Dark matter going nuclear:
the role of bound states in thermal decoupling

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Dark matter production in the early universe

Why do we care?

Production depends on the couplings of DM to other particles, which are the very probes of the DM properties.
Thermal freeze-out

\[ T > m_{DM} \]
DM kept in chemical & kinetic equilibrium with the plasma, via
\[ X + \bar{X} \leftrightarrow f + \bar{f} \]
\[ n_{DM} \sim T^3 \quad \text{or} \quad Y_{DM} = \text{constant} \]

\[ T < m_{DM} \]
\[ Y_{DM} \propto \exp(-m_{DM}/T), \text{ while still in equilibrium} \]

\[ T < m_{DM} / 25 \]
Density too small, annihilations stall \( \Rightarrow \) Freeze-out!

\[ \Omega \simeq 0.26 \times \left( \frac{1 \text{pb} \cdot c}{\sigma_{\text{ann}} v_{\text{rel}}} \right) \]

1 pb \( \sim \sigma_{\text{Weak}} \)

WIMP miracle!
WIMPs and variations

Weakly coupled to SM via $W^\pm, Z, H$
E.g. LSP in SUSY

Or

Weakly coupled to SM via non-SM interactions,
E.g. $\delta L = \frac{\bar{X} \gamma^\mu X \bar{q} \gamma_\mu q}{\Lambda^2}$

Or

Weakly coupled to light dark-sector particles that couple (feebly) to SM,
E.g. DM coupled to dark photon kinetically mixed with Hypercharge
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Thermal equilibrium in early universe

Indirect Detection:
radiation from DM annihilations

Production at Colliders

Direct Detection:
scattering on targets

Significant constraints.
No discovery so far.
What now?

Diversify dark matter searches

- **Heavier DM**
  
  Particles with $m \gtrsim \text{TeV}$ coupled to SM via the Weak or other interactions not constrained by collider experiments
  
  $\rightarrow$ existing and upcoming telescopes observing multi-TeV sky with increasing sensitivity, e.g. HESS, IceCube, CTA, Antares

- **Lighter DM**
  
  Particles with $m \sim \text{few GeV}$, possibly coupled to SM via a portal interaction, not constrained by older direct detection experiments
  
  $\rightarrow$ development of new generation of direct detection experiments

see e.g. 2008.00692: WIMP prospects with CTA Rinchiuso, Macias, Moulin, Rodd, Slatyer
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  $\rightarrow$ development of new generation of direct detection experiments

- Simple thermal-relic WIMP models live in the (multi-)TeV scale.
- Thermal-rellic DM can be as heavy as few $\times 100\text{ TeV}$.

**How heavy can thermal-rellic DM be, and what are the underlying dynamics of heavy ($\gtrsim \text{TeV}$) thermal-rellic DM?**
Long-range interactions

\[ \lambda_B \sim \frac{1}{\mu v_{\text{rel}}}, \frac{1}{\mu \alpha} \lesssim \frac{1}{m_{\text{mediator}}} \sim \text{interaction range} \]

\[ \mu: \text{ reduced mass } (m_{\text{DM}}/2) \]
Long-range interactions

Motivation

\[ \lambda_B \sim \frac{1}{\mu v_{rel}}, \quad \frac{1}{\mu \alpha} \lesssim \frac{1}{m_{\text{mediator}}} \sim \text{interaction range} \]

\[ \mu: \text{ reduced mass} \ (m_{\text{DM}}/2) \]

- Self-interacting DM
- DM explanations of astrophysical anomalies, e.g. galactic positrons, IceCube PeV neutrinos
- Sectors with stable particles in String Theory

- WIMP DM with \( m_{\text{DM}} > \text{few TeV.} \) \ [Hisano et al. 2002]
- WIMP DM with \( m_{\text{DM}} < \text{TeV,} \)
  in scenarios of DM co-annihilation with coloured partners.
Implications of long-range interactions

**Sommerfeld effect**
distortion of scattering-state wavefunctions

- Affects freeze-out ⇒ changes correlation of parameters (mass – couplings)
- Affects indirect detection signals
  - rates
  - velocity dependence
  - parametric resonances
Implications of long-range interactions

**Sommerfeld effect**
- distortion of scattering-state wavefunctions
  - Affects freeze-out ⇒ changes correlation of parameters (mass – couplings)
  - Affects indirect detection signals
    - rates
    - velocity dependence
    - parametric resonances

