First-principles-based equations of state for QCD at finite temperature and density

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Outline

I. Motivation for QCD modeling with all conserved charges

II. Four-dimensional BQS equations of state

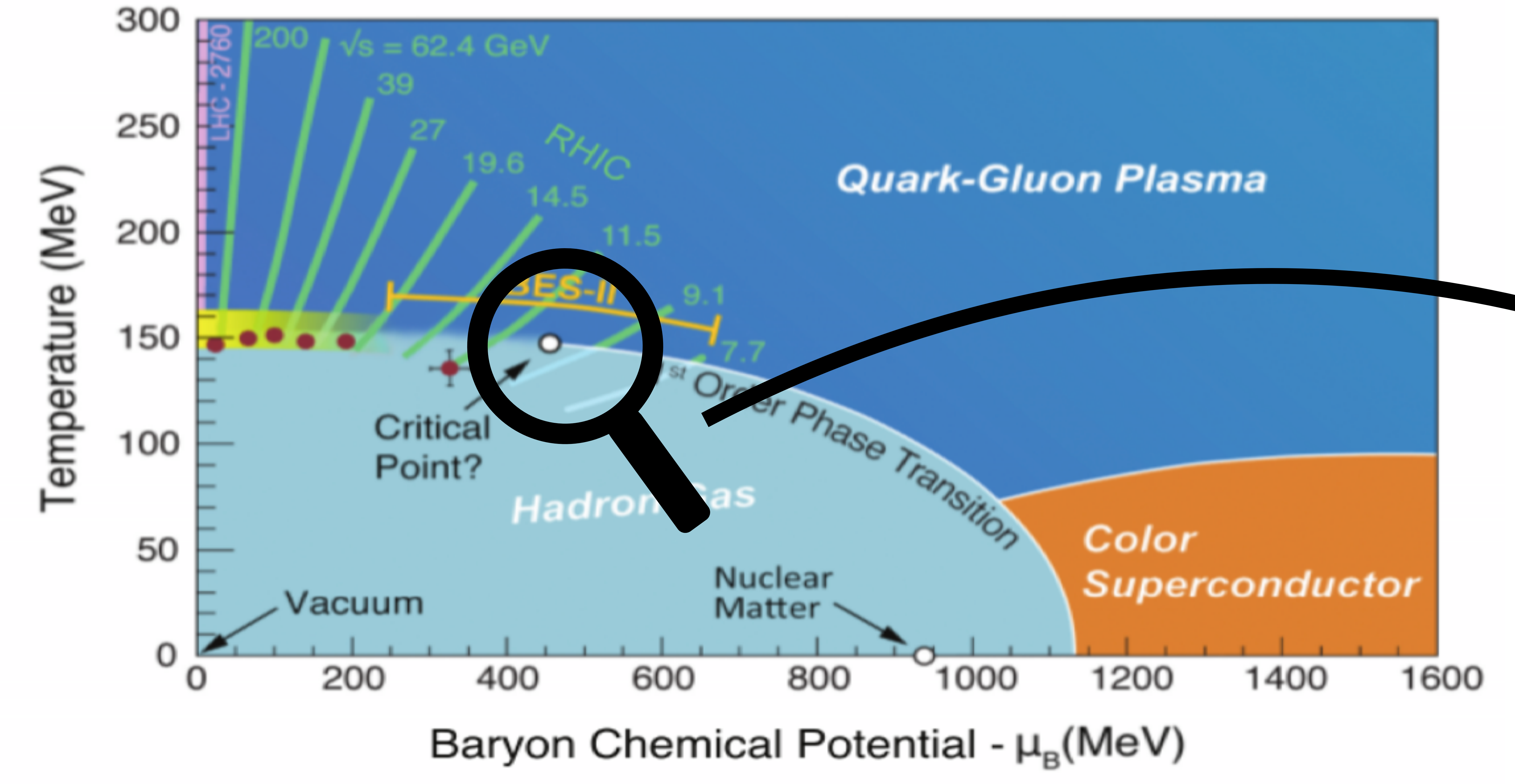
III. Strangeness-neutral equation of state with a critical point

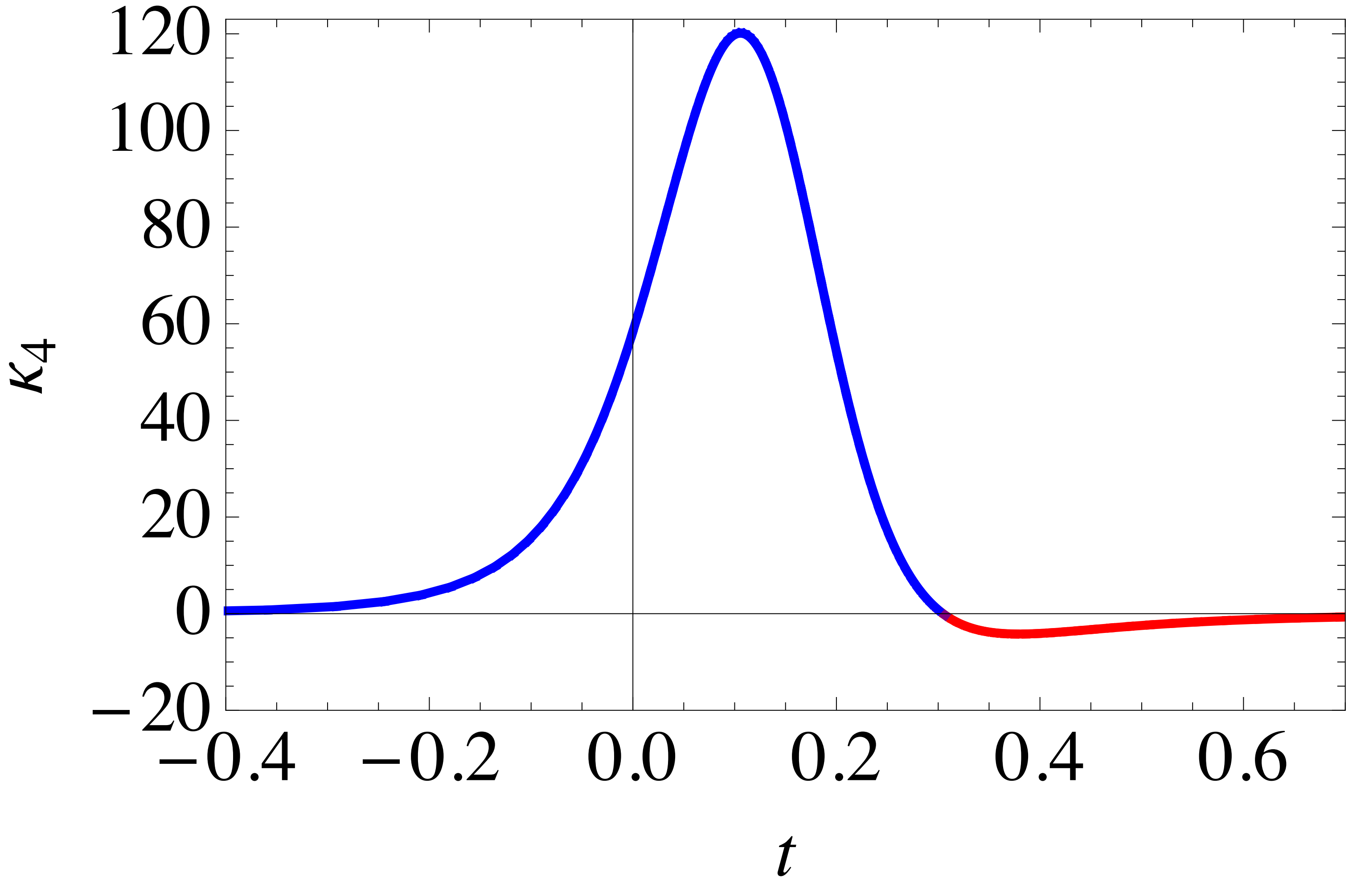
IV. Conclusions

I. Motivation

Search for Criticality

- Ongoing search for critical point requires support from theory community to provide candidates for criticality-carrying observables



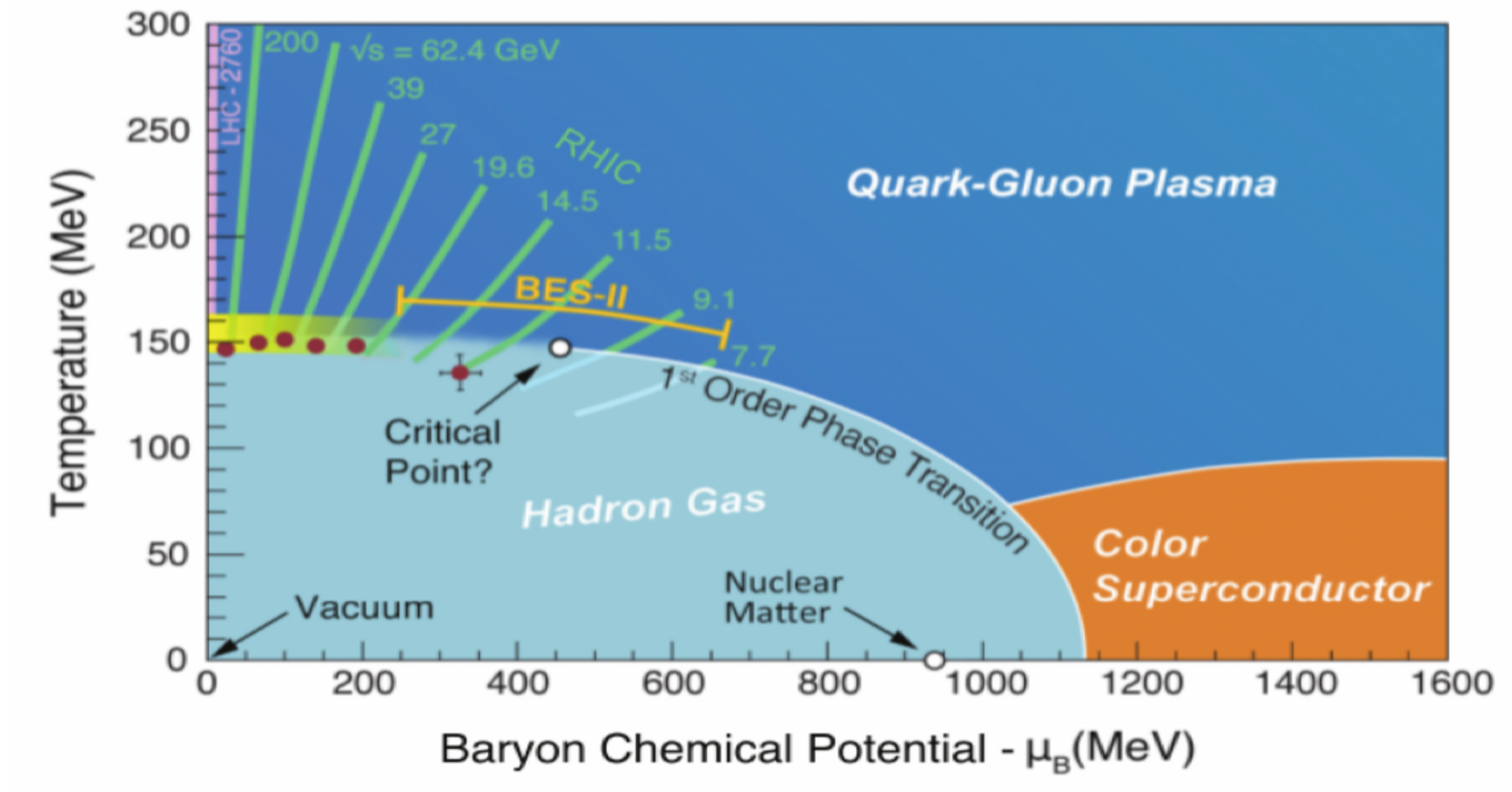


NSAC 2015 Long Range Plan for Nuclear Physics
M. Stephanov, K. Rajagopal and E. Shuryak, PRD (1999)
M. Stephanov, PRL (2011)

QCD Phase Diagram

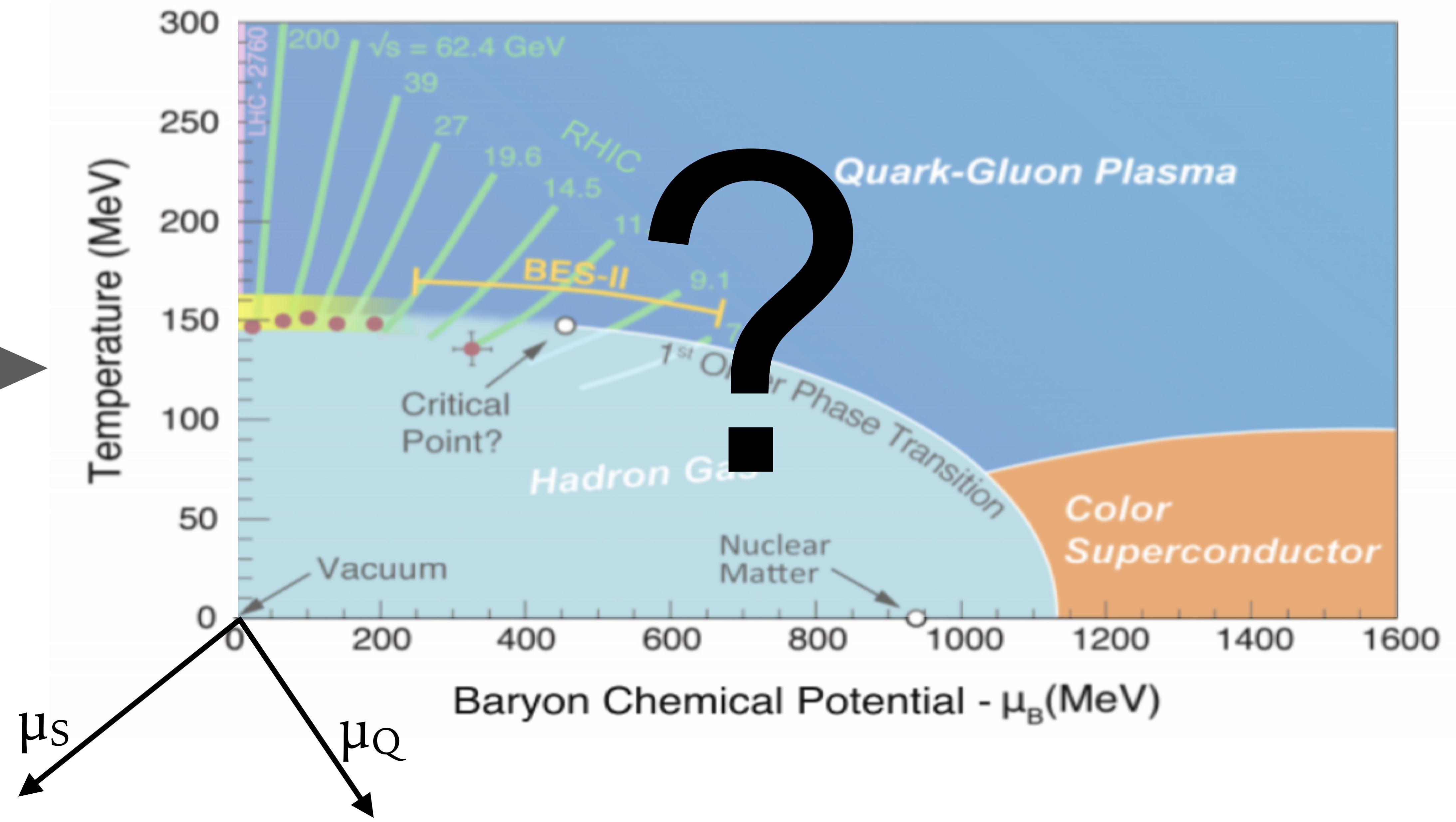
- The strongly interacting matter present in heavy-ion collisions carries a multitude of conserved quantum numbers: baryon number, strangeness and electric charge
    - This effects thermodynamics since each charge has an associated chemical potential

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln Z(V, T, \mu_B, \mu_S, \mu_Q)$$



$\mu_Q(T, \mu_B)$

$\mu_S(T, \mu_B)$



Lattice QCD Predictions

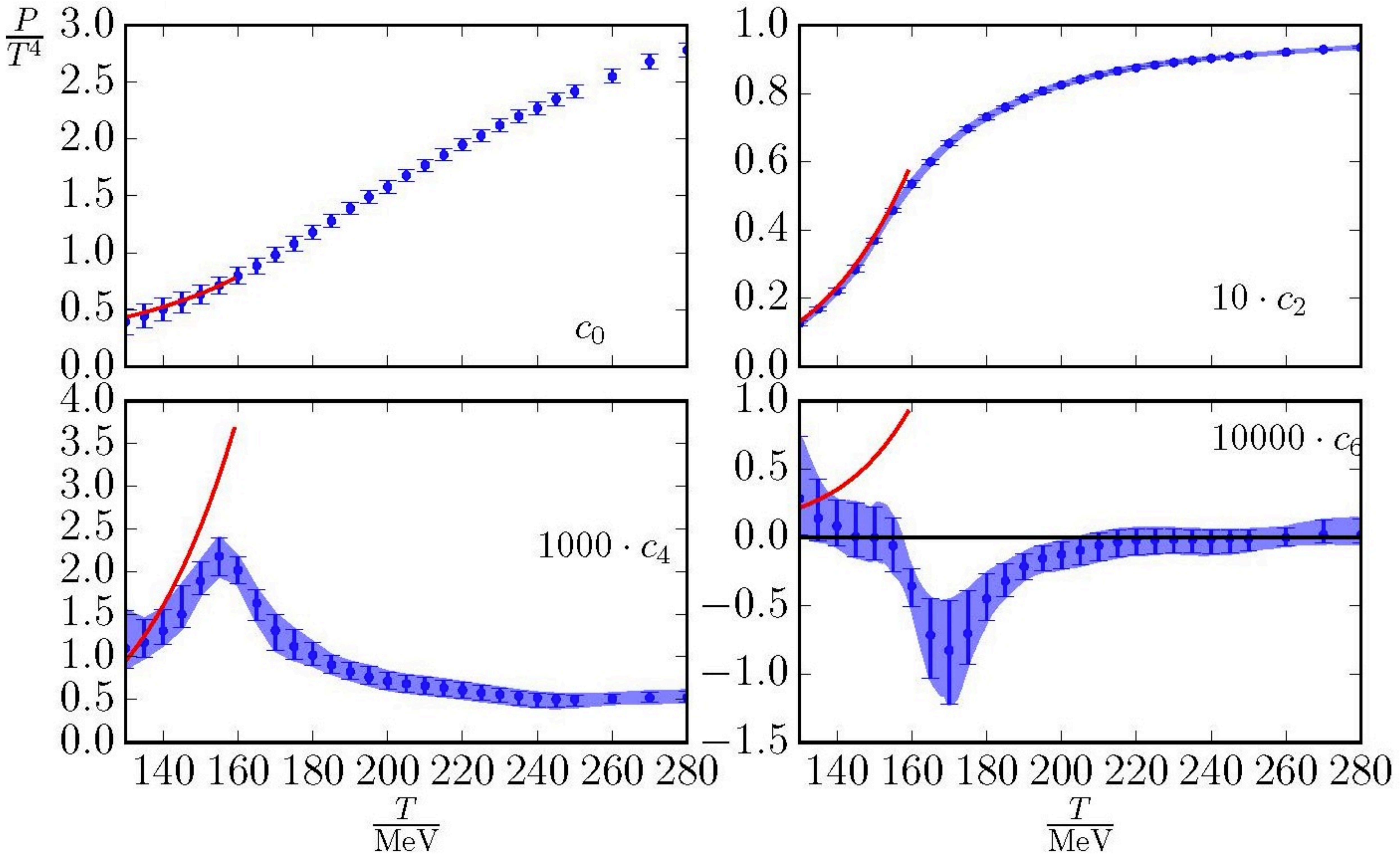
- The equation of state (EoS) for QCD has been calculated on the lattice under strangeness neutrality and fixed ratio of baryon number to electric charge, matching the heavy-ion situation

$$\frac{p(T,\mu_B)}{T^4} = \frac{p(T,0)}{T^4} + \sum_{n=1}^{\infty} \frac{1}{(2n)!} \frac{\mathrm{d}^{2n}(p/T^4)}{d(\frac{\mu_B}{T})^{2n}}\Bigg|_{\mu_B=0} \left(\frac{\mu_B}{T}\right)^{2n}$$

$$= \sum_{n=0}^{\infty} c_{2n}(T) \left(\frac{\mu_B}{T}\right)^{2n}$$

$$\langle n_s \rangle = 0$$

$$\langle n_Q \rangle = 0.4 \langle n_B \rangle$$



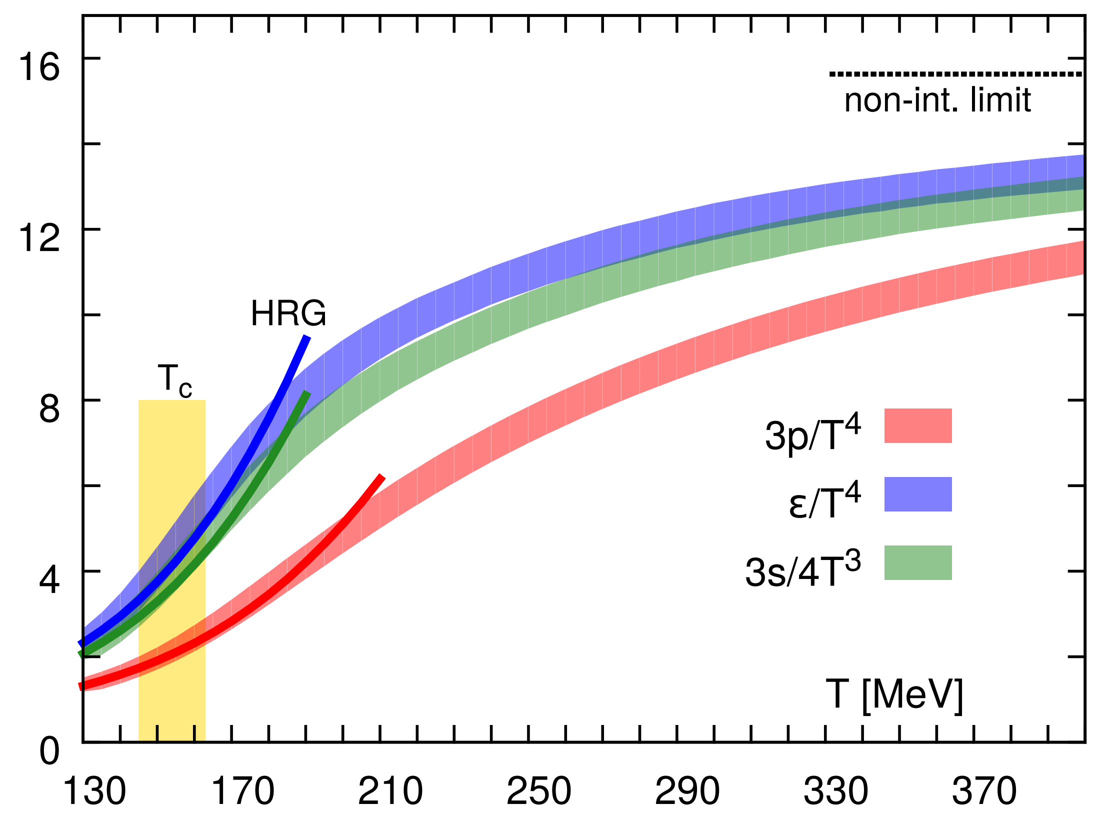
For $\mu_S = \mu_Q = 0$ see: S. Borsanyi et al, JHEP (2018)

WB: J. Guenther et al, NPA (2017)
HotQCD: A. Bazavov et al, PRD (2017)

Hadron Resonance Gas Model and Lattice QCD

➤ The HRG model agrees well with lattice QCD results for a range of quantities

Equation of state (pressure, entropy density, energy density, etc.)



