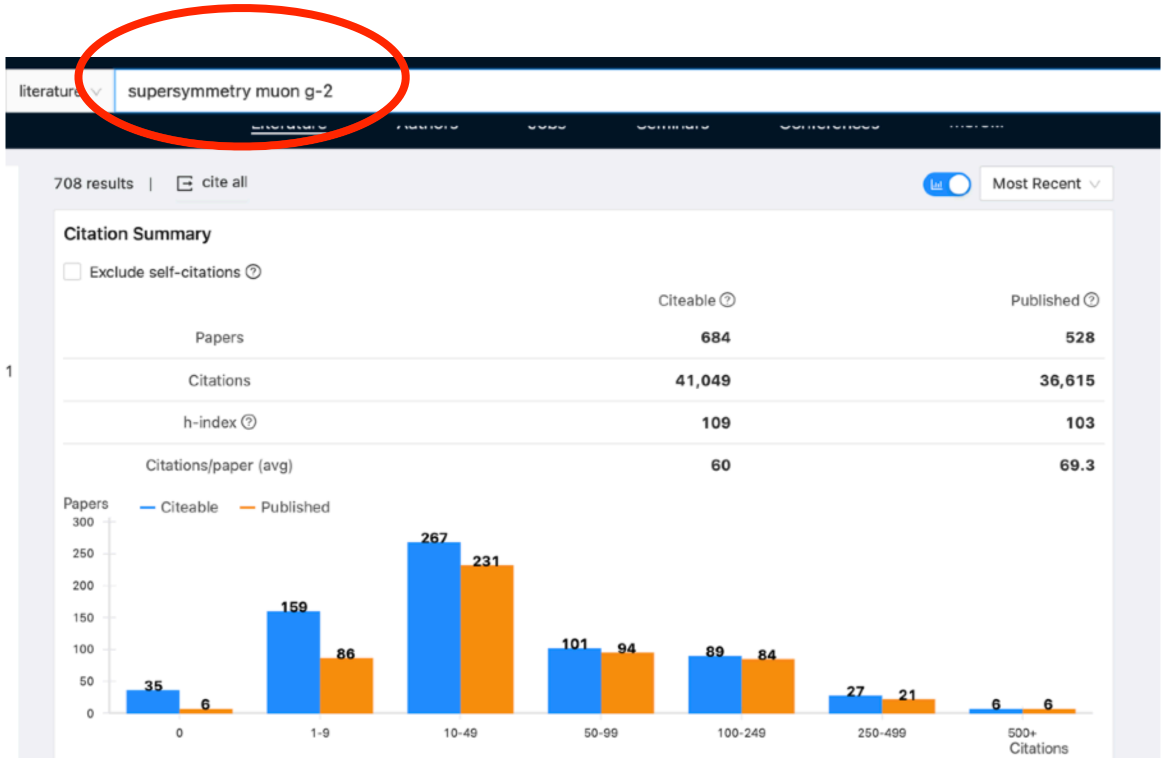
Connection information:

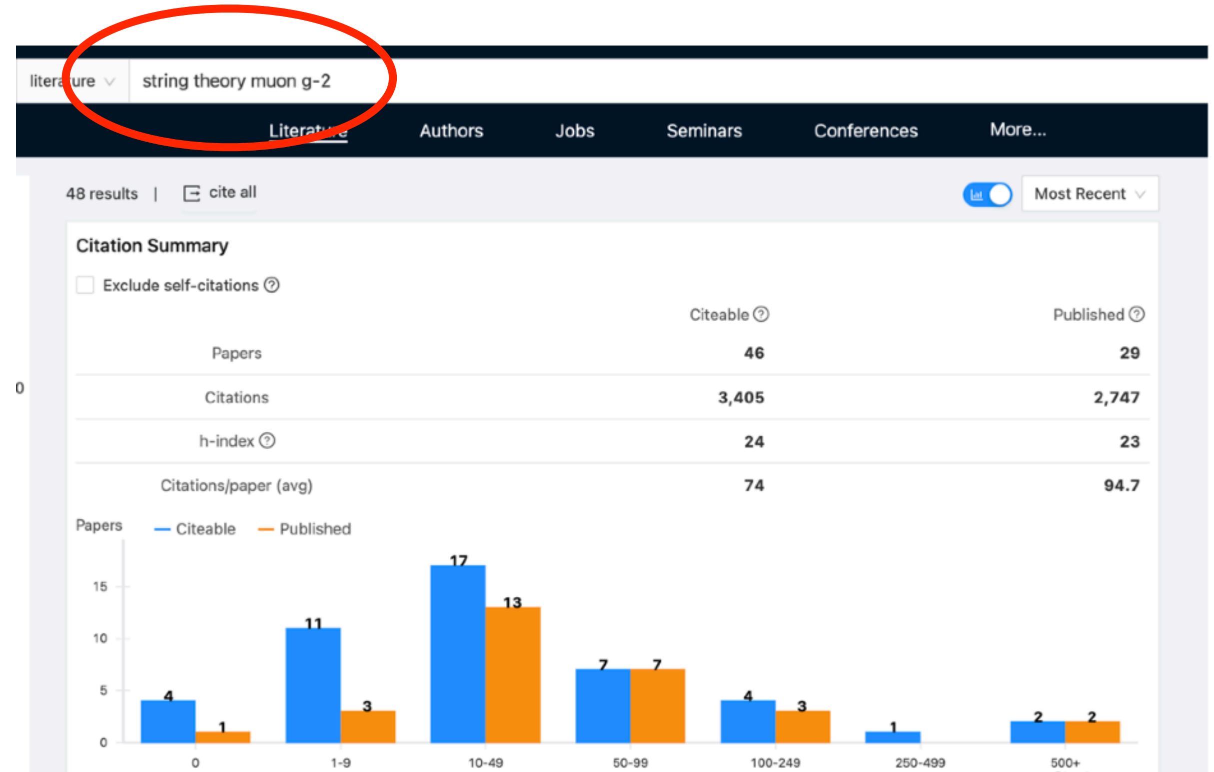
Zoom information to appear at this website. Live captions will be available.

Let's assume that the anomaly is real. (E.g. FNAL finds same central value as BNL, theory calculations are correct.) What can we learn?

Many models

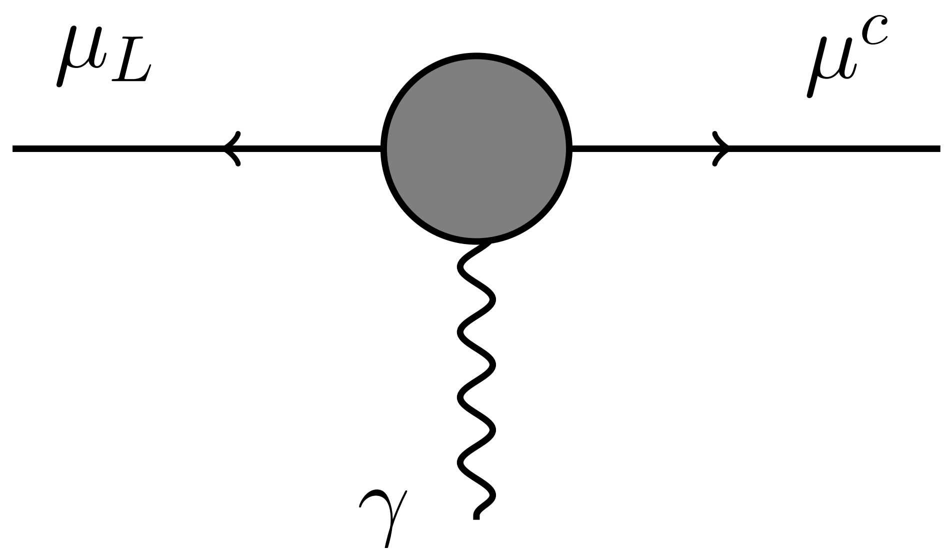
Pick your favorite theoretical framework, it can probably explain g-2:





This is NOT the focus of this talk.
Want a model-independent framework
to evaluate searches for new physics which explain g-2

Model-independent EFT analysis

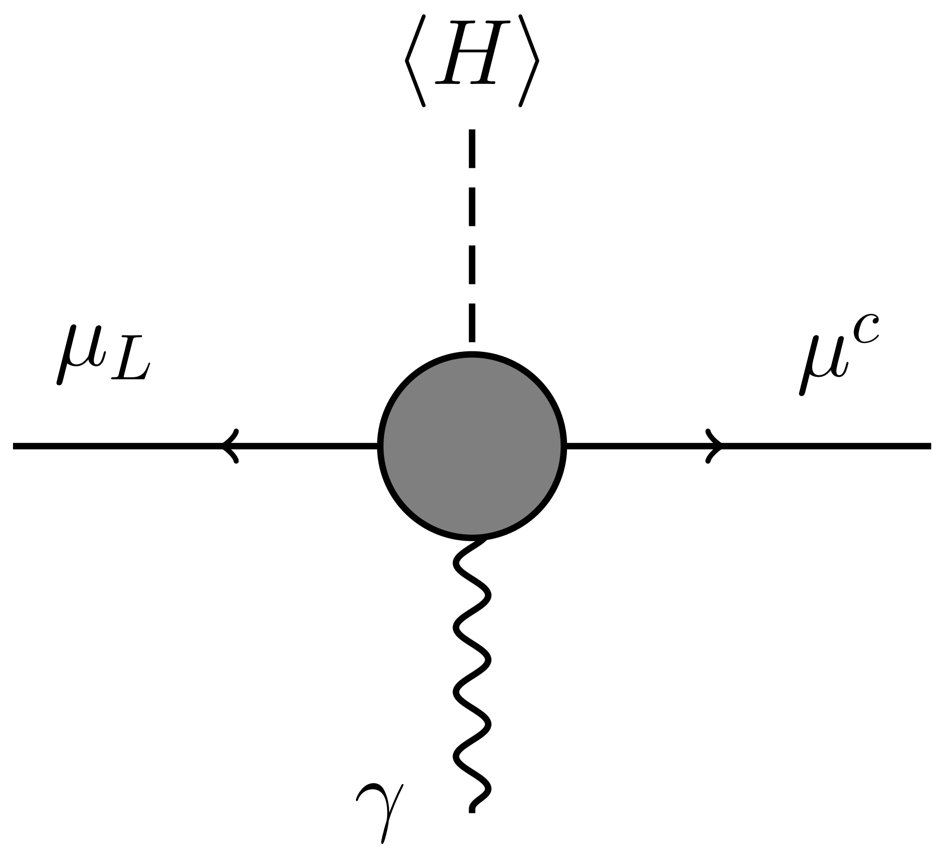


$$\frac{1}{M}(\mu_L \sigma^{\nu\rho} \mu^c) F_{\nu\rho}$$

Generated by a dimension-5 operator. What is the largest M can be?

Answer: much larger than the EW scale, so have to write the SM gauge-invariant version of this operator

Model-independent EFT analysis



$$\frac{1}{M^2} H^\dagger (L \sigma^{\nu\rho} \mu^c) F_{\nu\rho}$$

Measured value of g-2 $\implies M \lesssim 250 \text{ TeV}$

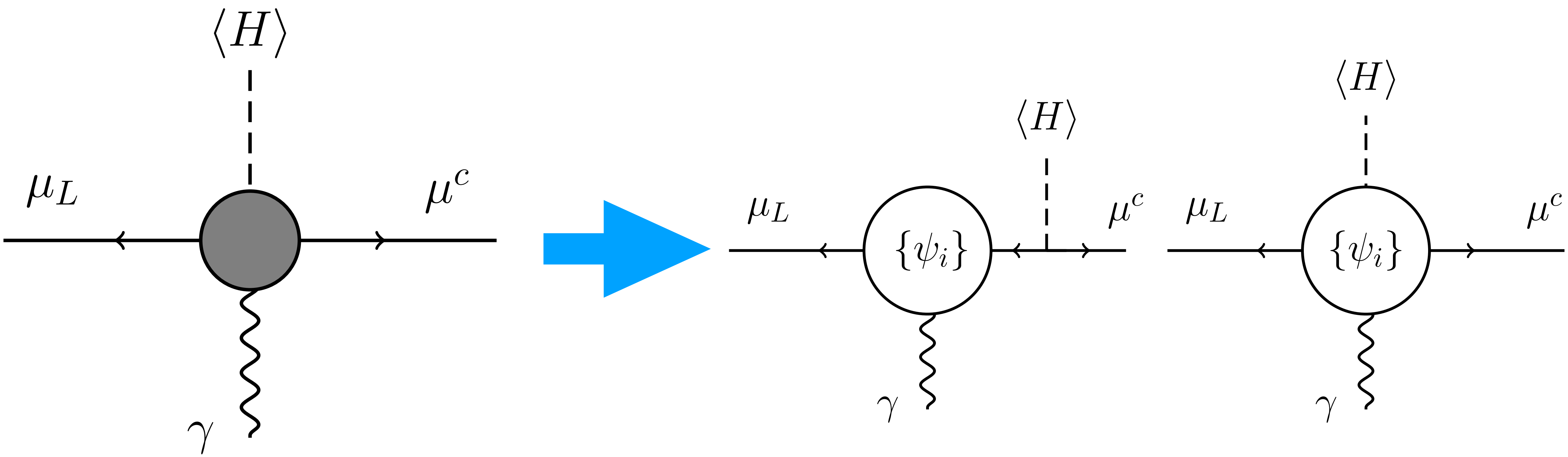
But, there must also be a new contribution to $\mu\mu \to h\gamma$

Can probe this at much lower energy scales: a 10 TeV muon collider!

