PRECISION PREDICTIONS FOR HIGGS BOSON PRODUCTION

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A Remarkable Success Story...

Standard Model Production Cross Section Measurements

Status: May 2020

ATLAS Preliminary
Run 1,2 $\sqrt{s} = 5,7,8,13$ TeV

“stairway to heaven”
This equation neatly sums up our current understanding of fundamental particles and forces.

...But not the full story

- origin of dark matter
- hierarchy problem
- matter anti-matter asymmetry
- hierarchy of scales (generations)
- unification with gravity
- ...

- is it really the SM Higgs? (properties)
- what is the Higgs potential?
- establish the Yukawa’s $Y_{ij}$
- ... scrutinise the Higgs sector
Experiments are Moving Fast

Discovery

ATLAS

Data

Slg+Bkg Fit (m_H=126.5 GeV)

Bkg (4th order polynomial)

1s=7 TeV, \( \int dt=4.8 \text{fb}^{-1} \)

1s=8 TeV, \( \int dt=5.9 \text{fb}^{-1} \)

H\to\gamma\gamma

2012

< 10 years

Today

ATLAS Preliminary

\( \sqrt{s} = 13 \text{ TeV}, 139 \text{ fb}^{-1} \)

\( H \to \gamma\gamma, m_H = 125.09 \text{ GeV} \)

ATLAS Preliminary

\( H \to \gamma\gamma, \sqrt{s} = 13 \text{ TeV}, 139 \text{ fb}^{-1} \)

Data, tot. unc.  syst. unc.

\( \text{gg}\to H \text{ default MC + XH} \)

\( \text{NNLOJET \& SCET NNLO \& NLL + XH} \)

\( \text{XH} = \text{VBF} + \text{VH} + \text{H+H} + \text{bH} \)

\( p_T^H \)

Higgs for precision phenomenology

- signal strengths \( \rightarrow \) differential spectra
- Higgs properties (couplings, potential, …)

A tool for New Physics searches

- scalar (\( \leftrightarrow \) sensitive to high scales), composite?
- extended EW symmetry breaking sector, portal, …?
How Much Precision?

ATLAS and CMS

HL-LHC Projection

\( \sqrt{s} = 14 \) TeV, 3000 fb\(^{-1}\) per experiment

<table>
<thead>
<tr>
<th></th>
<th>Total</th>
<th>Statistical</th>
<th>Experimental</th>
<th>Theory</th>
</tr>
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<td>0.7</td>
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<td>VBF</td>
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<td>4.3</td>
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Expected relative uncertainty

[Acronym Report '19]

theory uncertainties
scaled down by factor 2

[hi.lumihc.web.cern.ch]
How do we Predict this from Theory?
High-Precision Theory Predictions!

- (HL-)LHC — per-cent level!
- **Focus** — clean signature & high momentum transfer
  - perturbative QCD
  - with $\alpha_s \sim 0.1$
    - $NLO \sim \mathcal{O}(10\%), \ NNLO \sim \mathcal{O}(1\%)$
    - exceptions: Higgs
- predictions as close as possible to the experiment
  - fiducial cross sections & differential distributions
High-Precision Theory Predictions!

- (HL-)LHC — per-cent level!
- **FOCUS** — clean signature & high momentum transfer
  - perturbative QCD
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    - $\text{NLO} \sim \mathcal{O}(10\%), \text{NNLO} \sim \mathcal{O}(1\%)$
    - exceptions: Higgs
- predictions as close as possible to the experiment
  - *fiducial cross sections* & *differential distributions*
Theory Predictions for the LHC

\[ \sigma_{AB} = \sum_{ab} \int_0^1 dx_a \int_0^1 dx_b \ f_{a|A}(x_a) f_{b|B}(x_b) \ \hat{\sigma}_{ab}(x_a, x_b) \left(1 + \mathcal{O}(\Lambda_{QCD}/Q)\right) \]

- **Parton distribution functions** (non-perturbative, universal)
- **Non-perturbative effects** (no good understanding) ultimately, limiting factor?
- **Hard scattering** (perturbation theory)
**Hard Scattering — Perturbation Theory**

\[
\hat{\sigma}_{ab} = \hat{\sigma}_{ab}^{(0)} + \left( \frac{\alpha_s}{2\pi} \right)^1 \hat{\sigma}_{ab}^{(1)} + \left( \frac{\alpha_s}{2\pi} \right)^2 \hat{\sigma}_{ab}^{(2)} + \ldots
\]

---

**Gluon-fusion production:**
- \( \sim 90\% \) of all Higgs
- “heavy top limit”

**Leading order (LO)**
- “tree level”

---

**HTL**

\( m_t \rightarrow \infty \)
Hard Scattering — Perturbation Theory

\[ \hat{\sigma}_{ab} = \hat{\sigma}^{(0)}_{ab} + \left( \frac{\alpha_s}{2\pi} \right) \hat{\sigma}^{(1)}_{ab} + \left( \frac{\alpha_s}{2\pi} \right)^2 \hat{\sigma}^{(2)}_{ab} + \ldots \]

next-to-leading order (NLO)