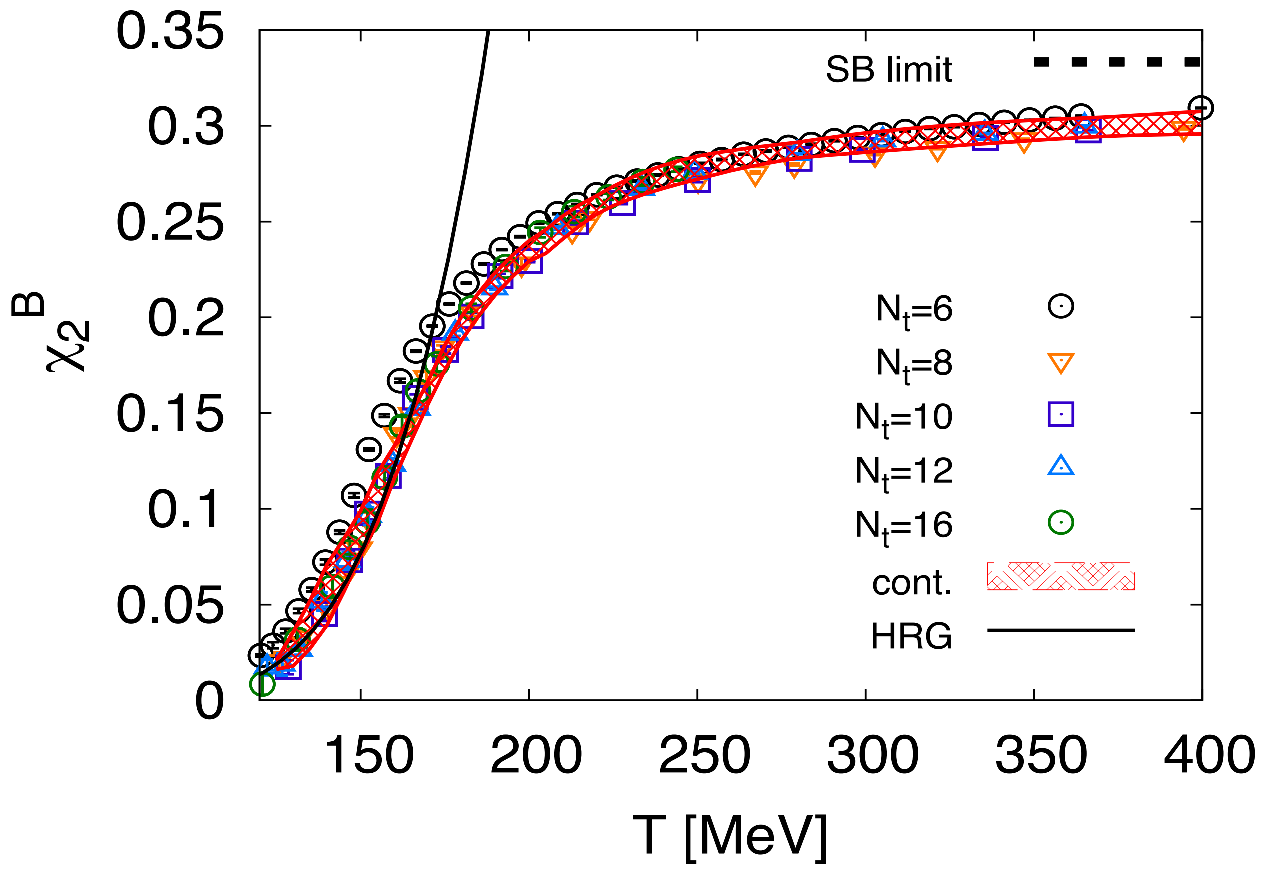
Particle density: $\frac{n_i}{T^3} = \frac{1}{T^3}\left(\frac{\partial p}{\partial \mu_i}\right)\Big|_{T,\mu_j}$

Entropy density: $\frac{s}{T^3} = \frac{1}{T^3}\frac{\partial p}{\partial T}\Big|_{\mu_i}$

Energy density: $\frac{\epsilon}{T^4} = \frac{s}{T^3} - \frac{p}{T^4} + \sum_i \frac{\mu_i}{T}\frac{n_i}{T^3}$

Trace anomaly: $\frac{I}{T^4} = \frac{1}{T^4}(\epsilon - 3P)$

Susceptibilities/fluctuations of conserved charges



$$\chi_{lmn}^{BSQ} = \frac{\partial^{l+m+n} p/T^4}{\partial(\mu_B/T)^l \partial(\mu_S/T)^m \partial(\mu_Q/T)^n}.$$

$\mathrm{mean}: M = \chi_1$ $\mathrm{variance}: \sigma^2 = \chi_2$

$\mathrm{skewness}: S = \chi_3/\chi_2^{3/2}$ $\mathrm{kurtosis}: \kappa = \chi_4/\chi_2^2$

A. Bazavov et al, PRD (2014); S. Borsanyi et al, JHEP (2012)

Hadron Resonance Gas Model

- In the low-temperature regime, the system is well-described by a gas of hadrons:
  - Treat as non-interacting system of resonant states
  - Grand Canonical
  - Match experimental cuts by transforming to p_T and y

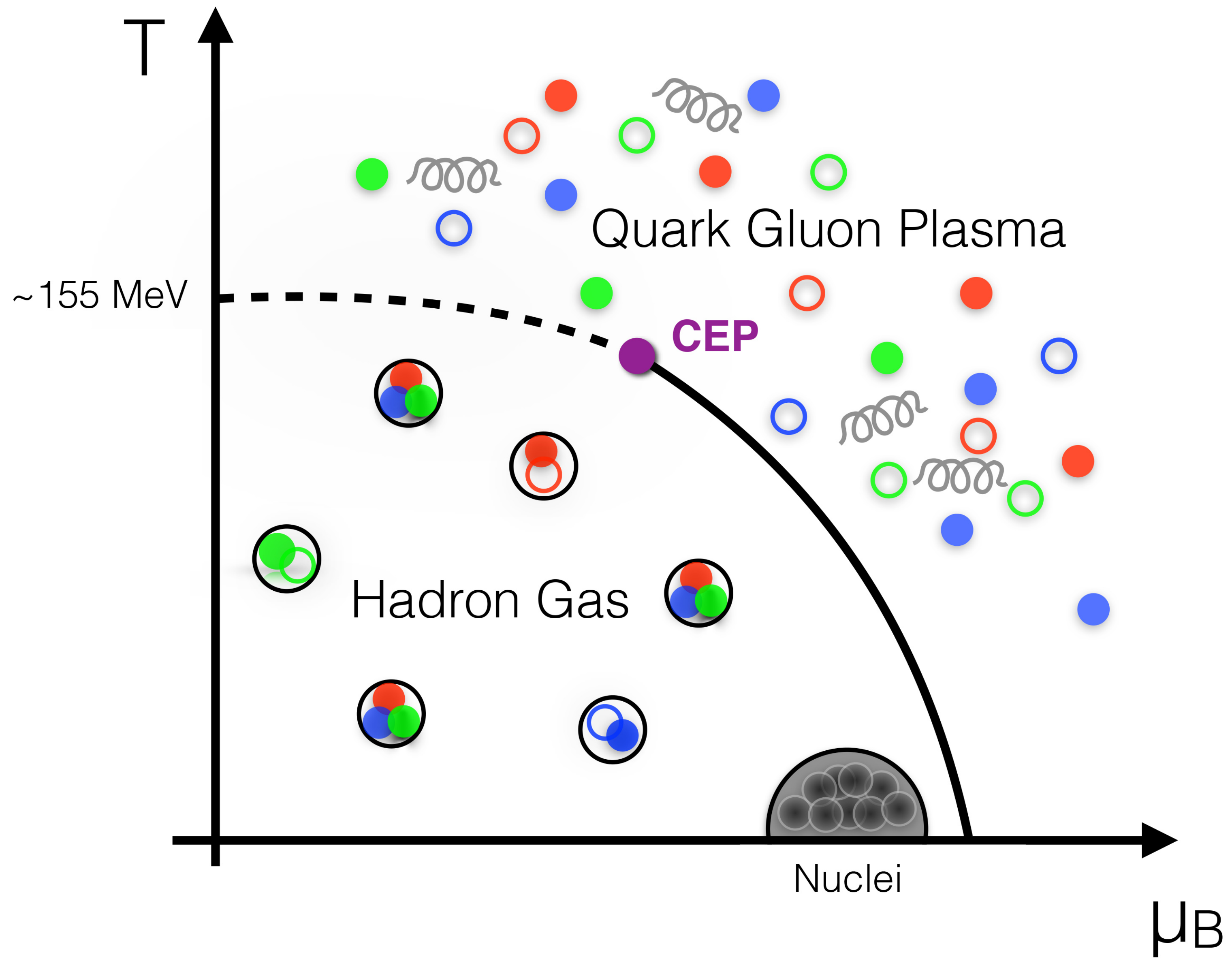
$$\frac{P}{T^4} = \frac{1}{VT^3} \sum_i \ln Z_i(T, V, \vec{\mu})$$

where: $$\ln Z_i^{M/B} = \mp \frac{V d_i}{(2\pi)^3} \int d^3p \, \ln\left(1 \mp \exp\left[-\left(\epsilon_i - \mu_a X_a^i\right)/T\right]\right)$$

energy $\epsilon_i = \sqrt{p^2 + m_i^2}$

conserved charges $\vec{X}_i = (B_i, S_i, Q_i)$

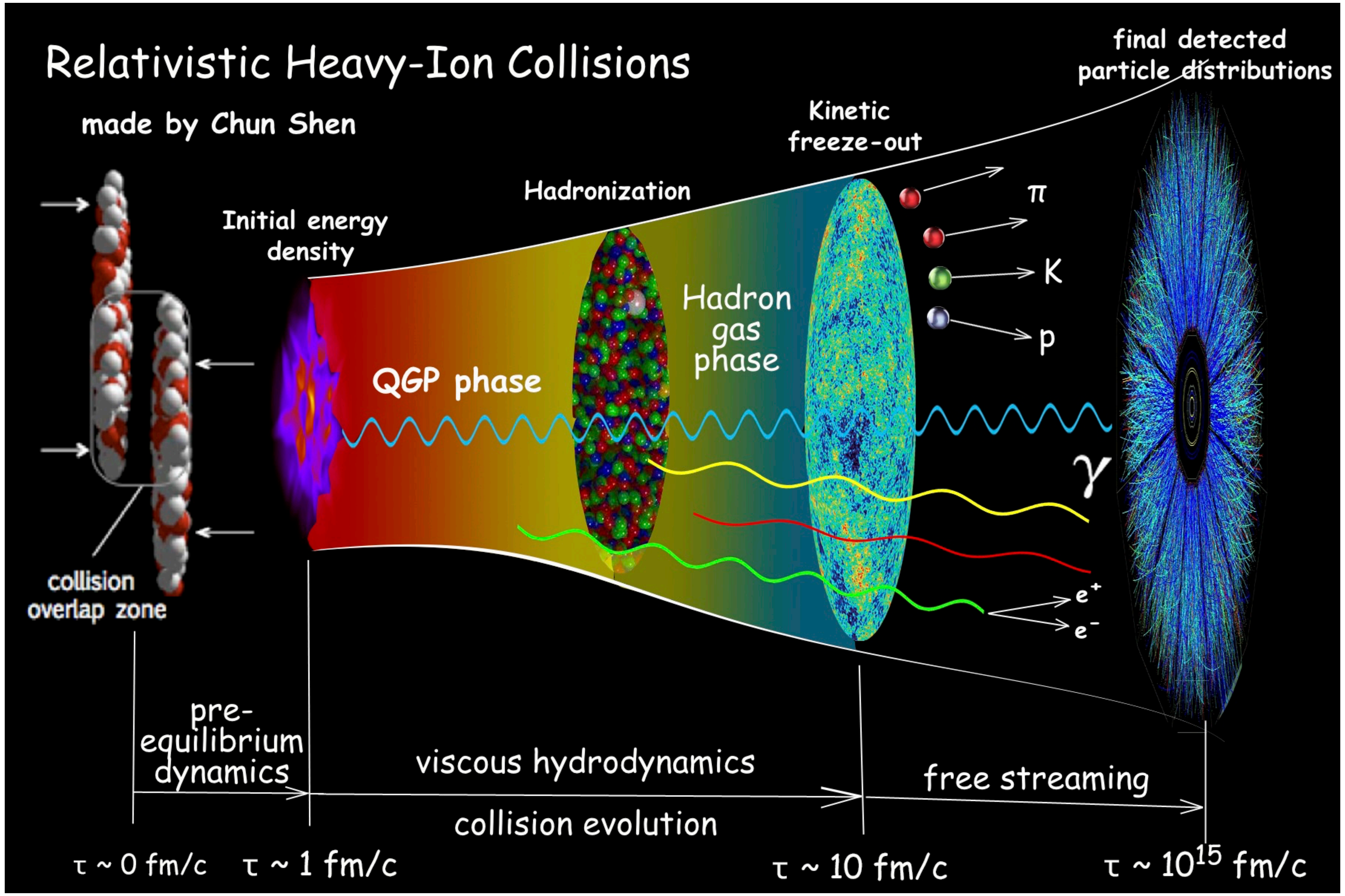
degeneracy d_i, mass m_i, volume V



H.T. Ding et al, Int. J. Mod Phys. (2015)

HIC Phenomenology

- Modeling should mimic experimental conditions in all stages in order to provide robust comparisons and estimates



ICCING: M. Martinez et al, arXiv: 1911.12454 & 1911.10272
BQS diffusion: J. Fotakis et al, PRD (2020)
MUSIC: G. Denicol et al, PRC (2018)
BEShydro: L. Du and U. Heinz, Comp. Phys. Comm (2020)
BQS fluctuations: V. Vovchenko et al, PLB (2021)

II. Four-dimensional BQS equations of state

Equation of State with Three Conserved Charges

- During HICs the system is not only confined to the T-μ_B plane: determine the equations that depend on μ_B, μ_Q, μ_S

$$\frac{P(T,\mu_B,\mu_Q,\mu_S)}{T^4} = \sum_{i,j,k} \frac{1}{i!j!k!} \chi_{ijk}^{BQS}(T) \left(\frac{\mu_B}{T}\right)^j \left(\frac{\mu_Q}{T}\right)^k \left(\frac{\mu_S}{T}\right)^i$$

where: $$\chi_{ijk}^{BQS}(T) = \left. \frac{\partial^{i+j+k}(p/T^4)}{\partial(\frac{\mu_B}{T})^i \partial(\frac{\mu_Q}{T})^j \partial(\frac{\mu_S}{T})^k} \right|_{\mu_B,\mu_Q,\mu_S=0}$$

Lattice results only between T $\sim$ 135 - 220 MeV for all 22 coefficients
- Utilize HRG for low T
- Impose Stefan-Boltzmann limit at high T

See also: A. Monnai et al, PRC (2019)

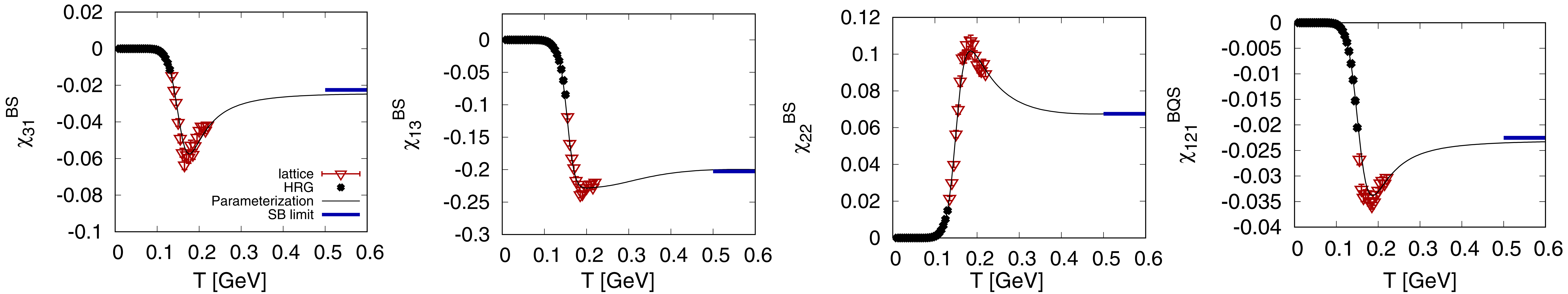
J. Noronha-Hostler, JS et al, PRC (2019)

Parametrized Taylor Coefficients

- Fit all 22 coefficients over a broad range of temperatures

$$\chi_{ijk}^{BQS}(T) = \frac{a_0^i + a_1^i/t + a_2^i/t^2 + a_3^i/t^3 + a_4^i/t^4 + a_5^i/t^5 + a_6^i/t^6 + a_7^i/t^7}{b_0^i + b_1^i/t + b_2^i/t^2 + b_3^i/t^3 + b_4^i/t^4 + b_5^i/t^5 + b_6^i/t^6 + b_7^i/t^7} + c_0$$

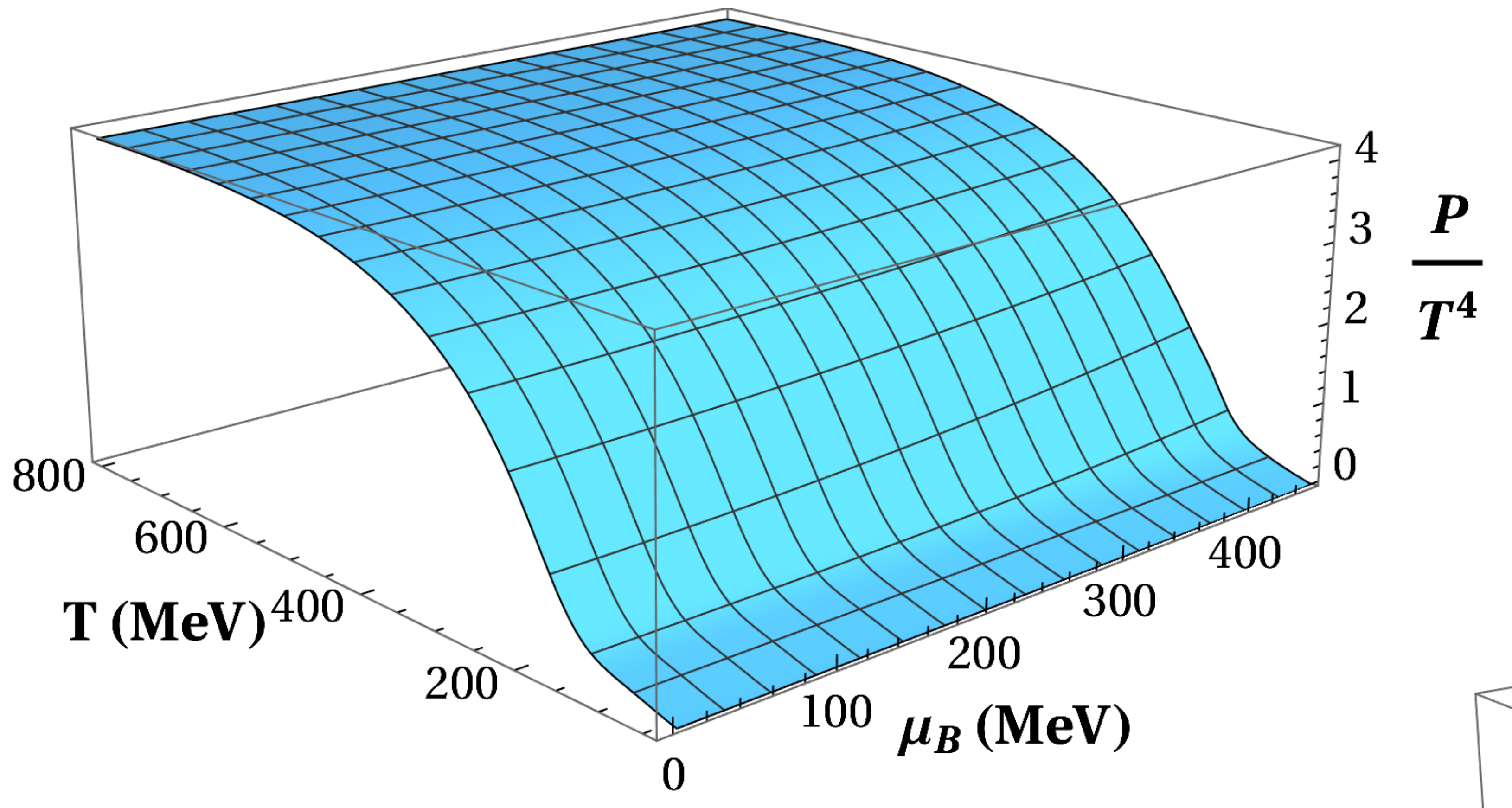
$$\chi_2^B(T) = e^{-h_1/t' - h_2/t'^2} \cdot f_3 \cdot (1 + \tanh(f_4 t' + f_5))$$

