Probing spatial parton distributions of the proton via hard processes in pp collisions

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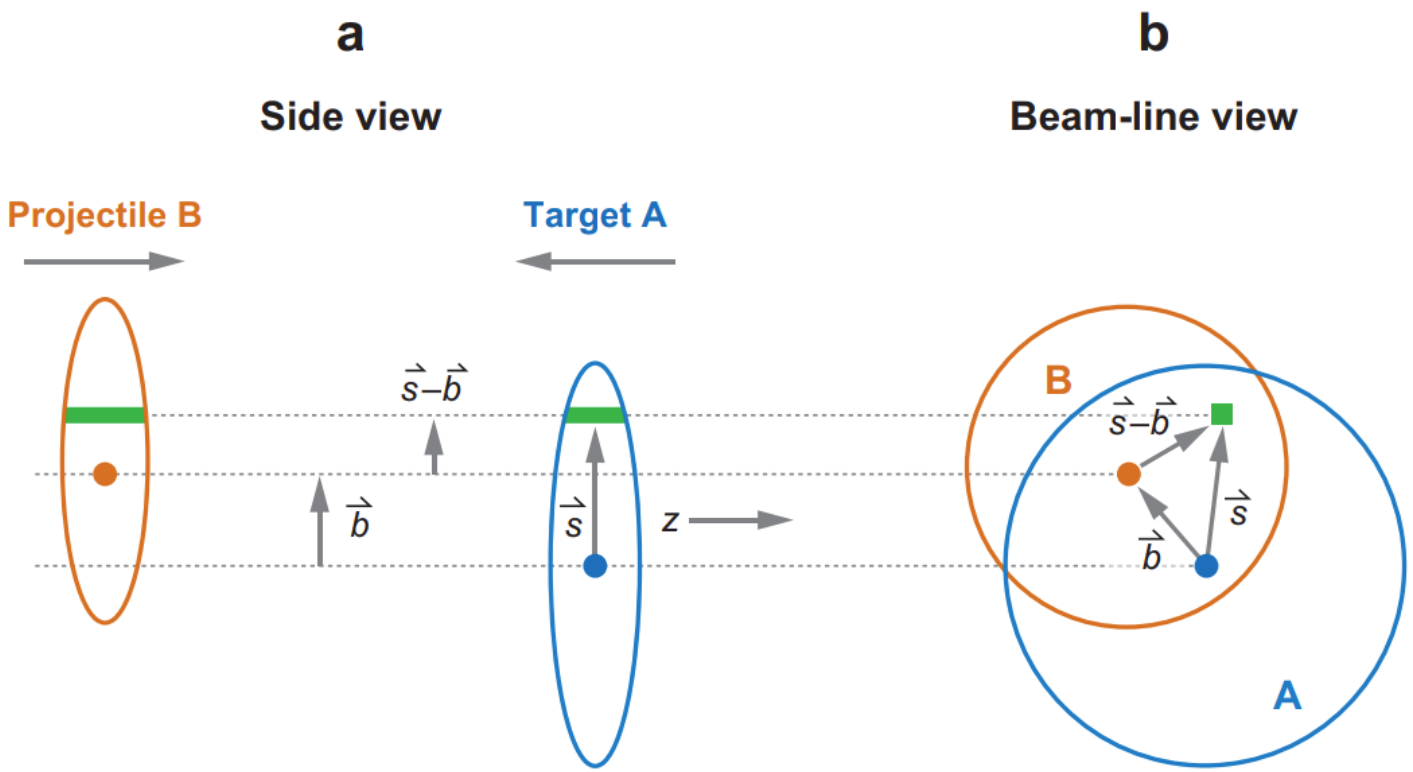
based on arXiv:2102.12916.

Motivation:

Aim: *a unified description of hard processes in pp, pA and AA*

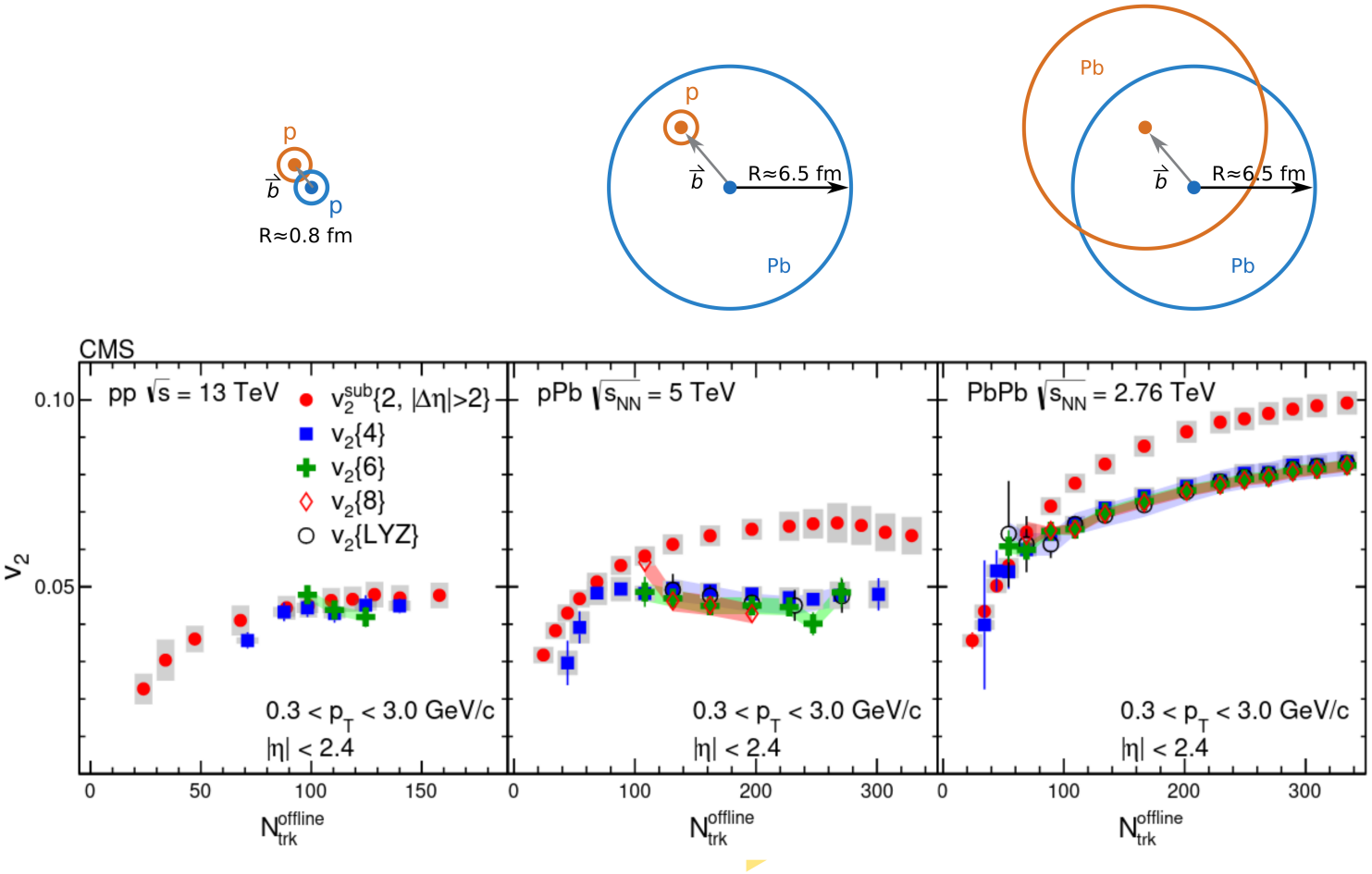
The standard picture in AA collisions

The CLASSICAL picture of collision geometry in the Glauber model:



Miller, Reygers, Sanders and Steinberg, *Ann. Rev. Nucl. Part. Sci.* **57** (2007) 205.

Flow in pp, pA and AA collisions



CMS collaboration, *Phys. Lett. B* **765** (2017) 193 [1606.06198].

Is this picture still valid for pp collisions?

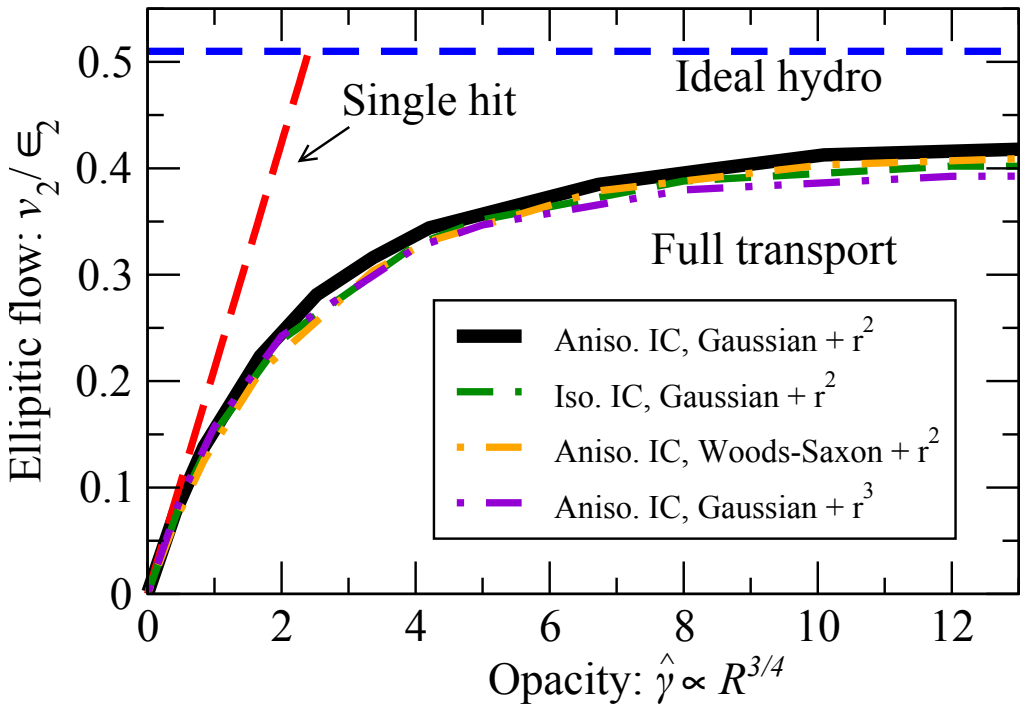
Flow in pp, pA and AA collisions

Phenomenological success based on this generalized picture:

- **Hydro, etc:**

For a review: Nagle and Zajc, *Ann. Rev. Nucl. Part. Sci.* **68** (2018) 211 [1801.03477].

- **Kinetic theory:** interplay of hydro and non-hydro modes



Kurkela, Wiedemann and BW, *Eur. Phys. J. C* **79** (2019) 965 [1905.05139].

