Fluid velocity from transverse momentum spectra

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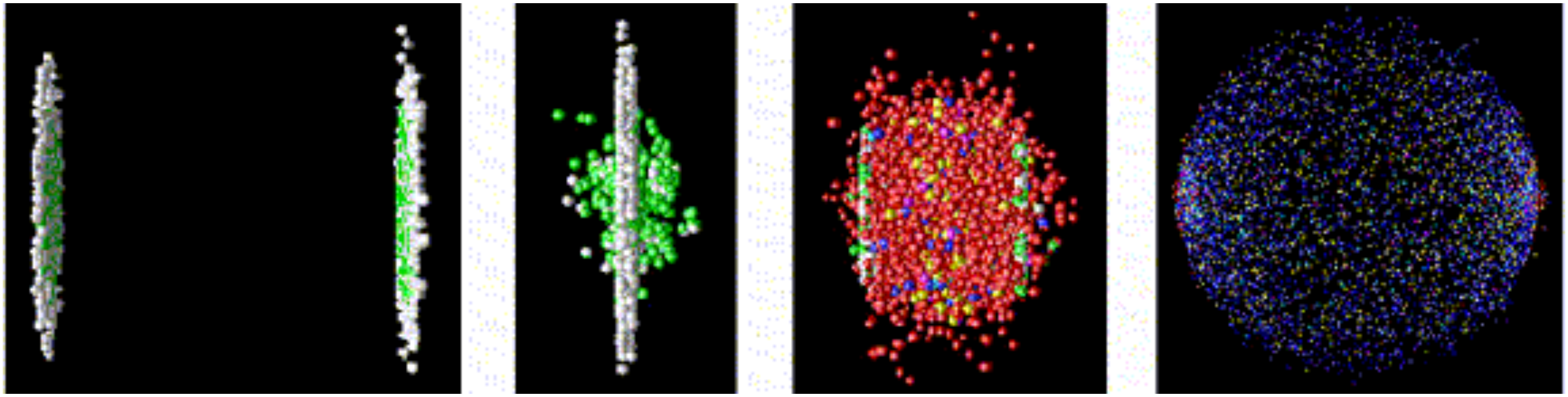
Based on 2012.07898, in collaboration with Anthony Guillen

Pb-Pb collisions at the LHC

I will discuss the interpretation of selected 2011 LHC data on identified hadron production in Pb+Pb collisions at 2.76 TeV.

Sketch of a Pb+Pb collision at LHC

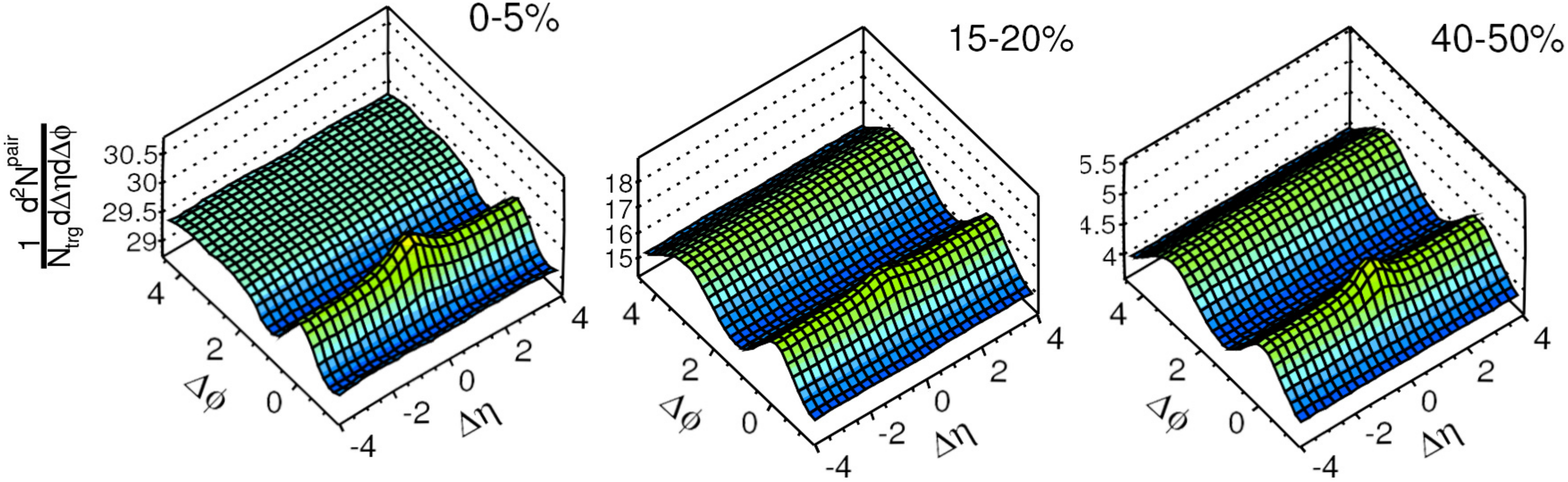


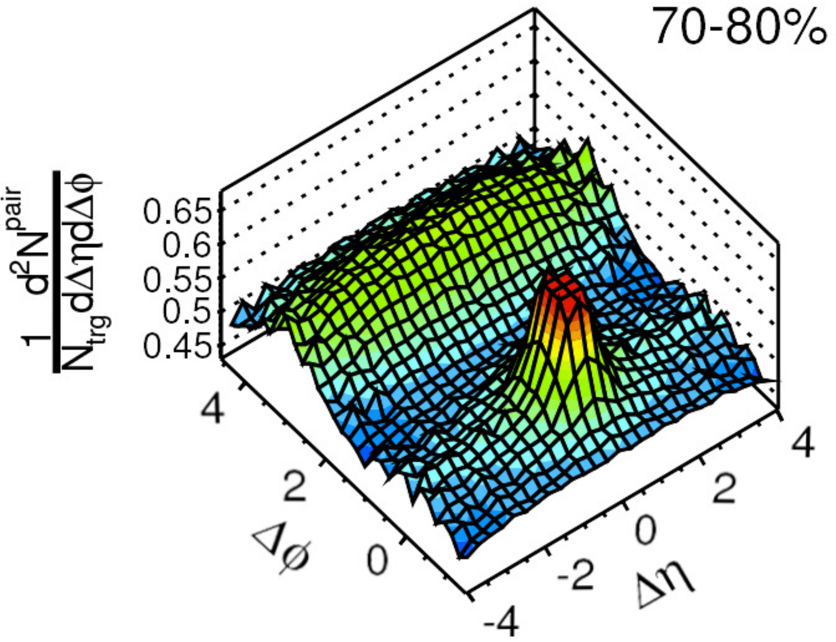
- Relativistic contraction of length by factor 5000: colliding thin pancakes
- The collision creates strongly-coupled quark-gluon matter, governed by strong interactions, which expands into the vacuum. ~30000 particles produced at the end.
- The best theoretical description is a macroscopic one: a small lump of fluid.

Outline

- How we see collective flow in p_t spectra
- What can we learn about the fluid just by analyzing these spectra, without any detailed hydrodynamic modeling?
- Generalization of the traditional blast-wave approach.
- Generic differences between blast wave and hydrodynamics
- Generalized blast-wave fits to LHC data
- Centrality dependence of p_t spectra

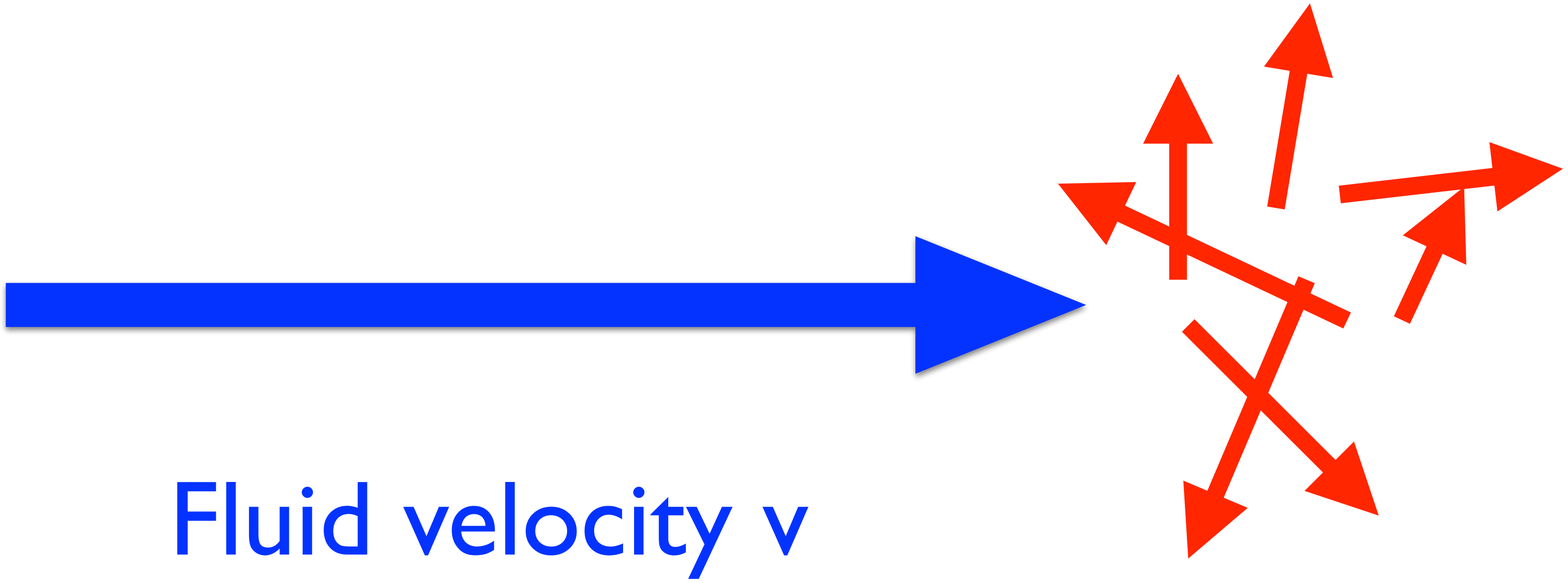
Evidence for collective motion: the *ridge*





In the most ~50% central collisions, pair correlations display a regular wave pattern, which is broken in more peripheral collisions

Collective motion



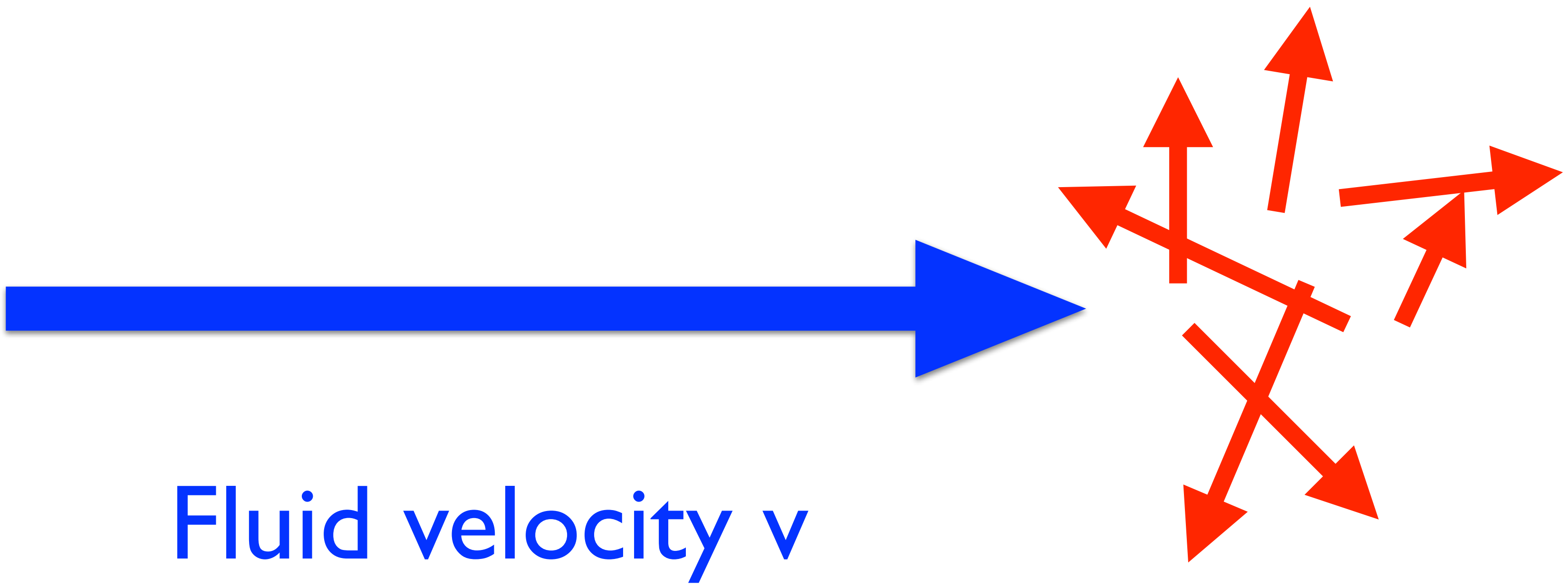
Fluid velocity v

Thermal motion

The velocity of a particle embedded in a fluid is the sum* of the fluid velocity v, which is the same for all particles around a given point, and a random thermal velocity of magnitude $\sim\sqrt{T/m}$.