[Buttazzo and Paridisi, 2012.02769]

“Model-exhaustive” analysis

Additional assumption of perturbativity means we can resolve the blob into individual loop diagrams:



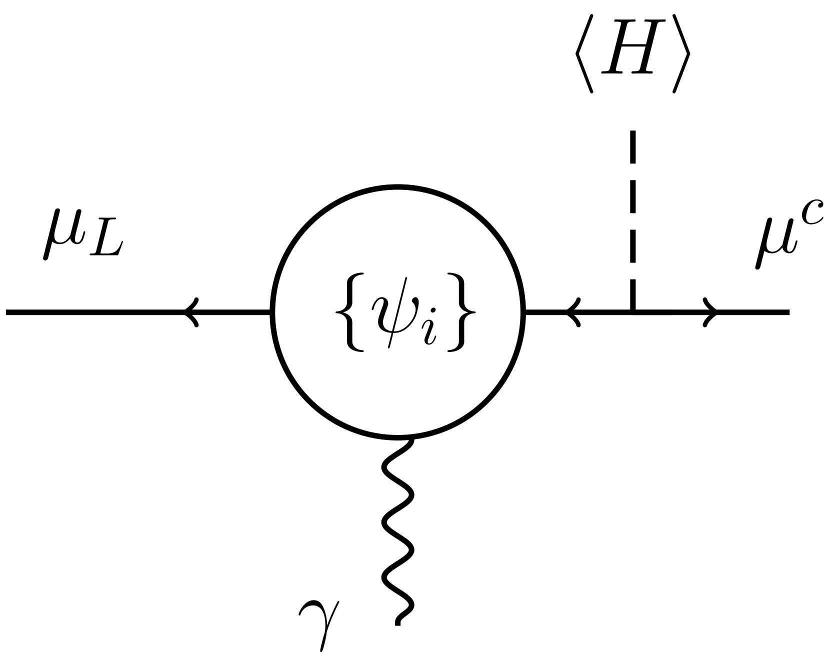
New fields $\{\psi_i\}$ have specific representations under SM gauge group. Each such model predicts a different signature, but all involve couplings to the SM (either directly to the muon, or through EW charges)

What muon collider parameters do we need to probe all such models?

Two classes of scenarios

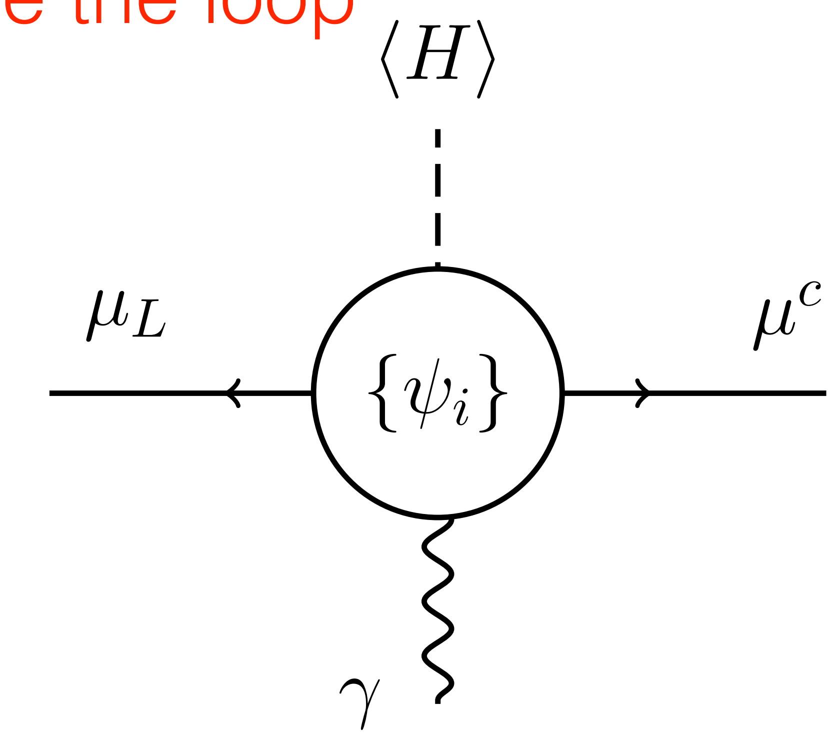
Need a Higgs insertion and a chirality flip, which can go:

outside the loop



"singlet scenarios"
(new physics can be SM neutral)

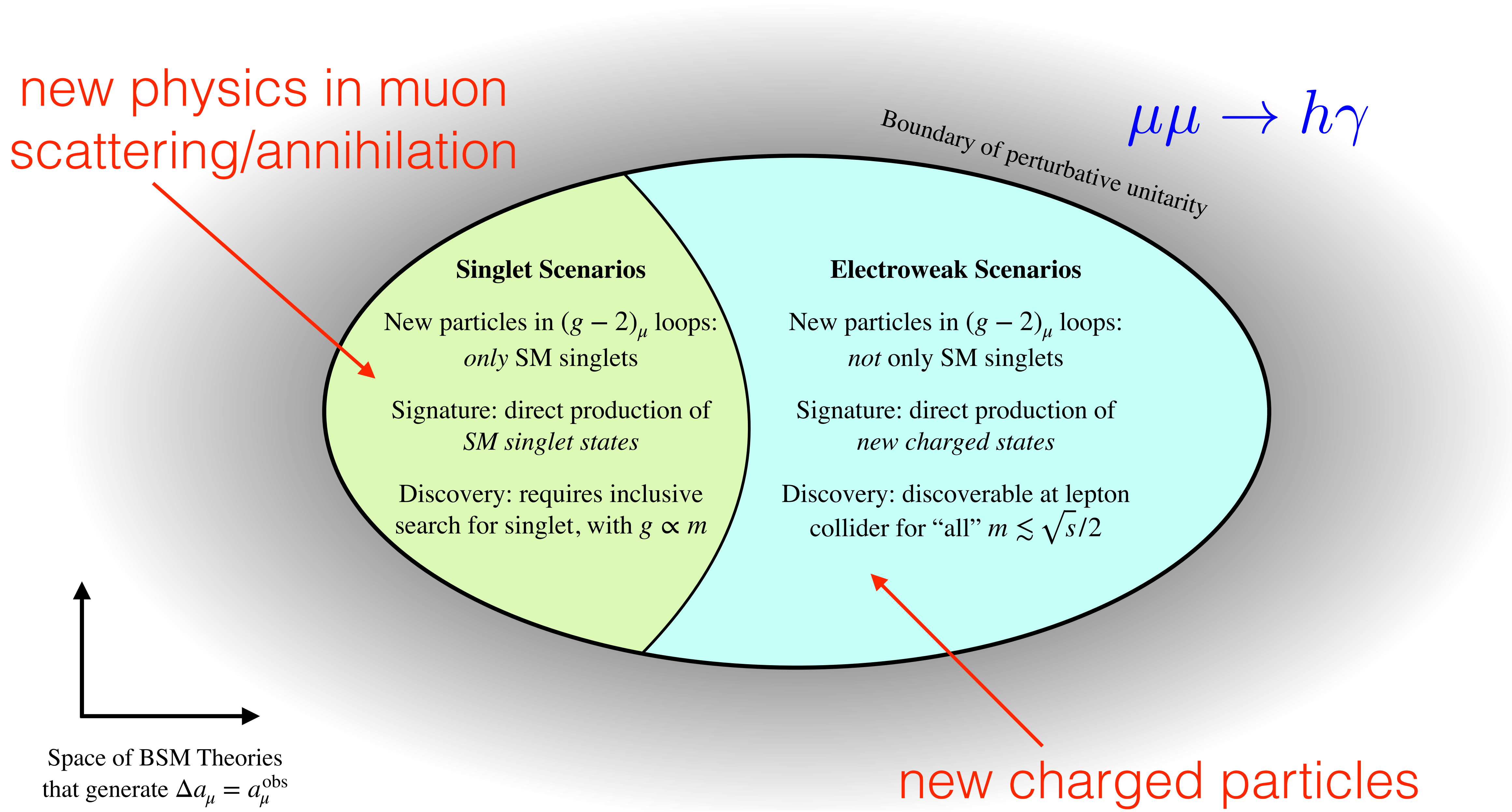
inside the loop



"electroweak scenarios"
(new physics has EW charges)

Small muon Yukawa: singlet scenarios usually give smaller contribution, so new physics scale is lower to match observed g-2

The general picture



Observed value of g-2 means this parameter space is bounded in BSM particle masses: how do we guarantee discovery?

A no-lose theorem

1. Assume the g-2 anomaly is real (fingers crossed for next week…)
2. Use fixed-target or B-factory searches to discover or exclude singlet models below the GeV scale. If no new physics…
3. Use a 215 GeV muon collider with 0.4 ab^{-1} luminosity, and/or 3 TeV with 1 ab^{-1}, to discover or exclude all remaining singlet scenarios. If no new physics…
4. Use a 30 TeV muon collider to discover EW scenarios. If no new physics…
5. Deviations in $\mu\mu \to h\gamma$ must be present, **and** there is an explicit fine-tuning in the Higgs mass and likely muon mass and flavor sectors as well. Nature is fine-tuned!