\( \left( \begin{array}{c}
\text{“virtual” (V)} \\
\text{“real” (R)}
\end{array} \right) \)
Hard Scattering — Perturbation Theory

\[
\hat{\sigma}_{ab} = \hat{\sigma}_{ab}^{(0)} + \left(\frac{\alpha_s}{2\pi}\right) \hat{\sigma}_{ab}^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2 \hat{\sigma}_{ab}^{(2)} + \ldots
\]

next-to-next-to-leading order (NNLO)

“double virtual” (VV) + “real-virtual” (RV) + “double real” (RR)
HARD SCATTERING — PERTURBATION THEORY

- Higgs in gluon fusion
  - notoriously slow convergence
- $N^3\text{LO}$ stabilises expansion!

\[ \sigma_{pp \to H} \]

$H \to \gamma\gamma$, $H \to ZZ^* \to 4l$ combined

$XH = \text{VBF} + \text{VH} + \text{ttH} + b\bar{b}H$

QCD scale uncertainty

Total uncertainty (scale, $\otimes$ PDF+$\alpha_s$)

Data

Theory

LO

NLO

NNLO

$N^3\text{LO}$

[Anastasiou et al. '15]

[Mistlberger '18]

“so, $N^3\text{LO}$ is a solved problem then?”

...not quite

next-to-next-to-next-to-leading order ($N^3\text{LO}$)

\[
\begin{pmatrix}
\text{VVV} & + & \text{RVV} & + & \text{RRV} & + & \text{RRR}
\end{pmatrix}
\]
A complete calculation of the order $\alpha_s^2$ correction to the Drell-Yan $K$-factor

$\sigma_{tot}$

R. Hamberg, W.L. van Neerven, T. Matsuura
What is the probability of producing a Higgs boson?

\[ \sigma_{pp\to H}^{N^3LO} = 48.68 \text{ pb}^{+2.07 \text{ pb}}_{-3.16 \text{ pb}} \]

- ✔ analytic integration over full phase space
- ✗ no information on final state

[Anastasiou et al. '15] [Mistlberger '18]
What is the probability of producing a Higgs boson?

\[ \frac{d\sigma_{pp \to H}}{dY} \]

... in direction

\[ Y = \frac{1}{2} \ln \left( \frac{E + p_z}{E - p_z} \right) \]

\[ Y \to 0 \iff \perp \] to beam

\[ Y \to \infty \iff \parallel \] to beam

- **Success**: analytic integration over QCD emissions
- **Failure**: partial information on final state
  - only \( y_H \) \( \leadsto \) no decay kinematics
  - no information on final-state partons

[Dulat, Möstlberger, Pelloni ’18]
What is the probability of producing a Higgs boson?

... in direction \( Y = \frac{1}{2} \ln \left( \frac{E + p_z}{E - p_z} \right) \)

\[
\begin{align*}
Y & \to 0 \quad \Leftrightarrow \quad \perp \quad \text{to beam} \\
Y & \to \infty \quad \Leftrightarrow \quad \parallel \quad \text{to beam}
\end{align*}
\]

... where the Higgs decays into a pair of photons, \( H \to \gamma \gamma \), and the leading and sub-leading photon have a transverse momentum that is larger than 35% and 25% of the Higgs boson mass, respectively, and are produced within the rapidity interval \( |y_\gamma| < 2.37 \), where the barrel-endcap region \( 1.37 < |y_\gamma| < 1.52 \) is excluded. Photons are further required to be isolated from additional QCD activity by requiring that the scalar sum of the transverse momenta of hadrons in a cone of \( \Delta R = 0.2 \) around the photons is less than 5% of the photon transverse energy \( E_T \).

Measurements are done within a fiducial volume:
What is the probability of producing a Higgs boson?

... in direction

\[ Y = \frac{1}{2} \ln \left( \frac{E + p_z}{E - p_z} \right) \]

\[ Y \to 0 \Leftrightarrow \perp \text{ to beam} \]
\[ Y \to \infty \Leftrightarrow \parallel \text{ to beam} \]

ask any* question!

Fully differential:
- numerical integration of phase space
- complete final-state information (decay, isol., ...)

*infrared safe
Infrared subtraction:

- retain full final-state information
- reshuffle singularities
  - extract singularities of $R$
  - cancel against $V$ (+ factorization)
Subtractions — NNLO

\[
(VV) + (RV) + (RR)
\]

- \(1/\varepsilon^4, 1/\varepsilon^3, 1/\varepsilon^2, 1/\varepsilon\)
- \(1/\varepsilon^2, 1/\varepsilon\)
- single unresolved

\[
\text{single unresolved} \simeq H + \text{jet} @ \text{NLO}
\]

\[
\text{fully unresolved} \simeq H @ \text{NNLO}
\]
tremendous progress:
  1. 2 → 2 done for most relevant processes
  2. 2 → 3 next frontier (first results...)

