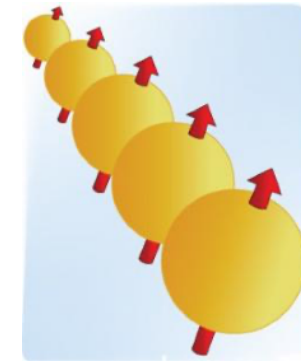
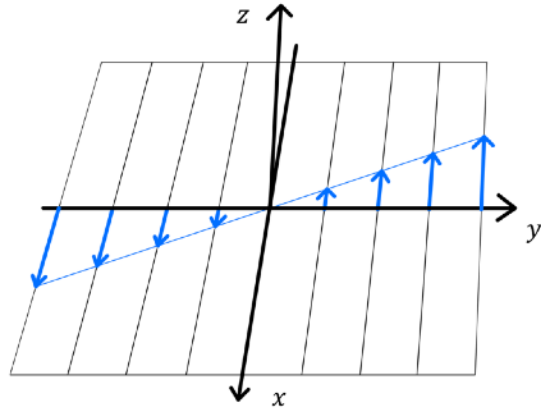


# Shear-induced polarization in heavy-ion collisions



$$\sigma^{ij} = \partial^i u^j + \partial^j u^i - \frac{2}{3} \delta^{ij} \nabla \cdot \vec{u}$$

*shear strength*

Yi Yin



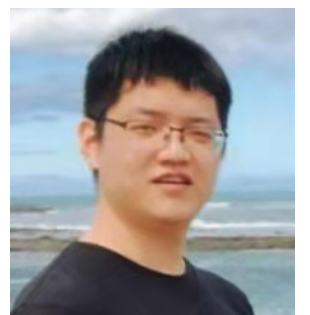
Institute of Modern Physics  
(IMP), Lanzhou, Chinese  
academy of sciences

Baochi Fu, Shuai Liu, Longgang Pang,  
Huichao Song, YY, 2103.10403;

BNL, Apr. 27th, 2021



Shuai Liu,  
postdoc@IMP



Baochi Fu,  
graduate@Peking  
U.

## Motivation

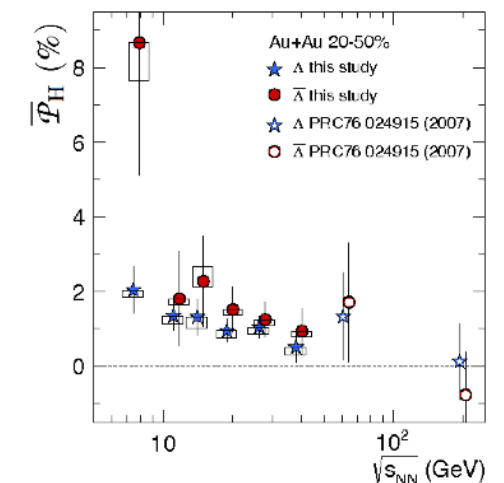
- Spin transport phenomena probe intriguing properties of quantum materials.

*Han et al, Nature Material, 19'*

- e.g.: probing excitation of fractional quantum Hall state with polarized Raman scattering.

*Nguyen, Dam Son, 2101.02213*

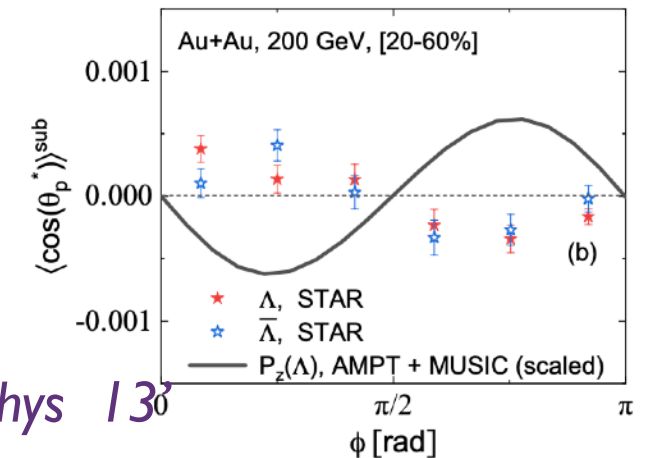
- **New frontier:**  $\Lambda$  hyperon spin polarization and non-trivial  $\phi$  meson spin alignment have been observed in heavy-ion collisions.



*STAR, Nature 17'.*

- other examples: CVE; magnetic-vorticity coupling ...
- Opportunities: exploring and understanding spin structure of QGP/ nuclear matter.
- c.f. “proton spin puzzle”.

- Vorticity effects in heavy-ion collisions:
  - describe the trends of global (phase-space averaged)  $\Lambda$  polarization.  
*Xin-Nian Wang, Zuo-Tang Liang, PRL 05'; Becattini et al, Annals Phys 13'*
  - predict behavior **qualitatively different** from the differential measurements. (“sign puzzle”) *STAR PRL 19'*



*Baochi Fu et. al,  
PRC21'*

However, qualitative discrepancies between theory and experiment may indicate that some fundamental feature of the dynamics itself (encoded hydro or transport) is misunderstood or unaccounted for. Alternatively, we may misunderstand the interface (“Cooper-

— Becattini, Lisa, *Annals Phys.* 2020

theory and phenomenology. Indeed, at this time, after having played the leading role, theory appears to have been surpassed by the experiments which have proved to be able to mea-

— *Becattini, Lisa, Annals Phys. 2020*

## Outline

- Response theory
- **New!** Shear-induced spin polarization (SIP)
- The discovery of SIP?

# Response theory

## Spin polarization generation

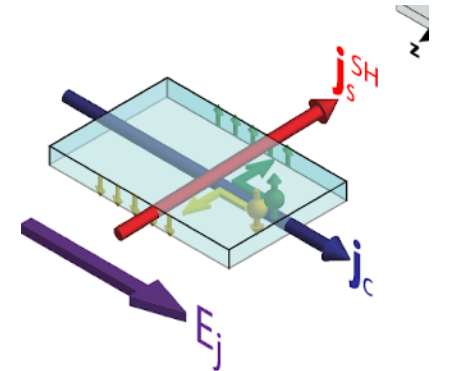
- Rotation (independent of the direction of  $\vec{p}$ ):

$$\Delta\epsilon = -\vec{s} \cdot \vec{\Omega} \rightarrow \vec{s} \parallel \vec{\Omega}$$

- Spin Hall effect:

$$\vec{s} \propto \vec{p} \times \vec{E}$$

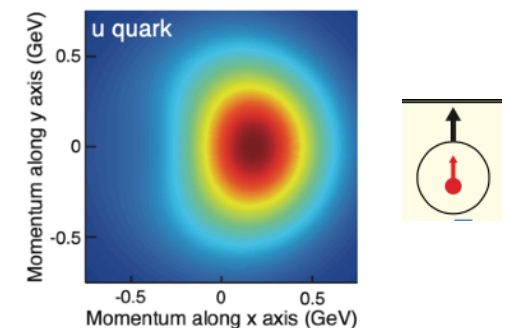
*Landau-Lifshitz volume 5*



*Illustration of spin Hall effect,  
Meyer et al, Nature material 17'*

- More generally: **spin-momentum correlation**.

- e.g. transverse momentum dependent quark distribution in a proton.

