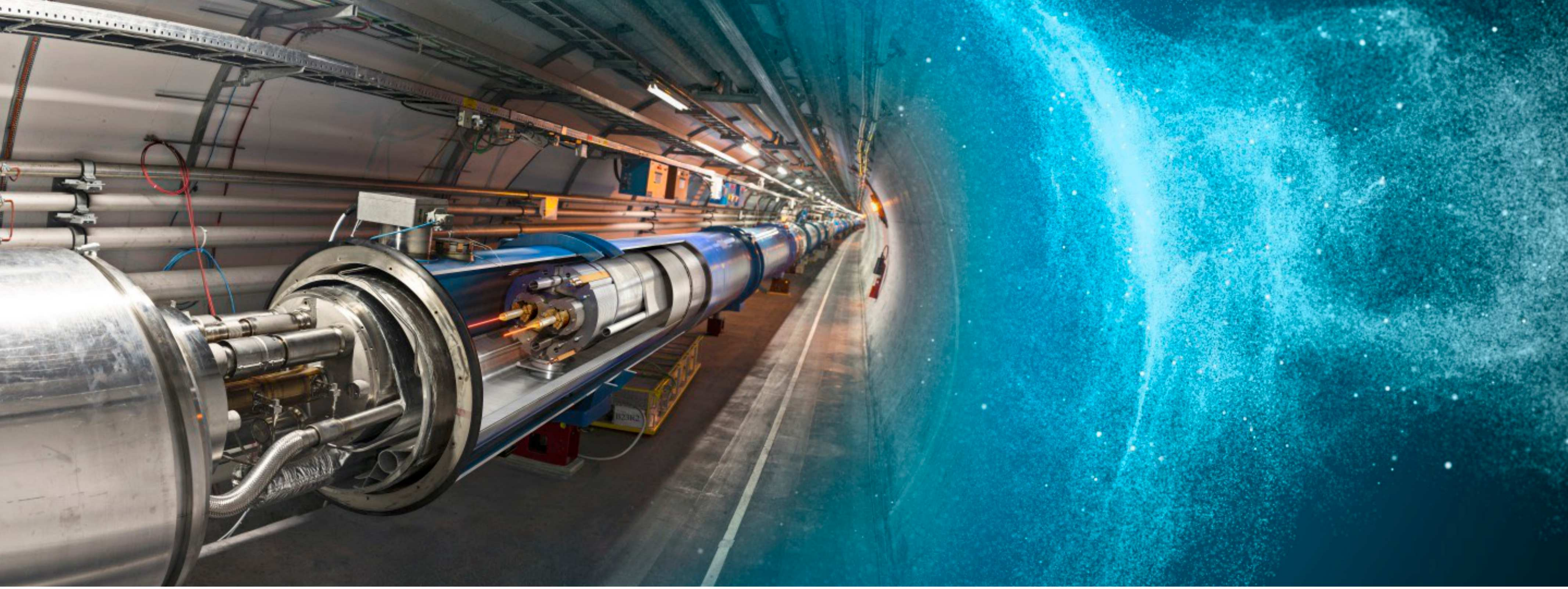


PRECISION PREDICTIONS FOR HIGGS BOSON PRODUCTION

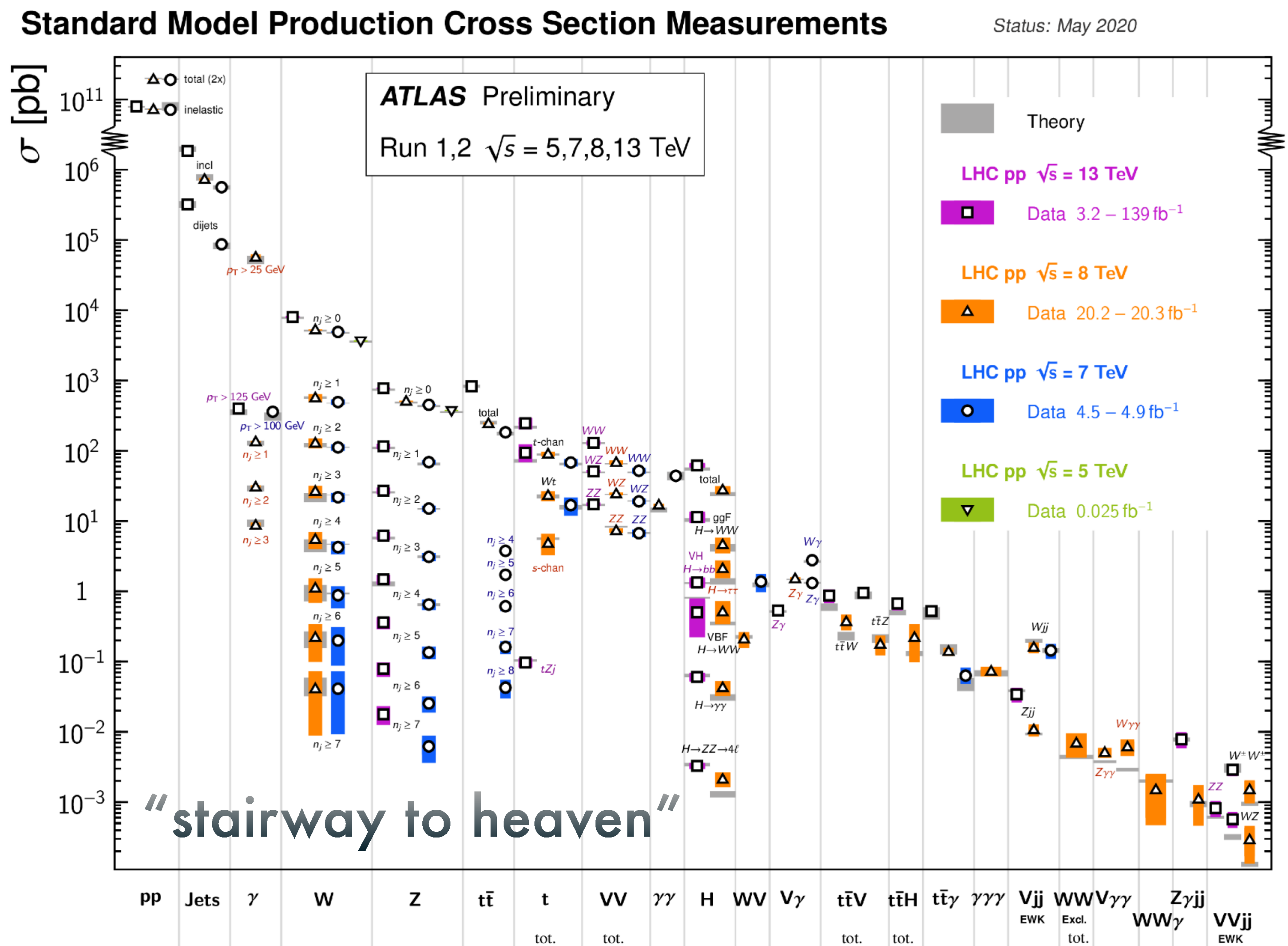
Alexander Huss

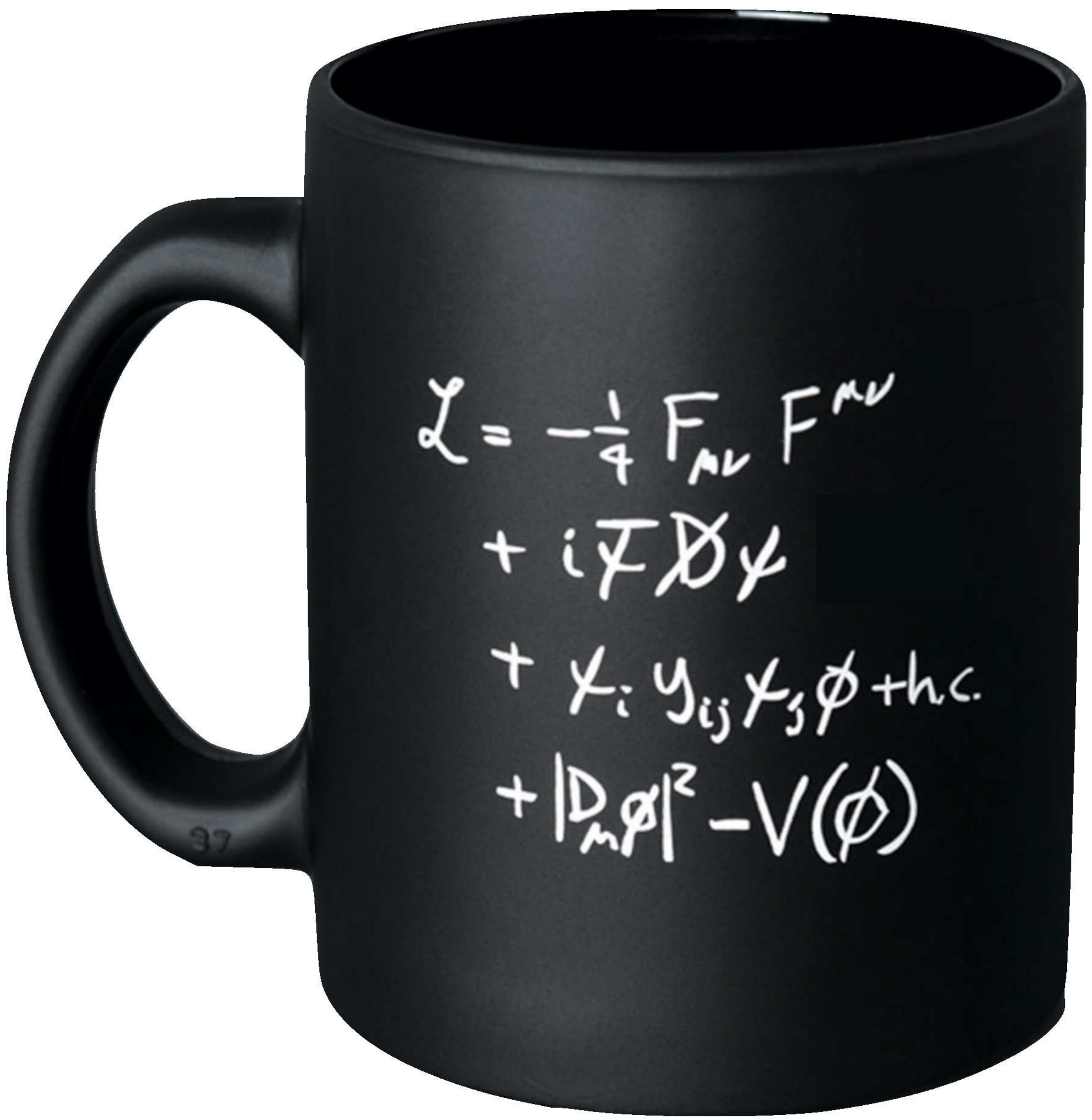
Work with: X. Chen, T. Gehrmann, N. Glover, B. Mistlberger, A. Pelloni

arXiv:2102.07607 [hep-ph]

HEP Seminar — Brookhaven National Laboratory, 29th April 2021

A Remarkable Success Story...





This equation neatly sums up our current understanding of fundamental particles and forces.

...But not the Full Story

- origin of dark matter
- hierarchy problem
- matter anti-matter asymmetry
- hierarchy of scales (generations)
- unification with gravity
- ...

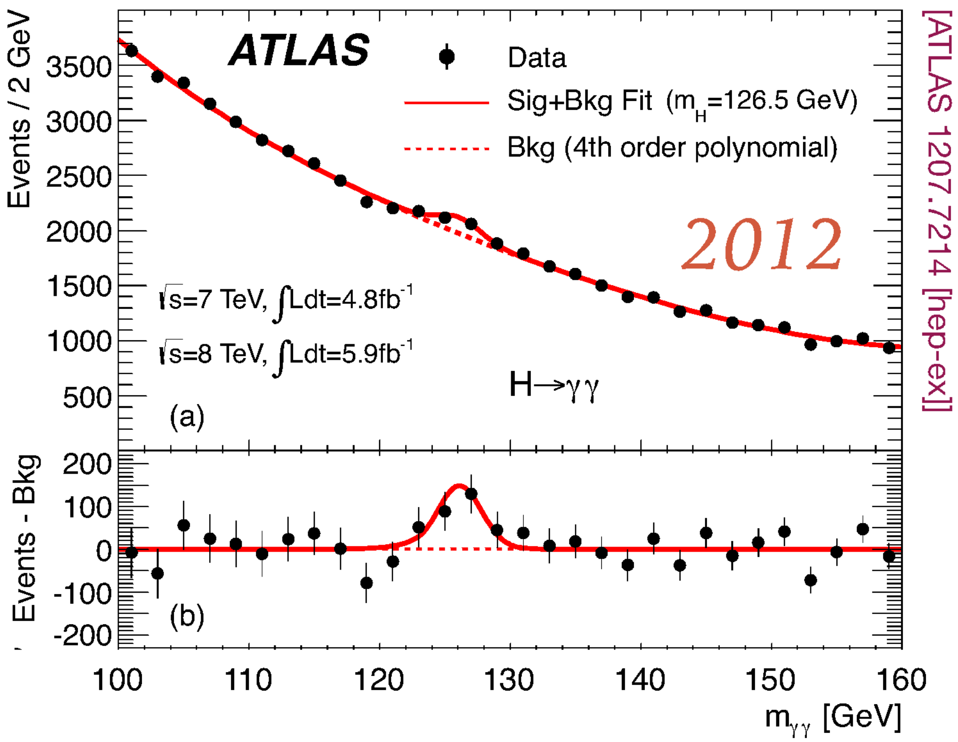
- is it *really* the SM Higgs? (properties)
- what is the Higgs potential?
- establish the Yukawa's Y_{ij}
- ...

scrutinise the Higgs sector

Experiments are Moving Fast

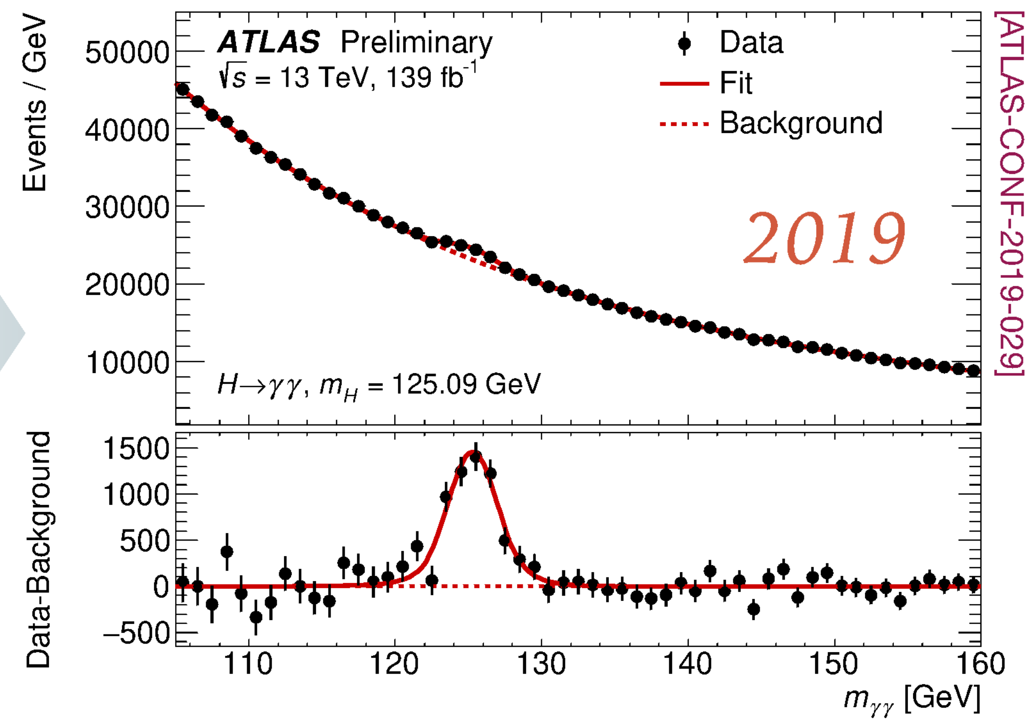
precision is the *name of the game!*

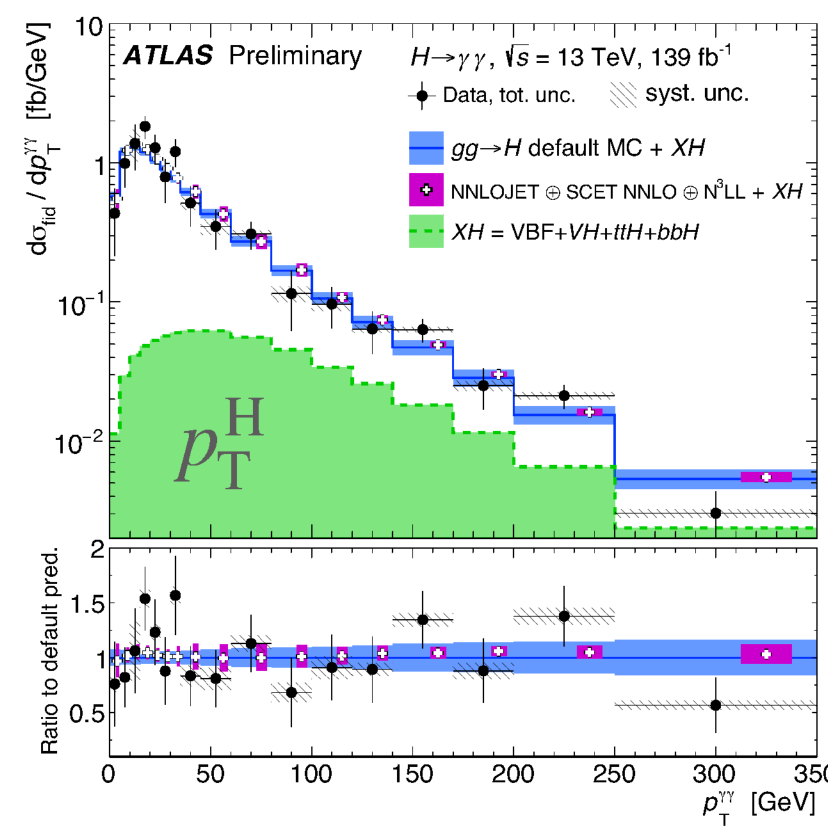
Discovery



< 10 years

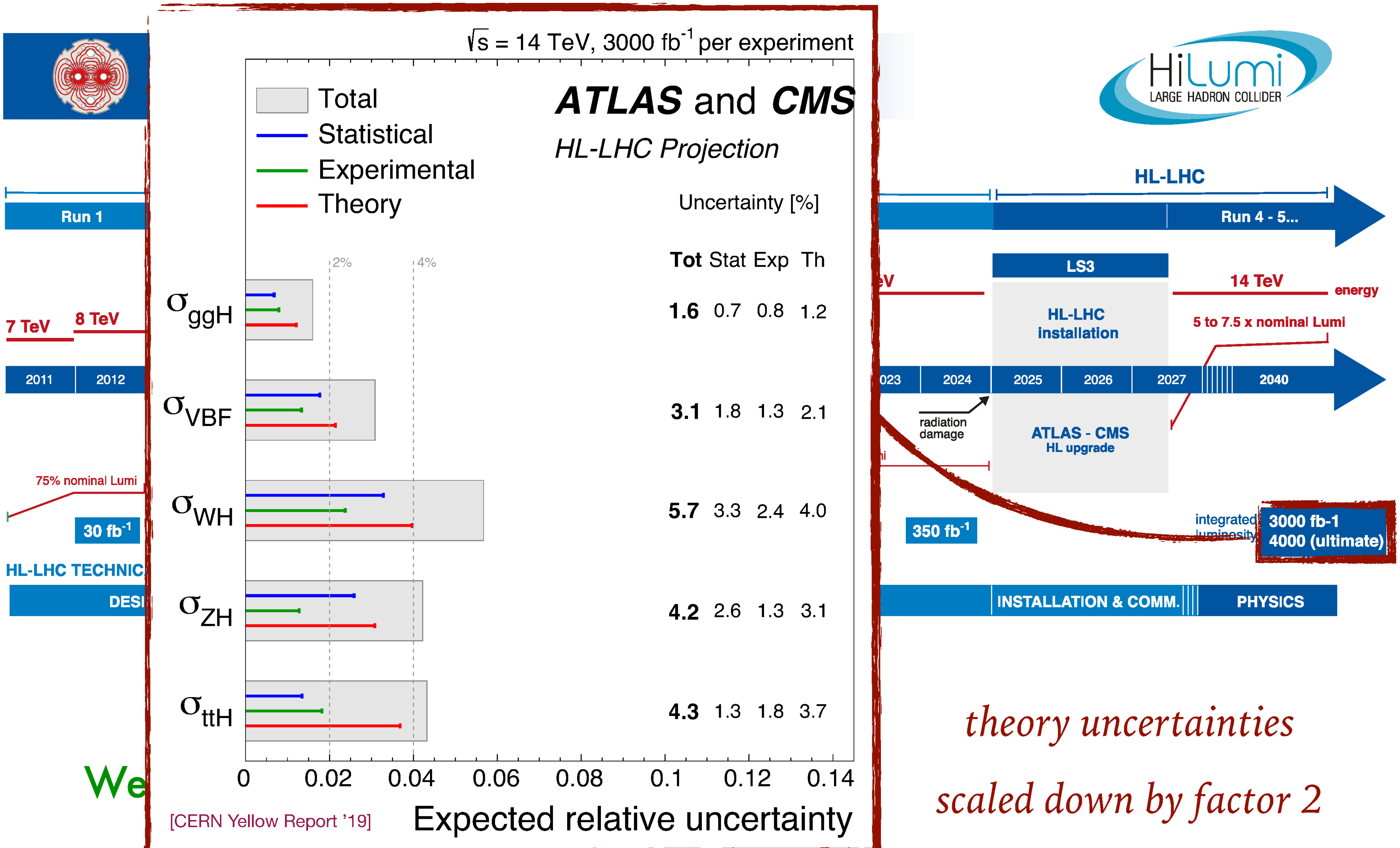
Today





- Higgs for precision phenomenology
  - signal strengths ⇝ differential spectra
  - Higgs properties (couplings, potential, …)
- A tool for New Physics searches
  - scalar (↔ sensitive to high scales), composite?
  - extended EW symmetry breaking sector, portal, …?

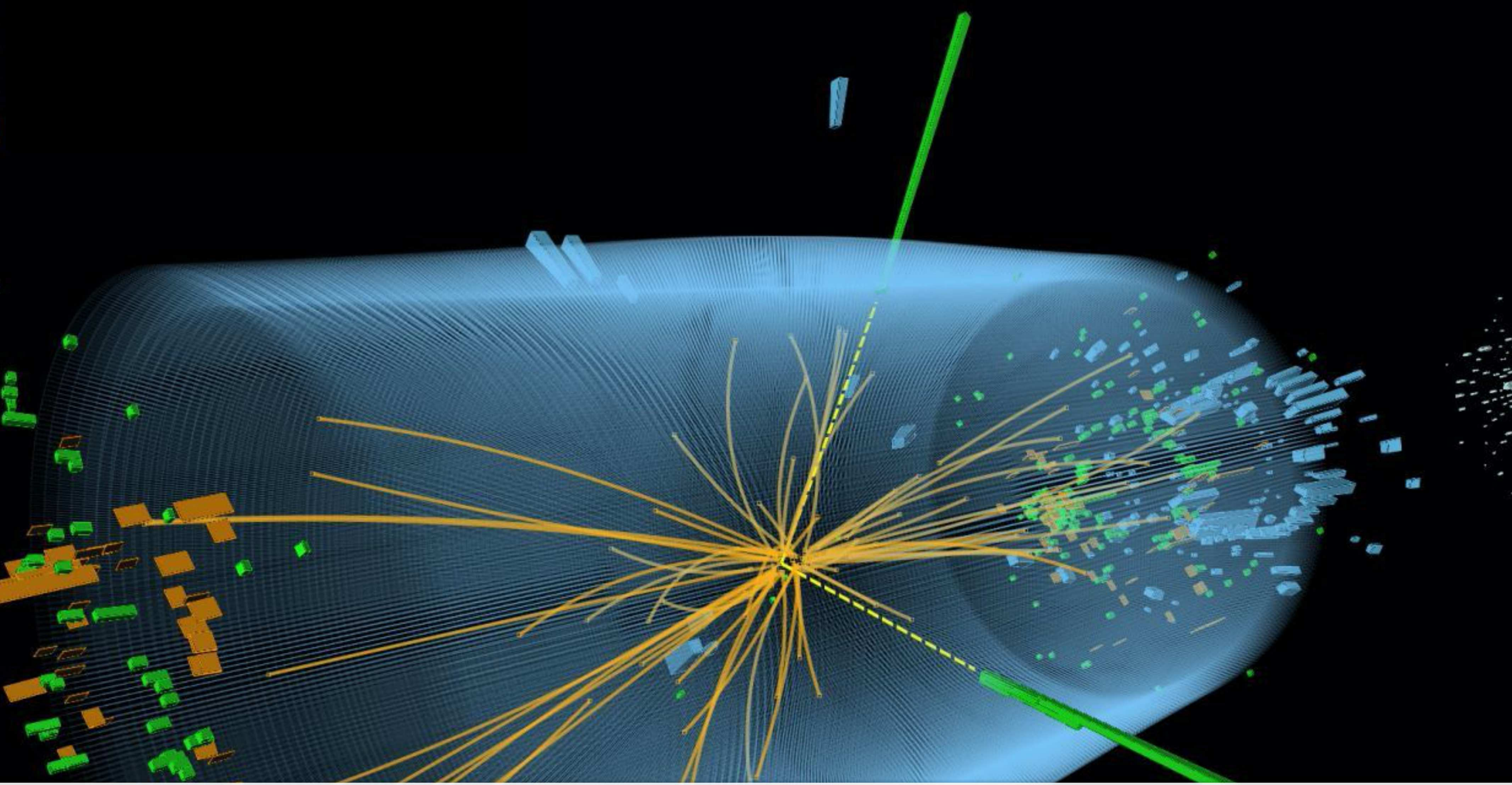
How Much Precision?