⇒ an optimal method has yet to emerge

**Different Methods**

- **Antenna** [Gehrmann-De Ridder, Gehrmann, Glover '05]
- **CoLoRful** [Del Duca, Somogyi, Trocsanyi '05]
- **qT-subtraction** [Catani, Grazzini '07; MATRIX]
- **STRIPPER (sector-improved residues)** [Czakon '10]
- **nested soft-collinear** [Caola, Melnikov, Röntsch '17]
- **N-jettiness** [Gaunt, Stahlhoven, Tackmann, Walsh '15; Boughezal, Focke, Liu, Petriello '15; MCFM]
- **Projection-to-Born** [Cacciari, et al. '15]
- **Geometric, Local analytical Sectors** [Herzog '18; Magnea et al. '18]

* Subtraction & Slicing
**Higgs + Jet @ NNLO — 3 Calculations!**

- **Residue subtraction**
  - [Caola, Melnikov, Schulze ’15]
  - ![Graph of residue subtraction](image)

- **Antenna subtraction**
  - [Chen, Cruz-Martinez, Gehrmann, Glover, Jaquier ’16]
  - ![Graph of antenna subtraction](image)

- **£: Very complex calculations**
  - ![Graph of $\tau_1$ jettiness subtraction](image)
  - [Boughezal, Focke, Giele, Liu, Petriello ’15]
  - [Campbell, Ellis, Seth ’19]

- **Validation!**
  - ![Graph of $p_T$](image)

- **H+jet ⇆ agreement**

- **Benchmark approaches**
  - ![Graph of $y_H$](image)
  - NNLO, $\epsilon=2.5\times10^{-6}$
  - NNLO, $\epsilon=10^{-4}$
  - NLO
**Subtractions — $N^3LO$**

- $1/\epsilon^6, 1/\epsilon^5, \ldots$
- $1/\epsilon^4, 1/\epsilon^3, \ldots$
- $1/\epsilon^2, 1/\epsilon$
- single unresolved
- single unresolved
- double unresolved
- single unresolved
- double unresolved
- triple unresolved

**Two Methods for “2 → 1”**
- $q_T$-subtraction
- Projection-to-Born

Isolate “radiating” part

$$double\ unresolved \approx H + \text{jet} \at\ NNLO$$

$$fully\ unresolved \ (\leftrightarrow p_T^H \rightarrow 0) \approx H \at\ N^3LO$$
**$q_T$ Subtraction @ N$^3$LO**

$q_T$ resummation

- expand to fixed order
- error: \( \mathcal{O} \left( (q_T^{\text{cut}}/Q)^n \right) \)

\[
\frac{d\sigma}{dq_T} \begin{cases} \ln^n(q_T/Q) & \text{H+jet @ NNLO} \\
1/e^n & q_T^{\text{cut}} \end{cases}
\]

\[
\begin{align*}
\sigma_{N^3LO}^H &= d\sigma_{N^3LO}^H \bigg|_{q_T^{\text{cut}}} + d\sigma_{N^3LO}^H \bigg|_{q_T > q_T^{\text{cut}}} \\
&\simeq \mathcal{H}_{N^3LO}^H \otimes \sigma_{LO}^H + \left[ \sigma_{NNLO}^{H+\text{jet}} - \sigma_{N^3LO}^{H,\text{CT}} \right]_{q_T > q_T^{\text{cut}}}
\end{align*}
\]

[Catani, Grazzini '07]

Competing interests: $q_T^{\text{cut}}$ as small as possible $\leftrightarrow$ suppress power corrections $\leftrightarrow$ as large as possible $\leftrightarrow$ numerical stability & efficiency
Higgs Rapidity @ N^3LO

N^3LO ~ few %
- good convergence
- very flat & well-approximated by NNLO × K_{N^3LO}

in principle, fully differential
- in practice: very challenging going beyond y_H (incl.)
- CPU cost
  ~ few million core hours

*Numerical approx. for unknown coefficients*
$q_T$ Subtraction @ N$^3$LO

\[ d\sigma_{N^3LO}^H = d\sigma_{N^3LO}^H \bigg|_{q_T < q_T^{cut}} + d\sigma_{N^3LO}^H \bigg|_{q_T > q_T^{cut}} \]

\[ \simeq \mathcal{H}_{N^3LO}^H \otimes d\sigma_{LO}^H + \left[ d\sigma_{NNLO}^{H+jet} - d\sigma_{N^3LO}^{H, CT} \right] \bigg|_{q_T > q_T^{cut}} \]

[Catani, Grazzini '07]

\[ \sqrt{s} = 13 \text{ TeV} \quad p_T > 0.7 \text{ GeV} \quad m_H = 125 \text{ GeV} \quad \text{PDF4LHC15} \quad \mu_F = \mu_R = 1/2 \cdot m_H \]

Non-local cancellations

[Chen, Gehrmann, Glover, AH, Li, Neill, Schulze, Stewart, Zhu '18]
HIGGS RAPIDITY @ N³LO — ANALYTIC

[DuLat, Mistlberger, Pelloni '18]

- tailored to observable
  - function of $y^H$
  - very efficient evaluation
    - few minutes!

