DIS at low x and gluon saturation

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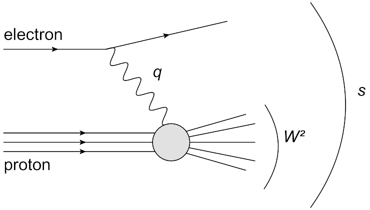
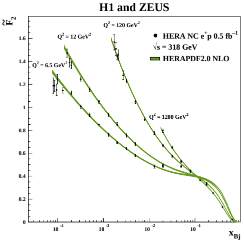
EIC PL Seminar, June 7th 2021

Deep inelastic scattering (DIS)





$$x_{Bj}\, Q^2\, \frac{d\sigma^{e+p\to e+X}}{dx_{Bj}\, d^2Q} = \frac{2\pi\, \alpha_{em}^2}{Q^2}\, [1+(1-y)^2] \Big\{ F_T(x_{Bj}, Q^2) + F_L(x_{Bj}, Q^2) - \frac{y^2}{[1+(1-y)^2]} F_L(x_{Bj}, Q^2) \Big\}$$

Photon virtuality : $Q^2 \equiv -q^2 > 0$

$F_2 = F_T + F_L$

Bjorken variable : $x_{Bj} \equiv \frac{Q^2}{2P\cdot q} \sim \frac{Q^2}{W^2}$

Inelasticity : $y \equiv \frac{Q^2}{x_{Bj}\, s}$

Bjorken and Regge limits

Squared energy of the $\gamma - p$ sub-collision:

$$(P+q)^2 = 2P.q - Q^2 + M_p^2 = Q^2\left(\frac{1}{x_{Bj}} - 1\right) + M_p^2$$

$\Rightarrow$ Two possible ways for $(P+q)^2$ to become large:

Bjorken limit: $Q^2 \to +\infty$ and x_{Bj} finite

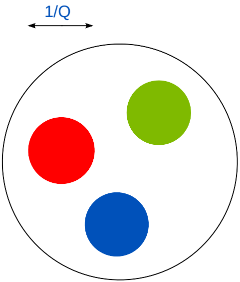
Energy $\sim$ transverse momenta : Hard process (rare)
$\to$ Validity range for the standard pQCD:
Parton model and collinear factorization $\to$ DGLAP evolution.

Regge limit: $x_{Bj} \to 0$ and Q^2 finite

Energy $\gg$ transverse momenta :
Typical high-energy process
$\to$ Main topic of this talk!

Note: Q^2 should be large enough to allow QCD perturbation theory: $\alpha_s(Q^2) \ll 1$.

DGLAP evolution: increasing Q^2



The hard scale Q^2 sets the transverse resolution to detect partons in the target proton.

Increasing $Q^2 \Rightarrow$ More substructures resolved

$\Rightarrow$ Target effectively containing more partons, but more dilute!

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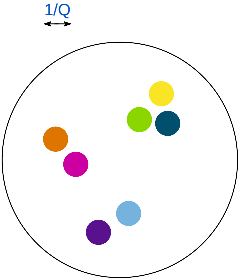

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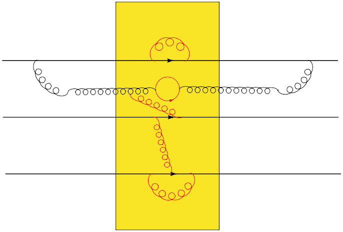
High-energy evolution in the Regge limit



Any process probes the content of an incoming hadron with a given time resolution.
$\rightarrow$ Too short-lived fluctuations not resolved as partons.

Increasing the energy of the collision $\Leftrightarrow$ boosting the incoming hadron and keeping the same probe.

Lorentz time dilation $\Rightarrow$ more and more fluctuations in the hadron resolved by the probe.

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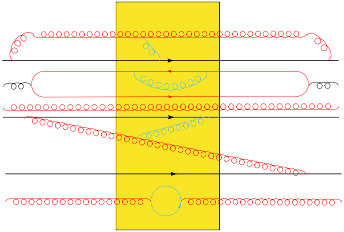

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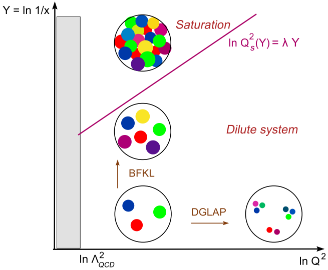
Kinematical regimes of DIS



- For $Q^2 \to +\infty$: target more and more dilute due to DGLAP evolution.
  $\Rightarrow$ QCD-improved parton model more and more valid.
- For $x_{Bj} \to 0$: target more and more dense
  $\Rightarrow$ Linear BFKL evolution eventually breaks down, as well as parton picture.