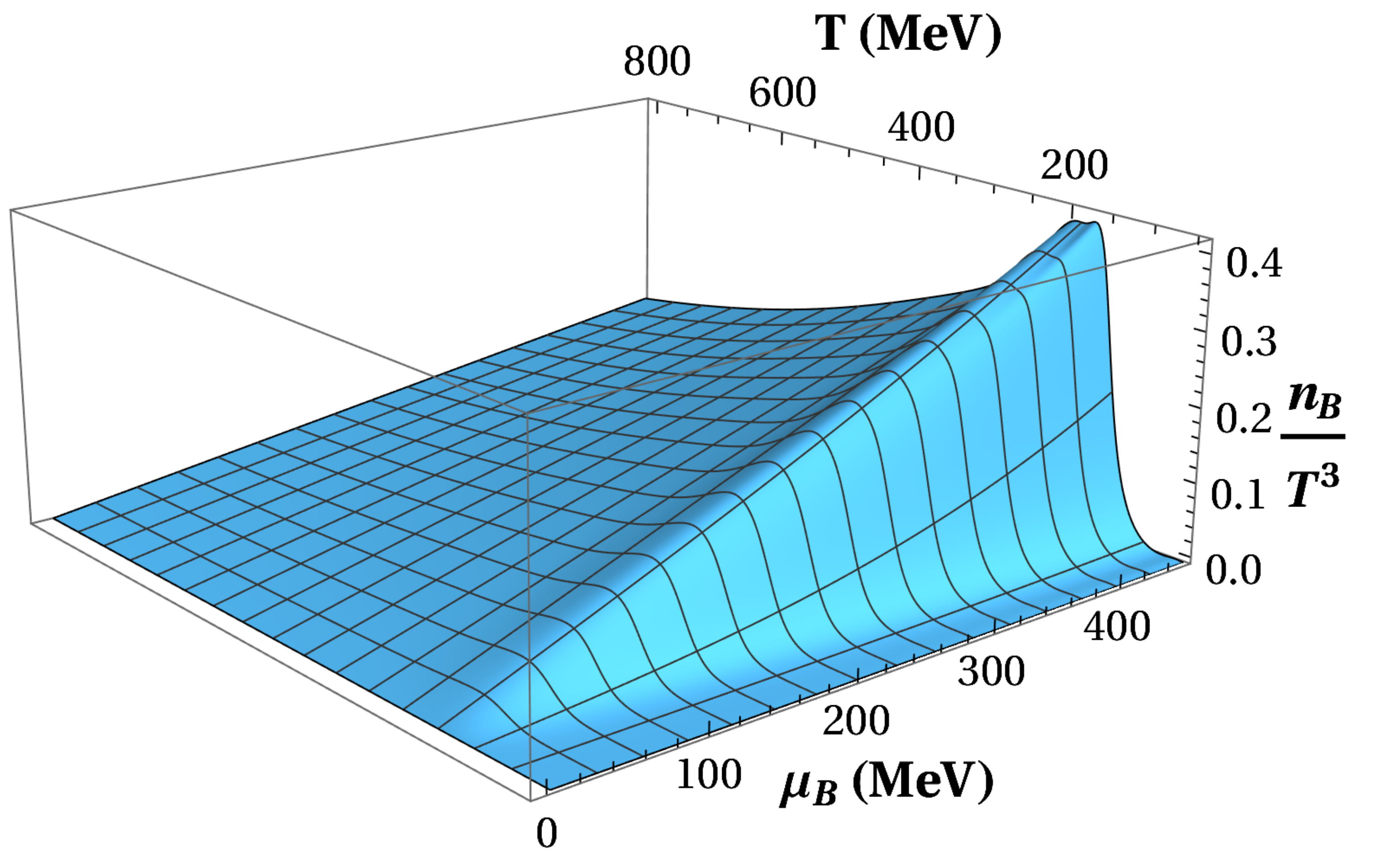


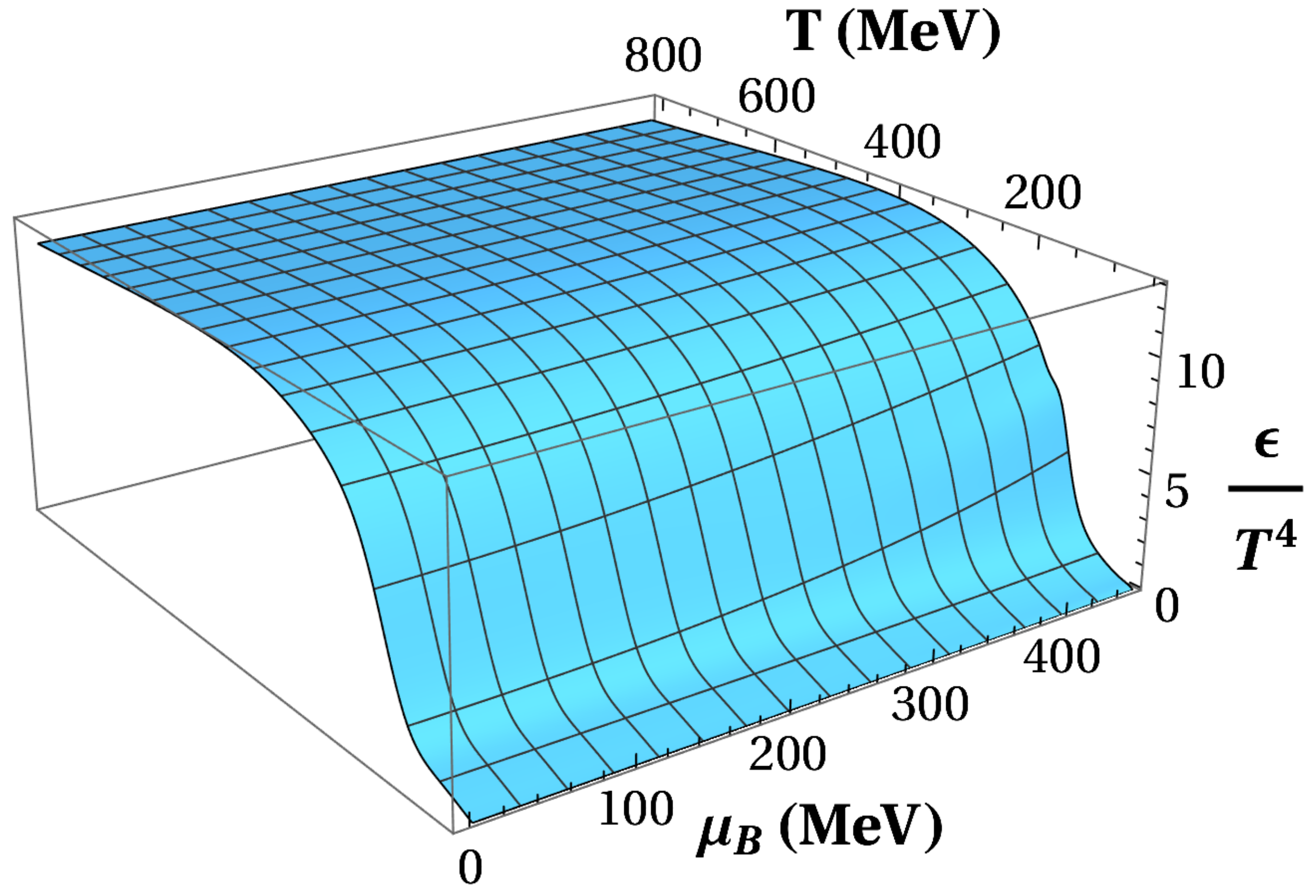
Reconstructed Taylor EoS

- Reconstruct the QCD equation of state from all diagonal and off-diagonal susceptibilities up to $\mathcal{O}(\mu_B^4)$

$$\frac{P(T,\mu_B,\mu_Q,\mu_S)}{T^4} = \sum_{i,j,k} \frac{1}{i!j!k!}\chi_{ijk}^{BQS}(T)\left(\frac{\mu_B}{T}\right)^j \left(\frac{\mu_Q}{T}\right)^k \left(\frac{\mu_S}{T}\right)^i$$





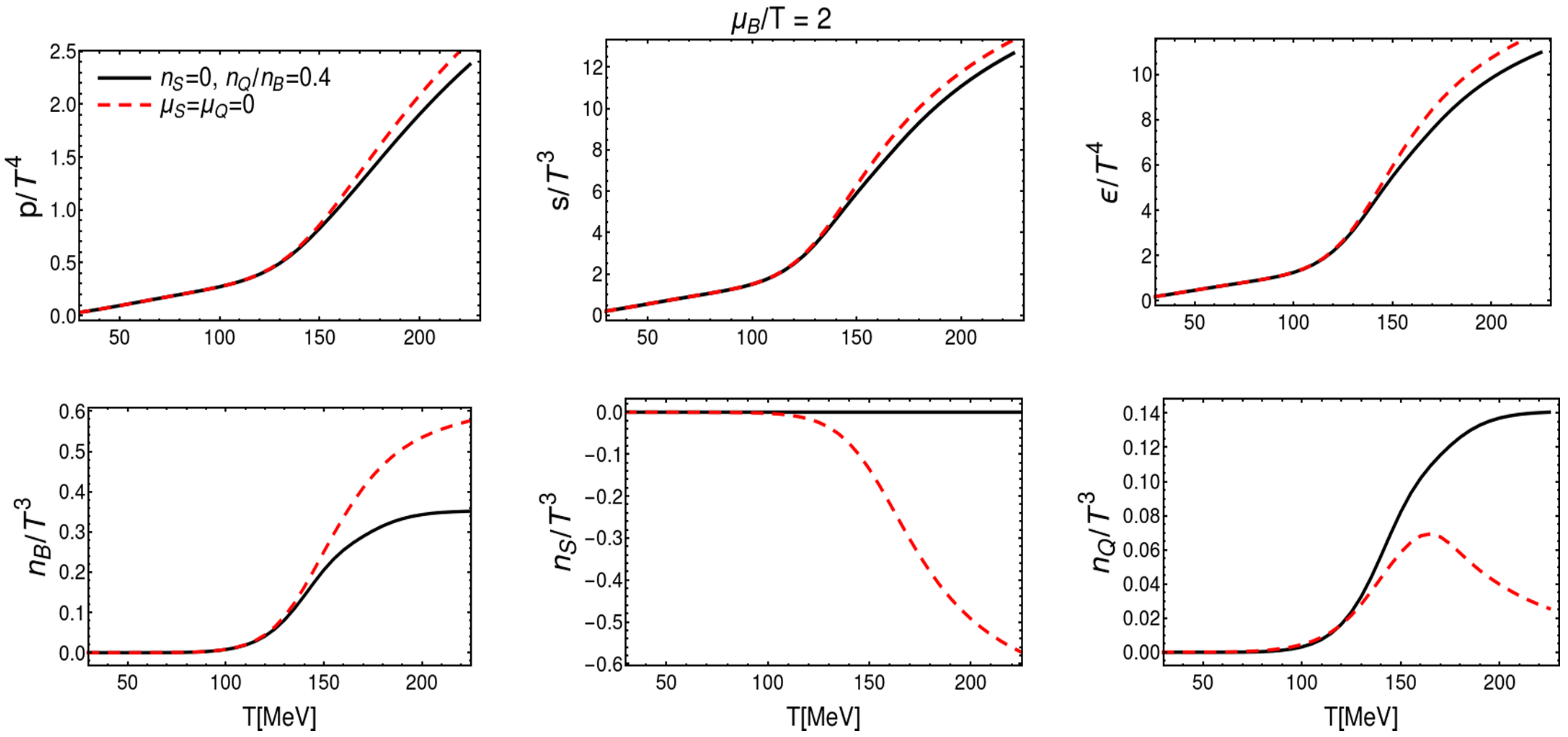


See also: A. Monnai et al, PRC (2019)

J. Noronha-Hostler, JS et al, PRC (2019)

EoS with Conserved Charge Constraints

- Observe the effect of imposing strangeness neutrality and a fixed ratio of baryon number to electric charge as expected for the densities

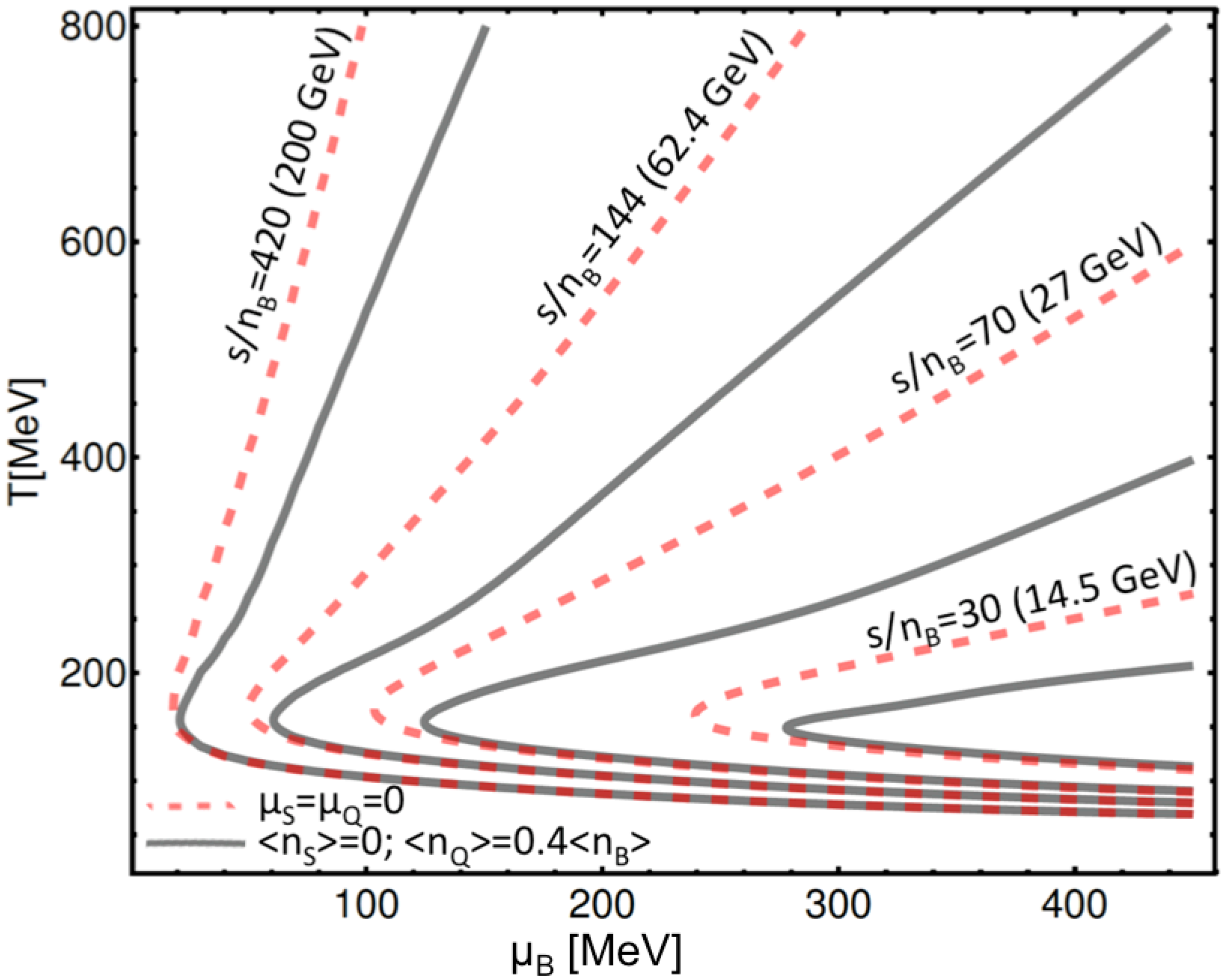


See also: A. Monnai et al, PRC (2019)

J. Noronha-Hostler, JS et al, PRC (2019)

Isentropic Trajectories

- Different paths through the phase diagram taken based on conserved charge conditions: isentropic trajectories stress importance of BQS modeling for heavy-ion phenomenology