**Bound states**
- **Unstable bound states**
  - ⇒ extra annihilation channel
    - **Freeze-out**
    - Indirect detection (different velocity dependence, resonances than annihilation)
    - Novel low-energy indirect detection signals
- **Stable bound states (particularly important for asymmetric DM)**
  - Novel low-energy indirect detection signals
  - Affect DM self-interactions (screening)
  - Inelastic scattering in direct detection experiments (?)
Outline

Bound states and density of thermal relic DM

- Dark U(1) sector
- Unitarity limit and long-range interactions
- Neutralino-squark coannihilation scenarios
- The 125 GeV Higgs as a light mediator
- Bound-state formation via emission of a charged scalar
- Bound-state formation via Higgs-doublet emission
1. Dark U(1) sector
Thermal freeze-out with long-range interactions

Dark U(1) model: Dirac DM $X, \bar{X}$ coupled to $\gamma_D$

Direct annihilation

\[
X + \bar{X} \to 2\gamma_D
\]

\[
\sigma_{\text{ann}} v_{\text{rel}} = \frac{\pi \alpha_D^2}{m_X^2} \times S_{\text{ann}}(\alpha_D/v_{\text{rel}})
\]

Radiative bound-state formation

\[
X + \bar{X} \to \mathcal{B}(X\bar{X}) + \gamma_D
\]

\[
\sigma_{\text{BSF}} v_{\text{rel}} = \frac{\pi \alpha_D^2}{m_X^2} \times S_{\text{BSF}}(\alpha_D/v_{\text{rel}})
\]

von Harling, KP: 1407.7874

\[
S = \sigma v_{\text{rel}} / \sigma_0
\]

\[
10^{-1} \quad 10^{-2} \quad 10^{-1} \quad 1 \quad 10 \quad 10^2
\]

\[
\zeta = \alpha_D / v_{\text{rel}}
\]

\[
S_{\text{ann}} \approx \left( \frac{2\pi \zeta}{1 - e^{-2\pi \zeta}} \right) \frac{\zeta \geq 1}{2\pi \zeta}
\]

\[
S_{\text{BSF}} \approx \left( \frac{2\pi \zeta}{1 - e^{-2\pi \zeta}} \right) \frac{2^9 \zeta^4 e^{-4\zeta \arccot \zeta}}{3(1 + \zeta^2)^2} \frac{\zeta \geq 1}{3.13 \times 2\pi \zeta}
\]
Thermal freeze-out with long-range interactions

Dark U(1) model: Dirac DM $X, \bar{X}$ coupled to $\gamma_D$

- **Direct Annihilation**: $X\bar{X} \rightarrow \gamma_D \gamma_D$
- **Bound-state formation and decay**: $X\bar{X} \rightarrow B(X\bar{X}) + 2\gamma_D$ or $3\gamma_D$

\[ m_X \text{ [GeV]} \]
\[ \alpha_D \]

von Harling, KP: 1407.7874
Baldes, KP: 1703.00478
Thermal freeze-out with long-range interactions

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- Direct Annihilation: $X \bar{X} \rightarrow \gamma_D \gamma_D$
- Bound-state formation: $X \bar{X} \rightarrow B(X \bar{X}) + \gamma_D$
- and decay: $B(X \bar{X}) \rightarrow 2\gamma_D$ or $3\gamma_D$

Important because it determines DM interactions today (direct, indirect detection)

Long-range effects indeed become at $m_{DM} \gtrsim$ few TeV.

Verifies expectation from unitarity arguments!

Dominant annihilation mode: $s$-wave.
Dominant BSF mode: $p$-wave

Same order!

Higher partial waves important / dominant in multi-TeV regime.

DM may be even heavier!
What just happened?
Making sense of the ladder diagrams

Every mediator exchange introduces an $\alpha = g^2/(4\pi)$ suppression in the amplitude.
How did we get an enhancement and bound states?

Bound-state ladder

\[ \begin{align*}
&\quad g \quad + \quad g \quad + \quad g \quad + \quad \cdots \\
&g \quad \quad \quad \quad g \quad \quad \quad \quad g
\end{align*} \]
What just happened?
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\( \alpha = g^2/(4\pi) \) suppression in the amplitude.
How did we get an enhancement and bound states?

Bound-state ladder
\[ \begin{array}{cccc}
g & \cdots & g \\
g & \cdots & g \\
g & \cdots & g \\
g & \cdots & g \\
\end{array} \]

Energy and momentum exchange scale with \( \alpha \)!