(6.) If you can build a ~100 TeV muon collider, you are guaranteed production of charged states or perturbative unitarity is violated

Why a muon collider isn't crazy

Input to the European Particle Physics Strategy Update

Muon Colliders

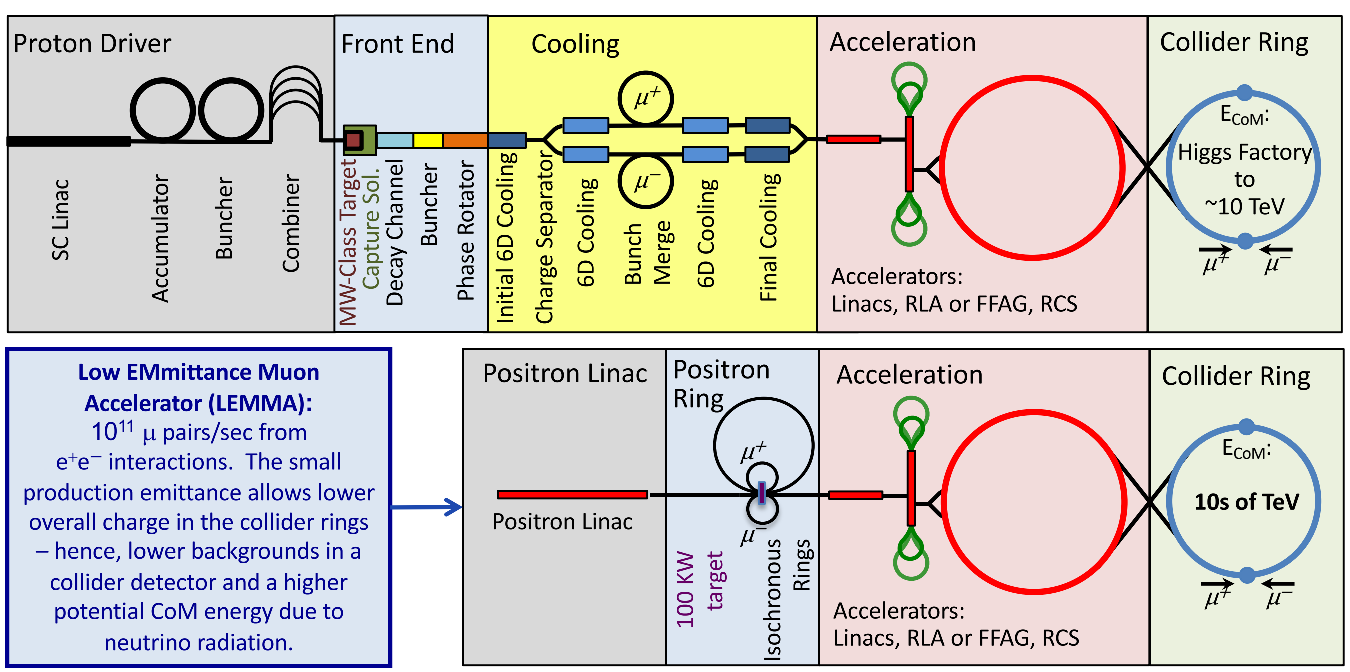
The Muon Collider Working Group
Jean Pierre Delahaye[1], Marcella Diemoz[2], Ken Long[3], Bruno Mansoulié[4], Nadia Pastrone[5] (chair), Lenny Rivkin[6], Daniel Schulte[1], Alexander Skrinsky[7], Andrea Wulzer[1,8]

The Muon Smasher's Guide

Hind Al Ali[1], Nima Arkani-Hamed[2], Ian Banta[1], Sean Benevedes[1], Dario Buttazzo[3], Tianji Cai[1], Junyi Cheng[1], Timothy Cohen[4], Nathaniel Craig[1], Majid Ekhterachian[5], JiJi Fan[6], Matthew Forslund[7], Isabel Garcia Garcia[8], Samuel Homiller[9], Seth Koren[10], Giacomo Koszegi[1], Zhen Liu[5,11], Qianshu Lu[9], Kun-Feng Lyu[12], Alberto Mariotti[13], Amara McCune[1], Patrick Meade[7], Isobel Ojalvo[14], Umut Oktem[1], Diego Redigolo[15,16], Matthew Reece[9], Filippo Sala[17], Raman Sundrum[5], Dave Sutherland[18], Andrea Tesi[16,19], Timothy Trott[1], Chris Tully[14], Lian-Tao Wang[10], and Menghang Wang[1]

Amazing recent progress on muon cooling problem:

Hot muons from pions



or cold muons from positrons

APS April Meeting 2021
Saturday–Tuesday, April 17–20, 2021; Virtual; Time Zone: Central Daylight Time, USA

Session Index

Session B08: Muon Collider Symposium I
Focus Live

Sponsoring Units: DPB DPF
Chair: Nadia Pastrone, INFN

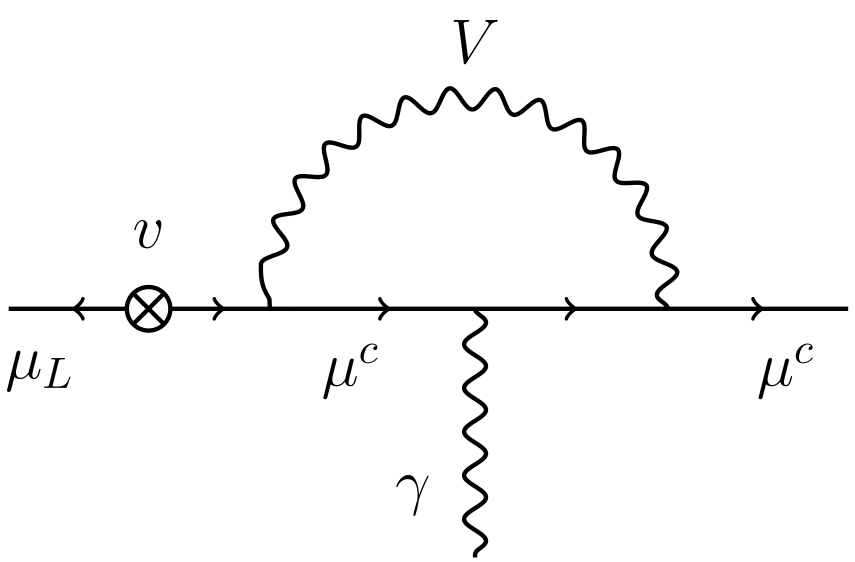
4 dedicated sessions at APS this year!
Interest is there, g-2 is more "value added" for physics motivation

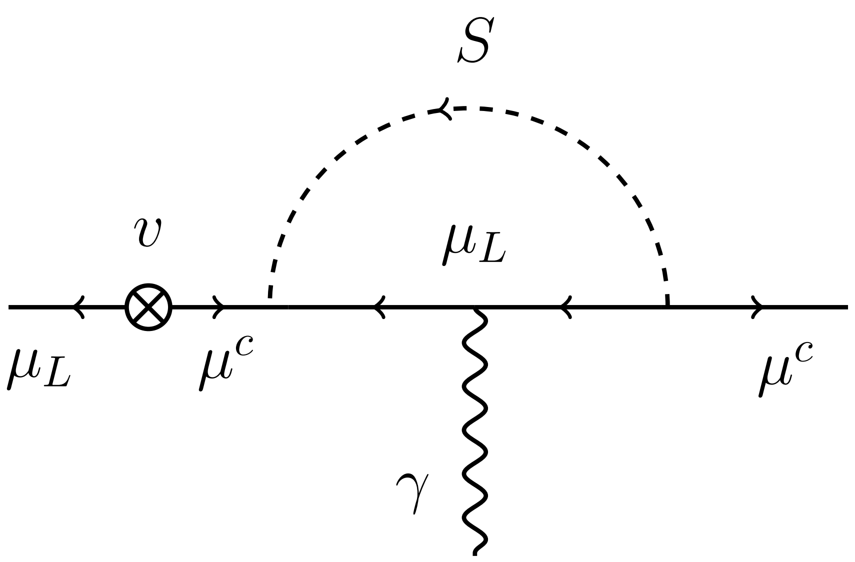
[Delahaye et al, 1901.06150, Al Ali et al, 2103.14043]

Singlet scenarios*

$$\mathcal{L}_{\rm V} \supset g_V V_\alpha(\mu_L^\dagger \bar\sigma^\alpha \mu_L + \mu^{c\dagger}\bar\sigma^\alpha \mu^c) + \frac{m_V^2}{2} V_\alpha V^\alpha$$

$$\mathcal{L}_{\rm S} \supset -(g_S S \mu_L \mu^c + \text{h.c.}) - \frac{1}{2} m_S^2 S^2$$





$$\Delta a_\mu^V / \Delta a_\mu \approx N_{\rm BSM} g_V^2 \left(\frac{200 \text{ GeV}}{m_V}\right)^2$$

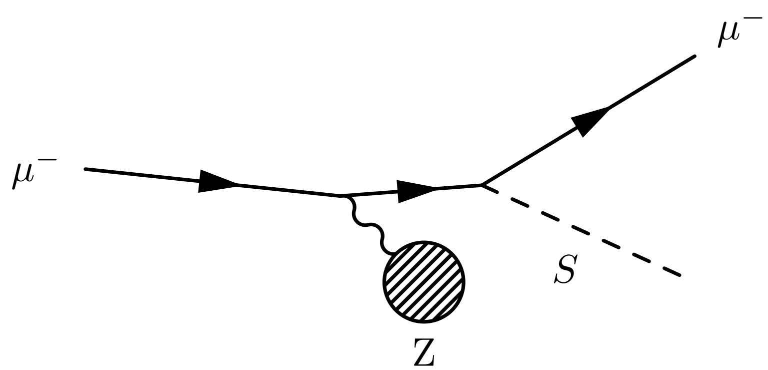
$$\Delta a_\mu^S / \Delta a_\mu \approx N_{\rm BSM} g_S^2 \left(\frac{700 \text{ GeV}}{m_S}\right)^2$$

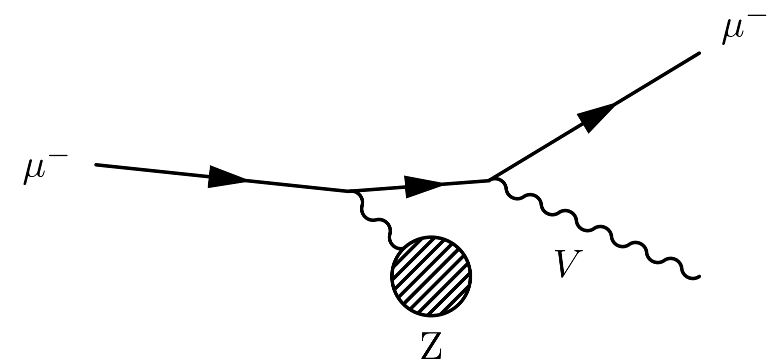
(allow for $N_{\rm BSM}$ copies of each, to be totally general)

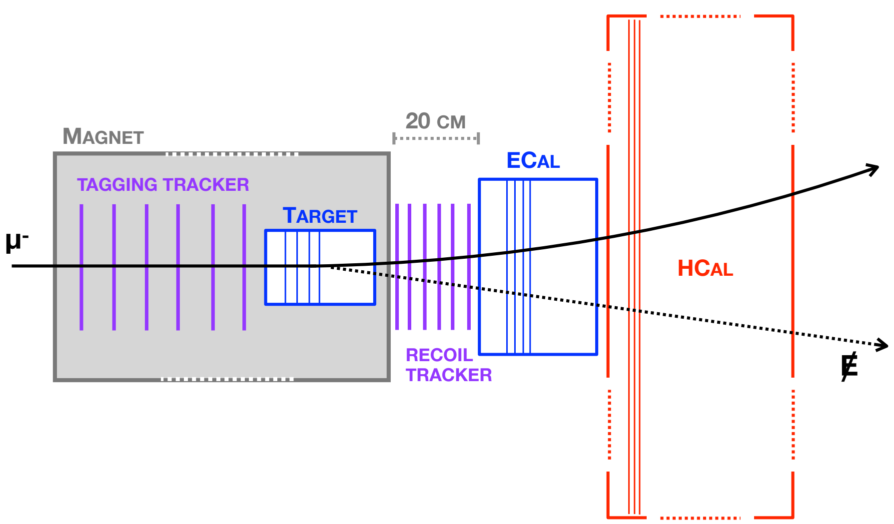
Higher loops give even lower BSM mass scale, easier to discover: focus on worst-case scenario (1-loop)