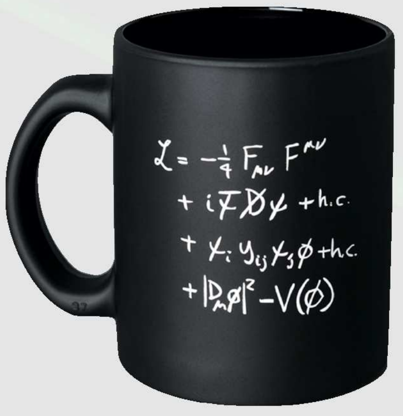
theory uncertainties

scaled down by factor 2

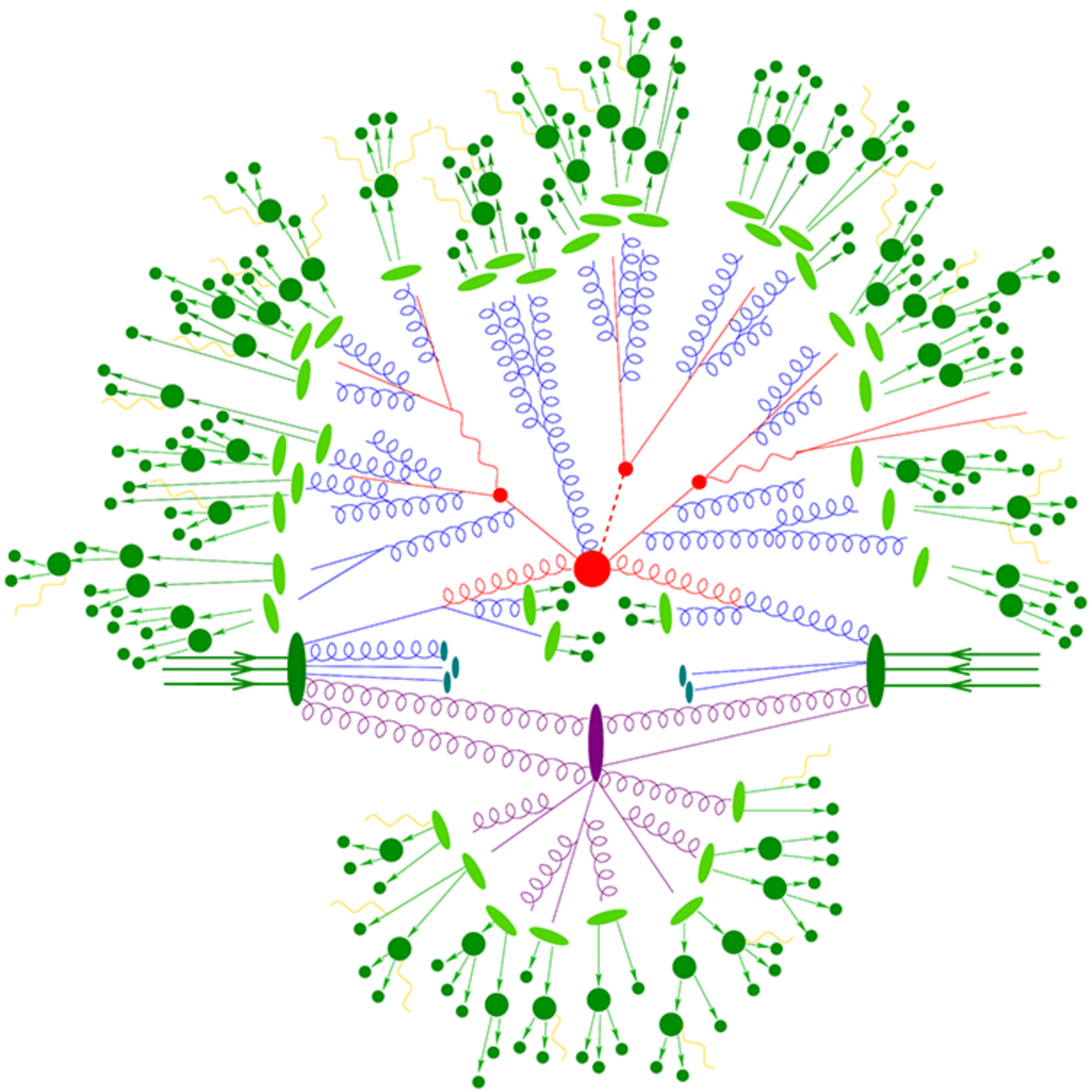
[hilumilhc.web.cern.ch]



How do we Predict this from THEORY?

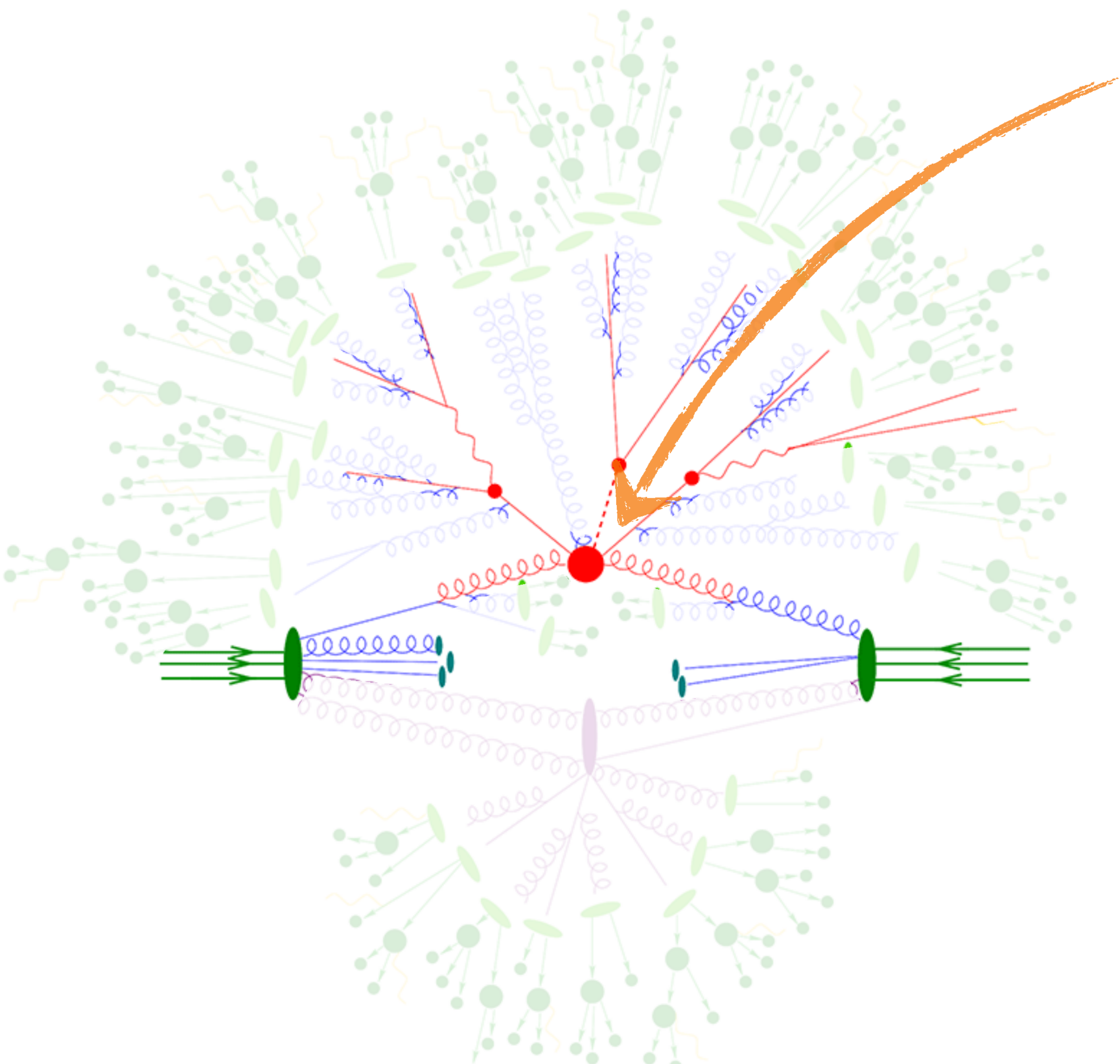


HIGH-PRECISION THEORY PREDICTIONS!



- (HL-)LHC — per-cent level!
- FOCUS — clean signature & high momentum transfer
  - *perturbative QCD*
- with $\alpha_s \sim 0.1$
  - *NLO* $\sim \mathcal{O}(10\%)$, *NNLO* $\sim \mathcal{O}(1\%)$
  - *exceptions: Higgs*
- predictions as close as possible to the experiment
  - *fiducial cross sections & differential distributions*

HIGH-PRECISION THEORY PREDICTIONS!



- (HL-)LHC — per-cent level!
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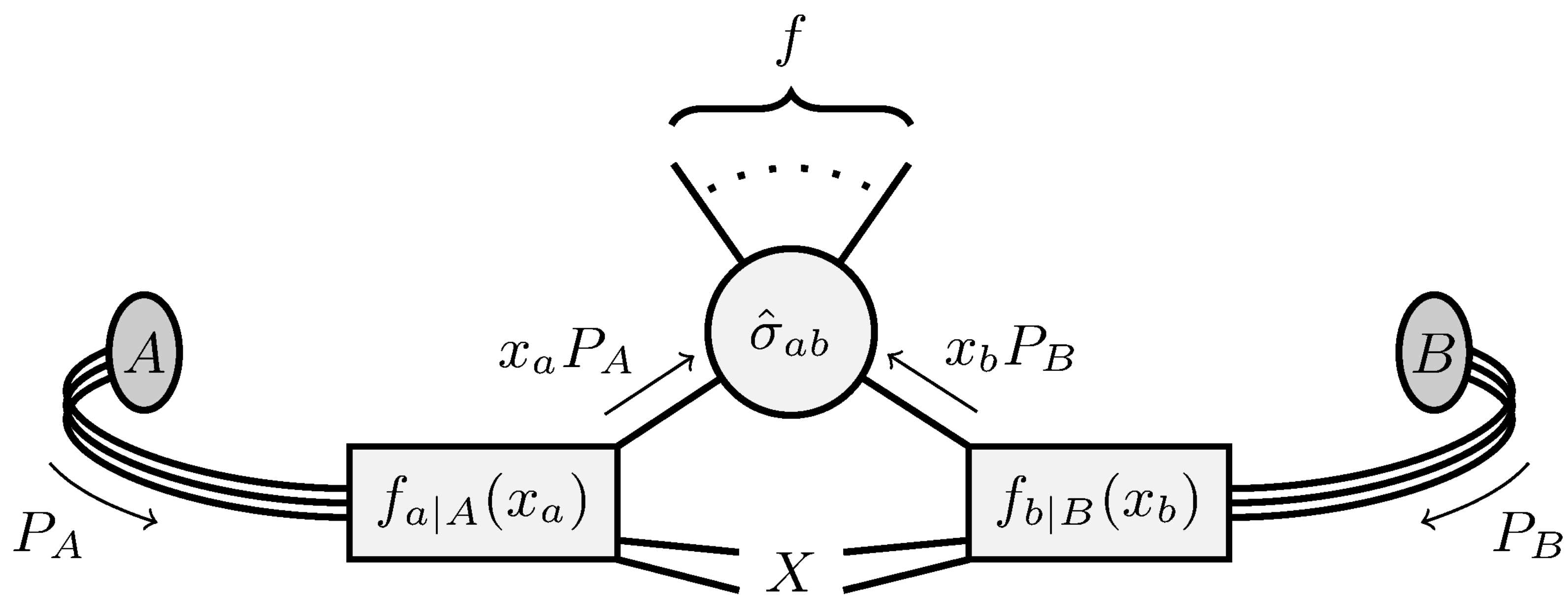
  - $NLO \sim \mathcal{O}(10\%),\ NNLO \sim \mathcal{O}(1\%)$
  - *exceptions: Higgs*
- predictions as close as possible to the experiment
  - *fiducial cross sections & differential distributions*

Theory Predictions for the LHC



$$\sigma_{AB} = \sum_{ab} \int_0^1 \mathrm{d}x_a \int_0^1 \mathrm{d}x_b \; f_{a|A}(x_a)\; f_{b|B}(x_b)\; \hat{\sigma}_{ab}(x_a, x_b)\; \left(1 + \mathcal{O}(\Lambda_{\mathrm{QCD}}/Q)\right)$$

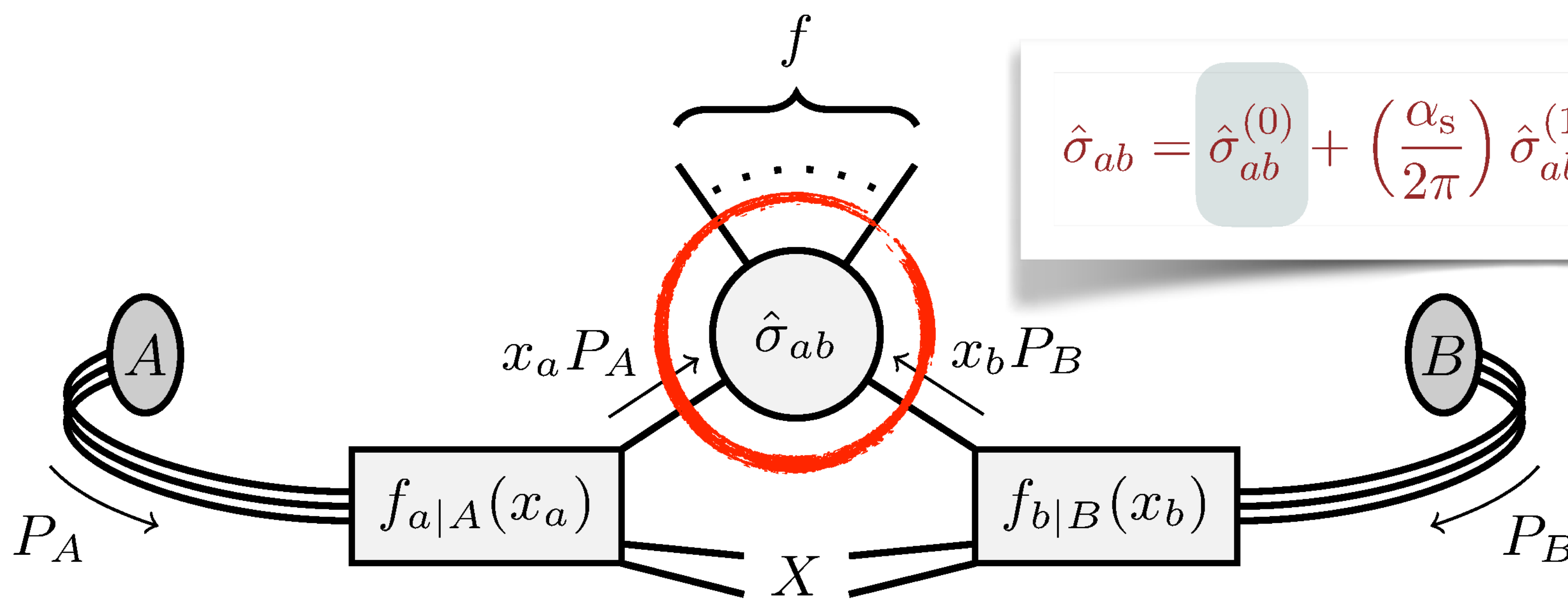
parton distribution functions
(non-perturbative, universal)

hard scattering
(perturbation theory)

non-perturbative effects
(no good understanding)
ultimately, limiting factor?

Hard Scattering — Perturbation Theory

$$\hat{\sigma}_{ab} = \hat{\sigma}^{(0)}_{ab} + \left(\frac{\alpha_s}{2\pi}\right)\hat{\sigma}^{(1)}_{ab} + \left(\frac{\alpha_s}{2\pi}\right)^2\hat{\sigma}^{(2)}_{ab} + \ldots$$

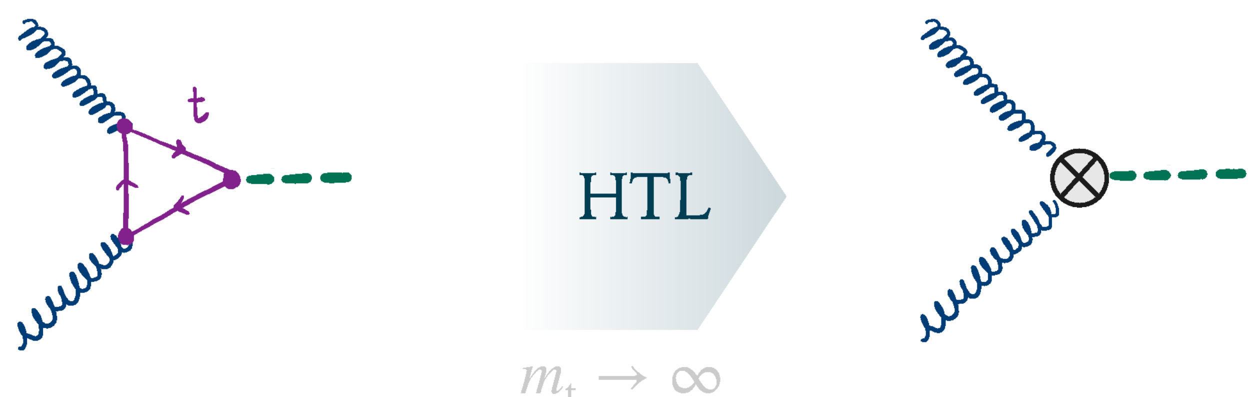


gluon-fusion production:

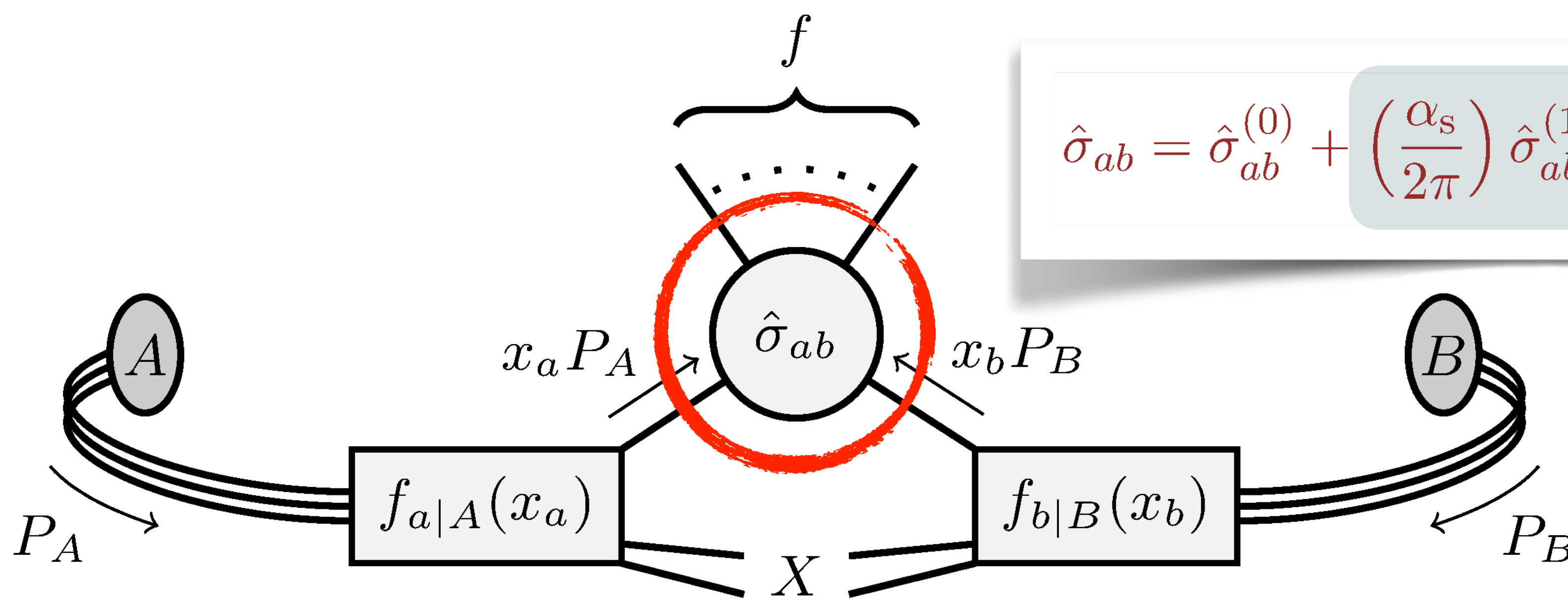
- $\sim 90\,\%$ of all Higgs
- "heavy top limit"

leading order (LO)

"tree level"