  Onset of nonlinear collective effects: Gluon saturation!

Regime of large gluon field, but weak coupling α_s

Approximations for high-energy scattering

Dilute projectile (ex: photon) : momentum $q^\mu \simeq \delta^{\mu +} q^+$

Dense target (ex: proton or nucleus) : momentum $P^\mu \simeq \delta^{\mu -} P^-$

High energy limit $\Rightarrow (P+q)^2 \sim 2P \cdot q \sim 2P^- \, q^+ \to +\infty$

Semi-classical approximation: Dense target has $A_a^\mu(x) = O(1/g)$: semi-classical field for small g.
$\Rightarrow$ Replace the target by a random background field, to be averaged over.

Eikonal approximation: Take the high-energy limit $s \to +\infty$ and drop power-suppressed contributions.

In the semi-classical approximation, the eikonal limit can be obtained by an infinite boost $P^- \to +\infty$ of the target field $A_a^\mu(x)$. Hence:

- Only the A_a^- component is relevant
- Infinite Lorentz dilation: $A_a^\mu(x)$ independent of x^-
- Infinite Lorentz contraction: $A_a^\mu(x) \propto \delta(x^+)$ (shockwave)

Eikonal dilute-dense scattering in LFPT

Method to calculate such *dilute-dense* processes at high-energy, following Bjorken, Kogut and Soper (1971):

- Decompose the projectile on a Fock basis at the time $x^+ = 0$, with appropriate Light-Front wave-functions.
- Each parton n scatters independently on the target via a light-like Wilson line $U_{\mathcal{R}_n}(\mathbf{x}_n)$ through the target:

$$U_{\mathcal{R}_n}(\mathbf{x}_n) = \mathcal{P}_+ \exp\left[-ig \int dx^+ \, T^a_{\mathcal{R}_n} \, A^-_a(x^+, \mathbf{x}_n)\right]$$

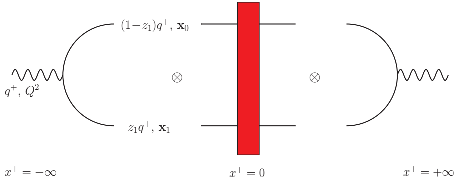
  with $\mathcal{R}_n = A$, F or $\bar{F}$ for g, q or $\bar{q}$ partons.

- Include final-state evolution of the projectile remnants.

Comments:

1. Light-cone gauge $A^+_a = 0$ strongly recommended!
2. At this stage, no apparent dependence on energy ...

Dipole factorization for DIS at LO



$$\sigma^{\gamma p \to X}_{T,L}(x_{Bj}, Q^2) = \frac{4N_c\,\alpha_{em}}{(2\pi)^2}\sum_f e_f^2 \int \mathrm{d}^2\mathbf{x}_0\,\mathrm{d}^2\mathbf{x}_1 \int_0^1 \mathrm{d}z_1 \times \mathcal{I}^{q\bar{q},LO}_{T,L}(x_{01}, z_1, Q^2)\left[1 - \left\langle \mathbf{S}_{01}\right\rangle_\eta\right]$$

Bjorken, Kogut, Soper (1971); Nikolaev, Zakharov (1990)

Dipole operator: $$\mathbf{S}_{01} = \frac{1}{N_c}\mathrm{Tr}\left(U_F(\mathbf{x}_0)\,U_F^\dagger(\mathbf{x}_1)\right)$$

η: regulator of rapidity divergence of light-like Wilson lines $U_F(\mathbf{x}_n)$.