- Momentum transfer: \(|\vec{q}| \sim \mu \alpha|.|.
- Energy transfer: \(q^0 \sim |\vec{q}|^2/\mu \sim \mu \alpha^2|.|.
- Off-shellness of interacting particles: \(q^0 \sim |\vec{q}|^2/\mu \sim \mu \alpha^2|.|.

one boson exchange \( \sim \alpha \times \frac{1}{(\mu \alpha)^2} \propto \frac{1}{\alpha} \)

each added loop
\( \sim \alpha \times \int dq^0 dq^1 dq^2 dq^3 \frac{1}{q_1 - m_1} \frac{1}{q_2 - m_2} \frac{1}{q_2^2} \)
\( \sim \alpha \times (\mu \alpha^2)(\mu \alpha)^3 \frac{1}{\mu \alpha^2} \frac{1}{\mu \alpha^2} \frac{1}{(\mu \alpha)^2} \)
\( \sim 1 \)
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- Off-shellness of interacting particles: \( q^0 \sim \frac{1}{(\mu \alpha)^2} \).

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\text{one boson exchange} \quad \sim \alpha \times \frac{1}{(\mu \alpha)^2} \propto \frac{1}{\alpha}
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each added loop

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\sim \alpha \times \int dq^0 d^3q \frac{1}{q_1 - m_1} \frac{1}{q_2 - m_2} \frac{1}{q_2^2} \\
\sim \alpha \times (\mu \alpha^2)(\mu \alpha)^3 \frac{1}{\mu \alpha^2} \frac{1}{\mu \alpha^2} \frac{1}{(\mu \alpha)^2} \\
\sim 1
\]

1/\( \alpha \) scaling responsible for non-perturbative effects (not largeness of coupling)
What just happened?
Making sense of the ladder diagrams

Every mediator exchange introduces an 
\( \alpha = \frac{g^2}{4\pi} \) suppression in the amplitude.
How did we get an enhancement and bound states?

Scattering state ladder

Energy and momentum exchange scale with both \( \alpha \) and \( \nu_{\text{rel}} \! \)!

\( \mu \nu_{\text{rel}} \) is the expectation value of the momentum in CM frame,
the quantum uncertainty scales with \( \alpha \).

The Sommerfeld effect appears when 
quantum uncertainty \( \sim \) expectation value.
2. Unitarity limit and long-range interactions
Partial-wave unitarity limit

\[ S^\dagger S = 1 \quad \xrightarrow{S=1+iT} \quad -i(T - T^\dagger) = T^\dagger T \]

Project on a partial wave and insert complete set of states on RHS

\[ \downarrow \]

\[ \sigma^{(\ell)}_{\text{inel}} \leftarrow \frac{\pi(2\ell + 1)}{k_{\text{cm}}^2} \xrightarrow{\text{non-rel}} \frac{\pi(2\ell + 1)}{\mu^2 v_{\text{rel}}^2} \quad \mu = \frac{M_{\text{DM}}}{2} \rightarrow \frac{4\pi(2\ell + 1)}{M_{\text{DM}}^2 v_{\text{rel}}^2} \]

[Griest, Kamionkowski (1990); Hui (2001)]

Physical meaning:
saturation of probability for inelastic scattering
Partial-wave unitarity limit

\[ \sigma_{inel}^{(\ell)} v_{rel} \leq \sigma_{uni}^{(\ell)} v_{rel} = \frac{4\pi (2\ell + 1)}{M_{DM}^2 v_{rel}} \]

Implies upper bound on the mass of thermal-relic DM

Griest, Kamionkowski (1990)

\[ \sigma_{ann} v_{rel} \simeq 2.2 \times 10^{-26} \text{ cm}^3/\text{s} \leq \frac{4\pi}{M_{DM}^2 v_{rel}} \]

\[ \langle v_{rel}^2 \rangle^{1/2} = \left( \frac{6T}{M_{DM}} \right)^{1/2} \text{ freeze-out} \]

\[ M_{DM}/T \approx 25 \rightarrow 0.49 \]

\[ M_{uni} \simeq \begin{cases} 117 \text{ TeV}, & \text{self-conjugate DM} \\ 83 \text{ TeV}, & \text{non-self-conjugate DM} \end{cases} \]
Partial-wave unitarity limit

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\langle v_{\text{rel}}^2 \rangle^{1/2} = (6T/M_{\text{DM}})^{1/2} \quad \text{freeze-out at } M_{\text{DM}}/T \approx 25 \quad 0.49
\]

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Two assumptions to be questioned

1. “one does not expect \( \sigma v_{\text{rel}} \propto 1/v_{\text{rel}} \) for annihilation channels in a non-relativistic expansion.”