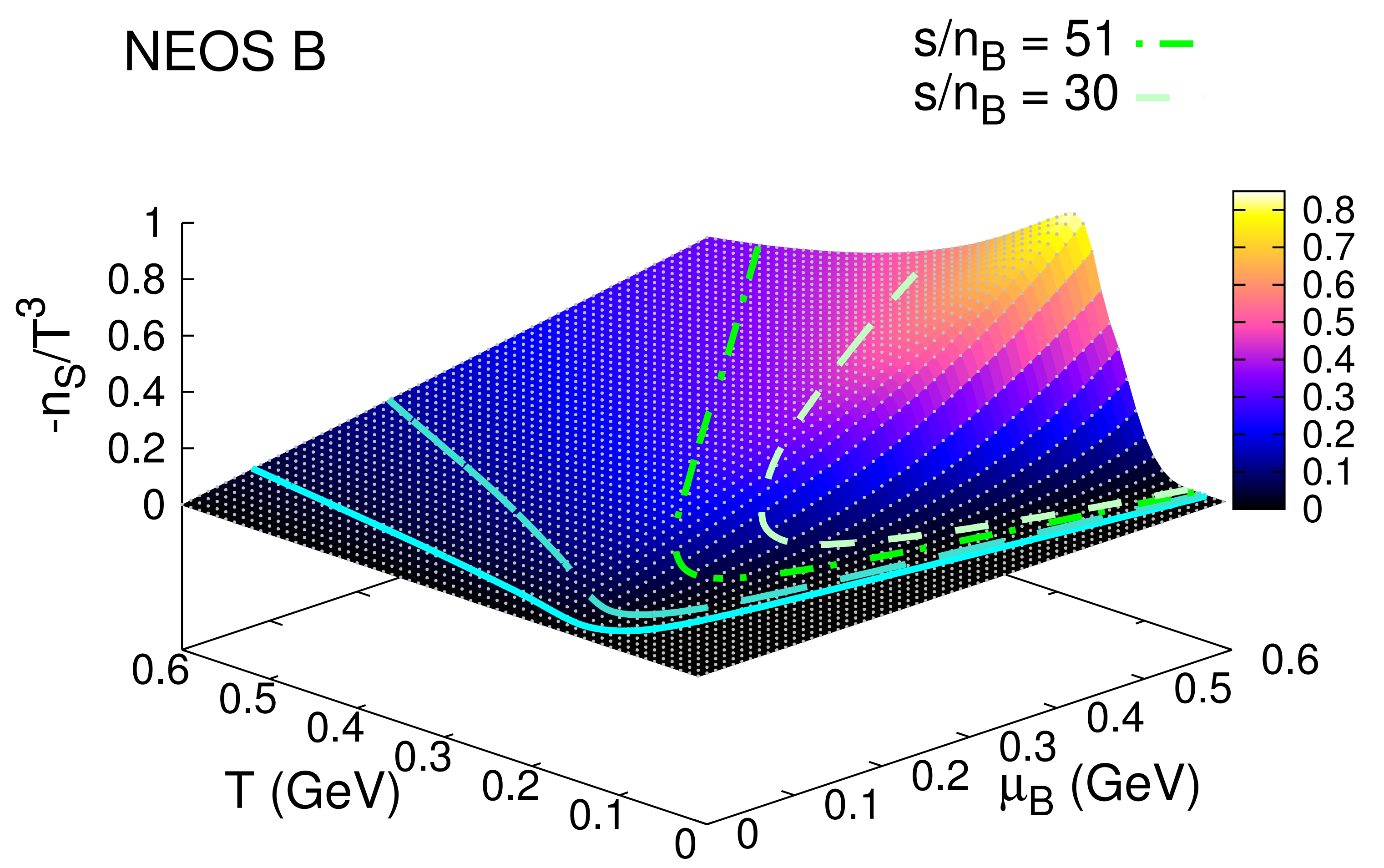


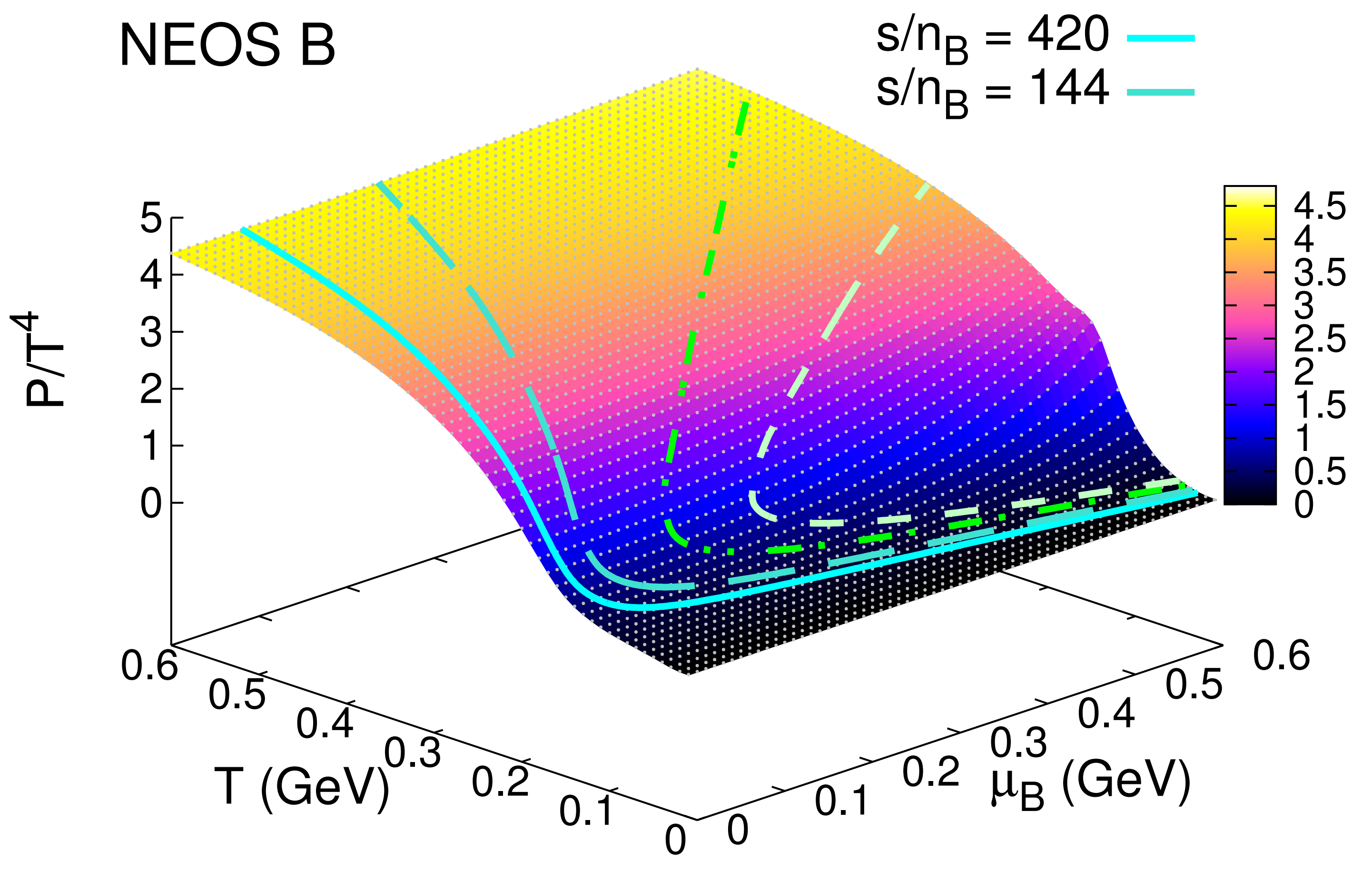
See also: *A. Monnai et al, PRC (2019)*

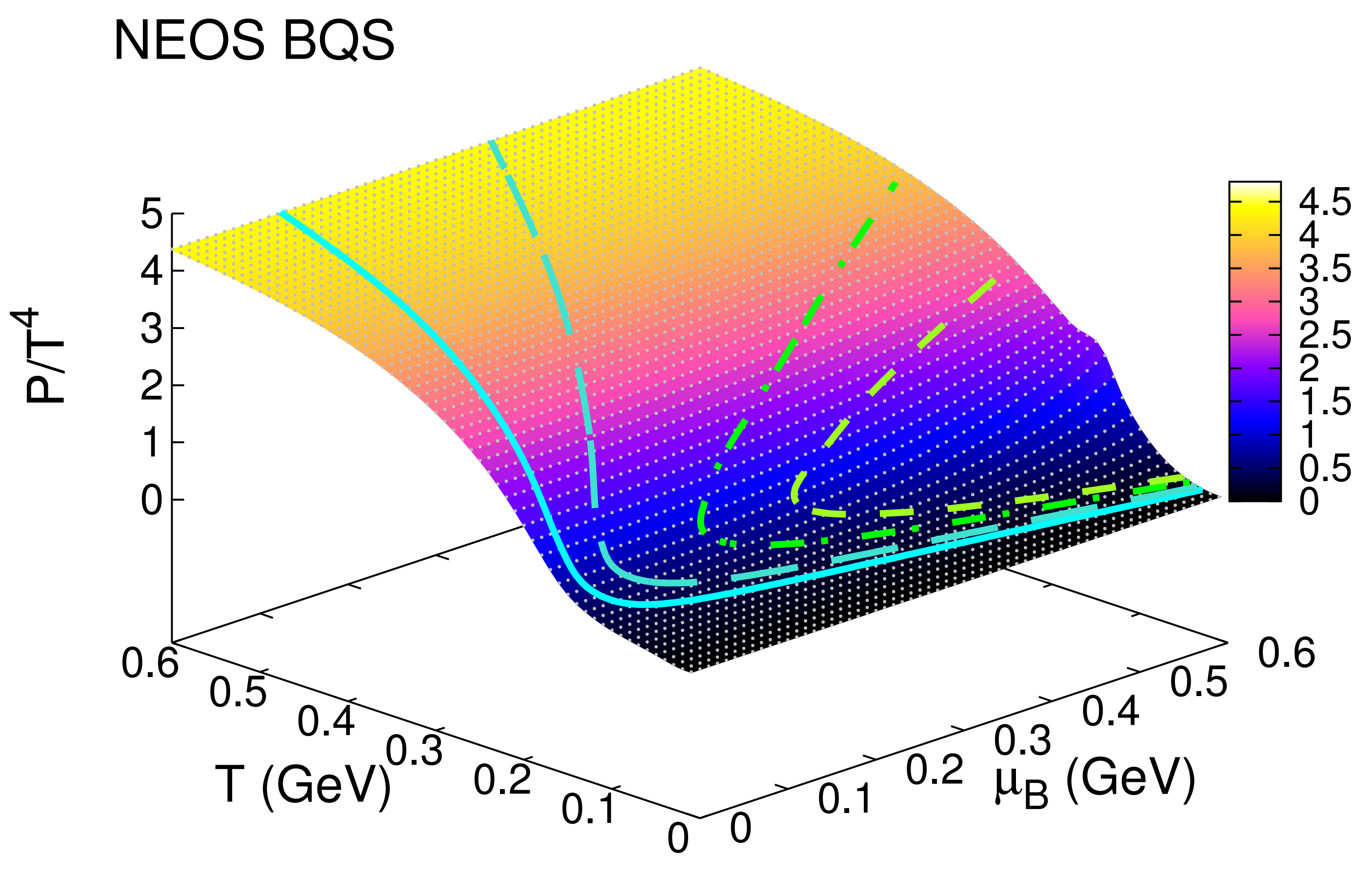
J. Noronha-Hostler, JS et al, PRC (2019)

NEOS

➤ Similar construction from Monnai, Schenke and Shen shows the same effects on the thermodynamic quantities

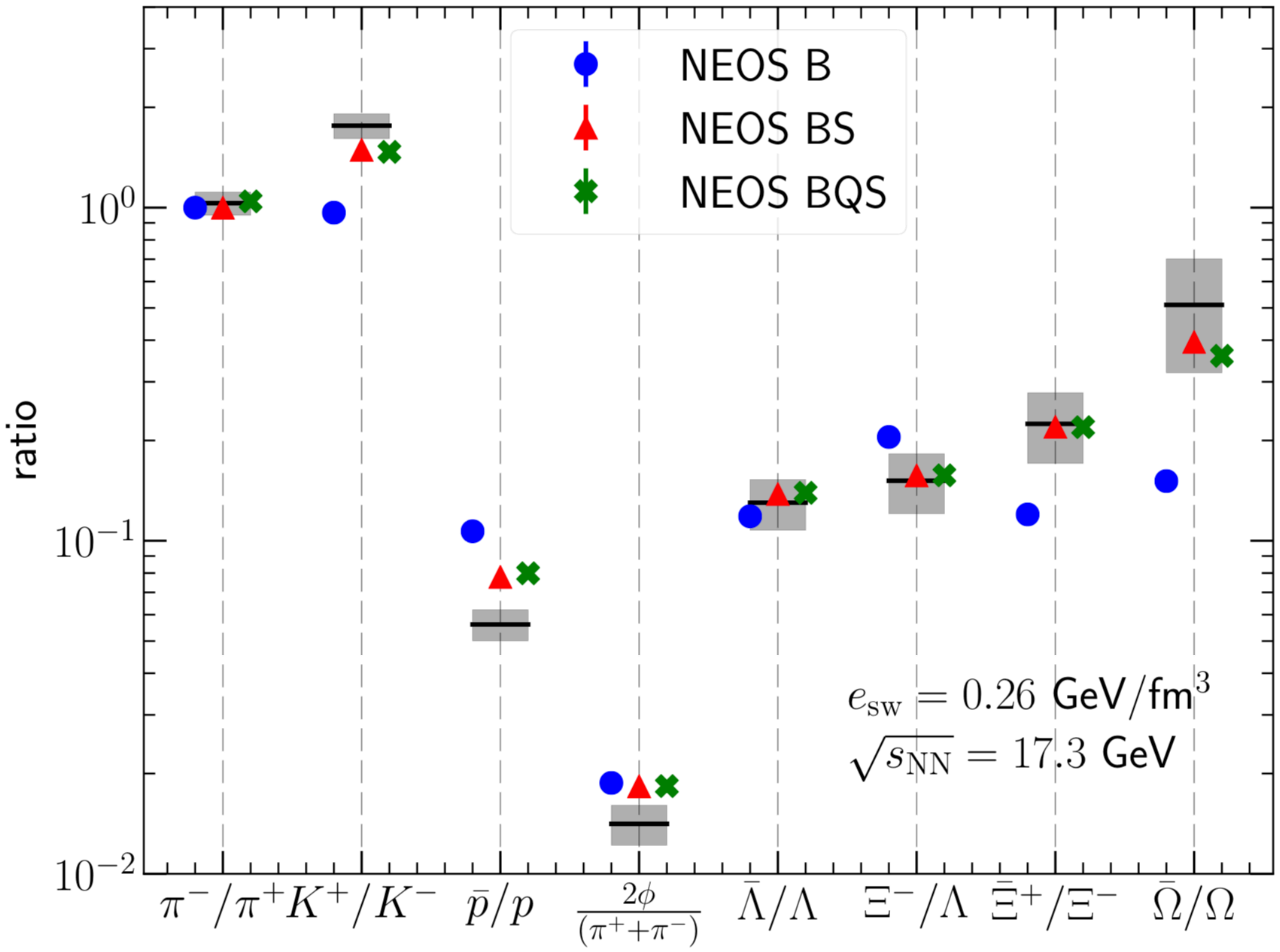






NEOS

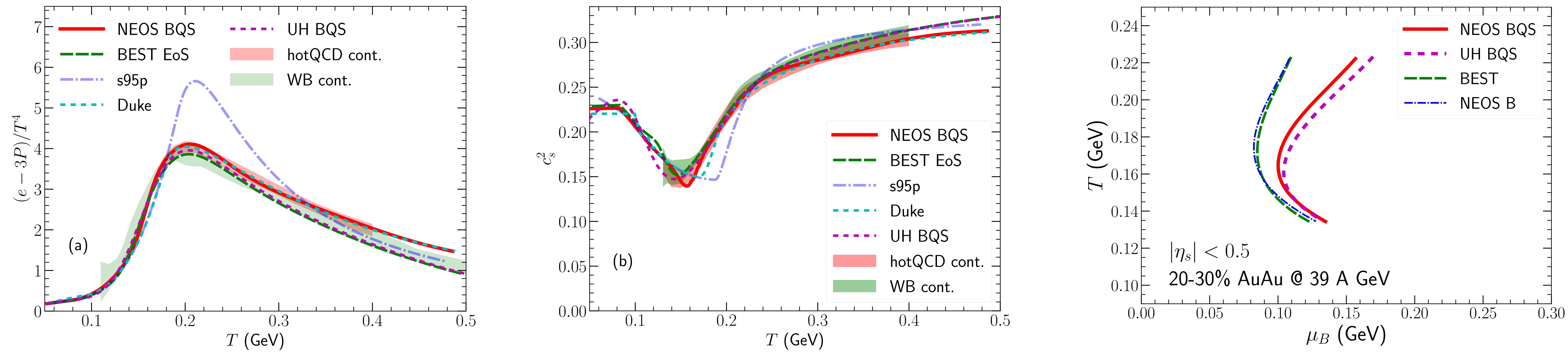
- Full dynamical modeling with successive additions of charge conservation conditions increases agreement of predicted particle ratios with experimental data
  - Including only baryon number is not sufficient
  - Strange baryons especially affected
  - Small difference after adding Q



A. Monnai et al, PRC (2019)

EoS Comparison

- Good qualitative agreement with both EoS's built from different lattice data, including the trace anomaly, speed of sound and HIC trajectories



III. Strangeness-neutral equation of state with a critical point

Equation of State with Criticality

➤ Update to the original EoS that first matched the Taylor expansion coefficients from Lattice QCD and implemented critical features based on universality arguments

PHYSICAL REVIEW C **101**, 034901 (2020)

QCD equation of state matched to lattice data and exhibiting a critical point singularity

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We construct a family of equations of state for QCD in the temperature range 30 MeV $\leqslant T \leqslant$ 800 MeV and in the chemical potential range $0 \leqslant \mu_B \leqslant$ 450 MeV. These equations of state match available lattice QCD results up to $O(\mu_B^4)$ and in each of them we place a critical point in the three-dimensional (3D) Ising model universality class. The position of this critical point can be chosen in the range of chemical potentials covered by the second Beam Energy Scan at the Relativistic Heavy Ion Collider. We discuss possible choices for the free parameters, which arise from mapping the Ising model onto QCD. Our results for the pressure, entropy density, baryon density, energy density, and speed of sound can be used as inputs in the hydrodynamical simulations of the fireball created in heavy ion collisions. We also show our result for the second cumulant of the baryon number in thermal equilibrium, displaying its divergence at the critical point. In the future, comparisons between RHIC data and the output of the hydrodynamic simulations, including calculations of fluctuation observables, built upon the model equations of state that we have constructed may be used to locate the critical point in the QCD phase diagram, if there is one to be found.

DOI: 10.1103/PhysRevC.101.034901

Code can be downloaded at:
https://bitbucket.org/bestcollaboration/eos_with_critical_point/src/master/

New (!) Equation of State with Criticality

- New version includes (but not limited to):
  - Imposing conditions on conserved charges as present in HICs
  - Hadronic species present in SMASH hadronic transport approach

$$\langle n_S \rangle = 0 \qquad\qquad \langle n_Q \rangle = 0.4 \langle n_B \rangle$$

Degrees of Freedom

N	Δ	Λ	Σ	Ξ	Ω	Unflavored				Strange
N_{938}	Δ_{1232}	Λ_{1116}	Σ_{1189}	Ξ_{1321}	Ω^-_{1672}	π_{138}	$f_{0\ 980}$	$f_{2\ 1275}$	$\pi_{2\ 1670}$	K_{494}
N_{1440}	Δ_{1620}	Λ_{1405}	Σ_{1385}	Ξ_{1530}	Ω^-_{2250}	π_{1300}	$f_{0\ 1370}$	$f_2'{}_{1525}$		K^*_{892}
N_{1520}	Δ_{1700}	Λ_{1520}	Σ_{1660}	Ξ_{1690}		π_{1800}	$f_{0\ 1500}$	$f_{2\ 1950}$	$\rho_{3\ 1690}$	$K_{1\ 1270}$
N_{1535}	Δ_{1900}	Λ_{1600}	Σ_{1670}	Ξ_{1820}			$f_{0\ 1710}$	$f_{2\ 2010}$		$K_{1\ 1400}$
N_{1650}	Δ_{1905}	Λ_{1670}	Σ_{1750}	Ξ_{1950}		η_{548}		$f_{2\ 2300}$	$\phi_{3\ 1850}$	K^*_{1410}
N_{1675}	Δ_{1910}	Λ_{1690}	Σ_{1775}	Ξ_{2030}		η'_{958}	$a_{0\ 980}$	$f_{2\ 2340}$		$K_0^*{}_{1430}$
N_{1680}	Δ_{1920}	Λ_{1800}	Σ_{1915}			η_{1295}	$a_{0\ 1450}$		$a_{4\ 2040}$	$K_2^*{}_{1430}$
N_{1700}	Δ_{1930}	Λ_{1810}	Σ_{1940}			η_{1405}		$f_{1\ 1285}$		K^*_{1680}
N_{1710}	Δ_{1950}	Λ_{1820}	Σ_{2030}			η_{1475}	ϕ_{1019}	$f_{1\ 1420}$	$f_{4\ 2050}$	$K_{2\ 1770}$
N_{1720}		Λ_{1830}	Σ_{2250}				ϕ_{1680}			$K_3^*{}_{1780}$
N_{1875}		Λ_{1890}				σ_{800}		$a_{2\ 1320}$		$K_{2\ 1820}$
N_{1900}		Λ_{2100}					$h_{1\ 1170}$			$K_4^*{}_{2045}$
N_{1990}		Λ_{2110}				ρ_{776}		$\pi_{1\ 1400}$		
N_{2060}		Λ_{2350}				ρ_{1450}	$b_{1\ 1235}$	$\pi_{1\ 1600}$		
N_{2080}						ρ_{1700}				
N_{2100}							$a_{1\ 1260}$	$\eta_{2\ 1645}$		
N_{2120}						ω_{783}				
N_{2190}						ω_{1420}		$\omega_{3\ 1670}$		
N_{2220}						ω_{1650}				
N_{2250}										

As of SMASH-1.7

- + corresponding antiparticles
- Perturbative treatment of photons and dileptons
- Isospin symmetry

smash

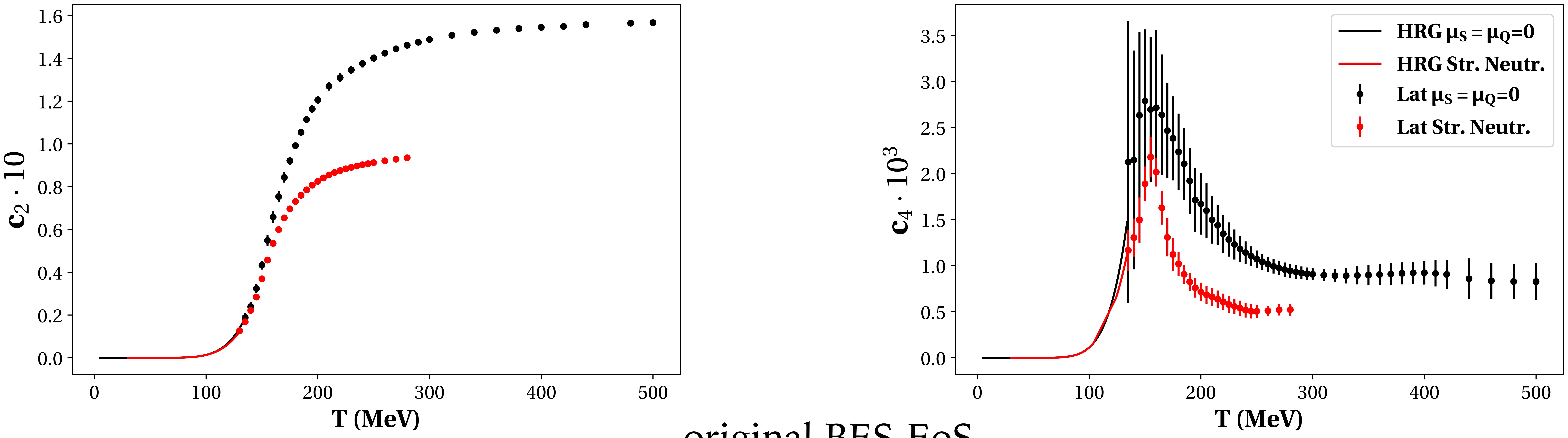
- Mesons and baryons according to particle data group
- Isospin multiplets and anti-particles are included