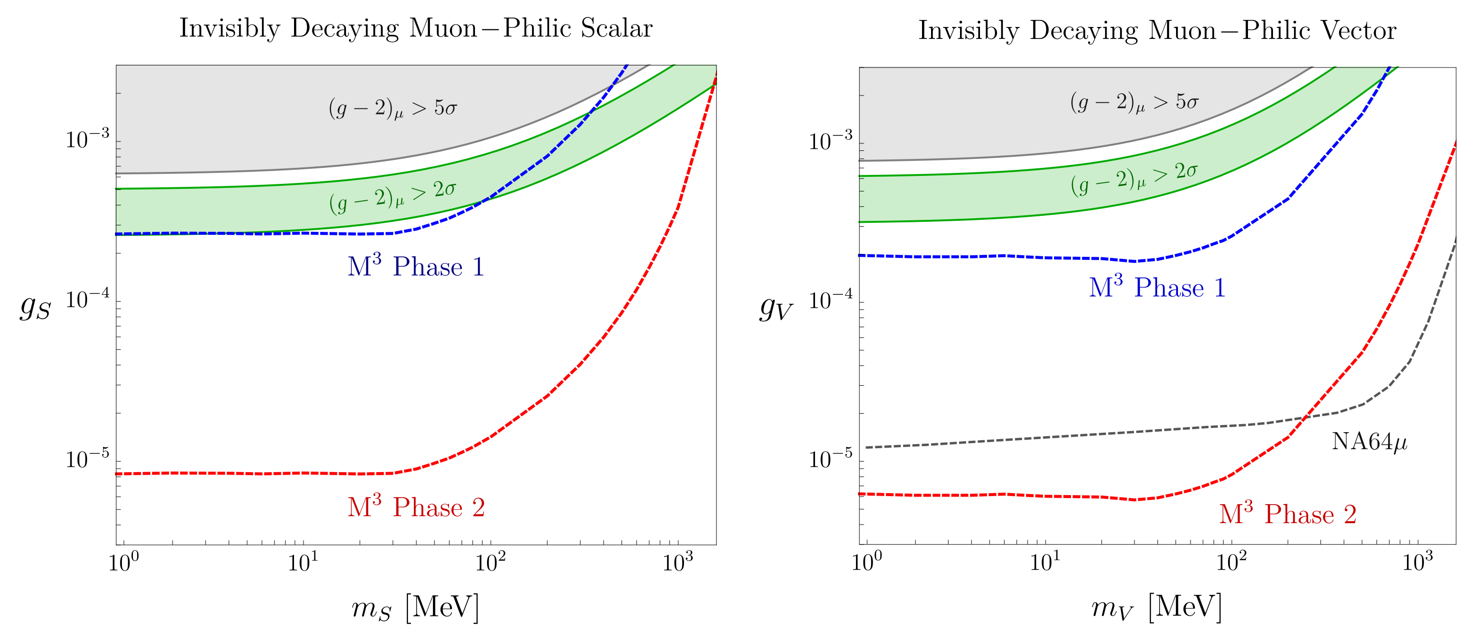
*(Remaining possibility: neutral RH fermion N in loop which mixes with muon neutrino, but suppressed by neutrino masses, too small)

Lowest masses: beam dump







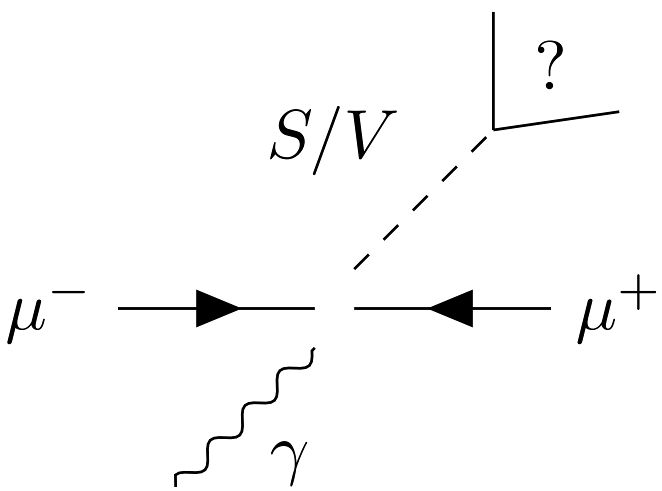


Complete coverage for 15 GeV muon beam, 10^{13} muons on 50 X_0 target

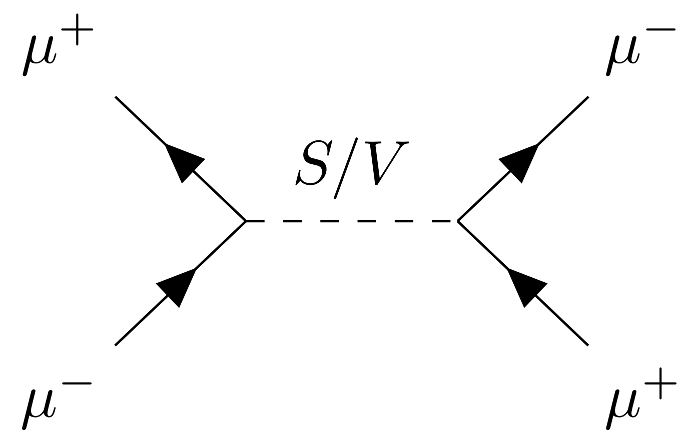
[**YK**, Krnjaic, Tran, Whitbeck, 1804.03144]

All other singlets: low-energy muon collider

Direct observation: mono-photon + anything



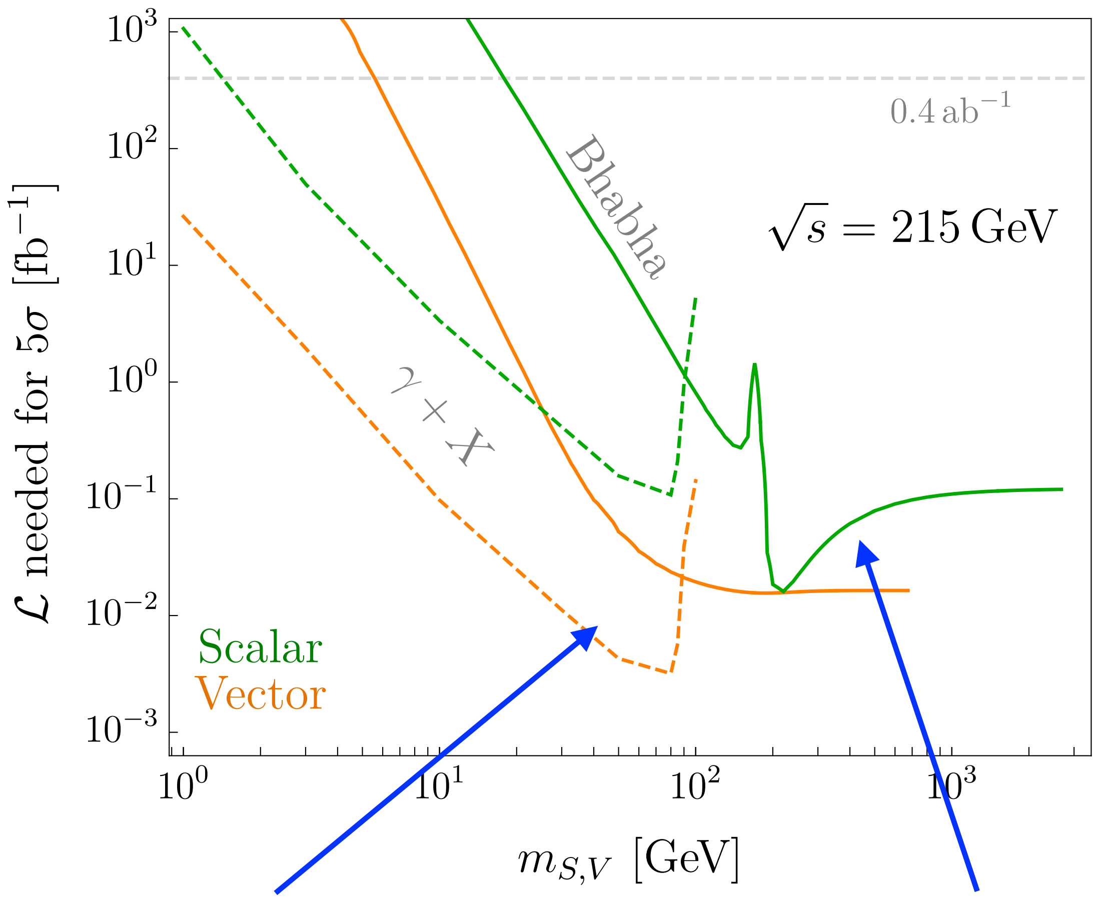
Indirect observation: corrections to Bhabha scattering



Amusing fact: heavier singlets are **easier** to discover, because couplings grow with masses for fixed g-2 contribution! Maximum mass ~ TeV reached at unitarity limit:

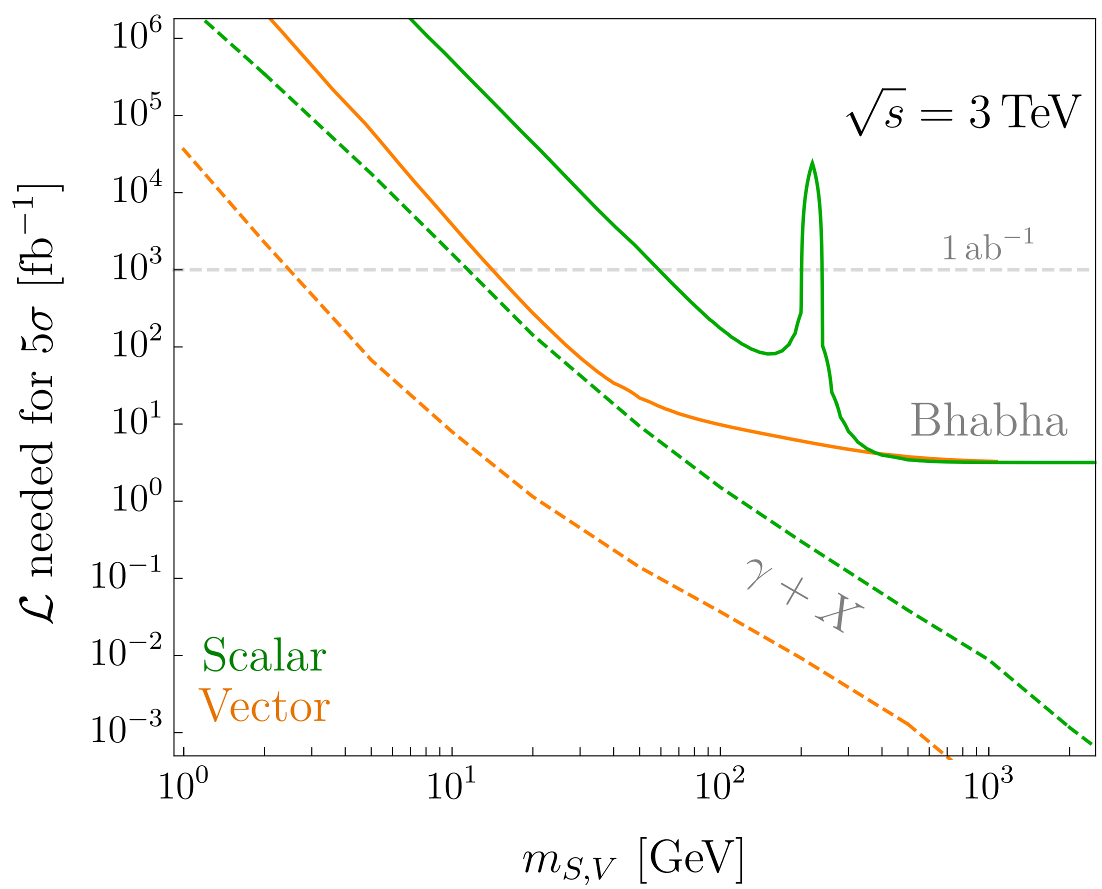
$$g_S^2 \leq \frac{4\pi}{N_{\rm BSM}} \quad , \quad g_V^2 \leq \frac{12\pi}{N_{\rm BSM}}$$

Singlet discovery criteria



reach improves up to kinematic threshold…

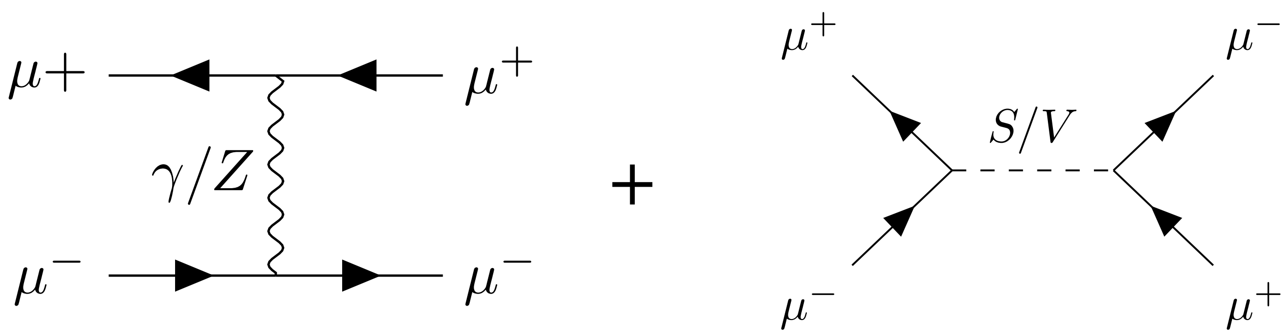
…where Bhabha covers remaining parameter space to unitarity limit