2. The s-wave yields the dominant contribution to the annihilation cross-section.
Partial-wave unitarity limit

\[ \sigma^{(\ell)}_{\text{inel}} v_{\text{rel}} \leq \sigma^{(\ell)}_{\text{uni}} v_{\text{rel}} = \frac{4\pi(2\ell + 1)}{M_{\text{DM}}^2 v_{\text{rel}}} \]

1) Velocity dependence of \( \sigma_{\text{uni}} \)

- Assuming \( \sigma_{\text{ann}} v_{\text{rel}} = \text{const.} \), setting it to maximal (inevitably for a fixed \( v_{\text{rel}} \)) and thermal averaging formally incorrect!
  \( \Rightarrow \) Unitarity violation at larger \( v_{\text{rel}} \), non-maximal cross-section at smaller \( v_{\text{rel}} \).

- Sommerfeld-enhanced inelastic processes exhibit exactly this velocity dependence at large couplings / small velocities, e.g. in QED

\[ \sigma^{\ell=0}_{\text{ann}} v_{\text{rel}} \sim \frac{\pi \alpha_D^2}{M_{\text{DM}}^2} \times \frac{2\pi \alpha_D / v_{\text{rel}}}{1 - \exp(-2\pi \alpha_D / v_{\text{rel}})} \quad \alpha_D \gg v_{\text{rel}} \rightarrow \frac{2\pi^2 \alpha_D^3}{M_{\text{DM}}^2 v_{\text{rel}}} \]

\( \Rightarrow \) Velocity dependence of \( \sigma_{\text{uni}} \) definitely not unphysical! 

Baldes, KP: 1703.00478
Partial-wave unitarity limit

\[ \sigma_{\text{inel}}(v_{\text{rel}}) \leq \sigma_{\text{uni}}(v_{\text{rel}}) = \frac{4\pi (2\ell + 1)}{M_{\text{DM}}^2 v_{\text{rel}}} \]

1) Velocity dependence of \( \sigma_{\text{uni}} \)

For a contact-type interaction, mediated by heavy particle with \( m_{\text{med}} \gtrsim M_{\text{DM}} \),

\[ \sigma_{\text{ann}}(v_{\text{rel}}) \sim \frac{\alpha_D^2 M_{\text{DM}}^2}{m_{\text{med}}^4} \lesssim \frac{4\pi}{M_{\text{DM}}^2 v_{\text{rel}}}. \]

Approaching unitarity limit requires large coupling (no surprise)

\[ \alpha_D \sim m_{\text{med}}^4 / M_{\text{DM}}^4 \gtrsim 1. \]

Calculation violates unitarity if

\[ m_{\text{med}} < \alpha_D^{1/2} M_{\text{DM}} \lesssim \alpha_D M_{\text{DM}}. \]

Comparison between physical scales \( \Rightarrow \) violation signals new effect at play!

What can we learn?

Baldes, KP: 1703.00478
Partial-wave unitarity limit

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Comparison between physical scales \( \Rightarrow \) violation signals new effect at play!

Including the Sommerfeld enhancement, for a light mediator, e.g. dark QED

\[ \sigma_{\text{ann}} v_{\text{rel}} \sim \frac{2\pi^2 \alpha_D^3}{M_{\text{DM}}^2 v_{\text{rel}}} \lesssim \frac{4\pi}{M_{\text{DM}}^2 v_{\text{rel}}}. \]

Unitarity indicates range of validity

\[ \alpha_D \lesssim 0.86 \]

Only numerical bound on a dimensionless coupling

\( \Rightarrow \) include (resummed) higher order corrections

Baldes, KP: 1703.00478
Partial-wave unitarity limit

$$\sigma_{\text{inel}}^{(\ell)} v_{\text{rel}} \leq \sigma_{\text{uni}}^{(\ell)} v_{\text{rel}} = \frac{4\pi(2\ell + 1)}{M_{\text{DM}}^2 v_{\text{rel}}}$$

1) Velocity dependence of $\sigma_{\text{uni}}$

Proper thermal average and taking into account delayed chemical decoupling

$$M_{\text{uni}} \approx \begin{cases} 117 \text{ TeV}, & \text{self-conjugate DM} \\ 83 \text{ TeV}, & \text{non-self-conjugate DM} \end{cases}$$

$$M_{\text{uni}} \approx \begin{cases} 198 \text{ TeV}, & \text{self-conjugate DM} \\ 138 \text{ TeV}, & \text{non-self-conjugate DM} \end{cases}$$

$s$-wave annihilation
Partial-wave unitarity limit

\[ \sigma_{\text{inel}} v_{\text{rel}} \leq \sigma_{\text{uni}} v_{\text{rel}} = \frac{4\pi(2\ell + 1)}{M_{DM}^2 v_{\text{rel}}} \]

2) Higher partial waves

In direct annihilation processes, s-wave dominates.