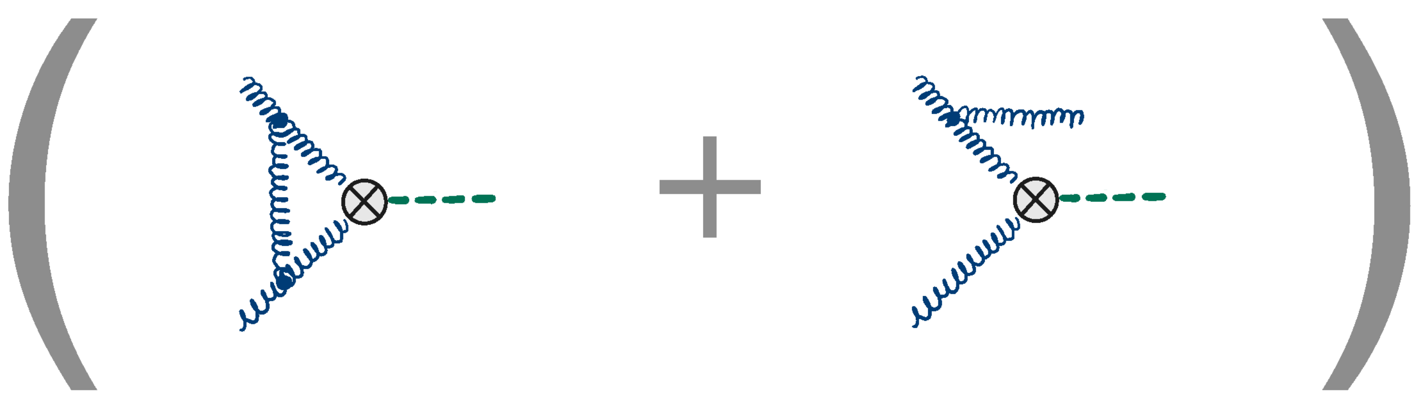


Hard Scattering — Perturbation Theory



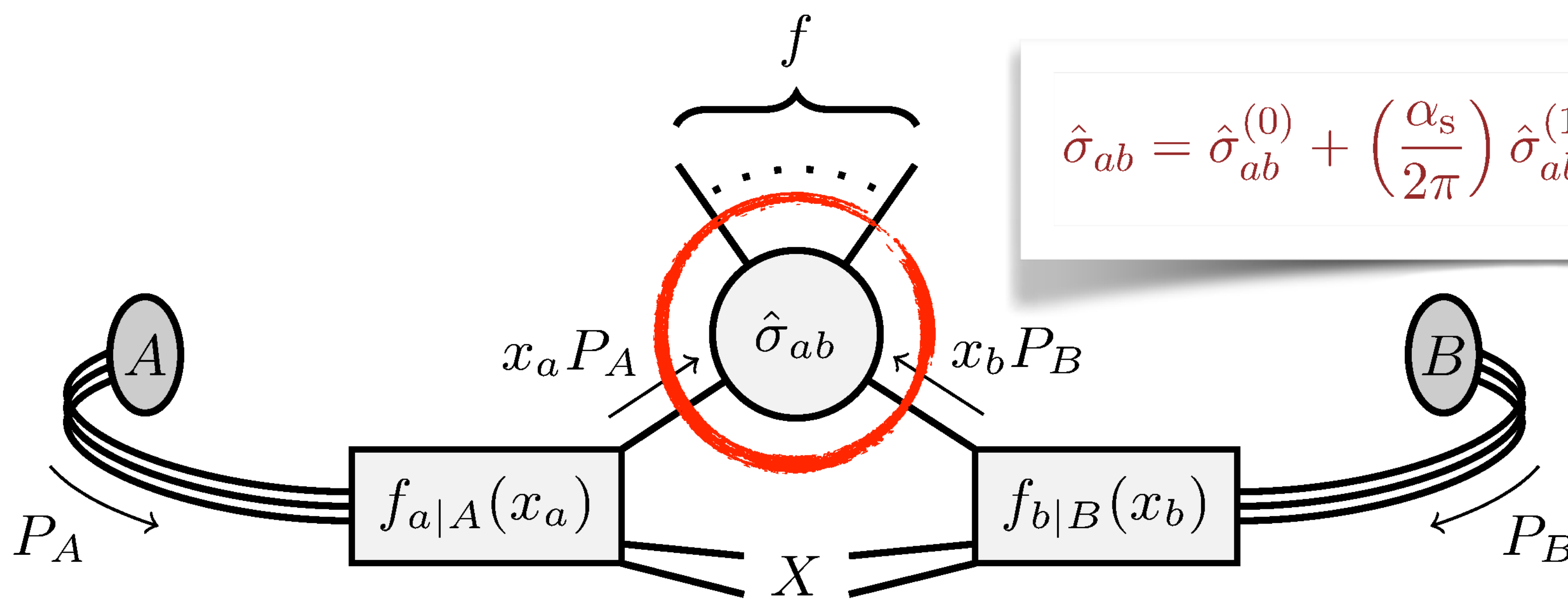
$$\hat{\sigma}_{ab} = \hat{\sigma}_{ab}^{(0)} + \left(\frac{\alpha_s}{2\pi}\right)\hat{\sigma}_{ab}^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2\hat{\sigma}_{ab}^{(2)} + \ldots$$

next-to-leading order (NLO)



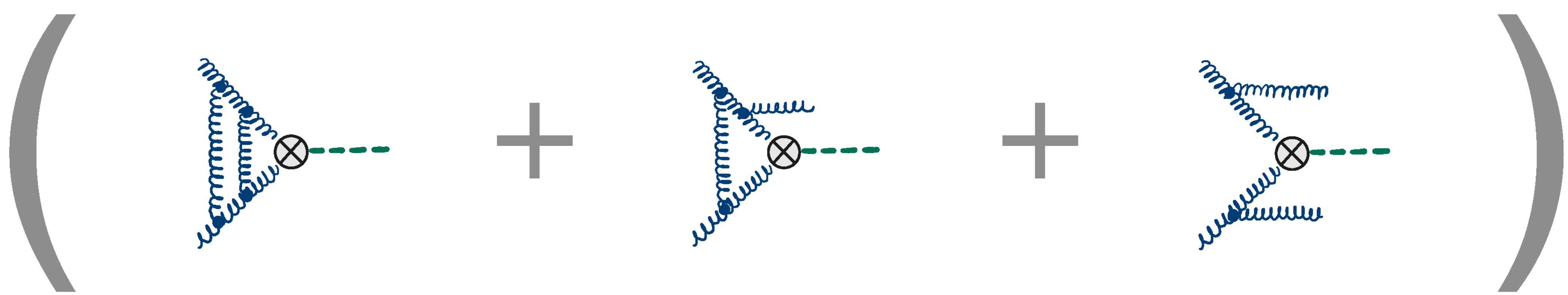
"virtual" (V) *"real" (R)*

Hard Scattering — Perturbation Theory



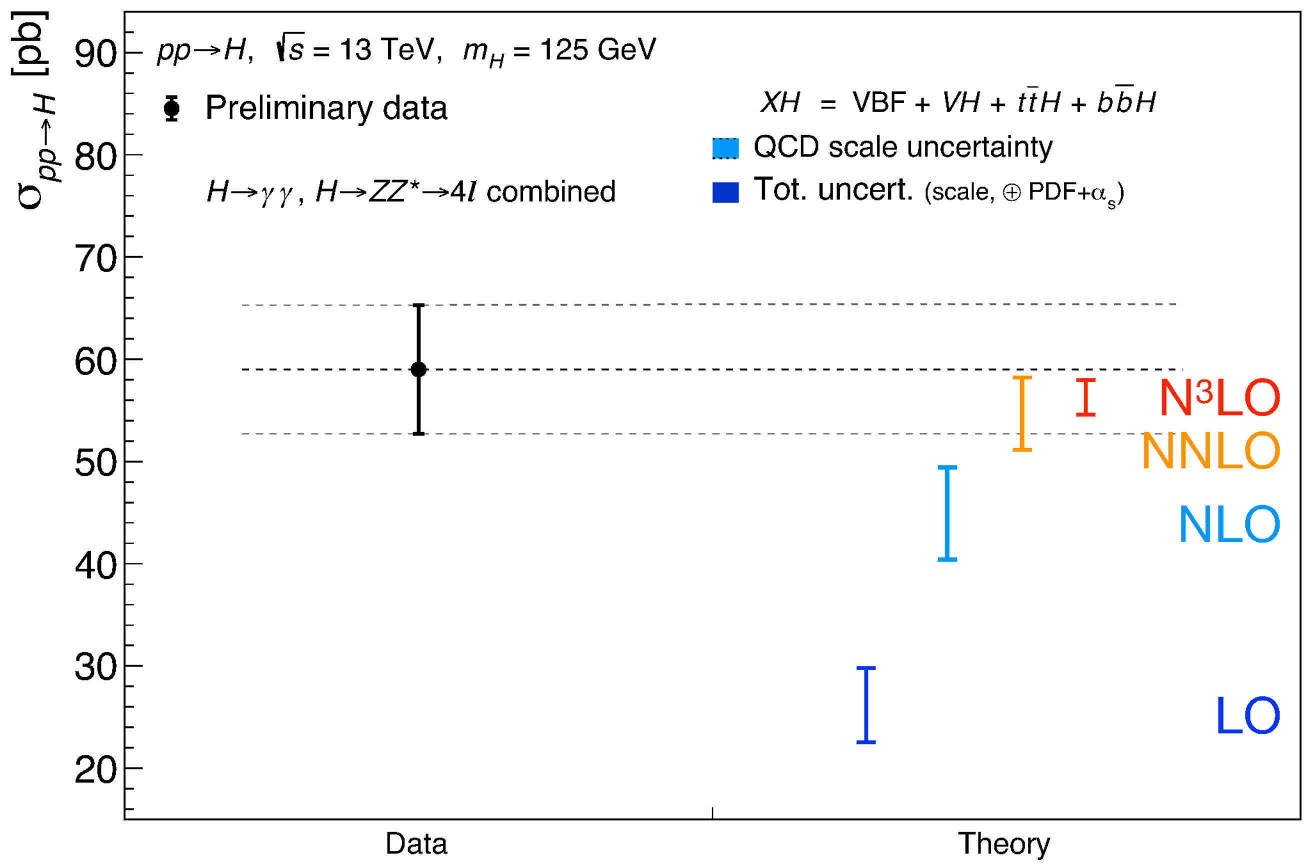
$$\hat{\sigma}_{ab} = \hat{\sigma}^{(0)}_{ab} + \left(\frac{\alpha_s}{2\pi}\right)\hat{\sigma}^{(1)}_{ab} + \left(\frac{\alpha_s}{2\pi}\right)^2\hat{\sigma}^{(2)}_{ab} + \dots$$

next-to-next-to-leading order (NNLO)



"double virtual" (VV) *"real-virtual" (RV)* *"double real" (RR)*

HARD SCATTERING — PERTURBATION THEORY

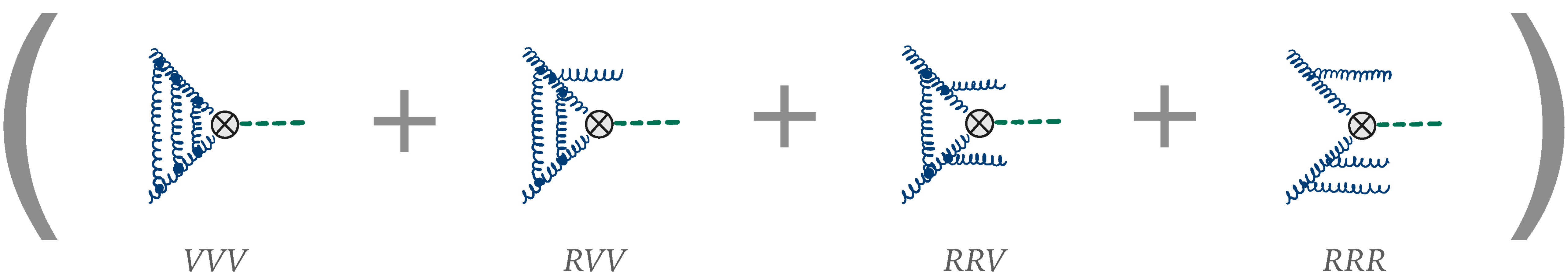


- Higgs in gluon fusion
  - notoriously slow convergence
- N^3LO stabilises expansion!

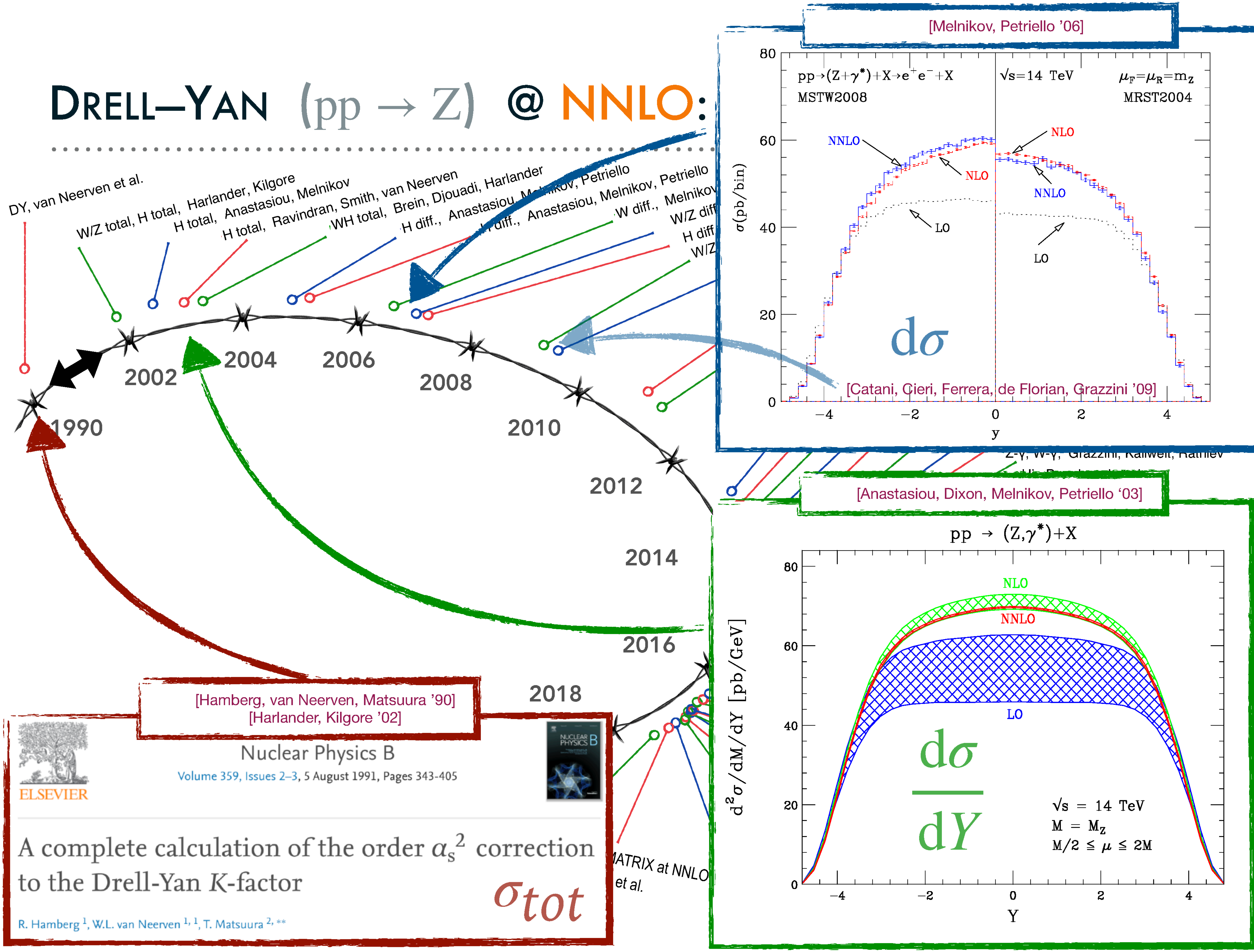
[Anastasiou et al. '15]
[Mistlberger '18]

> *"so, N^3LO is a solved problem then?"*
> *…not quite*

next-to-next-to-next-to-leading order (N^3LO)



Drell–Yan (pp → Z) @ NNLO:



HIGGS @ N³LO & GOING DIFFERENTIAL

$\sigma_{tot}^{pp\to H}$

- What is the probability of producing a Higgs boson?

inclusive

$$\sigma_{\rm tot}^{\rm N^3LO} = 48.68\ \text{pb}^{+2.07\ \text{pb}}_{-3.16\ \text{pb}}$$

✓ analytic integration over full phase space

✗ no information on final state

[Anastasiou et al. '15] [Mistlberger '18]

HIGGS @ N³LO & GOING DIFFERENTIAL

$\sigma_{tot}^{pp\to H}$ — What is the probability of producing a Higgs boson?

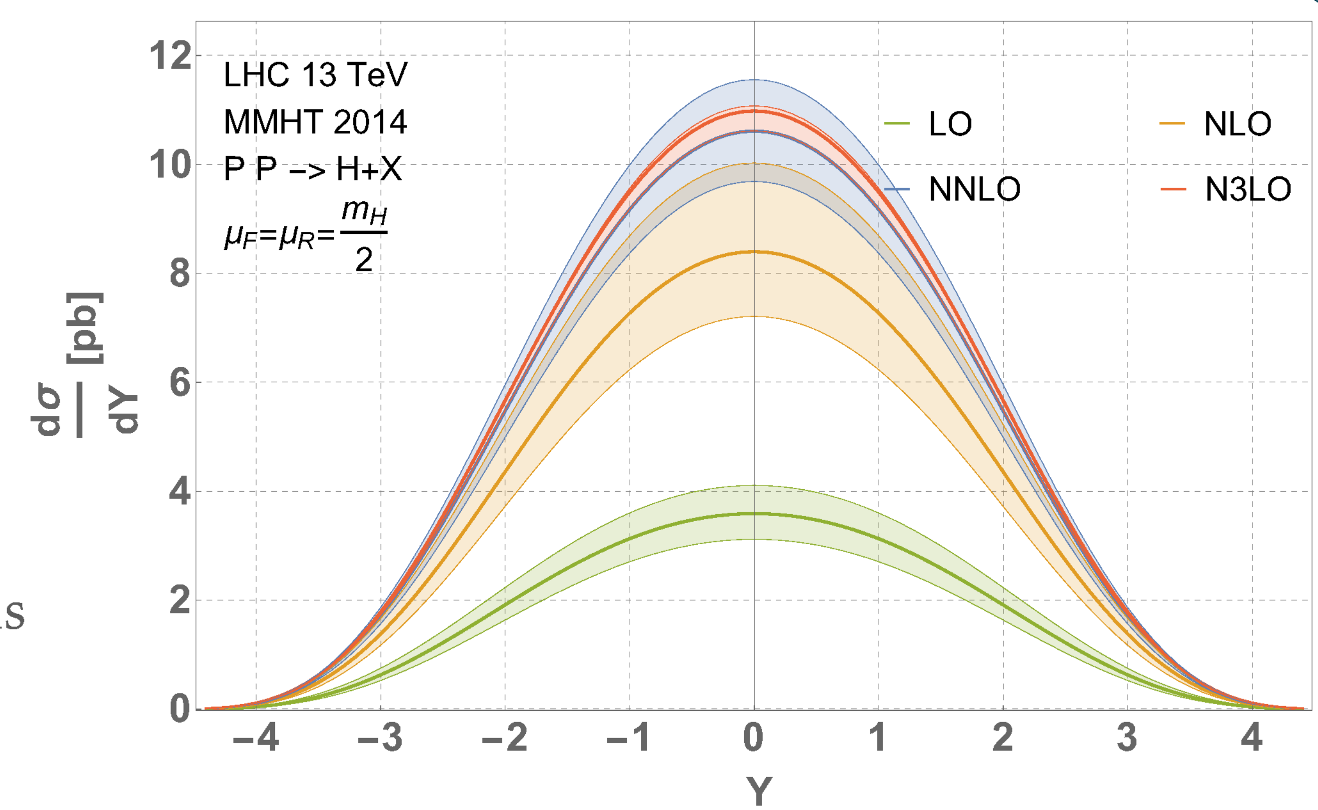
$\frac{d\sigma^{pp\to H}}{dY}$ … in direction $Y = \frac{1}{2}\ln\left(\frac{E+p_z}{E-p_z}\right)$ $\begin{cases} Y \to 0 & \Leftrightarrow \perp \text{ to beam} \\ Y \to \infty & \Leftrightarrow \parallel \text{ to beam} \end{cases}$

y_H differential

✔ analytic integration over QCD emissions

✘ partial information on final state

- only $y_H \rightsquigarrow$ no decay kinematics
- no information on final-state partons



[Dulat, Mistlberger, Pelloni '18]

Higgs @ N³LO & Going Differential

$\sigma^{pp\to H}_{tot}$

- What is the probability of producing a Higgs boson?

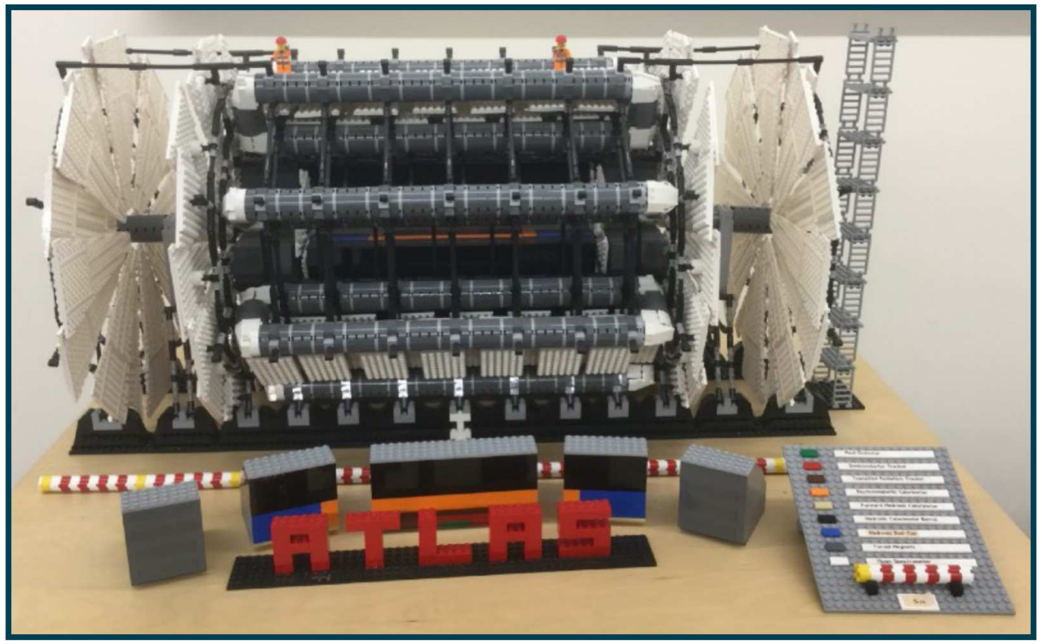
$\dfrac{d\sigma^{pp\to H}}{dY}$

... in direction $Y = \frac{1}{2}\ln\left(\frac{E+p_z}{E-p_z}\right)$ $\begin{cases} Y\to 0 & \Leftrightarrow \perp \text{ to beam} \\ Y\to\infty & \Leftrightarrow \parallel \text{ to beam}\end{cases}$

$d\sigma^{pp\to H}$

... where the Higgs decays into a pair of photons, $H\to\gamma\gamma$, and the leading and sub-leading photon have a transverse momentum that is larger than 35% and 25% of the Higgs boson mass, respectively, and are produced within the rapidity interval $|y_\gamma| < 2.37$, where the barrel-endcap region $1.37 < |y_\gamma| < 1.52$ is excluded. Photons are further required to be isolated from additional QCD activity by requiring that the scalar sum of the transverse momenta of hadrons in a cone of $\Delta R = 0.2$ around the photons is less than 5 % of the photon transverse energy E_T.

Measurements are done within a fiducial volume:



Higgs @ N³LO & Going Differential

$\sigma_{tot}^{pp\to H}$

- What is the probability of producing a Higgs boson?

