Adventures in the ALPs

Effective Lagrangians, Flavor Observables and Indirect Searches for Axion-Like Particles

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based on work with:
Martin Bauer, Anne Galda, Sophie Renner, Marvin Schnubel & Andrea Thamm
2012.12272, 2102.13112, 2105.01078 and in preparation

BROOKHAVEN NATIONAL LABORATORY

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mitp Mainz Institute for Theoretical Physics

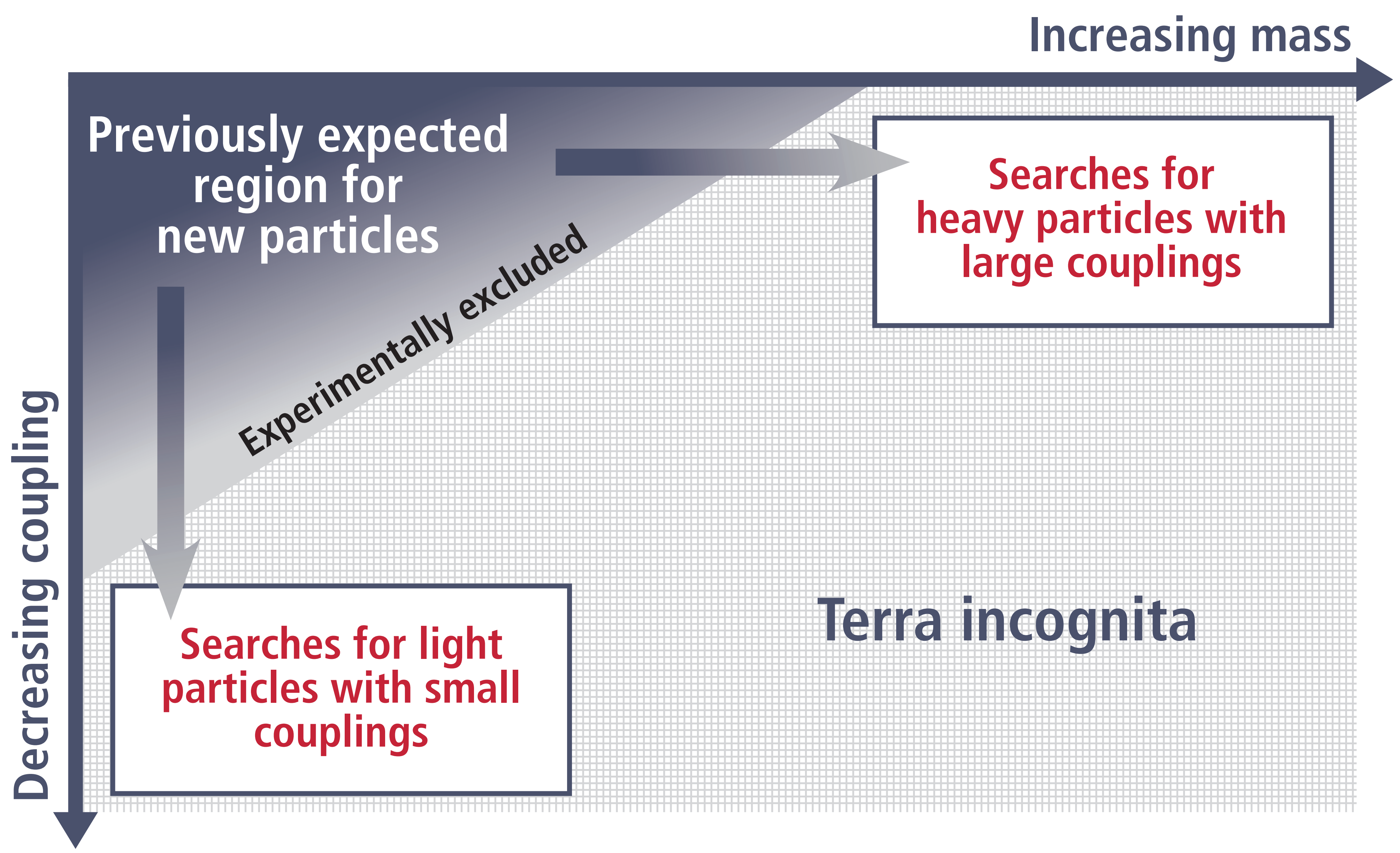
Outline:

- Matching and running for the ALP effective Lagrangian [2012.12272]
- Amusing facts about the rare decay $K \to \pi a$ [2102.13112]
- Flavor observables in eight benchmark scenarios [work in preparation]
- ALP-SMEFT interference [2105.01078]

Motivation

Axions and axion-like particles (ALPs) are well motivated theoretically:

- Peccei-Quinn solution to strong CP problem
  [Peccei, Quinn (1977); Weinberg (1978); Wilczek (1978)]
- ALPs as pseudo Nambu-Goldstone bosons
- Importance of low-energy processes in constraining ALP couplings
- Light but weakly-coupled new particles are an interesting alternative to heavy new particles and might provide hints about physics at energies scales out of the reach for direct searches at the LHC



Effective Lagrangian in the UV

Assume the scale of global symmetry breaking $\Lambda = 4\pi f$ is above the weak scale, and consider the most general effective Lagrangian for a pseudoscalar boson a coupled to the SM via classically shift-invariant interactions, broken only by a soft mass term: [Georgi, Kaplan, Randall (1986)]

hermitian matrices

$$\mathcal{L}_{\rm eff}^{D\le 5} = \frac{1}{2}\,(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2}\,a^2 + \frac{\partial^\mu a}{f}\sum_F \bar\psi_F\,\boldsymbol{c}_F\,\gamma_\mu\,\psi_F$$

$$+\, c_{GG}\,\frac{\alpha_s}{4\pi}\,\frac{a}{f}\,G_{\mu\nu}^a\,\tilde G^{\mu\nu,a} + c_{WW}\,\frac{\alpha_2}{4\pi}\,\frac{a}{f}\,W_{\mu\nu}^A\,\tilde W^{\mu\nu,A} + c_{BB}\,\frac{\alpha_1}{4\pi}\,\frac{a}{f}\,B_{\mu\nu}\,\tilde B^{\mu\nu}$$

Couplings to Higgs bosons only arise in higher orders: [Dobrescu, Landsberg, Matchev (2000); Bauer, MN, Thamm (2017)]

$$\mathcal{L}_{\rm eff}^{D\ge 6} = \frac{C_{ah}}{f^2}\,(\partial_\mu a)(\partial^\mu a)\,\phi^\dagger\phi + \frac{C'_{ah}}{f^2}\,m_{a,0}^2\,a^2\phi^\dagger\phi + \frac{C_{Zh}}{f^3}\,(\partial^\mu a)\left(\phi^\dagger\,iD_\mu\,\phi + \text{h.c.}\right)\phi^\dagger\phi + \dots$$

A redundant operator

- The only possible dimension-5 coupling to the Higgs doublet

$$\mathcal{L}_{\text{eff}}^{D\le 5} \supset c_\phi\, O_\phi = c_\phi\, \frac{\partial^\mu a}{f} \left(\phi^\dagger i D_\mu \phi + \text{h.c.} \right)$$

  is a redundant operator, which can be removed by means of the field redefinitions $\phi \to e^{i c_\phi\, a/f}\, \phi$ and $F \to e^{-i\beta_F\, c_\phi\, a/f}\, F$ as long as:

$$\beta_u - \beta_Q = -1\,, \qquad \beta_d - \beta_Q = 1\,, \qquad \beta_e - \beta_L = 1$$

- This adds $\boldsymbol{c}_F \to \boldsymbol{c}_F + \beta_F\, c_\phi\, \mathbb{1}$ to the ALP-fermion couplings, i.e.:

$$O_\phi = \mathcal{O}_\phi + \sum_F \beta_F\, O_F\,, \qquad \text{with} \quad O_F = \frac{\partial^\mu a}{f}\, \bar\psi_F^i \gamma_\mu \psi_F^i$$

vanishes by the EOMs

Alternative operator basis

A useful alternative form of the Lagrangian involves non-derivative couplings:

$$\mathcal{L}_{\text{eff}}^{D\le 5} = \frac{1}{2}\,(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2}\,a^2 - \frac{a}{f}\left(\bar{Q}\,\phi\,\tilde{\boldsymbol{Y}}_d\,d_R + \bar{Q}\,\tilde{\phi}\,\tilde{\boldsymbol{Y}}_u\,u_R + \bar{L}\,\phi\,\tilde{\boldsymbol{Y}}_e\,e_R + \text{h.c.}\right)$$

$$+ \tilde{c}_{GG}\,\frac{\alpha_s}{4\pi}\,\frac{a}{f}\,G_{\mu\nu}^a\,\tilde{G}^{\mu\nu,a} + \tilde{c}_{WW}\,\frac{\alpha_2}{4\pi}\,\frac{a}{f}\,W_{\mu\nu}^A\,\tilde{W}^{\mu\nu,A} + \tilde{c}_{BB}\,\frac{\alpha_1}{4\pi}\,\frac{a}{f}\,B_{\mu\nu}\,\tilde{B}^{\mu\nu}$$

where:

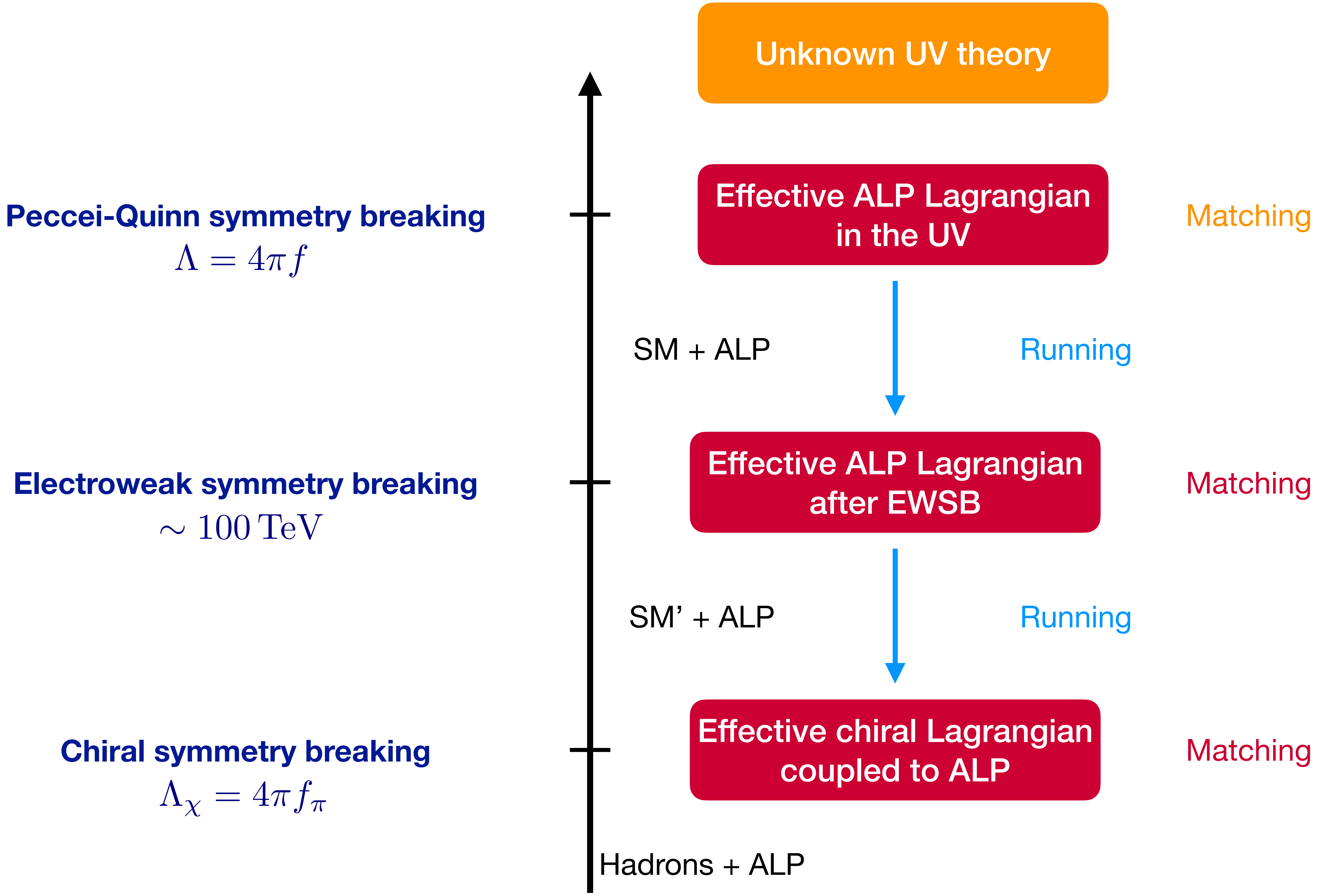
[Bauer, MN, Renner, Schnubel, Thamm (2020)]

$$\tilde{\boldsymbol{Y}}_d = i\left(\boldsymbol{Y}_d\,\boldsymbol{c}_d - \boldsymbol{c}_Q\boldsymbol{Y}_d\right), \qquad \tilde{\boldsymbol{Y}}_u = i\left(\boldsymbol{Y}_u\,\boldsymbol{c}_u - \boldsymbol{c}_Q\boldsymbol{Y}_u\right), \qquad \tilde{\boldsymbol{Y}}_e = i\left(\boldsymbol{Y}_e\,\boldsymbol{c}_e - \boldsymbol{c}_L\boldsymbol{Y}_e\right)$$

$$\tilde{c}_{GG} = c_{GG} + T_F\,\text{Tr}\left(\boldsymbol{c}_u + \boldsymbol{c}_d - N_L\,\boldsymbol{c}_Q\right)$$

$$\tilde{c}_{WW} = c_{WW} - T_F\,\text{Tr}\left(N_c\,\boldsymbol{c}_Q + \boldsymbol{c}_L\right)$$

$$\tilde{c}_{BB} = c_{BB} + \text{Tr}\left[N_c\left(\mathcal{Y}_u^2\,\boldsymbol{c}_u + \mathcal{Y}_d^2\,\boldsymbol{c}_d - N_L\,\mathcal{Y}_Q^2\,\boldsymbol{c}_Q\right) + \mathcal{Y}_e^2\,\boldsymbol{c}_e - N_L\,\mathcal{Y}_L^2\,\boldsymbol{c}_L\right]$$


Unknown UV theory
Peccei-Quinn symmetry breaking
$\Lambda = 4\pi f$
Effective ALP Lagrangian in the UV
Matching
SM + ALP
Running
Electroweak symmetry breaking
$\sim 100\,\mathrm{TeV}$
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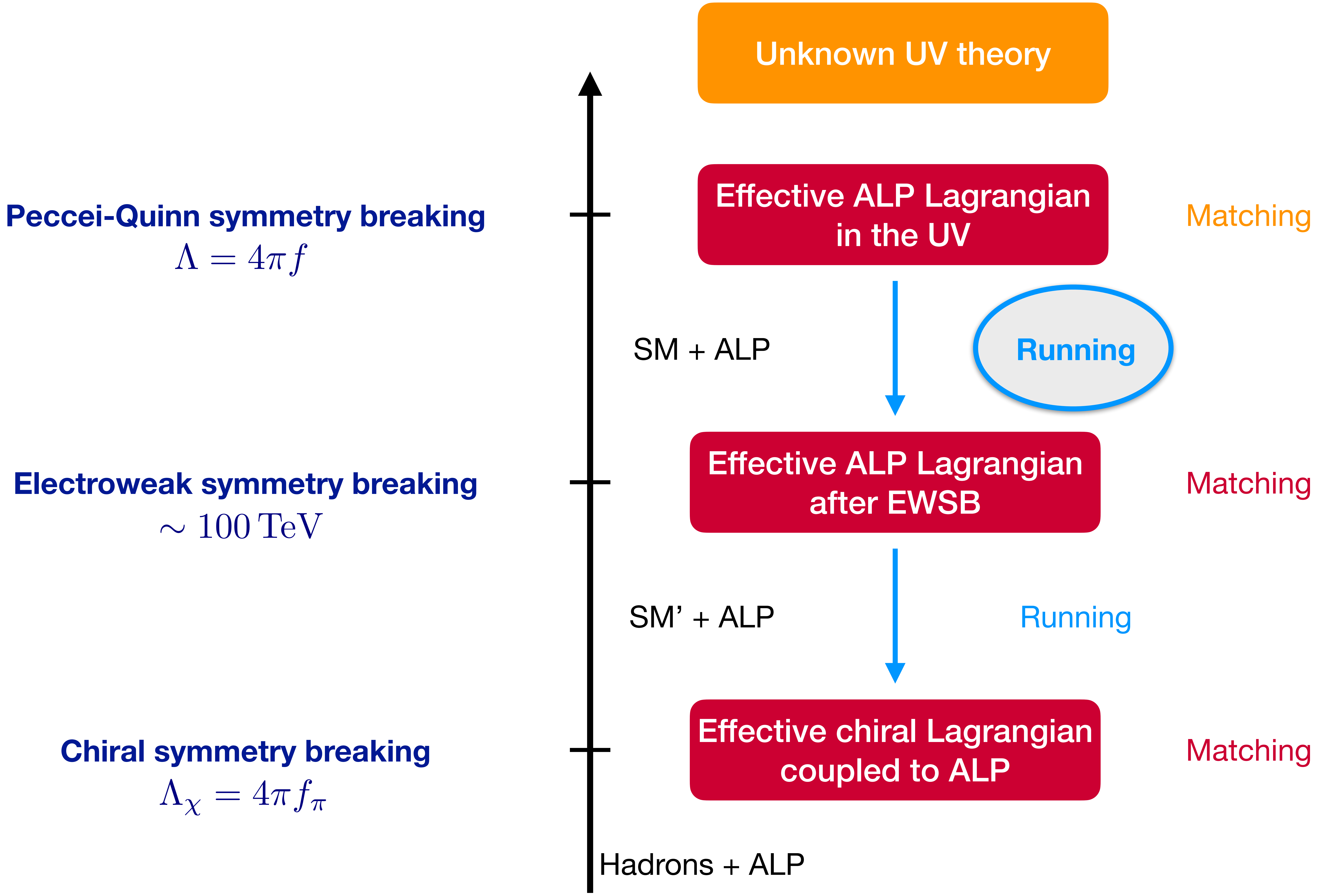
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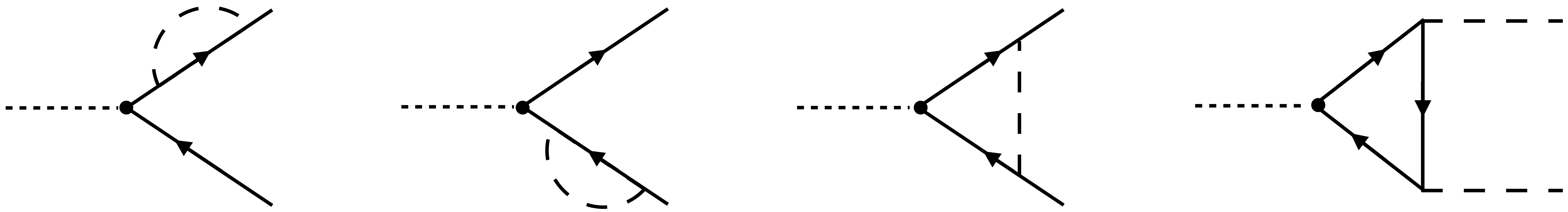
Evolution to the weak scale

Factoring out the gauge couplings from c_{VV} ensures that (at least to 2 loops):

$$\boxed{\frac{d}{d\ln\mu}\, c_{VV}(\mu) = 0\,; \quad V = G, W, B}$$

[Chetyrkin, Kniehl, Steinhauser, Bardeen (1998)]

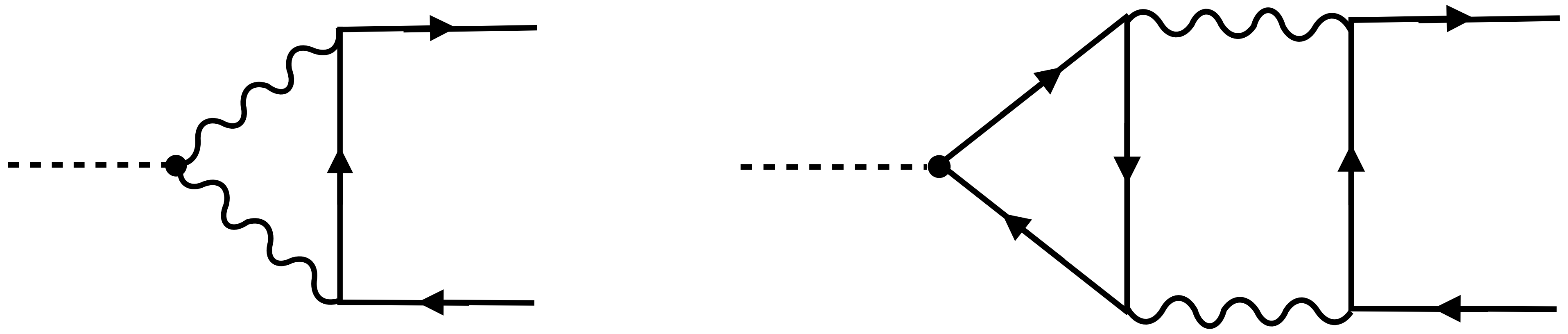
For the ALP-fermion couplings, we have computed:



1-loop Yukawa int.