H. Elfner, RHIC-BES Seminar (2020)
J. Weil et al, PRC (2016)

Taylor Coefficients from LQCD

➤ Lattice results for Taylor expansion of pressure around $\mu_B = 0$ up to $\mathcal{O}(\mu_B^4)$ are the backbone of the procedure for creating this equation of state

$$\frac{P(T,\mu_B)}{T^4} = \sum_n c_{2n}(T)\left(\frac{\mu_B}{T}\right)^{2n}$$

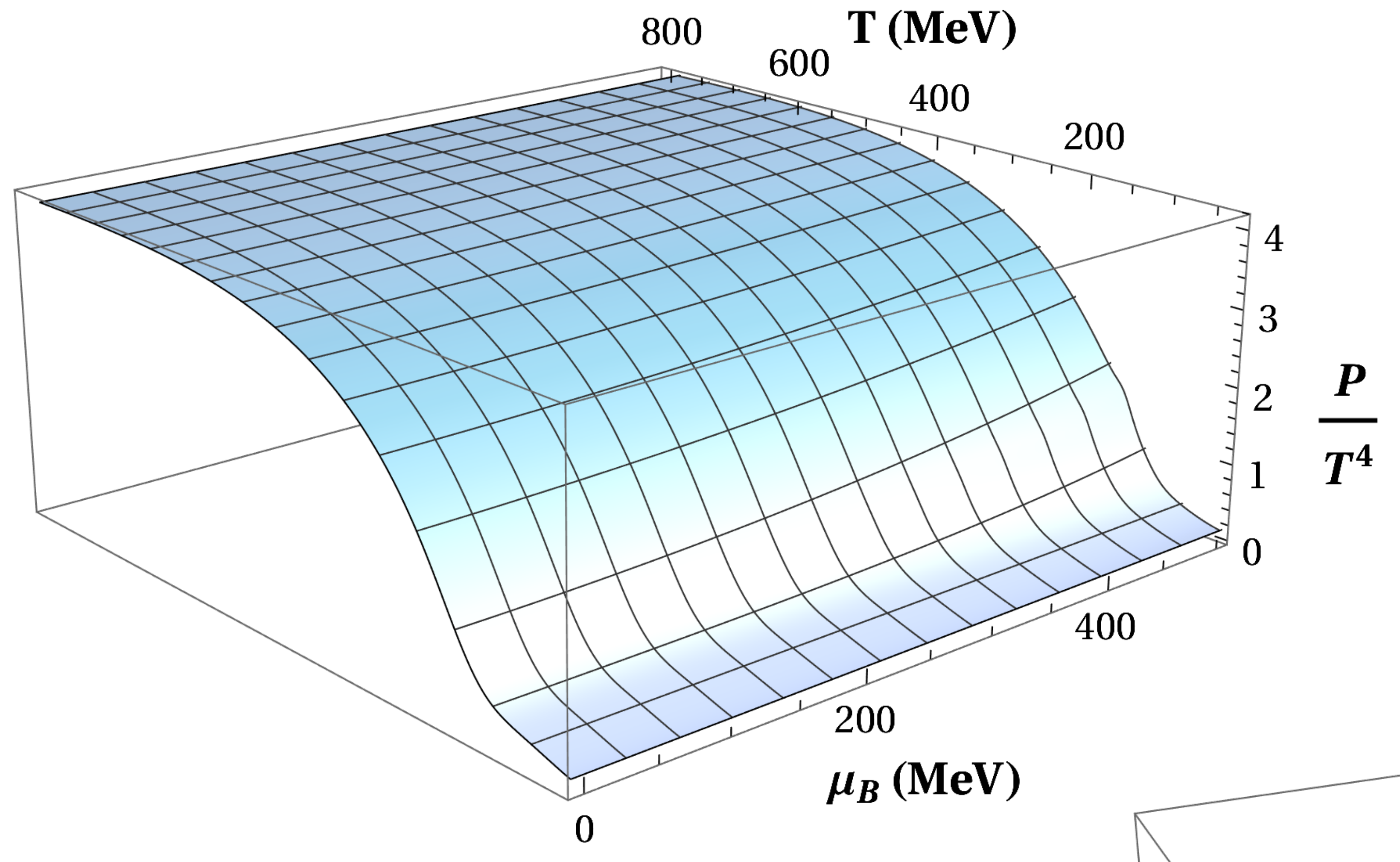


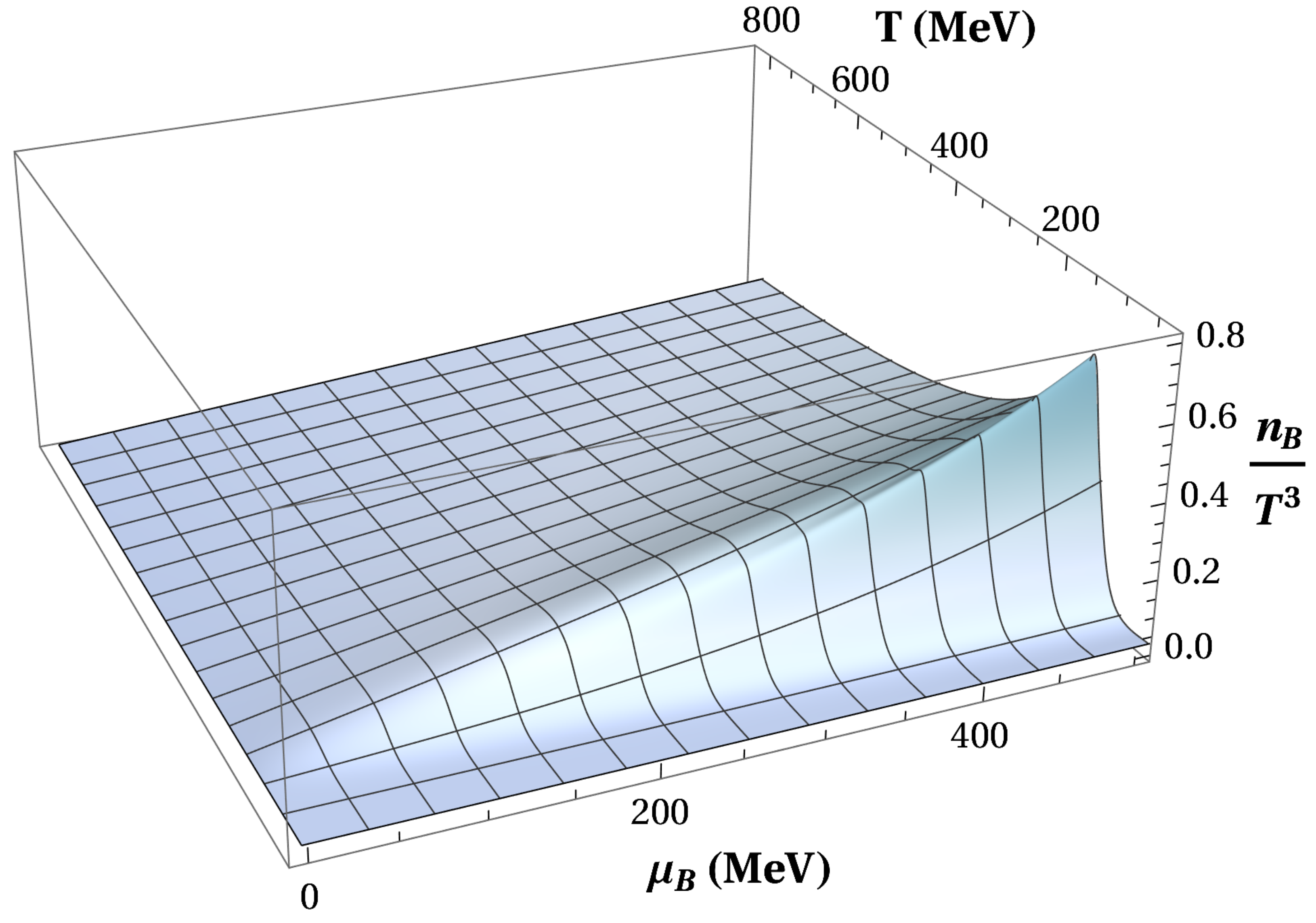
- original BES-EoS
- updated version

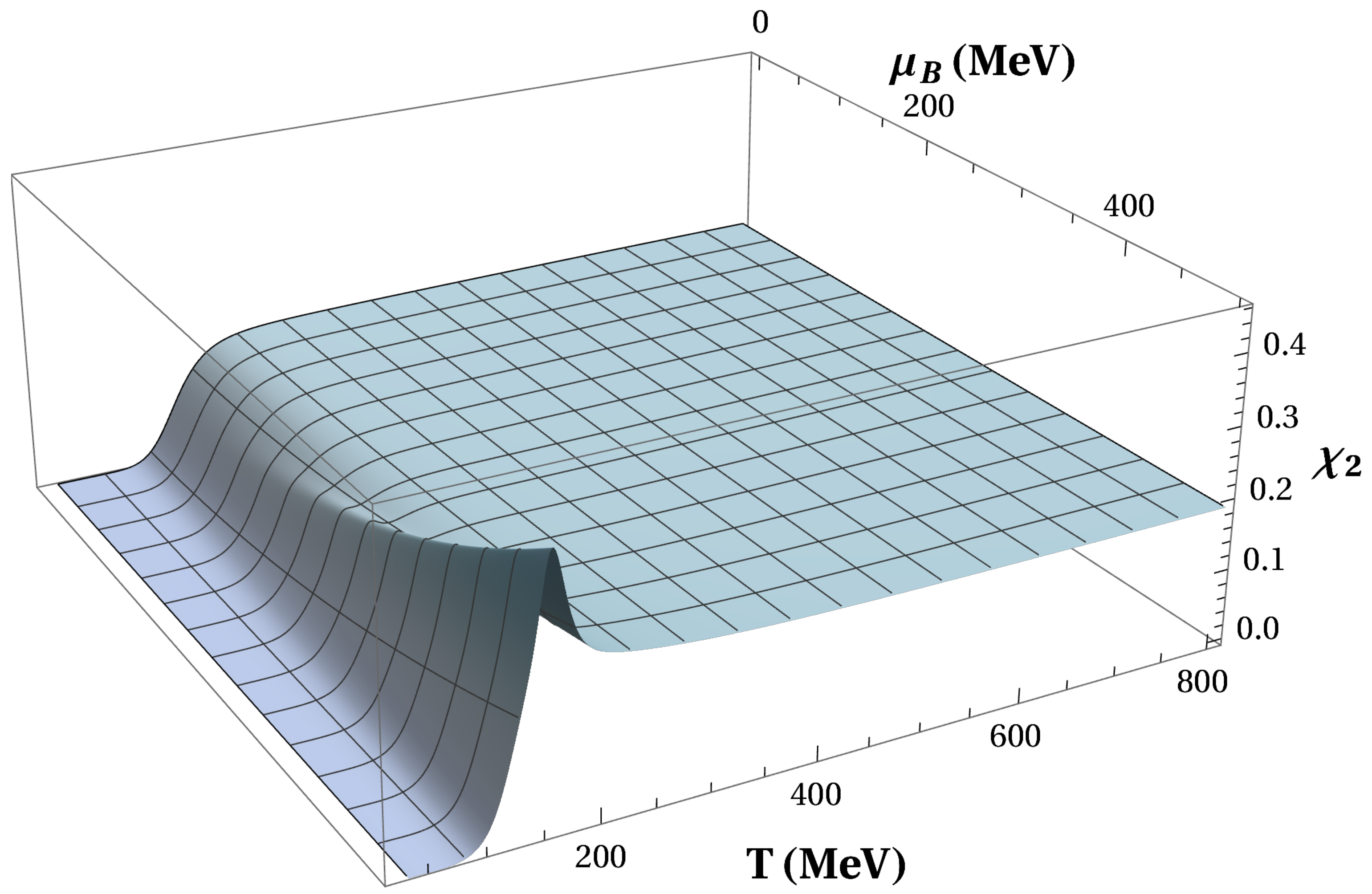
J. Guenther et al, NPA (2017)
R. Bellweid et al, PRD (2015)
See also: A. Bazavov et al, PRD (2017)

Lattice EoS at Finite T & μ_B

- Strangeness neutral equation of state from first-principles can be produced by running in "LAT" mode



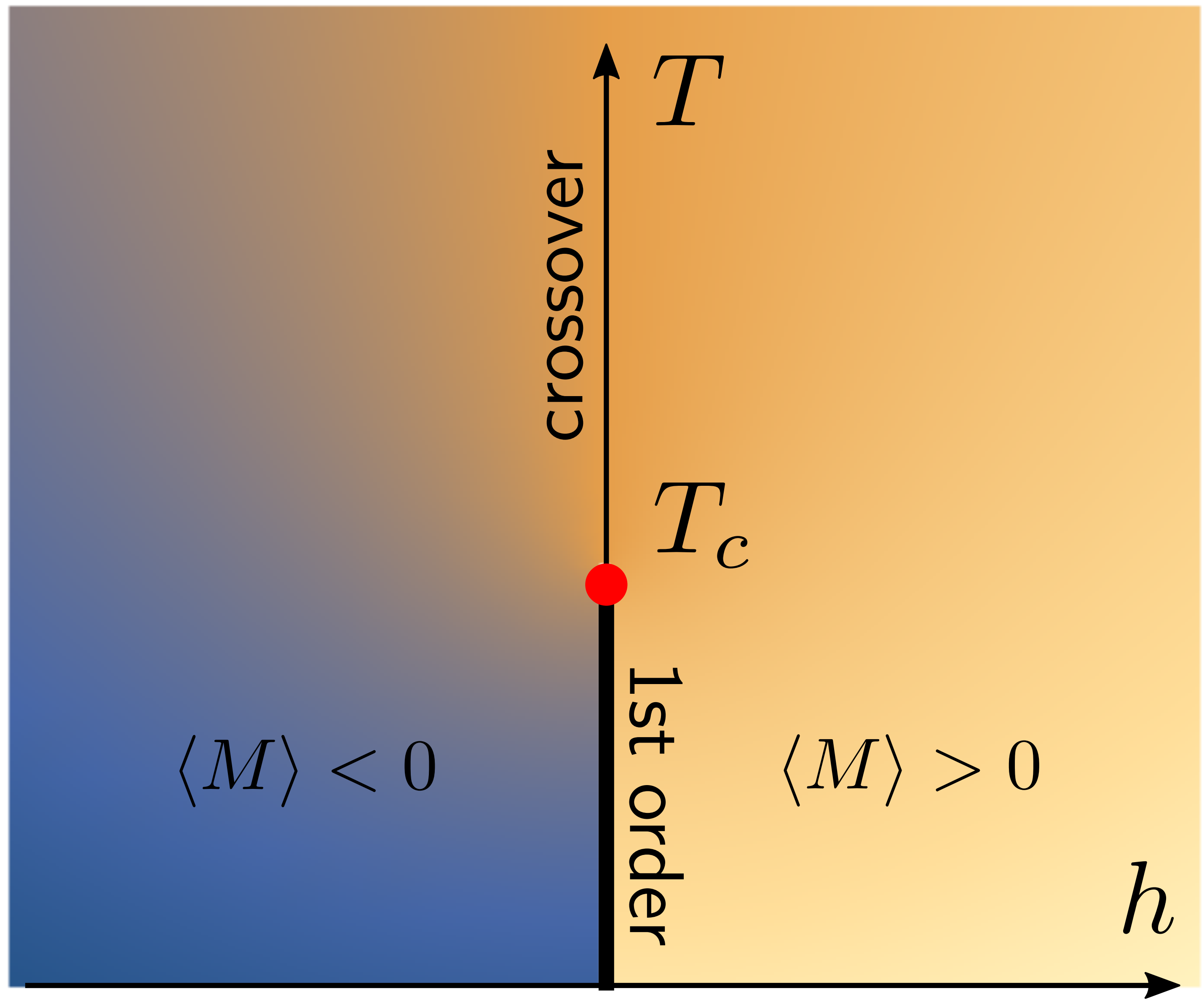




Universal Scaling EoS

- Criticality is implemented by mapping the critical point from the 3D Ising model onto the QCD phase diagram

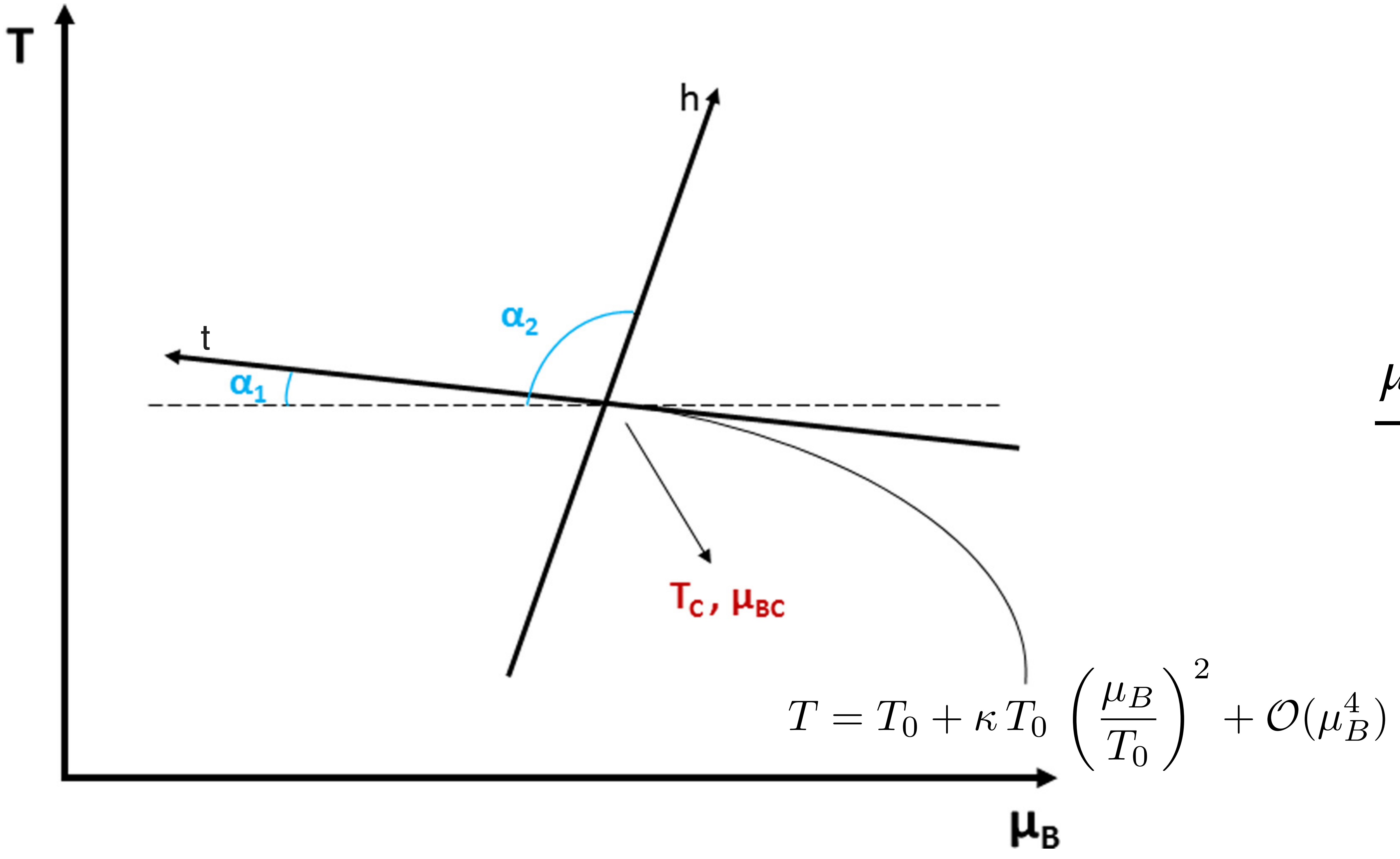
- Relevant and analogous quantities for Ising-QCD map:
  - Magnetic field, h ⟷ Baryon chemical potential, μ
  - Magnetization, M ⟷ Baryon density, n_B
  - Reduced temperature: $t = \dfrac{T - T_C}{T_C}$
  - Gibbs' free energy/thermodynamic potential = −Pressure



K. Rajagopal and F. Wilczek, Nucl. Phys. B (1993)
P. Parotto et al, PRC (2020)
A. Bzdak et al, Phys. Rep. (2020)
C. Nonaka, M. Asakawa, PRC (2005)

Mapping the 3D Ising Model onto QCD

- Phase transition along Ising temperature axis fixed onto QCD phase diagram along transition line from LQCD



$$\frac{T - T_C}{T_C} = w\ (t\ \rho\ sin\alpha_1 + h\ sin\alpha_2)$$

$$\frac{\mu_B - \mu_{B,C}}{T_C} = -\ w\ (t\ \rho\ cos\alpha_1 + h\ cos\alpha_2)$$

P. Parotto et al, PRC (2020)

3D Ising Model Parametrization

- Universal scaling behavior encoded in parameters (R, θ):
  - Magnetic field: $h = h_0 R^{\beta\delta} H(\theta)$
  - Reduced temperature: $t = R(1 - \theta^2)$
  - Magnetization: $M = M_0 R^{\beta} \theta$
  - Gibbs' free energy: $G = h_0 M_0 R^{2-\alpha}[g(\theta) - \theta H(\theta)]$

  where $\alpha = 0.11$, $\beta = 0.326$, $\delta = 4.8$ are 3D Ising critical exponents, H(θ) is a polynomial in odd powers of θ, and g(θ) is a polynomial in (1-θ^2).