- differential info lost
  - kinematics of the Higgs
    - decays
  - QCD radiation
    - isolation, veto, ...

Can we exploit this?

YES!
The Projection-to-Born Method

**Idea:** restore the *differential* information of an inclusive calculation!

- **genuine N³LO limits** ("fully unresolved")
  - Born kinematics
    - \( y^H \) only non-trivial variable!
    - \( p_H^\mu = M_H \left( \begin{array}{c} \cosh(y^H) \\ 0 \\ \sinh(y^H) \end{array} \right) \)

- **step 1:** evaluate inclusive prediction on Born phase space \( \widetilde{\Phi}_H \)
  - analysis: observables, fiducial cuts, histograms, ...

- **step 2:** fix what is wrong in step 1
  - compute the difference using a *projection to Born*:
    \[
    \Phi_{H+n} \xrightarrow{P2B} \widetilde{\Phi}_H
    \]
THE PROJECTION-TO-BORN METHOD

- **step 1:** evaluate inclusive prediction on Born phase space $\widetilde{\Phi}_H$

- $V V \checkmark$
- $V \checkmark$
- $LO \checkmark$

- $RV \times$
- $R \times$
- $RR \times$
- $R V \times$
- $V^n$
- $\Phi_H \equiv \widetilde{\Phi}_H$
- $\Phi_{H+1} \neq \widetilde{\Phi}_H$
- $\Phi_{H+2} \neq \widetilde{\Phi}_H$

**NNLO calculation for H+jet**

**Infrared finite but ... wrong**

(because we can reconstruct full Born)
The Projection-to-Born Method

- **step 2:** fix what is wrong in step 1

**Infrared finite & fully local**

- **real-emission phase space:** 
  \[ d\Phi_{H+n} \]
  \[ p_a + p_b \rightarrow p_H + k_1 + k_2 + \ldots + k_n \]

- **projection to Born:** 
  \[ d\tilde{\Phi}_H \]
  \[ \tilde{p}_a + \tilde{p}_b \rightarrow \tilde{p}_H \quad (\tilde{p}_a = \xi_a p_a, \quad \tilde{p}_b = \xi_b p_b) \]

  **on-shell:** 
  \[ \tilde{p}_H^2 \equiv \tilde{p}_H^2 = M_H^2 \Rightarrow \xi_a \xi_b = \frac{2p_a p_b - 2(p_a + p_b)k_{1\ldots n} + k_{1\ldots n}^2}{2p_a p_b} \]

  **rapidity:** 
  \[ \bar{y}_H \equiv y_H \Rightarrow \xi_a / \xi_b = \frac{2p_b p_H}{2p_a p_H} \]

  **→ decay products:** 
  \[ p_H \rightarrow p_1 + \ldots + p_m \quad (p_i^\mu \rightarrow \tilde{p}_i^\mu = \Lambda_{\mu \nu} p_i^\nu) \]

  \[ \Lambda_{\mu \nu}(p_H, \tilde{p}_H) = g_{\mu \nu} - \frac{2(p_H + \tilde{p}_H)\mu(p_H + \tilde{p}_H)\nu}{(p_H + \tilde{p}_H)^2} + \frac{2\tilde{p}_H p_H,\nu}{p_H^2} \]

- **sub-divergences**
  - dealt with H+jet @ NNLO
- **N^3LO divergences**
  - fully local prescription
  - P2B (“ideal” subtraction)
The Projection-to-Born Method — Master Formula

\[ \frac{d\sigma_N^{k, \text{LO}}}{d\mathcal{O}} = \frac{d\sigma_N^{k, \text{LO}}}{d\mathcal{O}_B} + \left\{ \frac{d\sigma_N^{k-1, \text{LO}}}{d\mathcal{O}} \right\} \bigg|_{\mathcal{O} \to \mathcal{O}_B} \]

Observables projected to Born fully local counter term

- Fully validated up to NNLO v.s. antenna subtraction
  - NLO coefficient: \( \sim \) per-mille
  - NNLO coefficient: \( \sim \) sub-per-cent
HIGGS @ N³LO USING PROJECTION-TO-BORN $d\sigma/dY^H$

[Chen, Gehrmann, Glover, AH, Mistlberger, Pelloni ’21]

**Inclusive**

**Fully Differential**

$\frac{d\sigma}{dy^H}$

$\frac{d\sigma_{pp\rightarrow H}}{dy}$

- $p_T^\gamma > 0.35 \cdot m_{\gamma\gamma}$
- $p_T^{\gamma*} > 0.25 \cdot m_{\gamma\gamma}$
- $|y^\gamma| < 2.37$
- **reject** $1.37 < |y^\gamma| < 1.52$ (barrel-endcap)
- **photon isolation** in $\Delta R < 0.2$
  \[ \sum_{\Delta R_{\gamma,i} < 0.2} p_T,i < 0.05 \cdot E_T^\gamma \]