$\frac{d\sigma^{pp\to H}}{dY}$

… in direction $Y = \frac{1}{2}\ln\left(\frac{E+p_z}{E-p_z}\right) \begin{cases} Y \to 0 & \Leftrightarrow \perp \text{ to beam} \\ Y \to \infty & \Leftrightarrow \parallel \text{ to beam} \end{cases}$

$d\sigma^{pp\to H}$

- ask *any** question!

fully differential

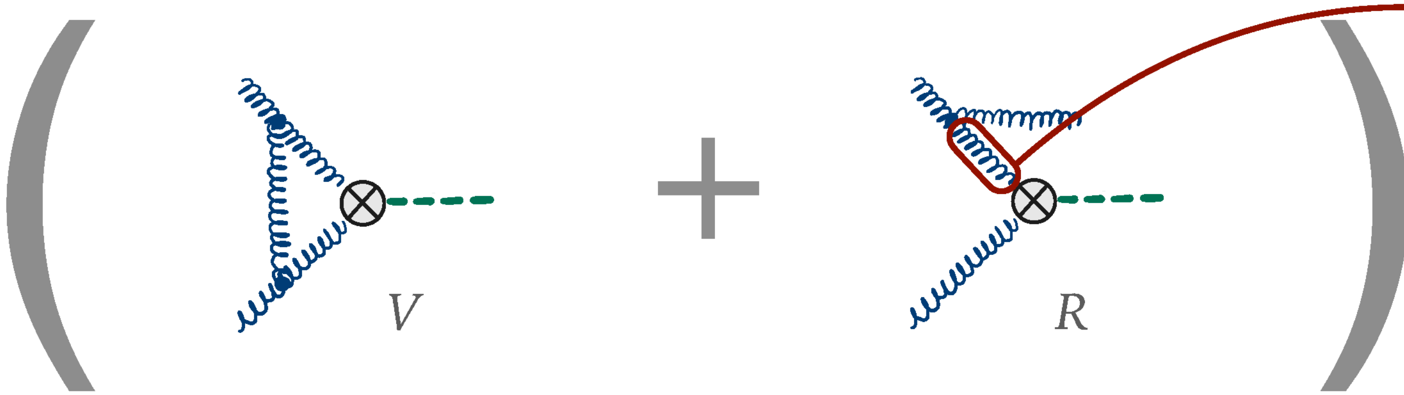
✔ numerical integration of phase space

✔ complete final-state information (decay, isol., …)

Infrared Singularities:
Subtractions

*infrared safe

SUBTRACTIONS — NLO

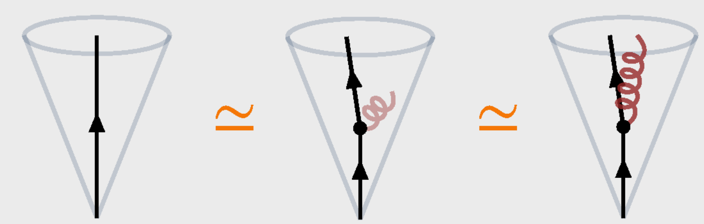


$$\sim \frac{1}{(p_a - k_1)^2} \sim \frac{1}{p_{\mathrm{T},1}}$$

- *$1/\varepsilon^2$, $1/\varepsilon$*
- *single unresolved*

$p_{\mathrm{T}}^{\mathrm{H}} = p_{\mathrm{T},1} \to 0$

indistinguishable:



soft: $E_g \to 0$

collinear: $\theta_{qg} \to 0$

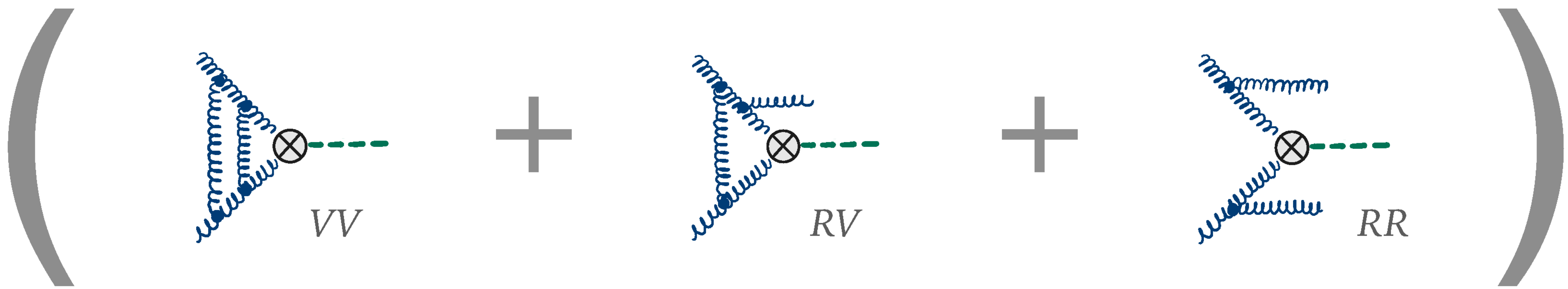
Infrared subtraction:

- retain *full* final-state information
- reshuffle singularities
  - extract singularities of R
  - cancel against V (+ factorization)

conceptually solved:

CS dipoles, FKS, …

SUBTRACTIONS — NNLO



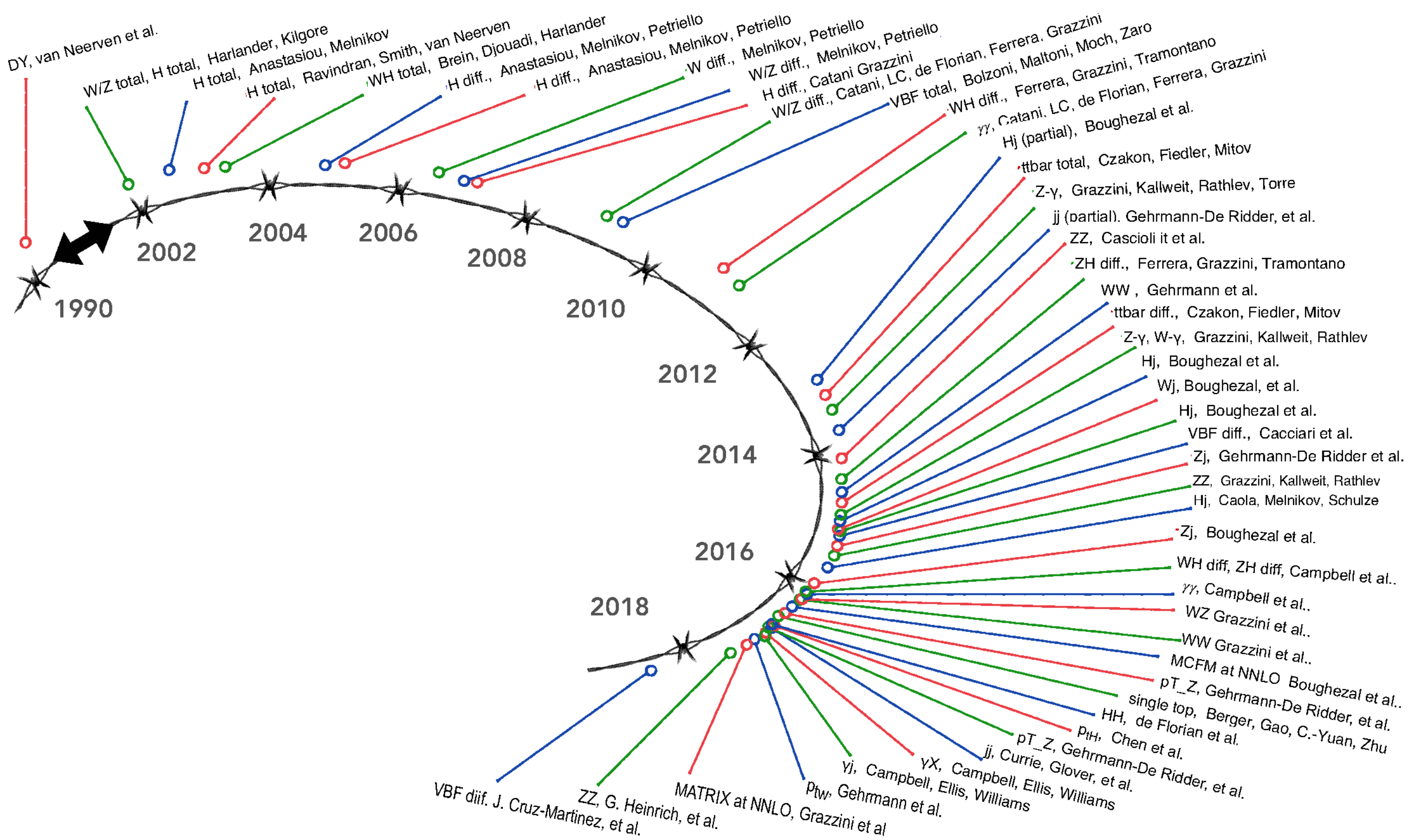
- $1/\varepsilon^4$, $1/\varepsilon^3$, $1/\varepsilon^2$, $1/\varepsilon$

- $1/\varepsilon^2$, $1/\varepsilon$
- *single unresolved*

- *single unresolved*
- *double unresolved*

single unresolved $\simeq$ H+jet @ NLO

fully unresolved $\simeq$ H @ NNLO



- tremendous progress:
  - $2 \to 2$ done for most relevant processes
  - $2 \to 3$ next frontier (first results…)
- ➡ an *optimal method* has yet to emerge

Different Methods*

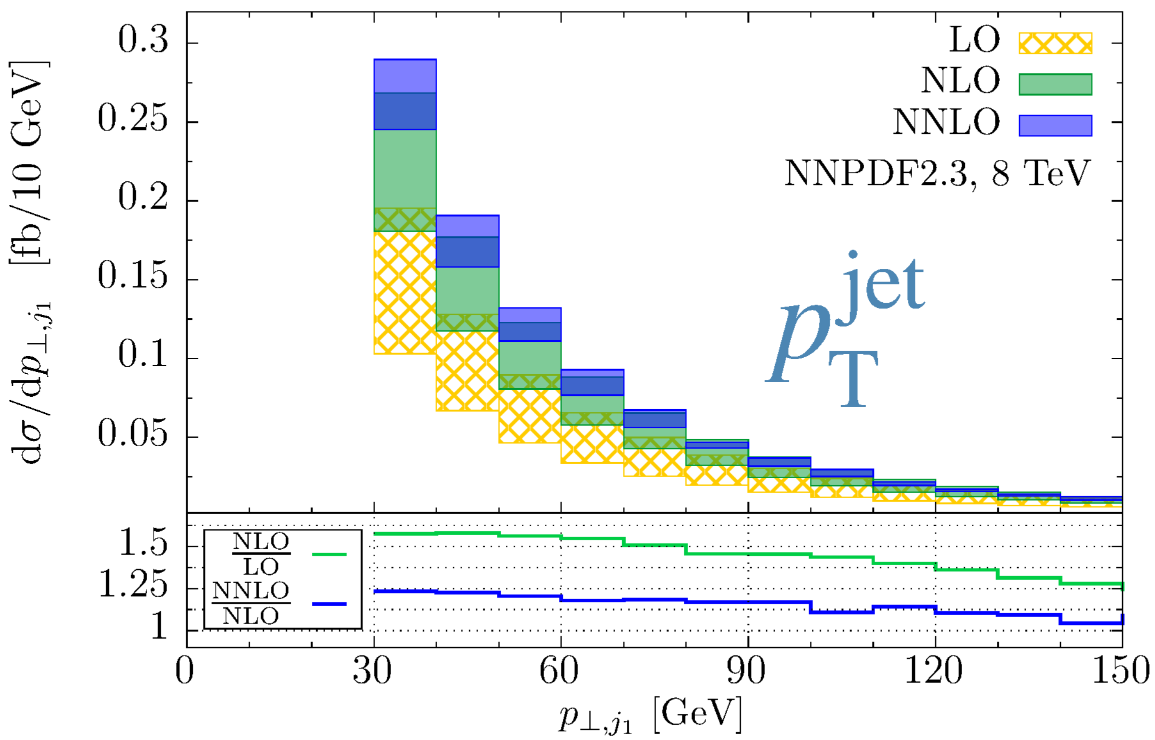
- Antenna [Gehrmann–De Ridder, Gehrmann, Glover '05]
- CoLorFul [Del Duca, Somogyi, Trocsanyi '05]
- qT-subtraction [Catani, Grazzini '07; MATRIX]
- STRIPPER (sector-improved residues) [Czakon '10]
- nested soft-collinear [Caola, Melnikov, Röntsch '17]
- N-jettiness [Gaunt, Stahlhofen, Tackmann, Walsh '15] [Boughezal, Focke, Liu, Petriello '15; MCFM]
- Projection-to-Born [Cacciari, et al. '15]
- Geometric, Local analytical Sectors
  [Herzog '18] [Magnea et al. '18]

* *Subtraction & Slicing*

Higgs + Jet @ NNLO — 3 Calculations!

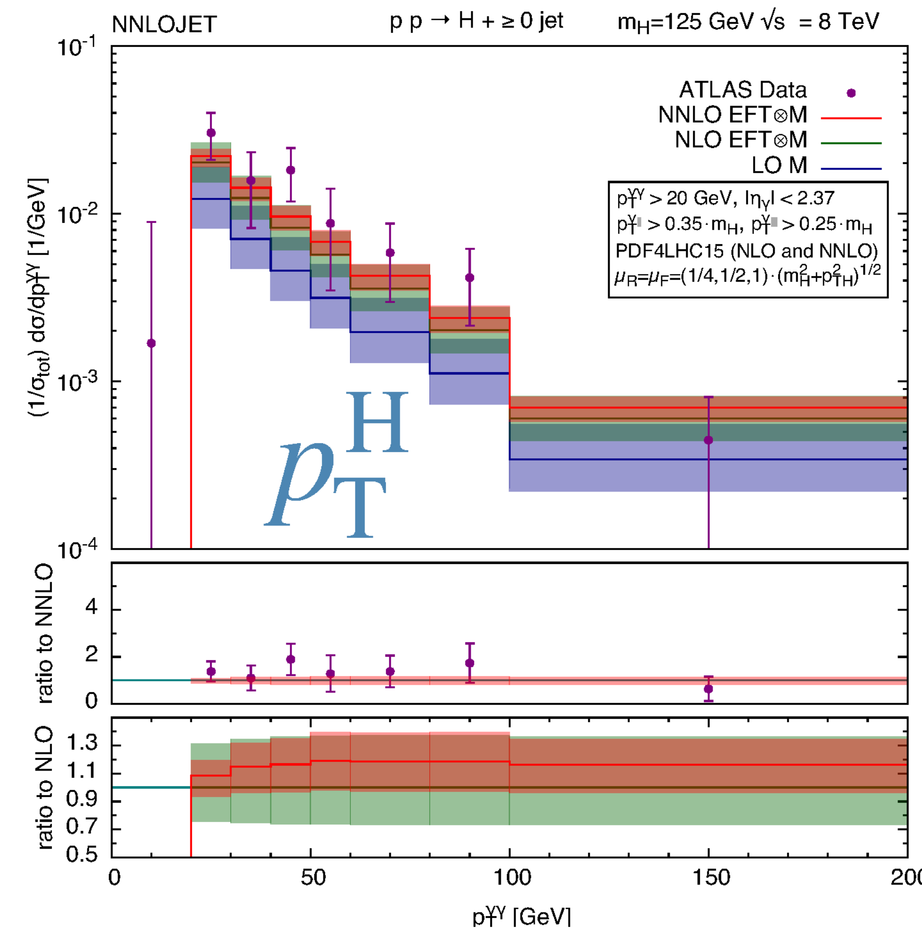
residue subtraction

[Caola, Melnikov, Schulze '15]



antenna subtraction

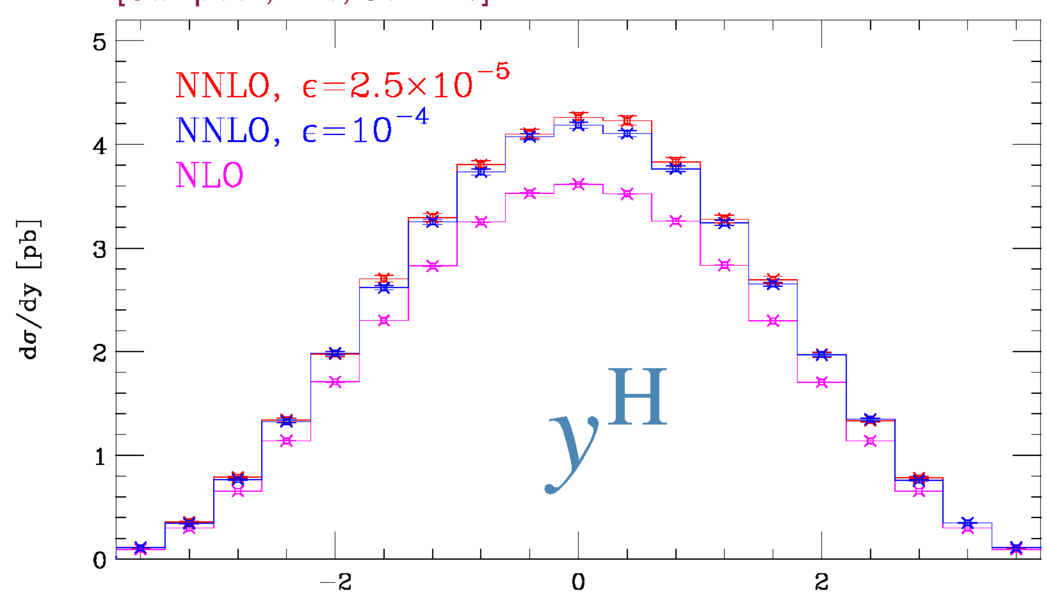
[Chen, Cruz-Martinez, Gehrmann, Glover, Jaquier '16]



τ_1 jettiness subtraction

[Boughezal, Focke, Giele, Liu, Petriello '15]

[Campbell, Ellis, Seth '19]



- *very complex* calculations
  - validation!
  - H+jet ⇝ agreement
- benchmark approaches

SUBTRACTIONS — N³LO

laregely unexplored

- *$1/\varepsilon^6$, $1/\varepsilon^5$, …*

- *$1/\varepsilon^4$, $1/\varepsilon^3$, …*
- *single unresolved*

- *$1/\varepsilon^2$, $1/\varepsilon$*
- *single unresolved*
- *double unresolved*

- *single unresolved*
- *double unresolved*
- *triple unresolved*

two methods for
"$2 \to 1$"

q_T-subtraction
Projection-to-Born

isolate "radiating" part

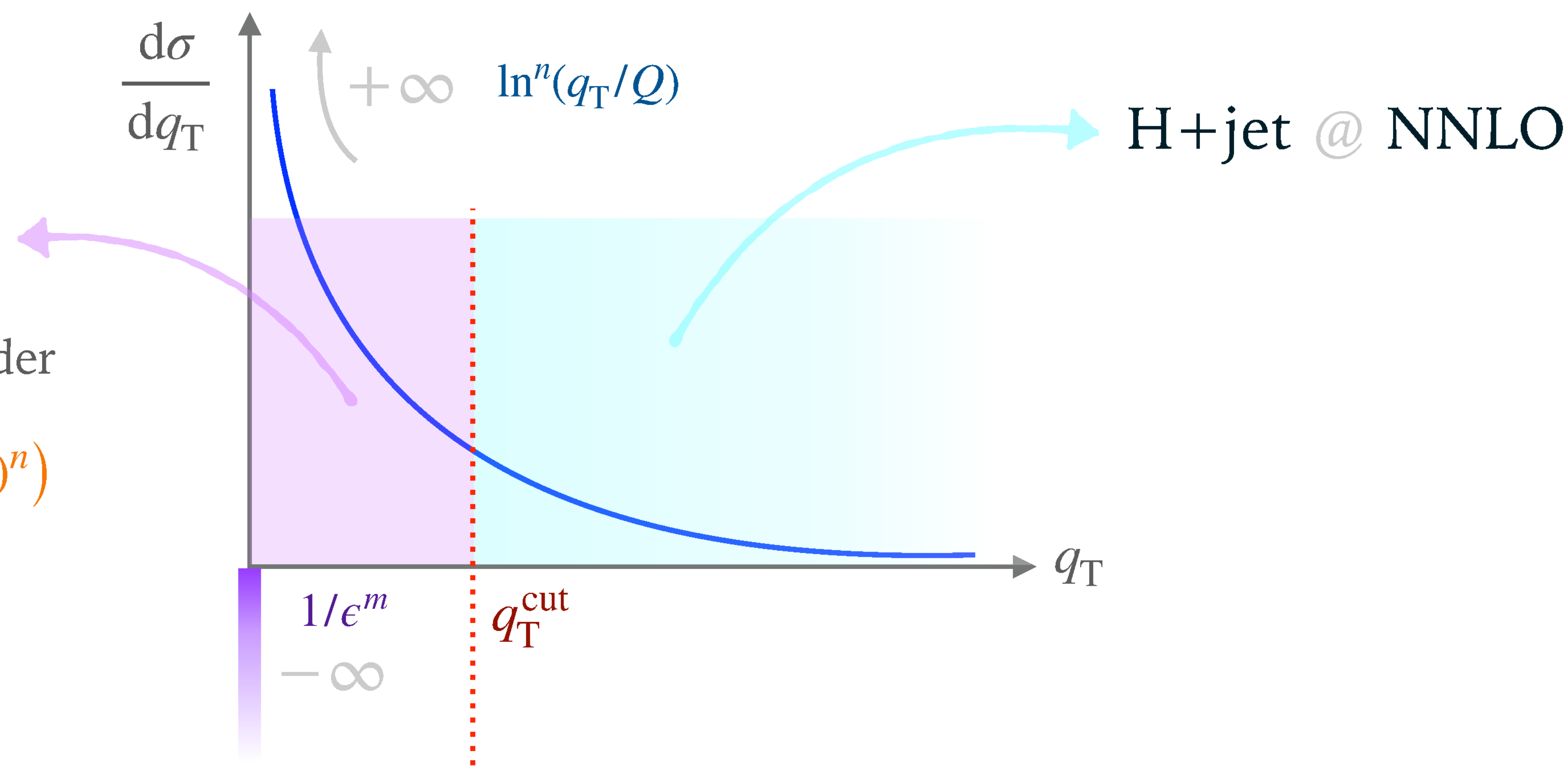
double unresolved $\simeq$ H+jet @ NNLO

fully unresolved $(\leftrightsquigarrow\ p_T^H \to 0)$ $\simeq$ H @ N³LO

q_T Subtraction @ N^{3}LO

q_T resummation

- expand to fixed order
- error: $\mathcal{O}\left((q_T^{\rm cut}/Q)^n\right)$



[Catani, Grazzini '07]

$$d\sigma^{\rm H}_{\rm N^3LO} = \underbrace{d\sigma^{\rm H}_{\rm N^3LO}\Big|_{q_T<q_T^{\rm cut}}}_{q_T\ \text{resummation}} + \underbrace{d\sigma^{\rm H}_{\rm N^3LO}\Big|_{q_T>q_T^{\rm cut}}}_{d\sigma^{\rm H+jet}_{\rm NNLO}}$$

$$\simeq \mathcal{H}^{\rm H}_{\rm N^3LO} \otimes d\sigma^{\rm H}_{\rm LO} + \left[d\sigma^{\rm H+jet}_{\rm NNLO} - d\sigma^{\rm H,\,CT}_{\rm N^3LO}\right]_{q_T>q_T^{\rm cut}}$$

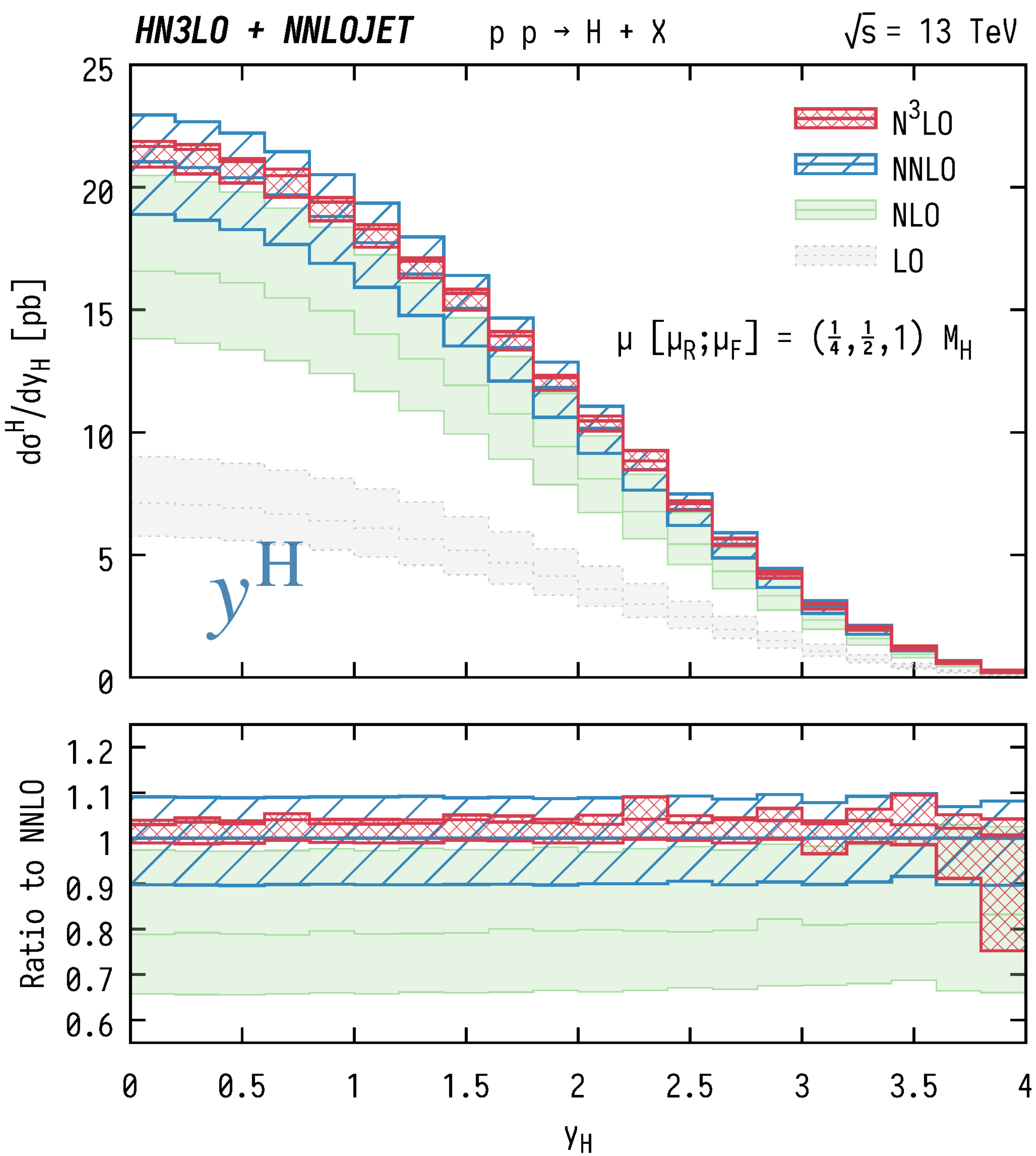
Competing interests: $q_T^{\rm cut}$ as small as possible $\leftrightsquigarrow$ $q_T^{\rm cut}$ as large as possible