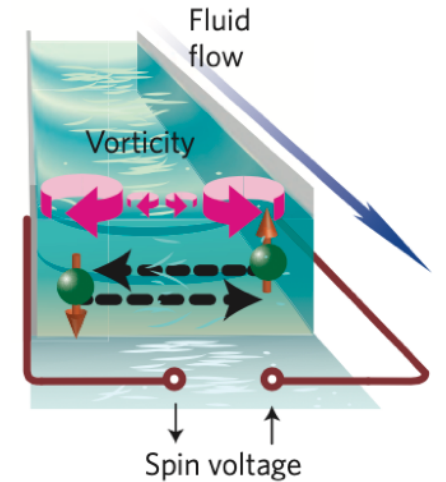


*EIC white paper*

*Differential spin polarizations: probing the spin-momentum correlation of the medium. (NB: differential spin polarization  $\neq$  local vorticity)*

## Hydro. gradient generates spin polarization

- The gradient of hydro. field (e.g. flow and energy/charge density) leads to spin polarization of fermions in a fluid.
- A familiar example: vorticity-induced polarization.



*Nature phys.,  
Takahashi et al, 16'*

- Nevertheless, vorticity is just one example of hydro. gradients. **Can we systematically analyze all possible effects of hydro. gradients?**

*The answer is yes based on the method of response theory*

## Response theory

- Response to hydro. gradients:
  - expansion in gradient.
  - relating expansion coefficients to correlators  $\langle O(x)T^{\mu\nu}(x') \rangle$ .
- E.g.: viscous stress-tensor and viscosities.

$$(T^{\mu\nu})_{\text{vis}} \propto \eta \sigma^{\mu\nu} \qquad q_{\text{heat}}^{\mu} \propto \kappa \partial_{\perp}^{\mu} T$$

- Applying similar procedure to spin polarization.



## Axial Wigner function

$$\mathcal{A}^\mu(t, \vec{x}, \vec{p}) = \int d^3\vec{y} e^{-i\vec{y}\cdot\vec{p}} \langle \bar{\psi}(t, \vec{x} - \frac{1}{2}\vec{y}) \gamma^\mu \gamma^5 \psi(t, \vec{x} + \frac{1}{2}\vec{y}) \rangle$$

- related to the phase space distribution of spin polarization vector.
- Building blocks for the gradient expansion:

$$\begin{aligned} \theta &= \partial_\perp \cdot u, \\ \omega^\mu &= \frac{1}{2} \epsilon^{\mu\nu\alpha\lambda} u_\nu \partial_\alpha^\perp u_\lambda, \quad \beta^{-1} \partial_\perp^\mu \beta, \\ \sigma^{\mu\nu} &= \frac{1}{2} (\partial_\perp^\mu u^\nu + \partial_\perp^\nu u^\mu) - \frac{1}{3} \Delta^{\mu\nu} \theta. \end{aligned}$$

*Tensors from gradient (focus on the neutral fluid)*

$$\begin{aligned} p^\mu &= \epsilon u^\mu + p_\perp^\mu, \\ Q^{\mu\nu} &= -\frac{p_\perp^\mu p_\perp^\nu}{p_\perp^2} - \frac{1}{3} \Delta^{\mu\nu}, \dots \end{aligned}$$

*Tensors formed by single particle momentum*

$$\text{e.g. : } \mathcal{A}^\mu \sim \epsilon^{\mu\nu\alpha\lambda} u_\nu Q_{\alpha\rho} \sigma_\lambda^\rho$$

- The most general expression consistent with symmetries (focus on the neutral fluid):

$$u \cdot \mathcal{A} = \tilde{c}_\omega p \cdot \omega ,$$

$$\mathcal{A}_\perp^\mu = c_\omega \omega^\mu + c_T \epsilon^{\mu\nu\alpha\lambda} u_\nu p_\alpha \partial_\lambda \log \beta + g_\sigma \epsilon^{\mu\nu\alpha\lambda} u_\nu Q_{\alpha\rho} \sigma^\rho_\lambda + g_\omega Q^{\mu\nu} \omega_\nu$$

vorticity effects

spin Nernst effect

shear-induced polarization

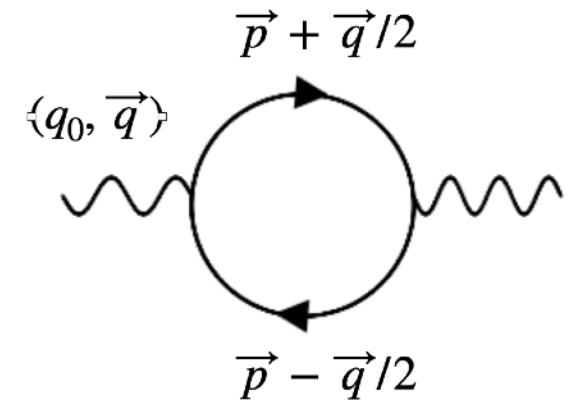
$$\vec{s} \propto \hat{p} \times \nabla \log T$$

Spin-momentum correlation

- The coupling between flow gradient and **momentum quadrupole**, although allowed by symmetry, has never been discussed before.
- The expansion coefficients  $\tilde{c}_\omega, c_\omega, c_T, g_\sigma, g_\omega$ , i.e., energy-spin and **momentum-spin correlation functions**, depend on  $T, p \cdot u$  and can be determined from microscopic theories.
- c.f. Sivers function etc.

- Computing retarded correlators:

$$\int_{\vec{y}} e^{i\vec{y}\cdot\vec{p}} \langle \bar{\psi}(t, \vec{x} - \frac{1}{2}\vec{y}) \gamma^\mu \gamma^5 \psi(t, \vec{x} + \frac{1}{2}\vec{y}) T^{\alpha\beta}(0,0) \rangle$$



- For general fermion mass at one-loop:

$$\mathcal{A}_\perp^\mu = (-n'_{FD}) \left[ \underbrace{\omega^\mu}_{\text{vorticity effects}} + \underbrace{\epsilon^{\mu\nu\alpha\lambda} u_\nu p_\alpha \partial_\lambda \log \beta}_{\text{spin Nernst effect}} + \frac{-p_\perp^2}{(p \cdot u)} \underbrace{\epsilon^{\mu\nu\alpha\lambda} u_\nu Q_{\alpha\rho} \sigma^\rho_\lambda}_{\text{shear-induced polarization}} \right] + 0 \times Q^{\mu\nu} \omega_\nu$$

(NB:  $p \cdot \mathcal{A} \sim \mathcal{O}(\text{higher loops})$ , see. Di-Lung Yang, Hattori, Yoshimasa, JHEP 20')

- Open question: dissipative or not?

## Chiral kinetic theory

- (analogous) magnetization current term contributes to axial Wigner function.

$$\vec{\mathcal{A}} = \sum_{\lambda=R,L} \left[ s_{\lambda} \hat{p} f_{\lambda} - \frac{\hat{p}}{2p} \times \nabla f_{\lambda} \right]$$

*Son, Yamamoto, PRD 12;*

*Chen, Son, Stephanov, Yee, YY, PRL 14;*

*Chen, Son, Stephanov, PRL 15;*

- Consider near equilibrium expansion:

$$\sum_{\lambda=R,L} \left[ -\frac{\hat{p}}{2p} \times \nabla n(\epsilon_p - \vec{p} \cdot \vec{u} + \Delta\epsilon_p) \right] \rightarrow \beta n(1-n) \epsilon^{ikj} Q_{jl} \sigma_k^l + (\sim \omega)$$

- Agrees with one loop calculations in the same settings.