$\frac{d\sigma_{pp\rightarrow H}}{dy}$

$\sqrt{s} = 13$ TeV
Higgs @ N^3LO using Projection-to-Born \( d\sigma/dy^H \)

- naive rescaling fails for \( |y^H| \lesssim 1.5 \)
  - \( \text{NNLO} \times K_{N^3LO} \)

- \( N^3\text{LO} \)
  - reduced uncertainties
  - bands largely overlap

- non trivial features
  - corrections larger \( |y^H| \lesssim 1.5 \)
  - rescaling works for \( |y^H| \gtrsim 1.5 \)
  - artefact @ \( y^H \sim 0.5 \)
**Higgs @ N^{3}LO using Projection-to-Born** \( H \to \gamma\gamma \)

[Chen, Gehrmann, Glover, AH, Mistlberger, Pelloni '21]

**Diagrams:**

- **NNLOJET + Rapidix**
  
  - \( p p \to H (\to \gamma \gamma) + X \)
  
  \( \sqrt{s} = 13 \text{ TeV} \)

- **Plots:**
  
  - \( |y_{\gamma_1}| \)
  
  - \( \Delta y(\gamma_1, \gamma_2) \)

- **Comparison:**
  
  - **N^{3}LO** systematically
  
  \( > \) **NNLO \times K_{N^{3}LO}**

- **Additional Notes:**
  
  - **N^{3}LO \sim NNLO \times K_{N^{3}LO}**
  
  - Instability in last bin
**IR Sensitivity & Potential Enhancements**

In the following: $p_T^H \sim 0$ (e.g. $p_T^{\gamma 1} \equiv p_T$)

- **Higgs rest frame** (back-to-back)
  - $\hat{y}^{\gamma 1} \equiv \hat{y} = - \hat{y}^{\gamma 2}$ & $p_T^{\gamma 1} \equiv p_T$
  - $E_{y/2}^{\gamma 1/2} \equiv \frac{M_H}{2} = p_T \cosh(\hat{y})$

  *boost by $y^H$*

- **Lab frame**
  - $y^{\gamma 1/2} = y^H \pm \hat{y}$ $\Rightarrow |\Delta y(y_1, y_2)|$

\[ |\Delta y(y_1, y_2)| \leq 2 \cosh^{-1} \left( \frac{M_H}{2p_{Tmin}} \right) \approx 1.8 \]
**Fiducial Acceptances & \( y^H \)**

- \( y'^1/2 \sim y^H \pm \hat{y} \)
  - \( |y'| \leq 2.37 \)
  - barrel-endcap: [1.37, 1.52]
- Sudakov shoulder @ \( |\hat{y}| \lesssim 0.9 \)

Born acceptance
**Fiducial Acceptances &** $y^H$

- $y^\gamma_{1/2} \sim y^H \pm \hat{y}$
  - $|y^\gamma| \leq 2.37$
  - barrel-endcap: [1.37, 1.52]
- Sudakov shoulder @ $|\hat{y}| \lesssim 0.9$

- **Sensitivity to the shoulder**
  - $y^H \lesssim y^\gamma_{\text{max}} - \hat{y}_{\text{shoulder}} \sim 1.49$
- **barrel-endcap v.s. shoulder**
  - $y^H \sim y^\gamma_{b-e} - \hat{y}_{\text{shoulder}} \sim [0.47, 0.62]$

1. $y^H \gtrsim 1.5$: NNLO $\times K_{\text{N}^3\Lambda \text{O}}$
2. $y^H \lesssim 1.5$: enhanced corrections
3. artefact @ $y^H \sim 0.5$ ?
1. $y^H \gtrsim 1.5$: NNLO × $K_{N^3LO}$

2. $y^H \lesssim 1.5$: enhanced corrections

3. artefact @ $y^H \sim 0.5$?

- Exposed infrared sensitivity
  - better cuts, resum (lin. power corr.), ...?
- NNLO: accidental cancellations
  - $\oplus$ NNLO $\ominus$ Sudakov
Consider the real-emission subtraction in the antenna subtraction formalism for $H + 0\text{jet (@ LC)}$:

$$
\int \left\{ d\sigma_{H+0\text{jet}}^R - d\sigma_{H+0\text{jet}}^{\text{SNLO}} \right\} = \int d\Phi_{H+1} \left\{ A3g0H(1_g, 2_g, 3_g, H) \, J(\Phi_{H+1}) - F_3^0(1_g, 2_g, 3_g) \, A2g0H(\tilde{1}_g, \tilde{2}_g, H) \, J(\tilde{\Phi}_{H+0}) \right\}
$$
Consider the real-emission subtraction in the antenna subtraction formalism for $H + 0\text{jet}$ (@ LC):

\[
\int \left\{ d\sigma_{H+0\text{jet}}^R - d\sigma_{H+0\text{jet}}^{\text{SNLO}} \right\} = \int d\Phi_{H+1} \left\{ A3g0H(1_g, 2_g, 3_g, H) \mathcal{J}(\Phi_{H+1}) - F_3^0(1_g, 2_g, 3_g) A2g0H(\tilde{1}_g, \tilde{2}_g, H) \mathcal{J}(\tilde{\Phi}_{H+0}) \right\}
\]