Transverse phase-space (TPS) PDFs

A coarse-grained picture of TPS PDFs in heavy nuclei

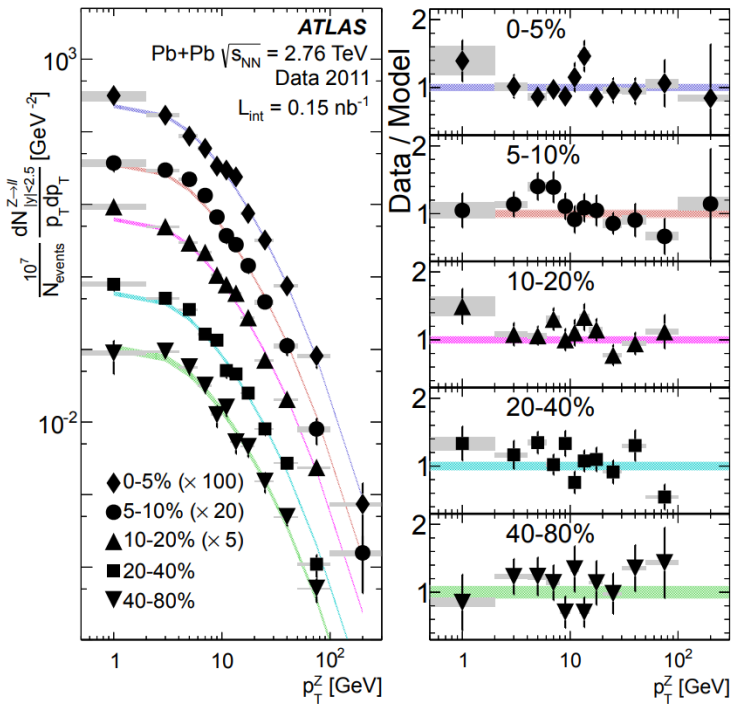
The Glauber model gives coarse-grained TPS PDFs:

$$f(\mathrm{r}, \mathrm{p}, z) \to T_A(\mathrm{r}) f(\mathrm{p}, z)$$

T_A: the nuclear thickness function

$f(\mathrm{p}, z)$: corresponding TMD PDF

r, p: transverse position, momentum

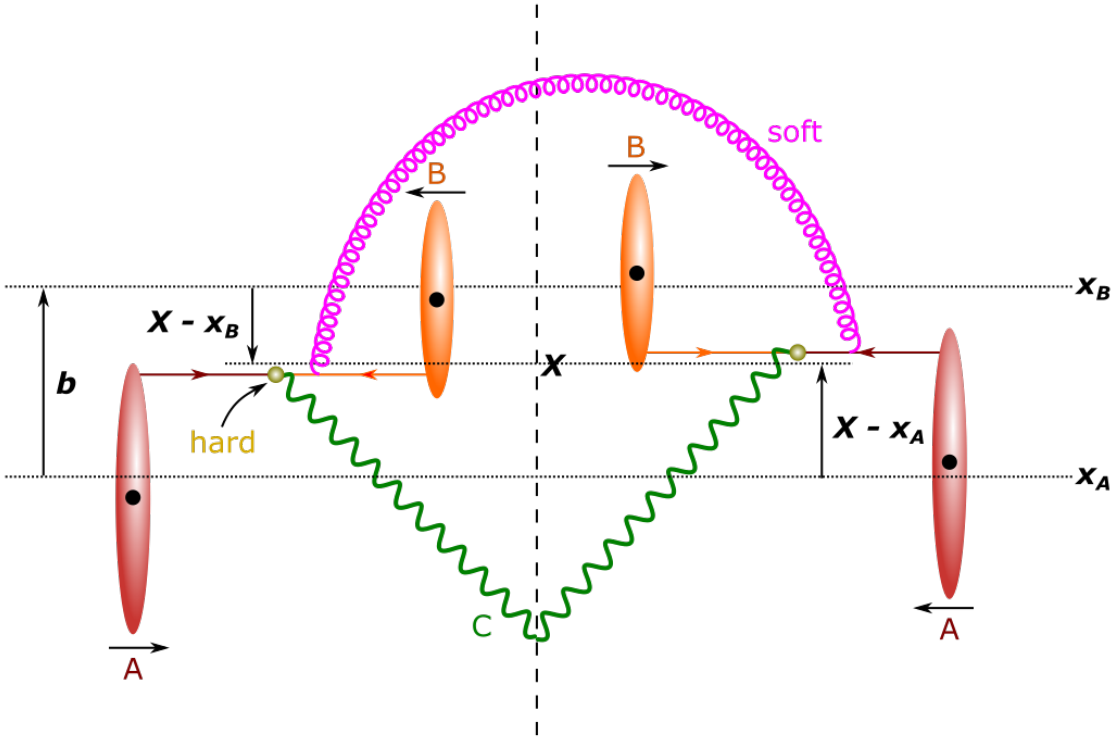


ATLAS, *Phys. Rev. Lett.* **110** (2013) 022301 [1210.6486].

TPS PDFs in the proton have not been measured yet!