*see also Hayata, Mameda, Yoshimasa, JHEP 20'*

## Comparison

- Summary of one-loop results:

see also Becattini et al,  
2103.10917

Spin polarization=[**Vorticity**]+[**T-gradient**]+[**Shear**]

- Popular: spin distribution in a specific hydro. configuration (no entropy production)

Becattini et al, *Annals Phys.* 323:2452 (08)  
*Annals Phys.* 338:32 (13) and follow-ups;

$$\partial_\mu(\beta u_\nu) + \partial_\nu(\beta u_\mu) = 0 \leftrightarrow \partial_\mu s^\mu = 0$$

$$\rightarrow \mathcal{A}^\mu \propto \epsilon^{\mu\nu\alpha\beta} \left[ \partial_\alpha(\beta u_\beta) - \partial_\beta(\beta u_\alpha) \right] p_\nu$$

- Without shear, agrees with one-loop calculations.
- Response theory analysis applies to general hydro. profile and can be improved systematically through higher-loop/non-perturbative calculations.

# Shear-induced polarization

## Interpretation

- In spatial components

$$(s^i)_{\text{SIP}} \propto \epsilon^{ikj} Q_{jl} \sigma_k^l = \epsilon^{ikj} \hat{p}_j \hat{p}_l \sigma_k^l, \quad Q_{ij} = \hat{p}_i \hat{p}_j - \frac{1}{3} \delta_{ij}$$

- c.f. Spin Hall effect:

$$\vec{s} \propto \hat{p} \times (\overrightarrow{\sigma \odot p})$$

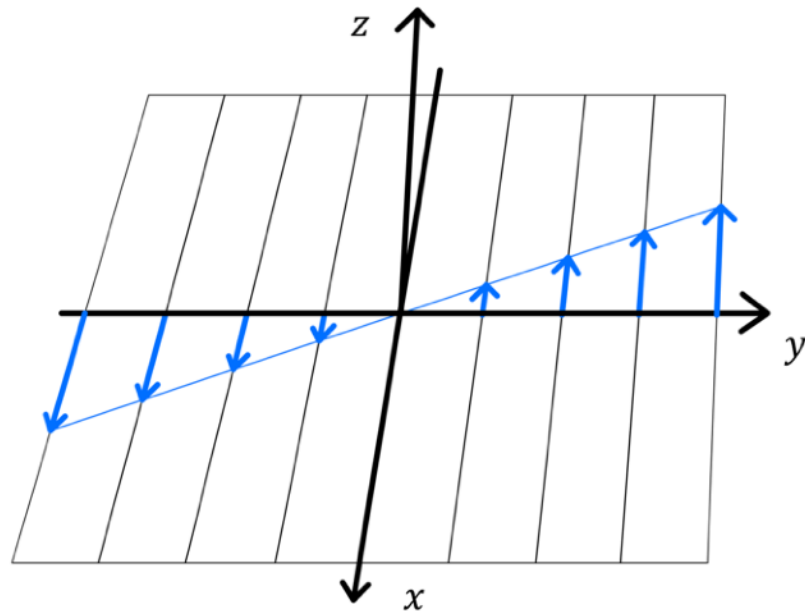
hydro. force

- $\sigma^{\mu\nu} \rightarrow T^{\mu\nu} \rightarrow M^{\mu\nu\alpha} \rightarrow$  spin polarization

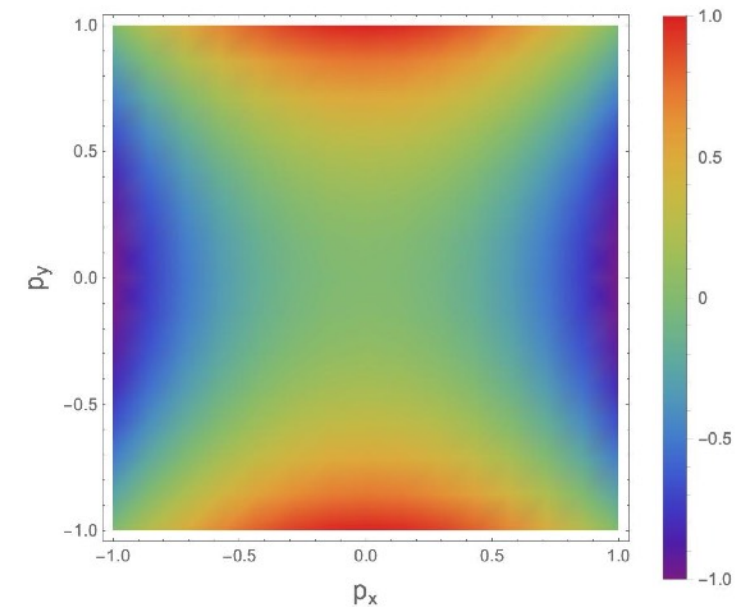
$$(M^{\mu\nu\alpha} = x^\nu T^{\mu\alpha} - x^\mu T^{\nu\alpha})$$

Belinfante, 1940

# Illustration



A standard shear flow profile:  
 $\omega^z \neq 0, \sigma^{xy} \neq 0$



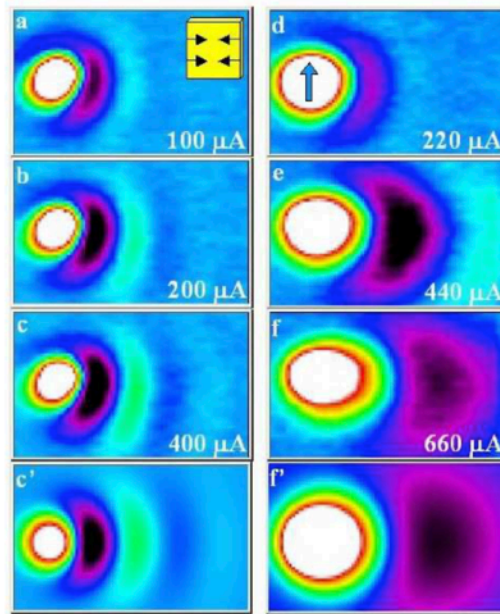
Spin polarization along z-direction in  
 phase space from SIP.

$$\mathcal{A}_{SIP}^i \propto \epsilon^{ikj} Q_{jl} \sigma_k^l, \quad Q_{ij} = \hat{p}_i \hat{p}_j - \frac{1}{3} \delta_{ij}$$

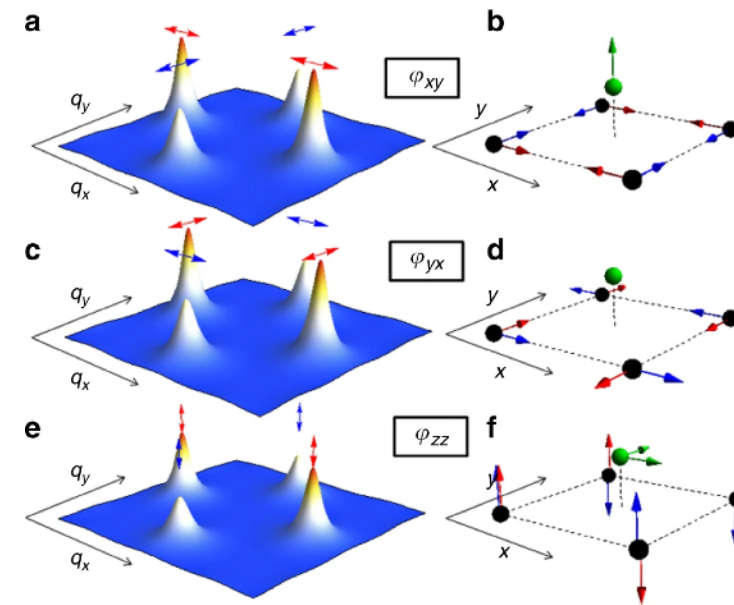
Shear-induced polarization (SIP): imaging anisotropy in a fluid into anisotropy in spin space.