- For contact-type interactions, higher \( \ell \) are \( v_{\text{rel}}^{2\ell} \) suppressed:

\[ \sigma_{\text{ann}} v_{\text{rel}} = \sum_{\ell} \sum_{r=0}^{\infty} c_{\ell r} v_{\text{rel}}^{2\ell + 2r} \]

- For long-range interactions:

\[ \sigma^{(\ell=0)}_{XX \rightarrow VV} v_{\text{rel}} \approx \frac{\pi \alpha_D^2}{M_{DM}^2} \times \left( \frac{2\pi \alpha_D / v_{\text{rel}}}{1 - e^{-2\pi \alpha_D / v_{\text{rel}}}} \right) \]

\[ \sigma^{(\ell=1)}_{XX \rightarrow SS} v_{\text{rel}} \approx \frac{3\pi \alpha_D^2}{8M_{DM}^2} v_{\text{rel}}^2 \times \left( \frac{2\pi \alpha_D / v_{\text{rel}}}{1 - e^{-2\pi \alpha_D / v_{\text{rel}}}} \right) \left( 1 + \frac{\alpha_D^2}{v_{\text{rel}}^2} \right) \]

\[ \lim_{\alpha_D \gg v_{\text{rel}}} \frac{2\pi^2 \alpha_D^3}{M_{DM}^2 v_{\text{rel}}} \]

\[ \lim_{\alpha_D \gg v_{\text{rel}}} \frac{6\pi^2 \alpha_D^5}{8M_{DM}^2 v_{\text{rel}}} \]

Same \( v_{\text{rel}} \) scaling (as expected from unitarity!), albeit \( v_{\text{rel}} \rightarrow \alpha_D^2 \) suppression.

Baldes, KP: 1703.00478
Partial-wave unitarity limit

\[ \sigma_{\text{inel}}^{(\ell)} v_{\text{rel}} \leq \sigma_{\text{uni}}^{(\ell)} v_{\text{rel}} = \frac{4\pi(2\ell + 1)}{M_{\text{DM}}^2 v_{\text{rel}}} \]

2) Higher partial waves

In direct annihilation processes, s-wave dominates.

However, DM may annihilate via formation and decay of bound states. The bound-state ladder reduces the order of the diagram!

\[ \sigma_{\text{ann}}^{(\ell=0)} v_{\text{rel}} \xrightarrow{\alpha_D \gg v_{\text{rel}}} \frac{2\pi^2 \alpha_D^3}{M_{\text{DM}}^2 v_{\text{rel}}} \]

\[ \sigma_{\text{BSF}}^{(\ell=1)} v_{\text{rel}} \xrightarrow{\alpha_D \gg v_{\text{rel}}} 3.13 \times \frac{2\pi^2 \alpha_D^3}{M_{\text{DM}}^2 v_{\text{rel}}} \]

Both s- and p- wave saturate their unitarity limit at \( \alpha_D \approx 0.86 \).

\[ \Rightarrow \text{Consider combined bound on DM mass, } M_{\text{uni}} \approx 276 \text{ TeV} \]
2) **Higher partial waves**

In direct annihilation processes, s-wave dominates.

However, DM may annihilate via formation and decay of bound states. The bound-state ladder reduces the order of the diagram.

Both s- and p-wave saturate their unitarity limit at $\alpha_D \approx 0.86$.

⇒ Consider combined bound on DM mass, $M_{\text{uni}} \approx 276$ TeV

\[
\sigma_{\text{inel}}^{(\ell)} \nu_{\text{rel}} \leq \sigma_{\text{uni}}^{(\ell)} \nu_{\text{rel}} = \frac{4\pi(2\ell + 1)}{M_{\text{DM}}^2 \nu_{\text{rel}}}
\]
Partial-wave unitarity limit

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\sigma_{\text{inel}}^{(\ell)} \nu_{\text{rel}} \leq \sigma_{\text{uni}}^{(\ell)} \nu_{\text{rel}} = \frac{4\pi (2\ell + 1)}{M_{\text{DM}}^2 \nu_{\text{rel}}}
\]

Can be approached or realised only in models with attractive long-range interactions

Baldes, KP: 1703.00478

Generic conclusion:
In viable thermal-relic DM scenarios, expect long-range behaviour at \( m_{\text{DM}} \gtrsim \) few TeV!