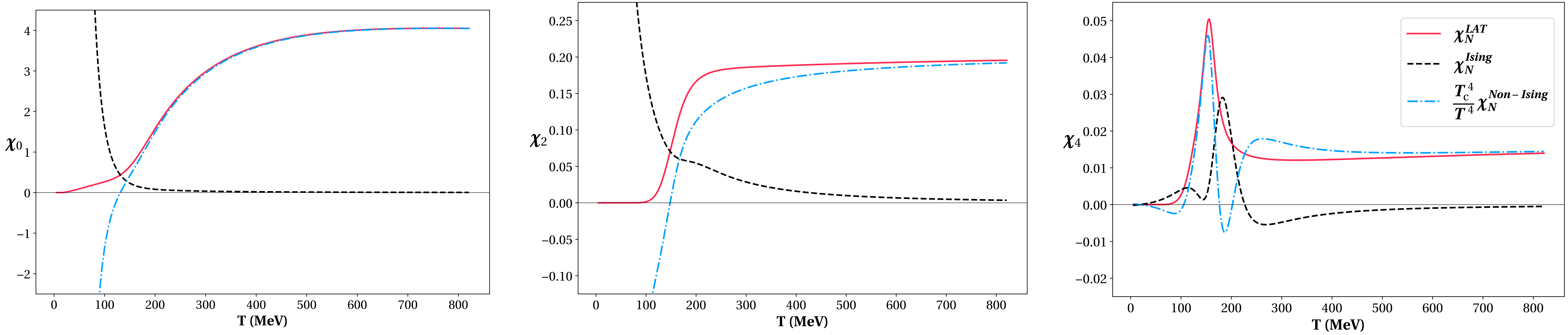
- Generally, free energy includes singular and non-singular contributions:
  - $P(T, \mu_B) = -G[R, \theta] + P_{bkg}(T, \mu_B)$

P. Parotto et al, PRC (2020)
A. Bzdak et al, Phys. Rep. (2020)
C. Nonaka, M. Asakawa, PRC (2005)
J. Zinn-Justin Quantum Field theory and Critical Phenomena

Singular and Non-singular Contributions

- Our construction requires that the total free energy (pressure) is the one from the lattice, so order-by-order we have:

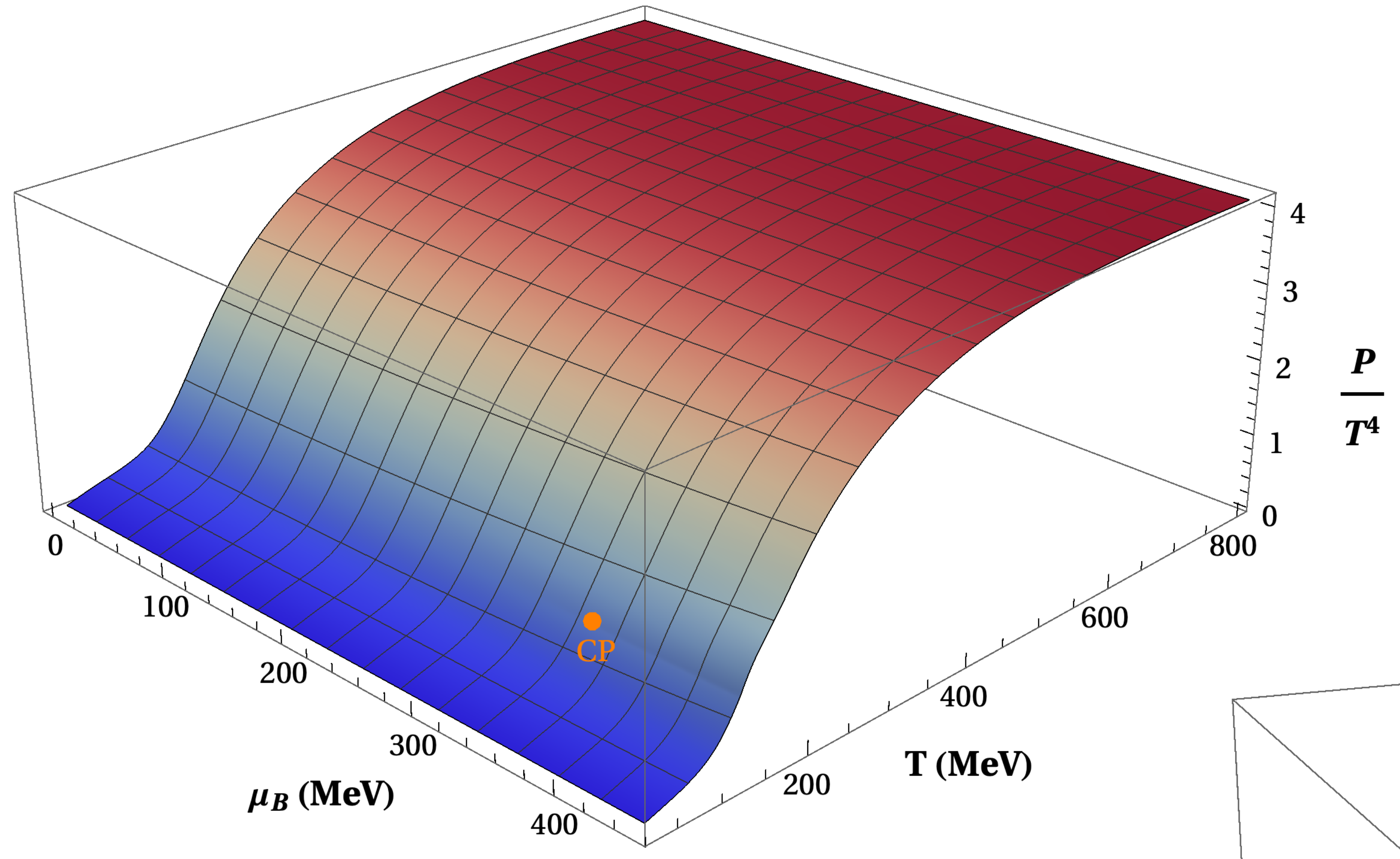
$$\chi_N^{Lat}(T) = \chi_N^{Ising}(T) + \chi_N^{Non-Ising}(T)$$



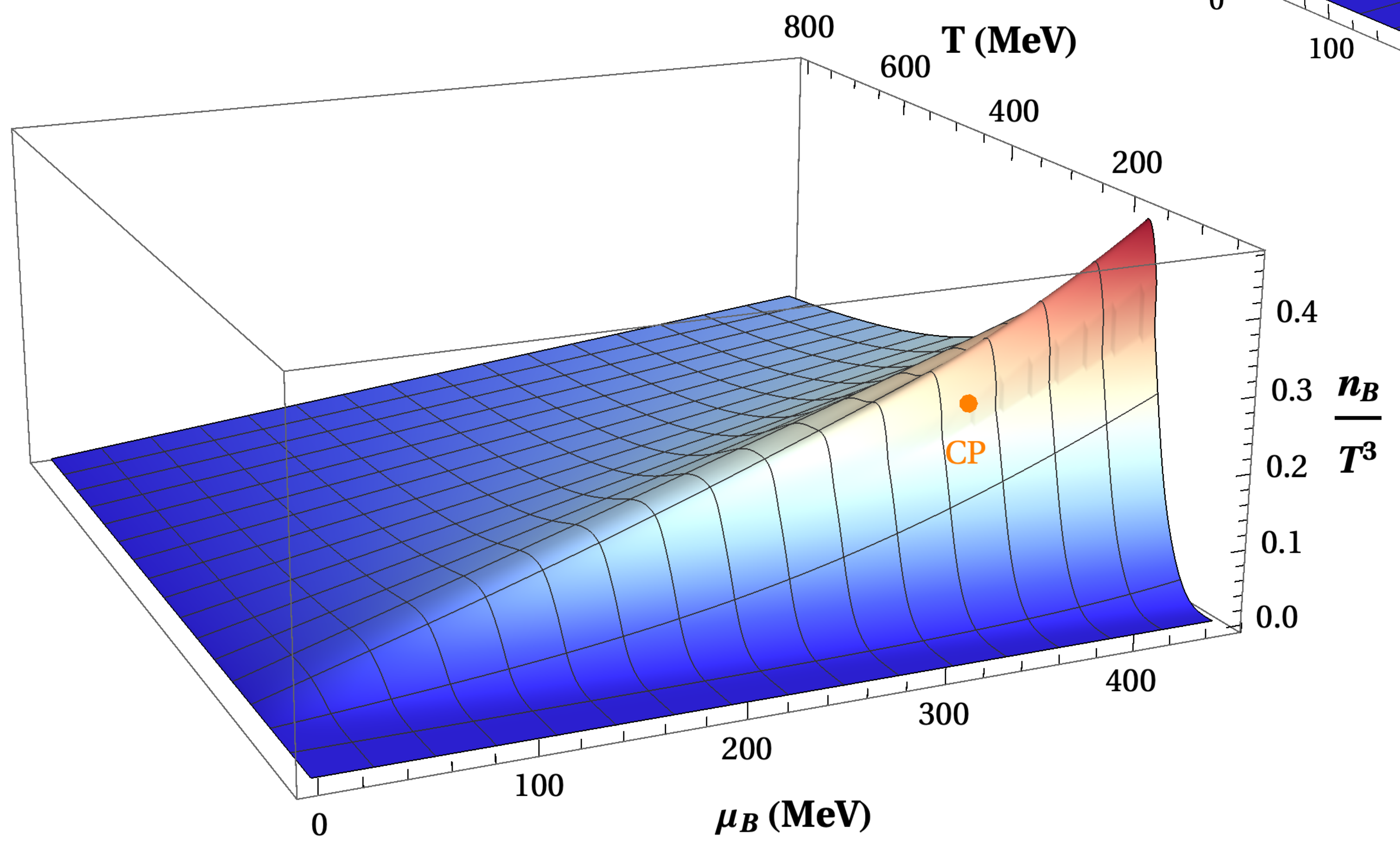
$$P(T,\mu_B) = T^4 \sum_{n=0}^{2} c_{2n}^{\text{non-Ising}}(T)\left(\frac{\mu_B}{T}\right)^{2n} + T_C^4\, P_{\text{symm}}^{\text{Ising}}(T,\mu_B)$$

EoS Thermodynamic Outputs

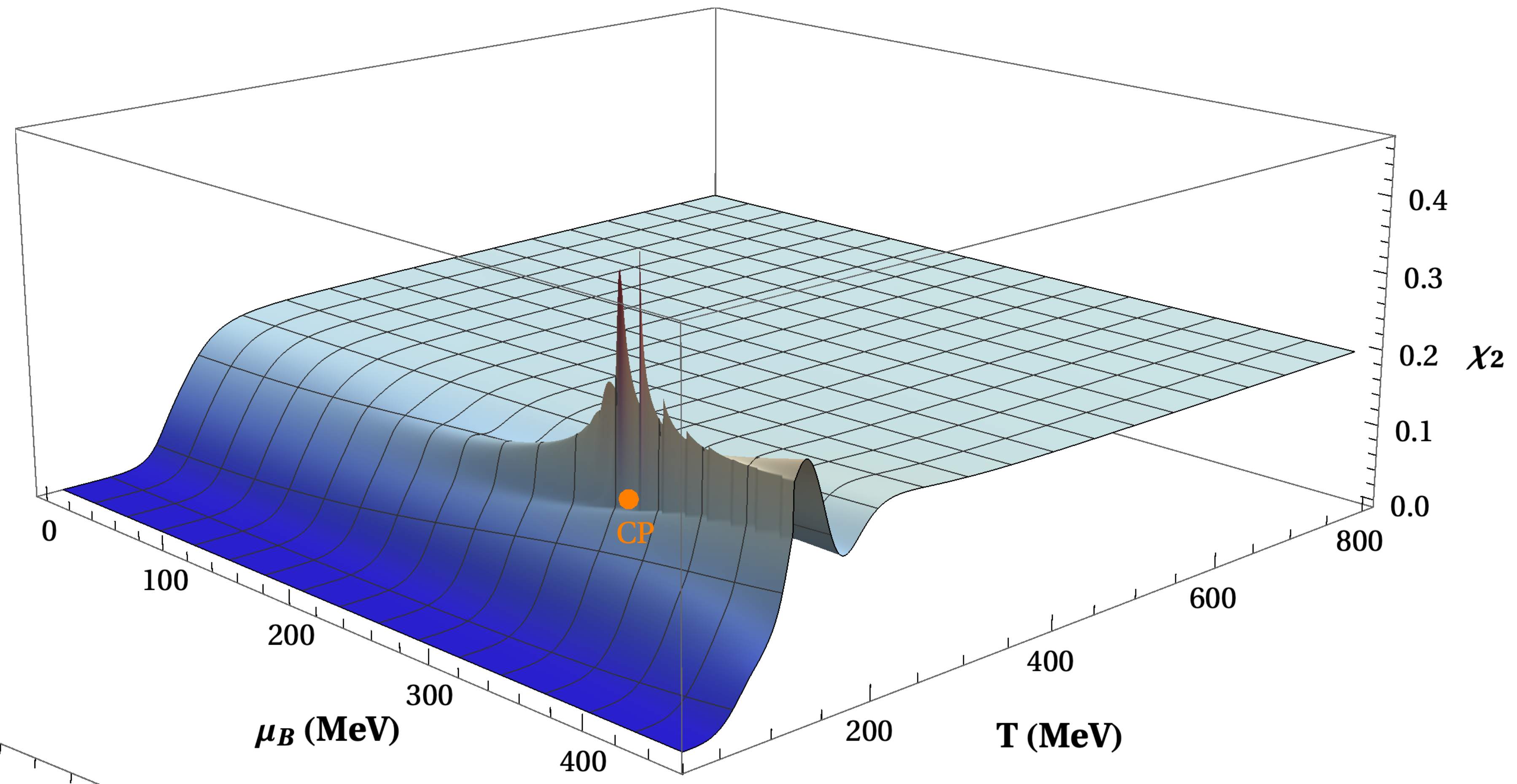
- Pressure and its derivatives show effects of critical region on these quantities: stronger effects with increasing derivatives



Pressure



Baryon density

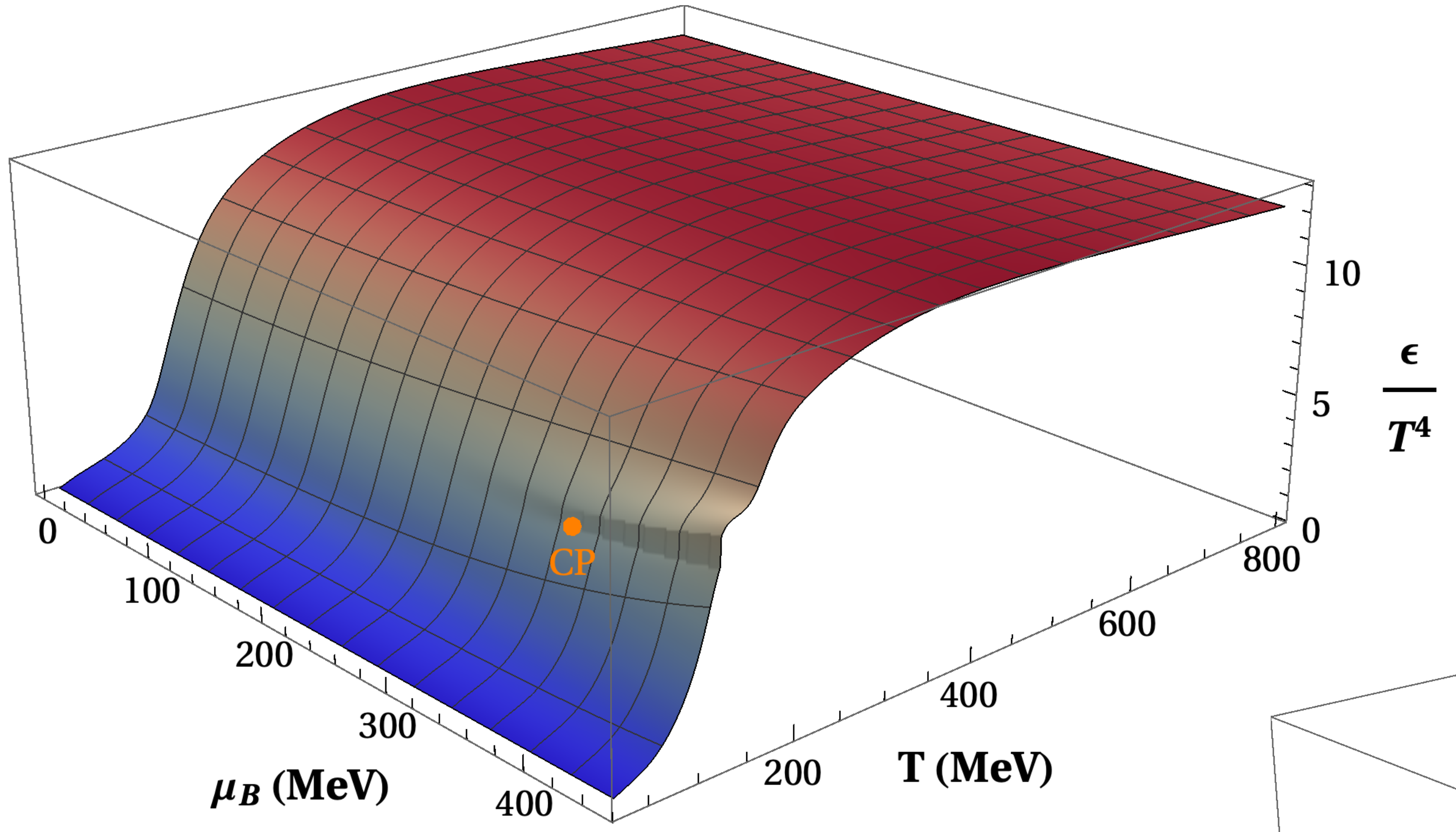


χ_2^B

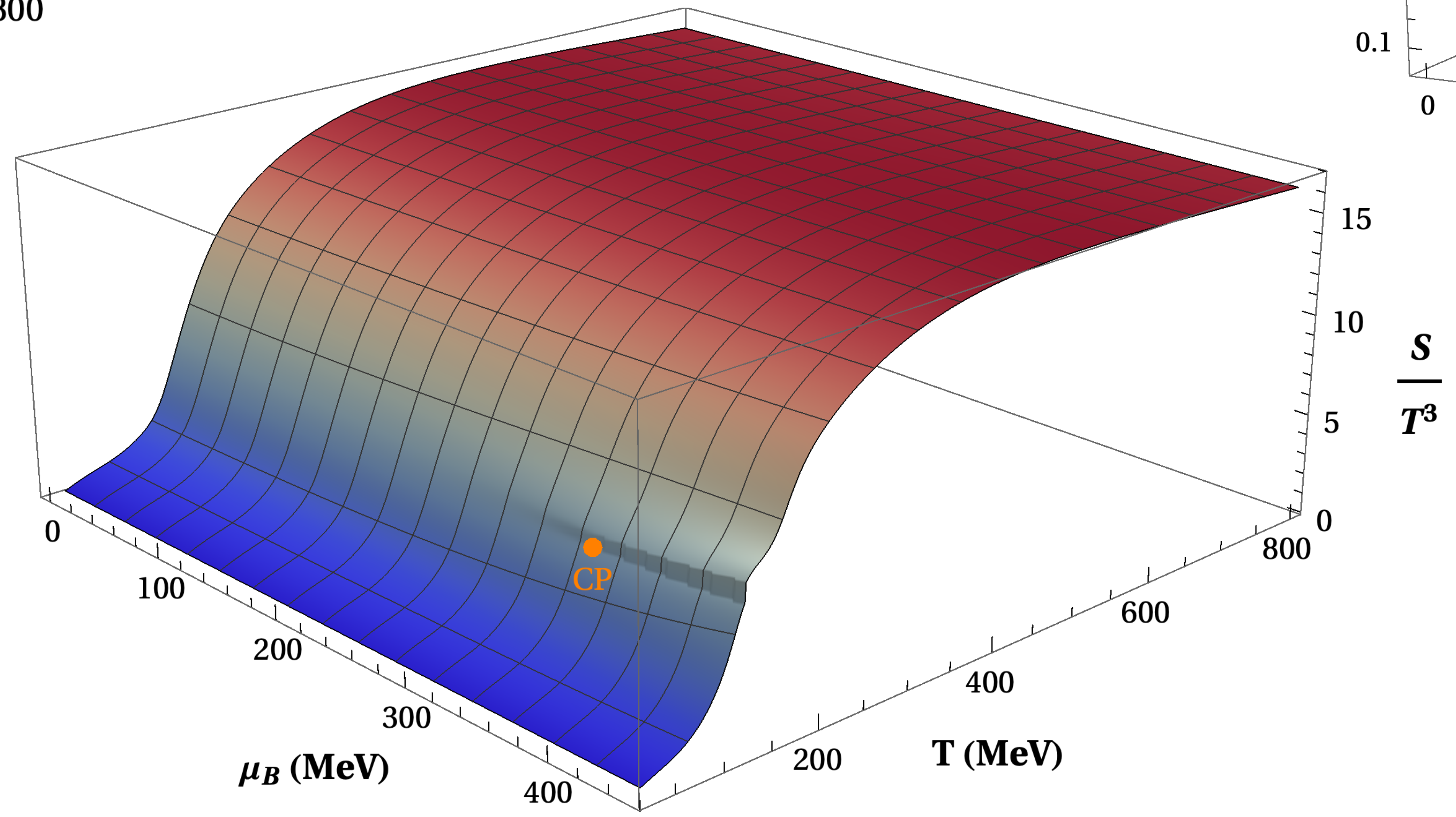
JS et al, arXiv:2103.08146

EoS Thermodynamic Outputs

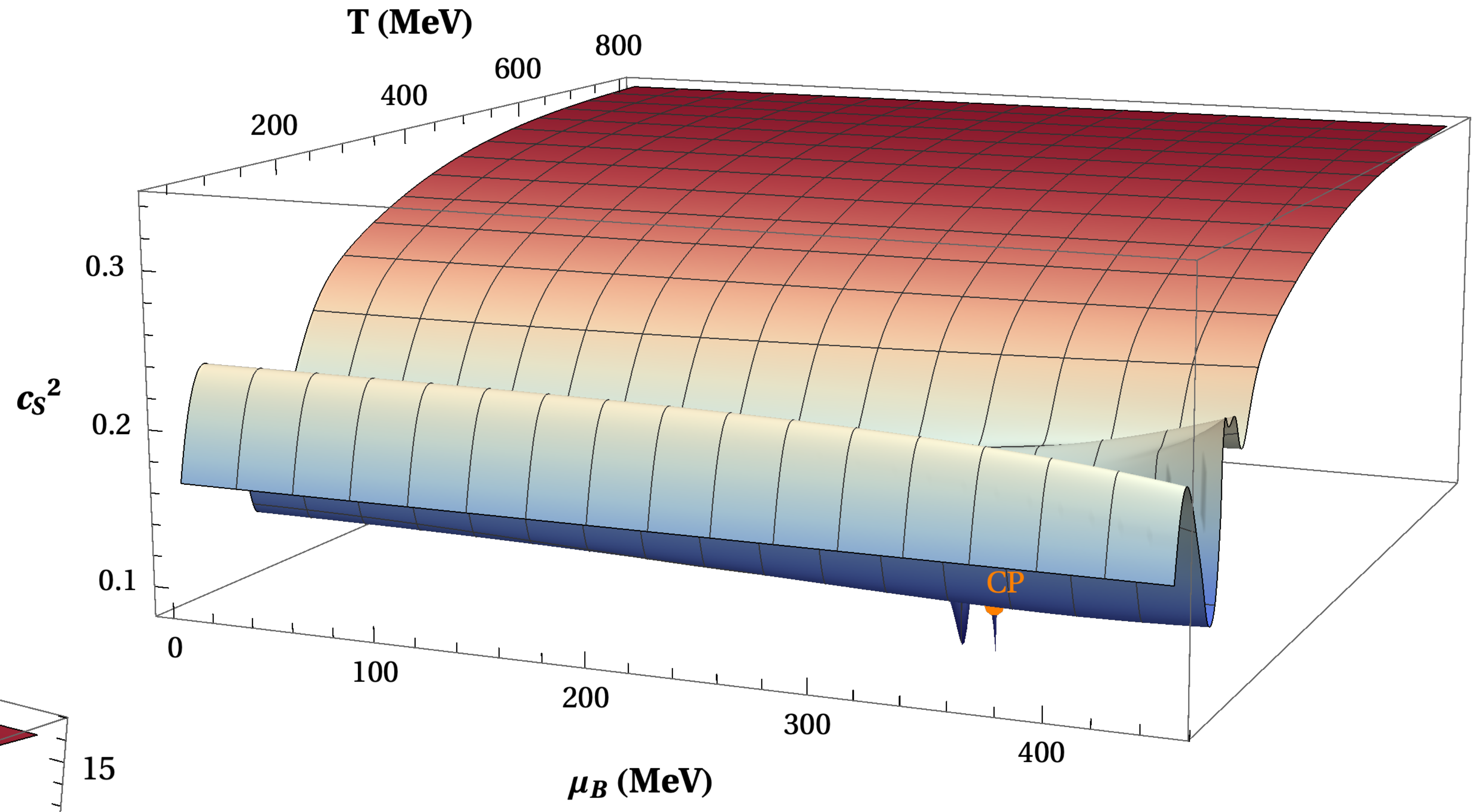
- Energy density and entropy exhibit discontinuities, while the speed of sound approaches zero at the critical point