**Antennae = ratios of physical Matrix Elements:**

\[
F_3^0(i_g, j_g, k_g) = \frac{A3g0H(i_g, j_g, k_g, H)}{A2g0H(i_g, k_g, H)}
\]
Consider the real-emission subtraction in the antenna subtraction formalism for $H + 0\text{jet}$ (@ LC):

$$
\int \left\{ d\sigma_{H+0\text{jet}}^R - d\sigma_{H+0\text{jet}}^{\text{SNLO}} \right\}
= \int d\Phi_{H+1} \left\{ A_3 g_0 H(1_g, 2_g, 3_g, H) \mathcal{J}(\Phi_{H+1}) 
- F_3^0(1_g, 2_g, 3_g) A_2 g_0 H(\tilde{1}_g, \tilde{2}_g, H) \mathcal{J}(\tilde{\Phi}_{H+0}) \right\}
= \int d\Phi_{H+1} A_3 g_0 H(1_g, 2_g, 3_g, H) \left\{ \mathcal{J}(\Phi_{H+1}) - \mathcal{J}(\tilde{\Phi}_{H+0}) \right\}
$$

⇒ Simple processes where antenna $\simeq$ real-emission Matrix Element

$\iff$ Projection-to-Born

Similarly at NNLO: $X_4^0$ & $X_3^0 \times X_3^0$ are “projections” of RR ME & NLO(+jet) subtraction term.

$$
d\sigma_{N^3\text{LO}}/dy_H \simeq \text{integrated antenna}: X_5^0, X_4^1, X_3^2
$$
CONCLUSIONS & OUTLOOK

- **exploration of the Higgs sector** — highest priority of LHC & future colliders
  
  ⇔ precision to scrutinise Standard Model & search for New Physics

- **fully differential N^3LO prediction for Higgs production (ggF)**
  
  ✤ fiducial cuts can induce *non-trivial* features in predictions
    
    ⇒ N^3LO ≠ NNLO × K_{N^3LO} (affects acceptances)
  
  ✤ IR sensitivity responsible for some of the features
    
    ⇒ possibility to avoid them? resum them?

- **future directions**
  
  ✤ other Higgs decays: H \rightarrow 4\ell, ...
  
  ✤ **Drell Yan**: more challenging (off-shell, lepton spin corr., threshold exp., ...)
  
  ✤ Projection-to-Born \approx Antennae

THANK YOU!
Backup.
SHOULDER AT NNLO

\[ \frac{1}{BR} \frac{d\sigma}{d\Delta\eta} \text{[pb/}\Delta\eta]\]

\[ pp \rightarrow H + X \rightarrow \gamma\gamma + X \]

LHC@13TeV

MMHT2014

\[ \mu = m_H/2 \]
NNLO using **Subtraction**

\[ \sigma_{NNLO} = \int_{\Phi_{Z+3}} (d\sigma_{NNLO}^{RR} - d\sigma_{NNLO}^{S}) \]

\[ + \int_{\Phi_{Z+2}} (d\sigma_{NNLO}^{RV} - d\sigma_{NNLO}^{T}) \]

\[ + \int_{\Phi_{Z+1}} (d\sigma_{NNLO}^{VV} - d\sigma_{NNLO}^{U}) \]

\[ \sum \text{ finite} \quad - \quad 0 \]

⇒ each line suitable for numerical evaluation in \( D = 4 \)
Antenna Factorization

- antenna formalism operates on colour-ordered amplitudes
- exploit universal factorisation properties in IR limits

\[ |A_{m+1}^0(\ldots, i, j, k, \ldots)|^2 \xrightarrow{j \text{ unresolved}} X_3^0(i, j, k) \xrightarrow{\text{antenna function}} |A_m^0(\ldots, \tilde{I}, \tilde{K}, \ldots)|^2 \]

\[ \text{colour-ordered amplitude} \quad \text{antenna function} \quad \text{reduced ME} \]

+ mapping \( \{p_i, p_j, p_k\} \rightarrow \{\tilde{p}_I, \tilde{p}_K\} \)

- captures multiple limits and smoothly interpolates between them

<table>
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<tr>
<th>limit</th>
<th>( X_3^0(i, j, k) )</th>
<th>mapping</th>
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<tbody>
<tr>
<td>( p_j \rightarrow 0 )</td>
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<td>( \tilde{p}_I \rightarrow p_i, \tilde{p}_K \rightarrow p_k )</td>
</tr>
<tr>
<td>( p_j \parallel p_i )</td>
<td>( \frac{1}{s_{ij}} P_{ij}(z) )</td>
<td>( \tilde{p}_I \rightarrow (p_i + p_j), \tilde{p}_K \rightarrow p_k )</td>
</tr>
<tr>
<td>( p_j \parallel p_k )</td>
<td>( \frac{1}{s_{jk}} P_{kj}(z) )</td>
<td>( \tilde{p}_I \rightarrow p_i, \tilde{p}_K \rightarrow (p_j + p_k) )</td>
</tr>
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* c.f. dipoles: \( X_3^0(i, j, k) \sim D_{i,j,k} + D_{k,j,i} \)
Antenna Factorization