Impact-parameter dependent collisions in QFT

The quantum picture (for A+B→ C + anything else):



BW, arXiv:2102.12916

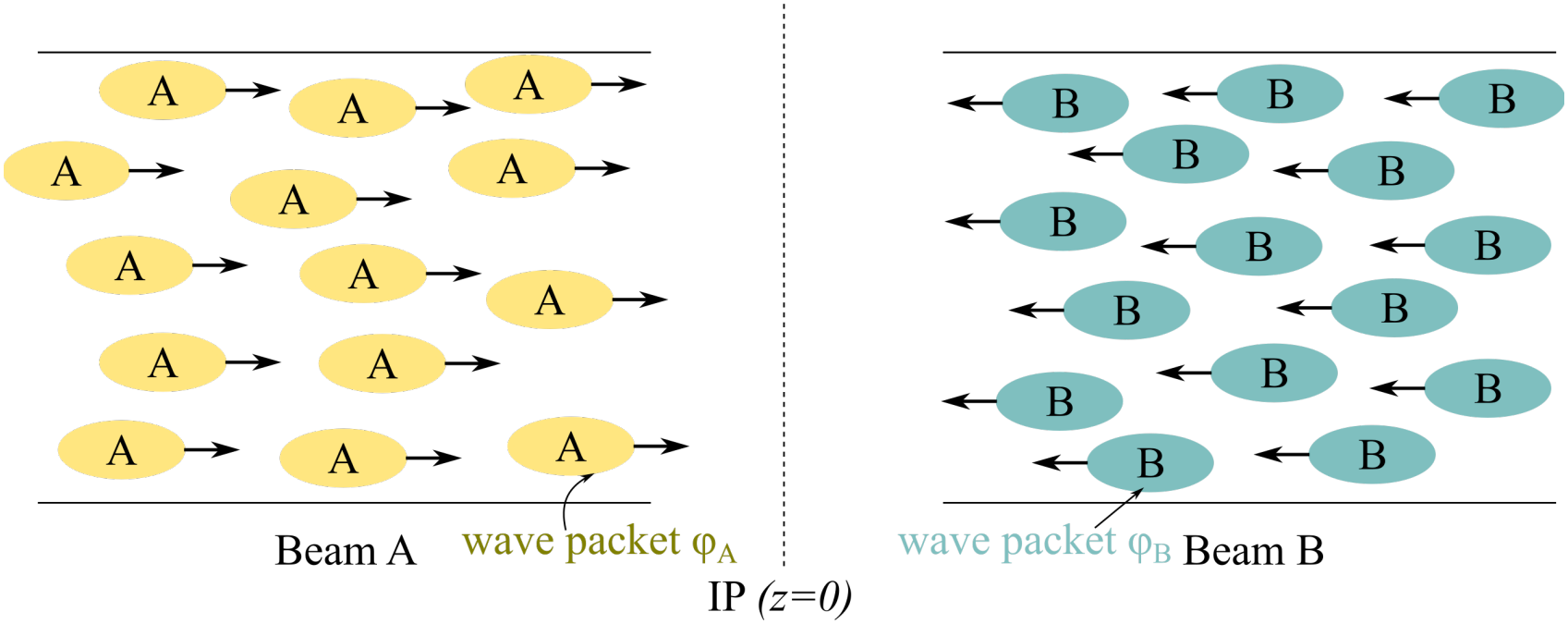
Two main issues:

1. Impact-parameter dependent cross section in QFT
2. The factorization formula with C being colorless, valid for pp, pA and AA

Impact-parameter dependent collisions

The cross section and collider physics

The quantum picture of colliding beams:



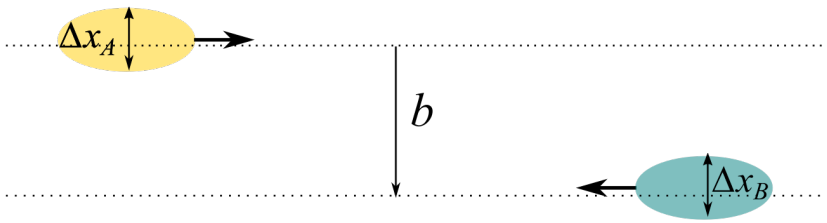
The cross section: the number of collision $N = \mathcal{L}\sigma$ with $\mathcal{L}$ the luminosity.

1. It is intrinsic to colliding particles.
2. Wave packets are not the same across experiments.

The wave packets should be able to be integrated out!

The impact parameter

Take two particles respectively from the two beams:



About defining b:

1. **It can be ambiguously defined if $b \gg \Delta x_A, \Delta x_B$.**
2. $b \gg \Delta x_A, \Delta x_B$ **barely change with time in high-energy limit.**

These are the conditions needed to define $d\sigma/d^2b$.

The impact-parameter dependent probability

- **In principle one can predict**

$$P_{\mathrm{b}}(\phi_A, \phi_B \to \{p_f\}) = \langle \phi_A \phi_B | \hat{S}^\dagger | \{p_f\} \rangle \langle \{p_f\} | \hat{S} | \phi_A \phi_B \rangle.$$

- **The issue:** ϕ_A and ϕ_B are not measured!

- $d\sigma/d^2b$ **is given from** P_b **if** ϕ_A, ϕ_B **can be integrated out:**

1. Condition 1 needed to identify transverse momenta in $\phi_i^* \times \phi_i$

2. Condition 2 needed to identify longitudinal momenta in $\phi_i^* \times \phi_i$

The impact-parameter dependent cross section

With the above two conditions imposed:

The colliding particles' wave packets are 1) localized but 2) not so localized ($p_z \gg p_T$) in transverse position space.