$\hookrightarrow$ suppress power corrections

$\hookrightarrow$ numerical stability & efficiency

Higgs Rapidity @ N³LO

[Cieri, Chen, Gehrmann, Glover, AH '18]



- N³LO ~ few %
  - good convergence
  - very flat & well-approximated by

$$\mathrm{NNLO} \times K_{\mathrm{N^3LO}}$$

- in principle, *fully differential*
  - in practice: very challenging going beyond y^H (incl.)
  - CPU cost

~ few *million* core hours

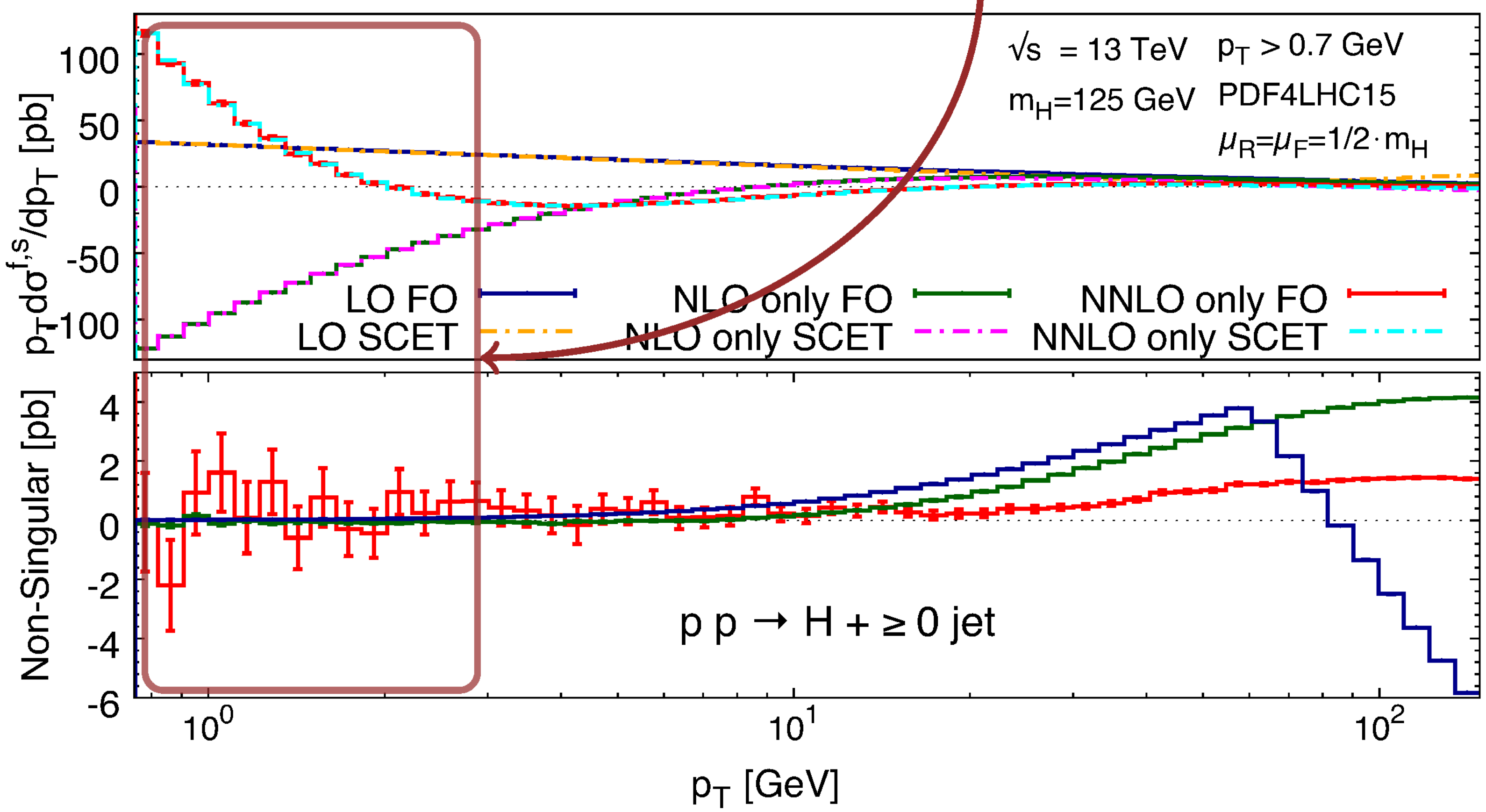
* *numerical approx. for unknown coefficients*

q_T Subtraction @ N^{3}LO

[Catani, Grazzini '07]

$$d\sigma^{H}_{N^3LO} = \underbrace{d\sigma^{H}_{N^3LO}\Big|_{q_T<q_T^{cut}}}_{q_T \text{ resummation}} + \underbrace{d\sigma^{H}_{N^3LO}\Big|_{q_T>q_T^{cut}}}_{d\sigma^{H+jet}_{NNLO}}$$

$$\simeq \mathcal{H}^{H}_{N^3LO} \otimes d\sigma^{H}_{LO} + \underbrace{\left[d\sigma^{H+jet}_{NNLO} - d\sigma^{H,\,CT}_{N^3LO}\right]_{q_T>q_T^{cut}}}$$

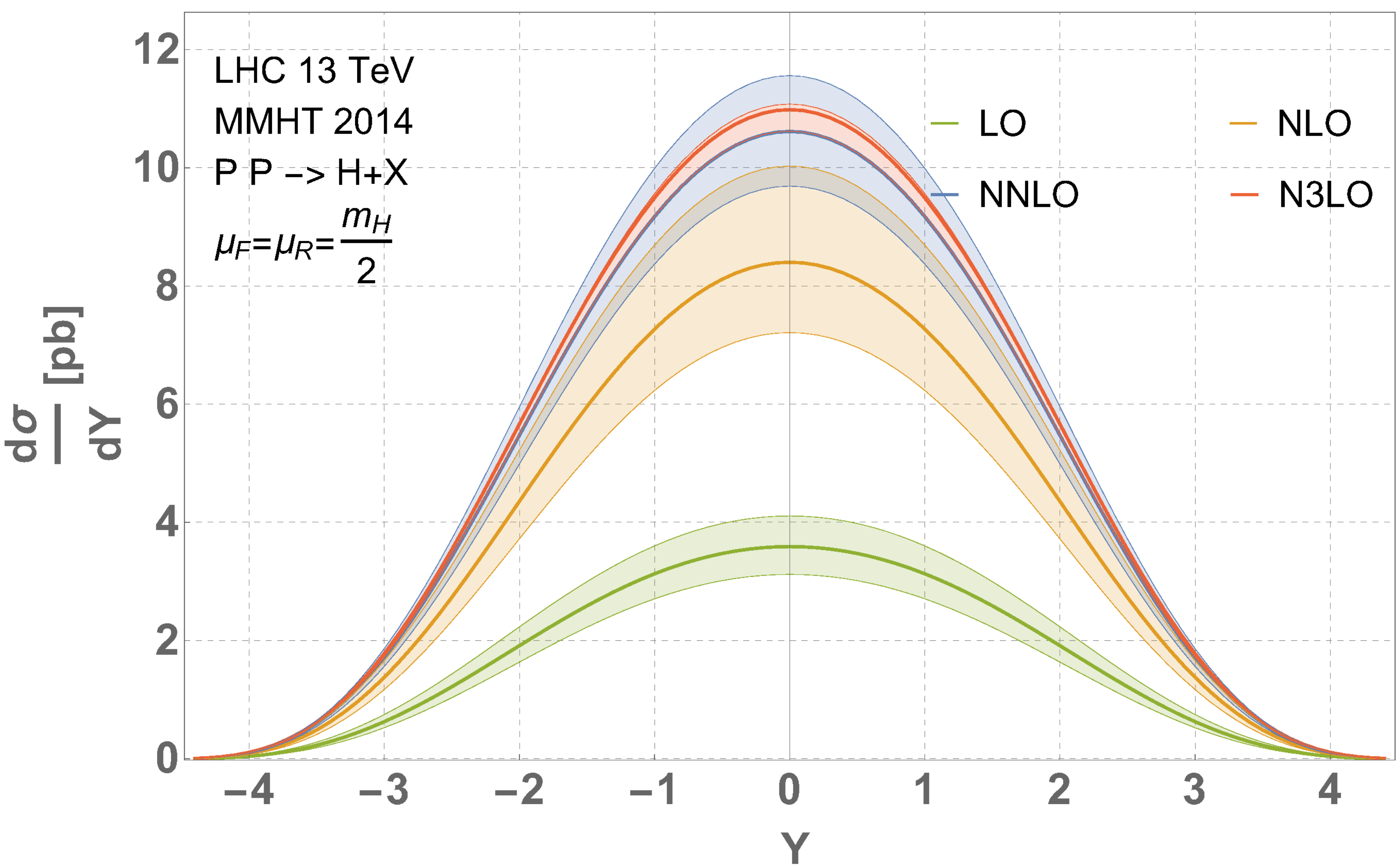


non-local cancellations

[Chen, Gehrmann, Glover, AH, Li, Neill, Schulze, Stewart, Zhu '18]

HIGGS RAPIDITY @ N^{3}LO — ANALYTIC

[Dulat, Mistlberger, Pelloni '18]



- tailored to observable
  - *function* of y^H
  - *very efficient* evaluation
    $\sim$ few *minutes*!
- *differential* info lost
  - kinematics of the Higgs
    ↪ decays
  - QCD radiation
    ↪ isolation, veto, …

Can we exploit this?

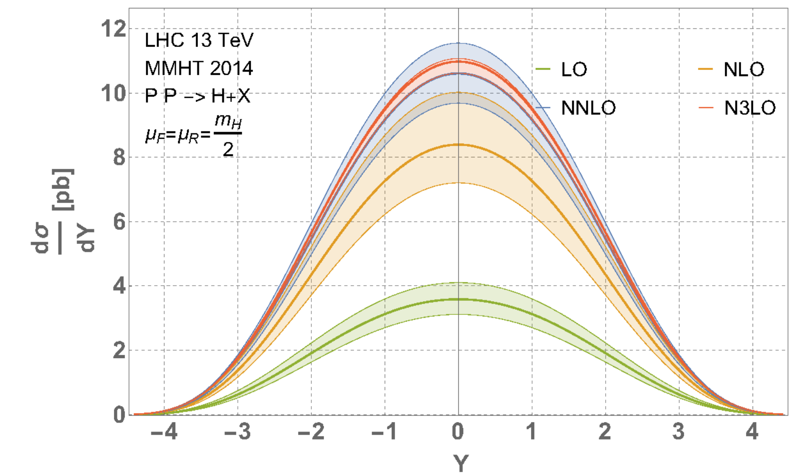
YES!

The Projection-to-Born Method

[VBF @ NNLO: Cacciari et al. '15]

Idea: restore the *differential* information of an inclusive calculation!

- *genuine* N^{3}LO limits ("fully unresolved")
  $\leftrightsquigarrow$ Born kinematics
  - y^{H} only non-trivial variable! $p_{\mathrm{H}}^{\mu} = M_{\mathrm{H}} \begin{pmatrix} \cosh(y^{\mathrm{H}}) \\ \mathbf{0}_{\mathrm{T}} \\ \sinh(y^{\mathrm{H}}) \end{pmatrix}$

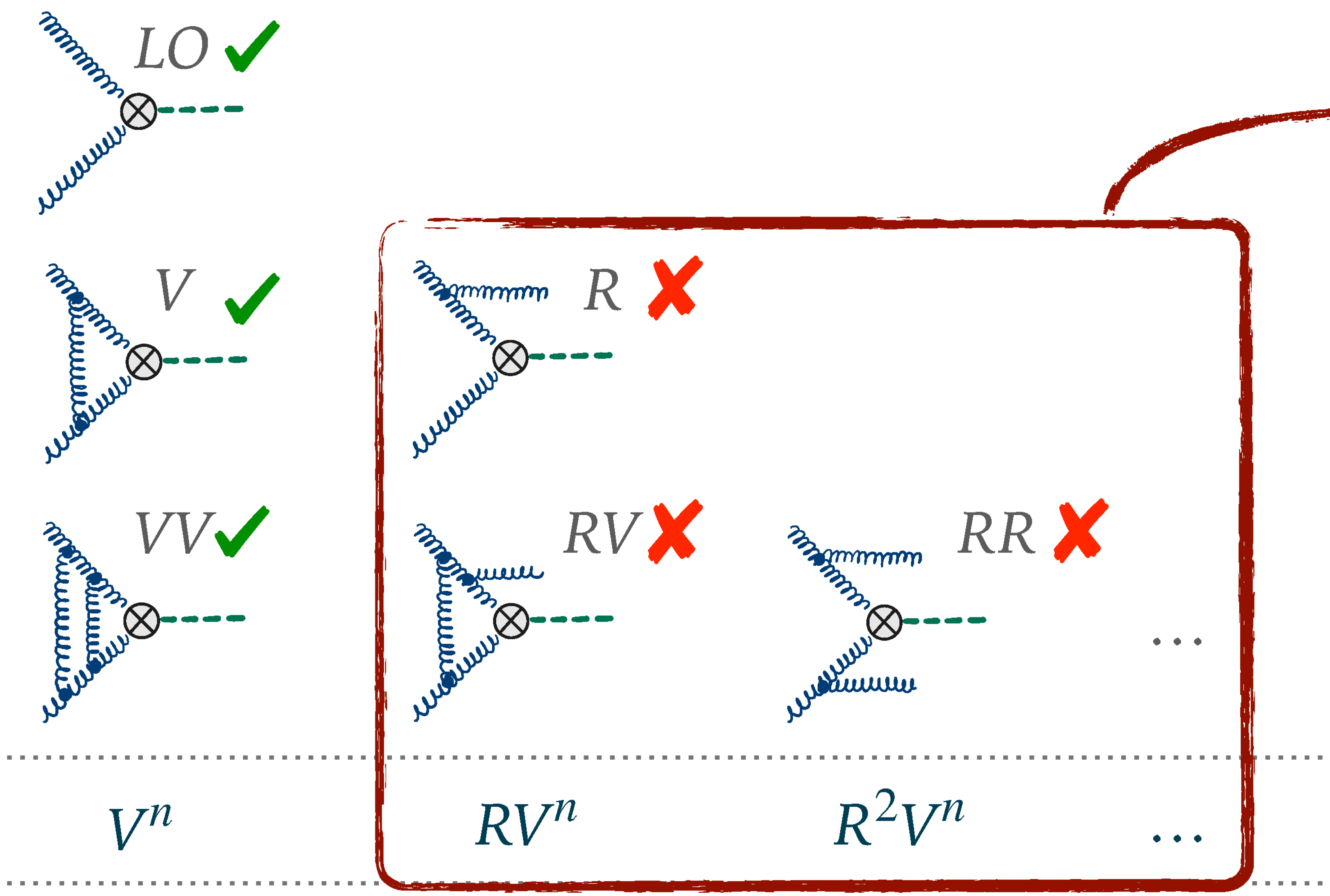


reconstructible

- step 1: evaluate inclusive prediction on Born phase space $\widetilde{\Phi}_{\mathrm{H}}$
  - analysis: observables, fiducial cuts, histograms, …
- step 2: fix what is wrong in step 1
  - compute the difference using a *projection to Born*: $\Phi_{\mathrm{H}+n} \xrightarrow{\text{P2B}} \widetilde{\Phi}_{\mathrm{H}}$

The Projection-to-Born Method

- step 1: evaluate inclusive prediction on Born phase space $\widetilde{\Phi}_{\mathrm{H}}$



NNLO calculation for
H+jet

$\Phi_{\mathrm{H}} \equiv \widetilde{\Phi}_{\mathrm{H}}$ $\quad\Phi_{\mathrm{H}+1} \neq \widetilde{\Phi}_{\mathrm{H}}$ $\quad\Phi_{\mathrm{H}+2} \neq \widetilde{\Phi}_{\mathrm{H}}$ $\quad\ldots$

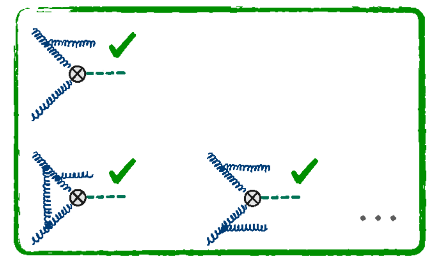
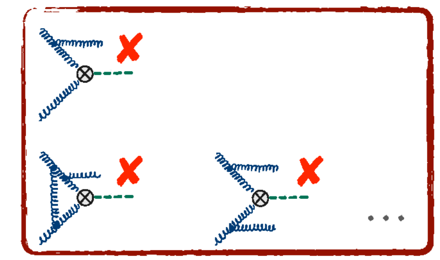
Infrared finite but … wrong

(because we can reconstruct *full* Born)

The Projection-to-Born Method

- step 2: fix what is wrong in step 1

Infrared finite & fully local

$\Big($  $-$  $\Big)$

H + jet $\qquad$ $(\mathrm{H}+\mathrm{jet})_{\mathcal{O}\to\mathcal{O}_B}$

- real-emission phase space: $\mathrm{d}\Phi_{\mathrm{H}+n}$

$$p_a + p_b \to p_{\mathrm{H}} + k_1 + k_2 + \ldots + k_n$$

- projection to Born: $\mathrm{d}\widetilde{\Phi}_{\mathrm{H}}$

$$\tilde{p}_a + \tilde{p}_b \to \tilde{p}_{\mathrm{H}} \qquad (\tilde{p}_a = \xi_a p_a, \quad \tilde{p}_b = \xi_b p_b)$$

on-shell: $\tilde{p}_{\mathrm{H}}^2 \equiv p_{\mathrm{H}}^2 = M_{\mathrm{H}}^2 \quad \Rightarrow \quad \xi_a\, \xi_b = \dfrac{2p_a p_b - 2(p_a+p_b)k_{1\ldots n} + k_{1\ldots n}^2}{2p_a p_b}$

rapidity: $\tilde{y}_{\mathrm{H}} \equiv y_{\mathrm{H}} \quad \Rightarrow \quad \xi_a / \xi_b = \dfrac{2p_b p_{\mathrm{H}}}{2p_a p_{\mathrm{H}}}$

$\hookrightarrow$ decay products: $p_{\mathrm{H}} \to p_1 + \ldots + p_m \qquad (p_i^\mu \to \tilde{p}_i^\mu = \Lambda^\mu{}_\nu\, p_i^\nu)$

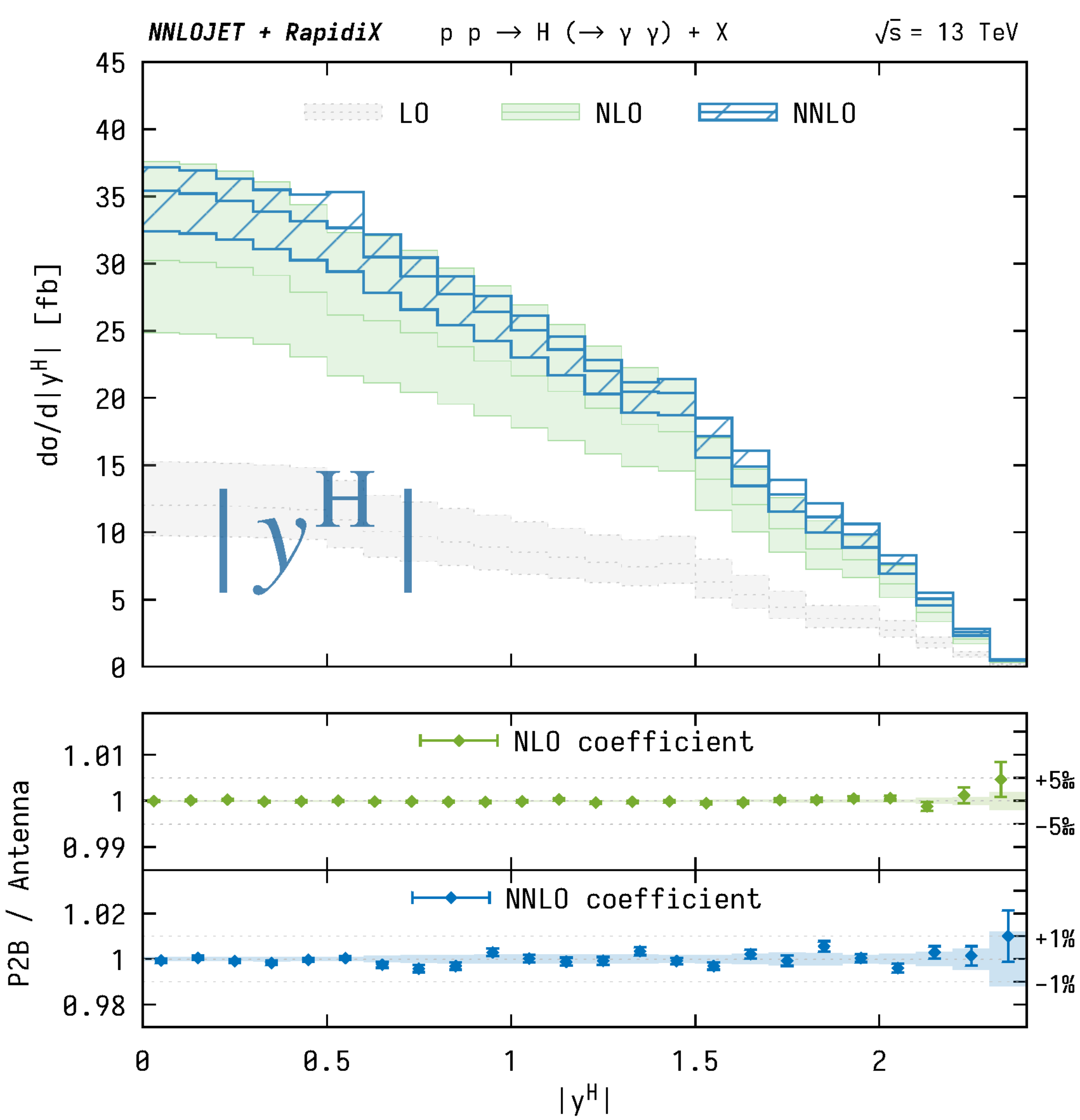
$$\Lambda^\mu{}_\nu(p_{\mathrm{H}}, \tilde{p}_{\mathrm{H}}) = g^\mu{}_\nu - \frac{2(p_{\mathrm{H}}+\tilde{p}_{\mathrm{H}})^\mu (p_{\mathrm{H}}+\tilde{p}_{\mathrm{H}})_\nu}{(p_{\mathrm{H}}+\tilde{p}_{\mathrm{H}})^2} + \frac{2\tilde{p}_{\mathrm{H}}^\mu p_{\mathrm{H},\nu}}{p_{\mathrm{H}}^2}$$

- sub-divergences
  - dealt with H+jet @ NNLO
- N^{3}LO divergences
  - fully local prescription
  - P2B (“ideal” subtraction)