**up to relativistic details*

Collective motion



Fluid velocity v

Thermal motion

The energy of a particle in the fluid can be decomposed as

$$E = \mathbf{m}/\sqrt{1-v^2} \quad + \quad O(T)$$

The collective motion has a larger effect on **heavy** particles.

Collective motion seen in m_t spectra

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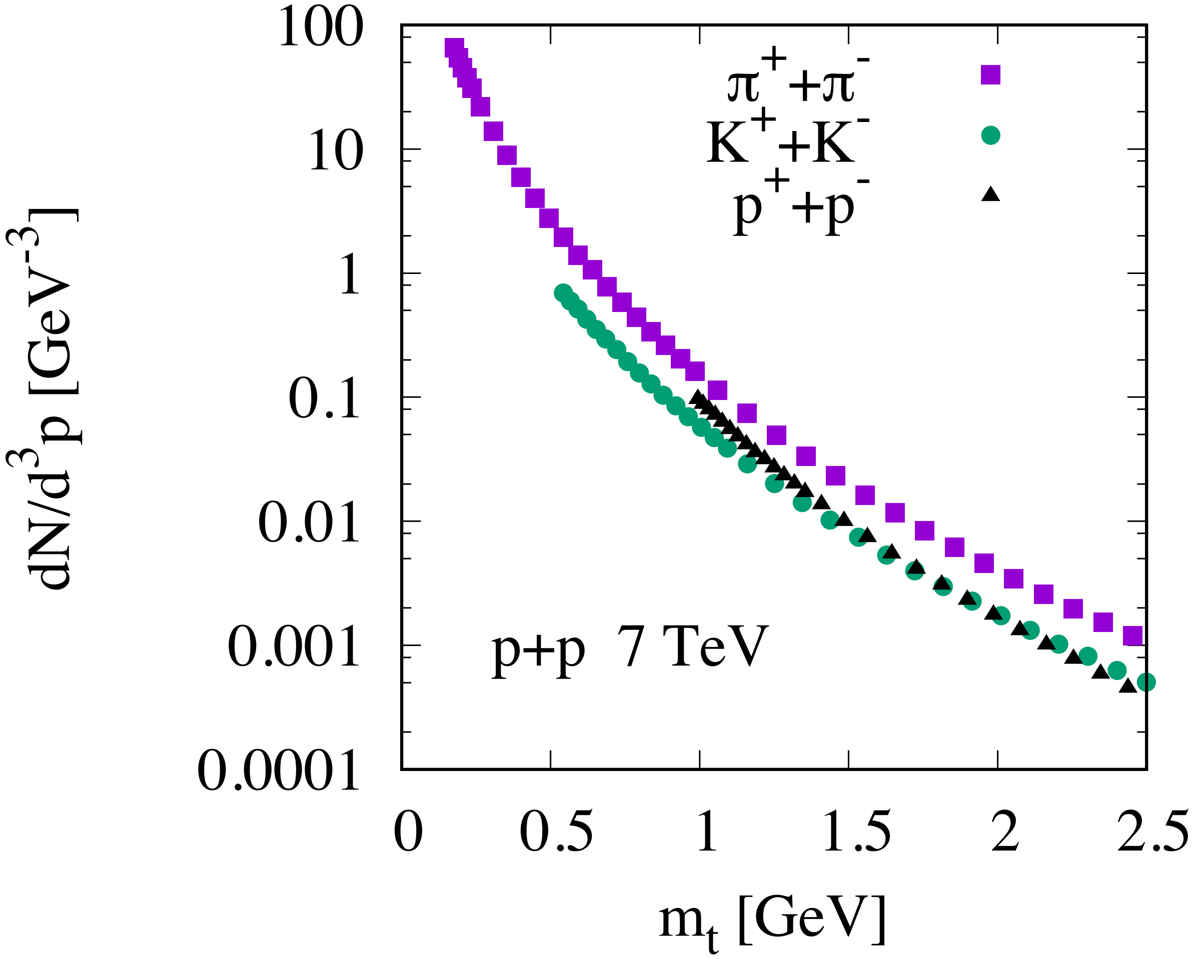

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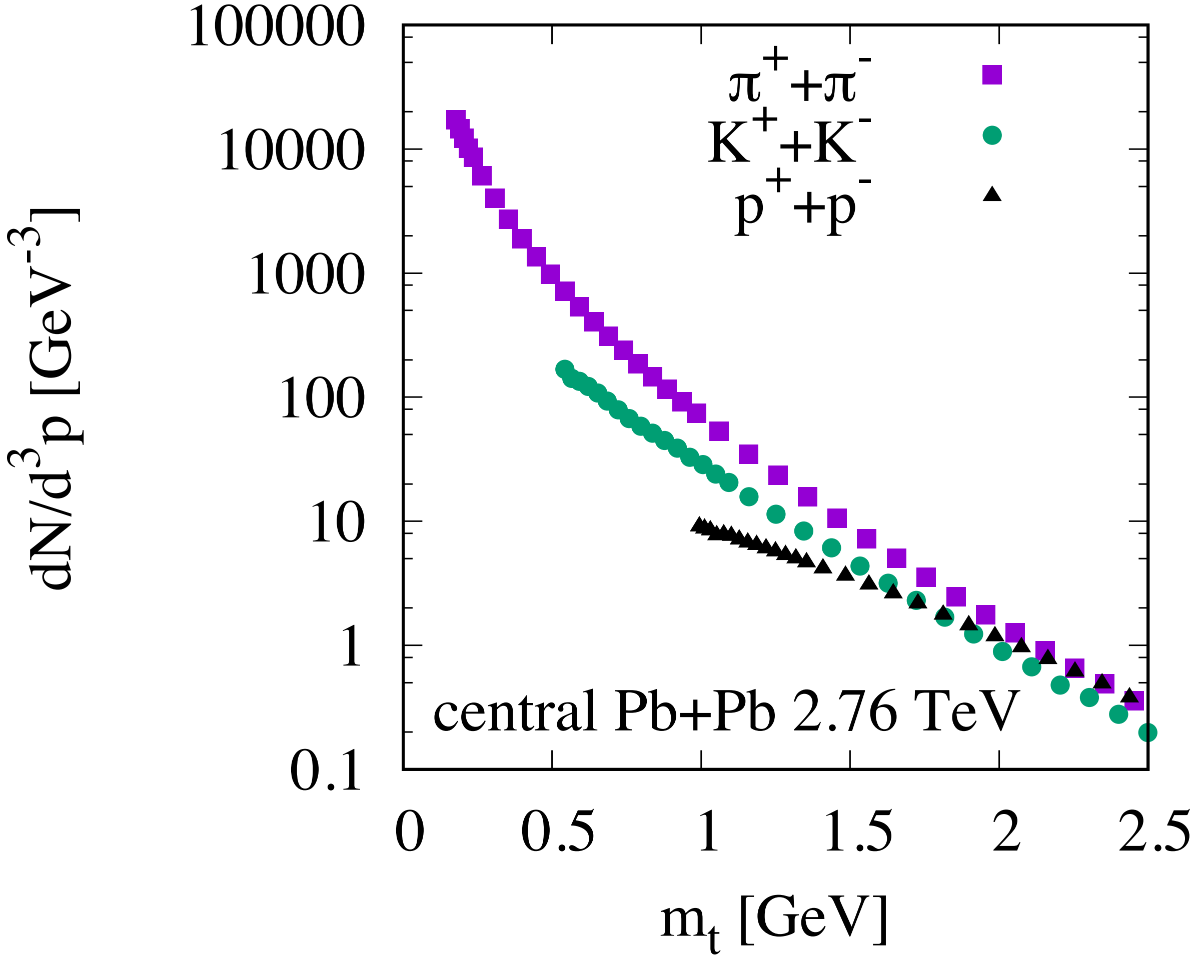

In proton-proton collisions, slopes are comparable for π, K, p
In Pb+Pb collisions, spectra are flatter for heavier particles.
Evidence for radial collective flow.

Collective motion seen in p_t spectra

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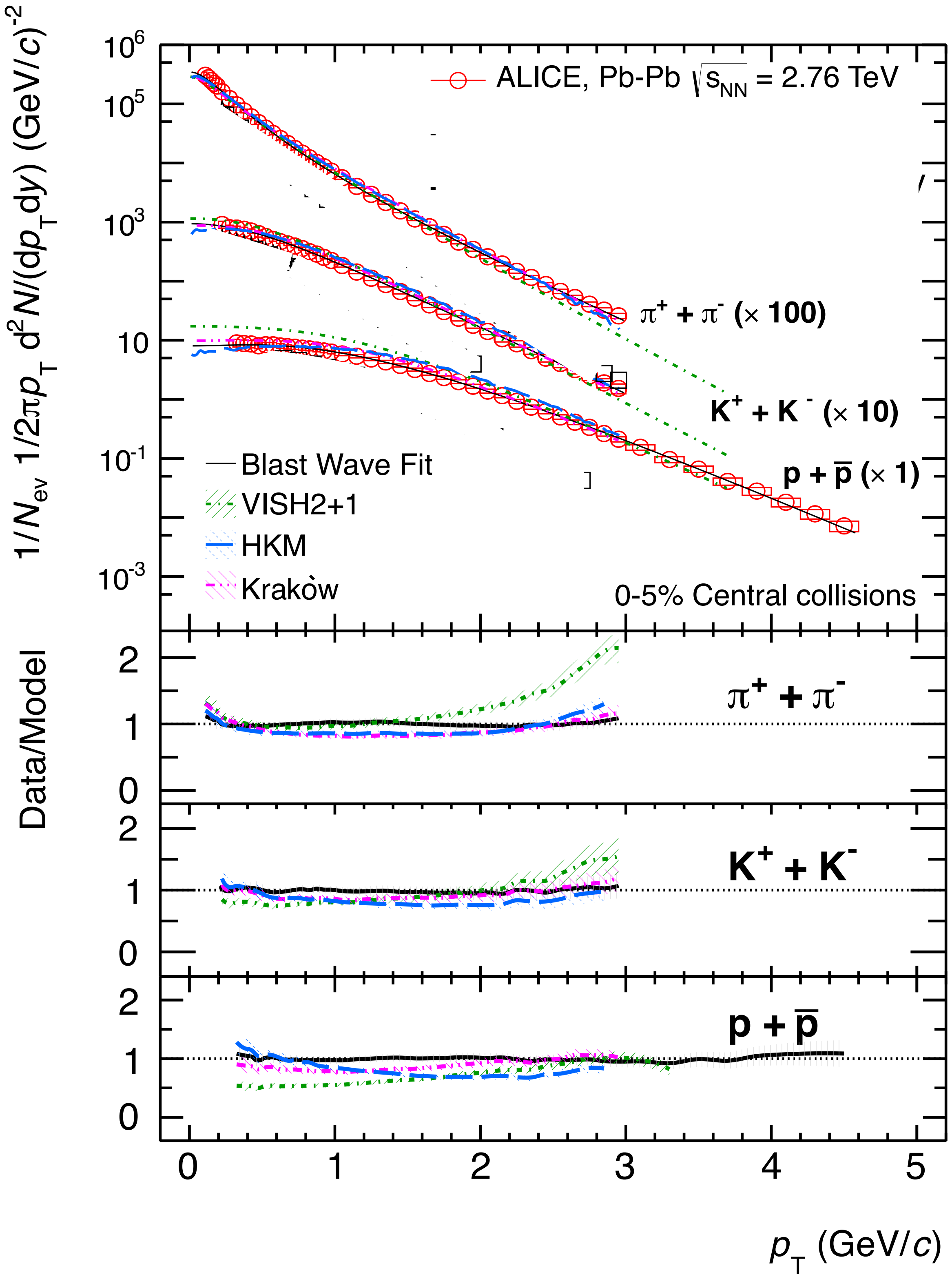


Various hydrodynamic models (VISH2+1, HKM, Krakow) were able to predict the p_t spectra reasonably well: they naturally capture the mass ordering.

Blast Wave fits, where the parameters are typically a fluid velocity and a temperature, describe the spectra very well.

Our goal

- What can we learn about the fluid directly from experimental data, without running a specific hydrodynamic simulation (whose results depend on initial conditions, equation of state, transport coefficients, treatment of hadronic phase)?
- Idea: Generalized blast wave fit to data, in a way that follows as closely as possible an actual hydrodynamic calculation.
- *Three differences* with the traditional blast-wave fit.