B-JIMWLK and BK evolutions

RG evolution for the dipole operator with respect to the regulator η:

$$\partial_\eta \left\langle \mathbf{S}_{01} \right\rangle_\eta = \frac{N_c \alpha_s}{\pi} \int \frac{\mathrm{d}^2 \mathbf{x}_2}{2\pi} \frac{x_{01}^2}{x_{02}^2\, x_{21}^2} \left\langle \mathbf{S}_{02}\mathbf{S}_{21} - \mathbf{S}_{01} \right\rangle_\eta$$

New operator $\left\langle \mathbf{S}_{02}\mathbf{S}_{21} \right\rangle_\eta$ appears $\Rightarrow$ only the first equation in Balitsky's infinite hierarchy. Balitsky (1996)

The whole hierarchy is equivalent to a functional equation, the JIMWLK equation
Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner, ... (1997-2002)

In practice: often truncate the hierarchy with the approx
$\left\langle \mathbf{S}_{02}\mathbf{S}_{21} \right\rangle_\eta \simeq \left\langle \mathbf{S}_{02} \right\rangle_\eta \left\langle \mathbf{S}_{21} \right\rangle_\eta$ to get the BK equation.

$$\partial_\eta \left\langle \mathbf{S}_{01} \right\rangle_\eta = \frac{N_c \alpha_s}{\pi} \int \frac{\mathrm{d}^2 \mathbf{x}_2}{2\pi} \frac{x_{01}^2}{x_{02}^2\, x_{21}^2} \left[\left\langle \mathbf{S}_{02} \right\rangle_\eta \left\langle \mathbf{S}_{21} \right\rangle_\eta - \left\langle \mathbf{S}_{01} \right\rangle_\eta \right]$$

Balitsky (1996); Kovchegov (1999)

Natural factorization scale choice: evolve over the typical rapidity interval available, like $\eta \sim \log(1/x_{Bj})$ for DIS

Solutions of the BK equation

Other notation: $N(\mathbf{r},\mathbf{b}) \equiv 1 - \mathbf{S}_{01}$, with: $\mathbf{r} \equiv \mathbf{x}_0 - \mathbf{x}_1$ and $\mathbf{b} \equiv (\mathbf{x}_0 + \mathbf{x}_1)/2$.

Inclusive DIS probes only $\int \mathrm{d}^2\mathbf{b}\, \langle N(\mathbf{r},\mathbf{b})\rangle_{\log(1/x_{Bj})}$

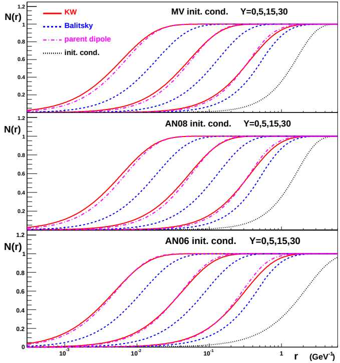
$\Rightarrow$ Common approximation: $\langle N(\mathbf{r},\mathbf{b})\rangle_Y \simeq \langle N(r)\rangle_Y$ with $r = |\mathbf{r}|$

Solutions of the BK equation can be obtained numerically, starting from appropriate initial conditions

- $\langle N(r)\rangle_Y \to 0$ for $r \to 0$: dilute regime
- $\langle N(r)\rangle_Y \to 1$ for $r \to +\infty$: gluon saturation
- Linear/Nonlinear transition at $r \sim 1/Q_s(Y)$, defining the saturation scale $Q_s(Y)$
- Increase of $Q_s(Y)$ with Y

Wave-front structure of the solution

$\Leftrightarrow$ geometric scaling $\langle N(r)\rangle_Y \simeq f(rQ_s(Y))$



Albacete and Kovchegov (2007)

Dipole factorisation : Universality

$\langle N(\mathbf{r}, \mathbf{b})\rangle_Y$ determines other observables

- in DIS at low x_{Bj} (HERA, EIC): diffractive, exclusive, semi-inclusive, ...
- in proton-proton (pp) or proton-nucleus (pA) collisions at high energy (LHC, RHIC)

Example: Inclusive hadron production at high rapidity y and moderate $\mathbf{p}_\perp$ in pp or pA

$$\frac{d\sigma}{dy\, d^2\mathbf{p}_\perp} = PDF \otimes \langle N(r)\rangle_Y \otimes FF$$

Hybrid factorization : collinear and dipole

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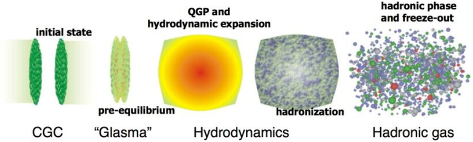
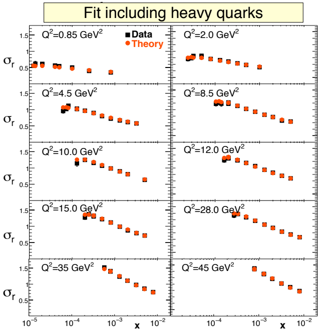
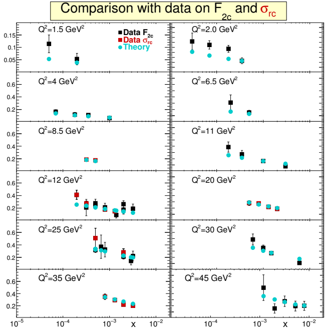
Hybrid factorization : collinear and dipole