THE PROJECTION-TO-BORN METHOD — MASTER FORMULA

$$\frac{\mathrm{d}\sigma_F^{\mathrm{N}^k\mathrm{LO}}}{\mathrm{d}\mathcal{O}} = \frac{\mathrm{d}\sigma_{F,\,\mathrm{inc.}}^{\mathrm{N}^k\mathrm{LO}}}{\mathrm{d}\mathcal{O}_B} + \left\{ \frac{\mathrm{d}\sigma_{F+\mathrm{jet}}^{\mathrm{N}^{k-1}\mathrm{LO}}}{\mathrm{d}\mathcal{O}} - \left.\frac{\mathrm{d}\sigma_{F+\mathrm{jet}}^{\mathrm{N}^{k-1}\mathrm{LO}}}{\mathrm{d}\mathcal{O}}\right|_{\mathcal{O}\to\mathcal{O}_B} \right\}$$

observables projected to Born
fully local counter term

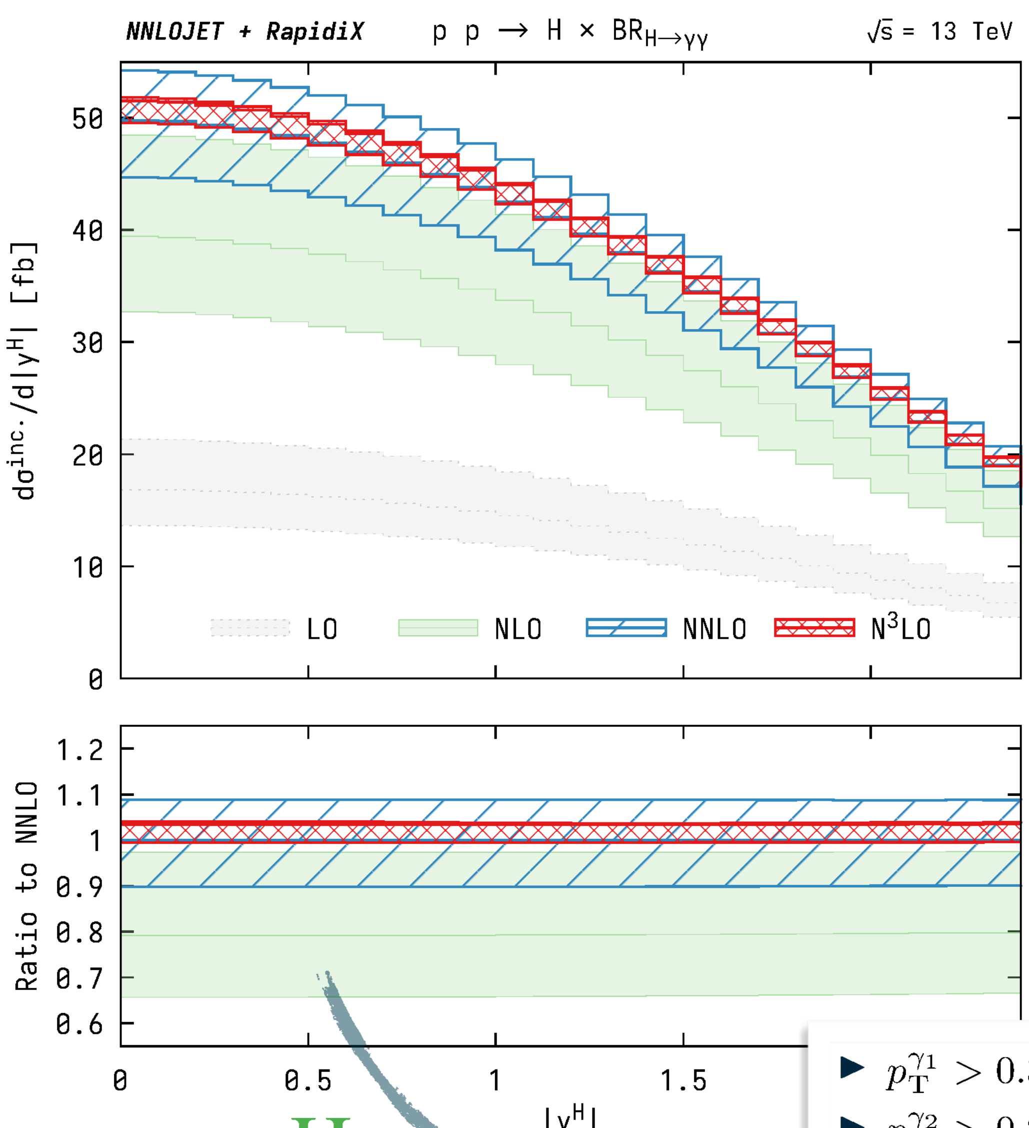


- Fully validated up to NNLO
  v.s. antenna subtraction
  - NLO *coefficient:* ~ per-mille
  - NNLO *coefficient:* ~ sub-per-cent

Higgs @ N^{3}LO using Projection-to-Born $d\sigma/dY^{H}$

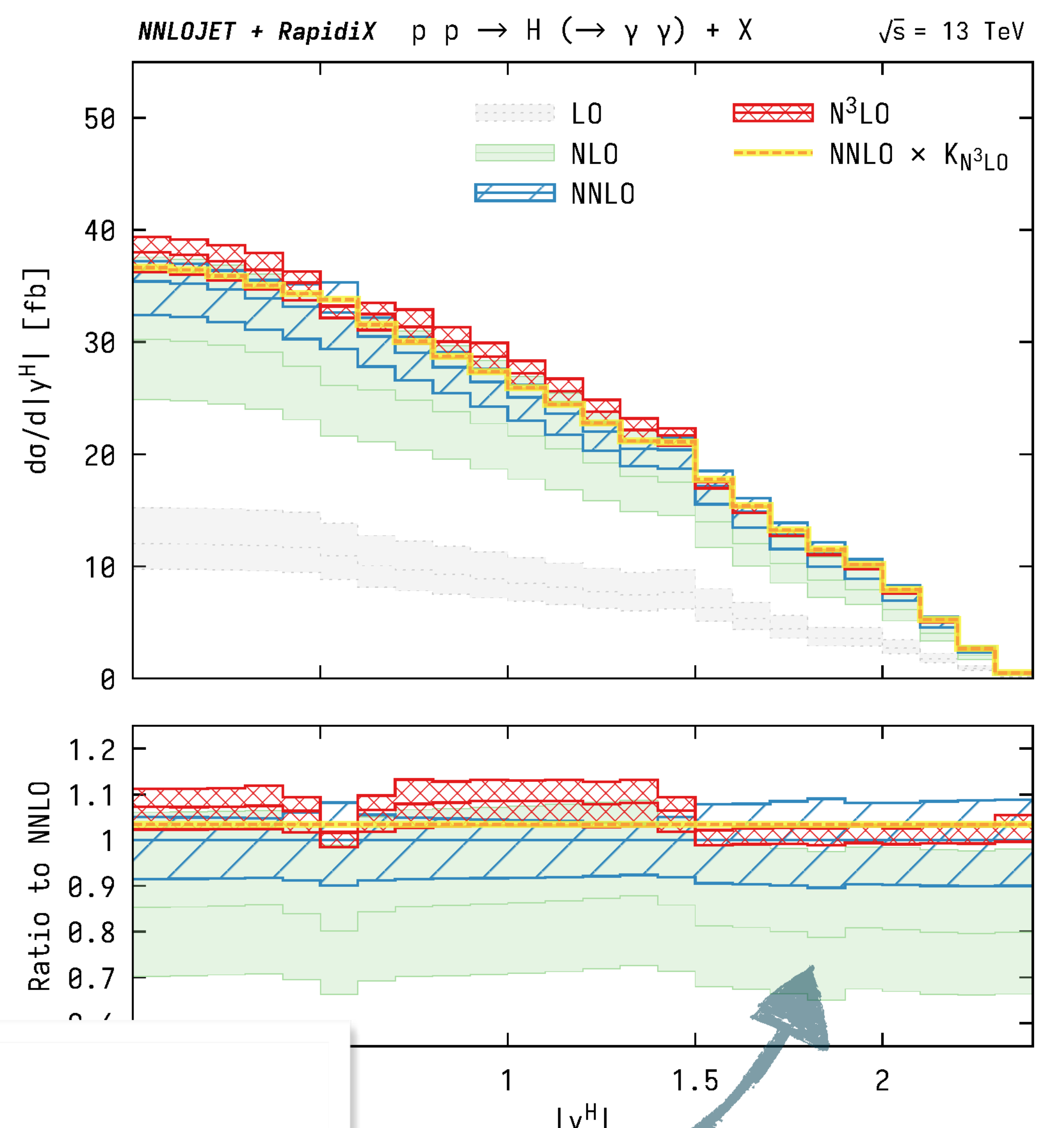
[Chen, Gehrmann, Glover, AH, Mistlberger, Pelloni '21]

Inclusive



$$\frac{d\sigma^{pp\to H}}{dY}$$

Fully Differential



$$d\sigma^{pp\to H}$$

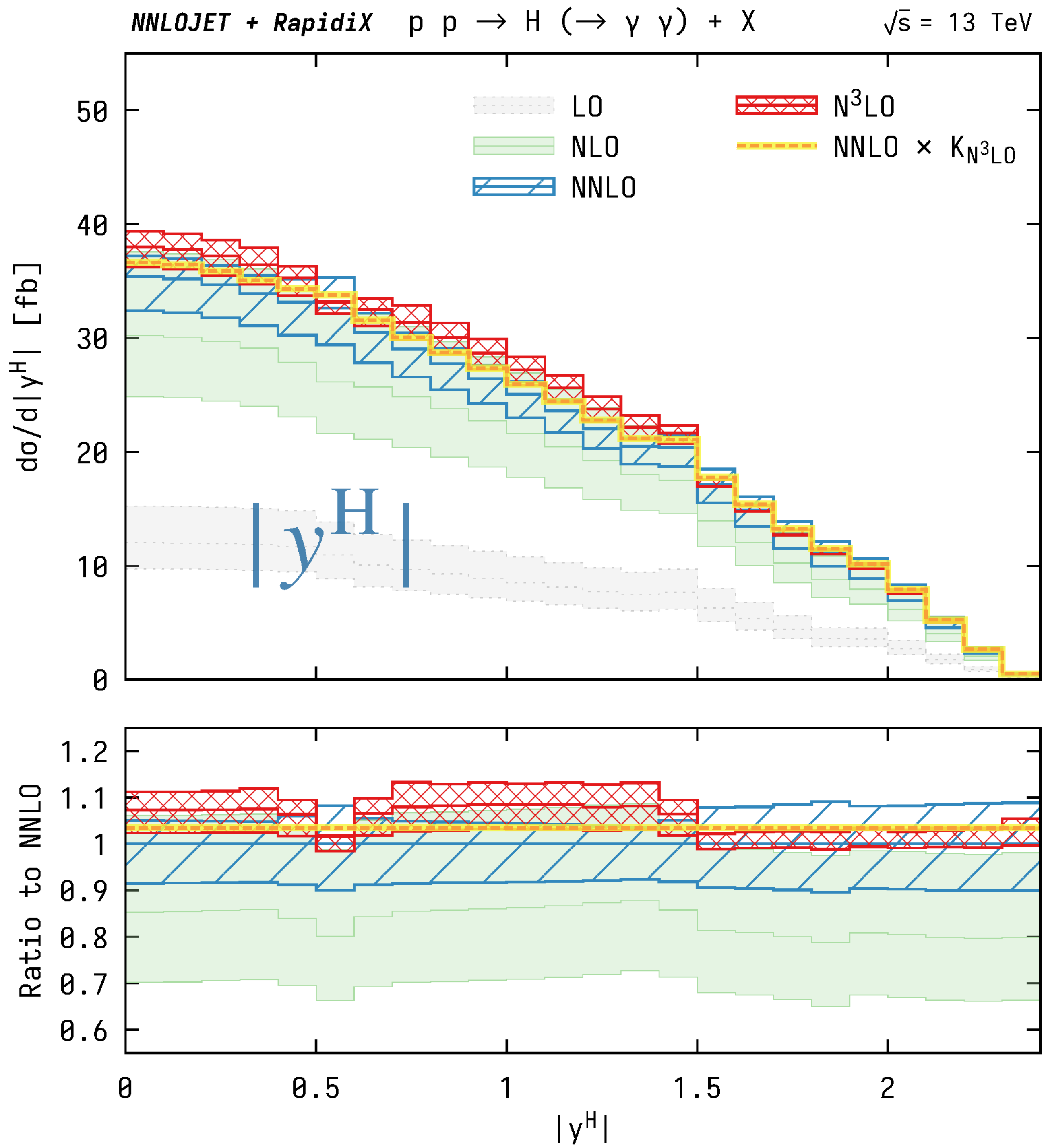
- $p_T^{\gamma_1} > 0.35 \cdot m_{\gamma\gamma}$
- $p_T^{\gamma_2} > 0.25 \cdot m_{\gamma\gamma}$
- $|y^{\gamma}| < 2.37$
- reject $1.37 < |y^{\gamma}| < 1.52$ (barrel-endcap)
- *photon isolation* in $\Delta R < 0.2$

  $\hookrightarrow \sum_{\Delta R_{i\gamma}<0.2} p_{T,i} < 0.05 \cdot E_T^{\gamma}$

HIGGS @ N³LO USING PROJECTION-TO-BORN $d\sigma/dY^H$

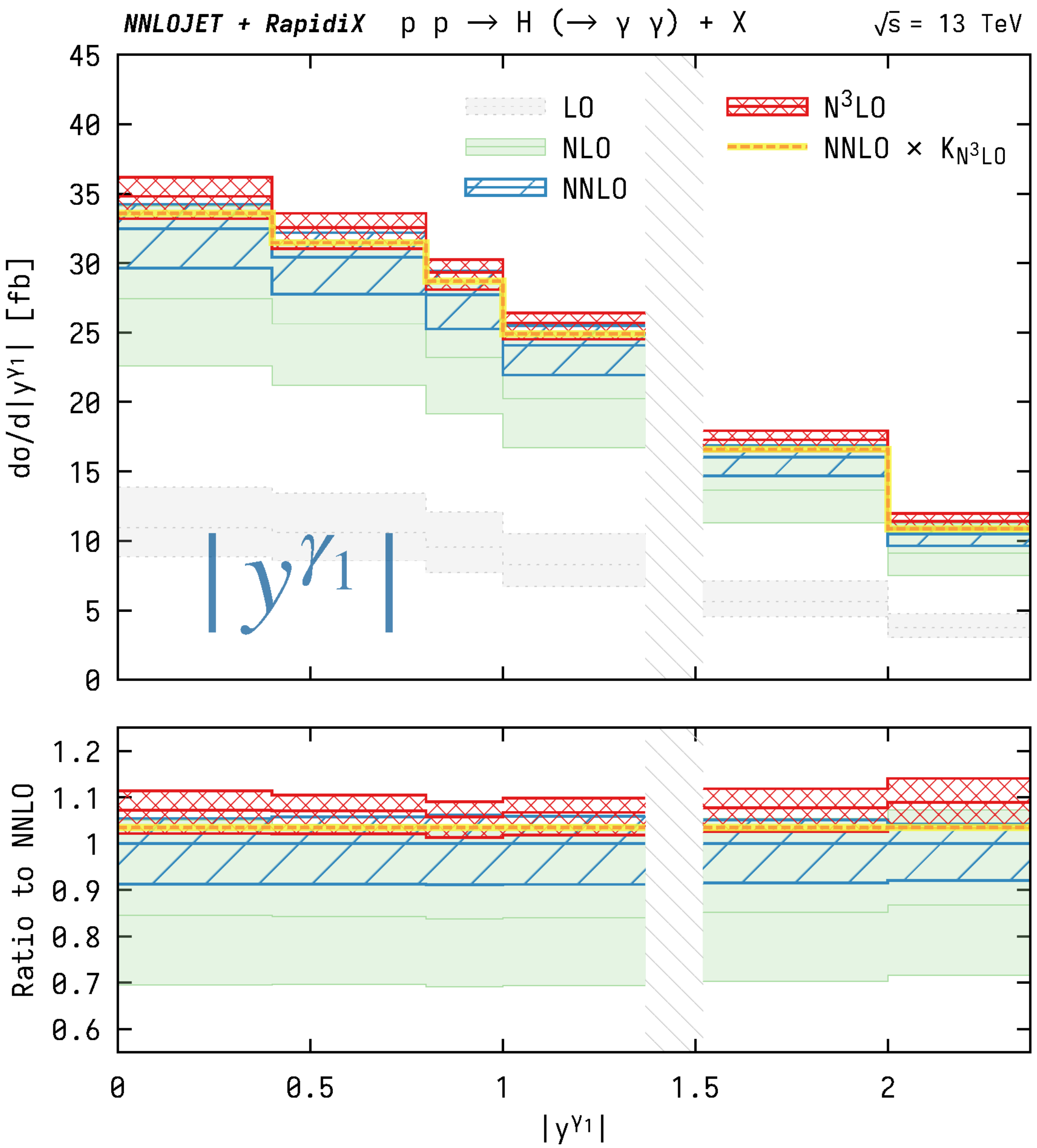
[Chen, Gehrmann, Glover, AH, Mistlberger, Pelloni '21]

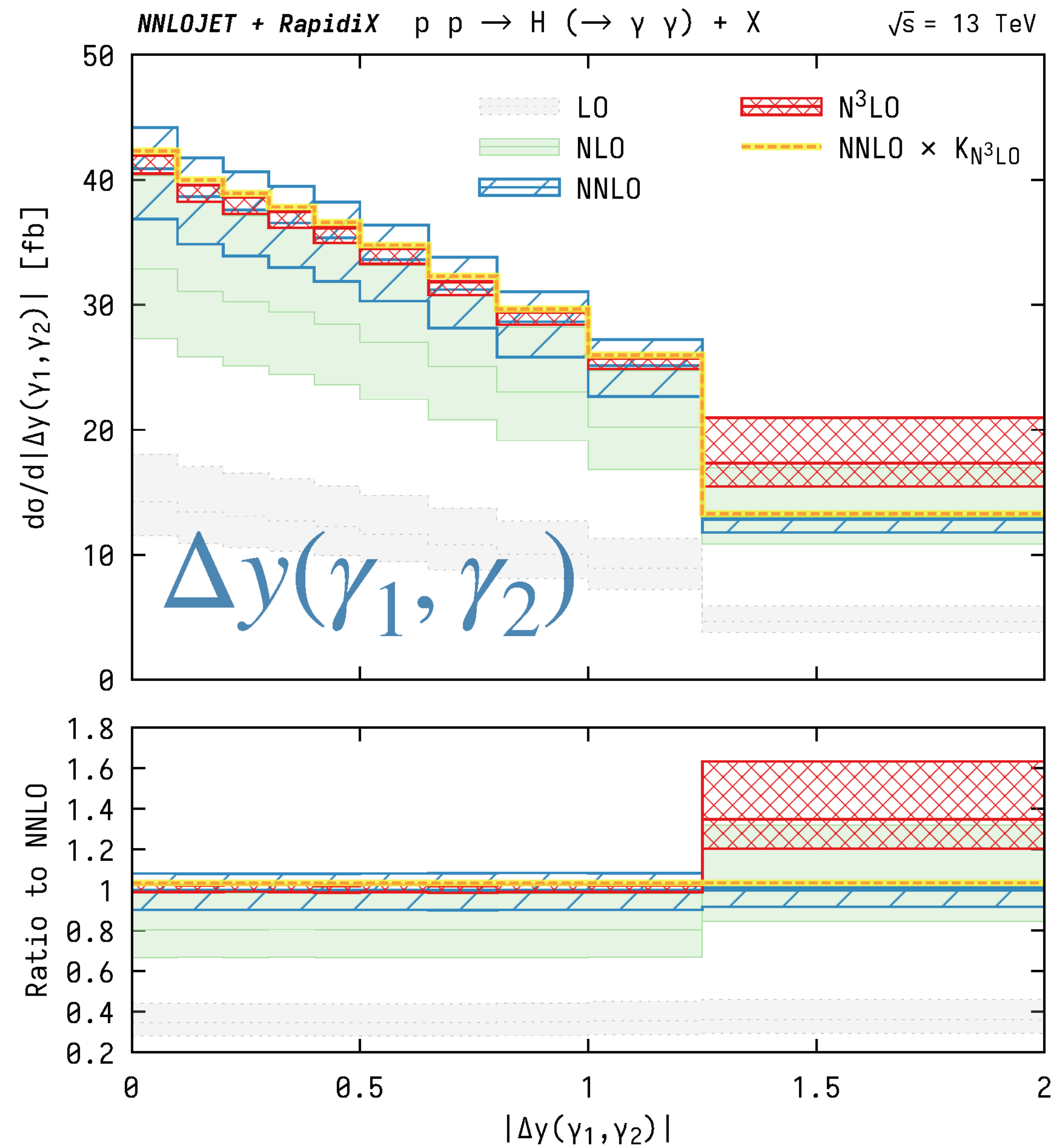
- naive rescaling fails for $|y^H| \lesssim 1.5$
  - NNLO × K_{N^3LO}
- N³LO
  - reduced uncertainties
  - bands largely overlap
- non trivial features
  - corrections larger $|y^H| \lesssim 1.5$
  - rescaling works for $|y^H| \gtrsim 1.5$
  - artefact @ $y^H \sim 0.5$



HIGGS @ N³LO USING PROJECTION-TO-BORN $H \to \gamma\gamma$

[Chen, Gehrmann, Glover, AH, Mistlberger, Pelloni '21]





- $\mathrm{N^3LO}$ systematically $> \mathrm{NNLO} \times K_{\mathrm{N^3LO}}$
- $\mathrm{N^3LO} \sim \mathrm{NNLO} \times K_{\mathrm{N^3LO}}$
- instability in last bin

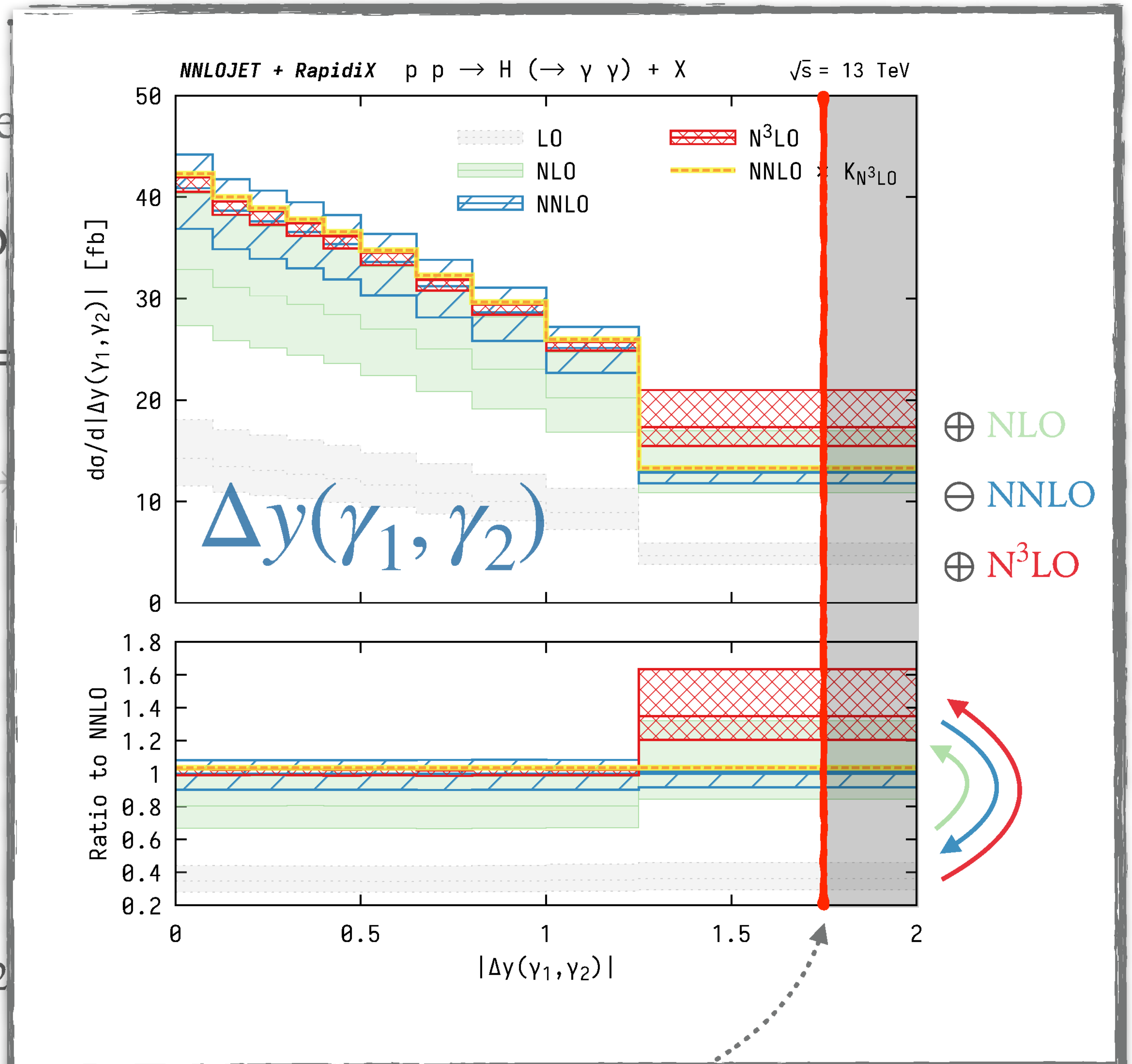
IR Sensitivity & Potential Enhancements

In the following: $p_T^H \sim 0$ (e

- Higgs rest frame (back-to
  - $\hat{y}^{\gamma_1} \equiv \hat{y} = -\hat{y}^{\gamma_2}$ & $p_T^{\gamma_1} \equiv p_T =$
  - $E^{\gamma_{1/2}} \equiv \frac{M_H}{2} = p_T \cosh(\hat{y})$