## Observation?



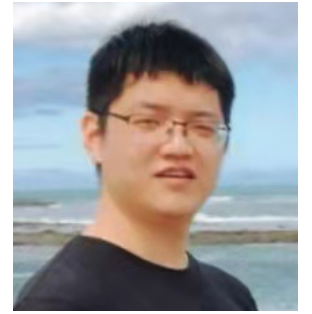
*n-type GaAs, Crooker and Smith, PRL, 04'*



*BaFe<sub>2</sub>As<sub>2</sub>, Kissikov et al, Nature communication, 20'*

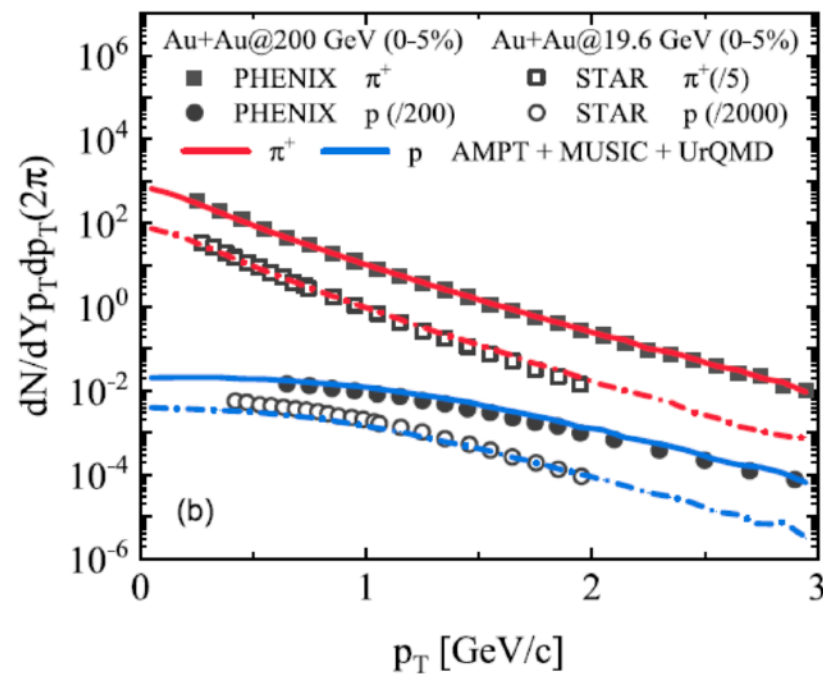
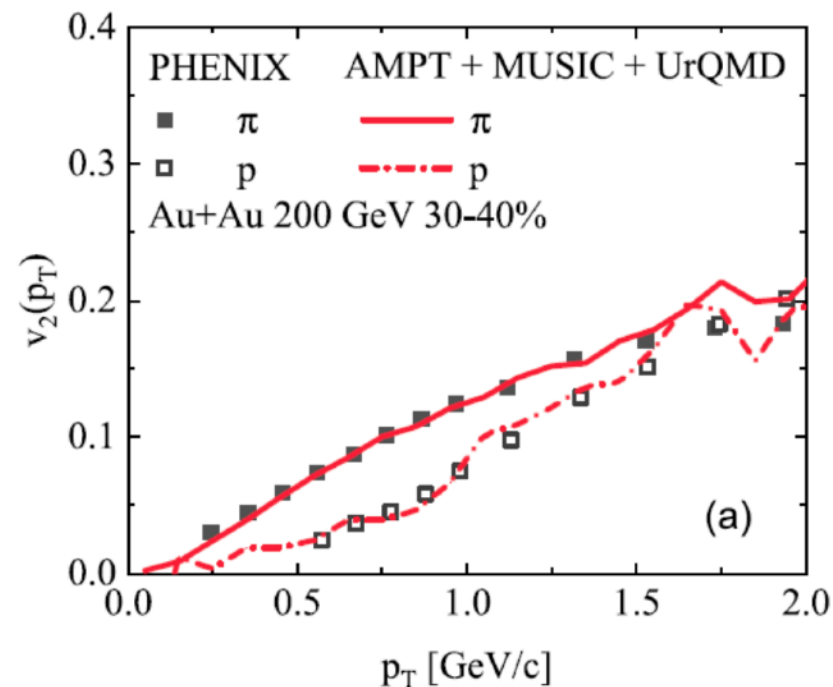
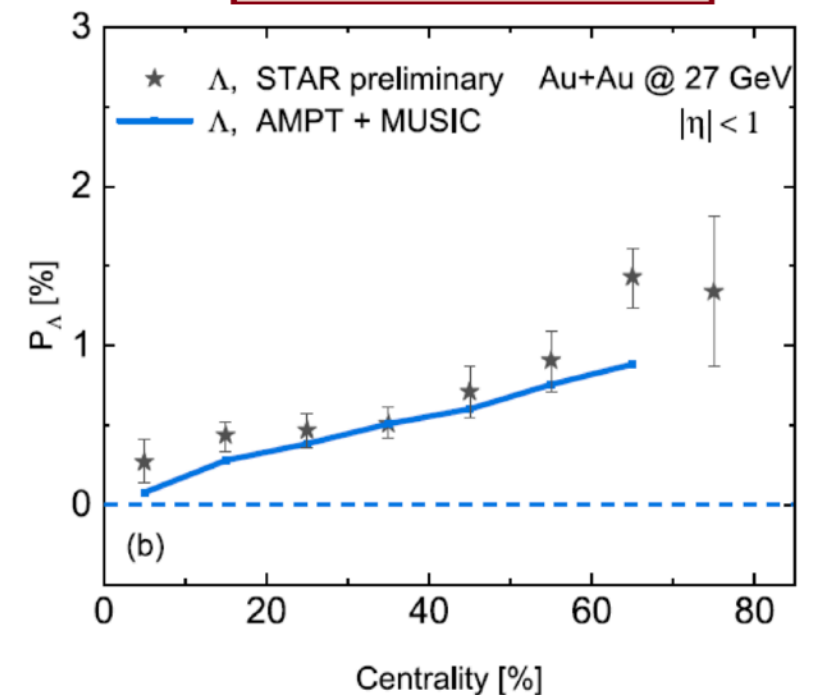
- The cousin effect, strain-induced polarization has been observed in crystals and liquid crystals.
- Shear-induced polarization (SIP): generic in fluids.
  - Can we/did we see SIP in heavy-ion collisions?
  - What can we learn ?