Gluon saturation and in particular $\langle N(\mathbf{r},\mathbf{b})\rangle_Y$ also determine the dynamics in the earliest stages of high-energy heavy ion collisions (LHC, RHIC)
$\Rightarrow$ Initial conditions for the formation and hydrodynamic evolution of the quark-gluon plasma



DIS phenomenology at LO





Fits of the reduced DIS cross-section σ_r and its charm contribution σ_{rc} at HERA data with numerical solutions of the running coupling BK equation.

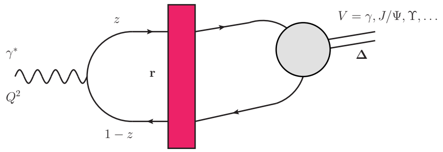
Albacete, Armesto, Milhano, Quiroga, Salgado (2011)

see also: Kuokkanen, Rummukainen, Weigert (2012);

Lappi, Mäntysaari (2013); . . .

Good fit, but require a big rescaling of Λ_{QCD} by an extra parameter, to slow down the BK evolution.

DVCS and exclusive vector mesons production



$$\mathcal{A}^{\gamma^* p \to Vp}(x_{\mathbb{P}}, Q^2, \boldsymbol{\Delta}) = \frac{i}{2\pi} \int \mathrm{d}^2\mathbf{r}\, \mathrm{d}^2\mathbf{b} \int_0^1 \mathrm{d}z\; e^{-i\boldsymbol{\Delta}\cdot[\mathbf{b}+(z-1/2)\mathbf{r}]} \left(\psi_V^* \psi\right)(\mathbf{r}, z, Q^2)\;\; N(\mathbf{r}, \mathbf{b})$$

Coherent contribution (intact target) $\Rightarrow$ target color average at the amplitude level :

$$\frac{\mathrm{d}\sigma^{\gamma^* p \to Vp}_{\mathrm{coh.}}}{\mathrm{d}t}(x_{\mathbb{P}}, Q^2, t) = \frac{1}{16\pi}\; \left| \left\langle \mathcal{A}^{\gamma^* p \to Vp}(x_{\mathbb{P}}, Q^2, \boldsymbol{\Delta}) \right\rangle_{\log(1/x_{\mathbb{P}})} \right|^2$$

Dependence on $t \equiv -\boldsymbol{\Delta}^2$ of the cross-section allows to probe the $|\mathbf{b}|$ dependence of $\langle N(\mathbf{r}, \mathbf{b})\rangle_Y$, meaning the transverse profile of the target

In principle: link with the physics of GPDs at low-x

DVCS and vector mesons: incoherent piece

In addition, there is an incoherent contribution to the cross section, with target break-up despite the colorless exchange:

$$\frac{\mathrm{d}\sigma^{\gamma^* p\to V p^*}_{\mathrm{incoh.}}}{\mathrm{d}t} = \frac{1}{16\pi}\left[\left\langle \left|\mathcal{A}^{\gamma^* p\to Vp}\right|^2\right\rangle_{\log(1/x_{\mathbb{P}})} - \left|\left\langle \mathcal{A}^{\gamma^* p\to Vp}\right\rangle_{\log(1/x_{\mathbb{P}})}\right|^2\right]$$

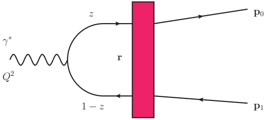
$\rightarrow$ Sensitive to fluctuations of $N(\mathbf{r},\mathbf{b})$

Sizable incoherent diffractive J/Ψ cross section at HERA!