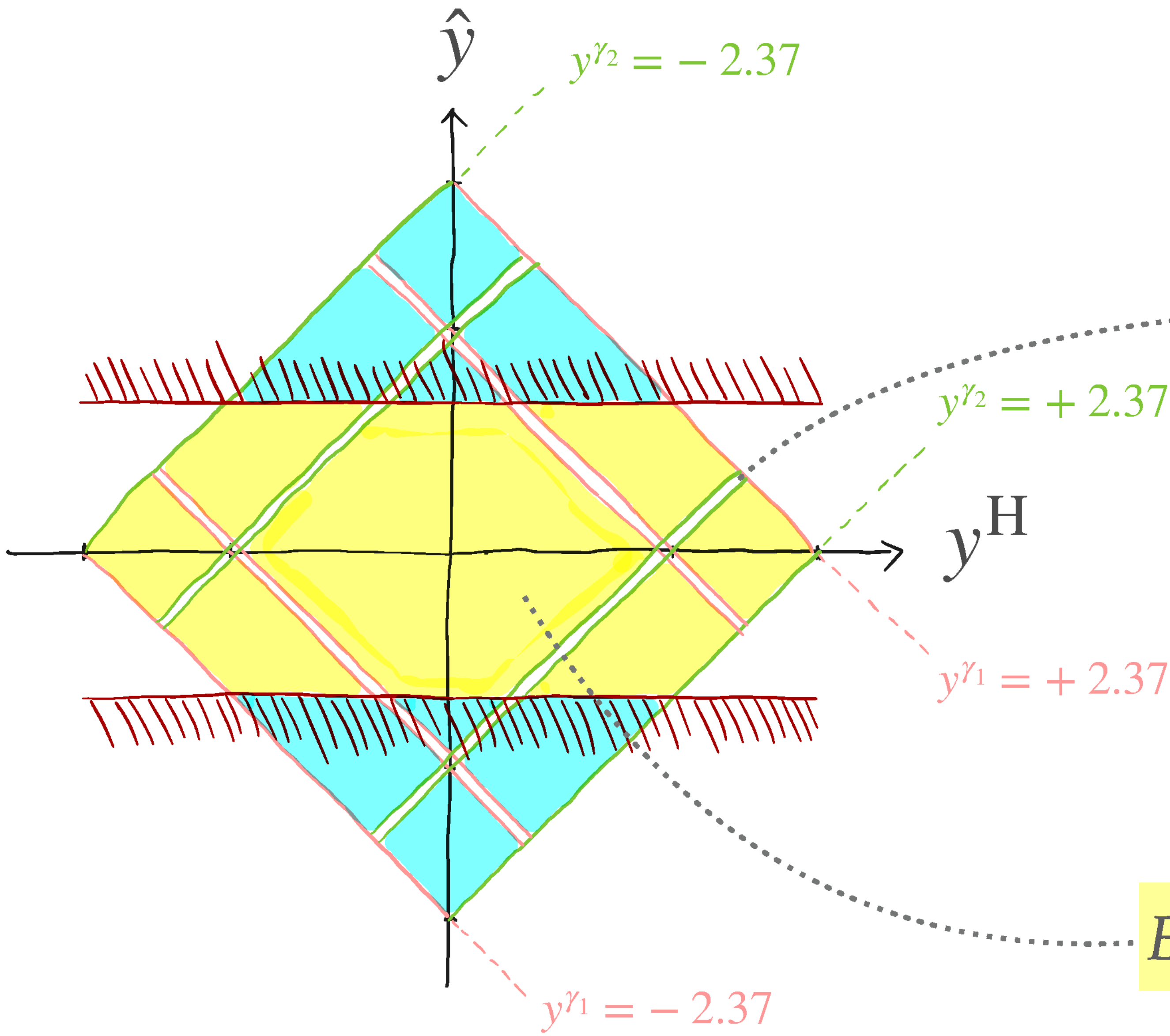
boost by y^H

- Lab frame
  - $y^{\gamma_{1/2}} = y^H \pm \hat{y}$ $\rightsquigarrow$ $|\Delta y(\gamma_1, \gamma_2$



$$|\Delta y(\gamma_1, \gamma_2)| \leq 2\cosh^{-1}\left(\frac{M_H}{2p_T^{\min}}\right) \approx 1.8$$

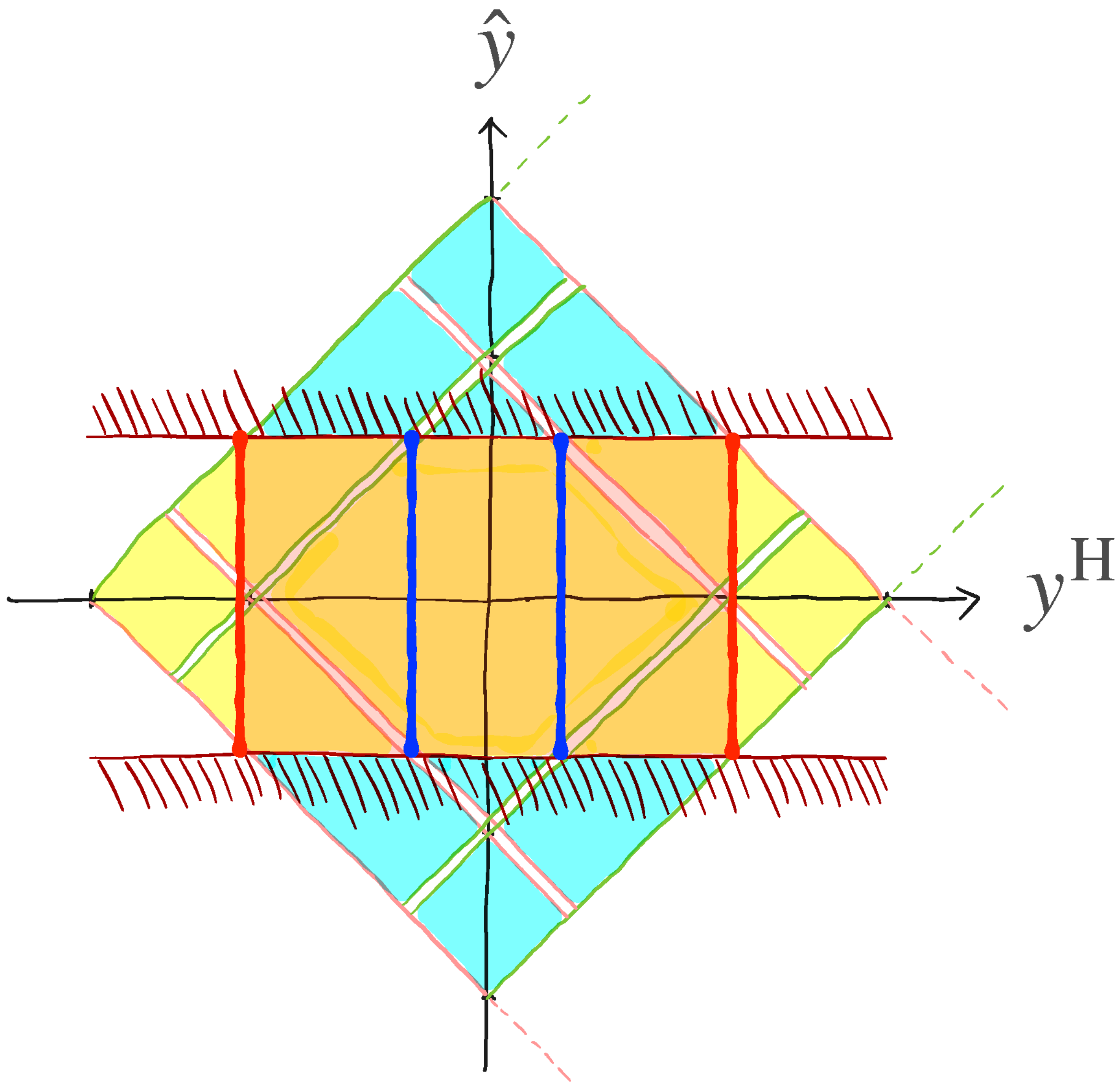
Fiducial Acceptances & y^{H}



- $y^{\gamma_{1/2}} \sim y^{\mathrm{H}} \pm \hat{y}$
  - $|\,y^{\gamma}\,| \leq 2.37$
  - barrel-endcap: [1.37, 1.52]
- Sudakov shoulder @ $|\,\hat{y}\,| \lesssim 0.9$

Born acceptance

Fiducial Acceptances & y^{H}



- $y^{\gamma_{1/2}} \sim y^{\mathrm{H}} \pm \hat{y}$
  - $|y^{\gamma}| \leq 2.37$
  - barrel-endcap: [1.37, 1.52]
- Sudakov shoulder @ $|\hat{y}| \lesssim 0.9$

★ Sensitivity to the shoulder

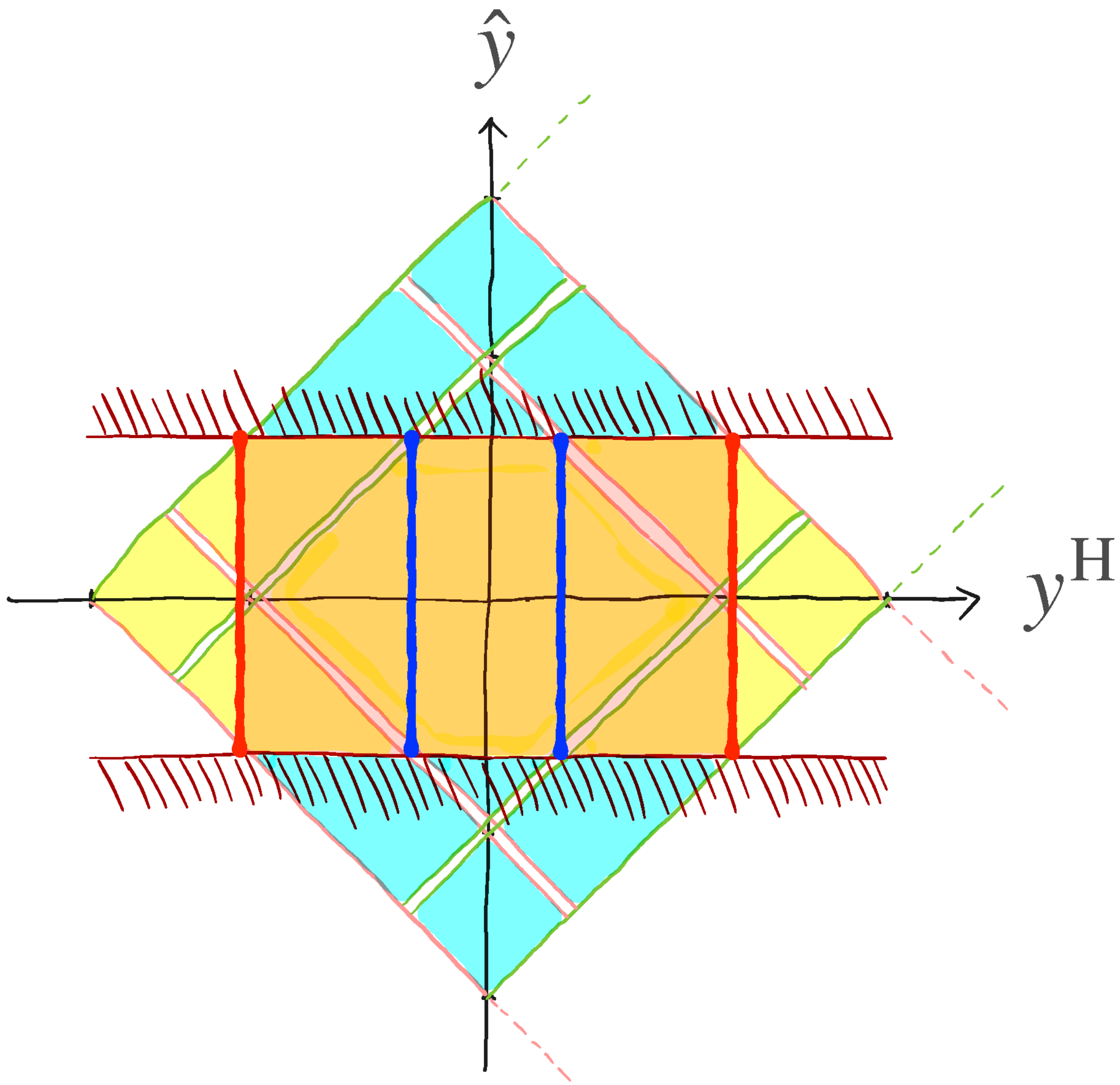
- $y^{\mathrm{H}} \lesssim y^{\gamma}_{\mathrm{max}} - \hat{y}_{\mathrm{shoulder}} \sim 1.49$

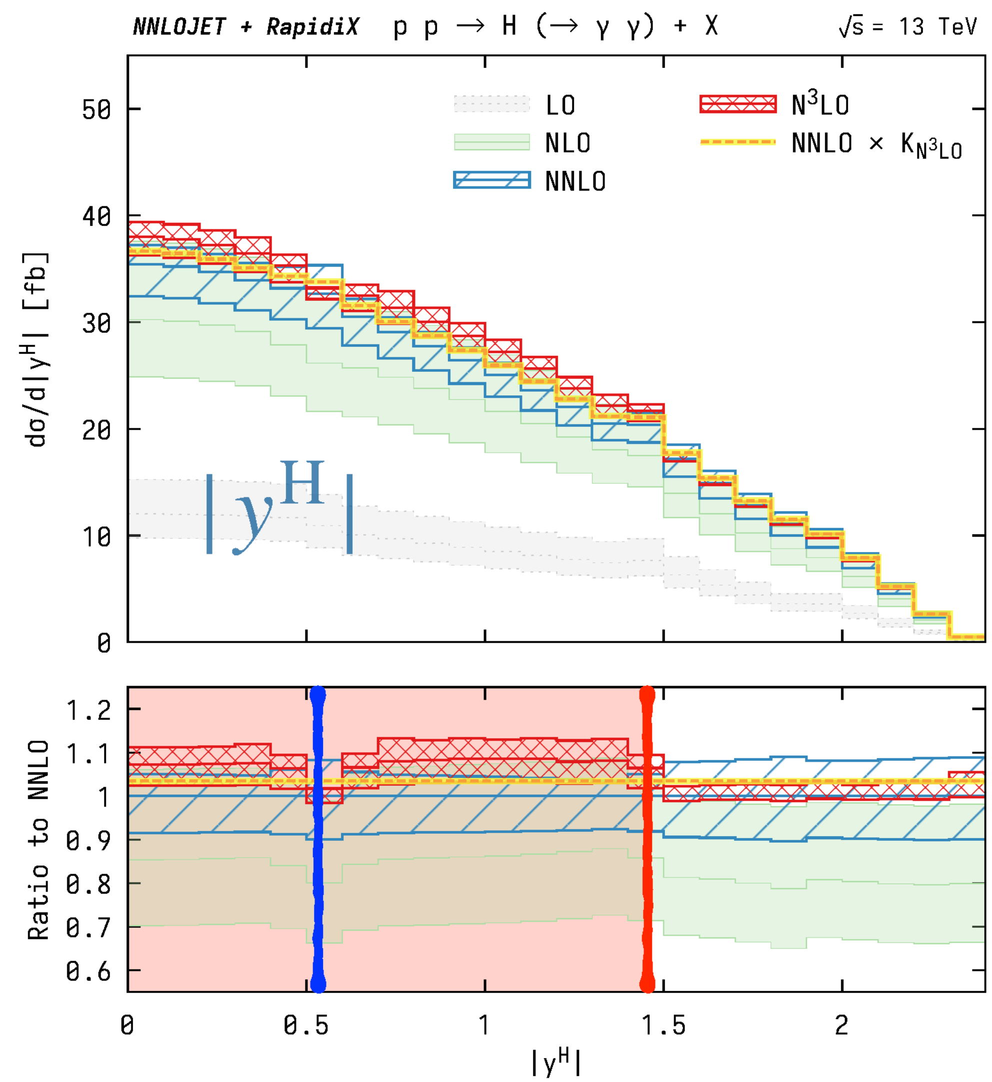
❖ barrel-endcap v.s. shoulder

- $y^{\mathrm{H}} \sim y^{\gamma}_{\mathrm{b-e}} - \hat{y}_{\mathrm{shoulder}} \sim [0.47, 0.62]$

1. $y^{\mathrm{H}} \gtrsim 1.5$: $\mathrm{NNLO} \times K_{\mathrm{N^3LO}}$ ✔
2. $y^{\mathrm{H}} \lesssim 1.5$: enhanced corrections
3. artefact @ $y^{\mathrm{H}} \sim 0.5$?

FIDUCIAL ACCEPTANCES & y^H



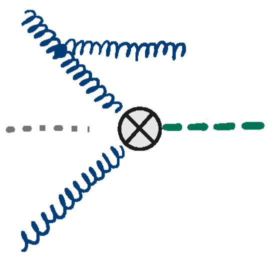


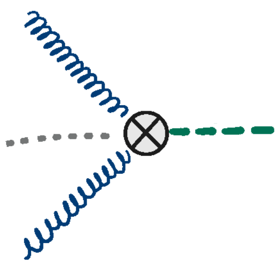
1. $y^H \gtrsim 1.5$: NNLO × K_{N^3LO} ✓
2. $y^H \lesssim 1.5$: enhanced corrections
3. artefact @ $y^H \sim 0.5$?

- Exposed infrared sensitivity
  - better cuts, resum (lin. power corr.), …?
- NNLO: accidental cancellations
  - ⊕ NNLO ⊖ Sudakov

Projection-to-Born — an "Antenna" View

Consider the real-emission subtraction in the antenna subtraction formalism
for $\mathrm{H}+0\mathrm{jet}$ (@ LC):



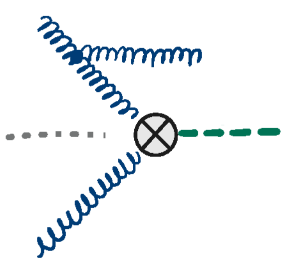


$$\int \left\{ \mathrm{d}\sigma^{\mathrm{R}}_{\mathrm{H+0jet}} - \mathrm{d}\sigma^{\mathrm{SNLO}}_{\mathrm{H+0jet}} \right\}$$

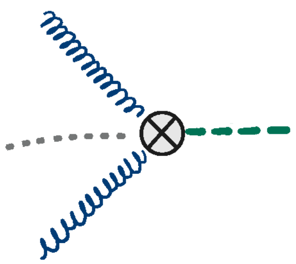
$$= \int \mathrm{d}\Phi_{\mathrm{H+1}} \Big\{ \mathtt{A3g0H}(1_{\mathrm{g}}, 2_{\mathrm{g}}, 3_{\mathrm{g}}, \mathrm{H})\ \mathcal{J}(\Phi_{\mathrm{H+1}}) - F_3^0(1_{\mathrm{g}}, 2_{\mathrm{g}}, 3_{\mathrm{g}})\ \mathtt{A2g0H}(\tilde{1}_{\mathrm{g}}, \tilde{2}_{\mathrm{g}}, \mathrm{H})\ \mathcal{J}(\widetilde{\Phi}_{\mathrm{H+0}}) \Big\}$$

Projection-to-Born — an "Antenna" View

Consider the real-emission subtraction in the antenna subtraction formalism
for $\mathrm{H}+0\mathrm{jet}$ (@ LC):



$$\int \left\{ \mathrm{d}\sigma^{\mathrm{R}}_{\mathrm{H+0jet}} - \mathrm{d}\sigma^{\mathrm{SNLO}}_{\mathrm{H+0jet}} \right\}$$



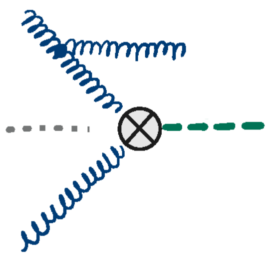
$$= \int \mathrm{d}\Phi_{\mathrm{H+1}} \Big\{ \mathtt{A3g0H}(1_{\mathrm{g}}, 2_{\mathrm{g}}, 3_{\mathrm{g}}, \mathrm{H})\ \mathcal{J}(\Phi_{\mathrm{H+1}}) - F_3^0(1_{\mathrm{g}}, 2_{\mathrm{g}}, 3_{\mathrm{g}})\ \mathtt{A2g0H}(\tilde{1}_{\mathrm{g}}, \tilde{2}_{\mathrm{g}}, \mathrm{H})\ \mathcal{J}(\widetilde{\Phi}_{\mathrm{H+0}}) \Big\}$$

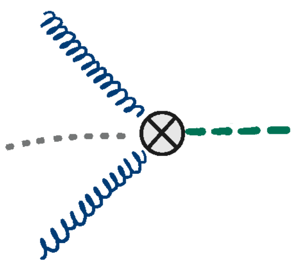
Antennae = ratios of *physical* Matrix Elements:

$$F_3^0(i_{\mathrm{g}}, j_{\mathrm{g}}, k_{\mathrm{g}}) \equiv \frac{\mathtt{A3g0H}(i_{\mathrm{g}}, j_{\mathrm{g}}, k_{\mathrm{g}}, \mathrm{H})}{\mathtt{A2g0H}(\tilde{i}_{\mathrm{g}}, \tilde{k}_{\mathrm{g}}, \mathrm{H})}$$

Projection-to-Born — an "Antenna" View

Consider the real-emission subtraction in the antenna subtraction formalism
for $\mathrm{H}+0\mathrm{jet}$ (@ LC):





$$\int \left\{ \mathrm{d}\sigma^{\mathrm{R}}_{\mathrm{H+0jet}} - \mathrm{d}\sigma^{\mathrm{SNLO}}_{\mathrm{H+0jet}} \right\}$$

$$= \int \mathrm{d}\Phi_{\mathrm{H+1}} \Big\{ \mathtt{A3g0H}(1_{\mathrm{g}}, 2_{\mathrm{g}}, 3_{\mathrm{g}}, \mathrm{H})\ \mathcal{J}(\Phi_{\mathrm{H+1}})$$

$$- F_3^0(1_{\mathrm{g}}, 2_{\mathrm{g}}, 3_{\mathrm{g}})\ \mathtt{A2g0H}(\tilde{1}_{\mathrm{g}}, \tilde{2}_{\mathrm{g}}, \mathrm{H})\ \mathcal{J}(\widetilde{\Phi}_{\mathrm{H+0}}) \Big\}$$

$$= \int \mathrm{d}\Phi_{\mathrm{H+1}}\ \mathtt{A3g0H}(1_{\mathrm{g}}, 2_{\mathrm{g}}, 3_{\mathrm{g}}, \mathrm{H}) \left\{ \mathcal{J}(\Phi_{\mathrm{H+1}}) - \mathcal{J}(\widetilde{\Phi}_{\mathrm{H+0}}) \right\}$$

$\Rightarrow$ Simple processes where antenna $\simeq$ real-emission Matrix Element
$\leftrightsquigarrow$ Projection-to-Born

Similarly at NNLO: X_4^0 & $X_3^0 \times X_3^0$ are "projections" of RR ME & NLO(+jet) subtraction term.

$\mathrm{d}\sigma_{\mathrm{N^3LO}}/\mathrm{d}y_{\mathrm{H}} \ \simeq$ integrated antenna: $\mathcal{X}_5^0$, $\mathcal{X}_4^1$, $\mathcal{X}_3^2$

Conclusions & Outlook

- **exploration of the Higgs sector** — highest priority of LHC & future colliders
  - $\leadsto$ precision to scrutinise Standard Model & search for New Physics

- *fully differential* N^{3}LO prediction for Higgs production (ggF)
  - *fiducial cuts* can induce *non-trivial* features in predictions
    - $\hookrightarrow$ $\mathrm{N^3LO} \neq \mathrm{NNLO} \times \mathrm{K}_{\mathrm{N^3LO}}$ (affects acceptances)
  - IR sensitivity responsible for some of the features
    - $\hookrightarrow$ possibility to avoid them? resum them?