Generalized blast wave fit (1/3)

Write the momentum distribution as an *arbitrary* superposition of boosted thermal distributions

$$\frac{dN}{d^3p} = \frac{2S+1}{(2\pi)^3} \int \frac{1}{e^{E^*/T_f} \pm 1} \Omega(\mathbf{u}) d\mathbf{u},$$

$E^* = p^\mu u_\mu$ = energy of particle in fluid frame
T_f = temperature

Volume of fluid with velocity **u** up to d**u**

Generalized blast wave fit (2/3)

Integrate over rapidity and azimuthal angle to obtain the transverse momentum distribution

$$\frac{dN}{dp_t} = \int_0^{\infty} f(p_t, u)\Omega(u)du$$

where $$f(p_t, u) \equiv \frac{2S+1}{(2\pi)^3} p_t \int_{-\infty}^{+\infty} dp_z \int_{-\pi}^{\pi} d\phi \frac{1}{e^{E^*/T_f} \pm 1}$$

u = radial component of 4-velocity = $v/\sqrt{1-v^2}$

In this talk, I call u the « fluid velocity », but it can be >1.

$\Omega(u)du$ = volume of fluid in fm^3 = *same for all particle species*

T_f = freeze-out temperature = *same for all particle species*

Generalized blast wave fit (3/3)

All hadron resonances are produced at temperature T_f following to the boosted thermal distribution. Resonances decay to stable hadrons which are measured.

We take into account the *feed-down from resonance decays* using the FastReso code of Mazeliauskas et al. 1809.11049 (see also 1907.11059). Amounts to replacing

$$\frac{1}{e^{E^*/T_f} \pm 1} \to f_1(E^*) + (f_2(E^*) - f_1(E^*)) \frac{E^* u^0}{p^0}$$

where $f_1(E^*)$ and $f_2(E^*)$ are functions which are computed by FastReso for each stable hadron.

Viscous or ideal hydro?

State-of-the-art hydrodynamic calculations include viscosity, which implies that the fluid is locally out of equilibrium.

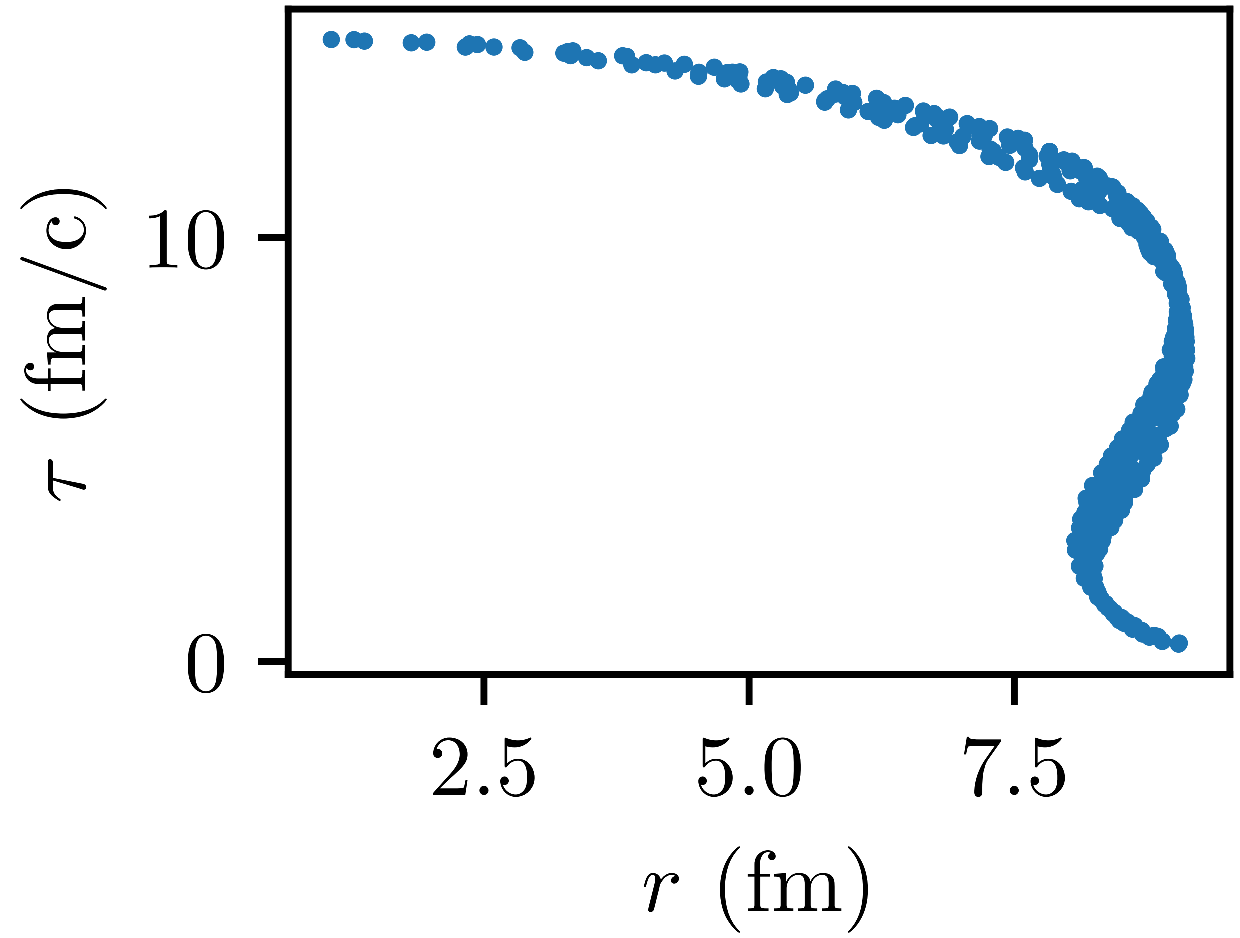
The resulting modifications of the equations of hydro are robust: Navier-Stokes+2nd order terms.

How the off-equilibrium correction is shared among the hadrons, and how it depends on momentum, is not known. It depends on the microscopic interactions at freeze-out (Dusling et al. 0909.0754).

Viscosity has a large effect on anisotropic flow, but a smaller effect on the spectra. We neglect it.

Blast-wave versus hydro

In a hydrodynamic calculation, one evaluates momentum distributions of outgoing particles by integrating over a freeze-out isotherm which is a curve in space-time.

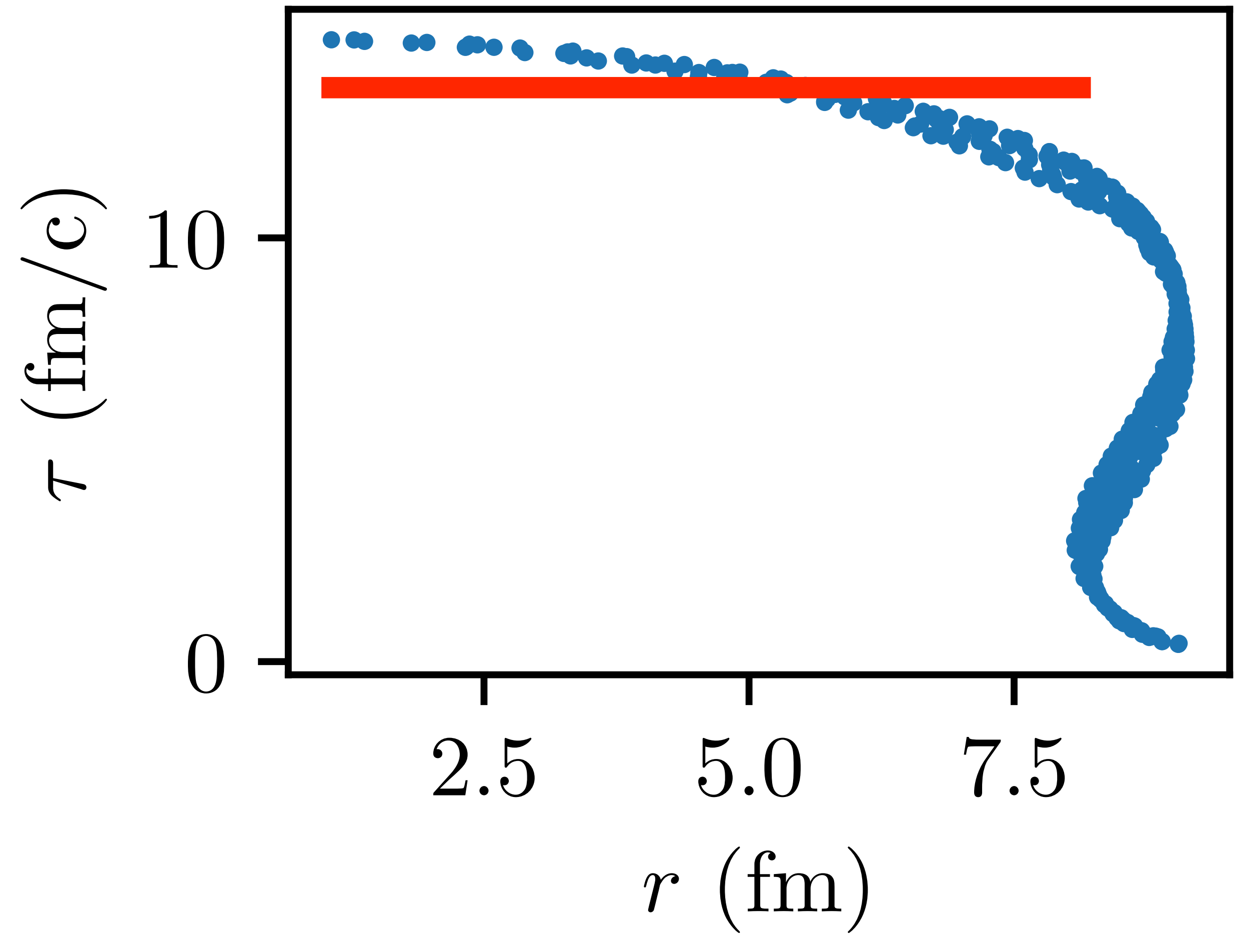


Blast-wave versus hydro

Space-like part of the isotherm.

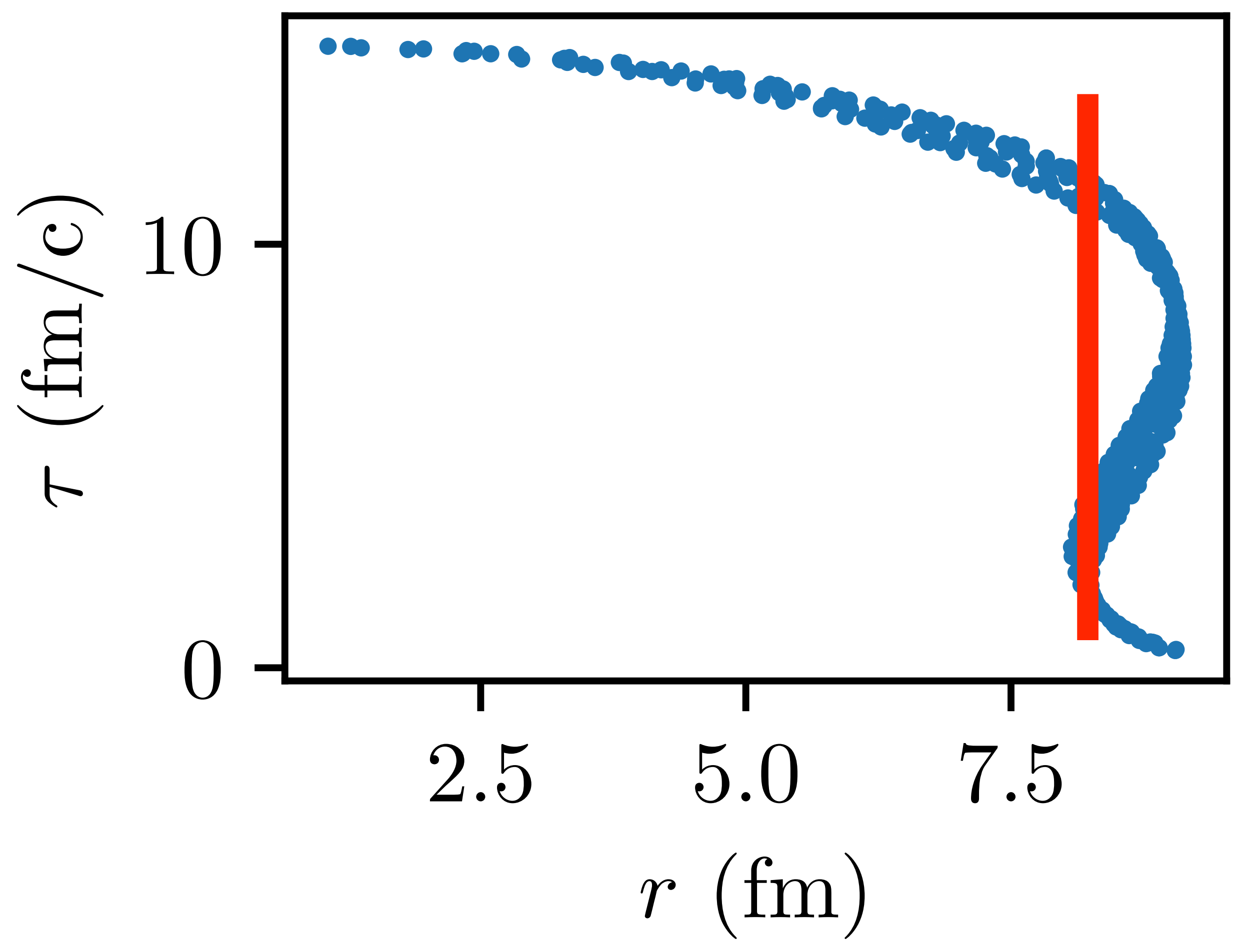
Evaluate the hadron content of the fluid at a given time =

At each point, a boosted thermal distributions = blast wave.