$\Rightarrow$ Large fluctuations of the proton density profile

Simultaneous description of coherent and incoherent cross section possible in models with three hotspots in a proton, with random positions.
Mäntysaari and Schenke (2016)

Diffractive dijets in DIS



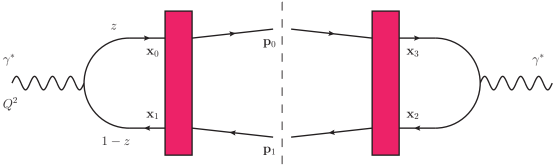
(Coherent) Diffractive dijet production in DIS $\Rightarrow$ target color average at the amplitude level:

$$\langle \mathcal{A}^{\gamma^* p \to jjp}(x_{\mathbb{P}}, Q^2, \mathbf{p}_0, \mathbf{p}_1) \rangle_{\log(1/x_{\mathbb{P}})} \propto \int \mathrm{d}^2\mathbf{r}\, \mathrm{d}^2\mathbf{b}\ e^{i\boldsymbol{\Delta}\cdot\mathbf{b}}\ e^{i\mathbf{P}\cdot\mathbf{r}}\ \psi(\mathbf{r}, z, Q^2)\ \langle N(\mathbf{r}, \mathbf{b}) \rangle_{\log(1/x_{\mathbb{P}})}$$

with the dijet transverse momentum $\boldsymbol{\Delta} \equiv \mathbf{p}_0 + \mathbf{p}_1$
and the jet typical transverse momentum $\mathbf{P} \equiv (\mathbf{p}_0 - \mathbf{p}_1)/2$.

$\Rightarrow$ Measuring the dependence of the cross section on the angle between $\boldsymbol{\Delta}$ and $\mathbf{P}$ probes the dependence of $\langle N(\mathbf{r}, \mathbf{b}) \rangle_Y$ on the angle between $\mathbf{r}$ and $\mathbf{b}$.
Altınoluk, Armesto, G.B., Rezaeian (2016) & Mäntysaari, Mueller and Schenke (2019)

Inclusive dijets in DIS



Inclusive dijet production : summation over the partons color and target average performed only at the cross section level

⇒ In addition to dipoles, the cross section involves the quadrupole operator

$$\mathbf{S}_{0123} = \frac{1}{N_c}\mathrm{Tr}\left(U_F(\mathbf{x}_0)\, U_F^\dagger(\mathbf{x}_1) U_F(\mathbf{x}_2)\, U_F^\dagger(\mathbf{x}_3)\right)$$

Requires the full B-JIMWLK evolution instead of the BK equation

Moreover, in the back-to-back correlation limit $|\mathbf{p}_0| \simeq |\mathbf{p}_1| \gg |\mathbf{p}_0 + \mathbf{p}_1|$:
→ link with the TMD formalism at low x
Dominguez, Marquet, Xiao, Yuan (2011)

Motivation for NLO corrections

So far, low x QCD phenomenology at LO (including LL resummation) with gluon saturation rather successful:
Qualitative or semi-quantitative agreement with the data across many observables.

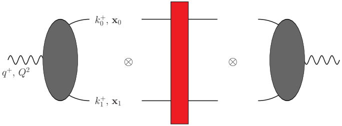
However, it has not been possible to observe the transition to the nonlinear regime in an unambigous way

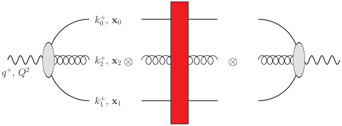
- No control over theoretical uncertainties
- Remaining issues like the necessity to artificially slow down the BK evolution

With a large increase in luminosity and lepton-nucleus scattering, the EIC is widely expected to test in a more stringent way gluon saturation physics than HERA.

$\Rightarrow$ It is urgent to push gluon saturation physics to NLO, bringing it to precision physics, in order to fully benefit from the EIC

NLO DIS calculation





$$
\begin{aligned}
\sigma_{T,L}(Q^2, x_{Bj}) \quad = \quad & \sum_{q\bar{q}\ \text{states}} \left|\widetilde{\Psi}^{\gamma^*_{T,L}}_{q\bar{q}}\right|^2 \Big[1 - \langle \mathbf{S}_{01}\rangle_0\Big] \\
& + \sum_{q\bar{q}g\ \text{states}} \left|\widetilde{\Psi}^{\gamma^*_{T,L}}_{q\bar{q}g}\right|^2 \Big[1 - \langle \mathbf{S}_{012}\rangle_0\Big] + O(\alpha_{em}\,\alpha_s^2)
\end{aligned}
$$