- antenna formalism operates on *colour-ordered* amplitudes
- exploit universal factorisation properties in IR limits

\[ |\mathcal{A}_{m+2}^0(\ldots, i, j, k, l, \ldots)|^2 \xrightarrow{j \text{ & } k \text{ unresolved}} X_4^0(i, j, k, l) \quad |\mathcal{A}_m^0(\ldots, \tilde{I}, \tilde{L}, \ldots)|^2 \]

*colour-ordered amplitude*

*antenna function*

*reduced ME + mapping*

\[ \{ p_i, p_j, p_k, p_l \} \rightarrow \{ \tilde{p}_I, \tilde{p}_L \} \]

- captures *multiple limits* and smoothly interpolates between them*

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- **double soft:** \( j, k \rightarrow 0 \)
- **triple-collinear:** \( i \parallel j \parallel k \) \& \( j \parallel k \parallel l \)
- **double collinear:** \( i \parallel j \), \( k \parallel l \)
- **soft-collinear:** \( i \parallel j \), \( k \rightarrow 0 \) \& \( k \parallel l \), \( j \rightarrow 0 \)
- **single-unresolved**
**Antenna Subtraction — Building Blocks**

- \( X(\ldots) \) based on physical matrix elements

\[
X^0_3(i, j, k) = \frac{|A^0_3(i, j, k)|^2}{|A^0_2(\tilde{I}, \tilde{K})|^2}, \quad X^0_4(i, j, k, l) = \frac{|A^0_4(i, j, k, l)|^2}{|A^0_2(\tilde{I}, \tilde{L})|^2},
\]

\[
X^1_3(i, j, k) = \frac{|A^1_3(i, j, k)|^2}{|A^0_2(\tilde{I}, \tilde{K})|^2} - X^0_3(i, j, k) \frac{|A^1_2(\tilde{I}, \tilde{K})|^2}{|A^0_2(\tilde{I}, \tilde{K})|^2},
\]

\[
A^0_3(i_q, j_g, k_{\bar{q}}) = \left| \begin{array}{c} \gamma^* \rightarrow \circ \leftarrow i_q \\ j_g \\ k_{\bar{q}} \end{array} \right|^2 \]

- integrating the antennae \( \longleftrightarrow \) phase-space factorization

\[
d\Phi_{m+1}(\ldots, p_i, p_j, p_k, \ldots) = d\Phi_m(\ldots, \tilde{p}_I, \tilde{p}_K, \ldots) \ d\Phi_{X^0_{ijk}}(p_i, p_j, p_k; \tilde{p}_I + \tilde{p}_K)
\]

\[
X^0_{3,1}(i, j, k) = \int d\Phi_{X^0_{ijk}} X^0_{3,1}(i, j, k), \quad X^0_4(i, j, k, l) = \int d\Phi_{X^0_{ijkl}} X^0_4(i, j, k, l)
\]
All building blocks known!

\( X_3^0, X_4^0, X_3^1 \) and integrated counterparts \( \chi_3^0, \chi_4^0, \chi_3^1 \) 
∀ configurations relevant at hadron colliders:

\[ \leftrightarrow \text{final–final} \quad e^+ e^- \]

[Gehrmann-De Ridder, Gehrmann, Glover '05]

\[ \leftrightarrow \text{initial–final} \quad e^+ p \]

[Daleo, Gehrmann-De Ridder, Gehrmann, Luisoni, Maitre '06,'09,'12]

\[ \leftrightarrow \text{initial–initial} \quad pp \]

[Boughezal, Daleo, Gehrmann-De Ridder, Gehrmann, Maitre, et al. '10,'11,'12]

\[ \chi_3^{0,1}(i, j, k) = \int d\Phi X_{i,j,k} X_3^{0,1}(i, j, k), \quad \chi_4^0(i, j, k, l) = \int d\Phi X_{i,j,k,l} X_4^0(i, j, k, l) \]
**Antenna Subtraction @ NLO** — $q\bar{q} \rightarrow ggZ$

\[ \int \left\{ d\sigma_{Z+1\text{jet}}^R - d\sigma_{Z+1\text{jet}}^S \right\} \]

\[ = \int d\Phi_{Z+2} \left\{ |A_{4}^{0}(1_{q}, 3_{g}, 4_{g}, 2_{\bar{q}}, Z)|^2 J(\Phi_{Z+2}) - d_{3}^{0}(1_{q}, 3_{g}, 4_{g}) \left| A_{3}^{0}(\bar{3}_{q}, (34)_{g}, 2_{\bar{q}}, Z) \right|^2 J(\bar{\Phi}_{Z+1}) - d_{3}^{0}(2_{\bar{q}}, 4_{g}, 3_{g}) \left| A_{3}^{0}(1_{q}, (34)_{g}, \bar{2}_{\bar{q}}, Z) \right|^2 J(\bar{\Phi}_{Z+1}) \right\} + (3 \leftrightarrow 4) \]