For producing any observable O

$$\frac{d\sigma}{d^2\mathrm{b}dO} = \int \frac{d^2\mathrm{q}}{(2\pi)^2} e^{i\mathrm{q}\cdot\mathrm{b}} \int \prod_f [d\Gamma_{p_f}]\, \delta(O - O(\{p_f\}))$$
$$\times \frac{1}{2s} M(p_A, p_B \to \{p_f\}) M^*(\bar{p}_A, \bar{p}_B \to \{p_f\}) (2\pi)^4 \delta^{(4)}(p_A + p_B - \sum p_f),$$

where $O(\{p_f\})$ defines the observable O as a function of the final-state momenta $\{p_f\}$, the incoming momenta

$$p_A^\mu = \bar{n}_A \cdot P_A \frac{n_A^\mu}{2} + \frac{q_T^\mu}{2} - \frac{q_T^2}{4\bar{n}_A \cdot P_A} \frac{\bar{n}_A^\mu}{2}, \qquad p_B^\mu = \bar{n}_B \cdot P_B \frac{n_B^\mu}{2} - \frac{q_T^\mu}{2} - \frac{q_T^2}{4\bar{n}_B \cdot P_B} \frac{\bar{n}_B^\mu}{2},$$
$$\bar{p}_A^\mu = \bar{n}_A \cdot P_A \frac{n_A^\mu}{2} - \frac{q_T^\mu}{2} - \frac{q_T^2}{4\bar{n}_A \cdot P_A} \frac{\bar{n}_A^\mu}{2}, \qquad \bar{p}_B^\mu = \bar{n}_B \cdot P_B \frac{n_B^\mu}{2} + \frac{q_T^\mu}{2} - \frac{q_T^2}{4\bar{n}_B \cdot P_B} \frac{\bar{n}_B^\mu}{2}.$$

Factorization and TPS PDFs

Factorization using SCET

The S-matrix element at leading order in $\lambda = \Lambda_{IR}/Q$

$$\langle p_C, \{p_X\}|\hat{S}|\phi_A\phi_B\rangle = \int d^4x e^{ip_C\cdot x}\langle\{p_X\}|i\hat{M}(x)|\phi_A\phi_B\rangle.$$

Here

$$i\hat{M}(x) = \overbrace{\int dt_A dt_B \underbrace{C^{a_A a_B}_{\alpha_A\alpha_B}(\epsilon, t_A, t_B)}_{\text{hard}} \underbrace{[S_{n_A}\phi_{n_A}(x+t_A\bar{n}_A)]^{\alpha_A}_{a_A}[S_{n_B}\phi_{n_B}(x+t_B\bar{n}_B)]^{\alpha_B}_{a_B}}_{\text{soft-collinear factorization}}}^{\text{hard-collinear factorization}},$$

S_{n_i}: the soft Wilson line along the collinear direction n_i

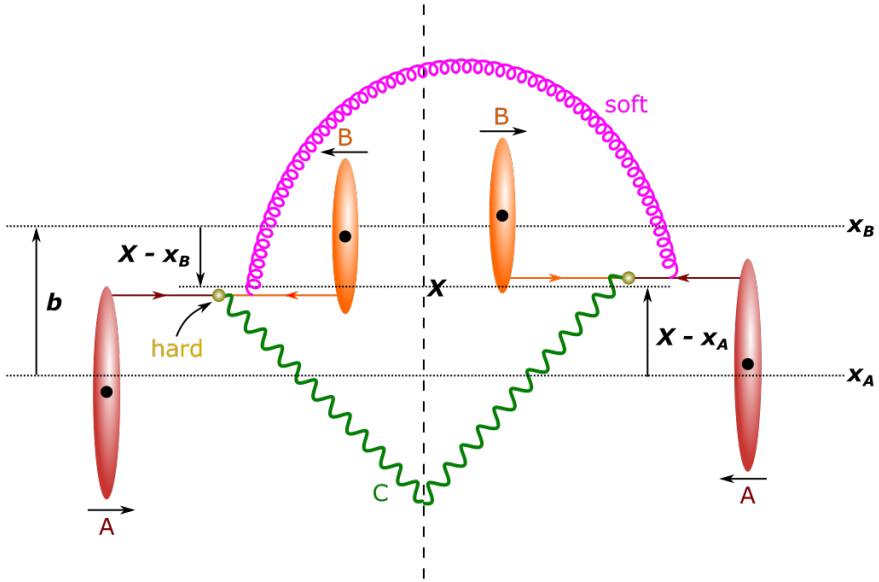
ϕ_{n_i} : $\chi_{n_i}(x) \equiv W^\dagger_{n_i}(x)\frac{n_i\bar{n}_i}{4}\psi_{n_i}(x), \qquad \bar{\chi}_{n_i}(x), \qquad \mathcal{B}^\mu_{n_i T} = \frac{1}{g_s}W^\dagger_{n_i}(x)iD^\mu_{n_i T}W_{n_i}(x)$

with $D^\mu_{n_i T} \equiv \partial^\mu_T - ig_s A^\mu_{n_i T}$ and W_{n_i} the n_i-collinear Wilson line.