[Choi, Im, Park, Yun (2017);
Martin Camalich, Pospelov, Vuong, Ziegler, Zupan (2020);
Heiles, König, MN (2020)]

requires the redundant Higgs operator as counterterm



2-loop gauge int.

[Altarelli, Ross (1988);
Chetyrkin, Kniehl, Steinhauser, Bardeen (1998)]

[Kodaira (1980); Larin (1993)]

Evolution to the weak scale

We find: [Bauer, MN, Renner, Schnubel, Thamm (2020); see also: Chala, Guedes, Ramos, Santiago (2020)]

$$\frac{d}{d\ln\mu}\,\boldsymbol{c}_Q(\mu) = \frac{1}{32\pi^2}\left\{\boldsymbol{Y}_u\boldsymbol{Y}_u^\dagger + \boldsymbol{Y}_d\boldsymbol{Y}_d^\dagger, \boldsymbol{c}_Q\right\} - \frac{1}{16\pi^2}\left(\boldsymbol{Y}_u\,\boldsymbol{c}_u\boldsymbol{Y}_u^\dagger + \boldsymbol{Y}_d\,\boldsymbol{c}_d\boldsymbol{Y}_d^\dagger\right)$$
$$+ \left[\boxed{\frac{\beta_Q}{8\pi^2}\,X} - \frac{3\alpha_s^2}{4\pi^2}\,C_F^{(3)}\,\tilde{c}_{GG} - \frac{3\alpha_2^2}{4\pi^2}\,C_F^{(2)}\,\tilde{c}_{WW} - \frac{3\alpha_1^2}{4\pi^2}\,\mathcal{Y}_Q^2\,\tilde{c}_{BB}\right]\mathbb{1}$$

$$\frac{d}{d\ln\mu}\,\boldsymbol{c}_q(\mu) = \frac{1}{16\pi^2}\left\{\boldsymbol{Y}_q^\dagger\boldsymbol{Y}_q, \boldsymbol{c}_q\right\} - \frac{1}{8\pi^2}\,\boldsymbol{Y}_q^\dagger\boldsymbol{c}_Q\boldsymbol{Y}_q + \left[\boxed{\frac{\beta_q}{8\pi^2}\,X} + \frac{3\alpha_s^2}{4\pi^2}\,C_F^{(3)}\,\tilde{c}_{GG} + \frac{3\alpha_1^2}{4\pi^2}\,\mathcal{Y}_q^2\,\tilde{c}_{BB}\right]\mathbb{1}$$

$$\frac{d}{d\ln\mu}\,\boldsymbol{c}_L(\mu) = \frac{1}{32\pi^2}\left\{\boldsymbol{Y}_e\boldsymbol{Y}_e^\dagger, \boldsymbol{c}_L\right\} - \frac{1}{16\pi^2}\,\boldsymbol{Y}_e\,\boldsymbol{c}_e\boldsymbol{Y}_e^\dagger + \left[\boxed{\frac{\beta_L}{8\pi^2}\,X} - \frac{3\alpha_2^2}{4\pi^2}\,C_F^{(2)}\,\tilde{c}_{WW} - \frac{3\alpha_1^2}{4\pi^2}\,\mathcal{Y}_L^2\,\tilde{c}_{BB}\right]\mathbb{1}$$

$$\frac{d}{d\ln\mu}\,\boldsymbol{c}_e(\mu) = \frac{1}{16\pi^2}\left\{\boldsymbol{Y}_e^\dagger\boldsymbol{Y}_e, \boldsymbol{c}_e\right\} - \frac{1}{8\pi^2}\,\boldsymbol{Y}_e^\dagger\boldsymbol{c}_L\boldsymbol{Y}_e + \left[\boxed{\frac{\beta_e}{8\pi^2}\,X} + \frac{3\alpha_1^2}{4\pi^2}\,\mathcal{Y}_e^2\,\tilde{c}_{BB}\right]\mathbb{1}$$

with:

$$X = \mathrm{Tr}\left[3\boldsymbol{c}_Q\left(\boldsymbol{Y}_u\boldsymbol{Y}_u^\dagger - \boldsymbol{Y}_d\boldsymbol{Y}_d^\dagger\right) - 3\boldsymbol{c}_u\boldsymbol{Y}_u^\dagger\boldsymbol{Y}_u + 3\boldsymbol{c}_d\boldsymbol{Y}_d^\dagger\boldsymbol{Y}_d - \boldsymbol{c}_L\boldsymbol{Y}_e\boldsymbol{Y}_e^\dagger + \boldsymbol{c}_e\boldsymbol{Y}_e^\dagger\boldsymbol{Y}_e\right]$$

Lagrangian at the weak scale

Effective Lagrangian in the broken phase:

$$\mathcal{L}_{\rm eff}(\mu_w) = \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2}\,a^2 + \mathcal{L}_{\rm ferm}(\mu_w) + c_{GG}\,\frac{\alpha_s}{4\pi}\,\frac{a}{f}\,G_{\mu\nu}^a\,\tilde{G}^{\mu\nu,a} + c_{\gamma\gamma}\,\frac{\alpha}{4\pi}\,\frac{a}{f}\,F_{\mu\nu}\,\tilde{F}^{\mu\nu}$$

$$+ c_{\gamma Z}\,\frac{\alpha}{2\pi s_w c_w}\,\frac{a}{f}\,F_{\mu\nu}\,\tilde{Z}^{\mu\nu} + c_{ZZ}\,\frac{\alpha}{4\pi s_w^2 c_w^2}\,\frac{a}{f}\,Z_{\mu\nu}\,\tilde{Z}^{\mu\nu} + c_{WW}\,\frac{\alpha}{2\pi s_w^2}\,\frac{a}{f}\,W_{\mu\nu}^+\,\tilde{W}^{-\mu\nu}$$

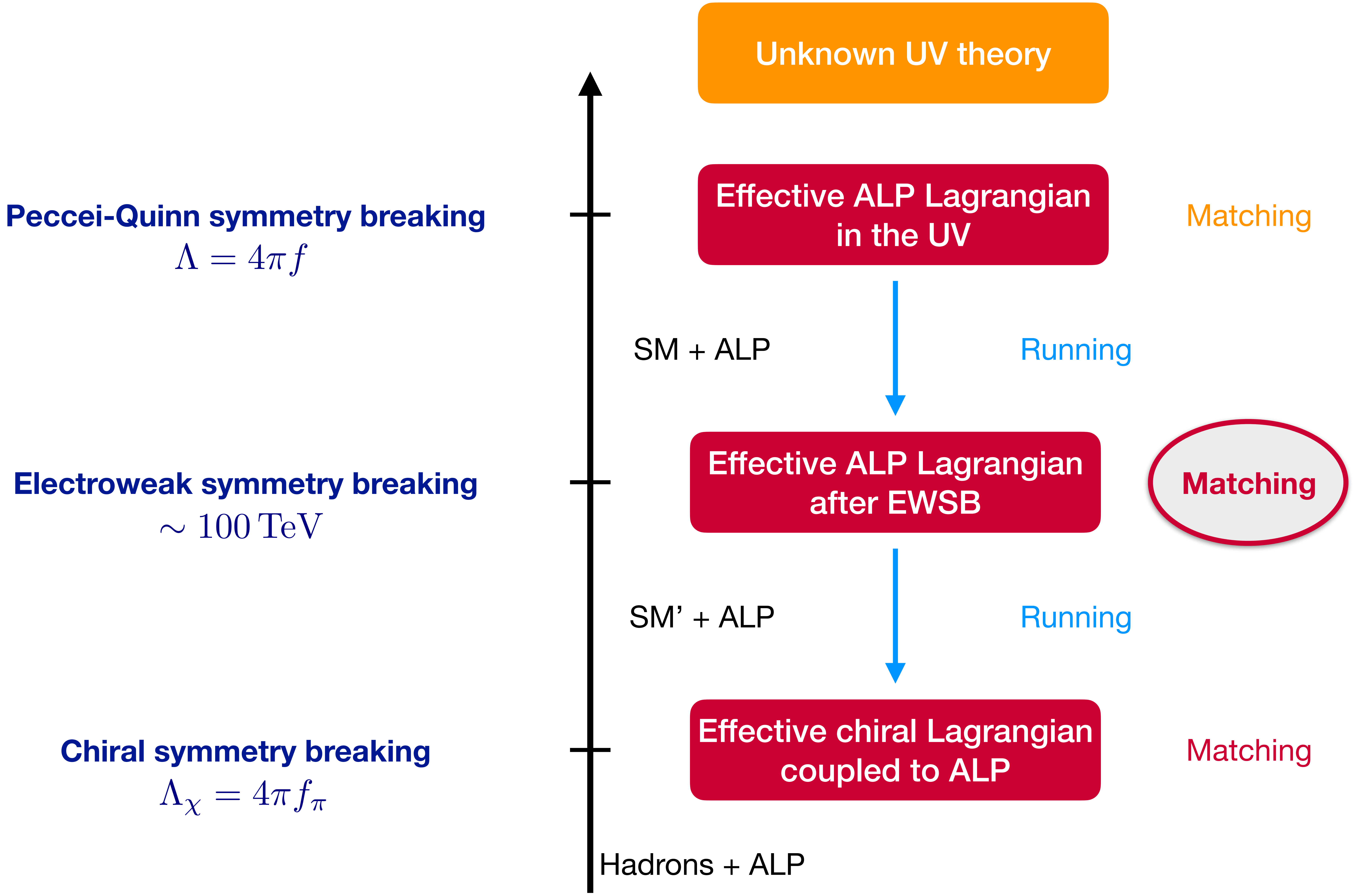
with:

matrices $\boldsymbol{c}_Q$, $\boldsymbol{c}_u$ etc. rotated to the mass basis

$$\mathcal{L}_{\rm ferm}(\mu_w) = \frac{\partial^\mu a}{f}\Big[\bar{u}_L\boldsymbol{k}_U\gamma_\mu u_L + \bar{u}_R\boldsymbol{k}_u\gamma_\mu u_R + \bar{d}_L\boldsymbol{k}_D\gamma_\mu d_L + \bar{d}_R\boldsymbol{k}_d\gamma_\mu d_R$$

$$+ \bar{\nu}_L\boldsymbol{k}_\nu\gamma_\mu \nu_L + \bar{e}_L\boldsymbol{k}_E\gamma_\mu e_L + \bar{e}_R\boldsymbol{k}_e\gamma_\mu e_R\Big]$$

In the next step, we integrate out the heavy particles *t*, *W*, *Z* and *h*.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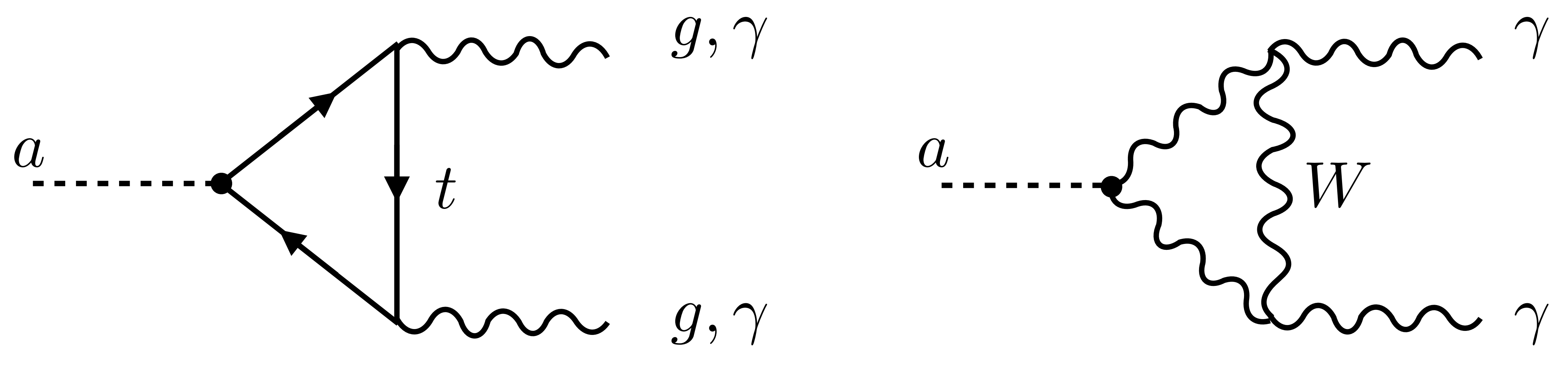
Weak-scale matching

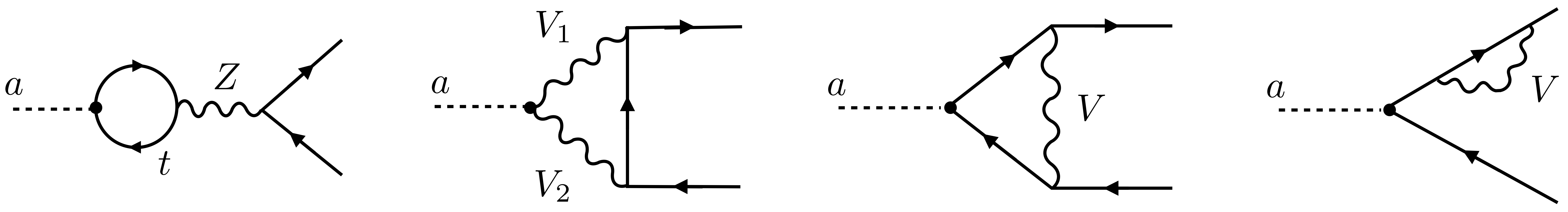
Matching contributions to the ALP-boson couplings are absent in the standard basis (for a light ALP):



$$\sim \frac{m_a^2}{m_t^2}\,, \quad \frac{m_a^2}{m_W^2}$$

[Bauer, MN, Thamm (2017)]

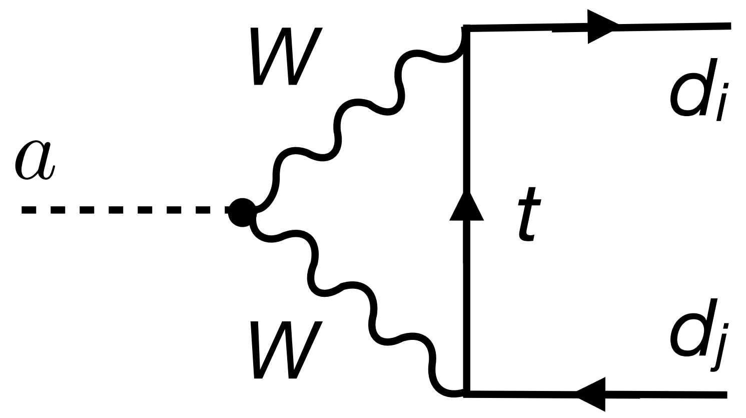
but there are non-trivial matching conditions to the ALP-fermion couplings:

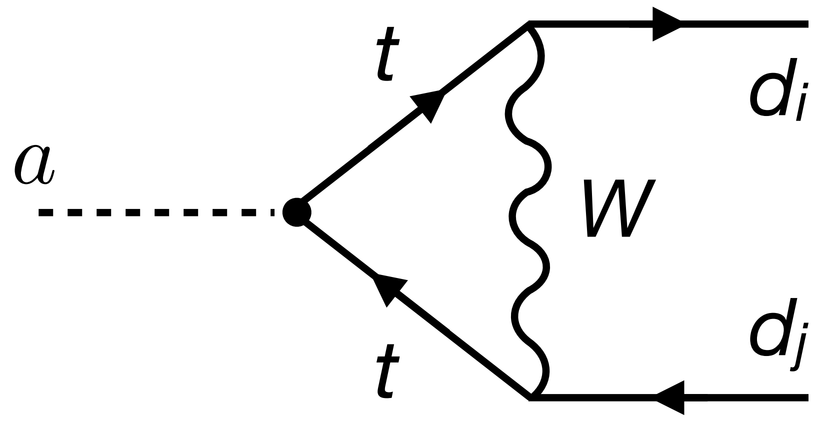


[Bauer, MN, Thamm (2017); Bauer, MN, Renner, Schnubel, Thamm (2020)]

Weak-scale matching

These include, in particular, flavor-violating contributions to k_D:





$$\begin{aligned}\left[\hat{\Delta}k_D(\mu_w)\right]_{ij} = \frac{y_t^2}{16\pi^2}\bigg\{ & V_{mi}^* V_{nj}\left[k_U(\mu_w)\right]_{mn}\left(\delta_{m3}+\delta_{n3}\right)\left[-\frac{1}{4}\ln\frac{\mu_w^2}{m_t^2}-\frac{3}{8}+\frac{3}{4}\frac{1-x_t+\ln x_t}{(1-x_t)^2}\right] \\ & + V_{3i}^* V_{3j}\left[k_U(\mu_w)\right]_{33} + V_{3i}^* V_{3j}\left[k_u(\mu_w)\right]_{33}\left[\frac{1}{2}\ln\frac{\mu_w^2}{m_t^2}-\frac{1}{4}-\frac{3}{2}\frac{1-x_t+\ln x_t}{(1-x_t)^2}\right] \\ & - \frac{3\alpha}{2\pi s_w^2}\, c_{WW}\, V_{3i}^* V_{3j}\,\frac{1-x_t+x_t\ln x_t}{(1-x_t)^2}\bigg\}\end{aligned}$$

[Bauer, MN, Renner, Schnubel, Thamm (2020)]

ALP couplings at the weak scale

Results for the flavor-diagonal couplings with $f = 1$ TeV and $\mu_w = m_t$:

$$\boxed{\mathcal{L}_{\text{ferm}}^{\text{diag}}(\mu) = \sum_{f \neq t} \frac{c_{ff}(\mu)}{2} \frac{\partial^\mu a}{f} \bar{f} \gamma_\mu \gamma_5 f}$$

with $\quad c_{f_i f_i}(\mu) = [k_f(\mu)]_{ii} - [k_F(\mu)]_{ii}$

We find:

$$c_{uu,cc}(\mu_w) \simeq c_{uu,cc}(\Lambda) - 0.116\, c_{tt}(\Lambda) - \left[6.35\, \tilde{c}_{GG}(\Lambda) + 0.19\, \tilde{c}_{WW}(\Lambda) + 0.02\, \tilde{c}_{BB}(\Lambda)\right] \cdot 10^{-3}$$

$$c_{dd,ss}(\mu_w) \simeq c_{dd,ss}(\Lambda) + 0.116\, c_{tt}(\Lambda) - \left[7.08\, \tilde{c}_{GG}(\Lambda) + 0.22\, \tilde{c}_{WW}(\Lambda) + 0.005\, \tilde{c}_{BB}(\Lambda)\right] \cdot 10^{-3}$$

$$c_{bb}(\mu_w) \simeq c_{bb}(\Lambda) + 0.097\, c_{tt}(\Lambda) - \left[7.02\, \tilde{c}_{GG}(\Lambda) + 0.19\, \tilde{c}_{WW}(\Lambda) + 0.005\, \tilde{c}_{BB}(\Lambda)\right] \cdot 10^{-3}$$

$$c_{e_i e_i}(\mu_w) \simeq c_{e_i e_i}(\Lambda) + 0.116\, c_{tt}(\Lambda) - \left[0.37\, \tilde{c}_{GG}(\Lambda) + 0.22\, \tilde{c}_{WW}(\Lambda) + 0.05\, \tilde{c}_{BB}(\Lambda)\right] \cdot 10^{-3}$$