Baldes, KP: 1703.00478

- Implications for all experimental probes: DM mass and/or couplings different than otherwise estimated.
- Indirect detection in the multi-TeV regime: non-perturbative effects (Sommerfeld, BSF) must be considered

Cirelli, Panci, KP, Sala, Taoso 1612.07295
Baldes, KP 1703.00478
Baldes, Cirelli, Panci, KP, Sala, Taoso 1712.07489
Cirelli, Gouttenoire, KP, Sala 1811.03608
What about WIMPs?

• Long-range condition: \( \alpha_2 M_{DM} \gtrsim m_W \Rightarrow M_{DM} \gtrsim 3 \text{ TeV} \)

• Long-range effects more important for DM in higher SU\(_L\)(2) multiplets
  – Pure 3-plet (Wino-like): \( M_{DM} \sim 3 \text{ TeV} \), only Sommerfeld important.
  – Pure 5-plet: \( M_{DM} \sim 14 \text{ TeV} \), Sommerfeld & bound states important.

• Mixed multiplet and coannihilation scenarios: more complexity
  – Neutralino-squark co-annihilation
  – Higgs-portal models
3. Neutralino-squark co-annihilation scenarios
Neutralino in SUSY models
Squark-neutralino co-annihilation scenarios

- Degenerate spectrum → soft jets → evade LHC constraints
- Large stop-Higgs coupling reproduces measured Higgs mass and brings the lightest stop close in mass with the LSP

⇒ DM density determined by “effective” Boltzmann equation

\[ \sigma_{\text{ann}}^{\text{eff}} = \left[ n_{\text{LSP}}^2 \sigma_{\text{ann}}^{\text{LSP}} + n_{\text{NLSP}}^2 \sigma_{\text{ann}}^{\text{NLSP}} \right] / n_{\text{tot}}^2 \]

Scenario probed in colliders. Important to compute DM density accurately!
→ QCD corrections
QCD corrections to stop annihilation

[Klasen+ (since 2014), DM@NLO]

QCD loop corrections

Gluon emission

Sommerfeld effect

broadly, the most important
QCD corrections to stop annihilation

[Klasen+ (since 2014), DM@NLO]

QCD loop corrections

Gluon emission

Strong coupling ($\alpha_s \sim 0.1$), massless mediators $\Rightarrow$ BSF important!
Stoponium formation

Sommerfeld effect

broadly, the most important
DM coannihilation with scalar colour triplet
MSSM-inspired toy model

\[ \mathcal{L} \supset \frac{1}{2} \chi^c \, i\not{\partial} \chi - \frac{1}{2} m_\chi \bar{\chi}^c \chi \\
+ \left[ (\partial_\mu + ig_s G_{\mu}^a T^a) X \right] \dagger \left[ (\partial^{\mu} + ig_s G^{a,\mu} T^a) X \right] - m_X^2 |X|^2 \\
+ (\chi \leftrightarrow X, X^\dagger) \text{ interactions in chemical equilibrium during freeze-out} \]
DM coannihilation with scalar colour triplet
MSSM-inspired toy model

Long-range interaction

\[ R \otimes \bar{R} = \sum_{\bar{\ell}} \hat{\ell} = 1 \oplus \text{adj} + \cdots \]

\[ V(r) = -\alpha_{g,[\bar{\ell}]} / r \]

\[ \alpha_{g,[\bar{\ell}]} = \alpha_s \times \left[ C_2(R) - C_2(\bar{R})/2 \right] \]

where \( \alpha_s = g_s^2/(4\pi) \)

\[ 3 \otimes \bar{3} = 1 \oplus 8 \]

\[ \alpha_{g,[1]} = + (4/3)\alpha_s \text{ attractive} \]

\[ \alpha_{g,[8]} = - (1/6)\alpha_s \text{ repulsive} \]

with \( \alpha_s \sim 0.1 \) at \( m_X \sim \text{TeV} \)
DM coannihilation with scalar colour triplet MSSM-inspired toy model

Bound-state formation and decay

Harz, KP 1805.01200: Cross-sections for radiative BSF in non-Abelian theories

In agreement with Brambilla, Escobedo, Ghiglieri, Vairo 1109.5826: Gluo-dissociation of quarkonium in pNRQCD
DM coannihilation with scalar colour triplet
MSSM-inspired toy model

Bound-state formation vs Annihilation

BSF can exceed Ann by more than an order of magnitude!