\[ \int \left\{ d\sigma_{Z+1\text{jet}}^V - d\sigma_{Z+1\text{jet}}^T \right\} \]

\[ = \int d\Phi_{Z+1} \left\{ |A_{3}^{1}(1_{q}, 3_{g}, 2_{\bar{q}}, Z)|^2 + \frac{1}{2} \left[ D_{3}^{0}(s_{13}) + D_{3}^{0}(s_{23}) \right] \left| A_{3}^{0}(1_{q}, 3_{g}, 2_{\bar{q}}, Z) \right|^2 \right\} J(\Phi_{Z+1}) \]
\[ \Delta \sigma_U : \]
\[ \Delta \sigma^U, A \sim -\frac{\epsilon}{2} \frac{\epsilon}{2} J_n^{(1)} M_n + J_n^{(1)} M_n^0 \]
\[ \Delta \sigma^U, B \sim -\frac{1}{2} J_n^{(1)} \otimes J_n^{(1)} M_n^0 \]
\[ \Delta \sigma^U, C \sim J_n^{(2)} M_n^0 \]

\[ \Delta \sigma_T : \]
\[ \Delta \sigma^T, a \sim J_n^{(1)} M_n + X_3^0 M_n^0 \]
\[ \Delta \sigma^T, b_1 \sim X_3^0 M_n^0 + \frac{\epsilon}{2} X_3^0 J_n^{(1)} M_n^0 \]
\[ \Delta \sigma^T, c \sim -\int \Delta \sigma^S, c + \Delta \sigma^T, c + \Delta \sigma^T, c \]
\[ \Delta \sigma^T, b_2 \sim X_3^0 M_n^0 + X_3^0 (J_n^{(1)} M_n^0) - M_3 X_3^0 J_n^{(1)} M_n^0 \]

\[ \Delta \sigma_S : \]
\[ \Delta \sigma^S, a \]
\[ \Delta \sigma^S, d \]
\[ \Delta \sigma^S, c \]
\[ \Delta \sigma^S, b_2 \]
\[ \Delta \sigma^S, b_1 \]

- **double real:** \( \Delta \sigma^S \sim X_3^0 |\mathcal{A}_{m+1}^0|^2, X_4^0 |\mathcal{A}_m^0|^2, X_3^0 X_3^0 |\mathcal{A}_m^0|^2 \)
- **real–virtual:** \( \Delta \sigma^T \sim X_3^0 |\mathcal{A}_{m+1}^0|^2, X_3^0 |\mathcal{A}_m^0|^2, X_3^1 |\mathcal{A}_m^0|^2 \)
- **double virtual:** \( \Delta \sigma^U = (\text{collect rest}) \sim \mathcal{X} |\mathcal{A}_m^{0,1}|^2 \)
**Antenna Subtraction — Checks of the Calculation**

**Analytic pole cancellation**
- \[ \text{Poles} \left( d\sigma^\text{RV} - d\sigma^\text{T} \right) = 0 \]
- \[ \text{Poles} \left( d\sigma^\text{VV} - d\sigma^\text{U} \right) = 0 \]

\[ \text{DimReg: } D = 4 - 2\epsilon \]

**Unresolved limits**
- \[ d\sigma^S \rightarrow d\sigma^\text{RR} \] (single- & double-unresolved)
- \[ d\sigma^T \rightarrow d\sigma^\text{RV} \] (single-unresolved)

\[ \text{bin the ratio: } \frac{d\sigma^S}{d\sigma^\text{RR}} \xrightarrow{\text{unresolved}} 1 \]

\[ q \bar{q} \rightarrow Z + g_3 \ g_4 \ g_5 \quad (g_3 \text{ soft} \ & \ g_4 \parallel \bar{q}) \]

![Image of Maple process output]

(approach singular limit: \( x_i = 10^{-7}, 10^{-8}, 10^{-9} \))

<table>
<thead>
<tr>
<th>Processes computed using the antenna subtraction method</th>
<th>NNLO subtraction set up for</th>
</tr>
</thead>
<tbody>
<tr>
<td>pp → V</td>
<td>&quot;colour neutral&quot; + 0,1,2 jets</td>
</tr>
<tr>
<td>pp → V + j</td>
<td></td>
</tr>
<tr>
<td>→ V → ℓℓ</td>
<td></td>
</tr>
<tr>
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<td></td>
</tr>
<tr>
<td>ep → eγ ℓγ</td>
<td></td>
</tr>
<tr>
<td>e⁺e⁻ → e⁺e⁻</td>
<td></td>
</tr>
<tr>
<td>pp → VH</td>
<td></td>
</tr>
<tr>
<td>→ H → bb</td>
<td></td>
</tr>
<tr>
<td>...</td>
<td></td>
</tr>
</tbody>
</table>