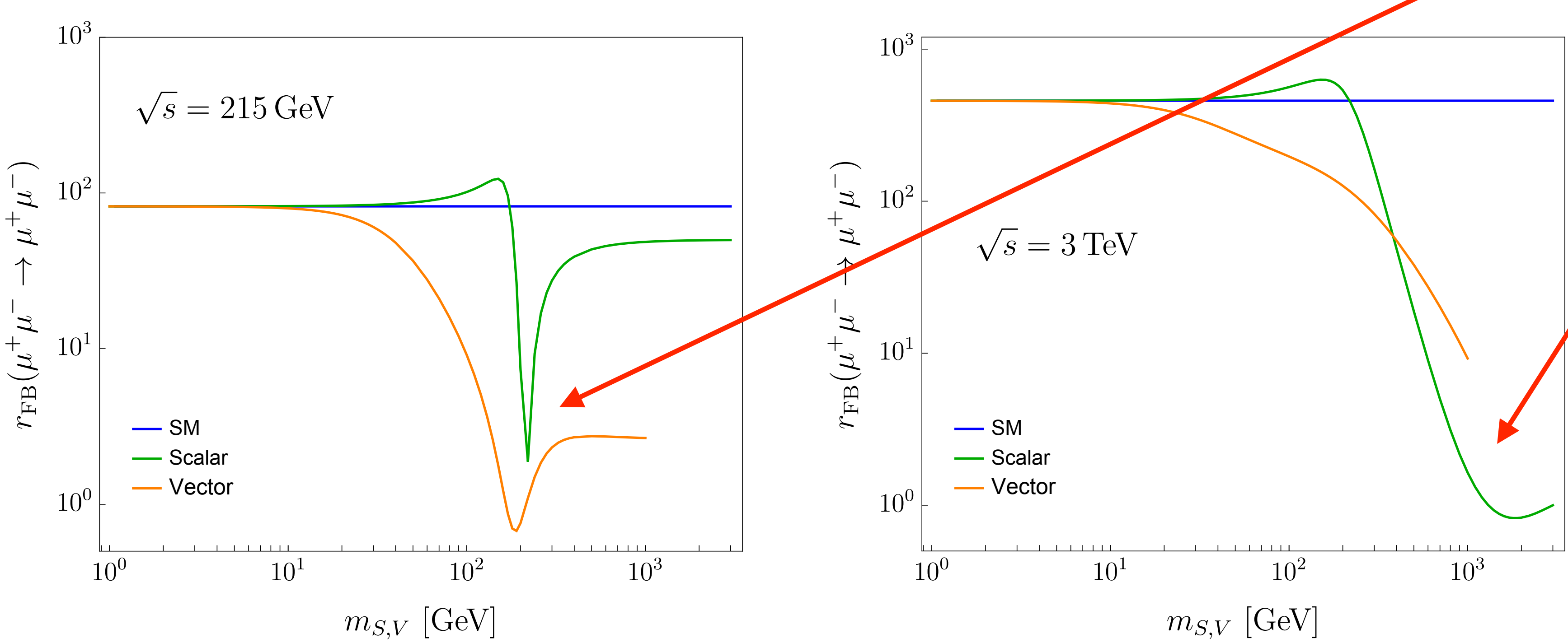
or just bump up the energy and keep the same luminosity to discover everything directly

Bhabha via FB

Muon colliders will likely use Bhabha rate as a luminosity monitor.
Exploit s-channel BSM structure against forward-peaked SM t-channel:



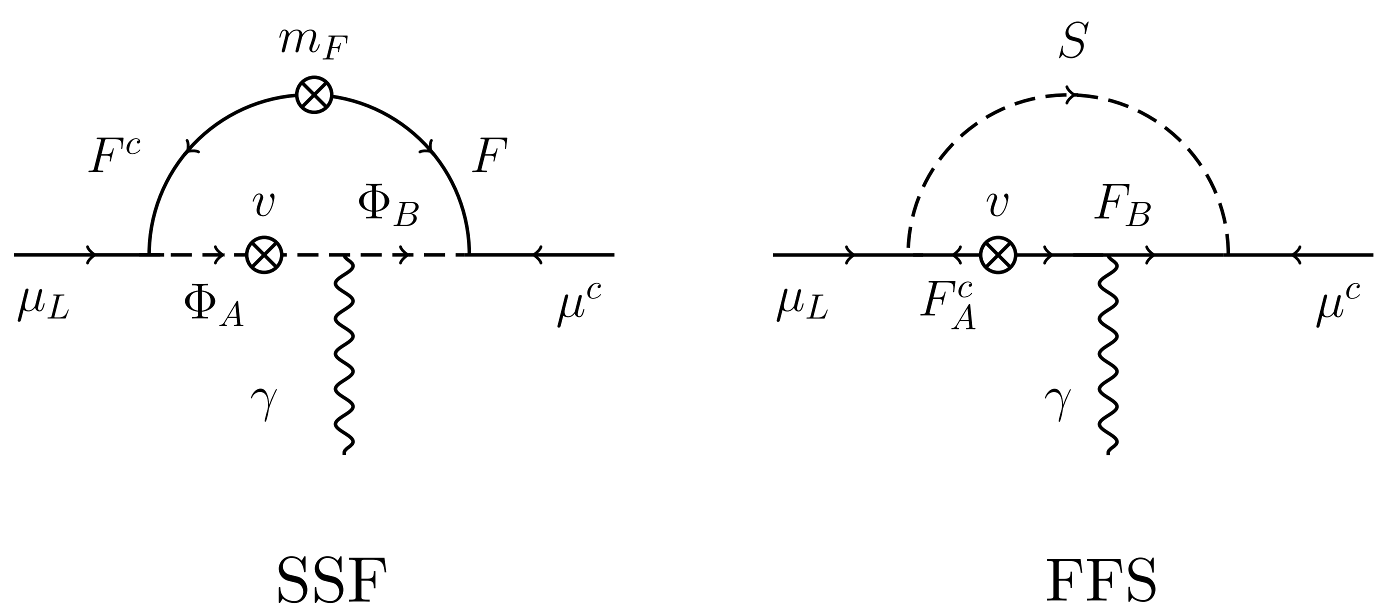
interference maximum at $m_{S,V} \simeq \sqrt{s}$



$$r_{\text{FB}} \equiv \frac{\displaystyle\int_0^{c_{\theta_0}} \frac{d\sigma}{dc_\theta} dc_\theta}{\displaystyle\int_{-c_{\theta_0}}^{0} \frac{d\sigma}{dc_\theta} dc_\theta}$$

Electroweak scenarios: general strategy

Minimal ingredients: new BSM scalars or fermions with SM EW charges.



Must have new charged particles: easy to see (stable, or decay visibly)

$$M_{\text{BSM,charged}}^{\text{max,unitarity}} \equiv \max_{\Delta a_\mu = \Delta a_\mu^{\text{obs}}\text{, perturbative unitarity}} \left\{ \min_{i\,\in\,\text{BSM spectrum}} \left(m_{\text{charged}}^{(i)} \right) \right\}$$

heaviest of the lightest charged states gives worst-case scenario

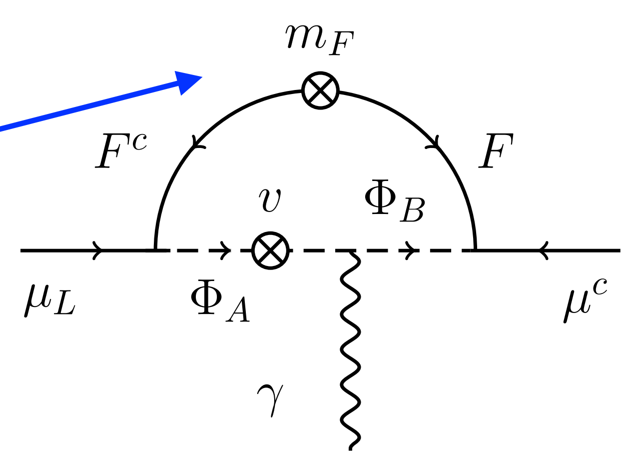
must explain g-2

for each theory and parameter values, find lightest charged particle

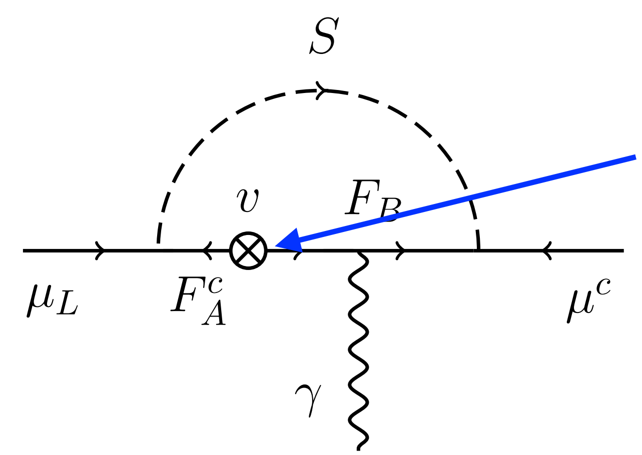
EW simplified models

As with singlets, focus on 1-loop contributions (highest mass scale)

chiral enhancement allows larger masses



SSF



FFS

large fermion-Higgs couplings allow larger masses

$$\mathcal{L}_{\text{SSF}} \supset -y_1 F^c L_{(\mu)} \Phi_A^* - y_2 F \mu^c \Phi_B - \kappa H \Phi_A^* \Phi_B - m_A^2 |\Phi_A|^2 - m_B^2 |\Phi_B|^2 - m_F F F^c + \text{h.c.}$$

$$\mathcal{L}_{\text{FFS}} \supset -y_1 F_A^c L_{(\mu)} \Phi^* - y_2 F_B \mu^c \Phi - y_{12} H F_A^c F_B - y'_{12} H^\dagger F_A F_B^c - m_A F_A F_A^c - m_B F_B F_B^c - m_S^2 |\Phi|^2 + \text{h.c.}$$

New fermions are in vector-like pairs to get large mass (and avoid anomalies)

SM gauge invariance:

$$\mathbf{1} \subset R^A \otimes R \otimes \mathbf{2}$$
$$R^B = \bar{R}$$
$$Y^A = -\frac{1}{2} - Y$$
$$Y^B = -1 - Y,$$

can just list off all such representations up to some maximum Q (we take Q <= 2)

The models

Model	R	R_A	R_B
SSF	1_{-1}	$2_{1/2}$	1_0
	1_{-2}	$2_{3/2}$	1_1
	1_0	$2_{-1/2}$	1_{-1}
	1_1	$2_{-3/2}$	1_{-2}
	$2_{-1/2}$	3_0	$2_{-1/2}$
	$2_{-3/2}$	3_1	$2_{1/2}$
	$2_{1/2}$	3_{-1}	$2_{-3/2}$
	$2_{-1/2}$	1_0	$2_{-1/2}$
	$2_{-3/2}$	1_1	$2_{1/2}$
	$2_{1/2}$	1_{-1}	$2_{-3/2}$
	3_{-1}	$2_{1/2}$	3_0
	3_0	$2_{-1/2}$	3_{-1}