ALP couplings at the weak scale

Corresponding results with $f = 10^9$ TeV:

$$c_{uu,cc}(m_t) \simeq c_{uu,cc}(\Lambda) - 0.350\, c_{tt}(\Lambda) - \Big[12.6\, \tilde{c}_{GG}(\Lambda) + 0.84\, \tilde{c}_{WW}(\Lambda) + 0.10\, \tilde{c}_{BB}(\Lambda) \Big] \cdot 10^{-3}$$

$$c_{dd,ss}(m_t) \simeq c_{dd,ss}(\Lambda) + 0.353\, c_{tt}(\Lambda) - \Big[16.8\, \tilde{c}_{GG}(\Lambda) + 1.30\, \tilde{c}_{WW}(\Lambda) + 0.07\, \tilde{c}_{BB}(\Lambda) \Big] \cdot 10^{-3}$$

$$c_{bb}(m_t) \simeq c_{bb}(\Lambda) + 0.294\, c_{tt}(\Lambda) - \Big[16.5\, \tilde{c}_{GG}(\Lambda) + 1.23\, \tilde{c}_{WW}(\Lambda) + 0.06\, \tilde{c}_{BB}(\Lambda) \Big] \cdot 10^{-3}$$

$$c_{e_i e_i}(m_t) \simeq c_{e_i e_i}(\Lambda) + 0.352\, c_{tt}(\Lambda) - \Big[2.09\, \tilde{c}_{GG}(\Lambda) + 1.30\, \tilde{c}_{WW}(\Lambda) + 0.38\, \tilde{c}_{BB}(\Lambda) \Big] \cdot 10^{-3}$$

Note that **all ALP couplings** enter via the matching conditions:

$$\boxed{\begin{aligned} \tilde{c}_{GG} &= c_{GG} + T_F \operatorname{Tr}\left(\boldsymbol{c}_u + \boldsymbol{c}_d - N_L\, \boldsymbol{c}_Q \right), \\ \tilde{c}_{WW} &= c_{WW} - T_F \operatorname{Tr}\left(N_c\, \boldsymbol{c}_Q + \boldsymbol{c}_L \right), \\ \tilde{c}_{BB} &= c_{BB} + \operatorname{Tr}\Big[N_c \left(\mathcal{Y}_u^2\, \boldsymbol{c}_u + \mathcal{Y}_d^2\, \boldsymbol{c}_d - N_L\, \mathcal{Y}_Q^2\, \boldsymbol{c}_Q \right) + \mathcal{Y}_e^2\, \boldsymbol{c}_e - N_L\, \mathcal{Y}_L^2\, \boldsymbol{c}_L \Big] \end{aligned}}$$

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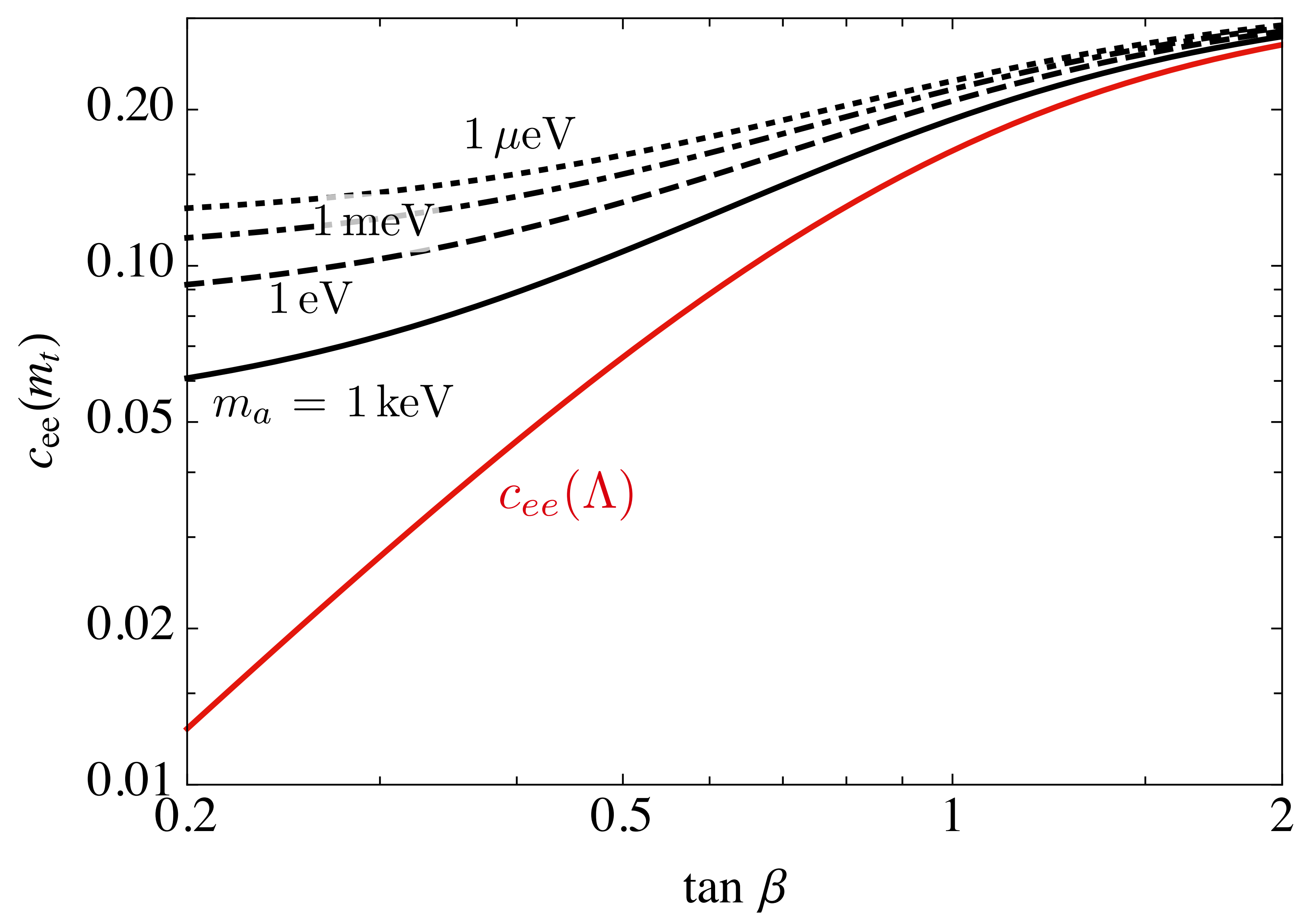
$$c_{e_ie_i}(m_t) \simeq c_{e_ie_i}(\Lambda) + 0.352\, c_{tt}(\Lambda) - \left[2.09\, \tilde{c}_{GG}(\Lambda) + 1.30\, \tilde{c}_{WW}(\Lambda) + 0.38\, \tilde{c}_{BB}(\Lambda)\right] \cdot 10^{-3}$$



ALP-electron coupling in the DFSZ model for different values of $\tan\beta = v_u/v_d$

The one-loop admixture of c_{tt} into all ALP-fermion couplings can have a very important effect, since it induces an ALP-lepton coupling even in lepto-phobic ALP models

[Bauer, MN, Renner, Schnubel, Thamm (2020)]

ALP couplings at the weak scale

Flavor off-diagonal coefficients with $f = 1$ TeV and $\mu_w = m_t$:

$$\boxed{\mathcal{L}_{\text{ferm}}^{\text{FCNC}}(\mu) = -\frac{ia}{2f} \sum_f \left[(m_{f_i} - m_{f_j})\,(k_f + k_F)_{ij}\, \bar{f}_i f_j + (m_{f_i} + m_{f_j})\,(k_f - k_F)_{ij}\, \bar{f}_i \gamma_5 f_j \right]}$$

with:

$$[k_u(\mu_w)]_{ij} = [k_u(\Lambda)]_{ij}\,; \quad i,j \neq 3\,,$$

$$[k_U(\mu_w)]_{ij} = [k_U(\Lambda)]_{ij}\,; \quad i,j \neq 3\,,$$

(top quark has been integrated out)

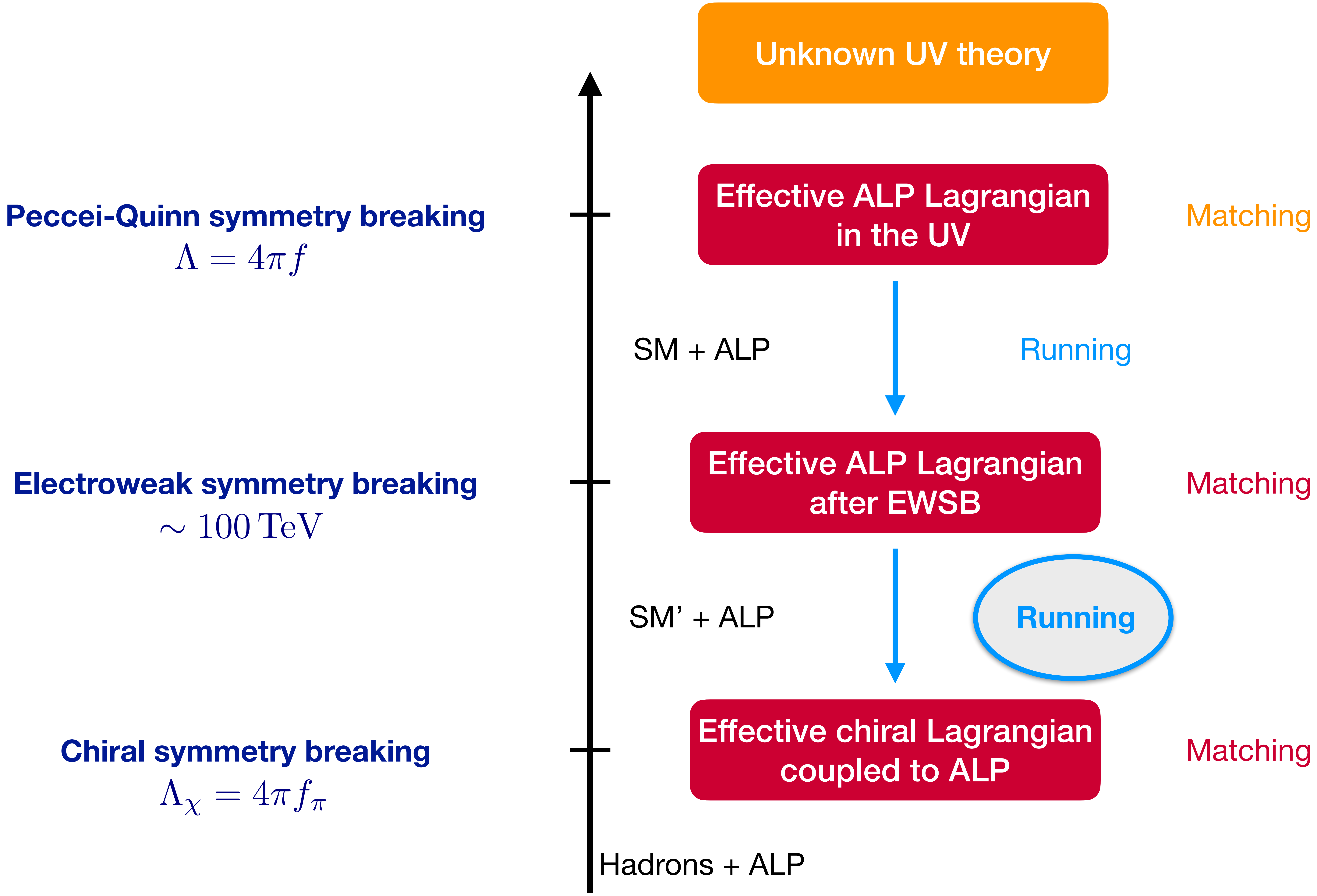
$$[k_d(\mu_w)]_{ij} = [k_d(\Lambda)]_{ij}\,,$$

$$[k_e(\mu_w)]_{ij} = [k_e(\Lambda)]_{ij}\,,$$

$$[k_L(\mu_w)]_{ij} = [k_L(\Lambda)]_{ij}\,.$$

RG running generates MFV-type flavor violation in the left-handed down-quark sector

$$[k_D(m_t)]_{ij} \simeq [k_D(\Lambda)]_{ij} + 0.019\, V_{ti}^* V_{tj} \left[c_{tt}(\Lambda) - 0.0032\, \tilde{c}_{GG}(\Lambda) - 0.0057\, \tilde{c}_{WW}(\Lambda) \right]$$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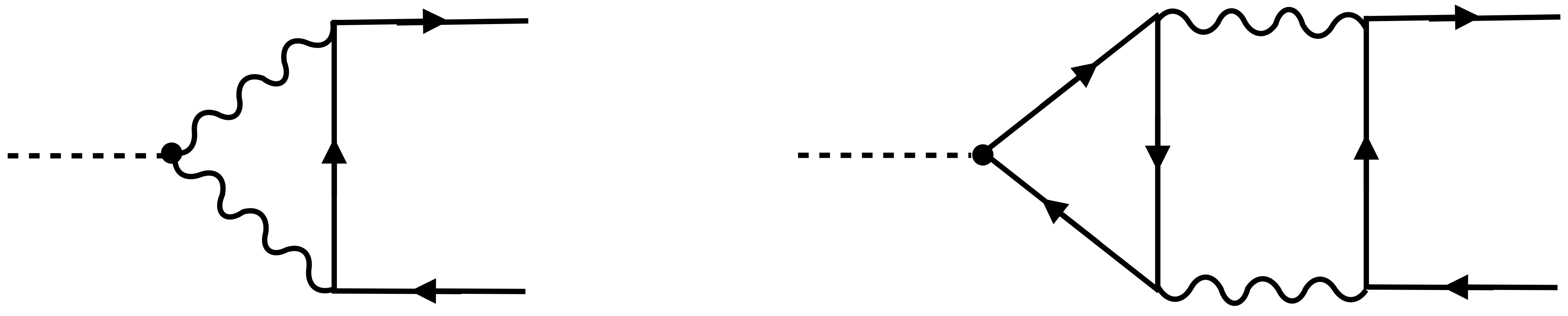
Evolution below the weak scale

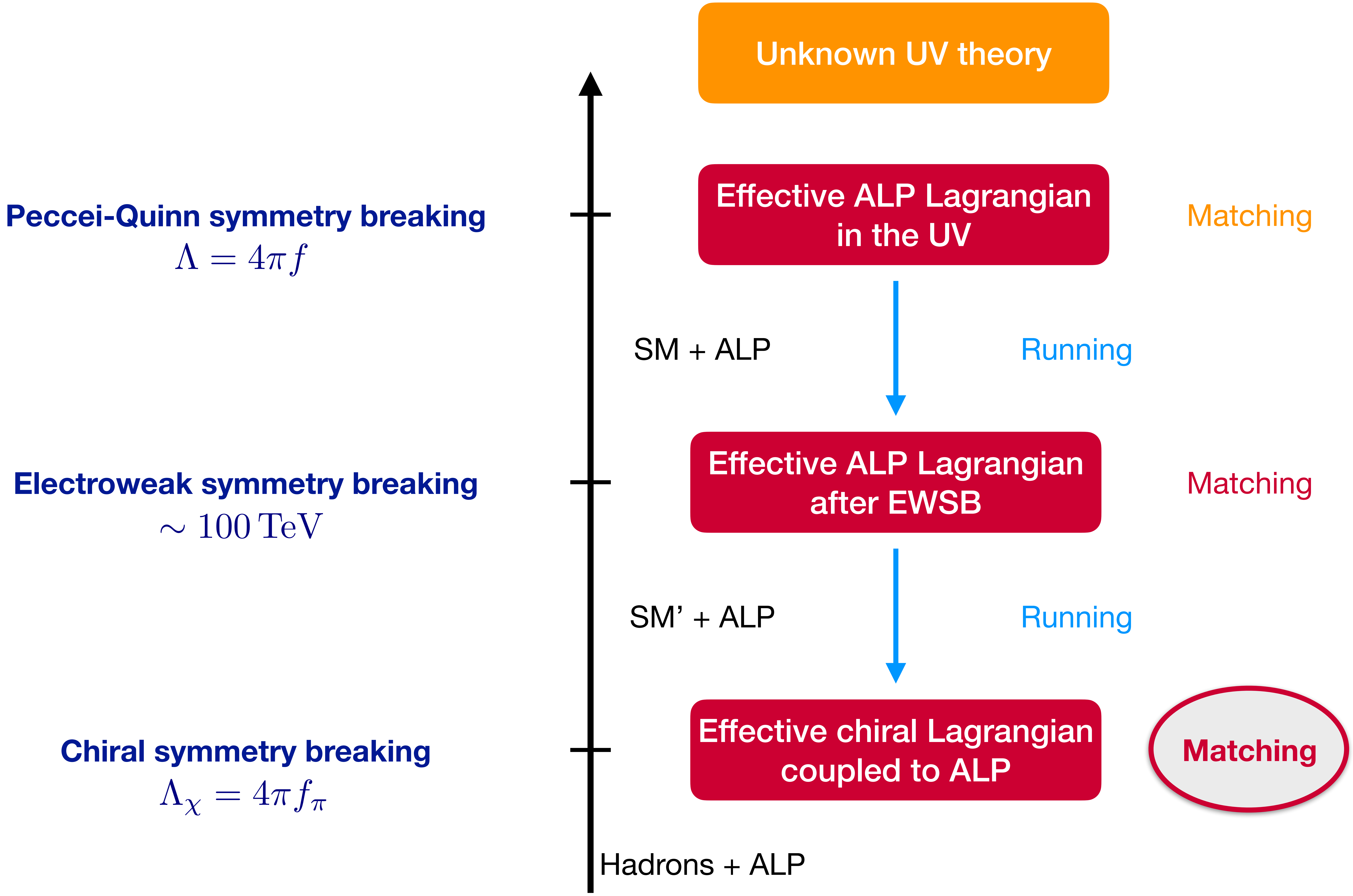
In this case only gluon and photon loops contribute:



We find numerically with $\mu_0 = 2$ GeV:

$$c_{qq}(\mu_0) = c_{qq}(m_t) + \Big[3.0\,\tilde{c}_{GG}(\Lambda) - 1.4\,c_{tt}(\Lambda) - 0.6\,c_{bb}(\Lambda)\Big]\cdot 10^{-2}$$
$$+\,Q_q^2\Big[3.9\,\tilde{c}_{\gamma\gamma}(\Lambda) - 4.7\,c_{tt}(\Lambda) - 0.2\,c_{bb}(\Lambda)\Big]\cdot 10^{-5}\,,$$
$$c_{\ell\ell}(\mu_0) = c_{\ell\ell}(m_t) + \Big[3.9\,\tilde{c}_{\gamma\gamma}(\Lambda) - 4.7\,c_{tt}(\Lambda) - 0.2\,c_{bb}(\Lambda)\Big]\cdot 10^{-5}\,.$$

[Bauer, MN, Renner, Schnubel, Thamm (2020)]


Unknown UV theory
Peccei-Quinn symmetry breaking
$\Lambda = 4\pi f$
Effective ALP Lagrangian in the UV
Matching
SM + ALP
Running
Electroweak symmetry breaking
$\sim 100\,\text{TeV}$
Effective ALP Lagrangian after EWSB
Matching
SM’ + ALP
Running
Chiral symmetry breaking
$\Lambda_\chi = 4\pi f_\pi$
Effective chiral Lagrangian coupled to ALP
Matching
Hadrons + ALP

Fun facts about $K \to \pi a$

Matching to the chiral Lagrangian

Georgi, Kaplan, Randall (1986) have developed a model-independent chiral Lagrangian approach valid for any ALP model

In the quark mass basis, the starting point is (at $\mu_\chi \approx 4\pi f_\pi$):

$$\begin{aligned}\mathcal{L}_{\rm eff} &= \mathcal{L}_{\rm QCD} + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2}\,a^2 \\ &\quad + c_{GG}\,\frac{\alpha_s}{4\pi}\,\frac{a}{f}\,G_{\mu\nu}^a\,\tilde{G}^{\mu\nu,a} + c_{\gamma\gamma}\,\frac{\alpha}{4\pi}\,\frac{a}{f}\,F_{\mu\nu}\,\tilde{F}^{\mu\nu} \\ &\quad + \frac{\partial^\mu a}{f}\left(\bar{q}_L\,\boldsymbol{k}_Q\,\gamma_\mu\,q_L + \bar{q}_R\,\boldsymbol{k}_q\,\gamma_\mu\,q_R + \dots\right)\end{aligned}$$

three light quarks *u*, *d*, *s*

Matching to the chiral Lagrangian

To bosonize this theory, one first eliminates the ALP-gluon coupling using the chiral rotation: [Srednicki (1985); Georgi, Kaplan, Randall (1986); Krauss, Wise (1986); Bardeen, Peccei, Yanagida (1987)]

$$q(x) \to \exp\left[-i\left(\boldsymbol{\delta}_q + \boldsymbol{\kappa}_q\gamma_5\right) c_{GG}\,\frac{a(x)}{f}\right] q(x) \qquad \text{with} \qquad \operatorname{Tr}\boldsymbol{\kappa}_q = \kappa_u + \kappa_d + \kappa_s = 1$$

diagonal in the quark mass basis

Modified quark mass matrix and ALP couplings:

$$\hat{\boldsymbol{m}}_q(a) = \exp\left(-2i\boldsymbol{\kappa}_q\,c_{GG}\,\frac{a}{f}\right)\boldsymbol{m}_q$$

$$\hat{c}_{\gamma\gamma} = c_{\gamma\gamma} - 2N_c\,c_{GG}\operatorname{Tr}\boldsymbol{Q}^2\,\boldsymbol{\kappa}_q$$

$$\left.\begin{aligned} \hat{\boldsymbol{k}}_Q(a) &= e^{i\boldsymbol{\phi}_q^-\,a/f}\left(\boldsymbol{k}_Q + \boldsymbol{\phi}_q^-\right)e^{-i\boldsymbol{\phi}_q^-\,a/f} \\ \hat{\boldsymbol{k}}_q(a) &= e^{i\boldsymbol{\phi}_q^+\,a/f}\left(\boldsymbol{k}_q + \boldsymbol{\phi}_q^+\right)e^{-i\boldsymbol{\phi}_q^+\,a/f} \end{aligned}\right\} \quad \text{with} \quad \boldsymbol{\phi}_q^\pm = c_{GG}\left(\boldsymbol{\delta}_q \pm \boldsymbol{\kappa}_q\right)$$