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Why is this important?

Effect on relic density: much much larger than obs uncertainty in $\Omega_{DM}$
4. The SM Higgs as a light mediator
Neutralino in SUSY models

Squark-neutralino co-annihilation scenarios

- Degenerate spectrum → soft jets → evade LHC constraints
- Large stop-Higgs coupling reproduces measured Higgs mass and brings the lightest stop close in mass with the LSP

⇒ DM density determined by "effective" Boltzmann equation

\[
\sigma_{\text{ann}}^{\text{eff}} = \left[ n_{\text{LSP}}^2 \sigma_{\text{ann}}^{\text{LSP}} + n_{\text{NLSP}}^2 \sigma_{\text{ann}}^{\text{NLSP}} + n_{\text{LSP}} n_{\text{NLSP}} \sigma_{\text{ann}}^{\text{LSP-NLSP}} \right] / n_{\text{tot}}^2
\]

Scenario probed in colliders. Important to compute DM density accurately!

→ QCD corrections
Higgs enhancement and relic density
MSSM-inspired toy model

DM co-annihilating with scalar colour-triplet that has a sizeable coupling to the Higgs
e.g. stop-neutralino co-annihilation scenarios with large $A$ terms

$$\mathcal{L} \supset \frac{1}{2} \bar{X}^c i\partial \chi - \frac{1}{2} m_X \bar{X}^c X$$

$$+ \left[ (\partial_\mu + i g_s G^a_\mu T^a) X \right]^\dagger \left[ (\partial_\mu + i g_s G^{a,\mu} T^a) X \right] - m_X^2 |X|^2$$

$$+ \frac{1}{2} \partial_\mu h \partial_\mu h - \frac{1}{2} m_h^2 h^2 - g_h m_X h |X|^2$$

$$+ (\chi \leftrightarrow X, X^\dagger) \text{ interactions in chemical equilibrium during freeze-out}$$

$$\alpha_s = \frac{g_s^2}{4\pi}$$

$$\alpha_h = \frac{g_h^2}{16\pi}$$
Higgs enhancement and relic density
MSSM-inspired toy model

$$V(r) = -\frac{\alpha_g}{r} - \frac{\alpha_h}{r} e^{-m_h r}$$

gluon exchange
Higgs exchange, typically thought to be too short-range

Enhancement of direct annihilation

Higgs-mediated bound states

Gluon potential influences the long-range effect of the Higgs!

Harz, KP: 1711.03552, 1901.10030
Higgs enhancement and relic density
MSSM-inspired toy model

Moderate coupling, appears in MSSM

$\alpha_h = 0.05$

Harz, KP: 1711.03552, 1901.10030
Higgs as a light mediator

- Sommerfeld enhancement of direct annihilation ✓
- Binding of bound states ✓

Harz, KP: 1711.03552
Harz, KP: 1901.10030

KP, Postma, Wiechers: 1505.00109
An, Wise, Zhang: 1606.02305
KP, Postma, de Vries: 1611.01394
Ko, Matsui, Tang: 1910.04311
Oncala, KP: 1911.02605
Harz, KP: 1711.03552
Harz, KP: 1901.10030
Higgs as a light mediator

- Sommerfeld enhancement of direct annihilation
  - Reference: Harz, KP: 1711.03552

- Binding of bound states
  - Reference: Harz, KP: 1901.10030

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- Formation of bound states via Higgs (doublet) emission?

  Capture via emission of neutral scalar suppressed, due to selection rules: quadruple transitions
  - Reference: KP, Postma, Wiechers: 1505.00109
  - Reference: An, Wise, Zhang: 1606.02305
  - Reference: KP, Postma, de Vries: 1611.01394

  Capture via emission of charged scalar [or its Goldstone mode] very very rapid: monopole transitions!
  - Reference: Ko, Matsui, Tang: 1910.04311
  - Reference: Oncala, KP: 1911.02605
  - Reference: Oncala, KP: 2101.08666/7

  Sudden change in effective Hamiltonian precipitates transitions. Akin to atomic transitions precipitated by $\beta$ decay of nucleus.
5. Bound-state formation via emission of a charged scalar
BSF via emission of a *charged* scalar

U(1) model:
scalar DM $X, X^\dagger$ coupled to doubly charged light scalar $\Phi$

\[ \mathcal{L} \supset -igX^\dagger V^\mu (\partial_\mu X) - i2g\Phi^\dagger V^\mu (\partial_\mu \Phi) - \frac{ym_x}{2} XX\Phi^\dagger + h.c. \]