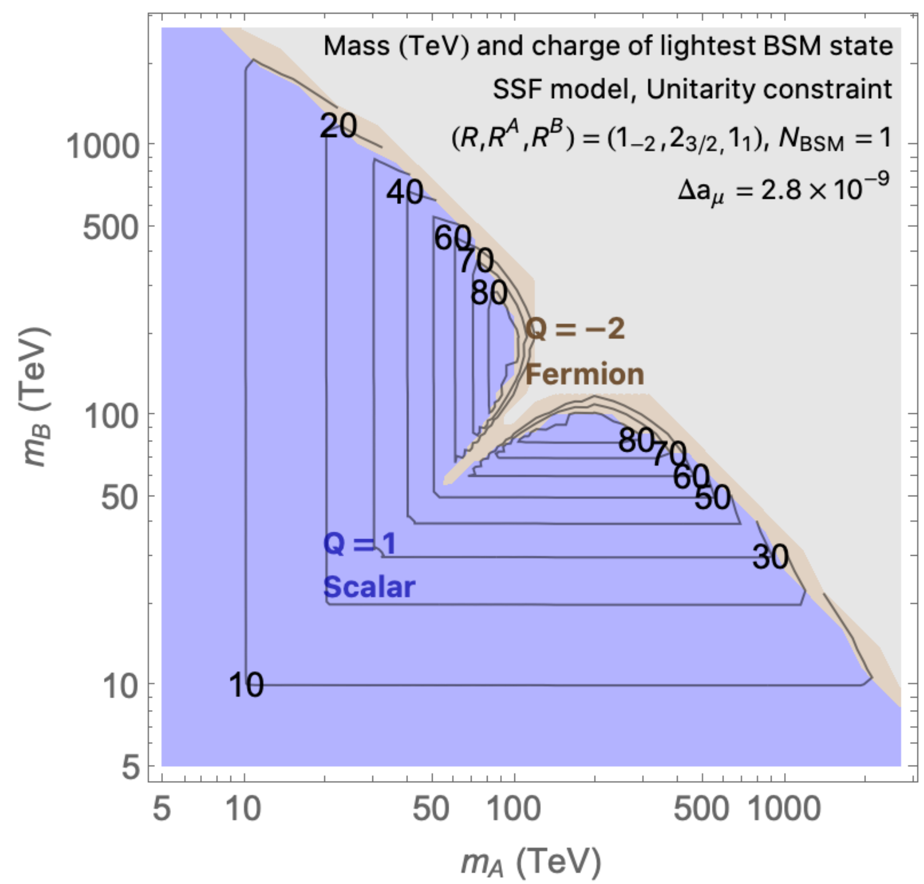
Model	R	R_A	R_B
FFS	1_{-1}	$2_{1/2}$	1_0
	1_{-2}	$2_{3/2}$	1_1
	1_0	$2_{-1/2}$	1_{-1}
	1_1	$2_{-3/2}$	1_{-2}
	$2_{-1/2}$	3_0	$2_{-1/2}$
	$2_{-3/2}$	3_1	$2_{1/2}$
	$2_{1/2}$	3_{-1}	$2_{-3/2}$
	$2_{-1/2}$	1_0	$2_{-1/2}$
	$2_{-3/2}$	1_1	$2_{1/2}$
	$2_{1/2}$	1_{-1}	$2_{-3/2}$
	3_{-1}	$2_{1/2}$	3_0
	3_0	$2_{-1/2}$	3_{-1}

As with singlets, allow for N_{BSM} copies of each.
No individual model means anything! Could be embedded in some nicer UV theory, e.g. SUSY, but "model-exhaustive" scans over all

Unitarity-only constraints

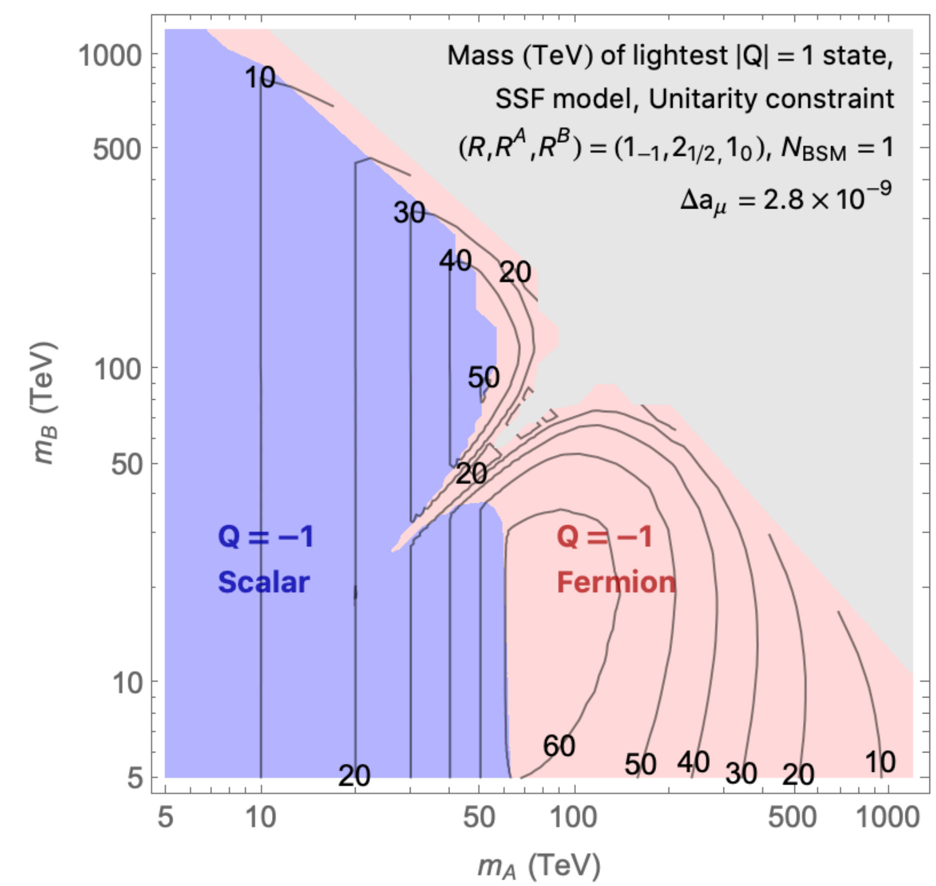
Let's look at two representative models:

SSF, all BSM fields charged



$$M^{\rm max,unitarity}_{\rm BSM,charged} = 86 \text{ TeV}$$

SSF, charged and neutral fields

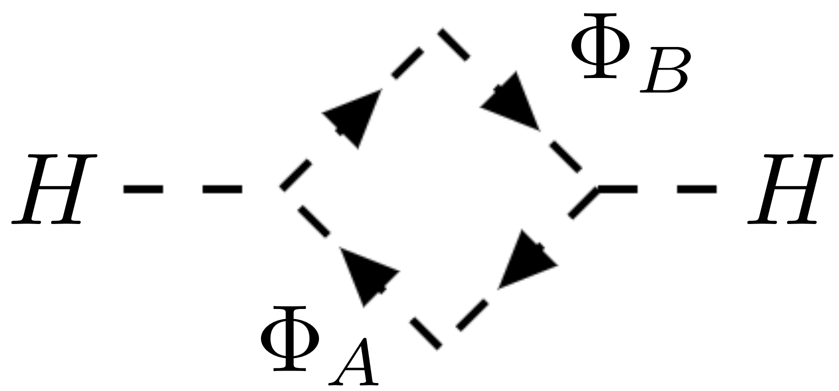


$$M^{\rm max,unitarity}_{\rm BSM,charged} = 65 \text{ TeV}$$

Consistent with parametric expectation: $\Delta a_\mu \propto \frac{m_\mu g_{\rm BSM} v}{M^2_{\rm BSM}}$, $M_{\rm BSM} \simeq 20$ TeV

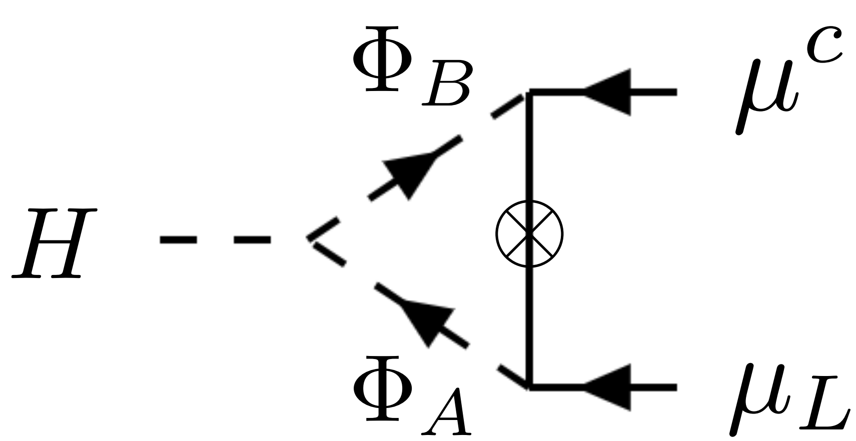
Fine-tuning problems

The unitarity-only models suffer from huge fine-tuning.



$$\Delta m_H^2 = C_1 N_{\rm BSM} \frac{\kappa^2}{16\pi^2}$$

Finite, calculable quadratic correction!



$$\Delta y_\mu \sim N_{\rm BSM} \frac{y_1 y_2}{16\pi^2} \, \frac{\kappa\, m_F}{M_{\rm BSM}^2}$$

Technically unnatural muon mass

Imposing e.g. 1% tuning on both will bring max mass scale down; equivalently, non-discovery implies a commensurate fine-tuning

Flavor constraints

SM gauge symmetries permit a flavor-anarchic set of models:

$$-\mathcal{L}_{\rm SSF} \supset y_1^i F^c L_i \Phi_A^* + y_2^i F \ell_i^c \Phi_B + \kappa H \Phi_A^* \Phi_B$$

$$-\mathcal{L}_{\rm FFS} \supset y_1^i F_A^c L_i S^* + y_2^i F_B \ell_i^c S + y_{12} H F_A F_B^c$$

Stringent CLFV constraints:

$$\mathrm{Br}(\mu \to e\gamma) < 4.2 \times 10^{-13}$$
$$\mathrm{Br}(\tau \to \mu\gamma) < 4.4 \times 10^{-8}$$
$$\mathrm{Br}(\tau \to e\gamma) < 3.3 \times 10^{-8}$$

$$\Rightarrow \quad \frac{y_{1,2}^e}{y_{1,2}^\mu} \lesssim 10^{-5} \quad , \quad \frac{y_{1,2}^\tau}{y_{1,2}^\mu} \lesssim 10^{-1} \quad , \quad \frac{y_{1,2}^\tau}{y_{1,2}^\mu}\frac{y_{1,2}^e}{y_{1,2}^\mu} \lesssim 10^{-1}$$

Use MFV ansatz as a stand-in for a solution to the CLFV constraints: imposing unitarity on largest coupling brings down max muon coupling

Similar to naturalness, non-discovery at lower mass scale implies fine-tuning between flavor gauge and mass eigenbases

How many copies?

Lots of new charged matter means gauge couplings run strong in UV

$$\beta_{Y,L} = \tfrac{1}{16\pi^2} b_{Y,L}\, g_{Y,L}^3$$

$$b_Y = \frac{41}{6} + \frac{1}{3}\sum_S Y_S^2 + \frac{2}{3}\sum_F Y_F^2,$$
$$b_L = -\frac{19}{6} + \frac{1}{3}\sum_S T(R_S) + \frac{2}{3}\sum_F T(R_F)$$

No Landau poles at 1 PeV: $b_Y < 249,\ b_L < 92 \implies N_{\mathrm{BSM}} \leq 571$

No Landau poles at Planck scale: $b_Y < 19,\ b_L < 7 \implies N_{\mathrm{BSM}} \leq 28$