[Bauer, MN, Renner, Schnubel, Thamm (2021)]

Matching to the chiral Lagrangian

- The light pseudoscalar mesons are described by $\boldsymbol{\Sigma}(x) = \exp\left[\frac{i\sqrt{2}}{f_\pi}\,\lambda^a\,\pi^a(x)\right]$
- The derivative ALP couplings to fermions are included in the covariant derivative:

$$iD_\mu\boldsymbol{\Sigma} = i\partial_\mu\boldsymbol{\Sigma} + e\,A_\mu\,[\boldsymbol{Q},\boldsymbol{\Sigma}] + \frac{\partial_\mu a}{f}\left(\hat{\boldsymbol{k}}_Q\,\boldsymbol{\Sigma} - \boldsymbol{\Sigma}\,\hat{\boldsymbol{k}}_q\right)$$

[Bauer, MN, Renner, Schnubel, Thamm (2021)]

- Leading-order effective chiral Lagrangian:

$$\mathcal{L}_{\text{eff}}^{\chi} = \frac{f_\pi^2}{8}\,\text{Tr}\left[\boldsymbol{D}^\mu\boldsymbol{\Sigma}\,(\boldsymbol{D}_\mu\boldsymbol{\Sigma})^\dagger\right] + \frac{f_\pi^2}{4}\,B_0\,\text{Tr}\left[\hat{\boldsymbol{m}}_q(a)\,\boldsymbol{\Sigma}^\dagger + \text{h.c.}\right]$$
$$+\,\frac{1}{2}\,\partial^\mu a\,\partial_\mu a - \frac{m_{a,0}^2}{2}\,a^2 + \hat{c}_{\gamma\gamma}\,\frac{\alpha}{4\pi}\,\frac{a}{f}\,F_{\mu\nu}\,\tilde{F}^{\mu\nu}$$

[Gasser, Leutwyler (1985)]

- Periodic potential breaks the shift symmetry and provides a mass for the axion (QCD instantons) [Weinberg (1978); Wilczek (1978)]

Weak decay $K \to \pi a$

- Strongest particle-physics constraint on ALP couplings for mass range $m_a < m_K - m_\pi \approx 354$ MeV

- Despite a 35-year history, we find that even nowadays most papers on this process are based on inconsistent equations

- The chiral implementation of the leading SU(3)-octet weak-interaction operator is: [Bernard, Draper, Soni, Politzer, Wise (1985); Crewther (1986); Kambor, Missimer, Wyler (1990)]

$$\mathcal{L}_{\text{weak}} = -\frac{4G_F}{\sqrt{2}}\, V_{ud}^* V_{us}\, g_8\, \left[L_\mu L^\mu\right]^{32}$$

where L_μ^{ij} is the chiral representation of the left-handed current $\bar{q}_L^i \gamma_\mu q_L^j$

Weak decay $K \to \pi a$

Georgi, Kaplan, Randall used:

$$L_\mu^{ij} = -\frac{i f_\pi^2}{4}\, e^{i(\phi_{q_i}^- - \phi_{q_j}^-)\, a/f}\, \big[\boldsymbol{\Sigma}\, \partial_\mu \boldsymbol{\Sigma}^\dagger\big]^{ij}$$

where the phase factor results from the chiral rotation, but the Noether theorem gives instead: [Bauer, MN, Renner, Schnubel, Thamm (2021)]

$$\boxed{\begin{aligned} L_\mu^{ji} &= -\frac{i f_\pi^2}{4}\, e^{i(\phi_{q_i}^- - \phi_{q_j}^-)\, a/f}\, \big[\boldsymbol{\Sigma}\, (\boldsymbol{D}_\mu \boldsymbol{\Sigma})^\dagger\big]^{ji} \\ &\ni -\frac{i f_\pi^2}{4} \left[1 + i(\delta_{q_i} - \delta_{q_j} - \kappa_{q_i} + \kappa_{q_j})\, c_{GG}\, \frac{a}{f}\right] \big[\boldsymbol{\Sigma}\, \partial_\mu \boldsymbol{\Sigma}^\dagger\big]^{ji} \\ &\quad + \frac{f_\pi^2}{4}\, \frac{\partial^\mu a}{f}\, \big[\hat{\boldsymbol{k}}_Q - \boldsymbol{\Sigma}\, \hat{\boldsymbol{k}}_q\, \boldsymbol{\Sigma}^\dagger\big]^{ji} \qquad \leftarrow \text{crucial extra terms!} \end{aligned}}$$

Weak decay $K \to \pi a$

Cancellation of auxiliary parameters:

$$D_1 \ni \frac{N_8}{2f}\, c_{GG} \,(\kappa_u - \kappa_d)\,(m_\pi^2 - m_a^2)$$

$$D_2 \ni -\frac{N_8}{6f}\, c_{GG}\,(2m_K^2 + m_\pi^2 - 3m_a^2)\,(\kappa_u + \kappa_d - 2\kappa_s)$$

$$D_3 \ni \frac{N_8}{2f}\, c_{GG} \Big[-(\delta_d - \delta_s - \kappa_d + \kappa_s)\,(m_K^2 + m_\pi^2 - m_a^2) + (\delta_u - \delta_d + \kappa_u + \kappa_s)\,(m_K^2 - m_\pi^2 + m_a^2) + (\delta_u - \delta_s + \kappa_u + \kappa_d)\,(m_K^2 - m_\pi^2 - m_a^2) \Big]$$

previously omitted contributions

$$D_4 \ni -\frac{N_8}{f}\, c_{GG}\, m_K^2\,(\delta_u - \delta_d)$$

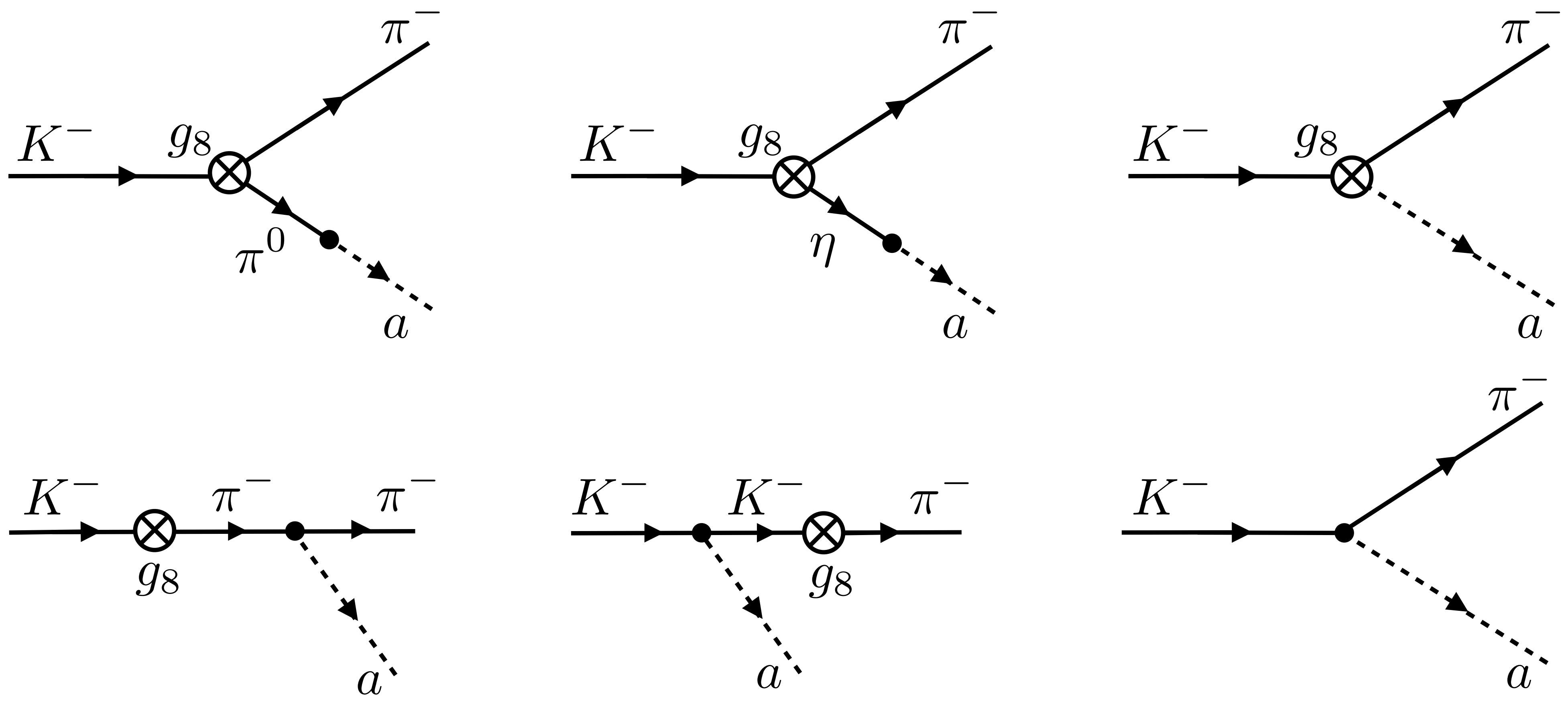
$$D_5 \ni \frac{N_8}{f}\, c_{GG}\, m_\pi^2\,(\delta_u - \delta_s)$$

with:

$$N_8 = -\frac{G_F}{\sqrt{2}}\, V_{ud}^* V_{us}\, g_8\, f_\pi^2$$

[Bauer, MN, Renner, Schnubel, Thamm (2021)]

ALP-pion mixing | ALP-η mixing | “direct” contribution



Final-state radiation | Initial-state radiation | “direct” flavor-changing ALP contribution

- Find that omitted contributions have a large effect (parametrically dominant terms)
- Including only the first two diagrams (ALP-meson mixing) gives an uncontrolled approximation (except in very special cases)

Weak decay $K \to \pi a$

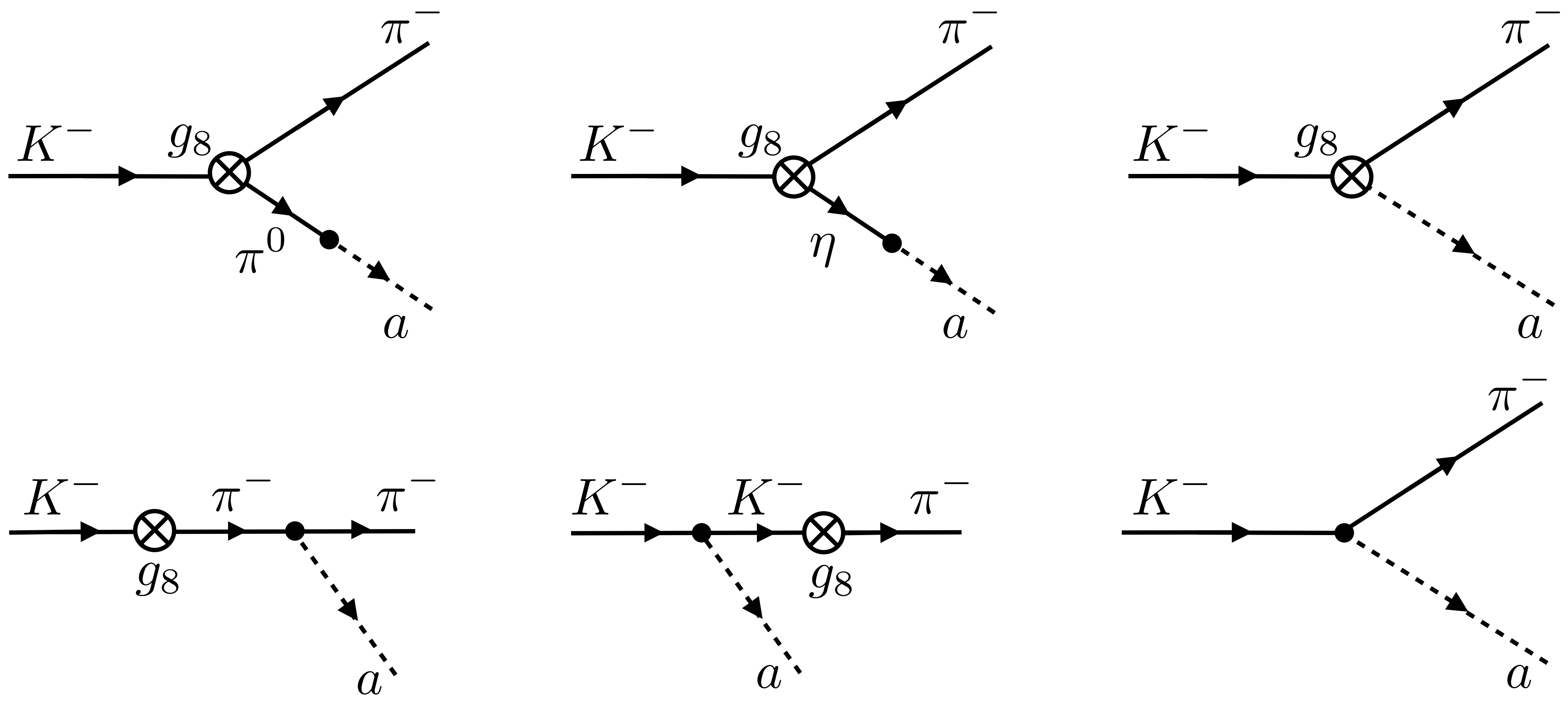
Decay amplitude:

$$i\mathcal{A}_{K^-\to\pi^- a} = \frac{N_8}{4f}\bigg[16\,c_{GG}\,\frac{(m_K^2-m_\pi^2)(m_K^2-m_a^2)}{4m_K^2-m_\pi^2-3m_a^2}$$
$$+\,6\,(c_{uu}+c_{dd}-2c_{ss})\,m_a^2\,\frac{m_K^2-m_a^2}{4m_K^2-m_\pi^2-3m_a^2}$$
$$+\,(2c_{uu}+c_{dd}+c_{ss})\,(m_K^2-m_\pi^2-m_a^2)+4c_{ss}m_a^2$$
$$+\,(k_d+k_D-k_s-k_S)\,(m_K^2+m_\pi^2-m_a^2)\bigg]$$
$$-\,\frac{m_K^2-m_\pi^2}{2f}\,[k_q+k_Q]^{23}$$

with:

$$N_8 = -\frac{G_F}{\sqrt{2}}\,V_{ud}^*V_{us}\,g_8\,f_\pi^2$$

[Bauer, MN, Renner, Schnubel, Thamm (2021)]



Georgi, Kaplan and Randall have only considered the axion-gluon coupling c_{GG} and find a result smaller by a factor

$$\frac{m_u}{2(m_u+m_d)} \approx 0.16$$

$K \to \pi a$ phenomenology

Expressing the ALP couplings in terms of the couplings at the scale $\Lambda = 4\pi f$ with $f = 1$ TeV, and assuming MFV, we find:

strong-interaction phase of g_8

$$|\mathcal{A}_{K^- \to \pi^- a}| \simeq 10^{-11}\,\text{GeV}\left[\frac{1\,\text{TeV}}{f}\right] \times \Big[e^{i\delta_8}\left(3.58\,c_{GG} + 1.79\,c_{uu}(\Lambda) + 1.81\,c_{dd}(\Lambda)\right)$$

$$+\, e^{i\alpha}\left(-\,65.8\,c_{uu}(\Lambda) + 0.32\,c_{dd}(\Lambda) + 0.21\,c_{GG} + 0.38\,c_{WW}\right)$$

weak phase of V_{td}^*

$$-\,1.12\cdot 10^{7}\,k_D^{12}(\Lambda)\Big]$$

← proportional to $V_{td}^* V_{ts}$ in MFV

The coefficients refer to $m_a = 0$, but they vary by less than 10% over the entire allowed mass range. Two "benchmarks": [see e.g.: Gori, Perez, Tobioka (2020)]

- only $c_{GG} \neq 0$: "indirect" contribution (g_8) dominates
- only $c_{WW} \neq 0$: "direct" contribution (from RG running) dominates

$K \to \pi a$ phenomenology

More generally, one can derive bounds $|c_{ii}|/f < \left[\Lambda_{ii}^{\mathrm{eff}}\right]^{-1}$ for all relevant ALP couplings using the NA62 upper limit $\mathrm{Br}(K^- \to \pi^- X) < 2.0 \cdot 10^{-10}$ (90% CL), which implies:

c_{ii}	c_{GG}	c_{WW}	c_{uu}	c_{dd}	k_D^{12}	$k_D^{12}/\lvert V_{td}V_{ts}\rvert$
$\Lambda_{ii}^{\mathrm{eff}}$ [TeV]	61.3	6.5	1126	31.0	$1.9 \cdot 10^8$	60 000

- very strong bounds on flavor-changing ALP couplings in the UV
- strong bounds on ALP couplings to fermions (c_u or c_Q)
- relatively strong bounds on ALP-boson couplings