# Heavy-ion collisions

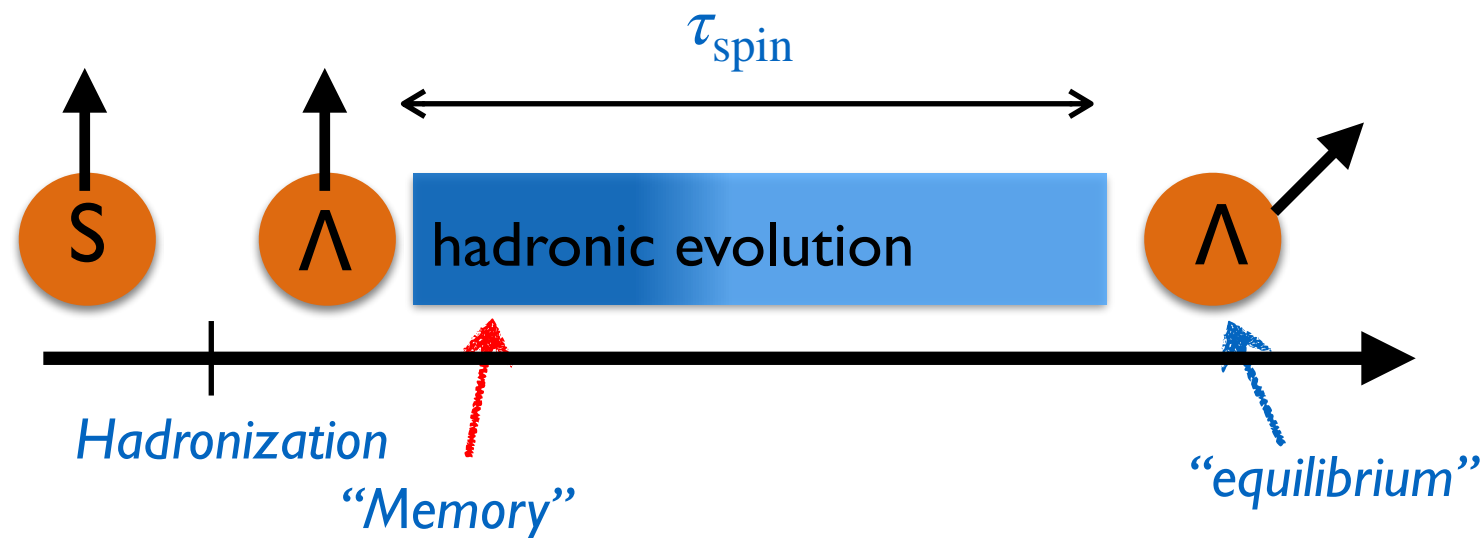
Hydro. Model

Baochi Fu, graduate@Peking U.

Transverse momentum spectra

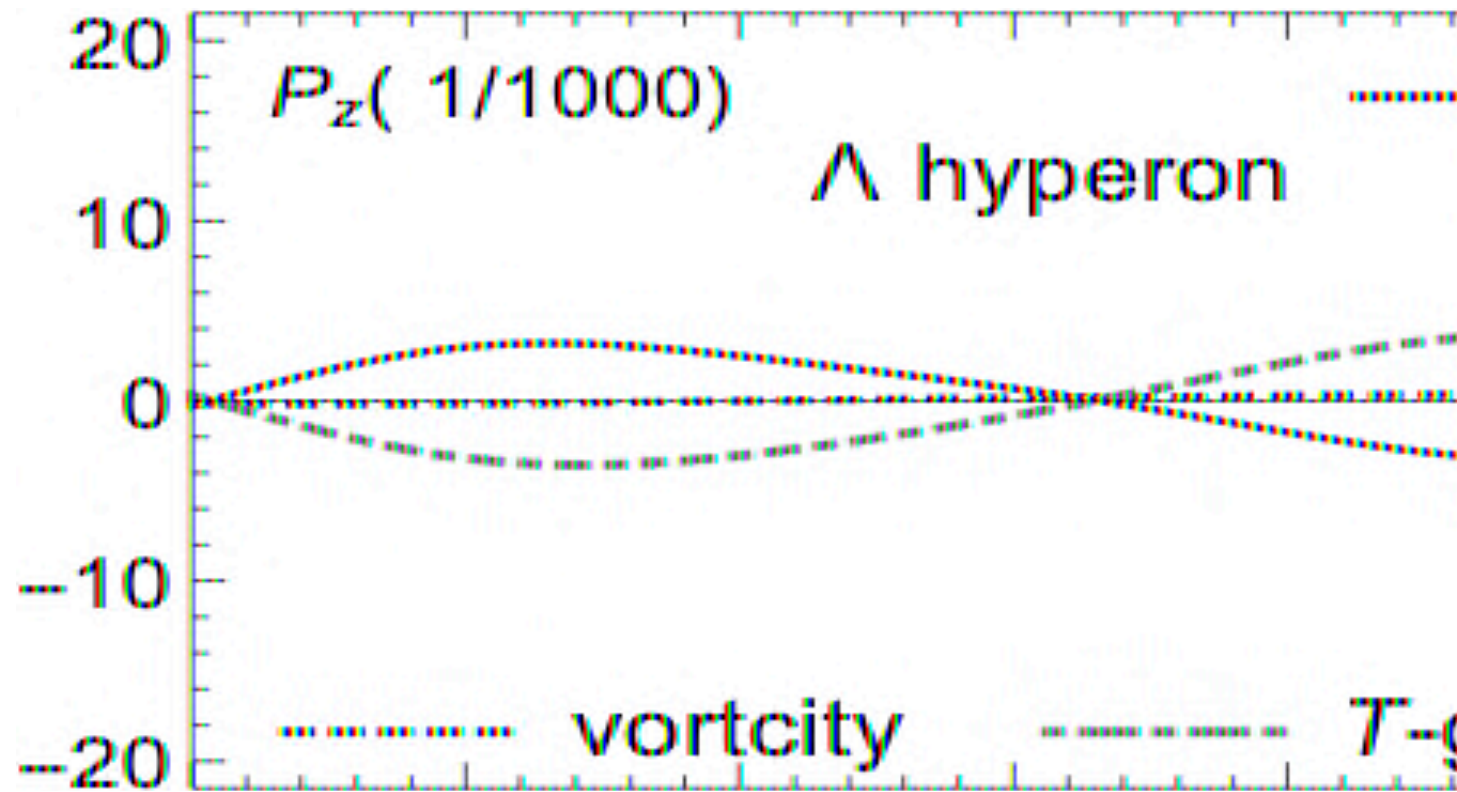
 $v_2(p_T)$ Global Polarization  
from thermal vorticity

- Hydro. profile from the data-calibrated hydro. modeling (AMPT+MUSIC).

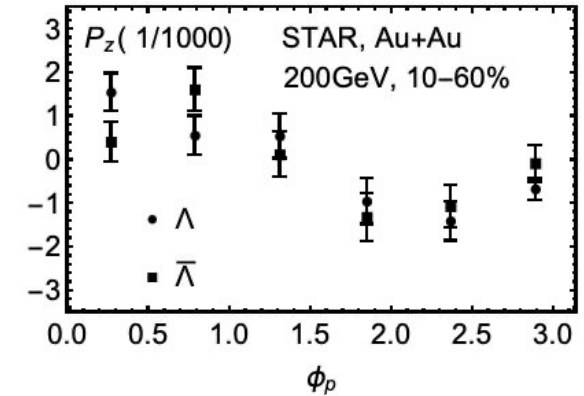


- “Local equilibrium”: expressible in terms of hydrodynamic field.
- Two benchmark scenarios:
  - “Lambda equilibrium”:  $\Lambda$  is born (shortly after hadronization) in equilibrium.
  - “**strange memory**”:  $\Lambda$  memorizes the polarization of strange quarks
- Focus on the qualitative feature.