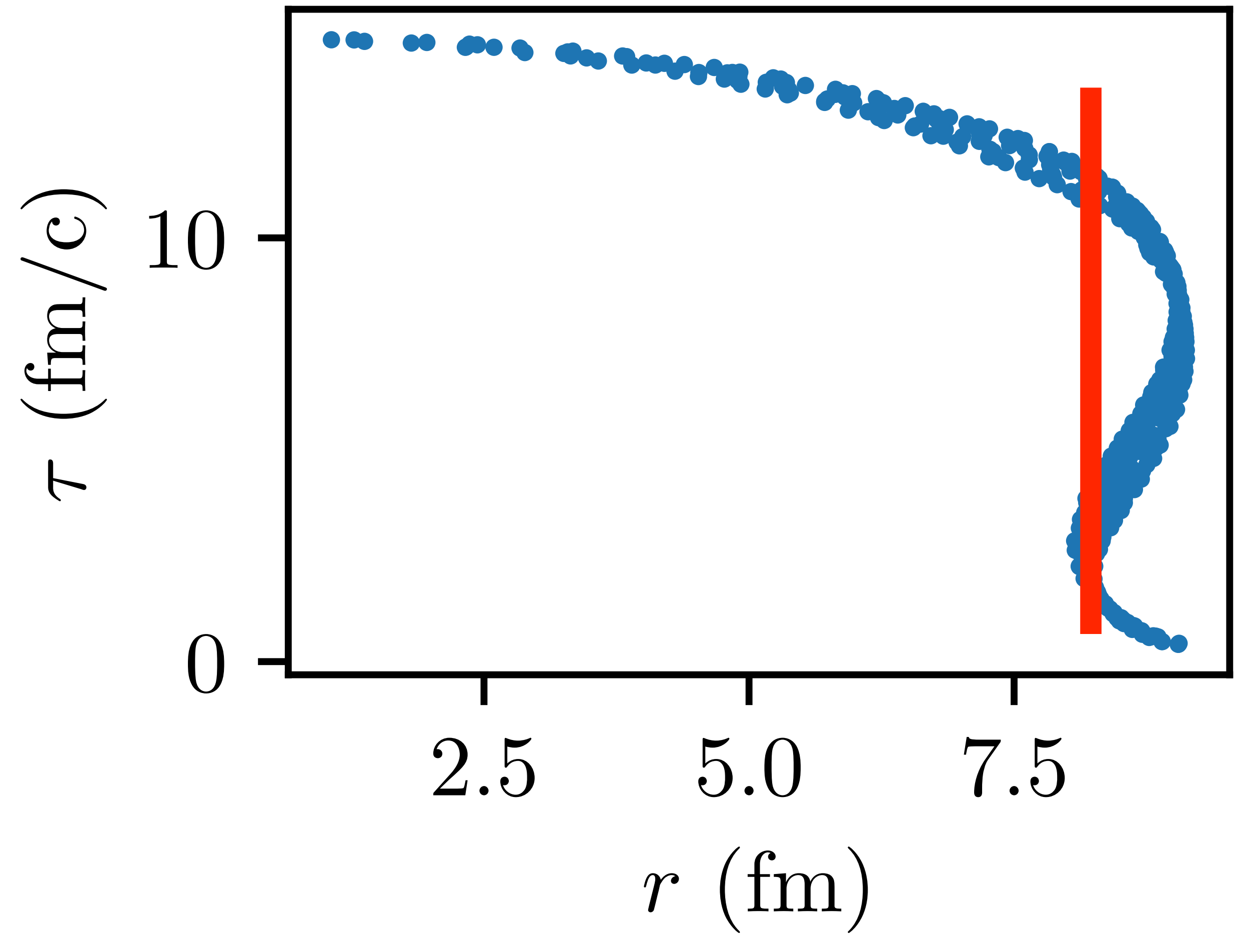
Blast-wave versus hydro

Time-like part of the isotherm. The particle flux through a fixed surface is proportional to the particle velocity.
This contribution is not just a boosted thermal distribution.

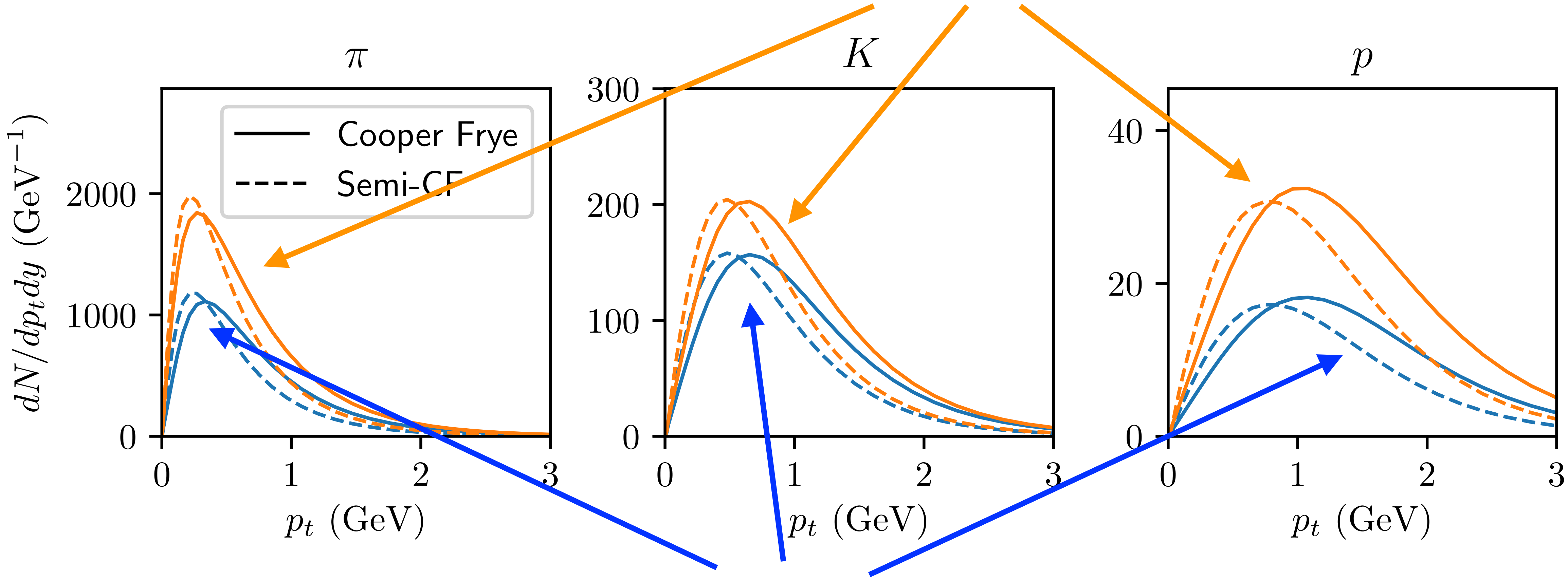


Blast-wave versus hydro

The blast-wave can be seen as an approximation where
particle velocity ≈ fluid velocity
We call this approximation semi-Cooper-Frye.



Test of semi-Cooper-Frye

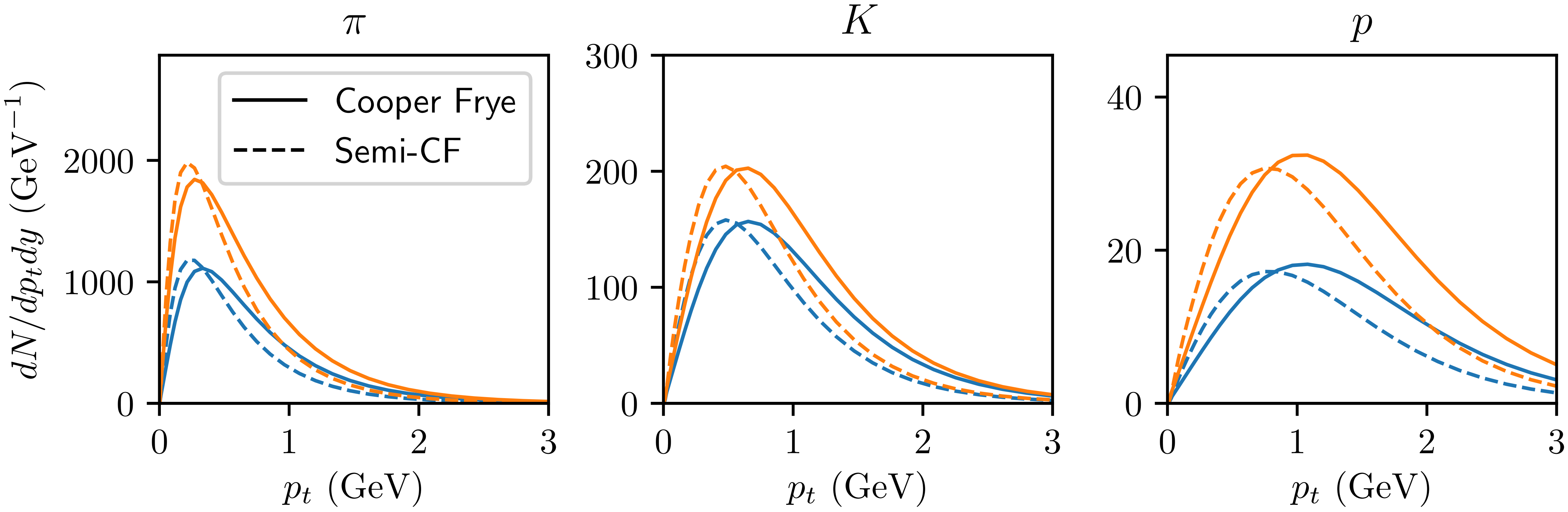


After feed-down from decays

Direct production (before decays)

Spectra from an ideal hydrodynamic simulation run with Music of one random central Pb+Pb collision at 2.76 TeV.
Initial conditions from TRENTo

Test of semi-Cooper-Frye



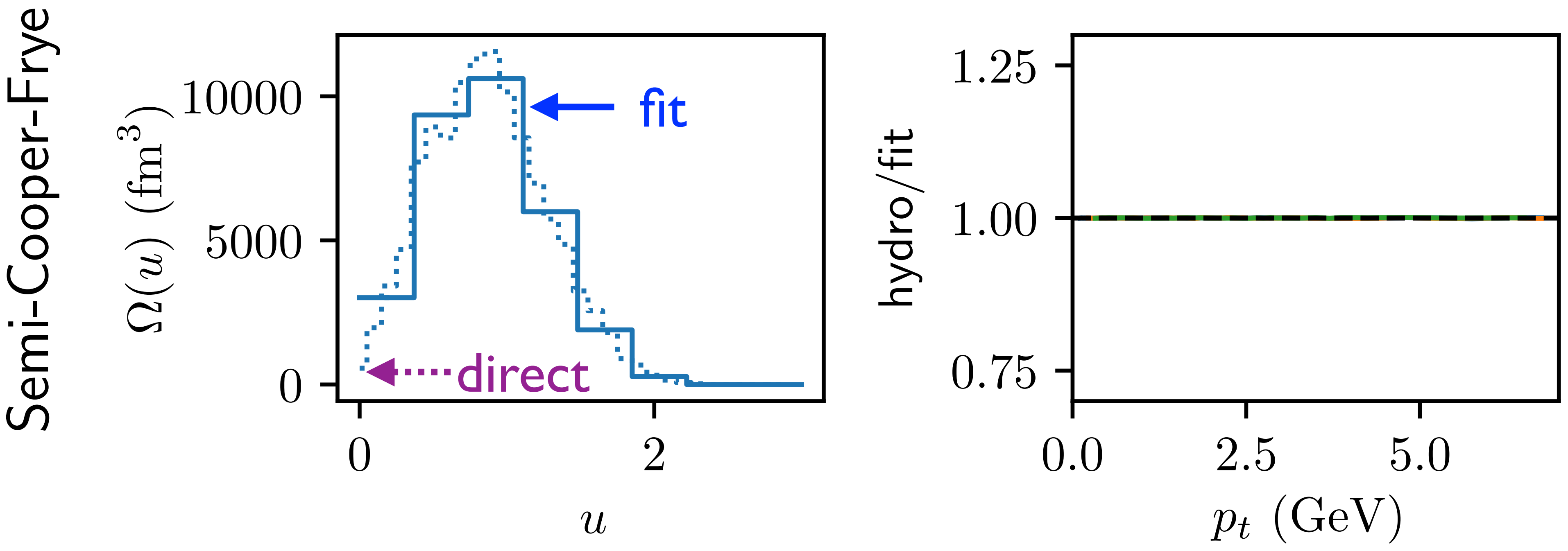
Approximation particle velocity ≈ fluid velocity