Conclusion: $N_{\mathrm{BSM}} \lesssim \mathcal{O}(10)$ is reasonable

Model-exhaustive results

Model	R	R_A	R_B	**Highest possible mass (TeV) of lightest charged BSM state**							
				Unitarity only		Unitarity + MFV		Unitarity + Naturalness		Unitarity + Naturalness + MFV	
				$N_{\rm BSM}$: 1	10	$N_{\rm BSM}$: 1	10	$N_{\rm BSM}$: 1	10	$N_{\rm BSM}$: 1	10
SSF	1_{-1}	$2_{1/2}$	1_0	65.2	241	12.9	47.1	11.5	11.5	6.54	10.1
	1_{-2}	$2_{3/2}$	1_1	85.9	321	18.1	64.8	19.2	19.2	8.41	12.3
	1_0	$2_{-1/2}$	1_{-1}	46.2	176	9.41	34.1	15.6	17.5	5.93	8.56
	1_1	$2_{-3/2}$	1_{-2}	81.8	302	17.1	63.7	19.3	19.3	8.38	12.1
	$2_{-1/2}$	3_0	$2_{-1/2}$	21.4	107	4.2	15.5	7.47	8.99	3.23	5.0
	$2_{-3/2}$	3_1	$2_{1/2}$	83.7	308	16.6	60.7	13.4	13.4	7.06	10.6
	$2_{1/2}$	3_{-1}	$2_{-3/2}$	95.5	356	18.3	67.8	15.6	15.6	7.75	11.3
	$2_{-1/2}$	1_0	$2_{-1/2}$	65.2	241	12.9	47.1	11.5	11.5	6.54	10.1
	$2_{-3/2}$	1_1	$2_{1/2}$	85.9	321	18.1	64.8	19.2	19.2	8.41	12.3
	$2_{1/2}$	1_{-1}	$2_{-3/2}$	44.8	155	8.8	32.3	10.9	10.9	5.64	8.56
	3_{-1}	$2_{1/2}$	3_0	95.4	359	19.4	73	20.1	30	7.75	11.5
	3_0	$2_{-1/2}$	3_{-1}	39.4	144	7.82	28.6	10.8	15.1	4.14	6.08
FFS	1_{-1}	$2_{1/2}$	1_0	37.3	118	8.87	28	12.3	18.7	4.6	7.04
	1_{-2}	$2_{3/2}$	1_1	67.3	213	15.8	50	13.5	18.8	4.86	6.93
	1_0	$2_{-1/2}$	1_{-1}	59.1	187	13.2	41.8	12.4	17.2	4.02	6.28
	1_1	$2_{-3/2}$	1_{-2}	73.2	231	17.4	55	13.9	19.7	5.04	7.25
	$2_{-1/2}$	3_0	$2_{-1/2}$	40	126	9.38	29.7	8.0	11.5	2.88	4.34
	$2_{-3/2}$	3_1	$2_{1/2}$	56.3	178	13.6	42.9	11.8	16.2	4.26	6.1
	$2_{1/2}$	3_{-1}	$2_{-3/2}$	82.3	260	19.2	60.6	13.6	19	4.93	7.0
	$2_{-1/2}$	1_0	$2_{-1/2}$	37.3	118	8.87	28	12.3	18.7	4.6	7.04
	$2_{-3/2}$	1_1	$2_{1/2}$	67.3	213	15.8	50	13.5	18.8	4.86	6.93
	$2_{1/2}$	1_{-1}	$2_{-3/2}$	46.2	146	11.2	35.4	9.83	13.8	3.49	5.18
	3_{-1}	$2_{1/2}$	3_0	71	225	17	53.6	13.1	18.1	4.04	6.97
	3_0	$2_{-1/2}$	3_{-1}	23.4	75	5.29	16.9	7.3	7.69	2.73	4.03
$M^{\rm max}_{\rm BSM,charged}$ (max in each column)				**95.5**	**359**	**19.4**	**73**	**20.1**	**30**	**8.41**	**12.3**

The no-lose theorem: EW scenarios

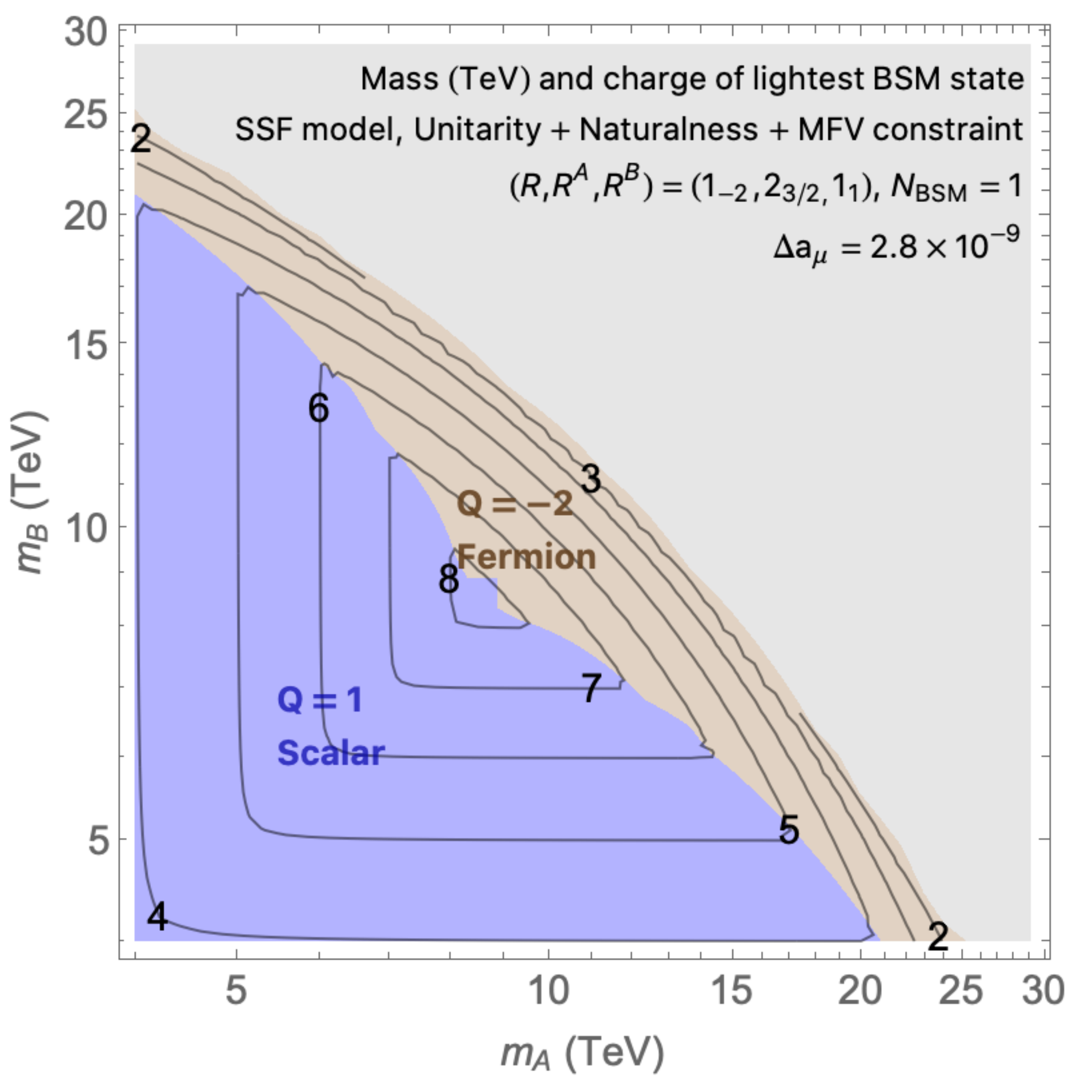
$$M_{\text{BSM,charged}}^{\text{max,X}} \equiv \max_{\Delta a_\mu = \Delta a_\mu^{\text{obs}}\,,\,X} \left\{ \min_{i\,\in\,\text{BSM spectrum}} \left(m_{\text{charged}}^{(i)} \right) \right\}$$

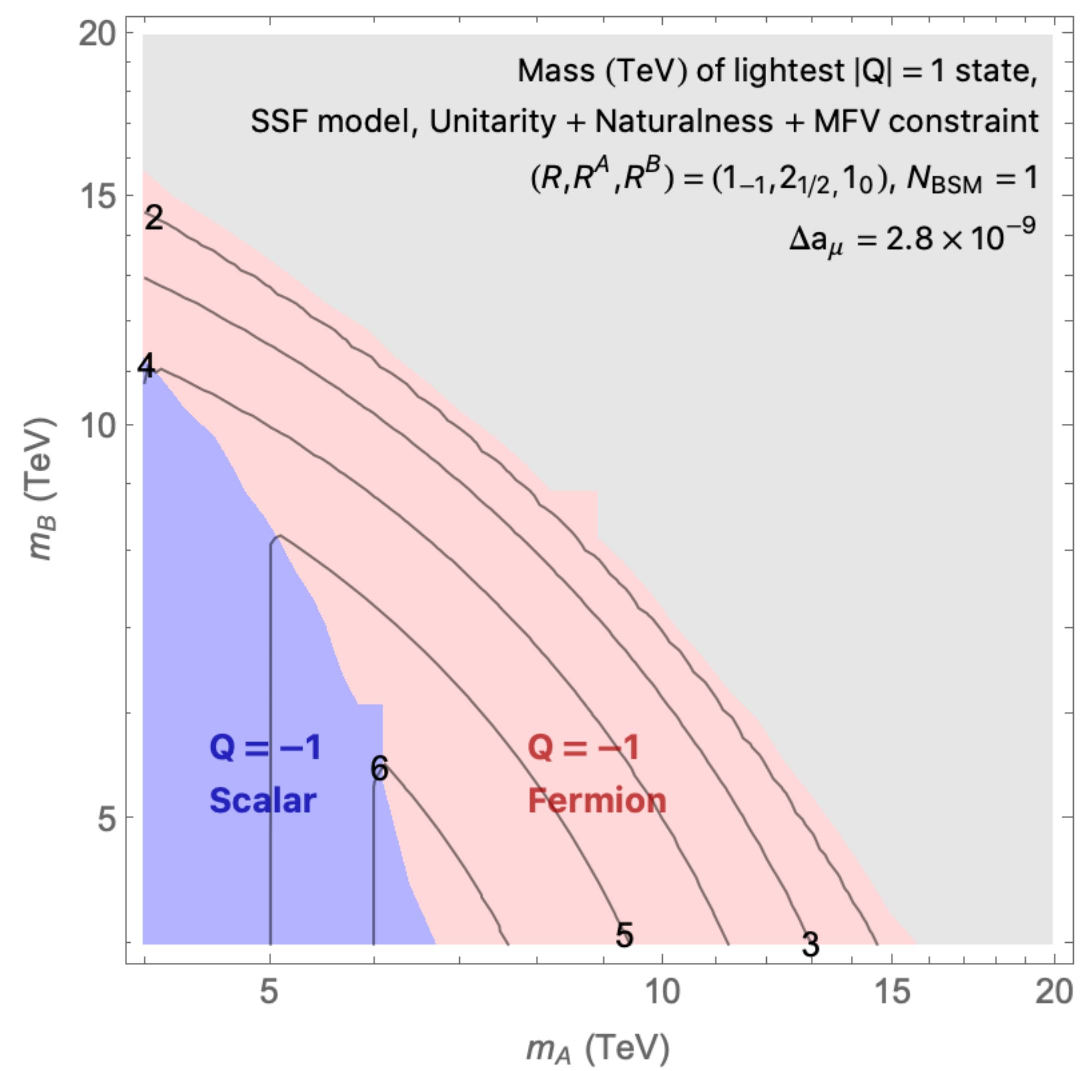
$$M_{\text{BSM,charged}}^{\text{max,X}} \approx \begin{cases} (100\,\text{TeV})\; N_{\text{BSM}}^{1/2} & \text{for} \quad X = (\text{unitarity*}) \\ (20\,\text{TeV})\; N_{\text{BSM}}^{1/2} & \text{for} \quad X = (\text{unitarity} + \text{MFV}) \\ (20\,\text{TeV})\; N_{\text{BSM}}^{1/6} & \text{for} \quad X = (\text{unitarity} + \text{naturalness*}) \\ (9\,\text{TeV})\; N_{\text{BSM}}^{1/6} & \text{for} \quad X = (\text{unitarity} + \text{naturalness} + \text{MFV}) \end{cases}$$

This is a lepton collider: no way to miss these guys!
(Vector boson fusion gives efficient EW production)

* = something very nontrivial is evading CLFV constraints.
Models with unitarity + naturalness + MFV are most
"theoretically reasonable," everything else is worst-case