## Lambda spin polarization along longitudinal direction



vs transverse azimuthal angle  $\phi_p$

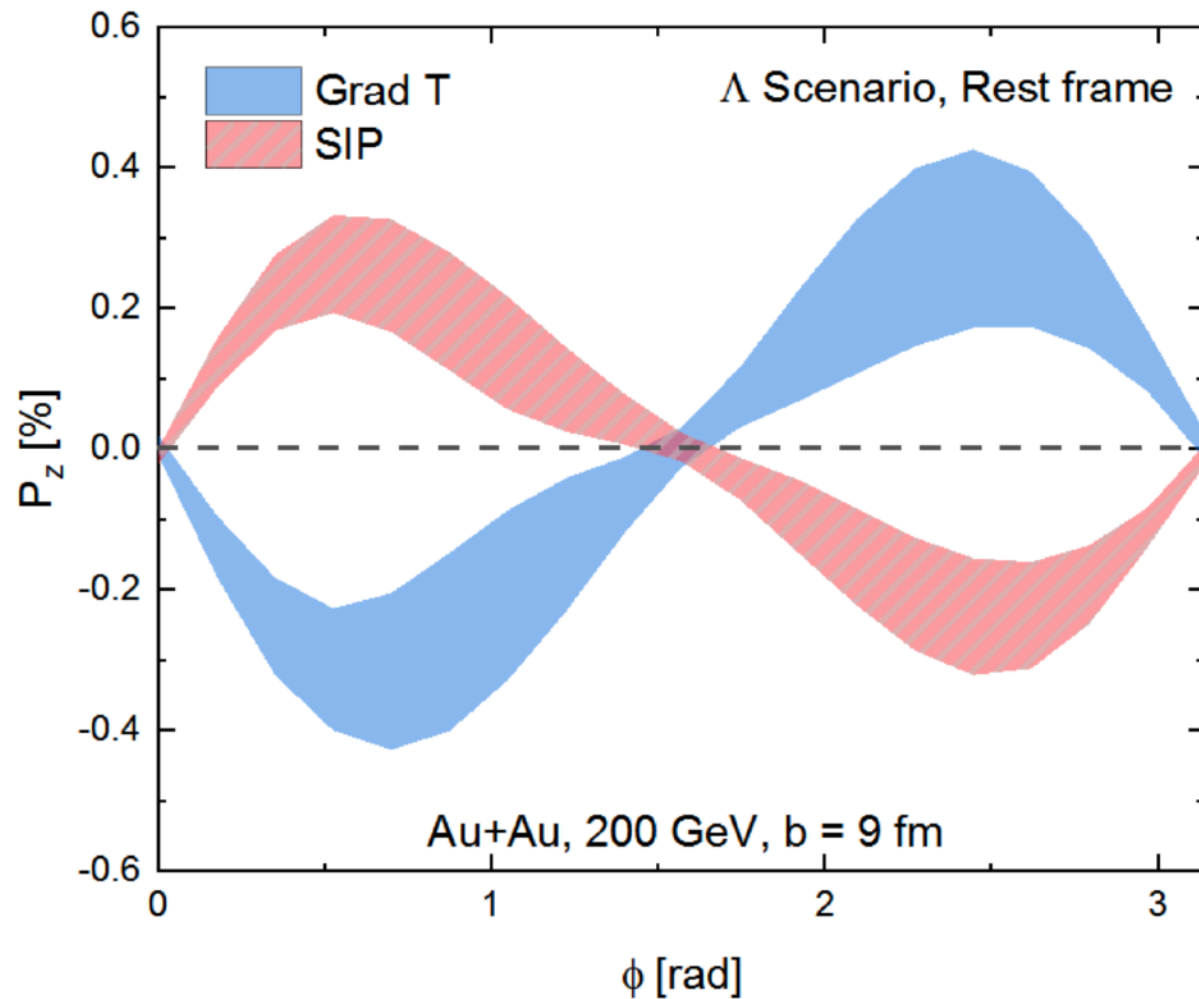


Spin polarization = [vorticity] + [T-gradient] + [Shear]

- SLP gives a “right sign” while the effect of T-gradient leads to “wrong sign”.

*also confirmed in 2103.14621 by Becattini et al from an independent hydro. simulation*

## Sensitivity to the inputs of hydro.

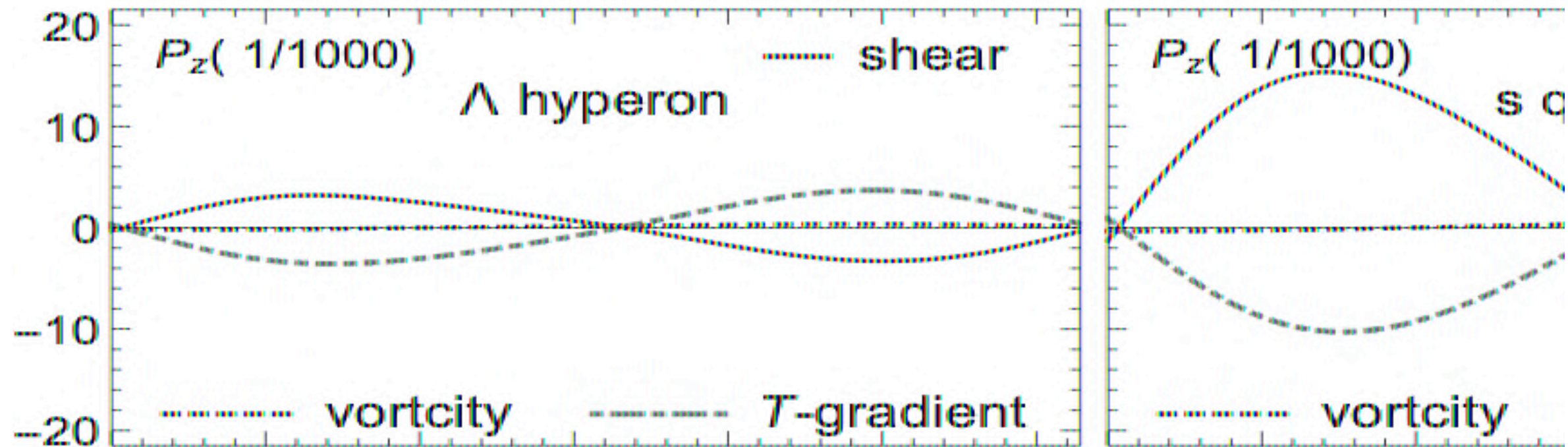


Band: possible flexibility of [Grad T] and [SIP]

- Initial flow: on  $\rightarrow$  off
- Initial condition: AMPT  $\rightarrow$  Glauber
- Shear viscosity: 0.08  $\rightarrow$  off
- Bulk viscosity:  $\zeta/s(T)$   $\rightarrow$  off
- Freeze-out temperature:  
167 MeV  $\rightarrow$  157 MeV