Overestimates yield at low p_t

Underestimates yield at high p_t

Test of the fitting algorithm

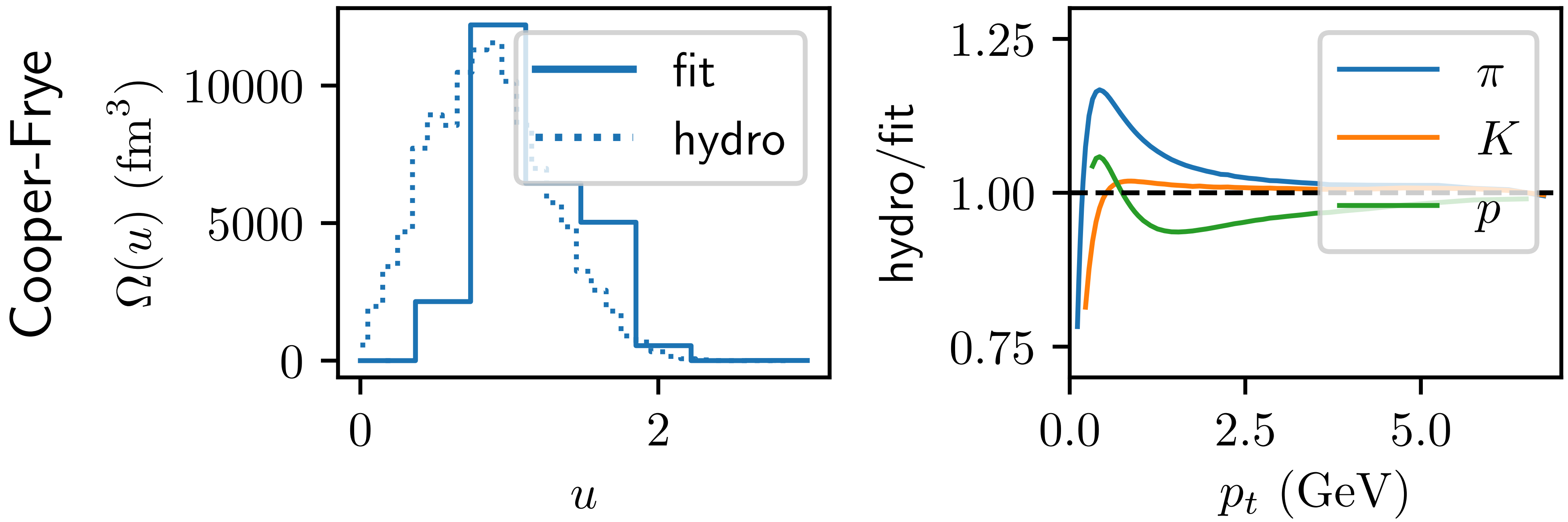
- The generalized blast-wave fit to the spectra returns the distribution of the fluid velocity Ω(u).
- In hydrodynamics, Ω(u) can be computed directly from the freeze-out isotherm.



We first fit spectra (combined fit of π, K, p) obtained within the blast-wave approximation, as a consistency check. OK.

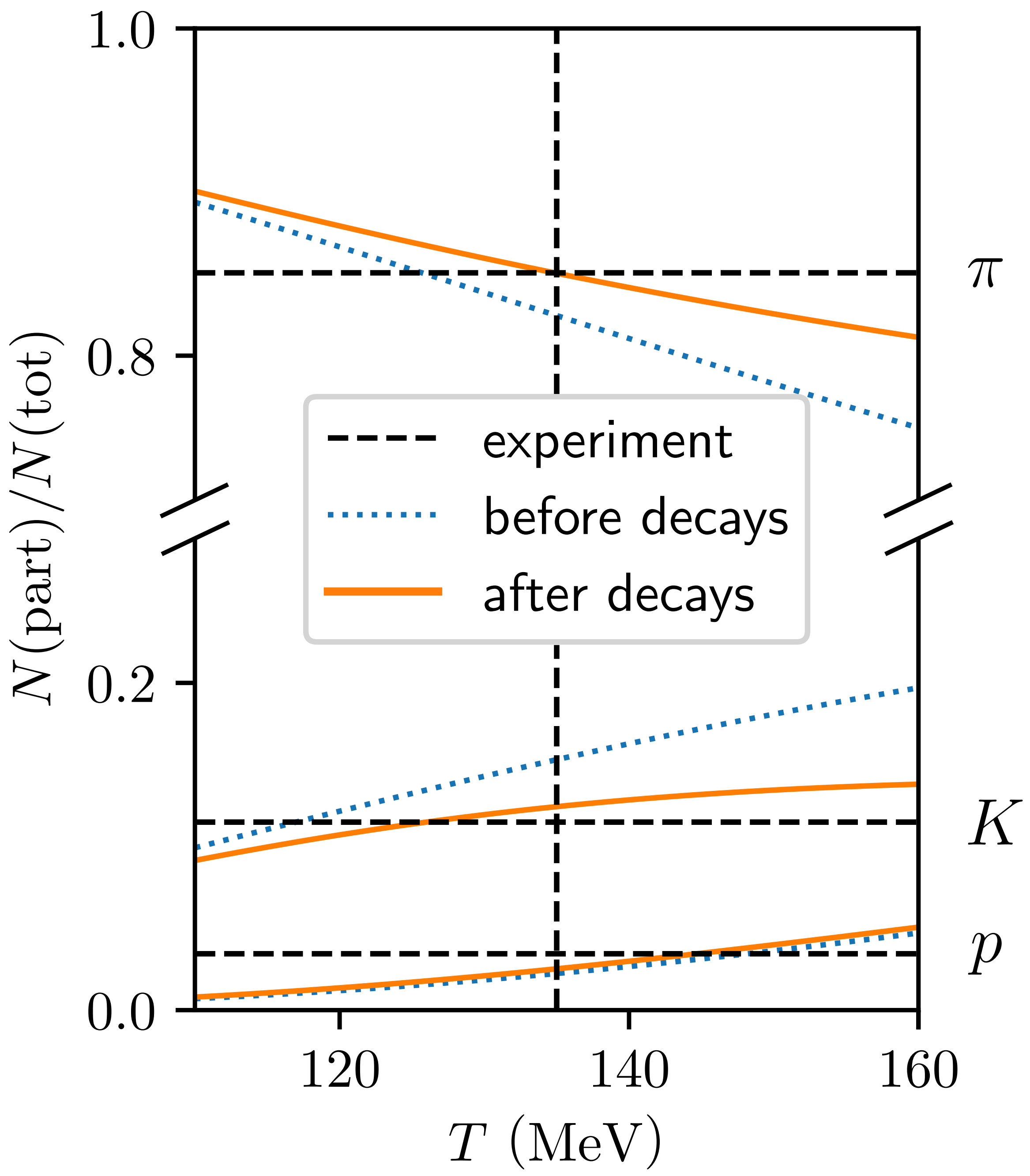
Blast-wave fit to hydrodynamics

We then fit the full hydrodynamic result
(p_t distributions computed using standard Cooper-Frye)



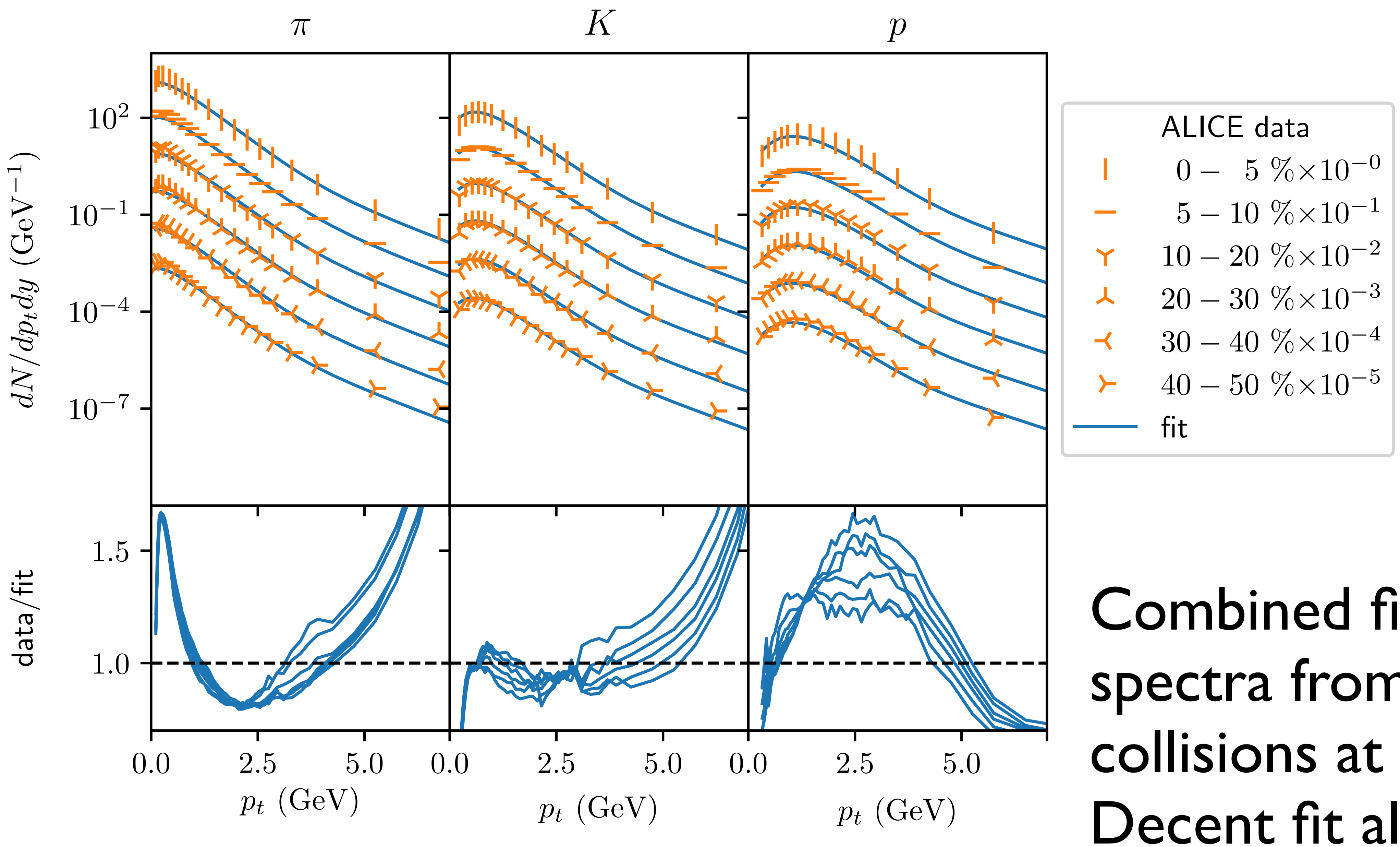
- A blast-wave fit, even generalized, does not give a perfect fit to an actual ideal hydrodynamic calculation
- The fit returns a distribution of fluid velocity $\Omega(u)$ which is shifted to the right and narrower than the true distribution

Application to LHC data



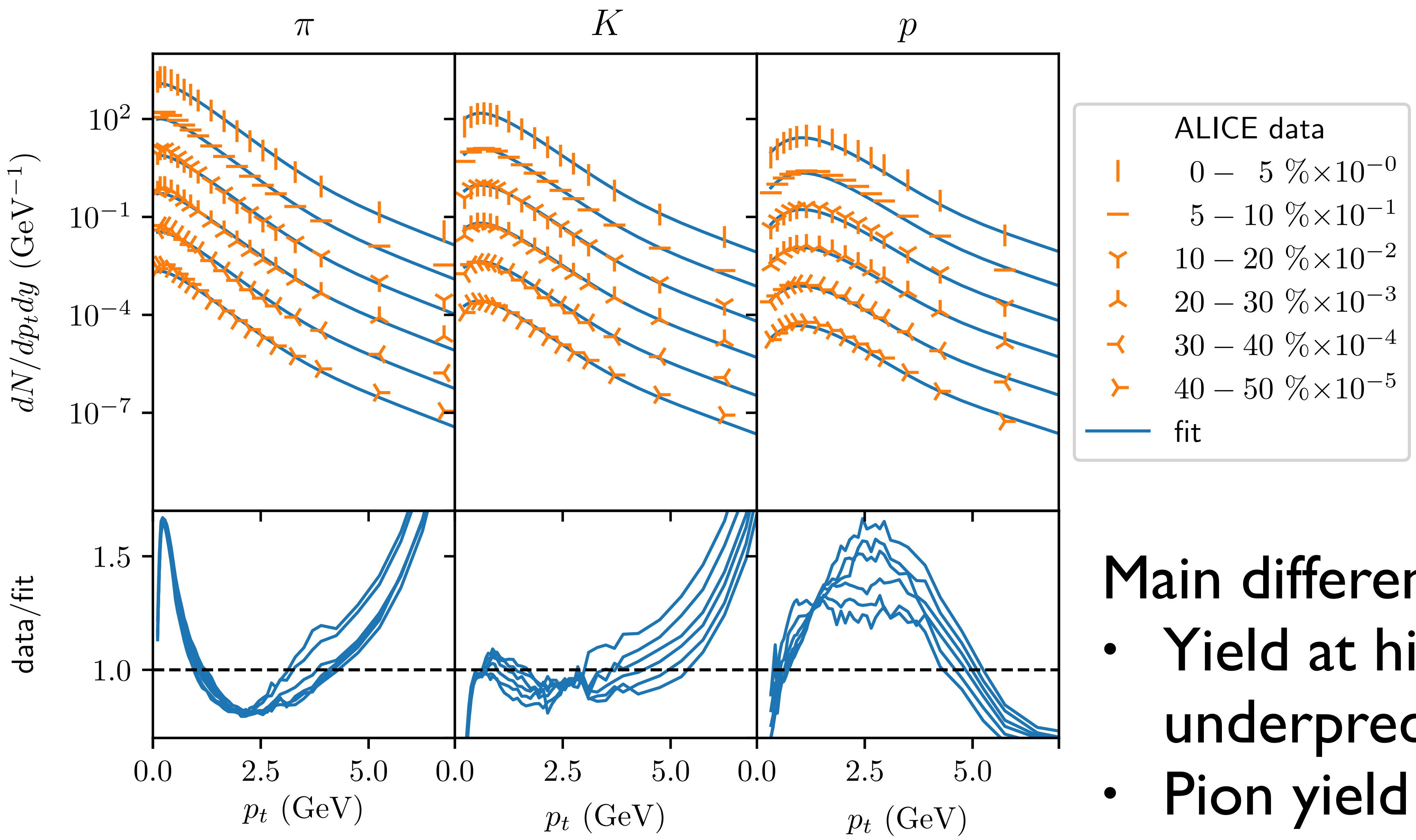
- The freeze-out temperature T_f is the same for all hadron species.
- It determines relative abundances of hadrons, rather than spectra.
- Preferred temperature for non-strange hadrons is ~135 MeV.