The impact-parameter dependent cross section

$$\frac{d\sigma}{d^2\mathrm{b}dy_C d^2\mathrm{p}_C} = \frac{1}{2(2\pi)^3}\int d^4X d^4x\ e^{-ip_C\cdot x}$$
$$\times \sum_{\{p_X\}}\langle\phi_A\phi_B|\hat{M}^\dagger\left(X+\frac{x}{2}\right)|\{p_X\}\rangle\langle\{p_X\}|\hat{M}\left(X-\frac{x}{2}\right)|\phi_A\phi_B\rangle.$$

The factorization formula



$$\frac{d\sigma_{AB}}{d^2\mathrm{b}\,dy_C\,d^2\mathrm{p}_C} = \frac{1}{4\pi s}\sum_{j,k}\int d^2\mathrm{X}\int d^2\mathrm{x}\,e^{i\mathrm{p}_C\cdot\mathrm{x}}\int_0^1 \frac{dz_A}{z_A}\frac{dz_B}{z_B}\underbrace{\mathcal{T}_{j/A}(\mathrm{X},z_A,\mathrm{x})\mathcal{T}_{k/B}(\mathrm{X}-\mathrm{b},z_B,\mathrm{x})}_{\text{Thickness beam functions}}$$

$$\times\int\prod_f[d\Gamma_{p_f}]\prod_{i=A,B}\delta(z_i\bar{n}_i\cdot P_i-\bar{n}_i\cdot p_C-\sum\bar{n}_i\cdot p_f)$$

$$\times\underbrace{H^{\bar{a}_A\bar{a}_B}_{a_Aa_B}(z_AP_A,z_BP_B\to p_C,\{p_f\})}_{\text{hard function}}\underbrace{\mathcal{S}^{\bar{a}_A\bar{a}_B}_{a_Aa_B}(\mathrm{x})}_{\text{soft function}}.$$

Valid for all pp, pA and AA!

Thickness beam functions

- **Definition:**

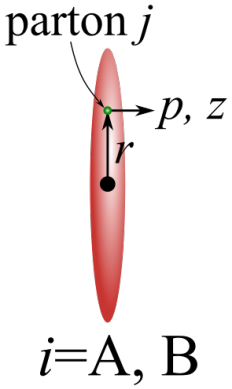
$$\mathcal{T}_{j/i}(\mathrm{r}, z, \mathrm{x}) = \int \frac{d^2\mathrm{q}}{(2\pi)^2} e^{i\mathrm{q}\cdot\mathrm{r}} \int \frac{dt}{2\pi} e^{-izt\bar{n}\cdot P}$$
$$\times \left\langle \bar{n}\cdot P, -\frac{\mathrm{q}}{2} \right| [\phi_n^\dagger]_a^{\alpha'} \left(\frac{t\bar{n}}{2} + \frac{x_T}{2}\right) \Gamma_{\alpha'\alpha} [\phi_n]_a^{\alpha} \left(-\frac{t\bar{n}}{2} - \frac{x_T}{2}\right) \left| \bar{n}\cdot P, \frac{\mathrm{q}}{2} \right\rangle$$

where Γ: some projector in spin/polarization.

Beam function at r!

- **Fourier transform of TPS PDF with respect to x:**

$f_{j/i}(\mathrm{r}, z, \mathrm{p}) =$



$= \int d^2\mathrm{x} e^{i\mathrm{p}\cdot\mathrm{x}} \mathcal{T}_{j/i}(\mathrm{r}, z, \mathrm{x}).$

TMD PDF at r!

Thickness beam functions

- **Connection:**

1. **Two-dimensional analogues of quantum phase-space distributions**

   Belitsky, Ji and Yuan, *Phys. Rev. D* **69** (2004) 074014 [hep-ph/0307383].

2. $\mathcal{T}_{j/i}(\mathrm{r}, z, 0)$**: the transverse spatial PDF at** r

3. **Generalized Parton Distributions (GPDs):**

   The off-diagonal matrix element needed for defining $\mathcal{T}_{j/i}(\mathrm{r}, z, 0)$ is the same as GPD with the incoming momenta differing only in the transverse plane.

   Diehl, *Phys. Rept.* **388** (2003) 41 [hep-ph/0307382].

Soft and hard functions

- **The soft function:**

$$\mathcal{S}^{\bar{a}_A \bar{a}_B}_{a_A a_B}(\mathrm{x}) = \langle 0| \bar{T}[S^{\dagger a'_B \bar{a}_B}_{n_B}(\mathrm{x}) S^{\dagger a'_A \bar{a}_A}_{n_A}(\mathrm{x})]\, T[S^{a_A a'_A}_{n_A}(0) S^{a_B a'_B}_{n_B}(0)]|0\rangle$$

- **The hard function:**

$$H^{\bar{a}_A \bar{a}_B}_{a_A a_B} \equiv \frac{P^{\bar{\alpha}_A \alpha_A}}{d_{c_A}} \frac{P^{\bar{\alpha}_B \alpha_B}}{d_{c_B}} \tilde{\mathcal{C}}^{*\bar{a}_A \bar{a}_B}_{\bar{\alpha}_A \bar{\alpha}_B} \tilde{\mathcal{C}}^{a_A a_B}_{\alpha_A \alpha_B}$$

with $\tilde{\mathcal{C}}$ given by

$$\tilde{\mathcal{C}}(\epsilon, z_A \bar{n}_A \cdot P_A, z_B \bar{n}_B \cdot P_B) = \int dt_A dt_B e^{i(t_A z_A \bar{n}_A \cdot P_A + t_B z_B \bar{n}_B \cdot P_B)} \mathcal{C}(\epsilon, t_A, t_B).$$

The same as for conventional cross section!