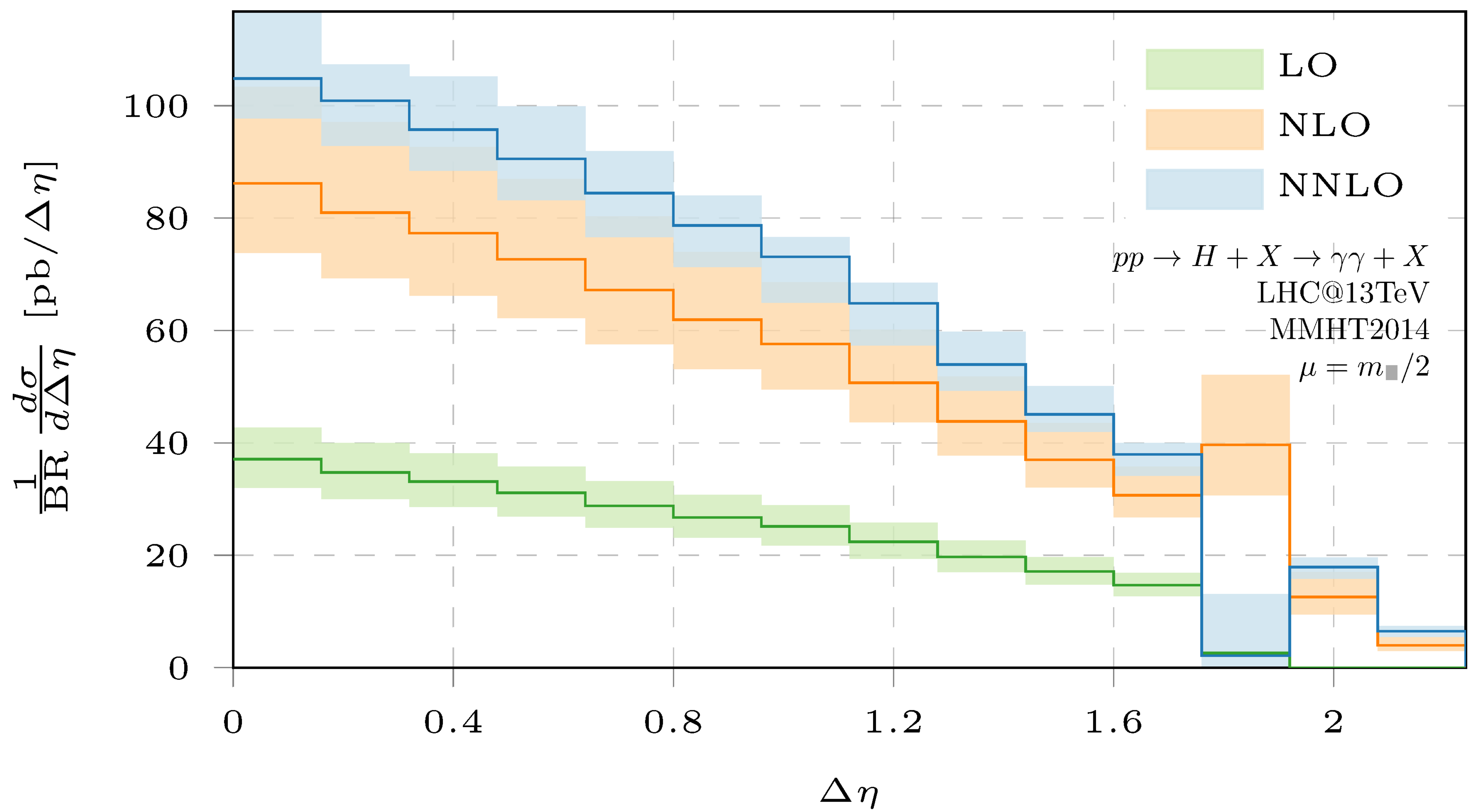
- *future directions*
  - other Higgs decays: $\mathrm{H} \to 4\ell$, …
  - Drell Yan: more challenging (off-shell, lepton spin corr., threshold exp., …)
  - Projection-to-Born $\simeq$ Antennae

Thank you!

Backup.

Shoulder at NNLO

[Dulat, Lionetti, Mistlberger, Pelloni, Specchia '17]



NNLO using Subtraction

$$
\begin{aligned}
\sigma_{\mathsf{NNLO}} = & \int_{\Phi_{Z+3}} \left(\mathrm{d}\sigma^{\mathrm{RR}}_{\mathsf{NNLO}} - \mathrm{d}\sigma^{\mathrm{S}}_{\mathsf{NNLO}} \right) \\
& + \int_{\Phi_{Z+2}} \left(\mathrm{d}\sigma^{\mathrm{RV}}_{\mathsf{NNLO}} - \mathrm{d}\sigma^{\mathrm{T}}_{\mathsf{NNLO}} \right) \\
& + \int_{\Phi_{Z+1}} \left(\mathrm{d}\sigma^{\mathrm{VV}}_{\mathsf{NNLO}} - \mathrm{d}\sigma^{\mathrm{U}}_{\mathsf{NNLO}} \right) \\
\hline
& \sum \qquad \text{finite} \qquad - 0
\end{aligned}
$$

Antenna subtraction

use simple processes $(1 \to 2)$
to reconstruct singularities of
arbitrary processes $(n \to m)$

colour ordering

$\Rightarrow$ each line suitable for numerical evaluation in $D = 4$

Antenna Factorization

- antenna formalism operates on *colour-ordered* amplitudes
- exploit universal factorisation properties in IR limits

$$\underbrace{|\mathcal{A}^0_{m+1}(\ldots,i,j,k,\ldots)|^2}_{\text{colour-ordered amplitude}} \xrightarrow{j\text{ unresolved}} \underbrace{X^0_3(i,j,k)}_{\text{antenna function}} \; \underbrace{|\mathcal{A}^0_m(\ldots,\widetilde{I},\widetilde{K},\ldots)|^2}_{\text{reduced ME}}$$

$+$ mapping
$\{p_i, p_j, p_k\} \to \{\widetilde{p}_I, \widetilde{p}_K\}$

- captures *multiple limits* and smoothly interpolates between them*

limit	$X^0_3(i,j,k)$	mapping
$p_j \to 0$	$\dfrac{2s_{ik}}{s_{ij}s_{jk}}$	$\widetilde{p}_I \to p_i,\ \ \widetilde{p}_K \to p_k$
$p_j \parallel p_i$	$\dfrac{1}{s_{ij}}P_{ij}(z)$	$\widetilde{p}_I \to (p_i+p_j),\ \ \widetilde{p}_K \to p_k$
$p_j \parallel p_k$	$\dfrac{1}{s_{jk}}P_{kj}(z)$	$\widetilde{p}_I \to p_i,\ \ \widetilde{p}_K \to (p_j+p_k)$

* c.f. dipoles: $X^0_3(i,j,k) \sim \mathcal{D}_{ij,k} + \mathcal{D}_{kj,i}$

Antenna Factorization

- antenna formalism operates on *colour-ordered* amplitudes
- exploit universal factorisation properties in IR limits

$$\underbrace{|\mathcal{A}^0_{m+2}(\ldots,i,j,k,l,\ldots)|^2}_{\text{colour-ordered amplitude}} \xrightarrow{j\ \&\ k\ \text{unresolved}} \underbrace{\boxed{X^0_4(i,j,k,l)}}_{\text{antenna function}} \underbrace{|\mathcal{A}^0_m(\ldots,\widetilde{I},\widetilde{L},\ldots)|^2}_{\text{reduced ME}}$$

+ mapping $\{p_i,p_j,p_k,p_l\} \to \{\widetilde{p}_I,\widetilde{p}_L\}$

- captures $\boxed{\textit{multiple limits}}$ and smoothly interpolates between them*

limit	$X^0_3(i,j,k)$
$p_j \to 0$	$\dfrac{2s_{ik}}{s_{ij}s_{jk}}$
$p_j \parallel p_i$	$\dfrac{1}{s_{ij}}P_{ij}(z)$
$p_j \parallel p_k$	$\dfrac{1}{s_{jk}}P_{kj}(z)$

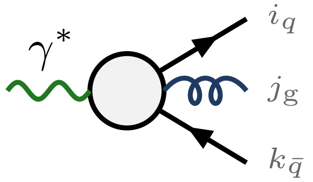
- double soft: $j,k \to 0$
- triple-collinear:
  $(i \parallel j \parallel k)$ & $(j \parallel k \parallel l)$
- double collinear: $(i \parallel j),(k \parallel l)$
- soft–collinear:
  $(i \parallel j), k \to 0$ & $(k \parallel l), j \to 0$
- single-unresolved

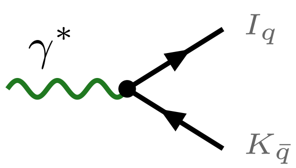
Antenna Subtraction — Building Blocks

- $X(\ldots)$ based on physical matrix elements $\quad X = \overbrace{A,\ B,\ C}^{q\bar{q}}, \overbrace{D,\ E}^{qg}, \overbrace{F,\ G,\ H}^{gg}$

$$X_3^0(i,j,k) = \frac{|\mathcal{A}_3^0(i,j,k)|^2}{|\mathcal{A}_2^0(\widetilde{I},\widetilde{K})|^2}, \qquad X_4^0(i,j,k,l) = \frac{|\mathcal{A}_4^0(i,j,k,l)|^2}{|\mathcal{A}_2^0(\widetilde{I},\widetilde{L})|^2},$$

$$X_3^1(i,j,k) = \frac{|\mathcal{A}_3^1(i,j,k)|^2}{|\mathcal{A}_2^0(\widetilde{I},\widetilde{K})|^2} - X_3^0(i,j,k)\frac{|\mathcal{A}_2^1(\widetilde{I},\widetilde{K})|^2}{|\mathcal{A}_2^0(\widetilde{I},\widetilde{K})|^2},$$

$A_3^0(i_q, j_g, k_{\bar{q}}) = \Bigg|$ 

$\Bigg|^2 \Bigg/ \Bigg|$ 

$\Bigg|^2$

- integrating the antennae $\longleftrightarrow$ phase-space factorization

$$\begin{aligned}\mathrm{d}\Phi_{m+1}(\ldots,p_i,p_j,p_k,\ldots) \\ = \mathrm{d}\Phi_m(\ldots,\widetilde{p}_I,\widetilde{p}_K,\ldots)\,\mathrm{d}\Phi_{X_{ijk}}(p_i,p_j,p_k;\widetilde{p}_I+\widetilde{p}_K)\end{aligned}$$

$$\mathcal{X}_3^{0,1}(i,j,k) = \int \mathrm{d}\Phi_{X_{ijk}} X_3^{0,1}(i,j,k), \quad \mathcal{X}_4^0(i,j,k,l) = \int \mathrm{d}\Phi_{X_{ijkl}} X_4^0(i,j,k,l)$$

Antenna Subtraction — Building Blocks

All building blocks known!

X_3^0, X_4^0, X_3^1 and integrated counterparts $\mathcal{X}_3^0, \mathcal{X}_4^0, \mathcal{X}_3^1$

$\forall$ configurations relevant at hadron colliders:

$\hookrightarrow$ final–final .... $\mathrm{e^+e^-}$

[Gehrmann-De Ridder, Gehrmann, Glover '05]

$\hookrightarrow$ initial–final .... $\mathrm{e^+p}$

[Daleo, Gehrmann-De Ridder, Gehrmann, Luisoni, Maitre '06,'09,'12]

$\hookrightarrow$ initial–initial .... pp

[Boughezal, Daleo, Gehrmann-De Ridder, Gehrmann, Maitre, et al. '10,'11,'12]

$$\mathcal{X}_3^{0,1}(i,j,k) = \int \mathrm{d}\Phi_{X_{ijk}} X_3^{0,1}(i,j,k), \quad \mathcal{X}_4^{0}(i,j,k,l) = \int \mathrm{d}\Phi_{X_{ijkl}} X_4^{0}(i,j,k,l)$$

Antenna Subtraction @ NLO — $q\bar{q} \to \mathrm{ggZ}$

$\mathrm{d}\hat{\sigma}^{T}$:

$$\boxed{\mathrm{d}\hat{\sigma}^{T} \sim \boldsymbol{J}_{n}^{(1)} M_{n}^{0}}$$

$\mathrm{d}\hat{\sigma}^{S}$:

$$\boxed{\mathrm{d}\hat{\sigma}^{S} \sim X_{3}^{0} M_{n}^{0}}$$

$$\int \left\{ \mathrm{d}\sigma^{\mathrm{R}}_{\mathrm{Z+1jet}} - \mathrm{d}\sigma^{\mathrm{S}}_{\mathrm{Z+1jet}} \right\}$$

$$= \int \mathrm{d}\Phi_{\mathrm{Z+2}} \Big\{ \left| \mathcal{A}_{4}^{0}(1_{q}, 3_{\mathrm{g}}, 4_{\mathrm{g}}, 2_{\bar{q}}, \mathrm{Z}) \right|^{2} \mathcal{J}(\Phi_{\mathrm{Z+2}})$$

$$- d_{3}^{0}(1_{q}, 3_{\mathrm{g}}, 4_{\mathrm{g}}) \left| \mathcal{A}_{3}^{0}(\widetilde{1}_{q}, \widetilde{(34)}_{\mathrm{g}}, 2_{\bar{q}}, \mathrm{Z}) \right|^{2} \mathcal{J}(\widetilde{\Phi}_{\mathrm{Z+1}})$$

$$- d_{3}^{0}(2_{\bar{q}}, 4_{\mathrm{g}}, 3_{\mathrm{g}}) \left| \mathcal{A}_{3}^{0}(1_{q}, \widetilde{(34)}_{\mathrm{g}}, \widetilde{2}_{\bar{q}}, \mathrm{Z}) \right|^{2} \mathcal{J}(\widetilde{\Phi}_{\mathrm{Z+1}}) \Big\} + (3 \leftrightarrow 4)$$

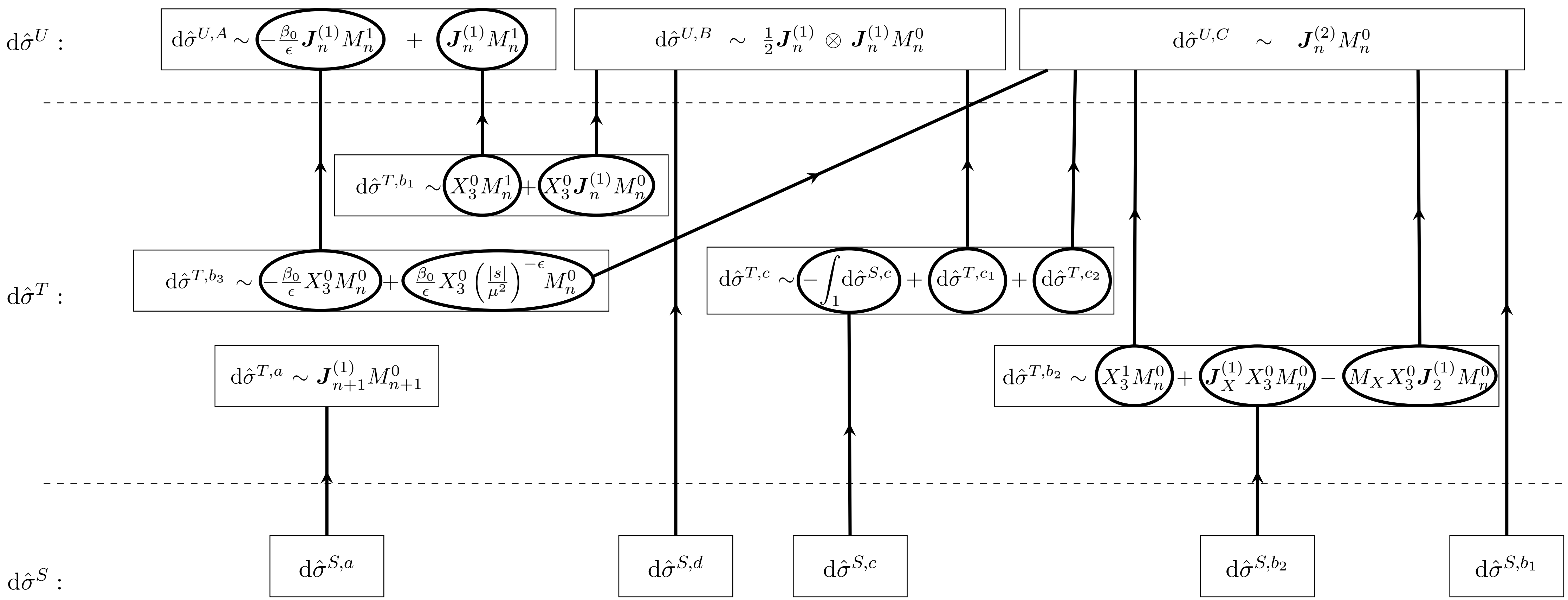
$$\int \left\{ \mathrm{d}\sigma^{\mathrm{V}}_{\mathrm{Z+1jet}} - \mathrm{d}\sigma^{\mathrm{T}}_{\mathrm{Z+1jet}} \right\}$$

$$= \int \mathrm{d}\Phi_{\mathrm{Z+1}} \Big\{ \left| \mathcal{A}_{3}^{1}(1_{q}, 3_{\mathrm{g}}, 2_{\bar{q}}, \mathrm{Z}) \right|^{2}$$

$$+ \frac{1}{2} \left[\mathcal{D}_{3}^{0}(s_{13}) + \mathcal{D}_{3}^{0}(s_{23}) \right] \left| \mathcal{A}_{3}^{0}(1_{q}, 3_{\mathrm{g}}, 2_{\bar{q}}, \mathrm{Z}) \right|^{2} \Big\} \mathcal{J}(\Phi_{\mathrm{Z+1}})$$

Antenna Subtraction @ NNLO

[J. Currie , E.W.N. Glover, S. Wells ’13]



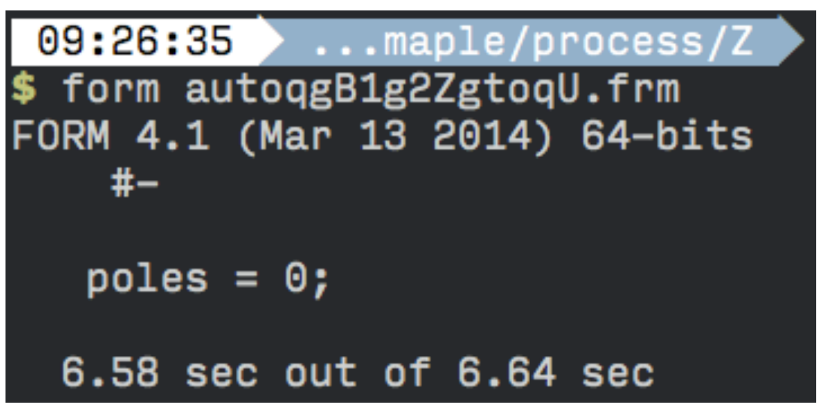
- **double real:** $\mathrm{d}\sigma^{\mathrm{S}} \sim X_3^0\ |\mathcal{A}_{m+1}^0|^2, \quad X_4^0\ |\mathcal{A}_m^0|^2, \quad X_3^0\ X_3^0\ |\mathcal{A}_m^0|^2$
- **real–virtual:** $\mathrm{d}\sigma^{\mathrm{T}} \sim \mathcal{X}_3^0\ |\mathcal{A}_{m+1}^0|^2, \quad X_3^0\ |\mathcal{A}_m^1|^2, \quad X_3^1\ |\mathcal{A}_m^0|^2$
- **double virtual:** $\mathrm{d}\sigma^{\mathrm{U}} =$ (collect rest) $\sim \mathcal{X}\ |\mathcal{A}_m^{0,1}|^2$

Antenna Subtraction — Checks of the Calculation

Analytic pole cancellation

- Poles $\left(\mathrm{d}\sigma^{\mathrm{RV}} - \mathrm{d}\sigma^{\mathrm{T}}\right) = 0$
- Poles $\left(\mathrm{d}\sigma^{\mathrm{VV}} - \mathrm{d}\sigma^{\mathrm{U}}\right) = 0$

DimReg: $D = 4 - 2\epsilon$

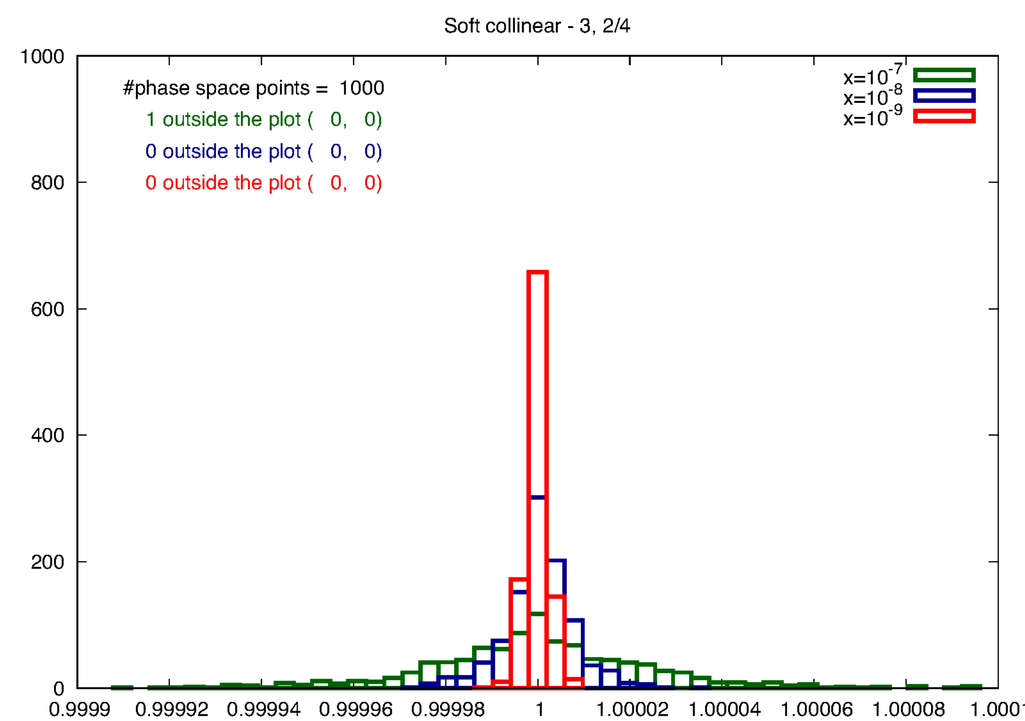


Unresolved limits

- $\mathrm{d}\sigma^{\mathrm{S}} \to \mathrm{d}\sigma^{\mathrm{RR}}$ (single- & double-unresolved)
- $\mathrm{d}\sigma^{\mathrm{T}} \to \mathrm{d}\sigma^{\mathrm{RV}}$ (single-unresolved)

bin the ratio: $\mathrm{d}\sigma^{\mathrm{S}}/\mathrm{d}\sigma^{\mathrm{RR}} \xrightarrow{\text{unresolved}} 1$

$q\,\bar{q} \to \mathrm{Z} + g_3\, g_4\, g_5$ (g_3 soft & $g_4 \parallel \bar{q}$)



(approach singular limit: $x_i = 10^{-7},\ 10^{-8},\ 10^{-9}$)

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Processes computed using the antenna subtra... m...

- $\mathrm{pp} \to V$ @ NNLO
- $\mathrm{pp} \to V + j$ @ ...
  - $\hookrightarrow V \to \ell\bar{\ell}$ (V...
- $\mathrm{pp} \to$ jets
- $\mathrm{pp} \to \gamma$ + ...
- ep ...
- ep ... @ NNLO
- $\mathrm{e^+e^-}$ ...s @ NNLO

... @ N^{3}LO

... @ NNLO

... (VBF) @ NNLO

... $\to \gamma\gamma,\ \tau\tau,\ V\gamma,\ VV$

- $\mathrm{pp} \to V\mathrm{H}$ @ NNLO
  - $\hookrightarrow \mathrm{H} \to \mathrm{bb}$
- ...

NNLO subtraction set up for

$\mathrm{p\,p} \to$ "colour neutral" $+\ 0{,}1{,}2$ jets