Application to LHC data



Combined fit to π, K, p spectra from Pb+Pb collisions at 2.76 TeV. Decent fit all the way up to p_t~5 GeV/c.

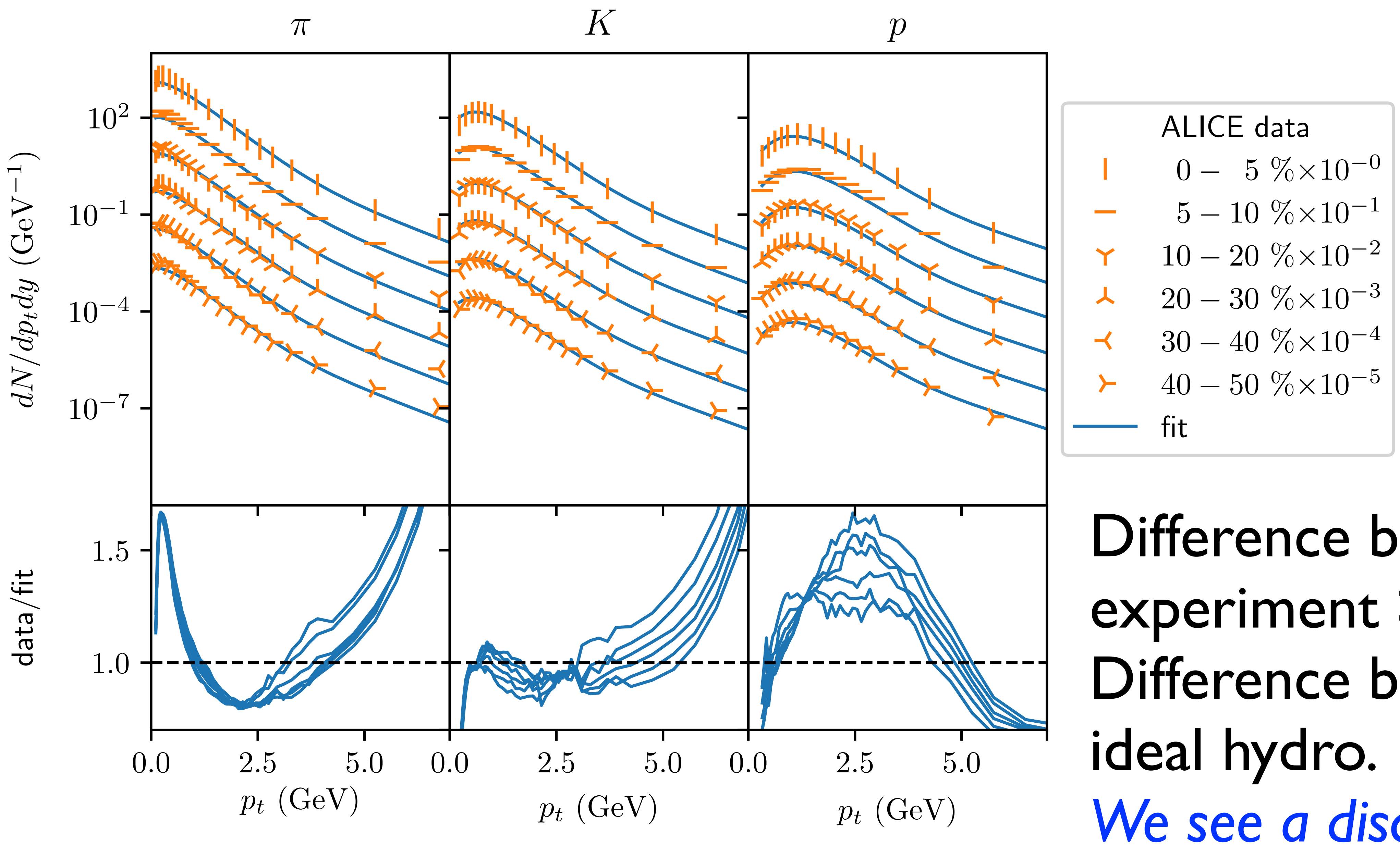
Application to LHC data



Main differences data/fit:

- Yield at high p_t underpredicted
- Pion yield at low p_t underpredicted
- Proton spectrum

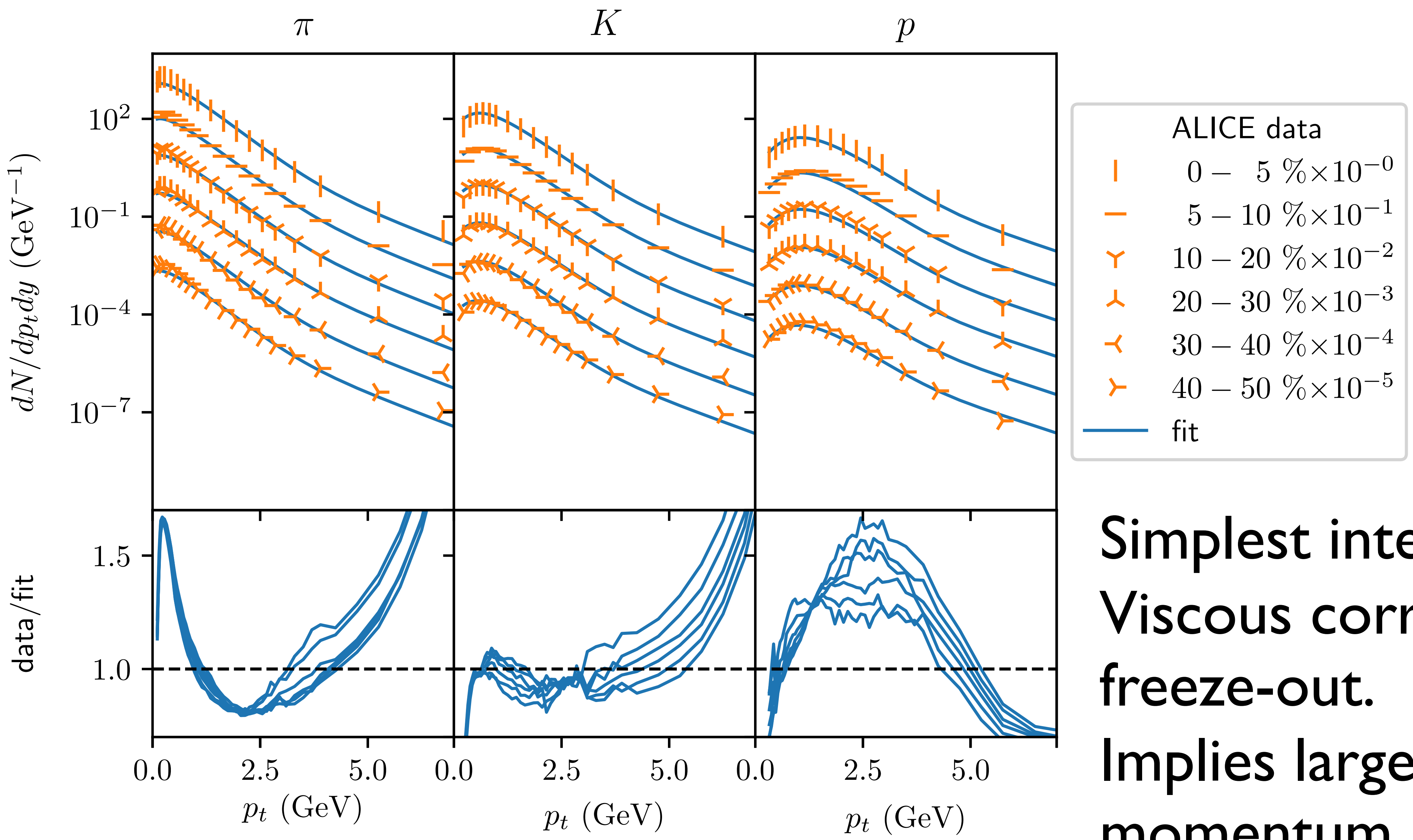
Application to LHC data



Difference blast wave / experiment > Difference blast wave / ideal hydro.

We see a discrepancy between experiment and ideal hydro.

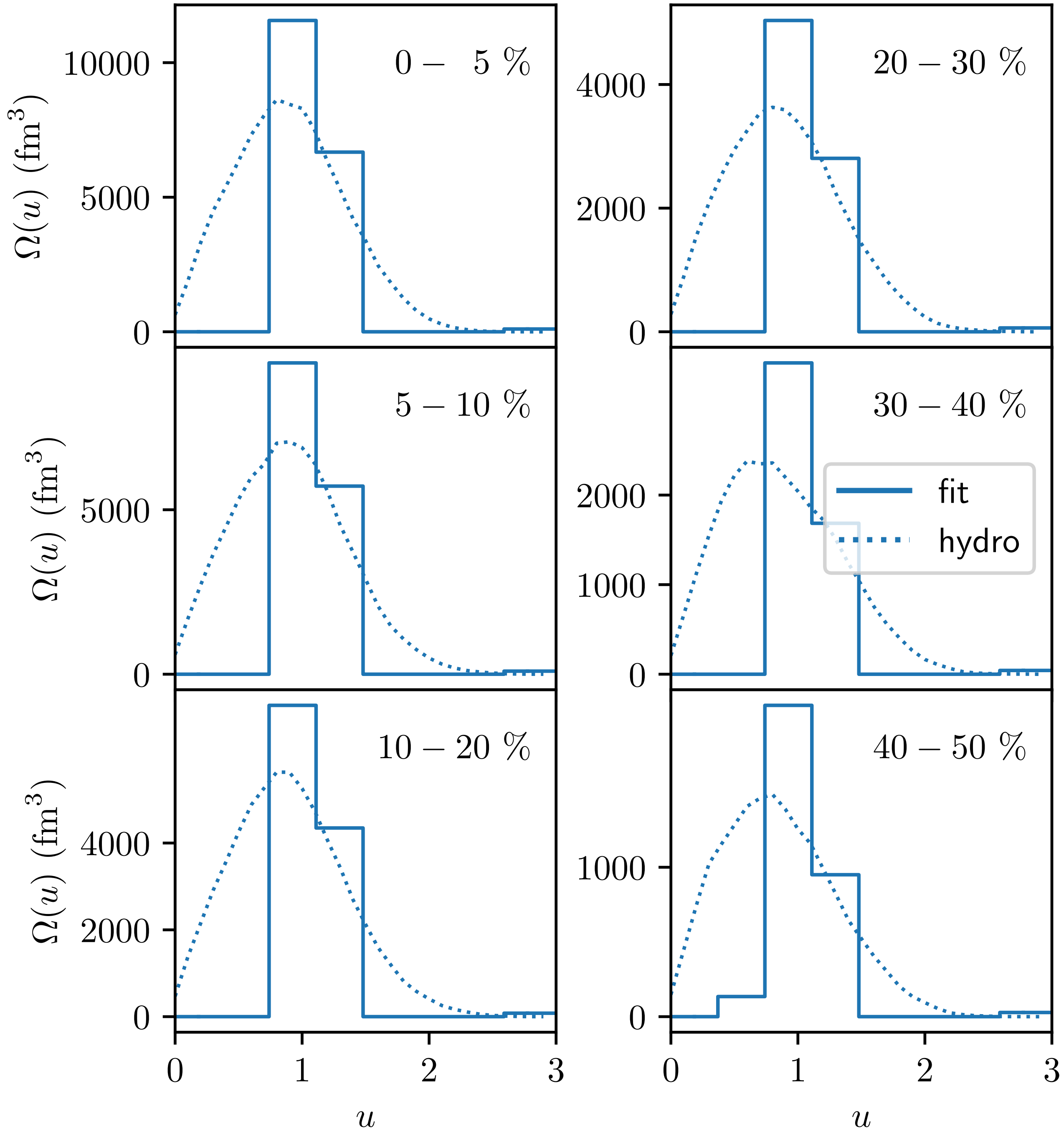
Application to LHC data



Simplest interpretation: Viscous correction δf at freeze-out.