Energy density



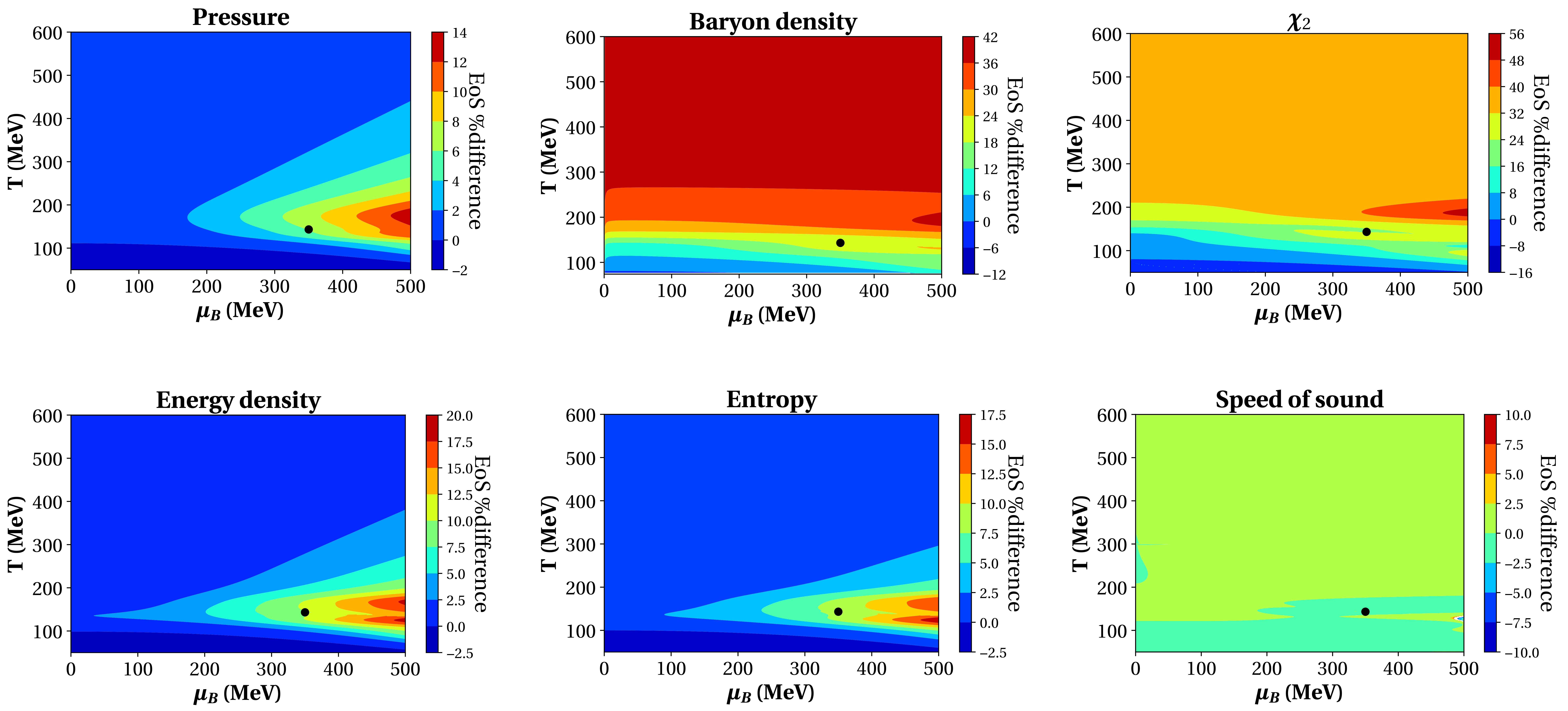
Entropy



Speed of sound

JS et al, arXiv:2103.08146

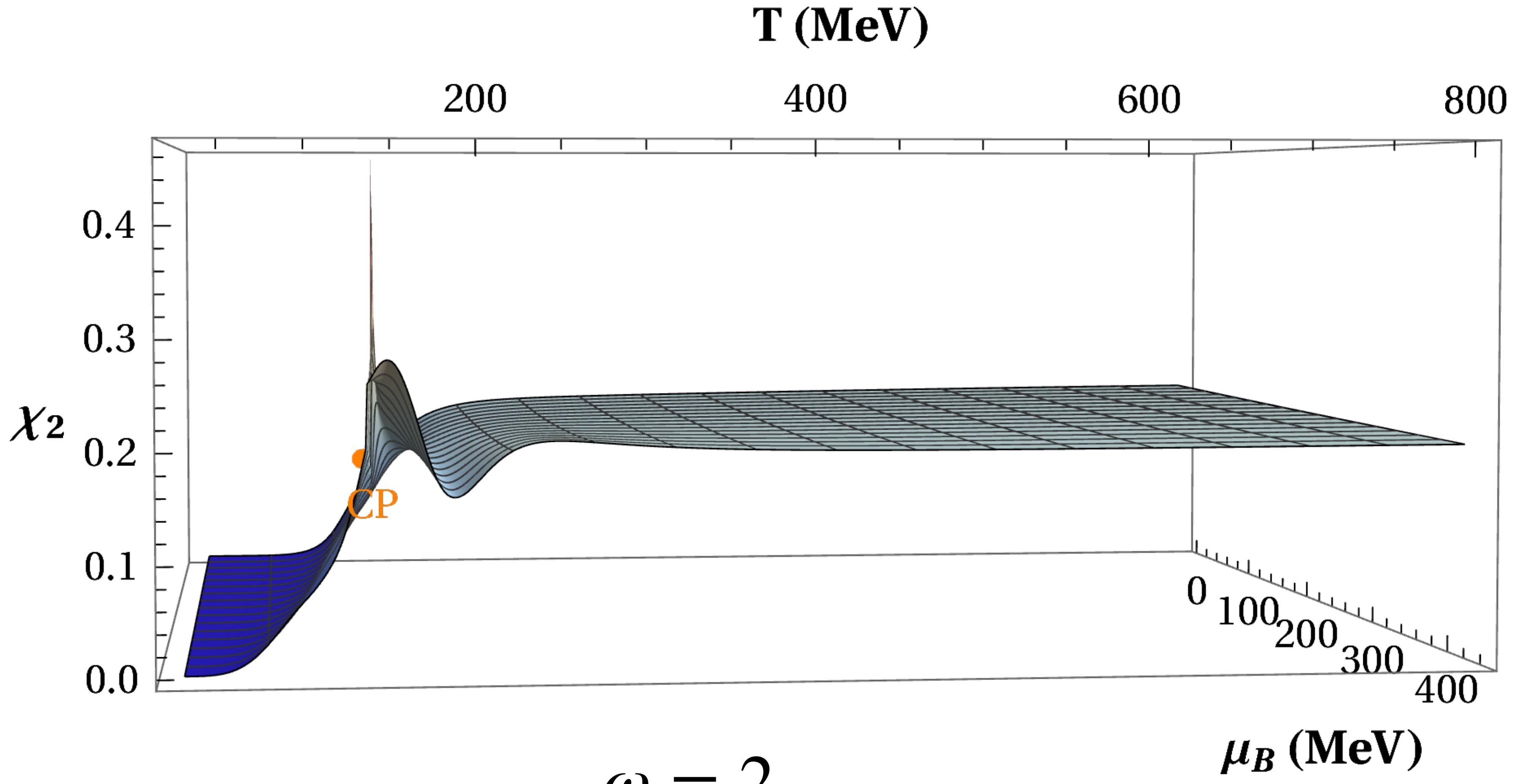
EoS Differences $\mu_S = \mu_Q = 0$ and Strangeness Neutrality



JS et al, arXiv:2103.08146

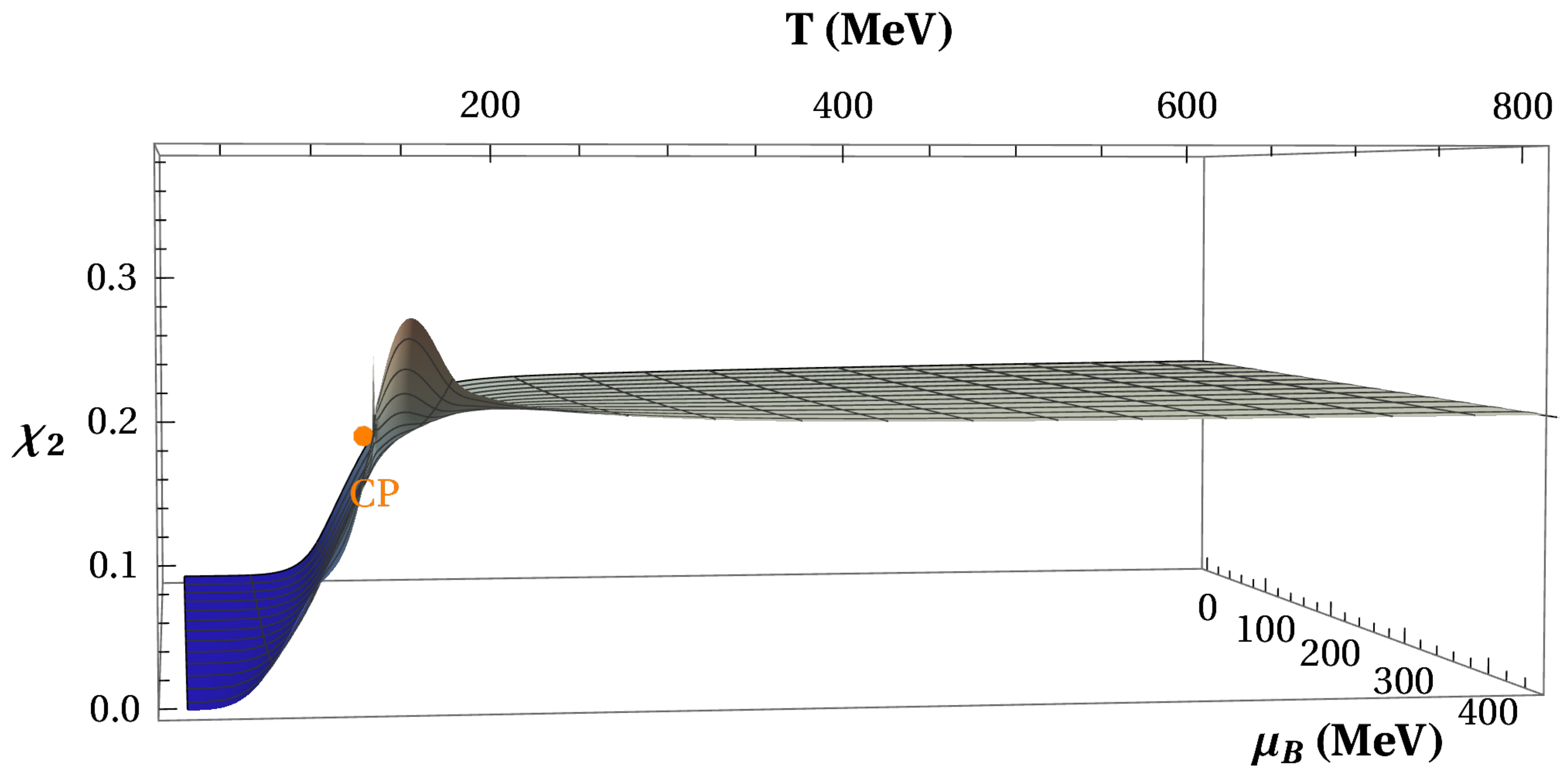
Size of Critical Region - Higher Order Susceptibilities

- By changing the parameters of the mapping we can control the critical contribution to the overall thermodynamics



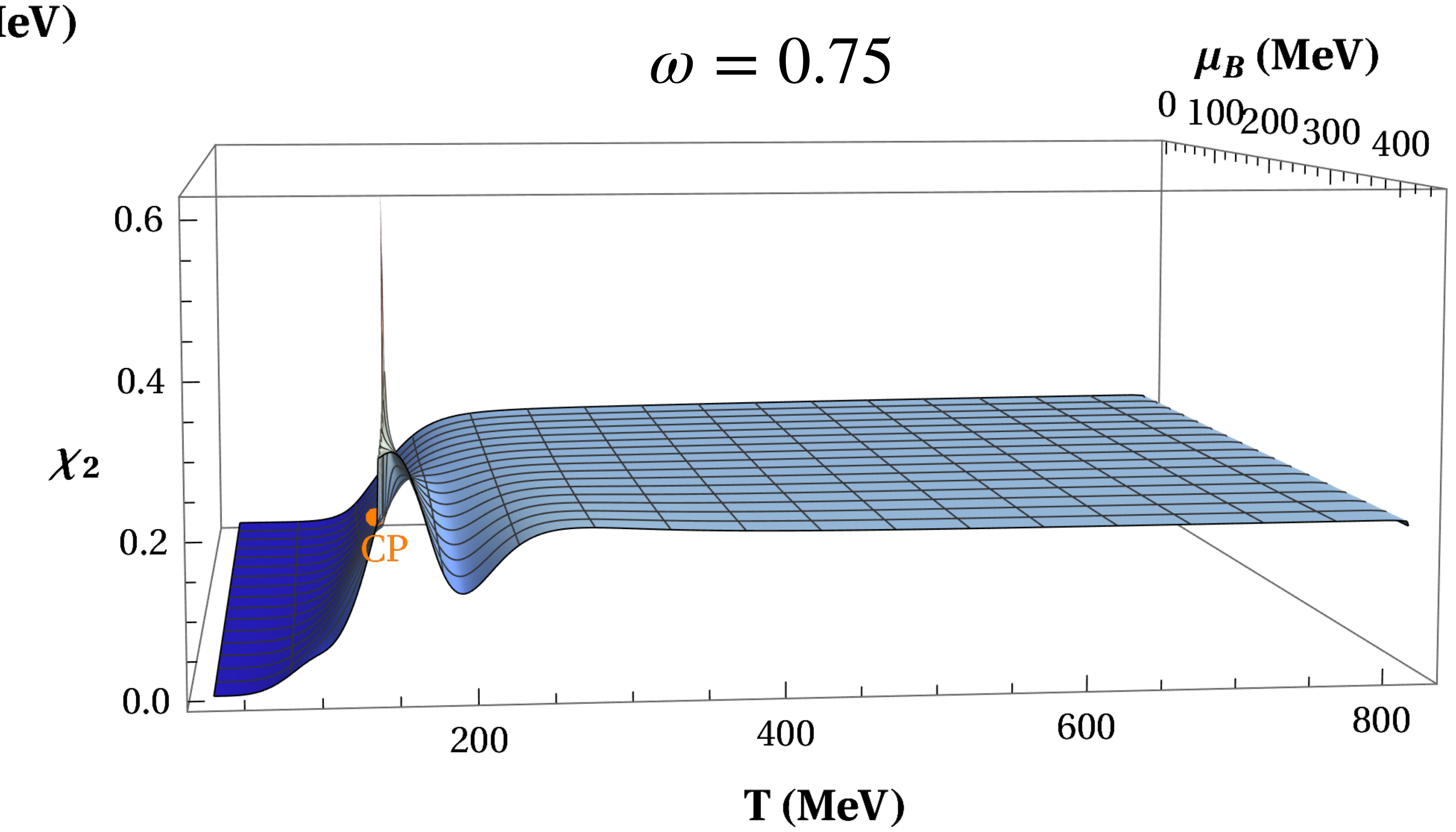
$\omega = 2$

(same as published version)



$\omega = 4$

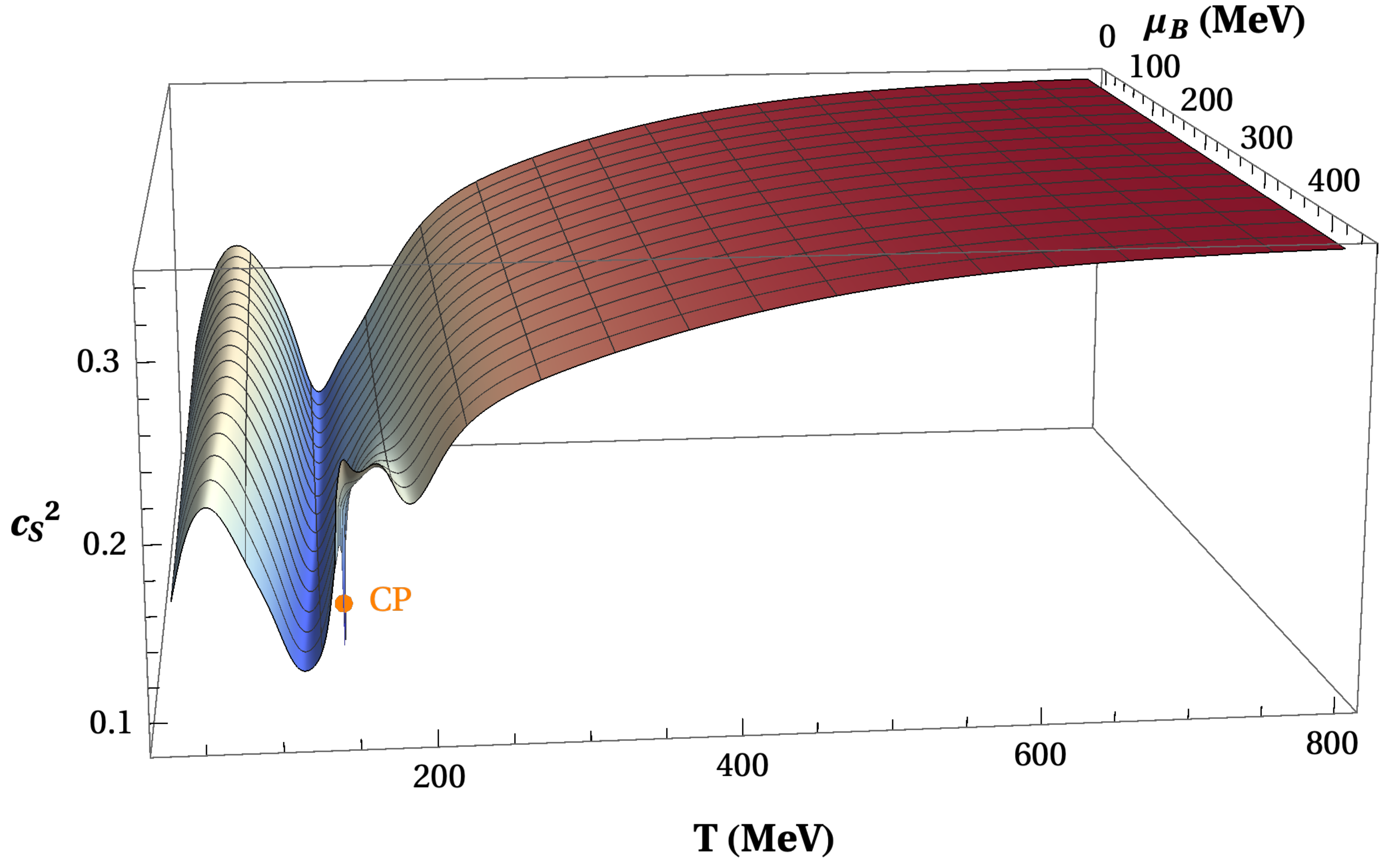
$\omega = 0.75$



M. Pradeep and M. Stephanov, PRD (2019)
P. Parotto et al, PRC (2020)
D. Mroczek et al, PRC (2021)
Wei-jie Fu et al, arXiv: 2101.06035

