Fluid velocity from transverse momentum spectra

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Based on 2012.07898, in collaboration with Anthony Guillen
I will discuss the interpretation of selected 2011 LHC data on identified hadron production in Pb+Pb collisions at 2.76 TeV.
Sketch of a Pb+Pb collision at LHC

• Relativistic contraction of length by factor 5000: colliding thin pancakes

• The collision creates strongly-coupled quark-gluon matter, governed by strong interactions, which expands into the vacuum. ~30000 particles produced at the end.

• The best theoretical description is a macroscopic one: a small lump of fluid.
Outline

• How we see collective flow in $p_t$ spectra
• What can we learn about the fluid just by analyzing these spectra, without any detailed hydrodynamic modeling?
• Generalization of the traditional blast-wave approach.
• Generic differences between blast wave and hydrodynamics
• Generalized blast-wave fits to LHC data
• Centrality dependence of $p_t$ spectra
Evidence for collective motion: the *ridge*

In the most ~50% central collisions, pair correlations display a regular wave pattern, which is broken in more peripheral collisions.
Collective motion

The velocity of a particle embedded in a fluid is the sum* of the fluid velocity \( v \), which is the same for all particles around a given point, and a random thermal velocity of magnitude \( \sim \sqrt{T/m} \).

*up to relativistic details
The energy of a particle in the fluid can be decomposed as

$$E = \frac{m}{\sqrt{1-v^2}} + O(T)$$

The **collective** motion has a larger effect on **heavy** particles.
Collective motion seen in $m_t$ spectra

In proton-proton collisions, slopes are comparable for $\pi$, $K$, $p$.

In Pb+Pb collisions, spectra are flatter for heavier particles. Evidence for radial collective flow.
Collective motion seen in $p_T$ spectra

Various hydrodynamic models (VISH2+1, HKM, Krakow) were able to predict the $p_T$ spectra reasonably well: they naturally capture the mass ordering.

Blast Wave fits, where the parameters are typically a fluid velocity and a temperature, describe the spectra very well.
Our goal

• What can we learn about the fluid directly from experimental data, without running a specific hydrodynamic simulation (whose results depend on initial conditions, equation of state, transport coefficients, treatment of hadronic phase)?

• Idea: Generalized blast wave fit to data, in a way that follows as closely as possible an actual hydrodynamic calculation.

• Three differences with the traditional blast-wave fit.
Generalized blast wave fit (1/3)

Write the momentum distribution as an *arbitrary* superposition of *boosted* thermal distributions

\[
\frac{dN}{d^3p} = \frac{2S + 1}{(2\pi)^3} \int \frac{1}{e^{E^*/T_f} \pm 1} \Omega(u) du.
\]

\(E^* = p^\mu u_\mu = \) energy of particle in fluid frame

\(T_f = \) temperature

Volume of fluid with velocity \(u\) up to \(du\)
Generalized blast wave fit (2/3)

Integrate over rapidity and azimuthal angle to obtain the transverse momentum distribution

\[
\frac{dN}{dp_t} = \int_0^\infty f(p_t, u) \Omega(u) du
\]

where

\[
f(p_t, u) = \frac{2S + 1}{(2\pi)^3} p_t \int_{-\infty}^{+\infty} dp_z \int_{-\pi}^{\pi} d\phi \frac{1}{e^{E^*/T_f} \pm 1}
\]

\[u = \text{radial component of 4-velocity} = \frac{v}{\sqrt{1-v^2}}\]

In this talk, I call \( u \) the « fluid velocity », but it can be >1.

\( \Omega(u) du = \text{volume of fluid in fm}^3 = \text{same for all particle species} \)

\( T_f = \text{freeze-out temperature} = \text{same for all particle species} \)
Generalized blast wave fit (3/3)

All hadron resonances are produced at temperature $T_f$ following to the boosted thermal distribution. Resonances decay to stable hadrons which are measured.

We take into account the feed-down from resonance decays using the FastReso code of Mazeliauskas et al. 1809.11049 (see also 1907.11059). Amounts to replacing

$$\frac{1}{e^{E^*/T_f} \pm 1} \rightarrow f_1(E^*) + (f_2(E^*) - f_1(E^*)) \frac{E^* u^0}{p^0}$$

where $f_1(E^*)$ and $f_2(E^*)$ are functions which are computed by FastReso for each stable hadron.
Viscous or ideal hydro?

State-of-the-art hydrodynamic calculations include viscosity, which implies that the fluid is locally out of equilibrium.

The resulting modifications of the equations of hydro are robust: Navier-Stokes+2nd order terms.

How the off-equilibrium correction is shared among the hadrons, and how it depends on momentum, is not known. It depends on the microscopic interactions at freeze-out (Dusling et al. 0909.0754).

Viscosity has a large effect on anisotropc flow, but a smaller effect on the spectra. We neglect it.
In a hydrodynamic calculation, one evaluates momentum distributions of outgoing particles by integrating over a freeze-out isotherm which is a curve in space-time.
Blast-wave versus hydro

Space-like part of the isotherm.
Evaluate the hadron content of the fluid at a given time =
At each point, a boosted thermal distributions = blast wave.
Blast-wave versus hydro

Time-like part of the isotherm. The particle flux through a fixed surface is proportional to the particle velocity. This contribution is not just a boosted thermal distribution.
Blast-wave versus hydro

The blast-wave can be seen as an approximation where 
particle velocity $\approx$ fluid velocity
We call this approximation semi-Cooper-Frye.
Test of semi-Cooper-Frye

Spectra from an ideal hydrodynamic simulation run with Music of one random central Pb+Pb collision at 2.76 TeV. Initial conditions from TRENTo
Test of semi-Cooper-Frye

Approximation $\text{particle velocity} \approx \text{fluid velocity}$
Overestimates yield at low $p_t$
Underestimates yield at high $p_t$
Test of the fitting algorithm

- The generalized blast-wave fit to the spectra returns the distribution of the fluid velocity $\Omega(u)$.
- In hydrodynamics, $\Omega(u)$ can be computed directly from the freeze-out isotherm.