## “Lambda equilibrium” vs “strange memory”

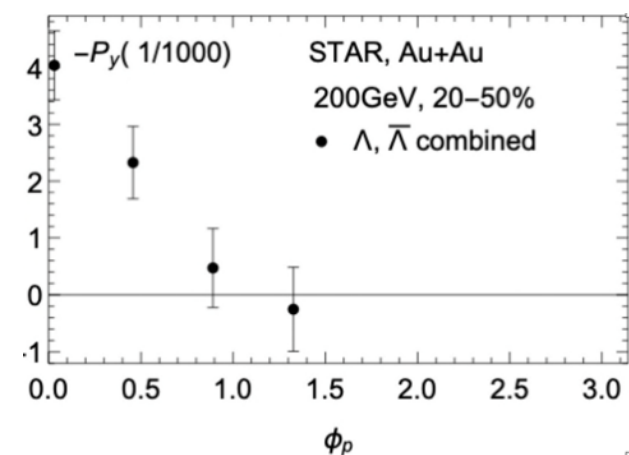
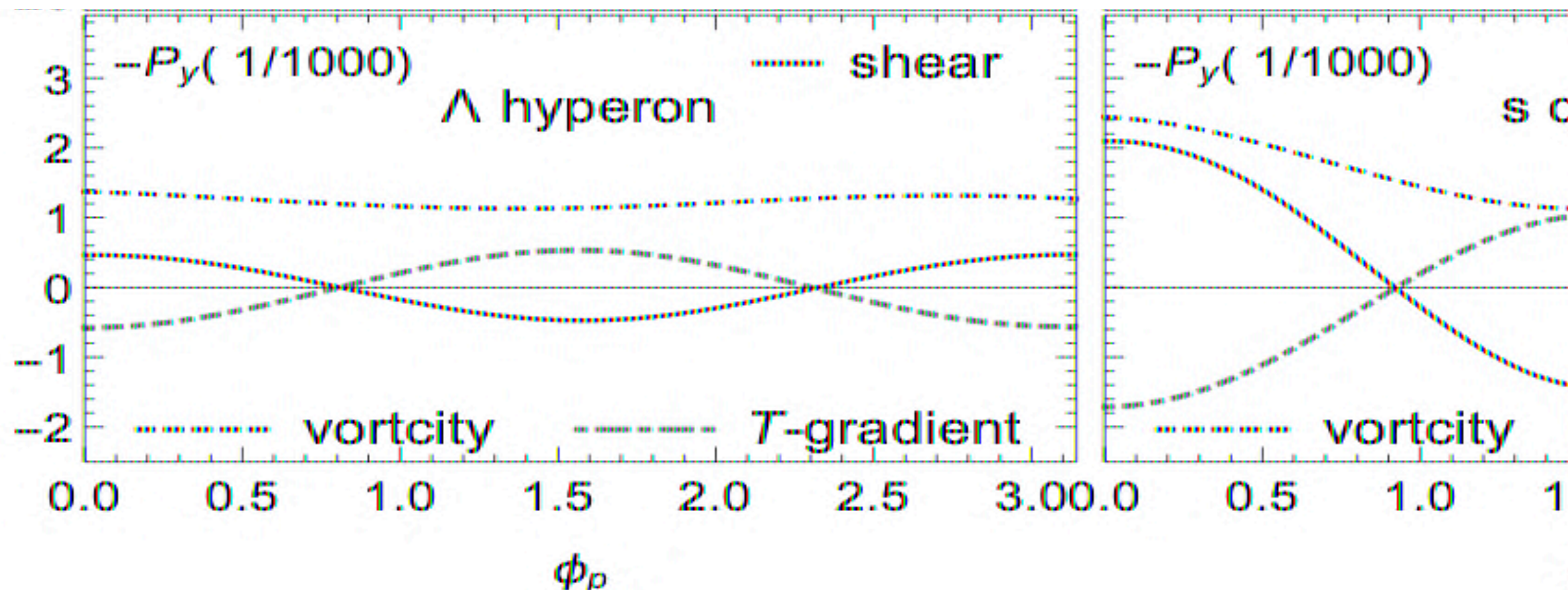


vs transverse azimuthal angle  $\phi_p$

$$\text{Spin polarization} = [\text{Vorticity}] + [\text{T-gradient}] + [\text{Shear}]$$

- SLP becomes more prominent when the mass of spin carrier becomes smaller.