Flavor benchmarks

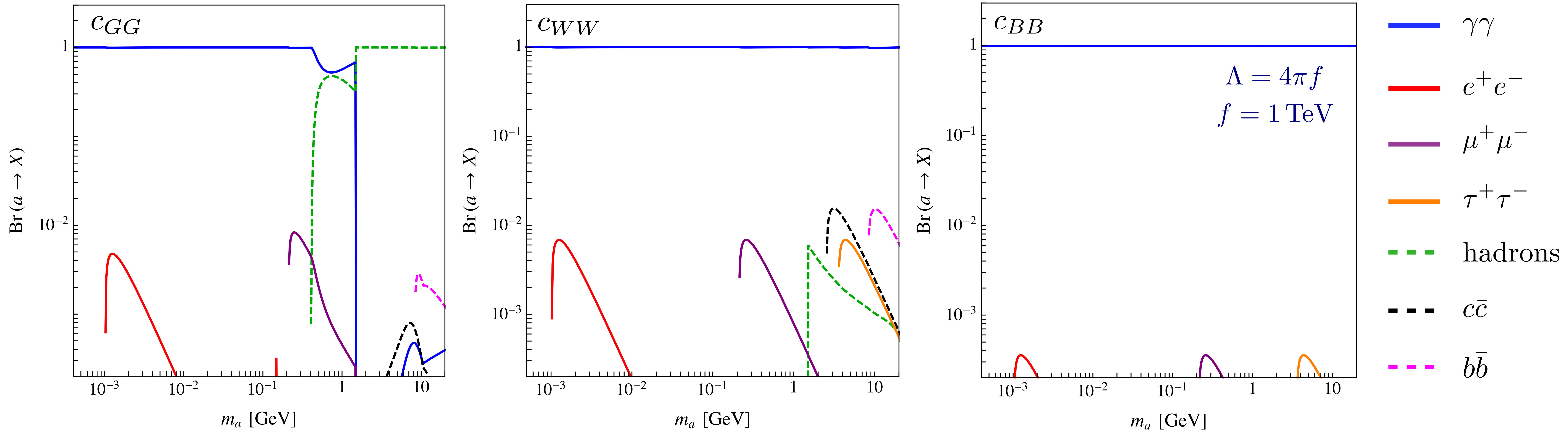
Flavor physics benchmarks

- RG evolution effects have a profound impact on phenomenology, for instance in flavor physics
- General lesson: no ALP couplings can be avoided !
- Below we consider **8 benchmark scenarios**, starting with a **single ALP coupling in the UV** (at $\Lambda = 4\pi f$) and assuming **flavor universality**
- We then calculate the contributions to various flavor observables and derived bounds on the UV couplings as a function of the ALP mass
- In this process, we carefully account for the effects of the ALP lifetime and its various decay modes

based on ongoing work with M. Bauer, S. Renner, M. Schnubel & A. Thamm

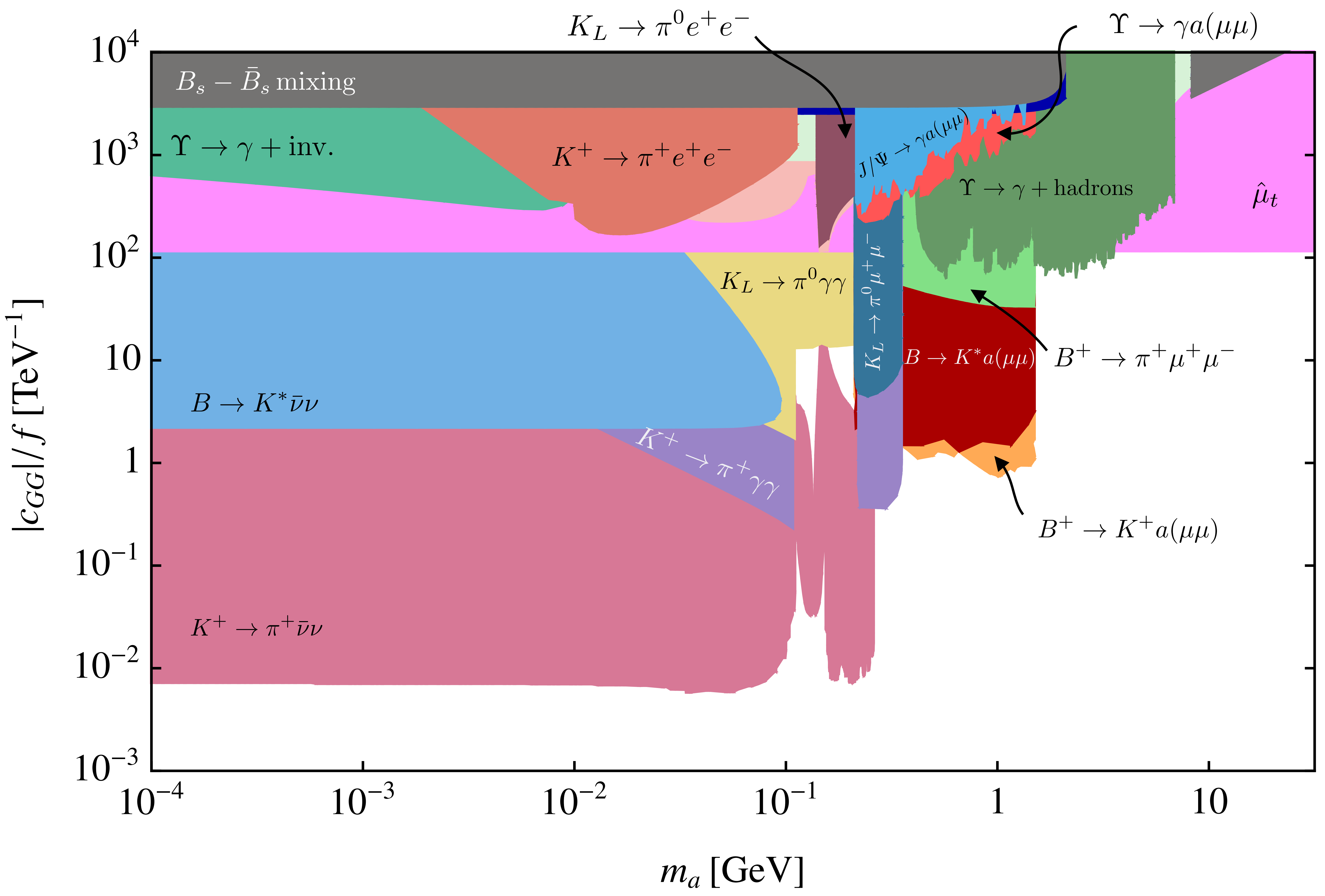
Flavor physics benchmarks

ALP branching fractions in the benchmarks with a single non-vanishing ALP-gauge boson coupling at the UV scale: [Bauer, MN, Thamm (2017)]



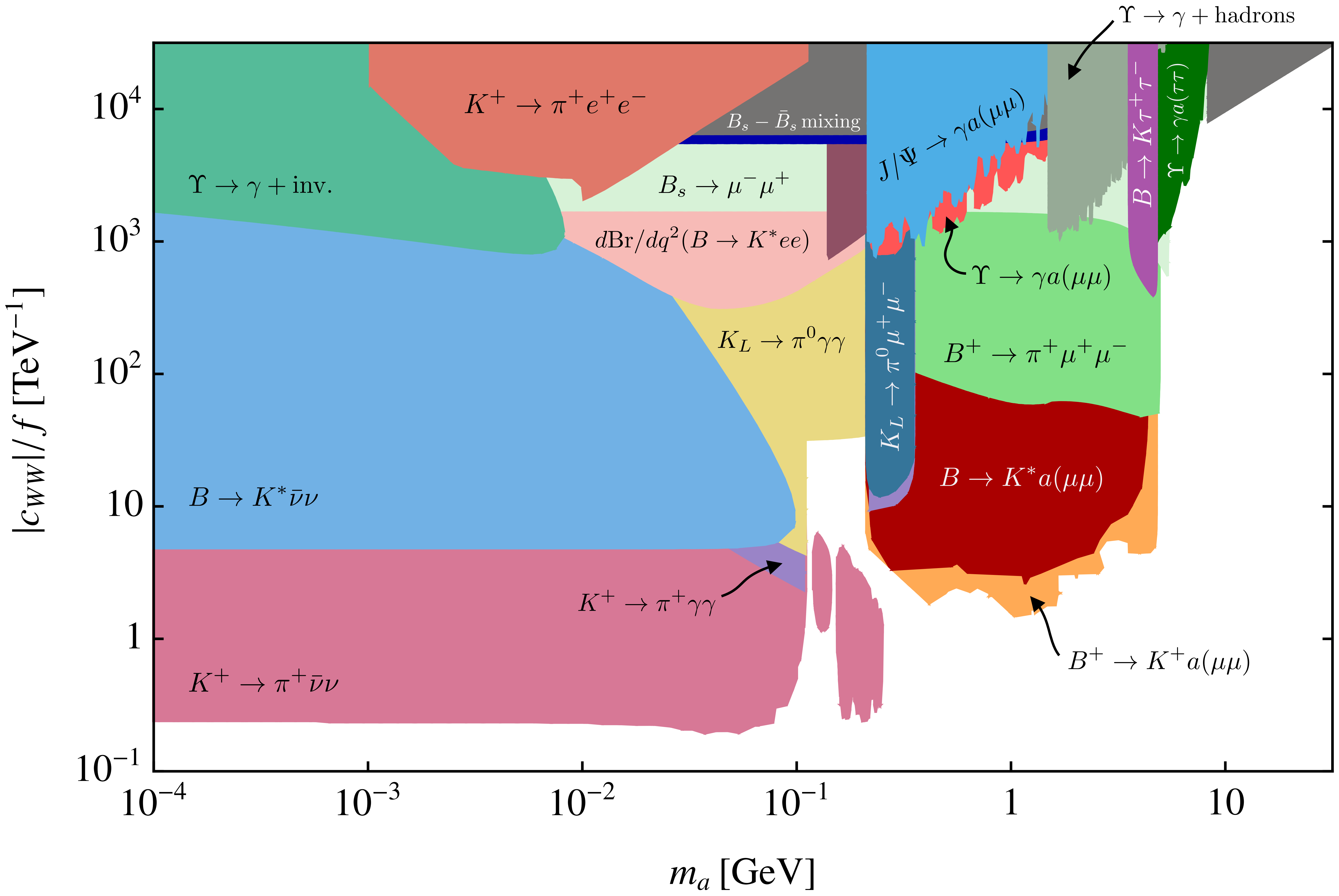
Flavor physics benchmarks

ALP-gluon coupling in the UV:



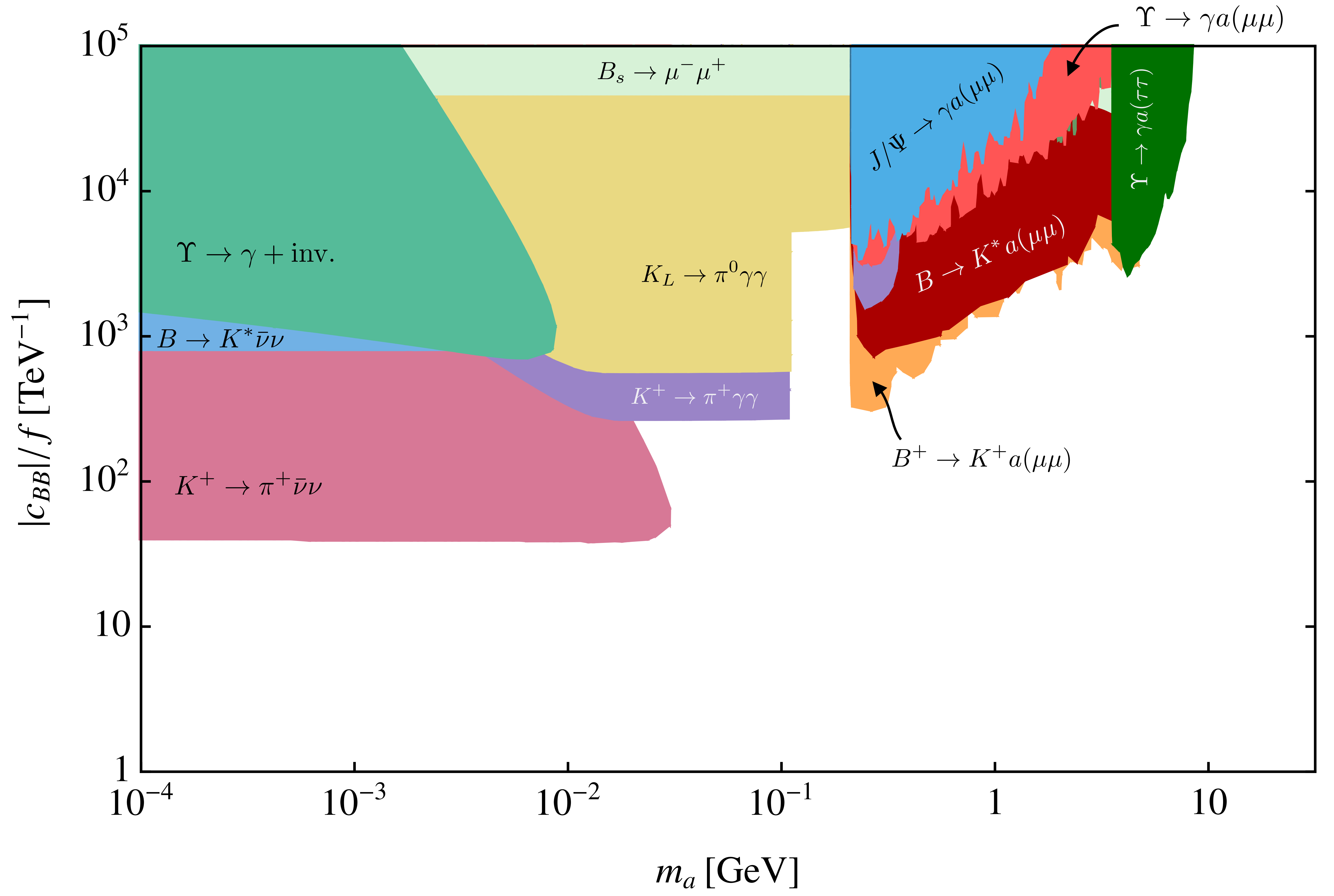
Flavor physics benchmarks

ALP-*W* coupling in the UV (note change in scale):



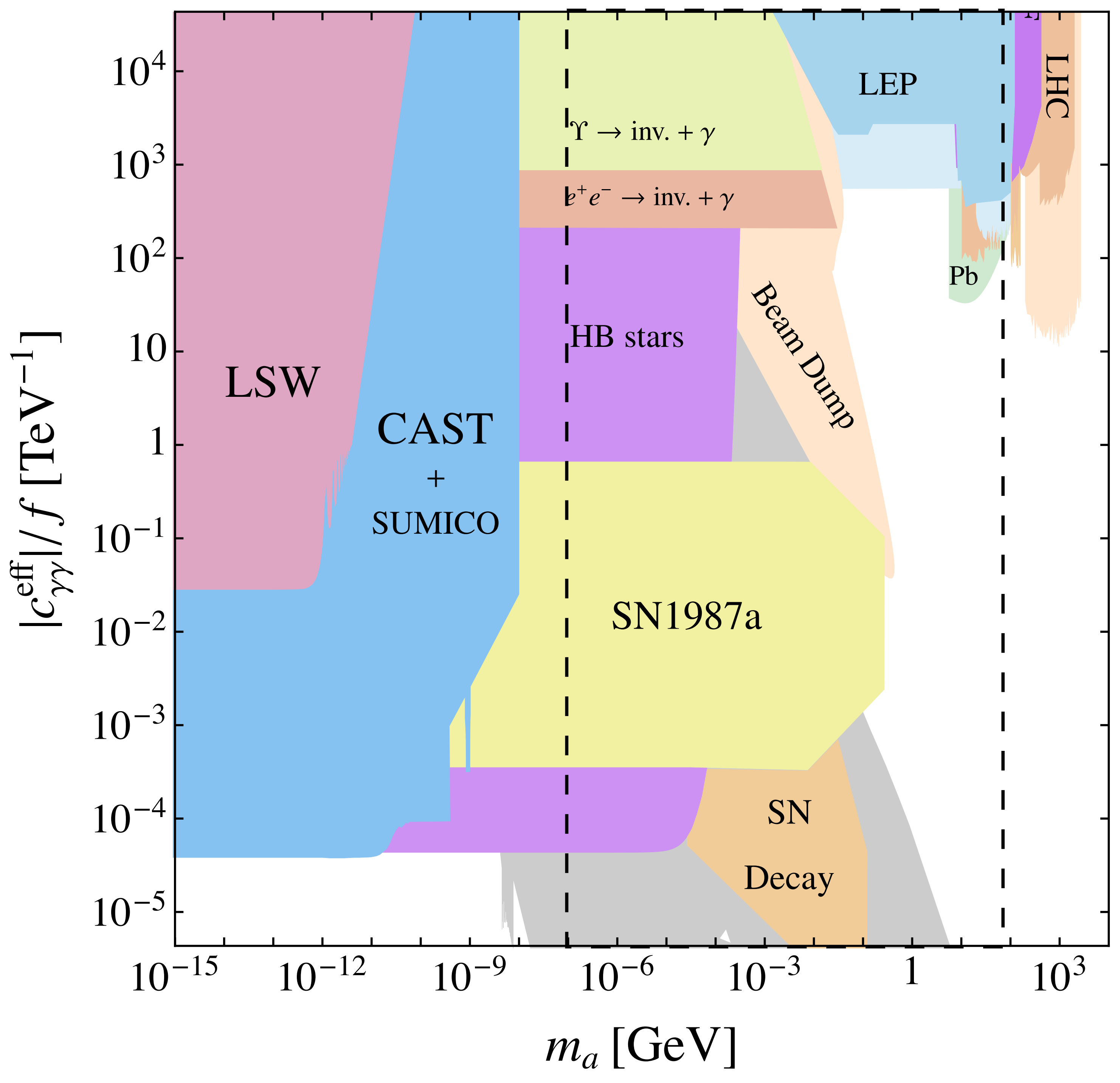
Flavor physics benchmarks

ALP-*B* coupling in the UV (note change in scale):

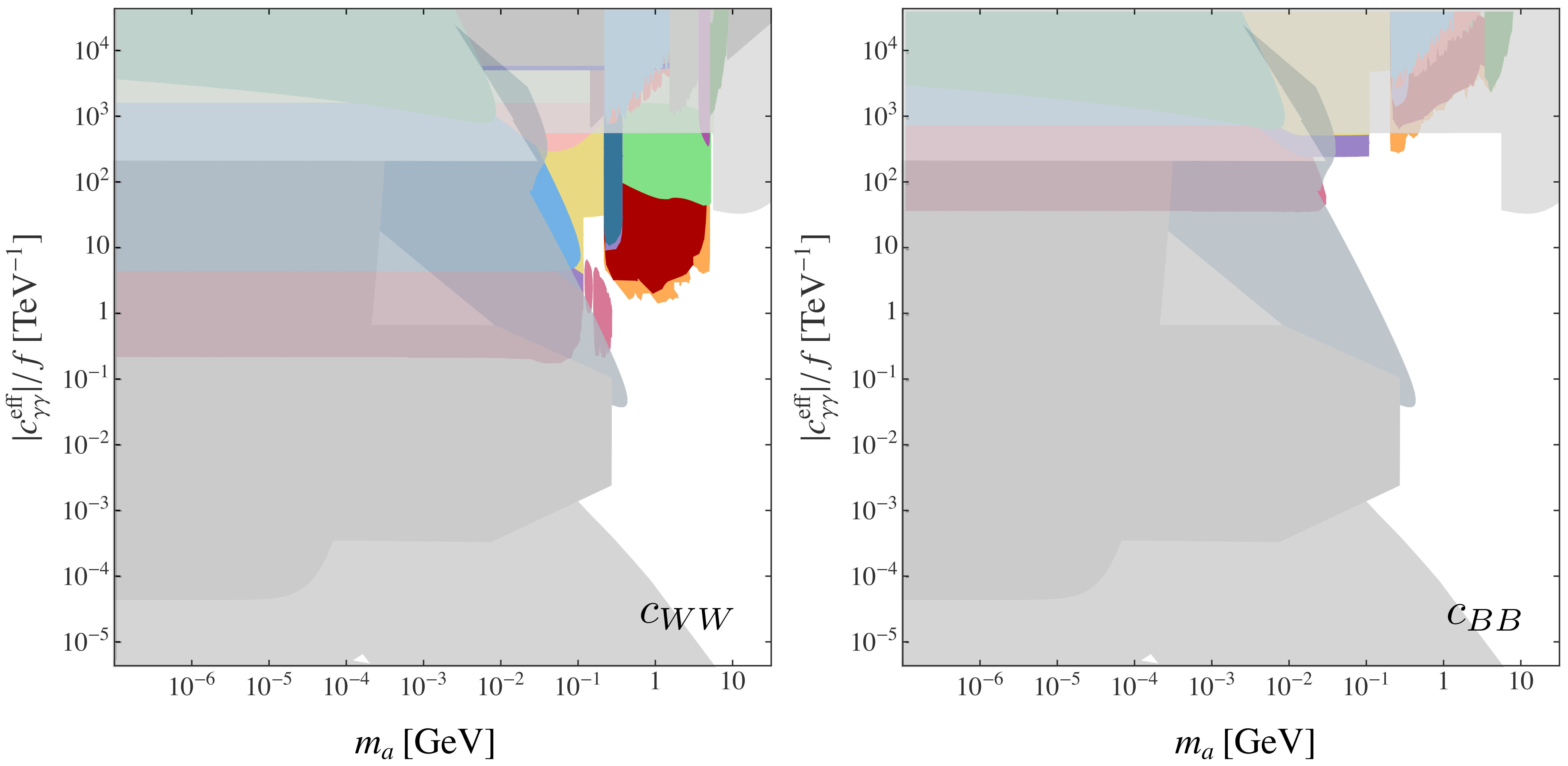


Flavor physics benchmarks

Impact on the chart for the ALP-photon coupling:



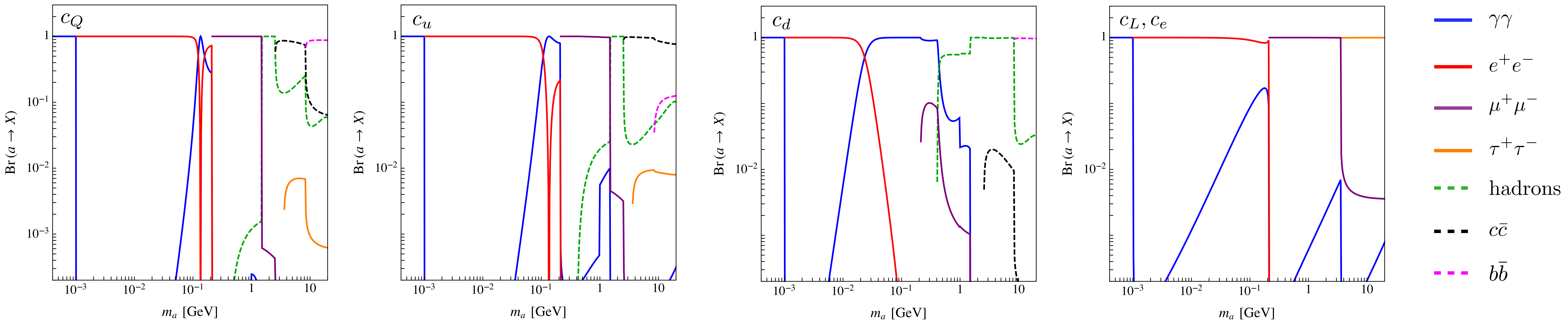
bounds from cosmology, astrophysics and collider physics