A verification of factorization

- **The impact-parameter dependent Drell-Yan cross section**

$$q\bar{q} \to \gamma^* + \text{anything else}$$

- **The verification procedure:**

  Its validity in such collisions can be verified order by order in perturbation theory by comparing to the results of the general formula for the impact-parameter dependent cross section, expended up to leading order in $\lambda = 1/(bQ) \ll 1$.

- **At the Born level**

$$\frac{d\sigma^{(0)}}{d^2\mathrm{b}} = \pi^2 e_q^2 \delta(s - Q^2)\delta^{(2)}(\mathrm{b}).$$

  The hard scattering is initiated by the quark and antiquark only when they pass very close to one another at a distance $\sim 1/Q$, which defines "point-like" in our calculation.

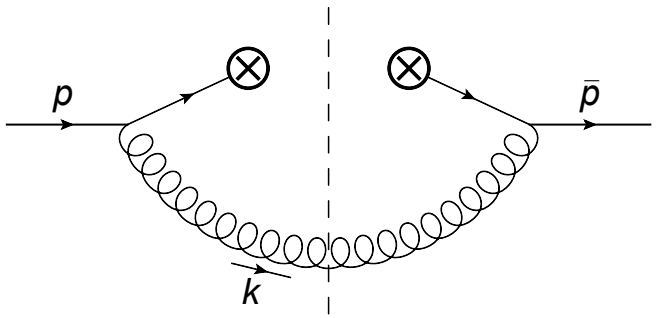
Spatial quark distribution in a quark

Perturbative series of the spatial quark distribution:

$$\mathcal{T}_{q/q}(\mathrm{r}, z) = \delta(1-z)\delta^{(2)}(\mathrm{r}) + \mathcal{T}^{(1)}_{q/q}(\mathrm{r}, z) + O(\alpha_s^2).$$

where

$$\mathcal{T}^{(1)}_{q/q}(\mathrm{r}, z) =$$



$$= \frac{\alpha_s C_F}{2\pi^2 r^2}\left[-\left(\frac{1}{\epsilon} + L_T\right)\delta(1-z) + \frac{1+z^2}{(1-z)_+}\right].$$

with $L_T \equiv 2\log\left(\frac{r\mu}{2e^{-\gamma_E}}\right)$.

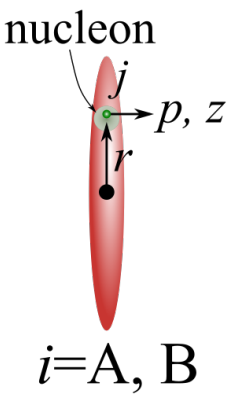
At finite b

$$\frac{d\sigma^{(1)}_{qq}}{d^2\mathrm{b}} \propto \alpha_s/(sb^2) + O(\alpha_s^2).$$

The Glauber model

For heavy nuclei, modelling modes $\sim \frac{1}{b} \ll \Lambda_{QCD}$

$\mathcal{T}_{j/i}(\mathrm{r}_i, z, \mathrm{x}) \rightarrow$



$=$ the thickness functions $\times$ the beam functions

The factorization formula gives

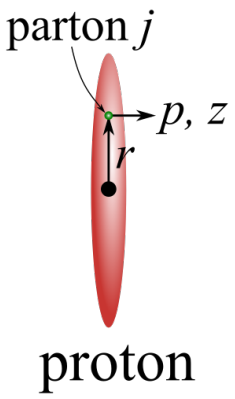
$$R_{AA} \equiv \frac{\frac{d\sigma_{AB}}{d^2\mathrm{b}dy_C d^2\mathrm{p}_C}}{T_{AB}(\mathrm{b})\frac{d\sigma_{nn}}{dy_C d^2\mathrm{p}_C}} = 1,$$

with $T_{AB}(\mathrm{b}) \equiv \int d^2\mathrm{X}\, T_A(\mathrm{X})\, T_B(\mathrm{X}-\mathrm{b})$.

TPS PDFs in the proton

For the proton, $r \sim b \sim 1/\Lambda_{QCD}$, **still non-perturbative!**

$\mathcal{T}_{j/p}(\mathrm{r}_i, z, \mathrm{x}) \rightarrow$



1. There are no first-principle calculations.
2. Opportunity to measure them in b-dependent pp collisions.
3. It is a prerequisite to study jet quenching in pp collisions.

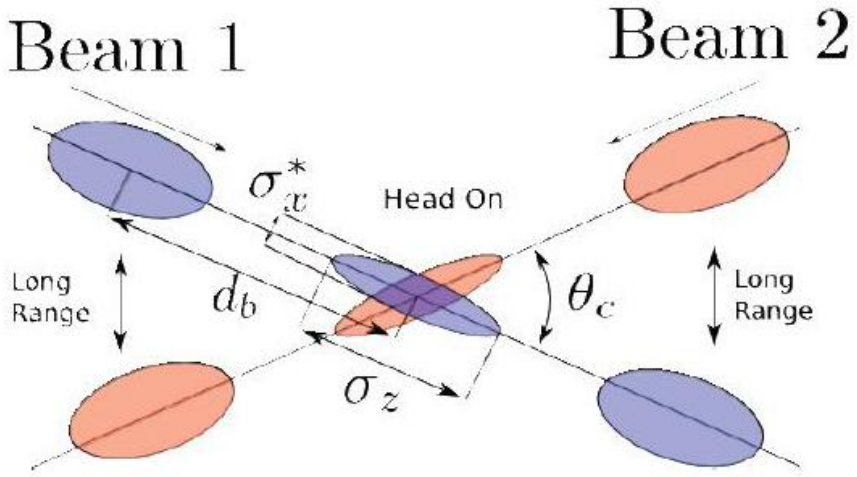
TPS PDFs in the proton

A curious issue:

1. Condition 1 for pp collisions:

$$r \sim b \sim \frac{1}{\Lambda_{QCD}} \gg \Delta x_T \geq \frac{1}{2\Delta p} \Rightarrow \Delta p \gg 100 \text{ MeV}.$$

2. An estimate of Δp at the LHC:

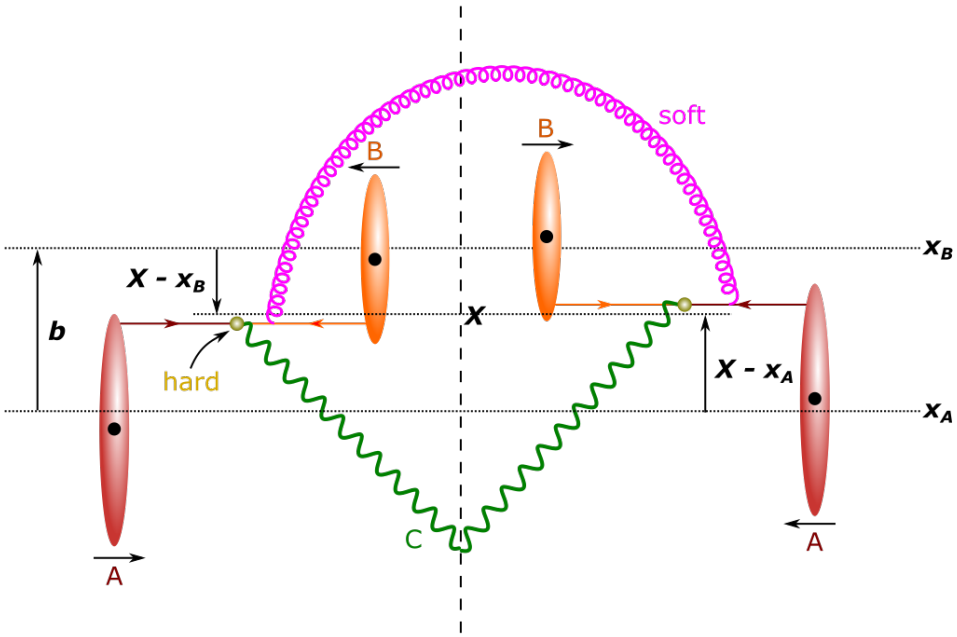


$$\Delta\theta_c \sim 10\mu\text{rad} \Rightarrow \Delta p \leq 70 \text{ MeV}$$

for $E = 7$ TeV proton beams.

Summary and Perspective

A unified framework for A+B→ C + anything else:



Issues solved:

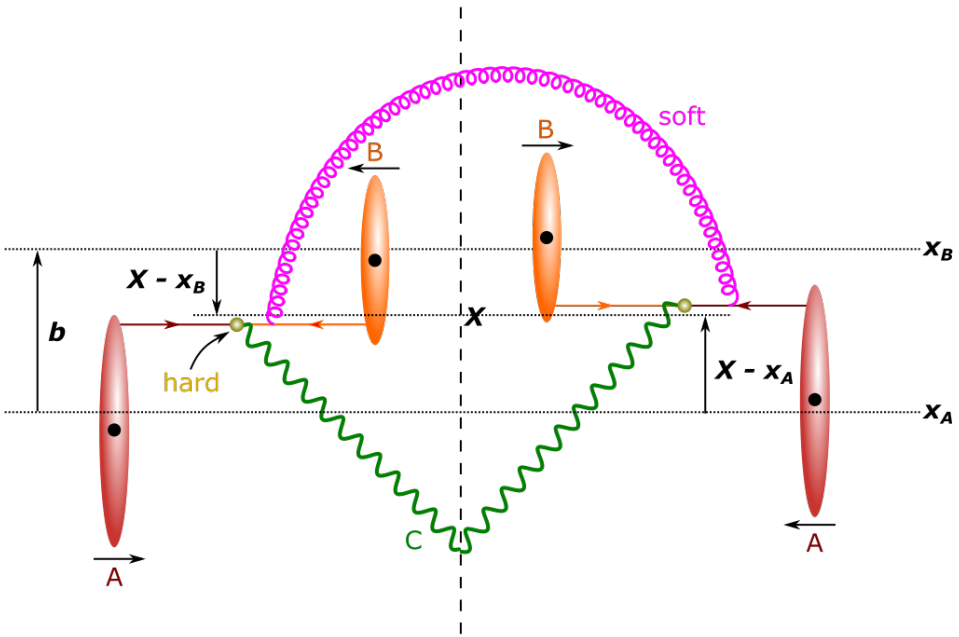
1. Impact-parameter dependent cross section in QFT
2. The factorization formula for $C = \gamma^*, W^{\pm}, Z^0, H^0$

Issues to be solved:

3. The factorization formula for $C =$ jets, etc.

Summary and Perspective

A unified framework for A+B→ C + anything else:



Issues to be solved (continued):

4. More theoretical studies needed for the measurement of proton TPS PDFs
   - Detailed analysis of the experimental conditions
   - Pin down the original of flow in pp collisions

     Most importantly, the relation between the impact parameter and $N_{\text{trk}}^{\text{offline}}$