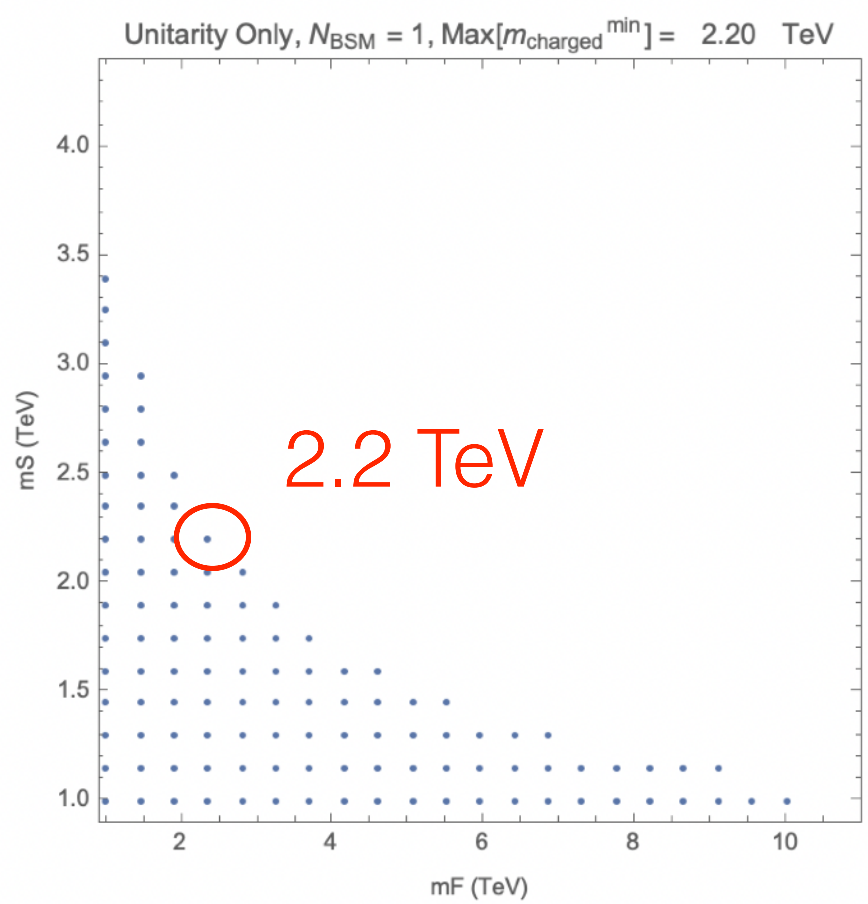
The space of "reasonable" models



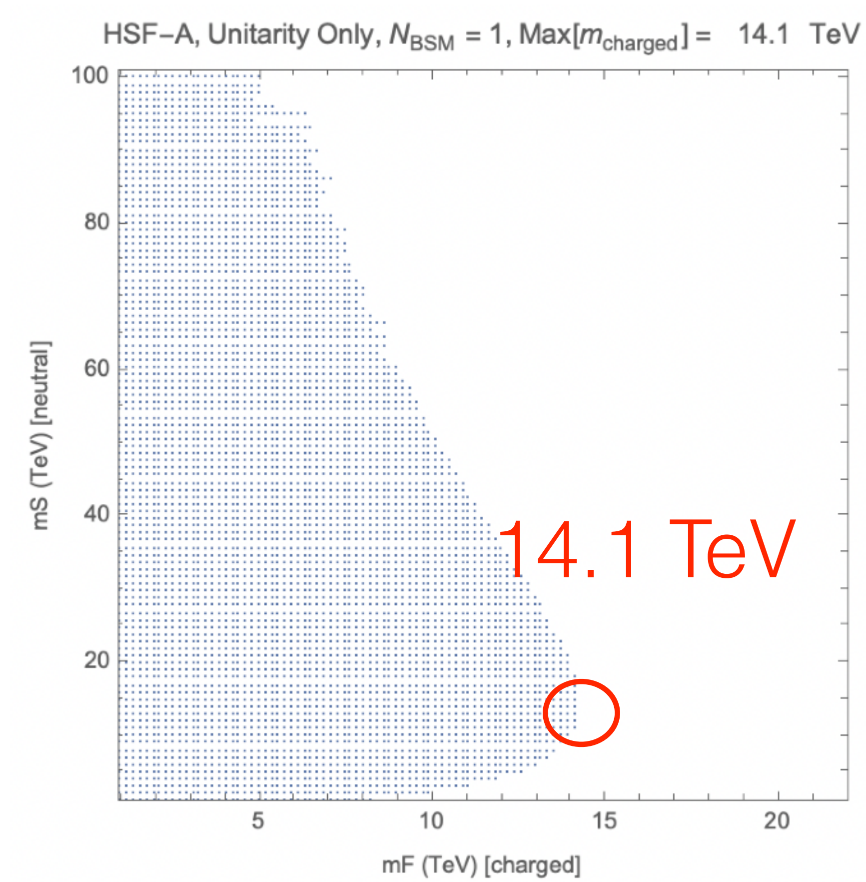


Some edge cases

"μFS" (replace one new fermion with muon):



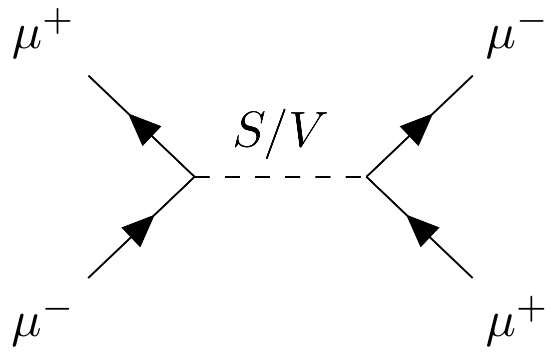
"HSF" (replace one new scalar with Higgs):



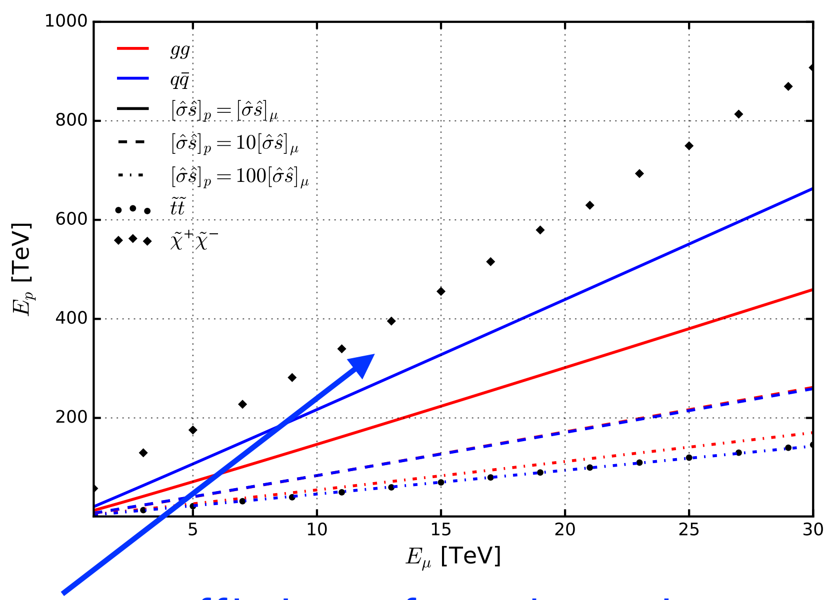
Still obey no-lose theorem!

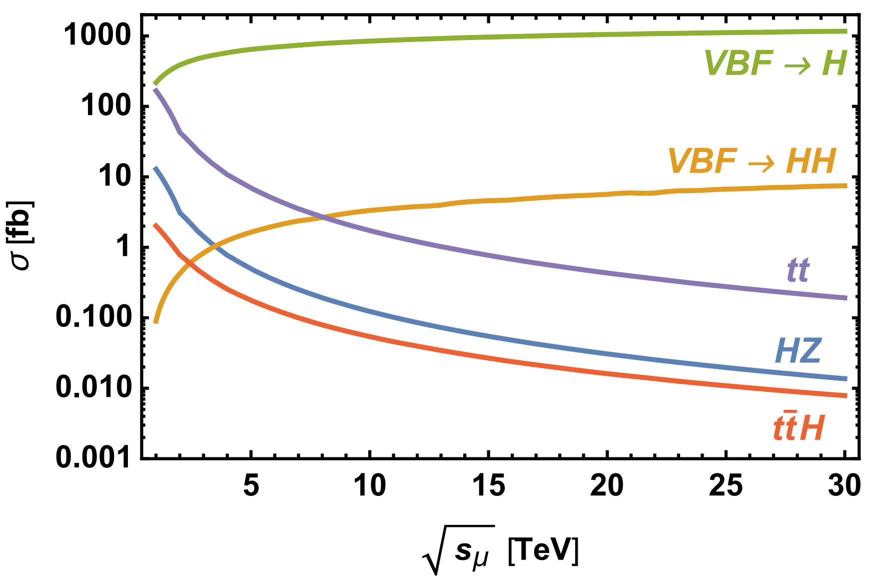
Why a muon collider?

Singlets: you collide the particles actually involved in g-2.
Reach is abysmal at a proton collider



EW models: full COM energy, **and** VBF enhances EW processes





30x more efficient for chargino production!

EW processes grow logarithmically

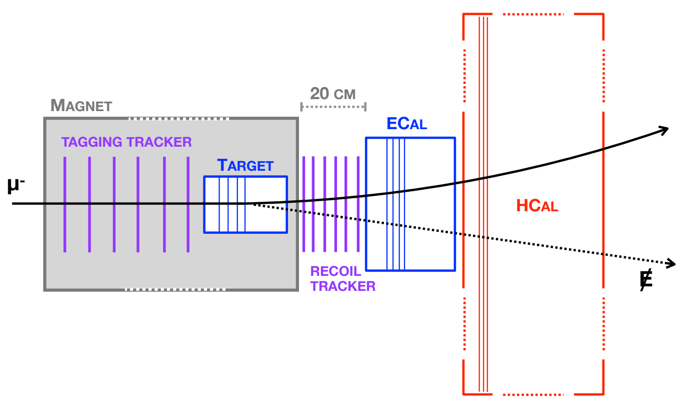
Muon g-2 provides strong motivation for a full muon collider **program:** discoveries possible at every step in energy and luminosity

[Costantini et al., 2005.10289; Al Ali et al., 2103.14043]

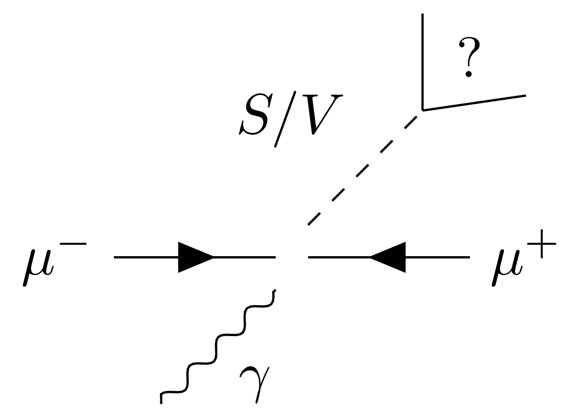
Conclusion

If g-2 is real, it is the strongest signal of BSM physics in the lab.

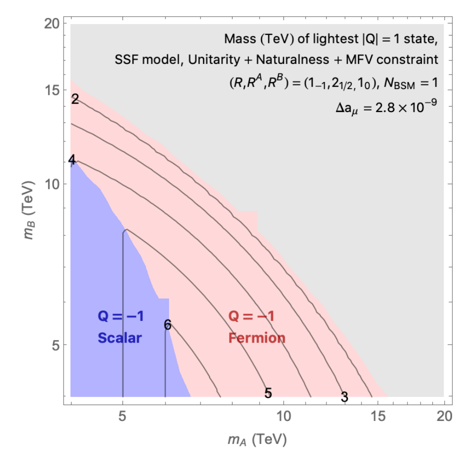
< 1 GeV: muon fixed-target

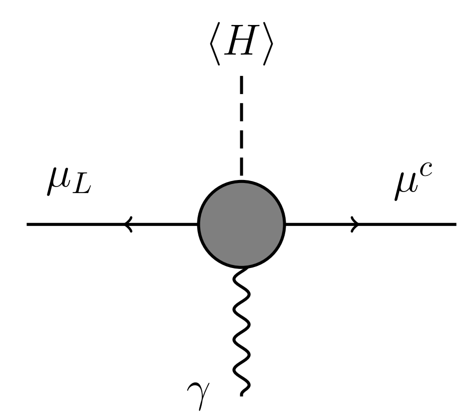


1 GeV - few TeV: singlets at 215 GeV or 3 TeV muon collider



100 GeV - 20 TeV: EW scenarios and $\mu^+\mu^- \to h\gamma$ at 30 TeV muon collider





Guaranteed signal, and if no new on-shell states, explicit experimental confirmation of a hierarchy problem.