$c_{\gamma\gamma} = c_{WW} + c_{BB}$

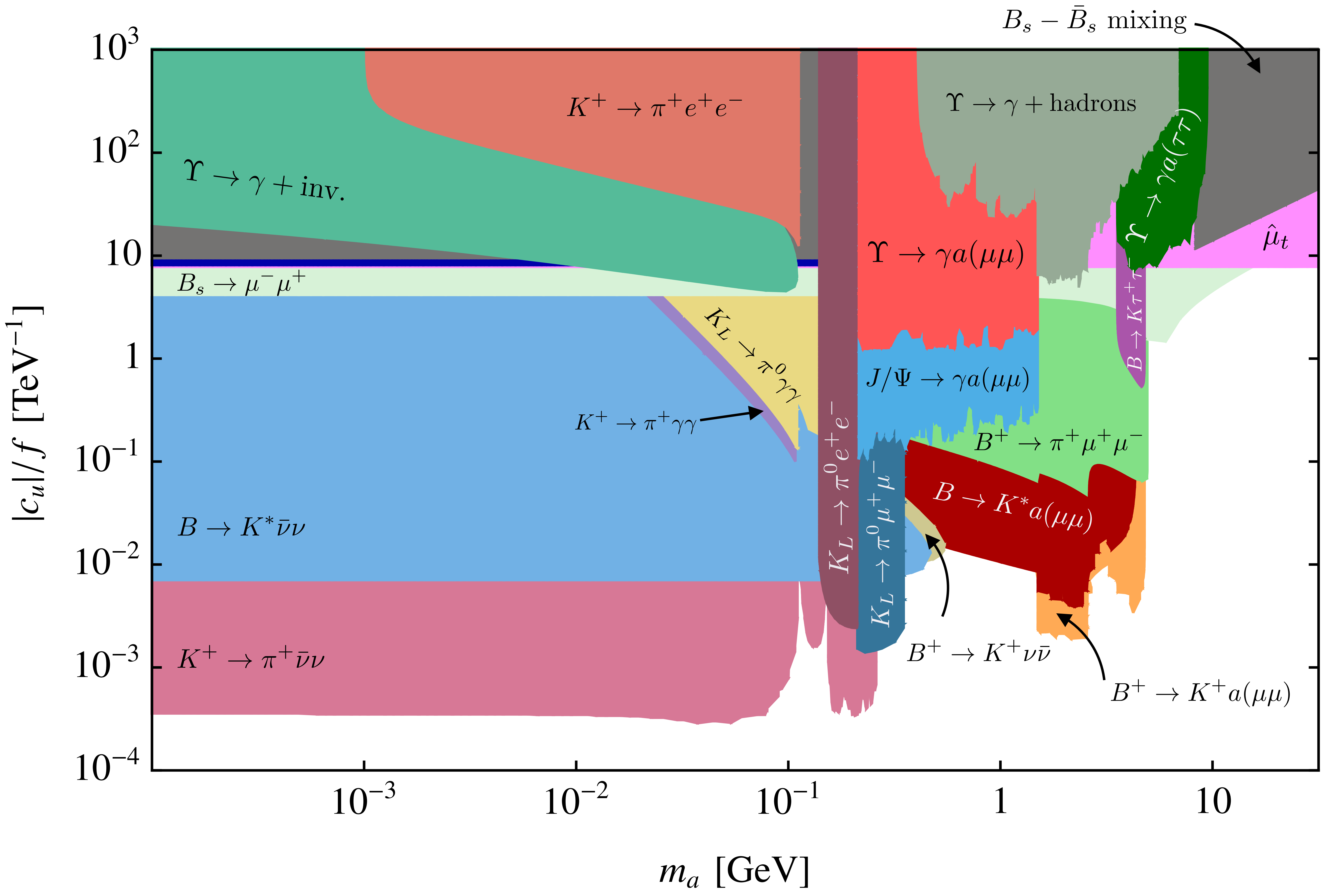
Flavor physics benchmarks

ALP branching fractions in the benchmarks with a single non-vanishing ALP-fermion coupling at the UV scale: [Bauer, MN, Thamm (2017)]



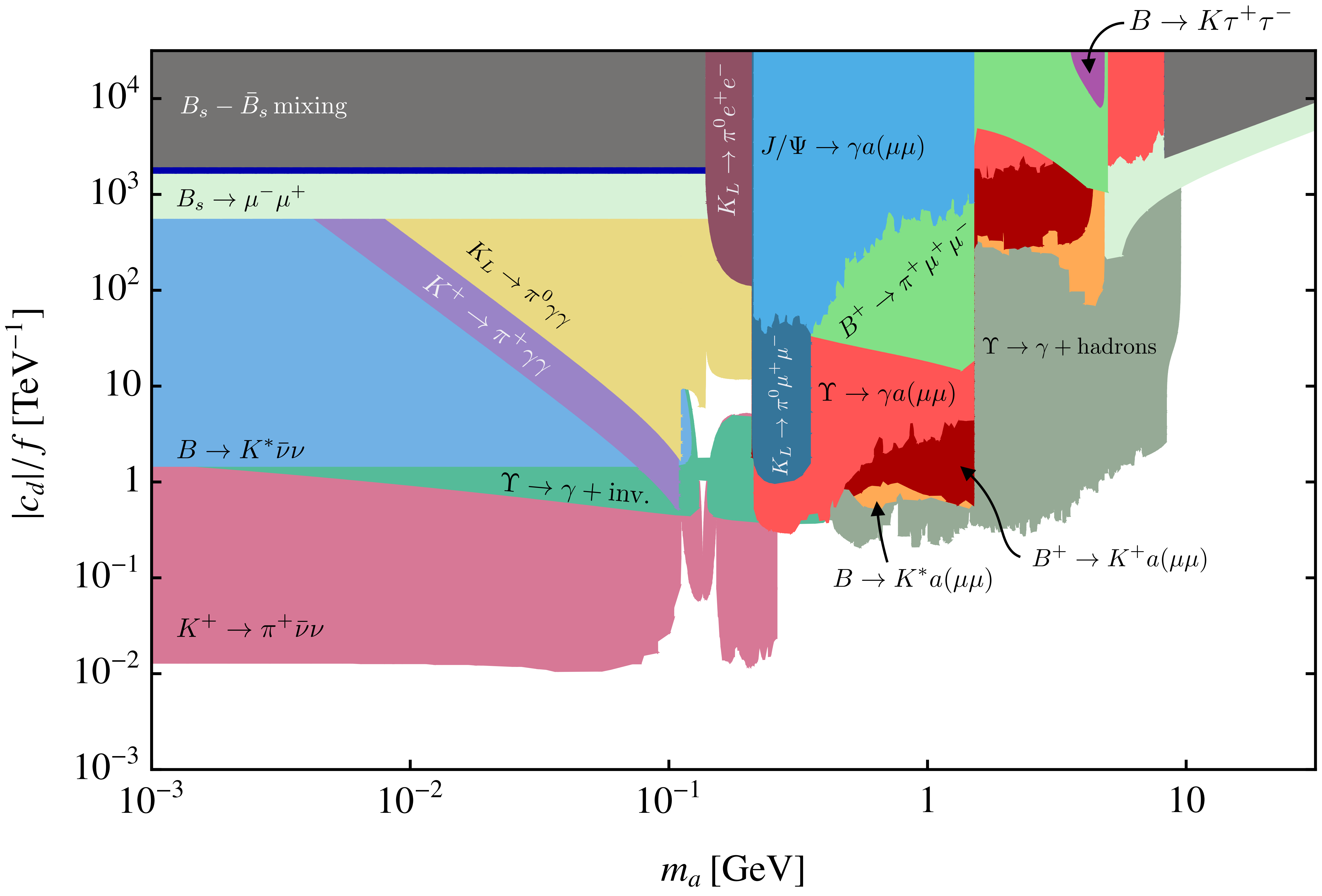
Flavor physics benchmarks

Flavor-universal ALP-u_R coupling in the UV:



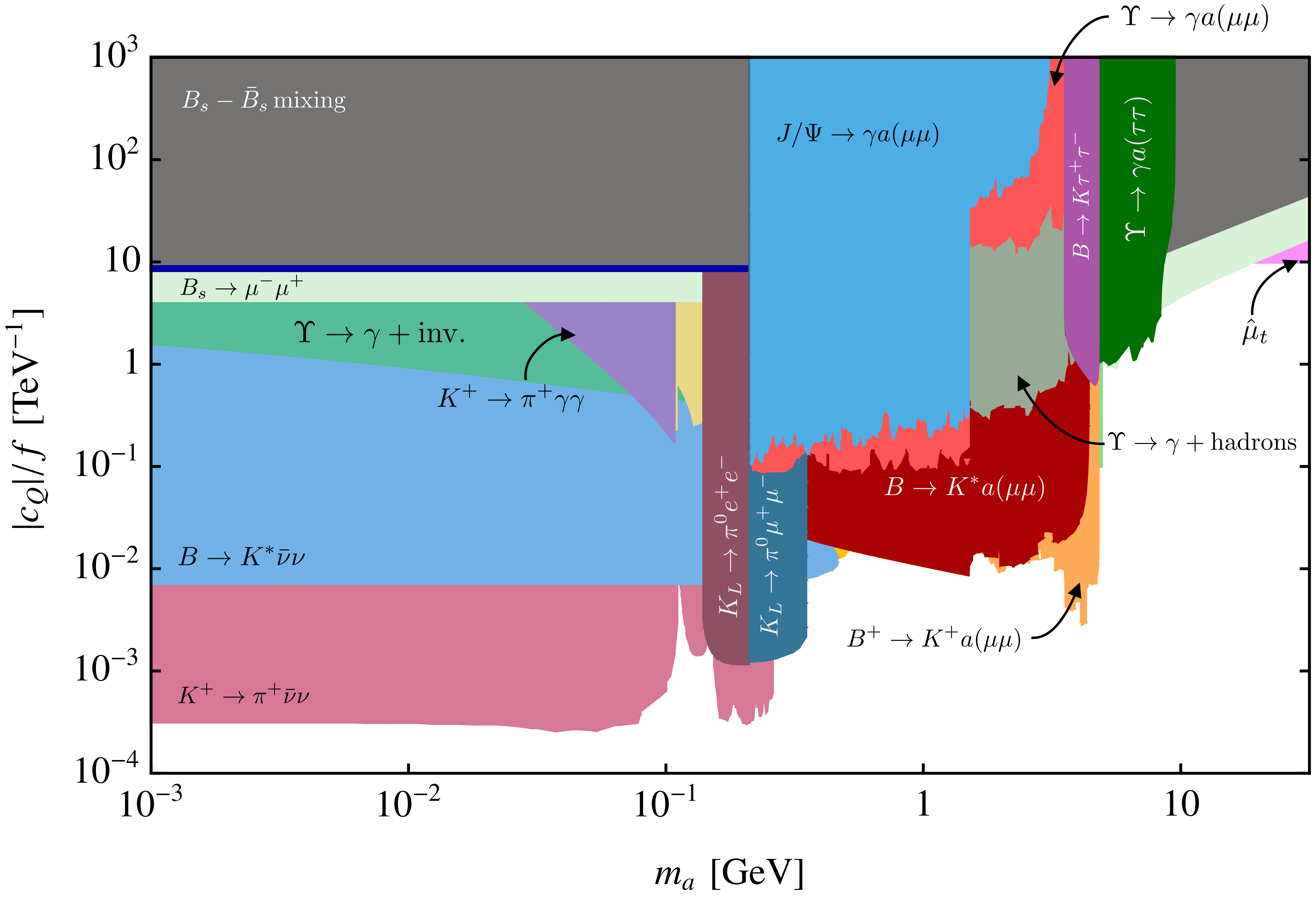
Flavor physics benchmarks

Flavor-universal ALP-d_R coupling in the UV:



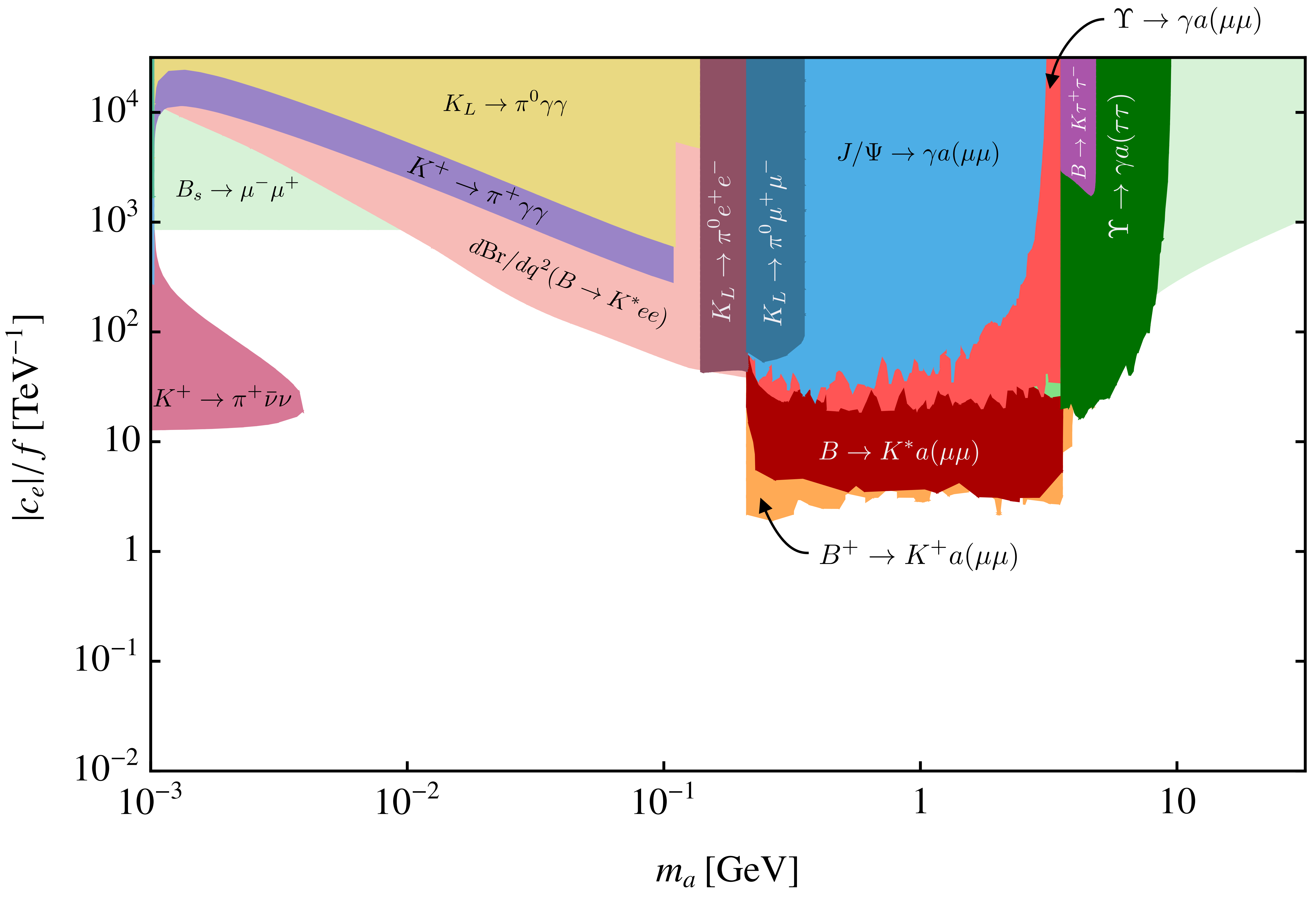
Flavor physics benchmarks

Flavor-universal ALP-Q_L coupling in the UV:



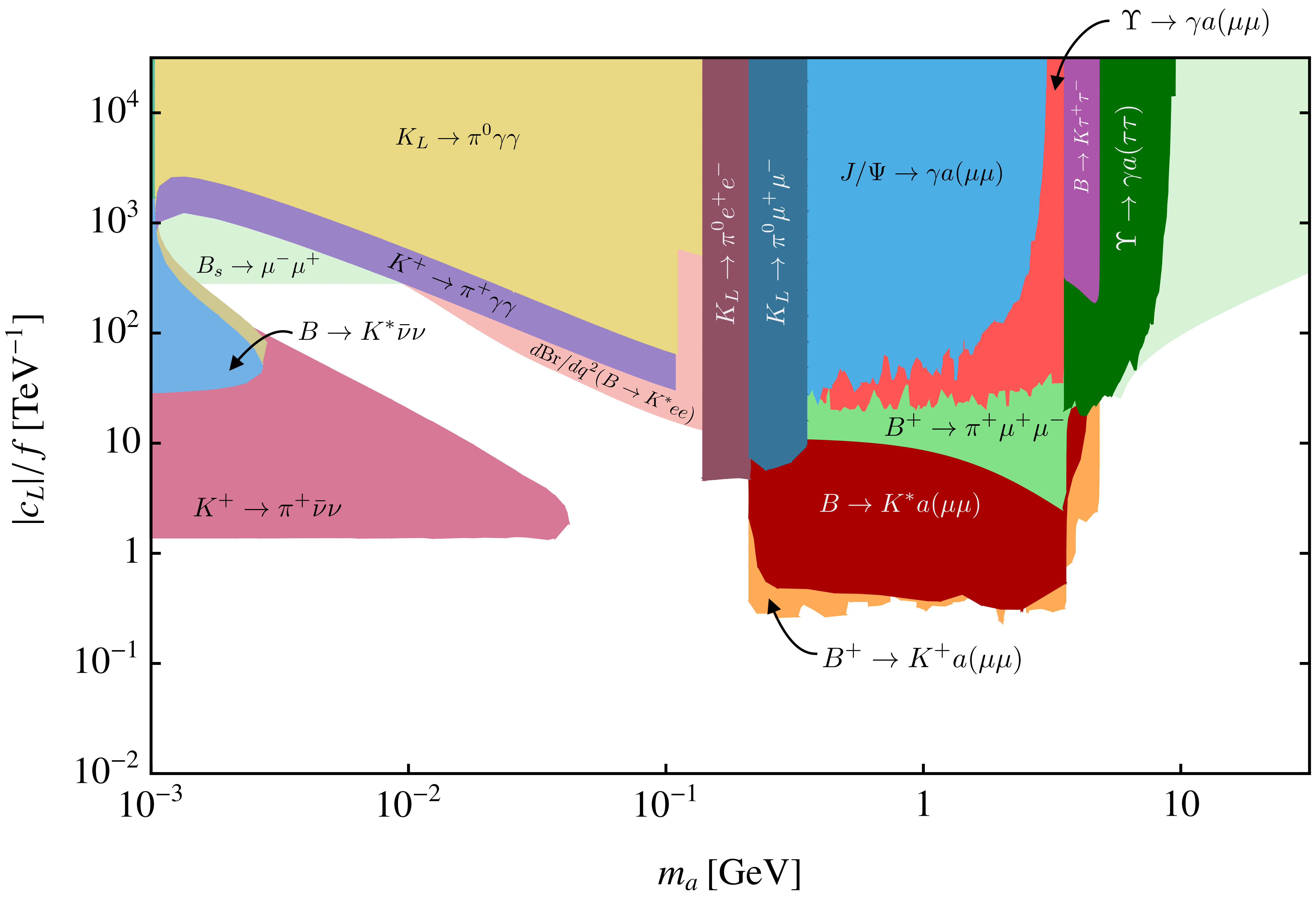
Flavor physics benchmarks

Flavor-universal ALP-e_R coupling in the UV:



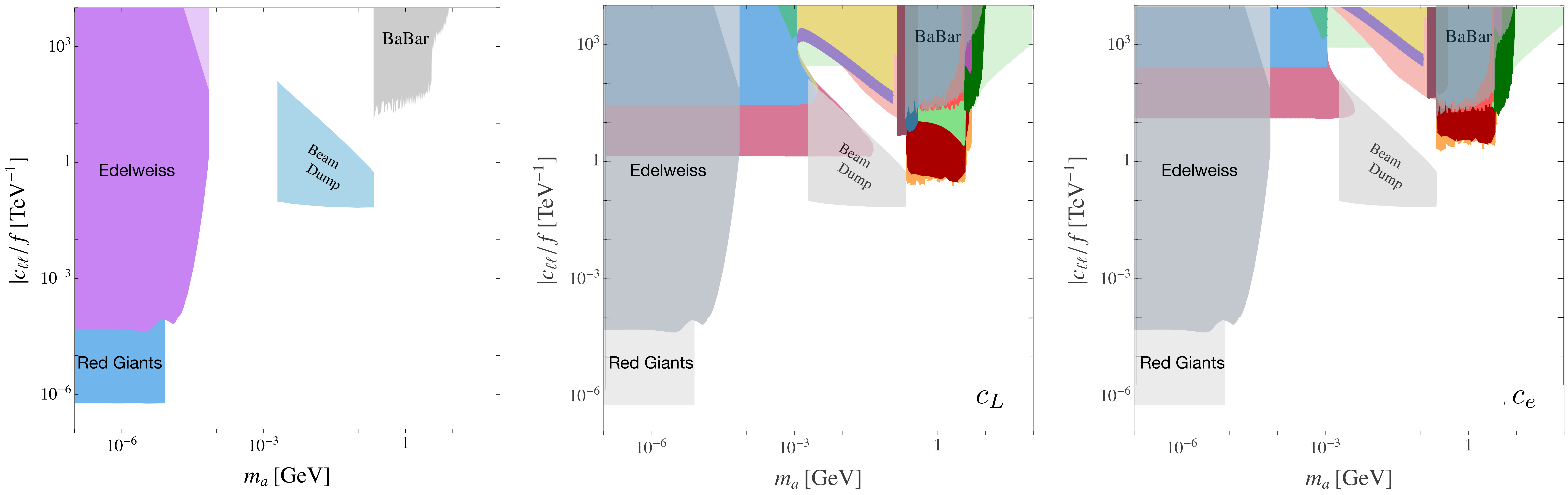
Flavor physics benchmarks

Flavor-universal ALP-L_L coupling in the UV:



Flavor physics benchmarks

Impact on the chart for the ALP-electron coupling:

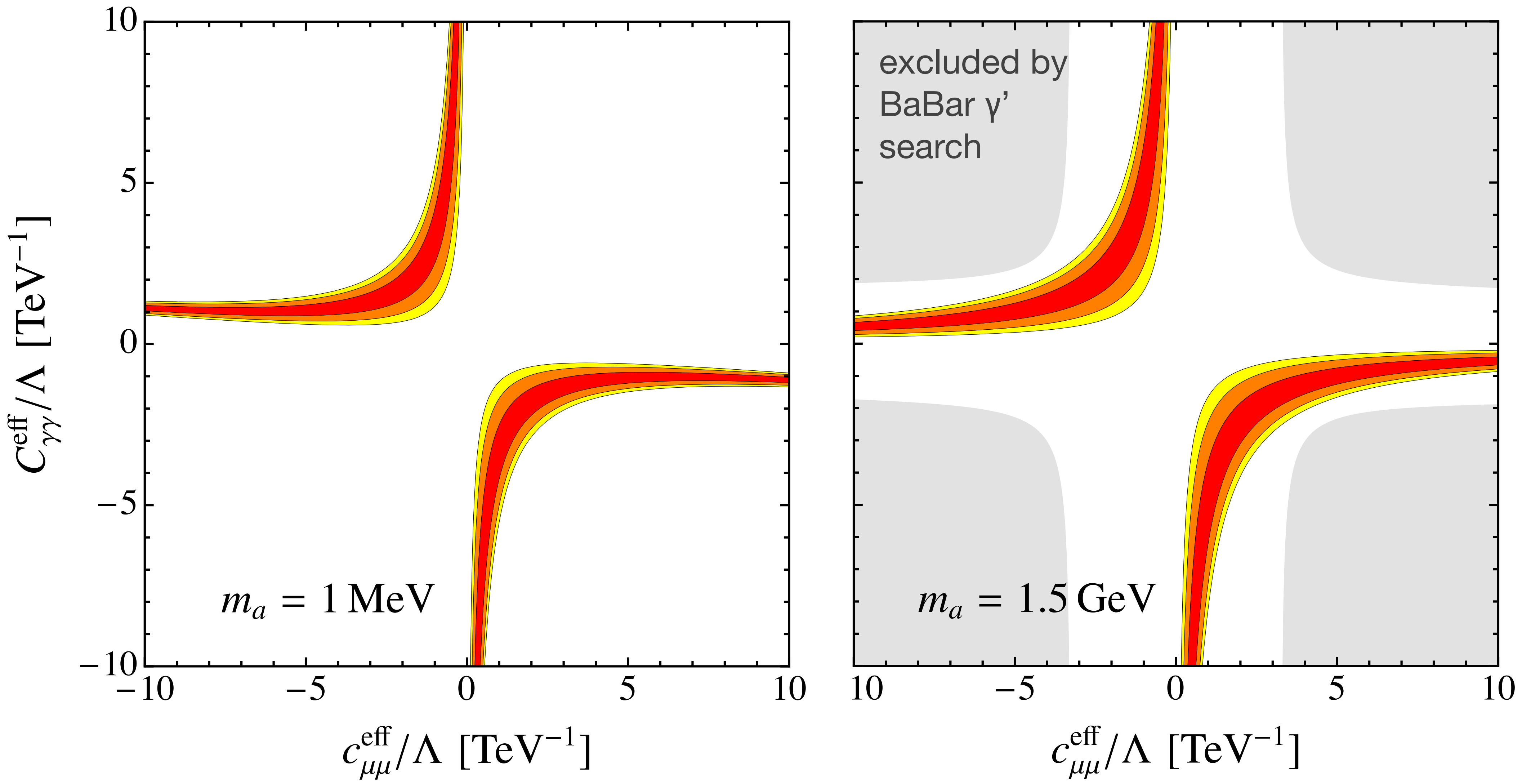


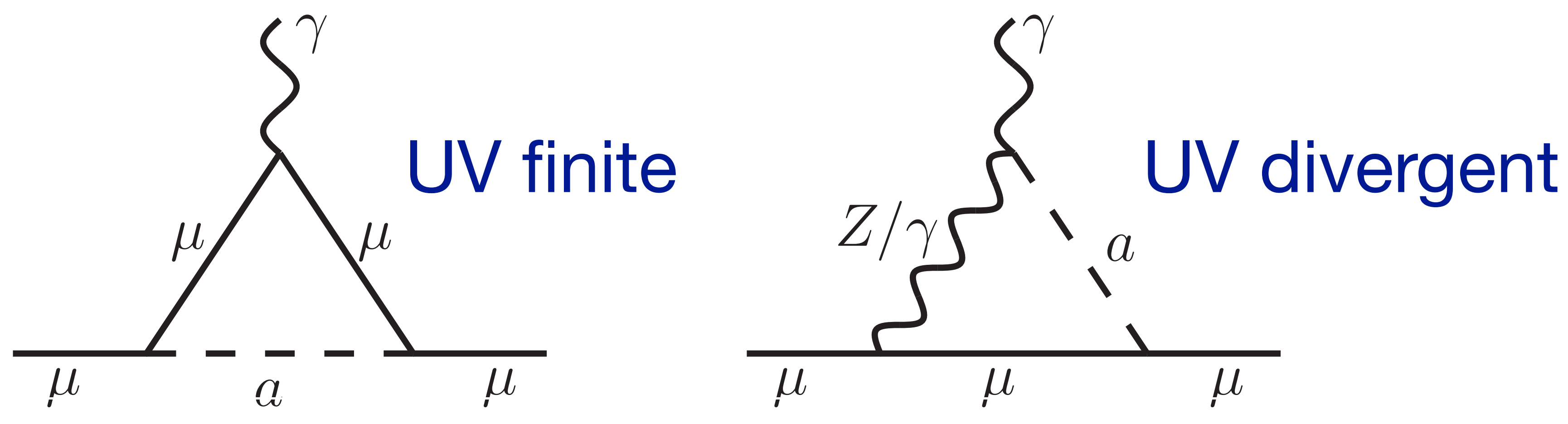
ALP—SMEFT interference

ALP—SMEFT interference

It is well-known that one-loop diagrams with virtual ALP exchange can be UV divergent. This was first studied in the context of $(g\text{-}2)_\mu$:

[Marciano, Masiero, Paradisi, Passera (2016); Bauer, MN, Thamm (2017)]





$$\delta a_\mu = \frac{m_\mu^2}{\Lambda^2}\left\{ K_{a_\mu}(\mu) - \frac{(c_{\mu\mu})^2}{16\pi^2}\, h_1\!\left(\frac{m_a^2}{m_\mu^2}\right) - \frac{2\alpha}{\pi}\, c_{\mu\mu}\, C_{\gamma\gamma}\left[\ln\frac{\mu^2}{m_\mu^2} + \delta_2 + 3 - h_2\!\left(\frac{m_a^2}{m_\mu^2}\right)\right] - \frac{\alpha}{2\pi}\,\frac{1-4s_w^2}{s_w c_w}\, c_{\mu\mu}\, C_{\gamma Z}\left(\ln\frac{\mu^2}{m_Z^2} + \delta_2 + \frac{3}{2}\right)\right\}.$$

needs a D=6 counterterm not contained in the ALP effective Lagrangian

ALP—SMEFT interference

A systematic treatment of these UV divergences requires an embedding of the ALP model in the SMEFT: [Buchmüller, Wyler (1986)]

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_a^2}{2}a^2 + \mathcal{L}_{\text{SM+ALP}} + \mathcal{L}_{\text{SMEFT}}$$

where:

$$\mathcal{L}_{\text{SM+ALP}}^{D=5} = C_{GG}\frac{a}{f}G_{\mu\nu}^a\tilde{G}^{\mu\nu,a} + C_{WW}\frac{a}{f}W_{\mu\nu}^I\tilde{W}^{\mu\nu,I} + C_{BB}\frac{a}{f}B_{\mu\nu}\tilde{B}^{\mu\nu} - \frac{a}{f}\left(\bar{Q}\tilde{H}\widetilde{\boldsymbol{Y}}_u u_R + \bar{Q}H\widetilde{\boldsymbol{Y}}_d d_R + \bar{L}H\widetilde{\boldsymbol{Y}}_e e_R + \text{h.c.}\right)$$

Irrespective of the existence of other new physics, the presence of a light ALP provides source terms S_i for the D=6 SMEFT Wilson coefficients:

$$\boxed{\frac{d}{d\ln\mu}C_i^{\text{SMEFT}} - \gamma_{ji}^{\text{SMEFT}}C_j^{\text{SMEFT}} = \frac{S_i}{(4\pi f)^2} \qquad (\text{for } \mu < 4\pi f)}$$

[Galda, MN, Renner: 2105.01078]

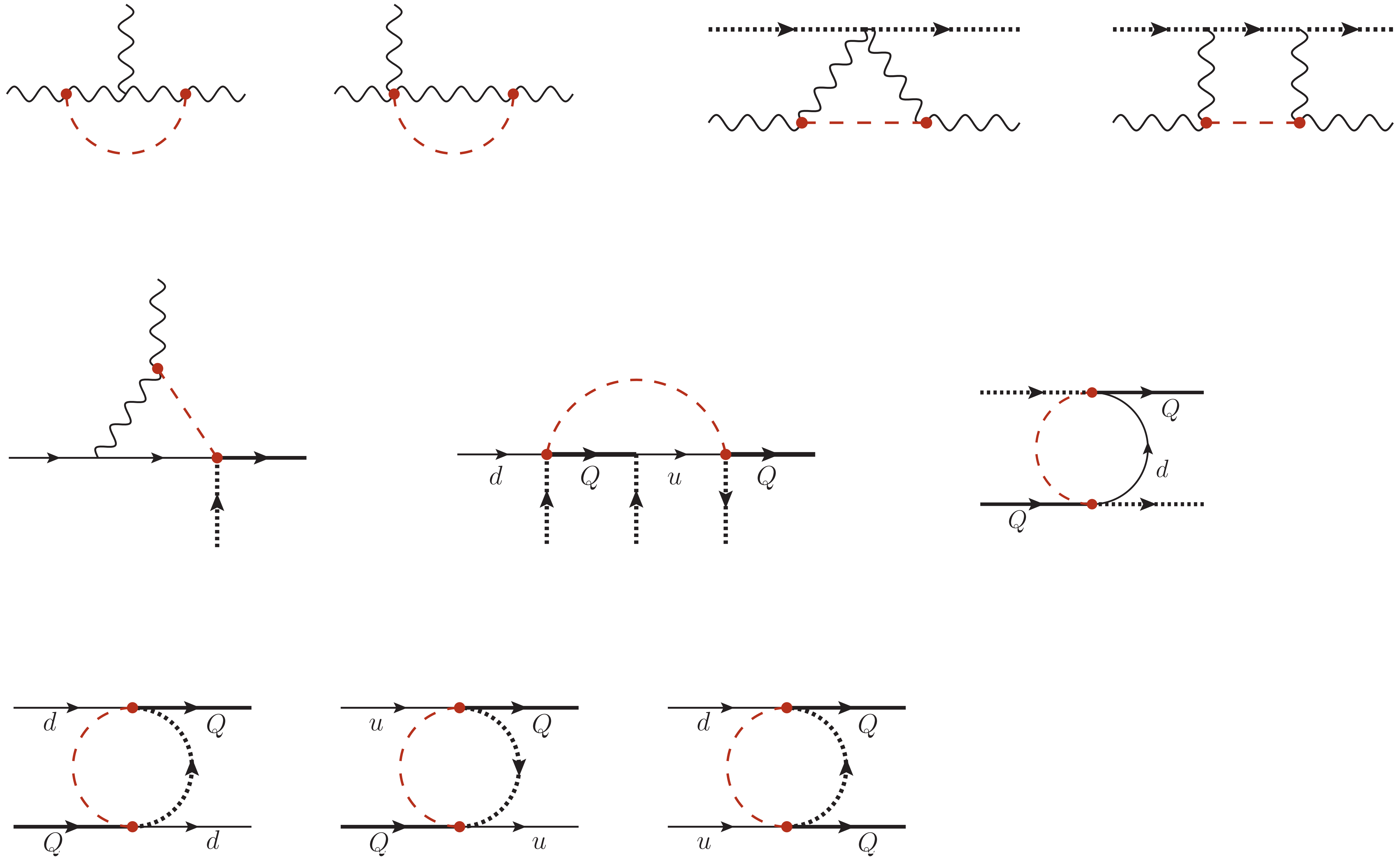
ALP—SMEFT interference

Systematic study of divergent Green's functions with ALP exchange:

[Galda, MN, Renner: 2105.01078]

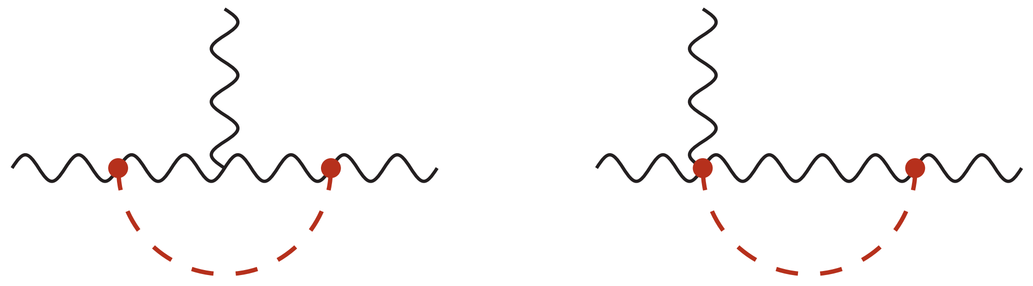
Operator class	Warsaw basis	Way of generation	
Purely bosonic			
X^3	yes	direct	—
X^2D^2	no	direct	
X^2H^2	yes	direct	—
XH^2D^2	no	—	
H^6	yes	—	EOM
H^4D^2	yes	—	EOM
H^2D^4	no	—	
Single fermion current			
ψ^2XD	no	—	
ψ^2D^3	no	—	
ψ^2XH	yes	direct	—
ψ^2H^3	yes	direct	EOM
ψ^2H^2D	yes	direct	EOM
ψ^2HD^2	no	—	
4-fermion operators			
$(\bar{L}L)(\bar{L}L)$	yes	—	EOM
$(\bar{R}R)(\bar{R}R)$	yes	—	EOM
$(\bar{L}L)(\bar{R}R)$	yes	direct	EOM
$(\bar{L}R)(\bar{R}L)$	yes	direct	—
$(\bar{L}R)(\bar{L}R)$	yes	direct	—
B-violating	yes	—	—

[Grzadkowski, Iskrzynski, Misiak, Rosiek (2010)]



ALP—SMEFT interference

Sample calculation: UV divergences of triple gauge-boson amplitudes



$$\mathcal{A}(gg(g)) = -\frac{C_{GG}^2}{\epsilon}\left[4g_s\langle Q_G\rangle + \frac{4}{3}\langle\widehat{Q}_{G,2}\rangle - 2m_a^2\langle G_{\mu\nu}^a G^{\mu\nu,a}\rangle\right] + \text{finite}$$

$$\mathcal{A}(WW(W)) = -\frac{C_{WW}^2}{\epsilon}\left[4g_2\langle Q_W\rangle + \frac{4}{3}\langle\widehat{Q}_{W,2}\rangle - 2m_a^2\langle W_{\mu\nu}^I W^{\mu\nu,I}\rangle\right] + \text{finite}$$

$$\mathcal{A}(BB) = -\frac{C_{BB}^2}{\epsilon}\left[\frac{4}{3}\langle\widehat{Q}_{B,2}\rangle - 2m_a^2\langle B_{\mu\nu}B^{\mu\nu}\rangle\right] + \text{finite}$$

Redundant operators:

$$\widehat{Q}_{G,2} = (D^\rho G_{\rho\mu})^a\,(D_\omega G^{\omega\mu})^a$$

$$\widehat{Q}_{W,2} = (D^\rho W_{\rho\mu})^I\,(D_\omega W^{\omega\mu})^I$$

$$\widehat{Q}_{B,2} = (D^\rho B_{\rho\mu})\,(D_\omega B^{\omega\mu})$$

Elimination of the redundant operators using the EOMs:

$$\begin{aligned}\widehat{Q}_{G,2} &\cong g_s^2\left(\bar{Q}\gamma_\mu T^a Q + \bar{u}\gamma_\mu T^a u + \bar{d}\gamma_\mu T^a d\right)^2\\ &= g_s^2\bigg[\frac{1}{4}\left([Q_{qq}^{(1)}]_{prrp} + [Q_{qq}^{(3)}]_{prrp}\right) - \frac{1}{2N_c}[Q_{qq}^{(1)}]_{pprr} + \frac{1}{2}[Q_{uu}]_{prrp} - \frac{1}{2N_c}[Q_{uu}]_{pprr}\\ &\qquad + \frac{1}{2}[Q_{dd}]_{prrp} - \frac{1}{2N_c}[Q_{dd}]_{pprr} + 2\,[Q_{qu}^{(8)}]_{pprr} + 2\,[Q_{qd}^{(8)}]_{pprr} + 2\,[Q_{ud}^{(8)}]_{pprr}\bigg]\end{aligned}$$

$$\begin{aligned}\widehat{Q}_{W,2} &\cong \frac{g_2^2}{4}\left(H^\dagger i\overleftrightarrow{D}_\mu^I H + \bar{Q}\gamma_\mu\sigma^I Q + \bar{L}\gamma_\mu\sigma^I L\right)^2\\ &= \frac{g_2^2}{4}\bigg[-4m_H^2\left(H^\dagger H\right)^2 + 4\lambda\,Q_H + 3Q_{H\Box} + 2\left([Q_{Hl}^{(3)}]_{pp} + [Q_{Hq}^{(3)}]_{pp}\right)\\ &\qquad + 2\Big[(Y_u)_{pr}[Q_{uH}]_{pr} + (Y_d)_{pr}[Q_{dH}]_{pr} + (Y_e)_{pr}[Q_{eH}]_{pr} + \text{h.c.}\Big]\\ &\qquad + 2\,[Q_{lq}^{(3)}]_{pprr} + 2\,[Q_{ll}]_{prrp} - [Q_{ll}]_{pprr} + [Q_{qq}^{(3)}]_{pprr}\bigg]\end{aligned}$$

ALP—SMEFT interference

One-loop results for the ALP source terms:

Operator class	Warsaw basis	Way of generation	
Purely bosonic			
X^3	yes	direct	—
X^2D^2	no	direct	
X^2H^2	yes	direct	—
XH^2D^2	no	—	
H^6	yes	—	EOM
H^4D^2	yes	—	EOM
H^2D^4	no	—	

$$S_G = 8 g_s C_{GG}^2 \,, \qquad S_{\widetilde{G}} = 0$$

$$S_W = 8 g_2 C_{WW}^2 \,, \qquad S_{\widetilde{W}} = 0$$

$$S_{HG} = 0 \,, \qquad S_{H\widetilde{G}} = 0$$

$$S_{HW} = -2 g_2^2 \, C_{WW}^2 \,, \qquad S_{H\widetilde{W}} = 0$$

$$S_{HB} = -2 g_1^2 \, C_{BB}^2 \,, \qquad S_{H\widetilde{B}} = 0$$

$$S_{HWB} = -4 g_1 g_2 \, C_{WW} C_{BB} \,, \qquad S_{H\widetilde{W}B} = 0$$

$$S_H = \frac{8}{3} \lambda g_2^2 \, C_{WW}^2 \,,$$

$$S_{H\Box} = 2 g_2^2 \, C_{WW}^2 + \frac{8}{3} g_1^2 \, \mathcal{Y}_H^2 \, C_{BB}^2$$

$$S_{HD} = \frac{32}{3} g_1^2 \, \mathcal{Y}_H^2 \, C_{BB}^2 \,.$$

ALP—SMEFT interference

One-loop results for the ALP source terms:

Operator class	Warsaw basis	Way of generation	
Single fermion current			
$\psi^2 XD$	no	—	
$\psi^2 D^3$	no	—	
$\psi^2 XH$	yes	direct	—
$\psi^2 H^3$	yes	direct	EOM
$\psi^2 H^2 D$	yes	direct	EOM
$\psi^2 HD^2$	no	—	

$$
\begin{aligned}
\boldsymbol{S}_{eW} &= -ig_2\, \widetilde{\boldsymbol{Y}}_e\, C_{WW} \\
\boldsymbol{S}_{eB} &= -2ig_1\, (\mathcal{Y}_L + \mathcal{Y}_e)\, \widetilde{\boldsymbol{Y}}_e\, C_{BB} \\
\boldsymbol{S}_{uG} &= -4ig_s\, \widetilde{\boldsymbol{Y}}_u\, C_{GG} \\
\boldsymbol{S}_{uW} &= -ig_2\, \widetilde{\boldsymbol{Y}}_u\, C_{WW} \\
\boldsymbol{S}_{uB} &= -2ig_1\, (\mathcal{Y}_Q + \mathcal{Y}_u)\, \widetilde{\boldsymbol{Y}}_u\, C_{BB} \\
\boldsymbol{S}_{dG} &= -4ig_s\, \widetilde{\boldsymbol{Y}}_d\, C_{GG} \\
\boldsymbol{S}_{dW} &= -ig_2\, \widetilde{\boldsymbol{Y}}_d\, C_{WW} \\
\boldsymbol{S}_{dB} &= -2ig_1\, (\mathcal{Y}_Q + \mathcal{Y}_d)\, \widetilde{\boldsymbol{Y}}_d\, C_{BB}
\end{aligned}
$$

ALP—SMEFT interference

One-loop results for the ALP source terms:

Operator class	Warsaw basis	Way of generation	
Single fermion current			
$\psi^2 XD$	no	—	
$\psi^2 D^3$	no	—	
$\psi^2 XH$	yes	direct	—
$\psi^2 H^3$	yes	direct	EOM
$\psi^2 H^2 D$	yes	direct	EOM
$\psi^2 HD^2$	no	—	

$$\boldsymbol{S}_{Hl}^{(1)} = \frac{1}{4}\,\widetilde{\boldsymbol{Y}}_e \widetilde{\boldsymbol{Y}}_e^\dagger + \frac{16}{3}\,g_1^2\,\mathcal{Y}_H \mathcal{Y}_L\, C_{BB}^2\,\mathbf{1}$$

$$\boldsymbol{S}_{Hl}^{(3)} = \frac{1}{4}\,\widetilde{\boldsymbol{Y}}_e \widetilde{\boldsymbol{Y}}_e^\dagger + \frac{4}{3}\,g_2^2\,C_{WW}^2\,\mathbf{1}$$

$$\boldsymbol{S}_{He} = -\frac{1}{2}\,\widetilde{\boldsymbol{Y}}_e^\dagger \widetilde{\boldsymbol{Y}}_e + \frac{16}{3}\,g_1^2\,\mathcal{Y}_H \mathcal{Y}_e\, C_{BB}^2\,\mathbf{1}$$

$$\boldsymbol{S}_{Hq}^{(1)} = \frac{1}{4}\left(\widetilde{\boldsymbol{Y}}_d \widetilde{\boldsymbol{Y}}_d^\dagger - \widetilde{\boldsymbol{Y}}_u \widetilde{\boldsymbol{Y}}_u^\dagger\right) + \frac{16}{3}\,g_1^2\,\mathcal{Y}_H \mathcal{Y}_Q\, C_{BB}^2\,\mathbf{1}$$

$$\boldsymbol{S}_{Hq}^{(3)} = \frac{1}{4}\left(\widetilde{\boldsymbol{Y}}_d \widetilde{\boldsymbol{Y}}_d^\dagger + \widetilde{\boldsymbol{Y}}_u \widetilde{\boldsymbol{Y}}_u^\dagger\right) + \frac{4}{3}\,g_2^2\,C_{WW}^2\,\mathbf{1}$$

$$\boldsymbol{S}_{Hu} = \frac{1}{2}\,\widetilde{\boldsymbol{Y}}_u^\dagger \widetilde{\boldsymbol{Y}}_u + \frac{16}{3}\,g_1^2\,\mathcal{Y}_H \mathcal{Y}_u\, C_{BB}^2\,\mathbf{1}$$

$$\boldsymbol{S}_{Hd} = -\frac{1}{2}\,\widetilde{\boldsymbol{Y}}_d^\dagger \widetilde{\boldsymbol{Y}}_d + \frac{16}{3}\,g_1^2\,\mathcal{Y}_H \mathcal{Y}_d\, C_{BB}^2\,\mathbf{1}$$

$$\boldsymbol{S}_{Hud} = -\widetilde{\boldsymbol{Y}}_u^\dagger \widetilde{\boldsymbol{Y}}_d$$

ALP—SMEFT interference

One-loop results for the ALP source terms:

Operator class	Warsaw basis	Way of generation	
Single fermion current			
$\psi^2 XD$	no	—	
$\psi^2 D^3$	no	—	
$\psi^2 XH$	yes	direct	—
$\psi^2 H^3$	yes	direct	EOM
$\psi^2 H^2 D$	yes	direct	EOM
$\psi^2 HD^2$	no	—	

$$\boldsymbol{S}_{eH} = -2\widetilde{\boldsymbol{Y}}_e \boldsymbol{Y}_e^\dagger \widetilde{\boldsymbol{Y}}_e - \frac{1}{2}\widetilde{\boldsymbol{Y}}_e \widetilde{\boldsymbol{Y}}_e^\dagger \boldsymbol{Y}_e - \frac{1}{2}\boldsymbol{Y}_e \widetilde{\boldsymbol{Y}}_e^\dagger \widetilde{\boldsymbol{Y}}_e + \frac{4}{3} g_2^2 \, C_{WW}^2 \, \boldsymbol{Y}_e$$

$$\boldsymbol{S}_{uH} = -2\widetilde{\boldsymbol{Y}}_u \boldsymbol{Y}_u^\dagger \widetilde{\boldsymbol{Y}}_u - \frac{1}{2}\widetilde{\boldsymbol{Y}}_u \widetilde{\boldsymbol{Y}}_u^\dagger \boldsymbol{Y}_u - \frac{1}{2}\boldsymbol{Y}_u \widetilde{\boldsymbol{Y}}_u^\dagger \widetilde{\boldsymbol{Y}}_u + \frac{4}{3} g_2^2 \, C_{WW}^2 \, \boldsymbol{Y}_u$$

$$\boldsymbol{S}_{dH} = -2\widetilde{\boldsymbol{Y}}_d \boldsymbol{Y}_d^\dagger \widetilde{\boldsymbol{Y}}_d - \frac{1}{2}\widetilde{\boldsymbol{Y}}_d \widetilde{\boldsymbol{Y}}_d^\dagger \boldsymbol{Y}_d - \frac{1}{2}\boldsymbol{Y}_d \widetilde{\boldsymbol{Y}}_d^\dagger \widetilde{\boldsymbol{Y}}_d + \frac{4}{3} g_2^2 \, C_{WW}^2 \, \boldsymbol{Y}_d$$

ALP—SMEFT interference

One-loop results for the ALP source terms:

Operator class	Warsaw basis	Way of generation	
4-fermion operators			
$(\bar{L}L)(\bar{L}L)$	yes	—	EOM
$(\bar{R}R)(\bar{R}R)$	yes	—	EOM
$(\bar{L}L)(\bar{R}R)$	yes	direct	EOM
$(\bar{L}R)(\bar{R}L)$	yes	direct	—
$(\bar{L}R)(\bar{L}R)$	yes	direct	—
B-violating	yes	—	—

$$\big[S_{ll}\big]_{prst} = \frac{2}{3}\, g_2^2\, C_{WW}^2 \left(2\delta_{pt}\,\delta_{sr} - \delta_{pr}\,\delta_{st}\right) + \frac{8}{3}\, g_1^2\, \mathcal{Y}_L^2\, C_{BB}^2\, \delta_{pr}\,\delta_{st}$$

$$\big[S_{qq}^{(1)}\big]_{prst} = \frac{2}{3}\, g_s^2\, C_{GG}^2 \left(\delta_{pt}\,\delta_{sr} - \frac{2}{N_c}\,\delta_{pr}\,\delta_{st}\right) + \frac{8}{3}\, g_1^2\, \mathcal{Y}_Q^2\, C_{BB}^2\, \delta_{pr}\,\delta_{st}$$

$$\big[S_{qq}^{(3)}\big]_{prst} = \frac{2}{3}\, g_s^2\, C_{GG}^2\, \delta_{pt}\,\delta_{sr} + \frac{2}{3}\, g_2^2\, C_{WW}^2\, \delta_{pr}\,\delta_{st}$$

$$\big[S_{lq}^{(1)}\big]_{prst} = \frac{16}{3}\, g_1^2\, \mathcal{Y}_L\, \mathcal{Y}_Q\, C_{BB}^2\, \delta_{pr}\,\delta_{st}$$

$$\big[S_{lq}^{(3)}\big]_{prst} = \frac{4}{3}\, g_2^2\, C_{WW}^2\, \delta_{pr}\,\delta_{st}$$

ALP—SMEFT interference

One-loop results for the ALP source terms:

Operator class	Warsaw basis	Way of generation	
4-fermion operators			
$(\bar{L}L)(\bar{L}L)$	yes	—	EOM
$(\bar{R}R)(\bar{R}R)$	yes	—	EOM
$(\bar{L}L)(\bar{R}R)$	yes	direct	EOM
$(\bar{L}R)(\bar{R}L)$	yes	direct	—
$(\bar{L}R)(\bar{L}R)$	yes	direct	—
B-violating	yes	—	—

$$\big[S_{ee}\big]_{prst} = \frac{8}{3}\, g_1^2\, \mathcal{Y}_e^2\, C_{BB}^2\, \delta_{pr}\, \delta_{st}$$

$$\big[S_{uu}\big]_{prst} = \frac{4}{3}\, g_s^2\, C_{GG}^2 \left(\delta_{pt}\, \delta_{sr} - \frac{1}{N_c}\, \delta_{pr}\, \delta_{st} \right) + \frac{8}{3}\, g_1^2\, \mathcal{Y}_u^2\, C_{BB}^2\, \delta_{pr}\, \delta_{st}$$

$$\big[S_{dd}\big]_{prst} = \frac{4}{3}\, g_s^2\, C_{GG}^2 \left(\delta_{pt}\, \delta_{sr} - \frac{1}{N_c}\, \delta_{pr}\, \delta_{st} \right) + \frac{8}{3}\, g_1^2\, \mathcal{Y}_d^2\, C_{BB}^2\, \delta_{pr}\, \delta_{st}$$

$$\big[S_{eu}\big]_{prst} = \frac{16}{3}\, g_1^2\, \mathcal{Y}_e \mathcal{Y}_u\, C_{BB}^2\, \delta_{pr}\, \delta_{st}$$

$$\big[S_{ed}\big]_{prst} = \frac{16}{3}\, g_1^2\, \mathcal{Y}_e \mathcal{Y}_d\, C_{BB}^2\, \delta_{pr}\, \delta_{st}$$

$$\big[S_{ud}^{(1)}\big]_{prst} = \frac{16}{3}\, g_1^2\, \mathcal{Y}_u \mathcal{Y}_d\, C_{BB}^2\, \delta_{pr}\, \delta_{st}$$

$$\big[S_{ud}^{(8)}\big]_{prst} = \frac{16}{3}\, g_s^2\, C_{GG}^2\, \delta_{pr}\, \delta_{st}$$

ALP—SMEFT interference

One-loop results for the ALP source terms:

Operator class	Warsaw basis	Way of generation	
4-fermion operators			
$(\bar{L}L)(\bar{L}L)$	yes	—	EOM
$(\bar{R}R)(\bar{R}R)$	yes	—	EOM
$(\bar{L}L)(\bar{R}R)$	yes	direct	EOM
$(\bar{L}R)(\bar{R}L)$	yes	direct	—
$(\bar{L}R)(\bar{L}R)$	yes	direct	—
B-violating	yes	—	—

$$
\begin{aligned}
\left[S_{le}\right]_{prst} &= \big(\tilde{\mathbf{Y}}_e\big)_{pt}\big(\tilde{\mathbf{Y}}_e^\dagger\big)_{sr} + \frac{16}{3}\, g_1^2\, \mathcal{Y}_L \mathcal{Y}_e\, C_{BB}^2\, \delta_{pr}\, \delta_{st} \\
\left[S_{lu}\right]_{prst} &= \frac{16}{3}\, g_1^2\, \mathcal{Y}_L \mathcal{Y}_u\, C_{BB}^2\, \delta_{pr}\, \delta_{st} \\
\left[S_{ld}\right]_{prst} &= \frac{16}{3}\, g_1^2\, \mathcal{Y}_L \mathcal{Y}_d\, C_{BB}^2\, \delta_{pr}\, \delta_{st} \\
\left[S_{qe}\right]_{prst} &= \frac{16}{3}\, g_1^2\, \mathcal{Y}_Q \mathcal{Y}_e\, C_{BB}^2\, \delta_{pr}\, \delta_{st} \\
\left[S_{qu}^{(1)}\right]_{prst} &= \frac{1}{N_c}\big(\tilde{\mathbf{Y}}_u\big)_{pt}\big(\tilde{\mathbf{Y}}_u^\dagger\big)_{sr} + \frac{16}{3}\, g_1^2\, \mathcal{Y}_Q \mathcal{Y}_u\, C_{BB}^2\, \delta_{pr}\, \delta_{st} \\
\left[S_{qu}^{(8)}\right]_{prst} &= 2\big(\tilde{\mathbf{Y}}_u\big)_{pt}\big(\tilde{\mathbf{Y}}_u^\dagger\big)_{sr} + \frac{16}{3}\, g_s^2\, C_{GG}^2\, \delta_{pr}\, \delta_{st} \\
\left[S_{qd}^{(1)}\right]_{prst} &= \frac{1}{N_c}\big(\tilde{\mathbf{Y}}_d\big)_{pt}\big(\tilde{\mathbf{Y}}_d^\dagger\big)_{sr} + \frac{16}{3}\, g_1^2\, \mathcal{Y}_Q \mathcal{Y}_d\, C_{BB}^2\, \delta_{pr}\, \delta_{st} \\
\left[S_{qd}^{(8)}\right]_{prst} &= 2\big(\tilde{\mathbf{Y}}_d\big)_{pt}\big(\tilde{\mathbf{Y}}_d^\dagger\big)_{sr} + \frac{16}{3}\, g_s^2\, C_{GG}^2\, \delta_{pr}\, \delta_{st}
\end{aligned}
$$

ALP—SMEFT interference

One-loop results for the ALP source terms:

Operator class	Warsaw basis	Way of generation	
4-fermion operators			
$(\bar{L}L)(\bar{L}L)$	yes	—	EOM
$(\bar{R}R)(\bar{R}R)$	yes	—	EOM
$(\bar{L}L)(\bar{R}R)$	yes	direct	EOM
$(\bar{L}R)(\bar{R}L)$	yes	direct	—
$(\bar{L}R)(\bar{L}R)$	yes	direct	—
B-violating	yes	—	—

$$\left[S_{ledq}\right]_{prst} = -2\big(\widetilde{\boldsymbol{Y}}_e\big)_{pr}\big(\widetilde{\boldsymbol{Y}}_d^\dagger\big)_{st}$$

$$\left[S_{quqd}^{(1)}\right]_{prst} = -2\big(\widetilde{\boldsymbol{Y}}_u\big)_{pr}\big(\widetilde{\boldsymbol{Y}}_d\big)_{st}$$

$$\left[S_{quqd}^{(8)}\right]_{prst} = 0 \qquad \text{(starts at 2 loops)}$$

$$\left[S_{lequ}^{(1)}\right]_{prst} = 2\big(\widetilde{\boldsymbol{Y}}_e\big)_{pr}\big(\widetilde{\boldsymbol{Y}}_u\big)_{st}$$

$$\left[S_{lequ}^{(3)}\right]_{prst} = 0 \qquad \text{(starts at 2 loops)}$$

With very few exceptions, all operators in the Warsaw basis are generated at one-loop order in the ALP model !

Top chromo-magnetic moment

Sample application: chromo-magnetic dipole moment of the top quark

$$\mathcal{L}_{tt\bar{g}} = g_s \left(\bar{t}\gamma^\mu T^a t\, G^a_\mu + \frac{\hat{\mu}_t}{2m_t}\, \bar{t}\,\sigma^{\mu\nu} T^a t\, G^a_{\mu\nu} + \frac{i\,\hat{d}_t}{2m_t}\, \bar{t}\,\sigma^{\mu\nu}\gamma_5\, T^a t\, G^a_{\mu\nu} \right)$$

with:

$$\hat{\mu}_t = \frac{y_t v^2}{g_s}\, \Re e\, C^{33}_{uG}\,, \qquad \hat{d}_t = \frac{y_t v^2}{g_s}\, \Im m\, C^{33}_{uG}$$

ALP-induced contribution follows from the solution of:

$$\frac{d}{d\ln\mu}\, \Re e\, C^{33}_{uG} = \boxed{\frac{S^{33}_{uG}}{(4\pi f)^2}} + \left(\frac{15\alpha_t}{8\pi} - \frac{17\alpha_s}{12\pi} \right) \Re e\, C^{33}_{uG} + \frac{9\alpha_s}{4\pi}\, y_t\, C_G + \frac{g_s y_t}{4\pi^2}\, C_{HG}$$

$$\frac{d}{d\ln\mu}\, C_G = \boxed{\frac{S_G}{(4\pi f)^2}} + \frac{15\alpha_s}{4\pi}\, C_G$$

$$\frac{d}{d\ln\mu}\, C_{HG} = \left(\frac{3\alpha_t}{2\pi} - \frac{7\alpha_s}{2\pi} \right) C_{HG} + \frac{g_s y_t}{4\pi^2}\, \Re e\, C^{33}_{uG}$$

Top chromo-magnetic moment

At lowest logarithmic order, one finds: [Galda, MN, Renner: 2105.01078]

$$\hat{\mu}_t \approx -\frac{8m_t^2}{(4\pi f)^2}\left[c_{tt}\,C_{GG}\,\ln\frac{4\pi f}{m_t} - \frac{9\alpha_s}{4\pi}\,C_{GG}^2\,\ln^2\frac{4\pi f}{m_t}\right]$$

$$\approx -\left(5.87\,c_{tt}\,C_{GG} - 1.98\,C_{GG}^2\right)\cdot 10^{-3}\times\left[\frac{1\,\text{TeV}}{f}\right]^2$$

Combined with experimental bounds from CMS (2019), we obtain:

$$-0.68 < \left(c_{tt}\,C_{GG} - 0.34\,C_{GG}^2\right)\times\left[\frac{1\,\text{TeV}}{f}\right]^2 < 2.38 \qquad (95\%\ \text{CL})$$

color dipole operator

Weinberg 3-gluon operator

Comparable to strongest bounds following from collider and flavor physics !

Summary

- Axions and axion-like particles appear in many well-motivated extensions of the SM, including those addressing the strong CP problem
- They are an interesting target for searches in high-energy physics, using flavor, collider and precision probes
- If the scale of global symmetry breaking is far above the weak scale, it is important to connect the low-energy ALP couplings in a systematic way with the couplings in the UV theory
- A correct implementation of the left-handed quark currents in the chiral Lagrangian is required to correctly obtain the $K \to \pi a$ decay amplitude
- ALP unavoidably provide source terms for D=6 SMEFT operators