- Perturbative building blocks for NLO DIS:
  $\widetilde{\Psi}^{\gamma^*_{T,L}}_{q\bar{q}}$ LFWF at one loop and $\widetilde{\Psi}^{\gamma^*_{T,L}}_{q\bar{q}g}$ LFWF at tree-level
- UV divergences shown to cancel between $q\bar{q}$ and $q\bar{q}g$ ($\rightarrow$ Dim. Reg.)
- High-energy resummation performed at the end

G.B. (2016-2017) & Hänninen, Lappi and Paatelainen (2017)
see also Balitsky and Chirilli (2011-2013)

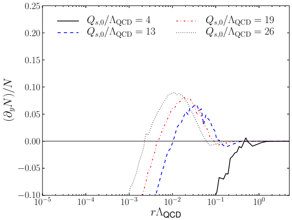
NLL Evolution

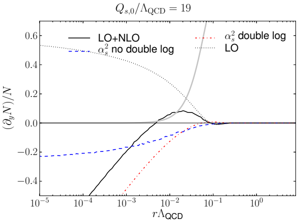
Both the BK and the JIMWLK equation are known at order α_s^2, allowing an NLL resummation

Balitsky and Chirilli (2008-2013) & Lublinsky and Mulian (2017)

However: large negative corrections in the NLL equations leading to inconsistencies.

Physically $0 < \langle N(r) \rangle_Y < 1$, but NLL BK makes it negative at small r, as observed numerically





Lappi, and Mäntysaari (2015)

Collinear resummation for the NLL evolution

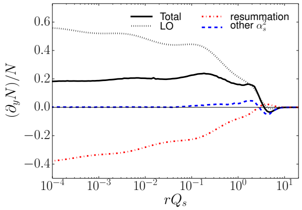
The problematic large collinear logs are artifacts from the kinematical approximations:

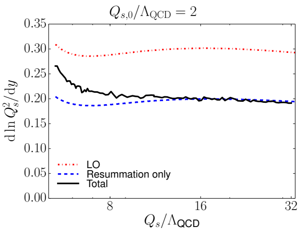
Physically, partons at each step in the evolution are ordered both in k^+ and in k^-.

But usual calculations maintain only one ordering.

$\Rightarrow$ Restoring the second ordering is equivalent to resum the large collinear logs
G. B. (2014) & Iancu *et al.* (2015-2019)

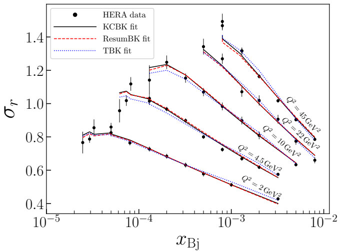
Performing such collinear resummation makes the solutions of NLL BK well-behaved numerically:

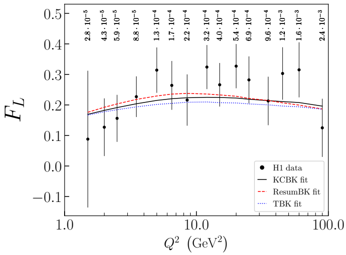




Lappi, and Mäntysaari (2016)

NLO DIS fit





Fit of $N(r, Y)$ on HERA data for σ_r ($\simeq F_2$) using :

- The NLO corrections to the cross section
- The BK equation at LL with running coupling and collinear resummation (various schemes)

$\rightarrow$ Successful fits, with weak dependence on collinear resummation scheme

$\rightarrow$ F_L obtained from the fit is consistent with HERA data

G.B., Hänninen, Lappi, and Mäntysaari (2020)

Other NLO results for DIS

The calculation of NLO correction to inclusive DIS is being extended to include quark masses:

- F_L with massive quarks
  G.B., Lappi and a Paatelainen (2021)

Other DIS processes have been calculated at NLO in the last few years:

- Exclusive dijet production
  Boussarie, Grabovsky, Szymanowski, and Wallon (2016-2019)
- Exclusive light vector meson production
  Boussarie, Grabovsky, Szymanowski, and Wallon (2017)
- Inclusive photon + dijet production
  Roy and Venugopalan (2020)
- Exclusive heavy vector meson production
  Mäntysaari and Penttala (2021)

Conclusions

- NLO revolution ongoing for low-x QCD with gluon saturation
- Precise predictions for most DIS observables of interest should be available before the start of the EIC

Other hot topic not mentioned in this talk:

Very active study of power-suppressed non-eikonal corrections in high-energy QCD

- Could be sizable at intermediate energies, like at the EIC
- Provides the leading behavior at low x_{Bj} for spin observables