Implies large δf for low-momentum pions, at variance with the usual quadratic ansatz in p^2

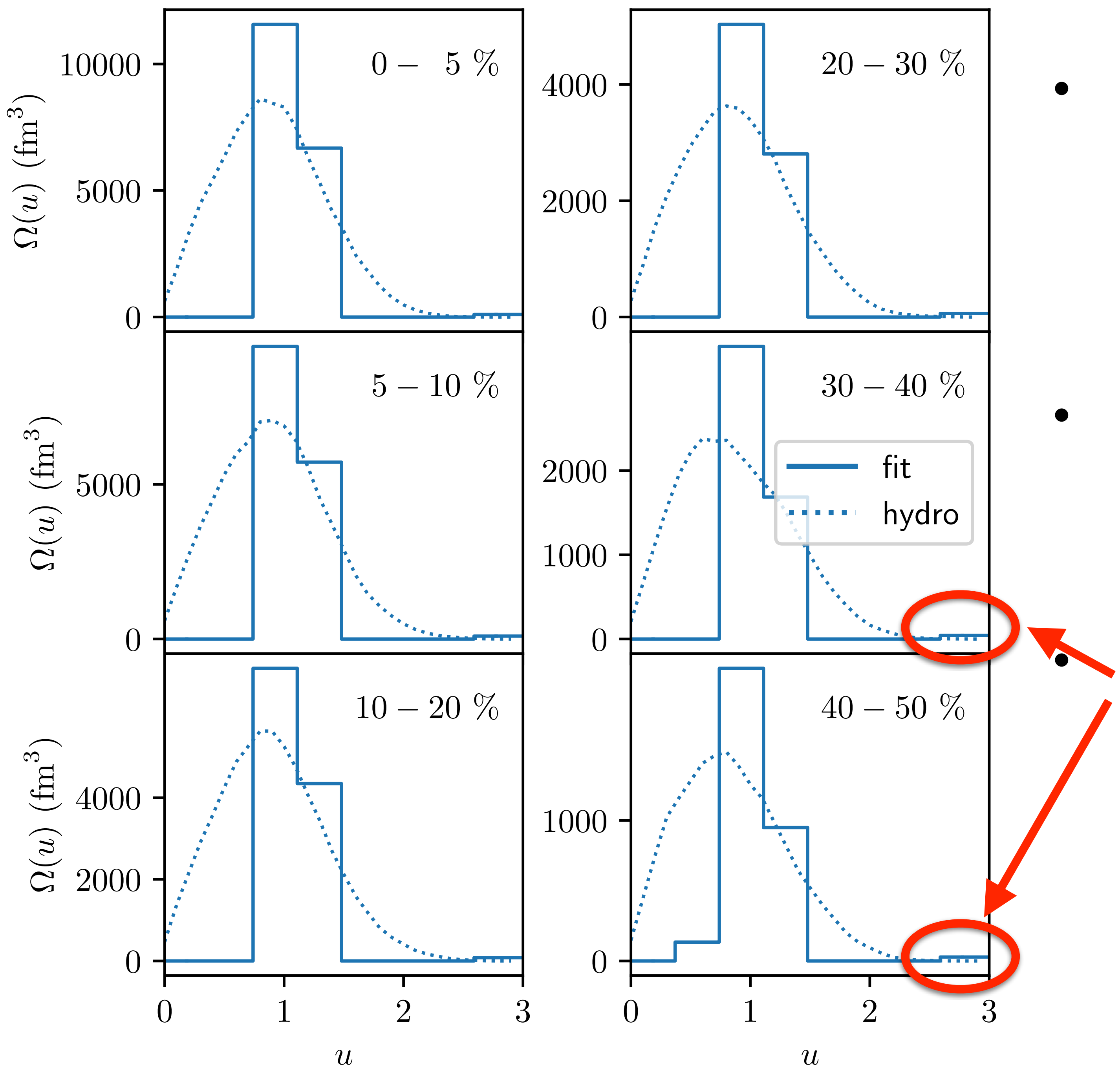
Distribution of fluid velocity from data



Total volume of the fluid per unit rapidity $\int \Omega(u)du$ extracted from experiment ≈ same as in a standard hydro calculation for all centralities.

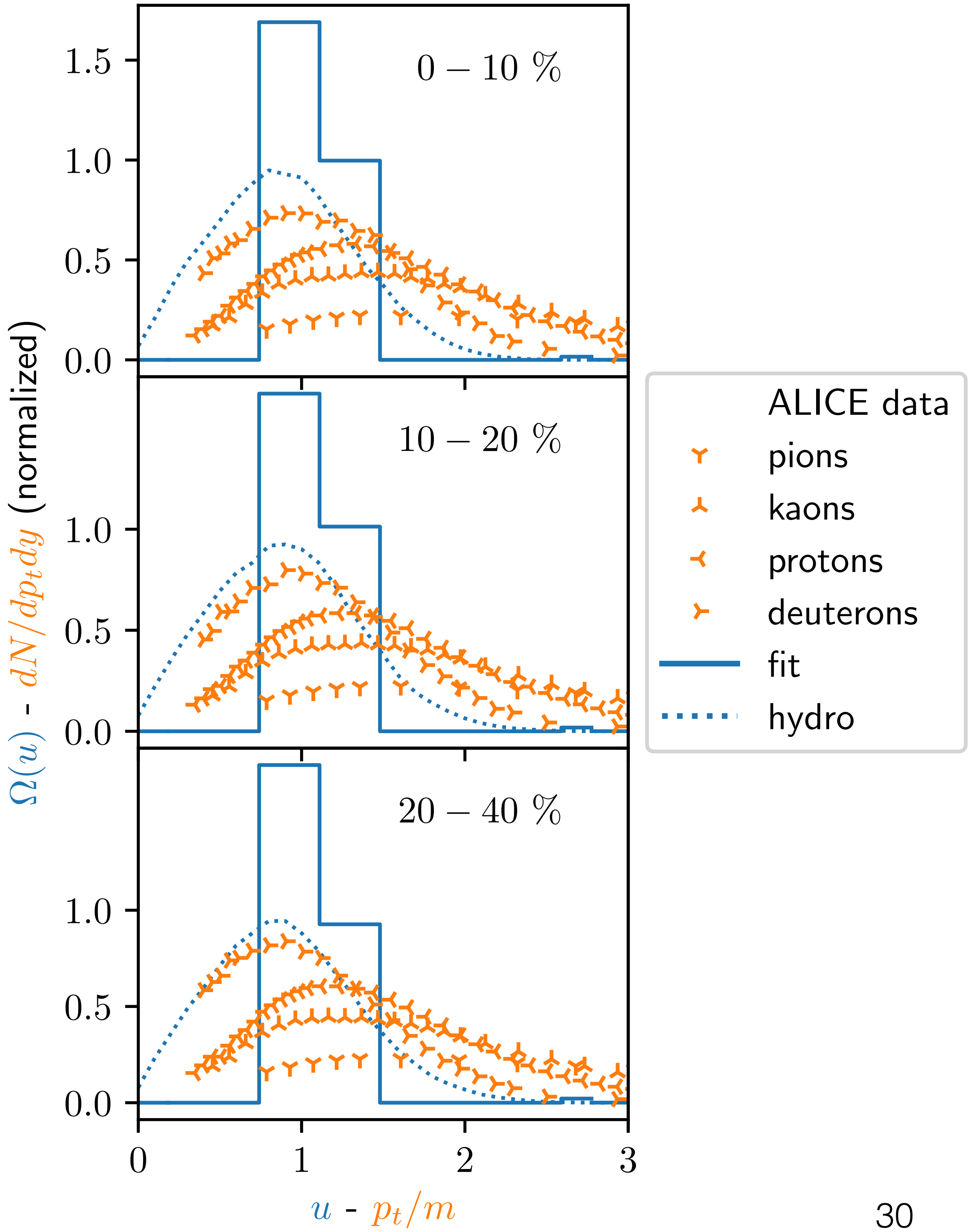
This volume determines the hadron multiplicity.

Distribution of fluid velocity from data



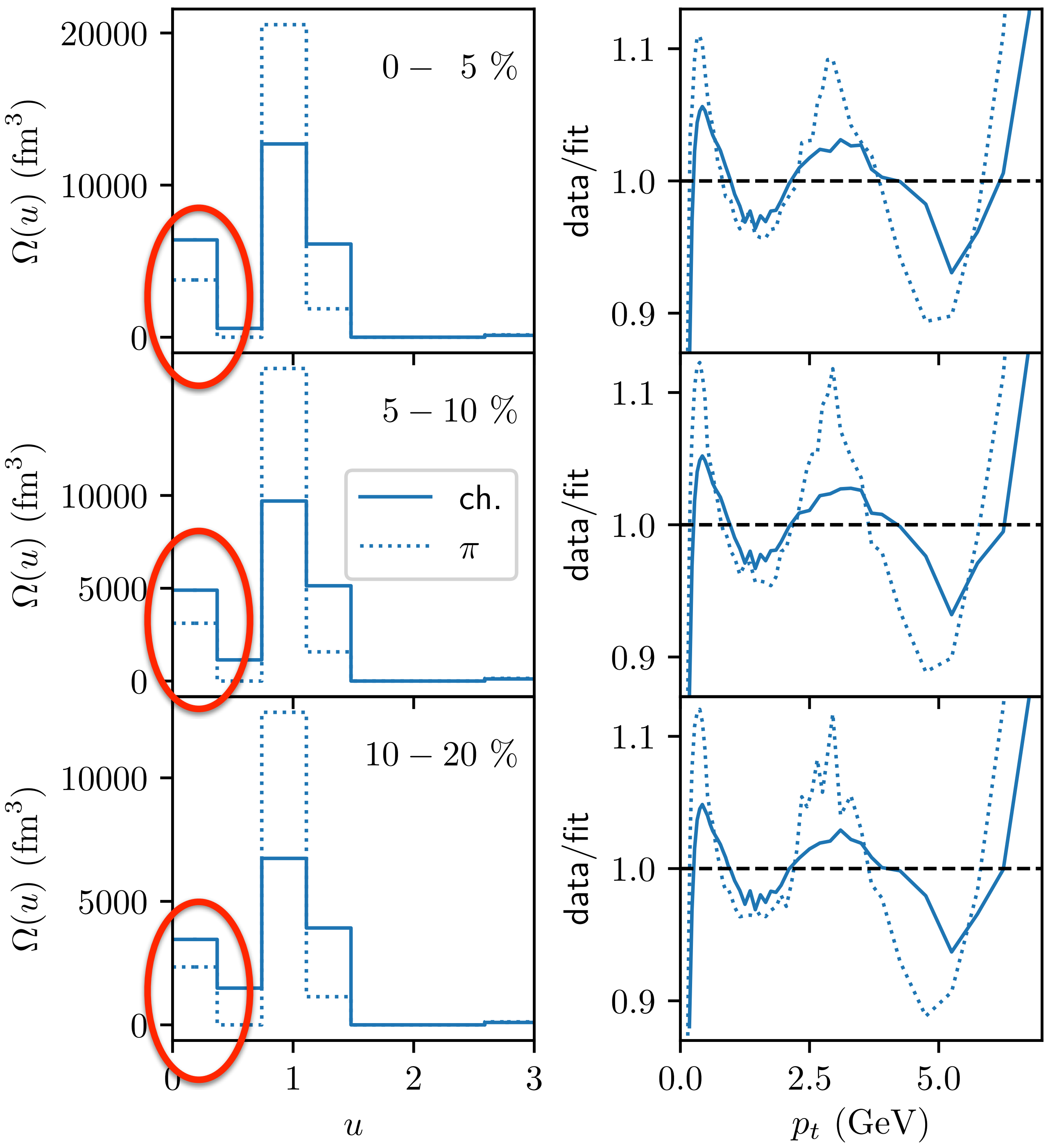
- Fluid velocity distribution from experiment: narrower than expected in hydro.
- Partially explained by the difference between blast wave and hydro.
- Large fluid velocities explain why the fit works up to p_t~5 GeV/c.