$m_x \gg m_\Phi$

\[ U_{XX^\dagger}(r) = -\frac{\alpha_V}{r} - (-1)^\ell \frac{\alpha_\Phi}{r} e^{-m_\Phi r} \]

\[ U_{XX}(r) = +\frac{\alpha_V}{r} \]

Oncala, KP: 1911.02605
BSF via emission of a *charged* scalar

**U(1) model**

\[ \alpha_{V} = 10^{-2} \quad \alpha_{\Phi} = 10^{-3} \]

**BSF \_\_ \_ very large, even for small values of \( \alpha_{\Phi}, \alpha_{V} \) !**

Oncala, KP: 1911.02605
BSF via emission of a charged scalar U(1) model

\[ \alpha_\nu = 10^{-2} \quad \alpha_\phi = 10^{-3} \]

At \( T \sim \text{binding energy} \ll m_\chi / 30 \)

\[ \Rightarrow \text{recoupling of DM destruction} \]

when BSF via charged scalar emission considered

\[ x = m_\chi / T \]
BSF via emission of a \textit{charged} scalar

U(1) model

\begin{align*}
\alpha_Y &= 0 \quad \text{← DM coupling to gauge vector boson (limit of global symmetry)} \quad \rightarrow \quad \alpha_Y &= 0
\end{align*}

Oncala, KP: 1911.02605
BSF via emission of a \textit{charged} scalar

\textbf{U(1) model}

\[ \alpha_V = 0 \quad \rightarrow \quad \text{DM coupling to gauge vector boson} \quad \text{(limit of global symmetry)} \quad \rightarrow \quad \alpha_V = 0 \]

\[ \begin{align*}
\text{AnnP} & \quad \text{AnnS} \\
\text{AnnS} + \text{BSF}_\phi
\end{align*} \]

\[ \begin{align*}
\text{AnnP} & \quad \text{AnnS} \\
\text{AnnS} + \text{BSF}_\phi
\end{align*} \]

\[ \begin{align*}
\Omega/\Omega_{DM} & \quad \Omega/\Omega_{DM}
\end{align*} \]

\[ \begin{align*}
\alpha & \quad \alpha \\
\alpha_{\phi} & \quad \alpha_{\phi}
\end{align*} \]

\textit{Very large effect!}

Generalisable to non-Abelian theories.

Potentially important for Higgs-portal models

Oncala, KP: 1911.02605
6. Bound-state formation via Higgs-doublet emission
Renormalisable Higgs-portal WIMP models

Mass-degenerate Singlet-Doublet coupled to the Higgs: \( L \supset -y \bar{D} H S \)
\( D \) & \( S \) co-annihilate; freeze-out begins before the EWPT if \( M_{DM} > 5 \text{TeV} \)

\[ M_{DM} = 20 \text{ TeV}, \quad \alpha_H = y^2 / (4\pi) = 0.2 \]

Oncala, KP: 2101.08666/7
Renormalisable Higgs-portal WIMP models

Mass-degenerate **Singlet-Doublet** coupled to the Higgs: \[ L \supset - y \bar{D} H S \]

\( D \) & \( S \) co-annihilate; freeze-out begins before the EWPT if \( M_{DM} > 5 \text{TeV} \)
Renormalisable Higgs-portal WIMP models

Mass-degenerate Singlet-Doublet coupled to the Higgs: \( \mathcal{L} \supset -y \bar{D} H S \)

\( D \) & \( S \) co-annihilate; freeze-out begins before the EWPT if \( M_{DM} > 5\text{TeV} \)

\[
\alpha_H = \frac{y^2}{(4\pi)}
\]

Effect only at large \( \alpha_H \), due to phase-space suppression in Higgs emission.

For higher multiplets, important effect expected also at lower \( \alpha_H \).

Huge effect!
Impels reconsideration of Higgs-portal models (incl. neutralino-squark coann scenarios)
Conclusion
(or my ten cents drachmas)

- **Bound states impel complete reconsideration of thermal decoupling at / above the TeV scale.**

  Unitarity limit can be approached / realised only by attractive long-range interactions ⇒ bound states play very important role!

  Baldes, KP: 1703.00478

- **Important experimental implications:**
  - **DM heavier than anticipated:** multi-TeV probes very important.
  - **Indirect detection**
    * Enhanced rates due to BSF
    * Novel signals: low-energy radiation emitted in BSF
    * Indirect detection of asymmetric DM
  - **Colliders:** improved detection prospects due increased mass gap in coannihilation scenarios