Size of Critical Region - Speed of Sound

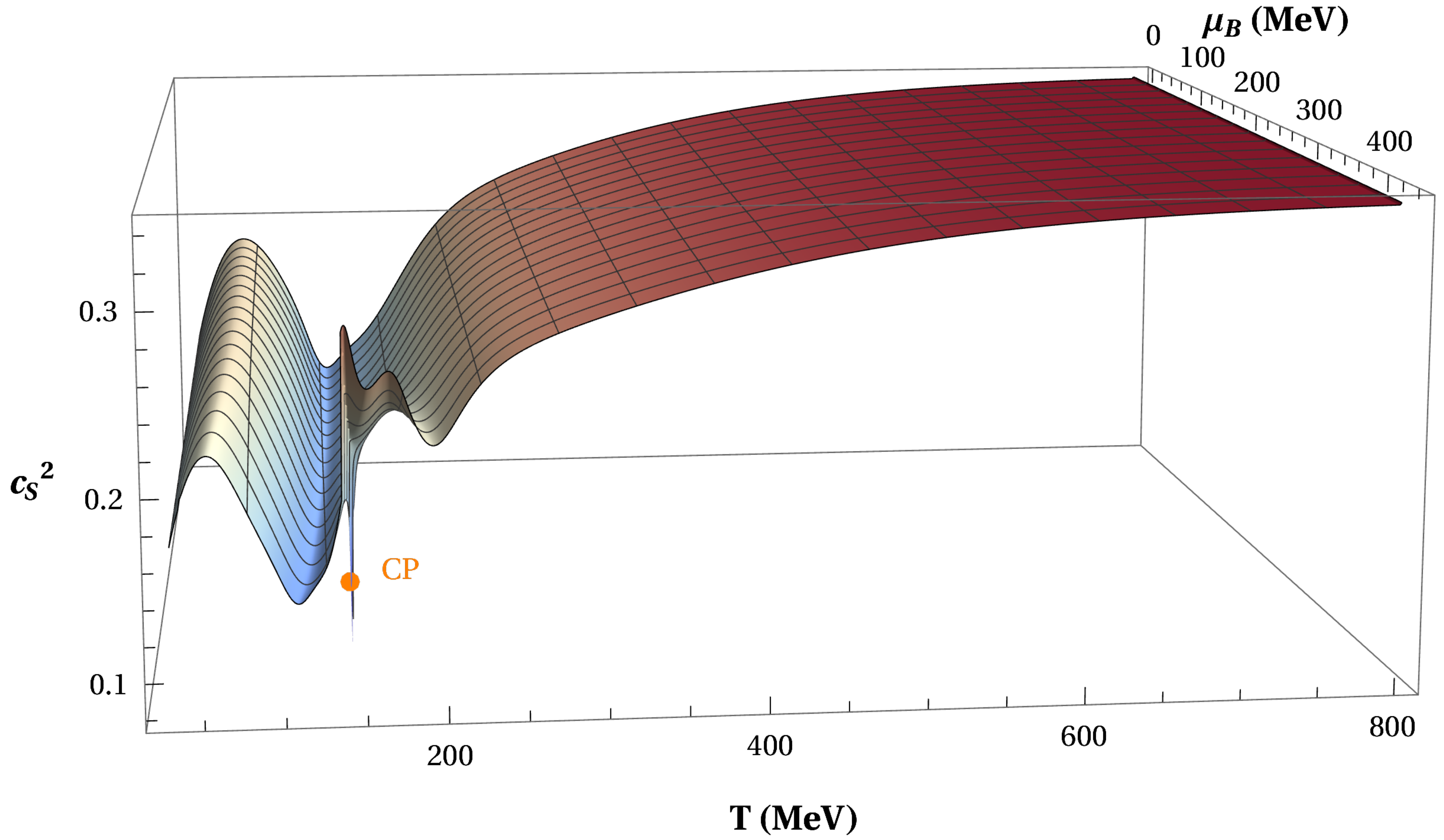
➤ By changing the parameters of the mapping we can control the critical contribution to the overall thermodynamics

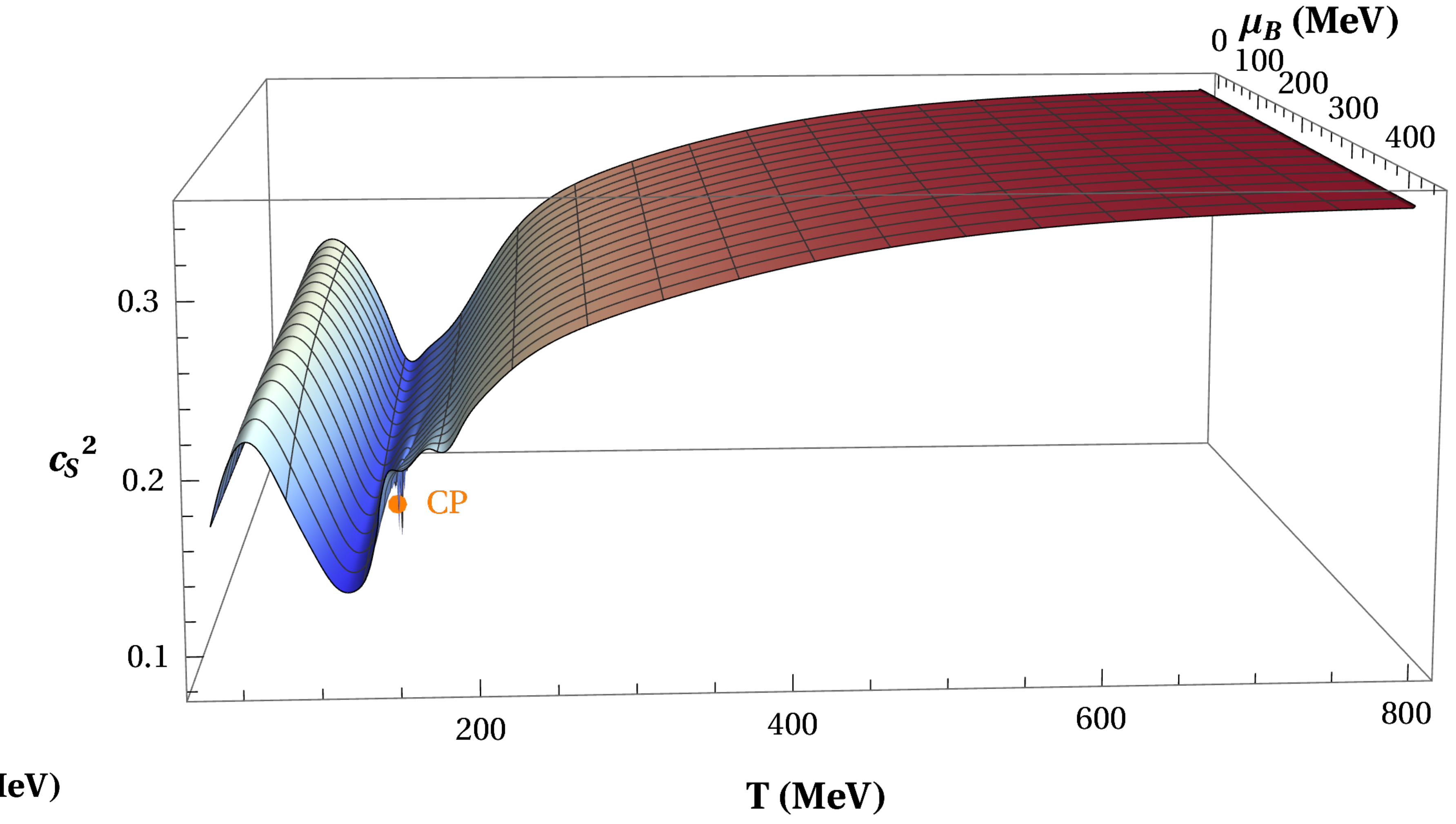


$\omega = 2$

(same as published version)

$\omega = 0.75$





$\omega = 4$

M. Pradeep and M. Stephanov, PRD (2019)
P. Parotto et al, PRC (2020)
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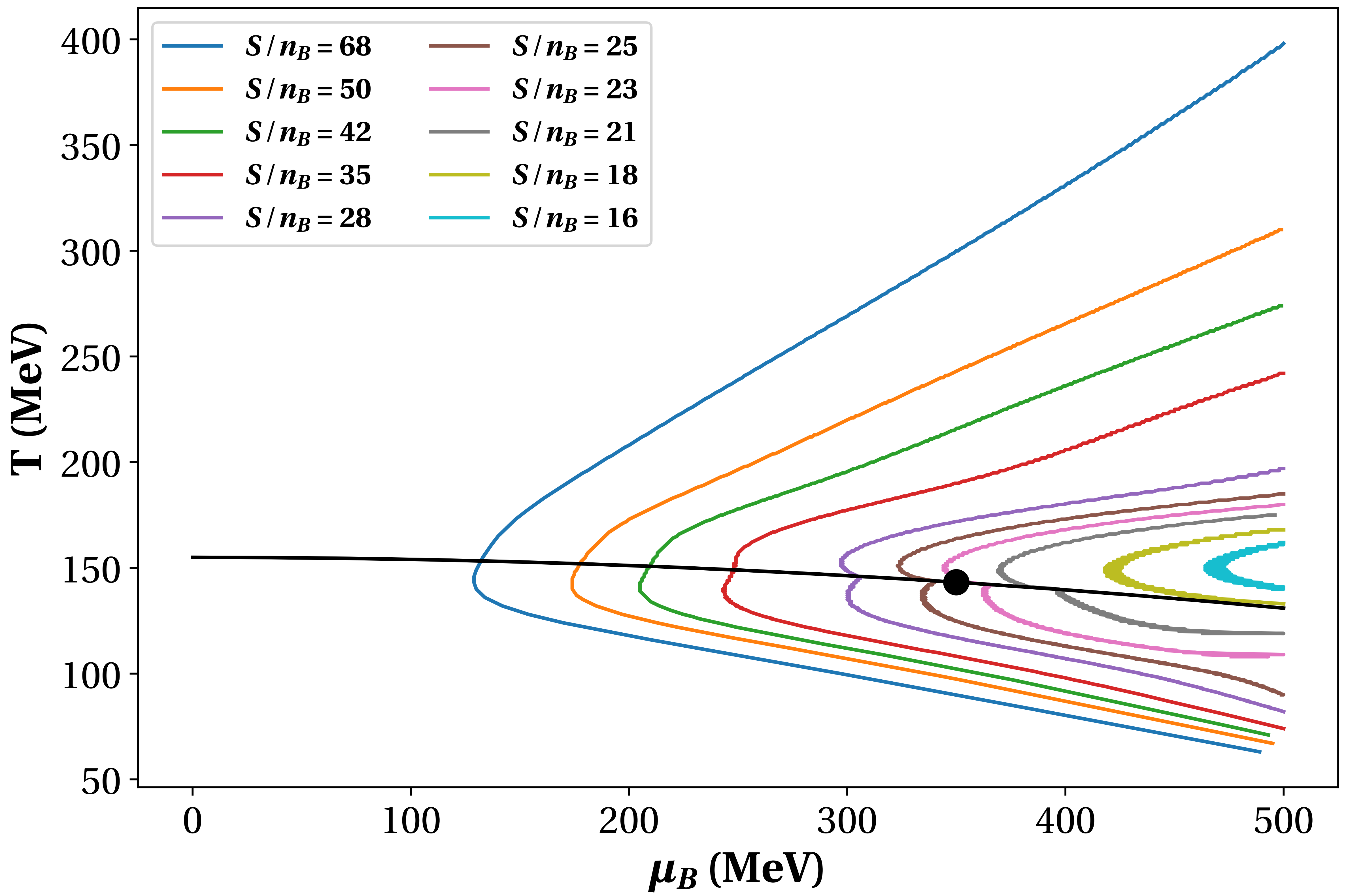
Wei-jie Fu et al, arXiv: 2101.06035

Isentropic Trajectories

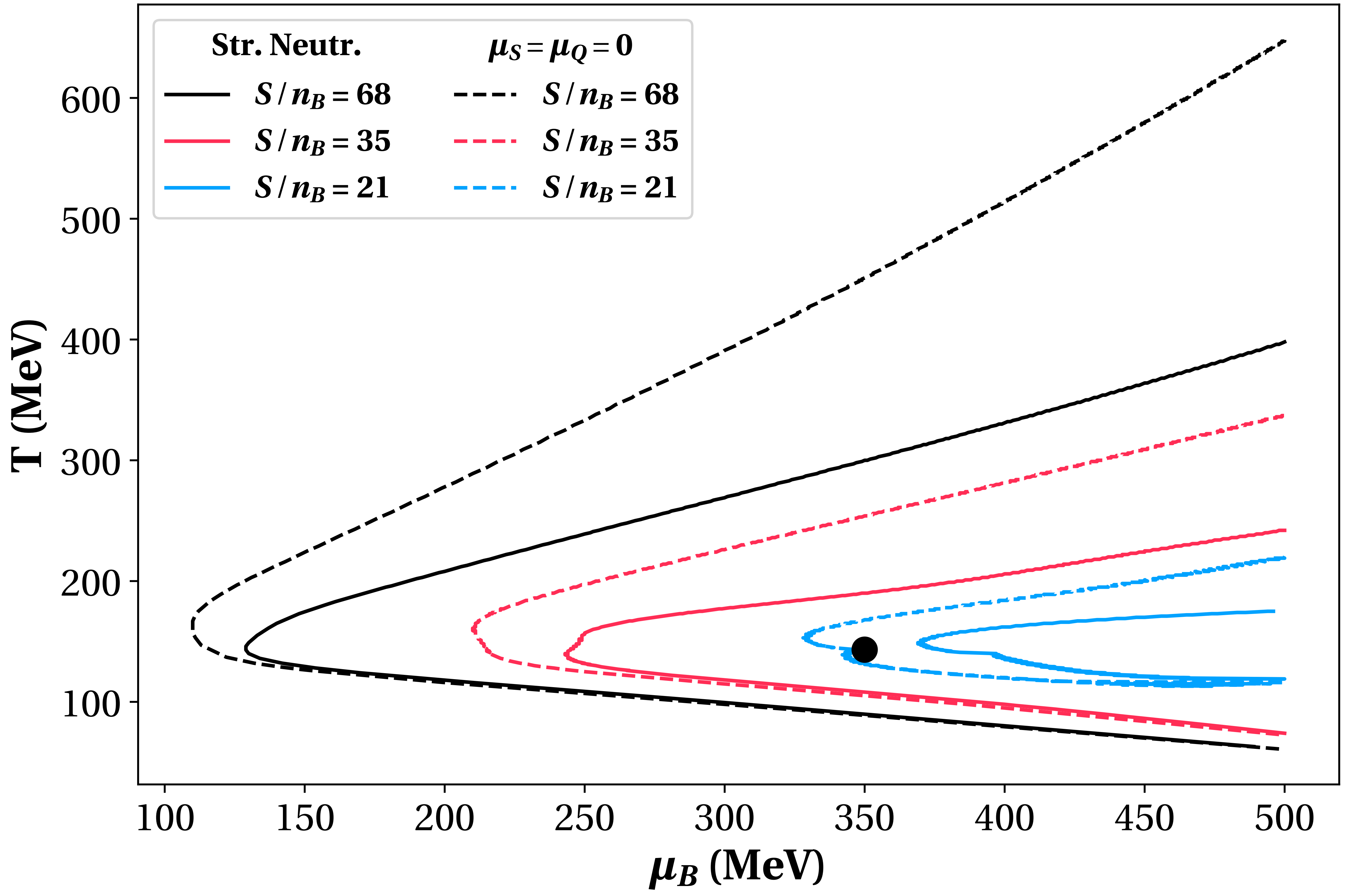
- Isentropes show the path of the HIC system through the phase diagram in the absence of dissipation



JS et al, arXiv:2103.08146

Isentropic Trajectories

- Isentropes show the path of the HIC system through the phase diagram in the absence of dissipation
  - Different path when conserved charge conditions applied



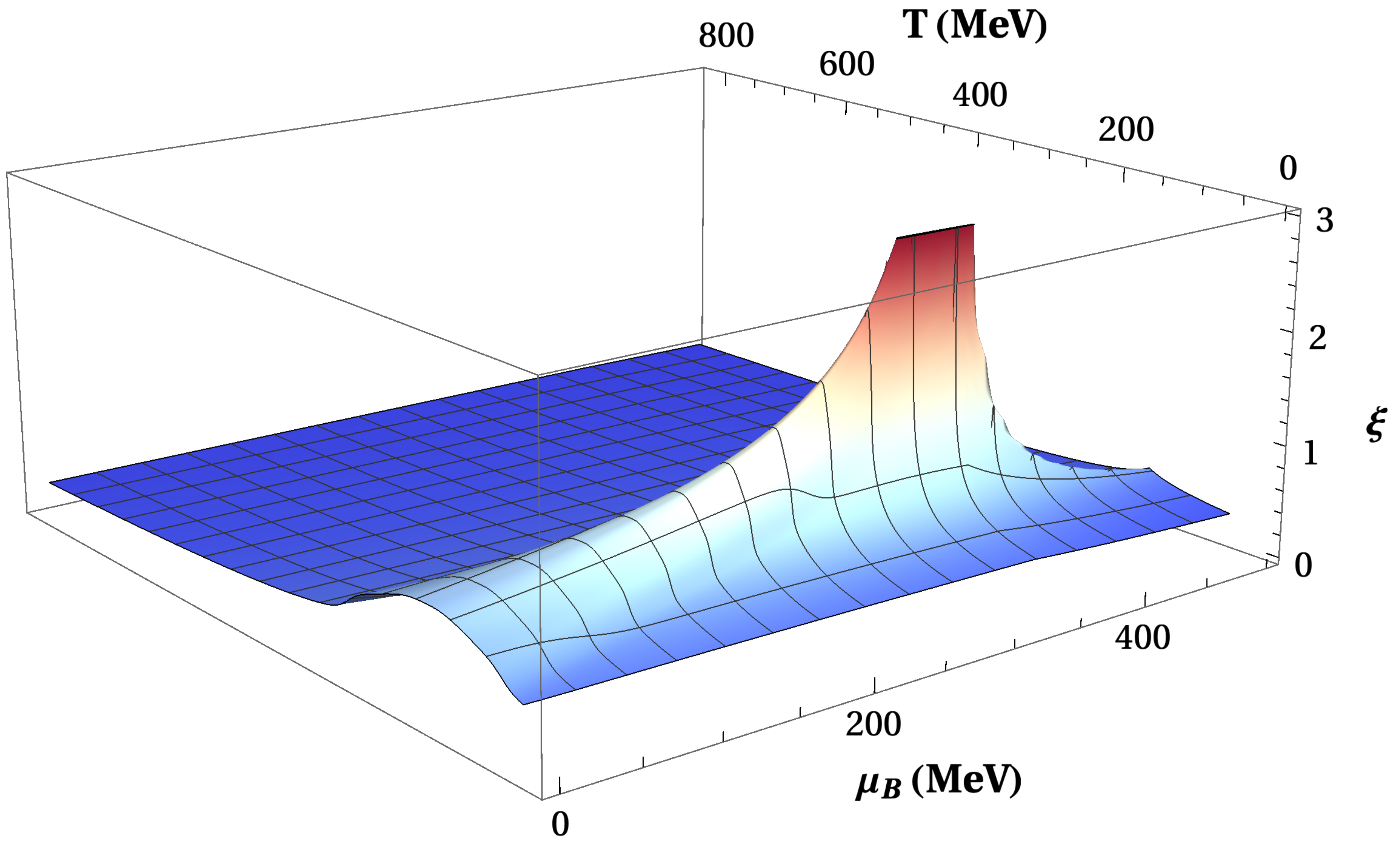
JS et al, arXiv:2103.08146

Correlation Length

➤ Additionally, calculate the correlation length in the 3D Ising model:

$$\xi^2(t, M) = f^2 |M|^{-2\nu/\beta} g(x)$$

where f = 1fm, $\nu = 0.63$ is the correlation length critical exponent, g(x) is the scaling function and the scaling parameter is $x = \dfrac{|t|}{|M|^{1/\beta}}$



Correlation length

B. Berdnikov and K. Rajagopal, PRD (2000)
C. Nonaka and M. Asakawa, PRC (2005)
JS et al, arXiv:2103.08146

Conclusions

- Realistic modeling of heavy-ion systems should involve constraints on the conserved charges.
- We provide an update to the BES-EoS that includes strangeness neutrality conditions, includes the same hadronic states as SMASH, and performs in a range of temperature and baryonic chemical potential relevant for BES-II.
- We see the expected critical features in the EoS and note a shift in the isentropic trajectories between the new and original versions.
- A calculation of the correlation length in the 3D Ising model is provided.