## Similar story for $P_y$

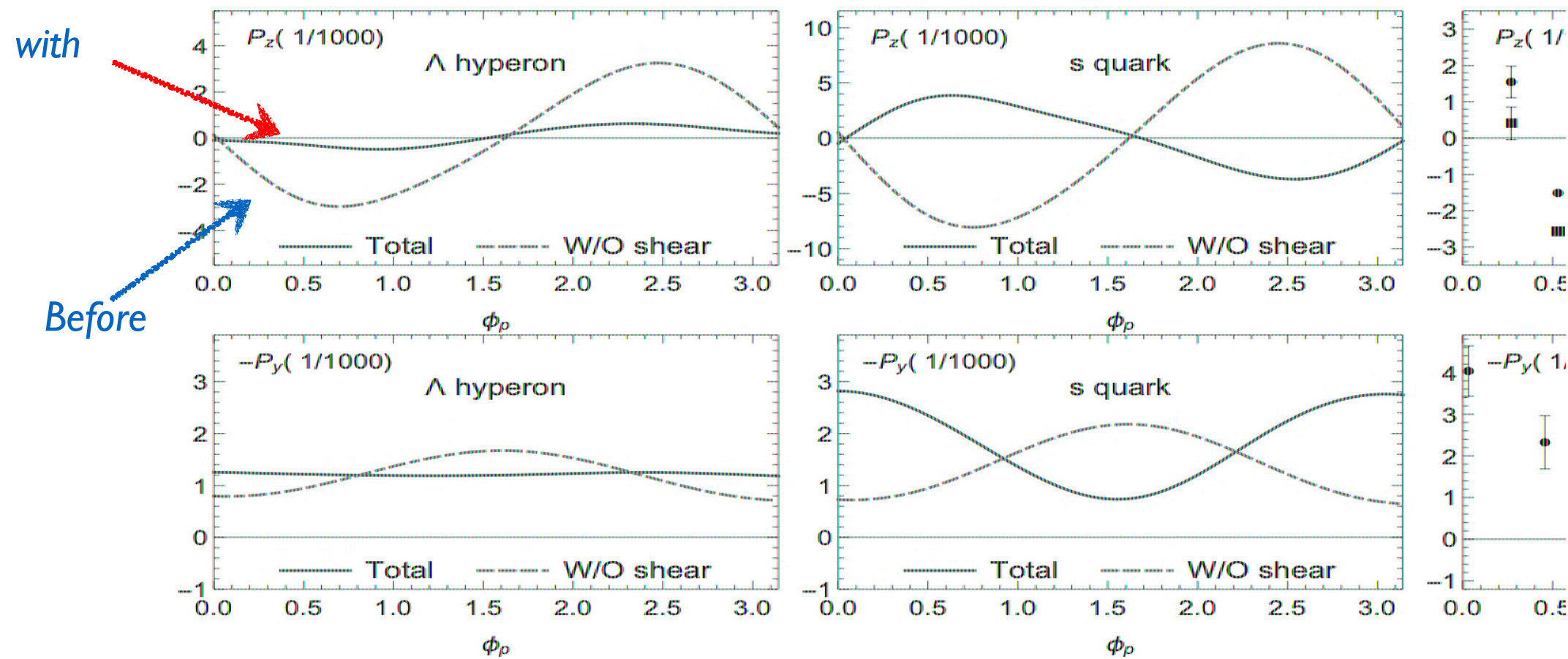


*STAR preliminary results*



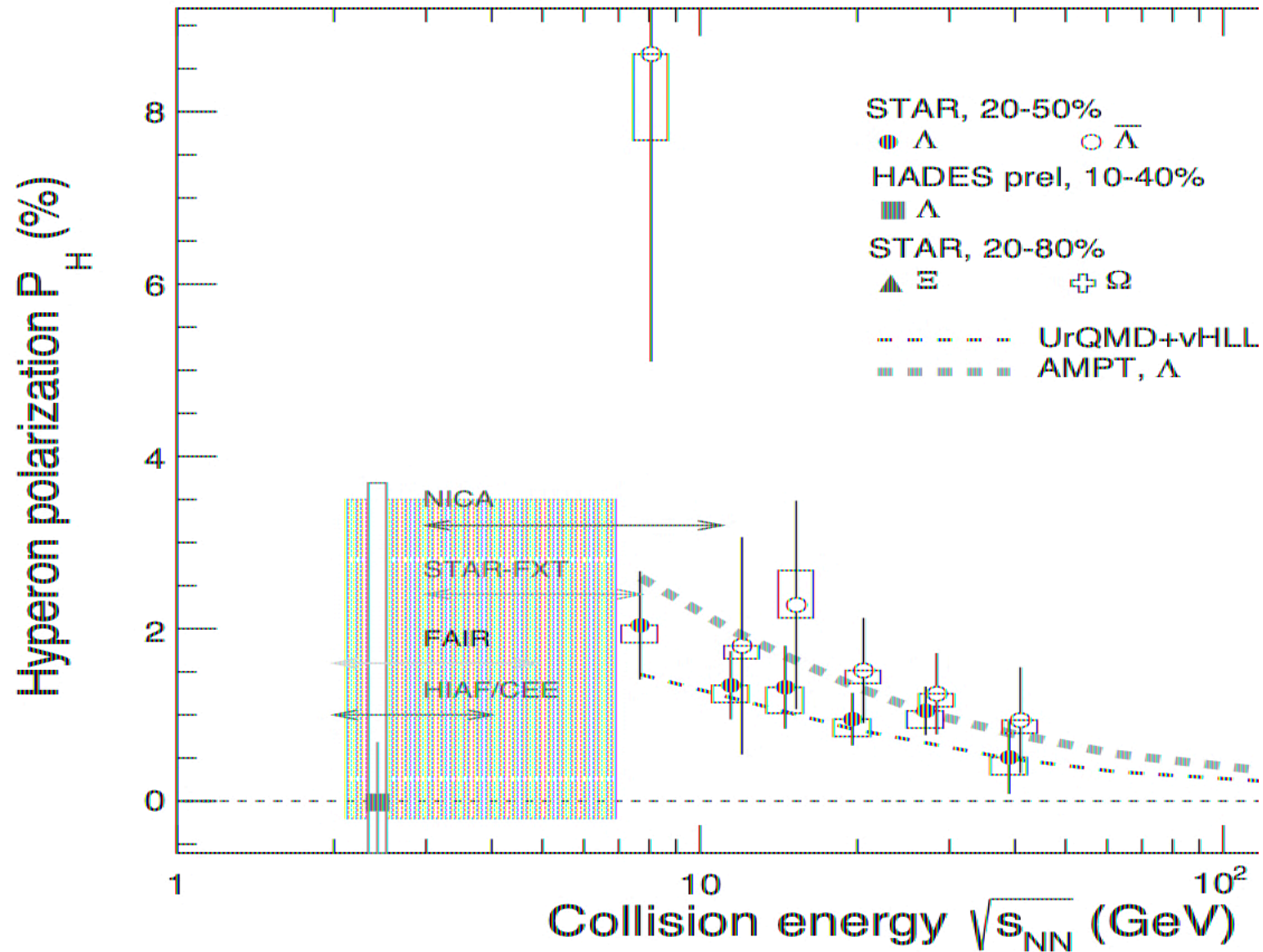
# Total spin polarization

Baochi Fu, Shuai Liu, Longgang Pang, Huichao Song and YY,  
2103.10403



- Shear-induced polarization (SIP) effects are indispensable.
- SIP determines the qualitative feature of differential polarization in the “strange memory” scenario .
- $\Lambda$  polarization may probe the properties of QGP.

## QGP signature



The suppression of global  $\Lambda$  polarization may tell us below which beam energy QGP “is switched” off.

# Summary and outlook

## Summary

- Differential spin polarization: probes the spin-momentum correlation of QCD matter
- Response theory analyses the effects of hydro. gradient on spin polarization systematically.
- **New!** Shear-induced polarization (SIP): spin polarization generation through shear flow.
- SIP and Lambda's memory of strange quark polarization: key to understand qualitative behavior of heavy-ion collisions data.
- For quantitative study, a comprehensive transport theory with spin is crucially needed.

## Outlook

- To claim the discovery of shear-induced polarization and spin Nerst effect (effects of T-gradient), what is the road map?
- Baryonic Spin Hall effect (future): exploring QCD matter at finite baryon density.

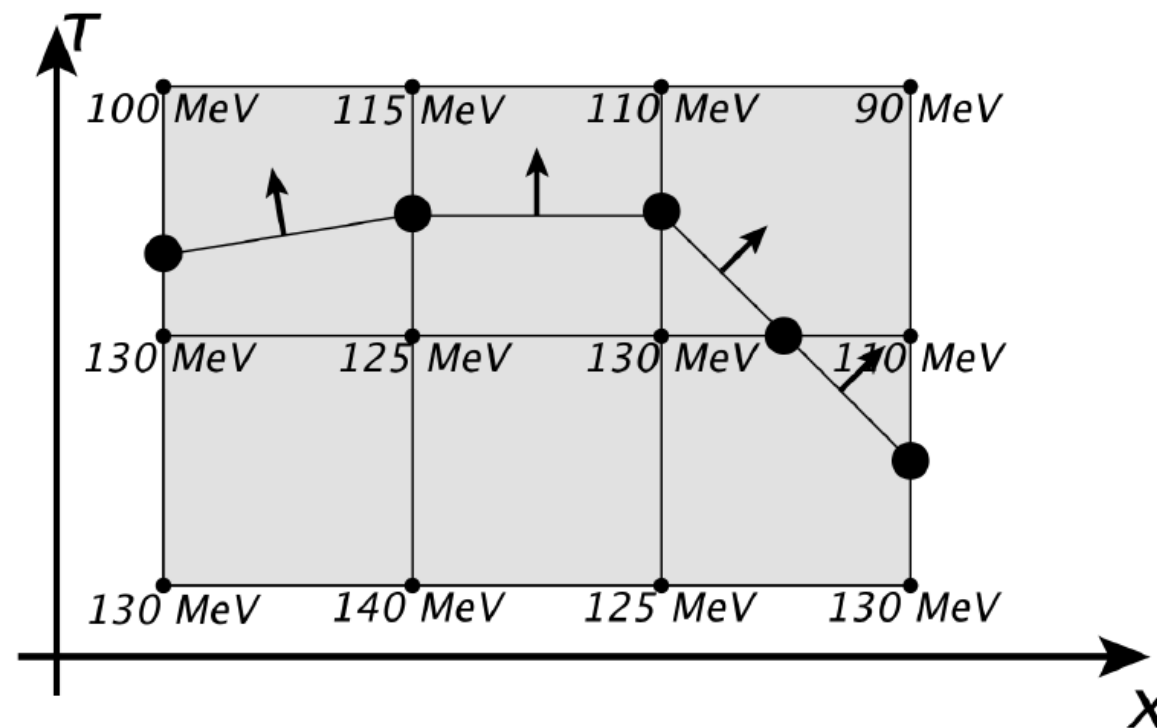
$$\vec{P}_{\pm} \propto \pm \hat{p} \times \nabla \mu_B$$

*Shuai Liu-YY, 2020.12421. Also, Son, Yamamoto, PRD 12; Di-Lung Yang, Hattori, Yoshimasa, PRD 19'*

Ultimate goal: spin structure of QCD matter.

# Back-up

## Comments on 2103.14621 by Becattini et al



*T-gradient on equal-T surface  
from hydro. simulation (Fig. by  
Schenke, 16')*

- confirms that SIP gives right sign contribution.
- Assuming that T-gradient effects can be ignored on the equal-T surface, they further argue that “Lambda equilibrium” scenario agrees with data with SIP.
- However, T-gradient is generically (and explicitly) non-zero on the equal-T surface.
- So, we stand for “strange memory” scenario.

## Can $\Lambda$ spin flipping rate be small?

Quark model+vector meson dominance, nucleon (N) and hyperon interaction is mediated by  $\omega$  which couples with constituent u and d quark.

*Jennings, PLB 1990; Cohen-Weber PRC 1991*

However, spin of  $\Lambda$  is carried by s quark. So

(spin-dependent) N- $\Lambda$  interaction  $\ll$  (spin-dependent) N-N interaction.

This picture explains the puzzling experimental results

$$N-\Lambda \approx \frac{1}{40} N-N$$

*S.Ajimura et al. PRL 2001*

Under this picture,  $\Lambda$  spin flip rate could be (much) smaller than its equilibration rate  $\Rightarrow$  worthy checking in future.