We first fit spectra (combined fit of $\pi$, $K$, $p$) obtained within the blast-wave approximation, as a consistency check. OK.
Blast-wave fit to hydrodynamics

We then fit the full hydrodynamic result
\( p_t \) distributions computed using standard Cooper-Frye

- A blast-wave fit, even generalized, does not give a perfect fit to an actual ideal hydrodynamic calculation
- The fit returns a distribution of fluid velocity \( \Omega(u) \) which is shifted to the right and narrower than the true distribution
Application to LHC data

- The freeze-out temperature $T_f$ is the same for all hadron species.
- It determines relative abundances of hadrons, rather than spectra.
- Preferred temperature for non-strange hadrons is $\sim 135$ MeV.
Application to LHC data

Combined fit to $\pi$, $K$, $p$ spectra from Pb+Pb collisions at 2.76 TeV. Decent fit all the way up to $p_t \sim 5$ GeV/c.
Application to LHC data

Main differences data/fit:
- Yield at high $p_t$ underpredicted
- Pion yield at low $p_t$ underpredicted
- Proton spectrum
Application to LHC data

We see a discrepancy between experiment and ideal hydro.

Difference blast wave / ideal hydro.

We see a discrepancy between experiment and ideal hydro.
Application to LHC data

Simplest interpretation: Viscous correction $\delta f$ at freeze-out.
Implies large $\delta f$ for low-momentum pions, at variance with the usual quadratic ansatz in $p^2$. 
Distribution of fluid velocity from data

Total volume of the fluid per unit rapidity $\int \Omega(u)du$ extracted from experiment $\approx$ same as in a standard hydro calculation for all centralities.

This volume determines the hadron multiplicity.
Distribution of fluid velocity from data

- Fluid velocity distribution from experiment: narrower than expected in hydro.
- Partially explained by the difference between blast wave and hydro.
- Large fluid velocities explain why the fit works up to $p_t \sim 5$ GeV/c.
Distribution of hadron velocities

- Heavy particles follow the fluid: for deuterons, the distribution of $p_t/m$ is close to the distribution of the fluid 4-velocity.
- Therefore, a combined blast-wave fit to identified particle spectra is dominated by the heaviest particles included in the fit.
Blast-wave fits to unidentified spectra

- We have also fitted the charged hadron spectra published by ALICE.
- The fit is now dominated by the pions, which represent $\sim 85\%$ of the hadron yield.
- Good fit all the way to $p_t \sim 5\text{GeV/c}$
- Pion yield at low $p_t$ « explained » by a a fraction of the fluid at rest: $u \sim 0$. 
Centrality dependence of the fluid velocity distribution

- We evaluate the mean $\langle u \rangle$ and the standard deviation $\sigma(u)$ of the fluid velocity distribution $\Omega(u)$ from LHC data, as a function of the collision centrality.
- We calculate $\langle u \rangle$ and $\sigma(u)$ in event-by-event ideal hydro.
Centrality dependence of the fluid velocity distribution

- $\Omega(u)$ becomes broader for more peripheral collisions: $\sigma(u)/<u>$ increases.
- A similar increase is found in our event-by-event hydro calculation.
- Event-by-event fluctuations naturally explain the observed centrality dependence of $p_t$ spectra.
Summary

• We have generalized the blast-wave fit in a way that follows as closely as possible an actual hydrodynamic calculation: arbitrary distribution of fluid velocity $\Omega(u)$, resonance decays included.
• Still, a blast-wave fit is not equivalent to a hydrodynamic calculation due to the time-like part of the freeze-out isotherm.
• We obtain good fits of ALICE data all the way up to $p_t \sim 5\text{GeV/c}$.
• The pion excess at low $p_t$ compared to hydro is generic.
• The mild centrality dependence of $p_t$ spectra is naturally explained in hydrodynamics. Its broadening is due to event-by-event fluctuations.
Supplementary material
Cooper-Frye vs semi-Cooper-Frye

\[
\frac{dN}{d^3p} = \frac{2S + 1}{(2\pi)^3} \int_{\sigma} \frac{1}{e^{E^*/T_f} \pm 1} \frac{p^\mu}{p^0} d\sigma_\mu
\]

\[
\frac{dN}{d^3p} = \frac{2S + 1}{(2\pi)^3} \int_{\sigma} \frac{1}{e^{E^*/T_f} \pm 1} \frac{u^\mu}{u^0} d\sigma_\mu
\]

\[
= \frac{2S + 1}{(2\pi)^3} \int_{\sigma} \frac{1}{e^{E^*/T_f} \pm 1} \Omega(u) du
\]

where

\[
\Omega(u) du = \int_{\sigma, u \text{ in } du} \frac{u^\mu}{u^0} d\sigma_\mu
\]

defines the distribution of fluid velocity in hydro.
Identified versus charged spectra

By summing the identified spectra of $\pi$, $K$, $p$, $\Sigma$, one recovers the unidentified charged spectra.