Distribution of hadron velocities



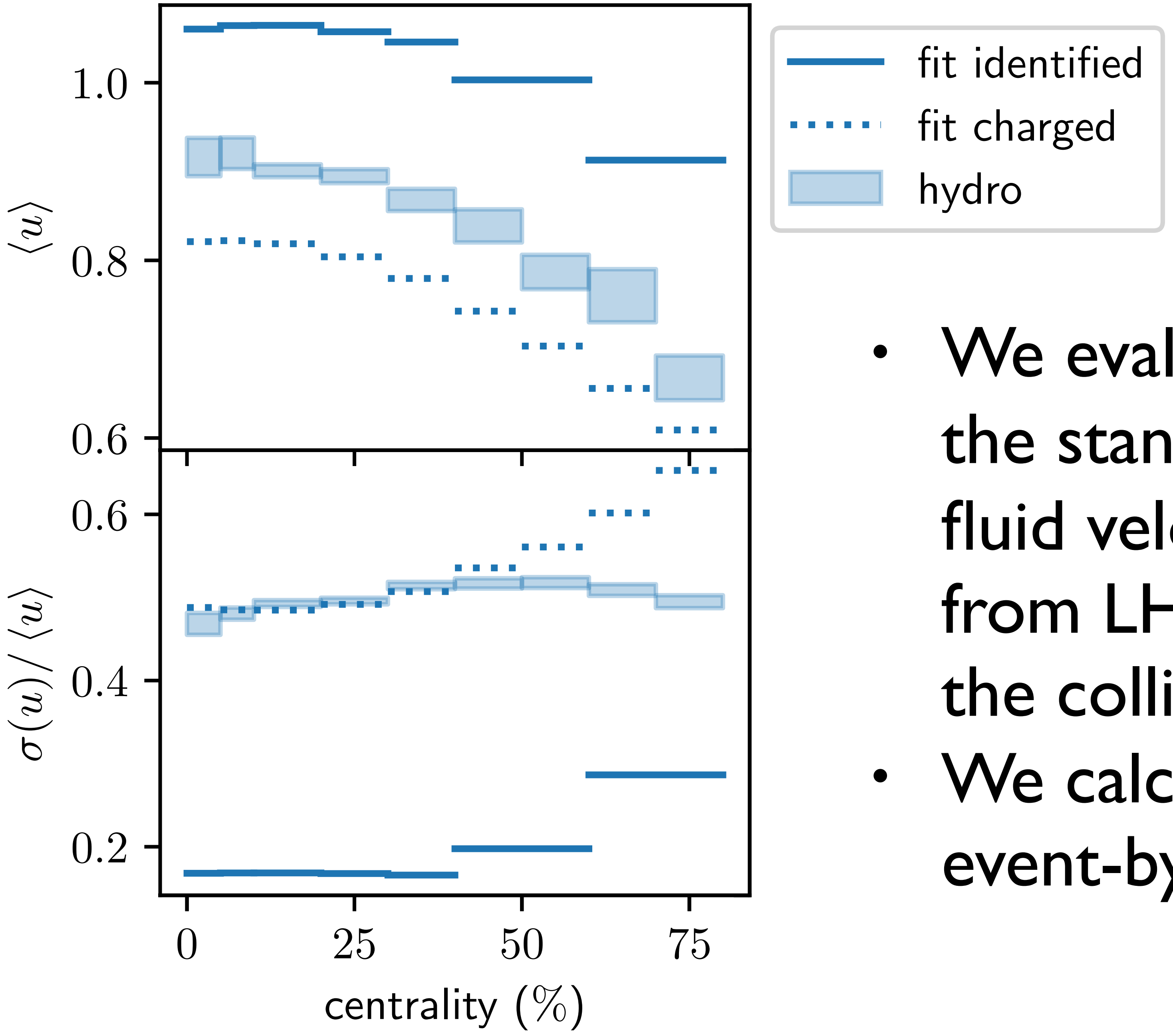
- Heavy particles follow the fluid: for deuterons, the distribution of p_t/m is close to the distribution of the fluid 4-velocity.
- Therefore, a combined blast-wave fit to identified particle spectra is dominated by the heaviest particles included in the fit.

Blast-wave fits to unidentified spectra



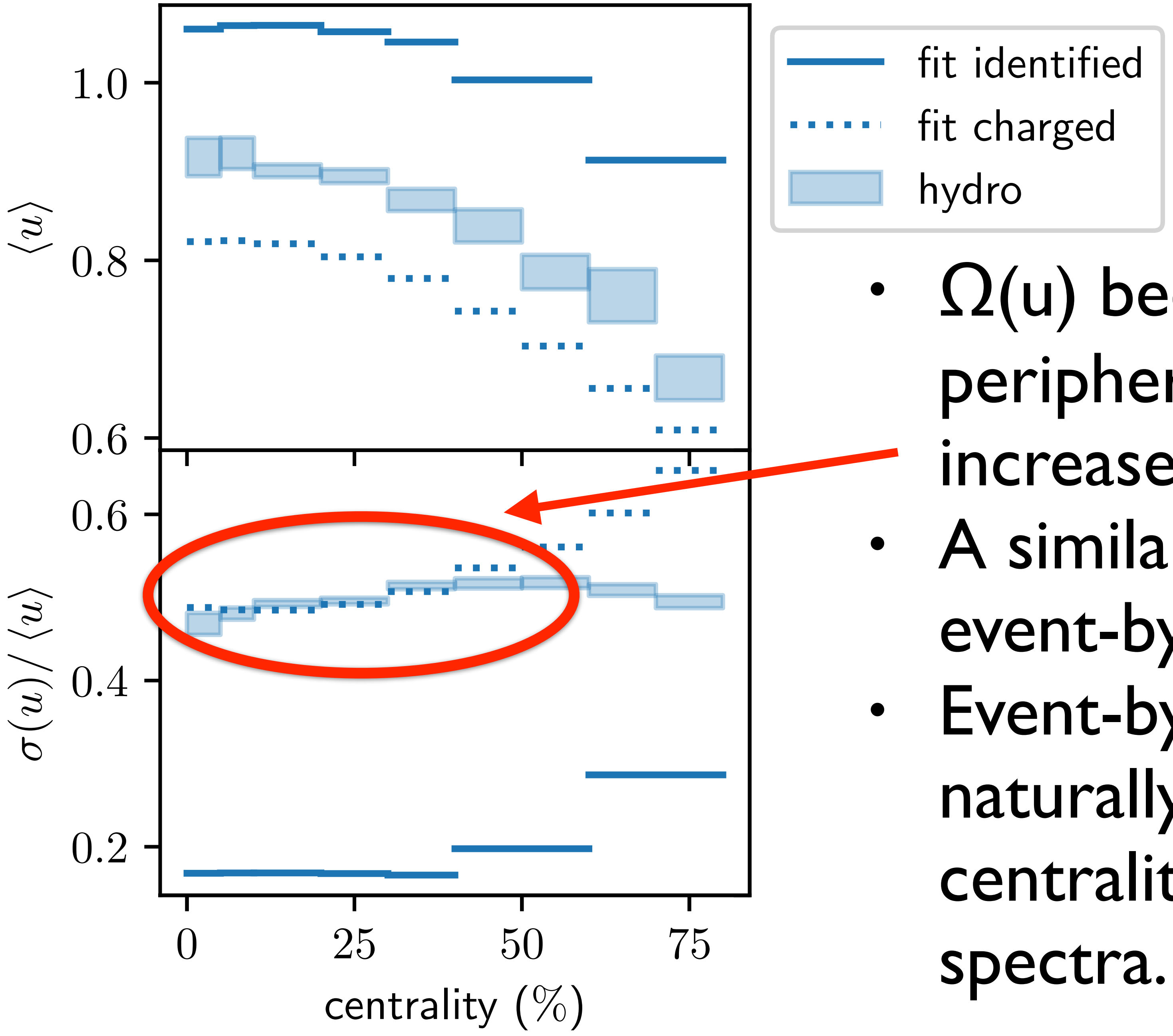
- We have also fitted the charged hadron spectra published by ALICE.
- The fit is now dominated by the pions, which represent ~85% of the hadron yield.
- Good fit all the way to p_t~5GeV/c
- Pion yield at low pt « explained » by a a fraction of the fluid at rest: u~0.

Centrality dependence of the fluid velocity distribution



- We evaluate the mean <u> and the standard deviation σ(u) of the fluid velocity distribution Ω(u) from LHC data, as a function of the collision centrality.
- We calculate <u> and σ(u) in event-by-event ideal hydro.

Centrality dependence of the fluid velocity distribution



- $\Omega(u)$ becomes broader for more peripheral collisions: $\sigma(u)/\langle u \rangle$ increases.
- A similar increase is found in our event-by-event hydro calculation.
- Event-by-event fluctuations naturally explain the observed centrality dependence of p_t spectra.

Summary

- We have generalized the blast-wave fit in a way that follows as closely as possible an actual hydrodynamic calculation: arbitrary distribution of fluid velocity $\Omega(u)$, resonance decays included.
- Still, a blast-wave fit is not equivalent to a hydrodynamic calculation due to the time-like part of the freeze-out isotherm.
- We obtain good fits of ALICE data all the way up to p_t~5GeV/c.
- The pion excess at low p_t compared to hydro is generic.
- The mild centrality dependence of p_t spectra is naturally explained in hydrodynamics. Its broadening is due to event-by-event fluctuations.

Supplementary material

Cooper-Frye vs semi-Cooper-Frye

Cooper-Frye

$$\frac{dN}{d^3p} = \frac{2S+1}{(2\pi)^3} \int_\sigma \frac{1}{e^{E^*/T_f} \pm 1} \frac{p^\mu}{p^0} d\sigma_\mu$$

semi-Cooper-Frye

$$\frac{dN}{d^3p} = \frac{2S+1}{(2\pi)^3} \int_\sigma \frac{1}{e^{E^*/T_f} \pm 1} \frac{u^\mu}{u^0} d\sigma_\mu$$

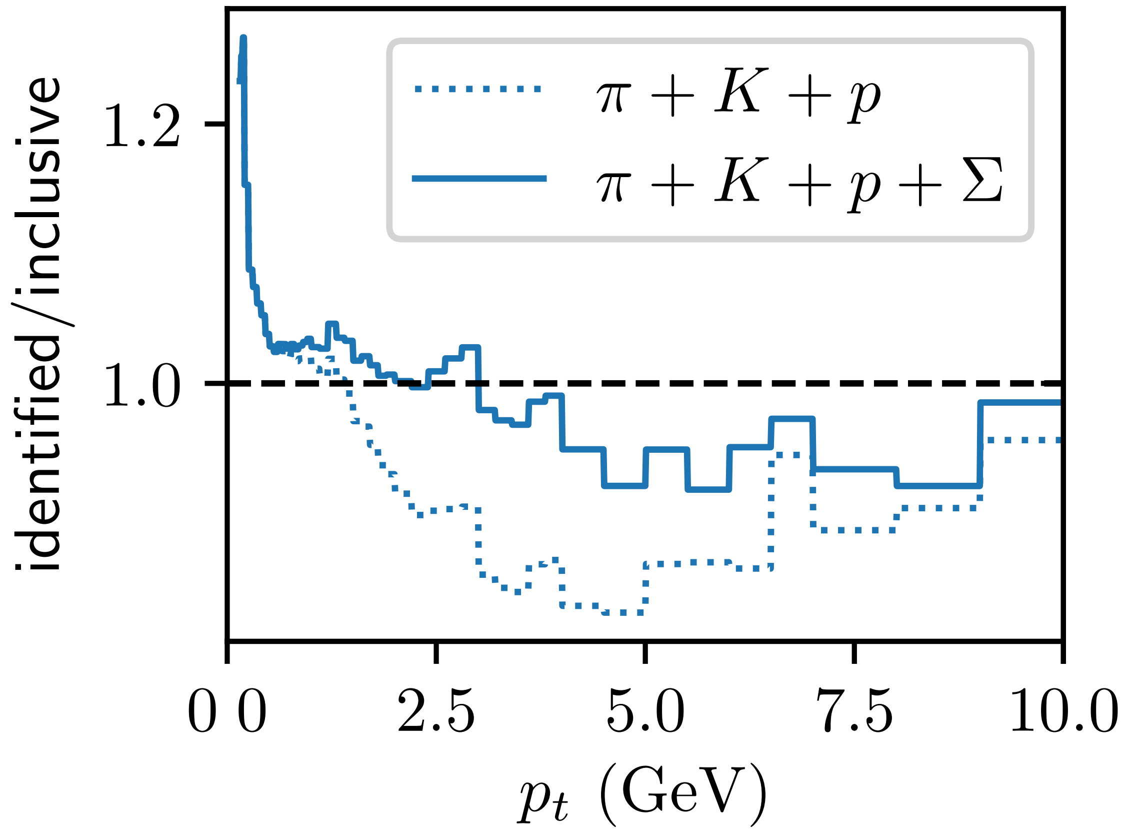
$$= \frac{2S+1}{(2\pi)^3} \int \frac{1}{e^{E^*/T_f} \pm 1} \Omega(\mathbf{u}) d\mathbf{u}$$

where

$$\Omega(\mathbf{u}) d\mathbf{u} = \int_{\sigma, \mathbf{u} \text{ in } d\mathbf{u}} \frac{u^\mu}{u^0} d\sigma_\mu$$

defines the distribution of fluid velocity in hydro.

Identified versus charged spectra



By summing the identified spectra of π, K, p, Σ, one recovers the unidentified charged spectra.