MUON MAGNETIC MOMENT: NEW PHYSICS OR NOT?

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Forschungszentrum Jülich & University of Wuppertal

Budapest-Marseille-Wuppertal collaboration

Borsanyi,Fodor,Guenther,Hoelbling,Katz,Lellouch,Lippert Miura,
Parato, Stokes, Toth, Torok, Varnhorst

[2002.12347,Nature (2021)] Leading-order hadronic vacuum polariz ...

Why so fascinating?

1. Experiment.

$$a_\mu^{\text{exp}} = 11659206.1(4.1) \times 10^{-10}$$

bathroom scale sensitive to weight of a single eyelash

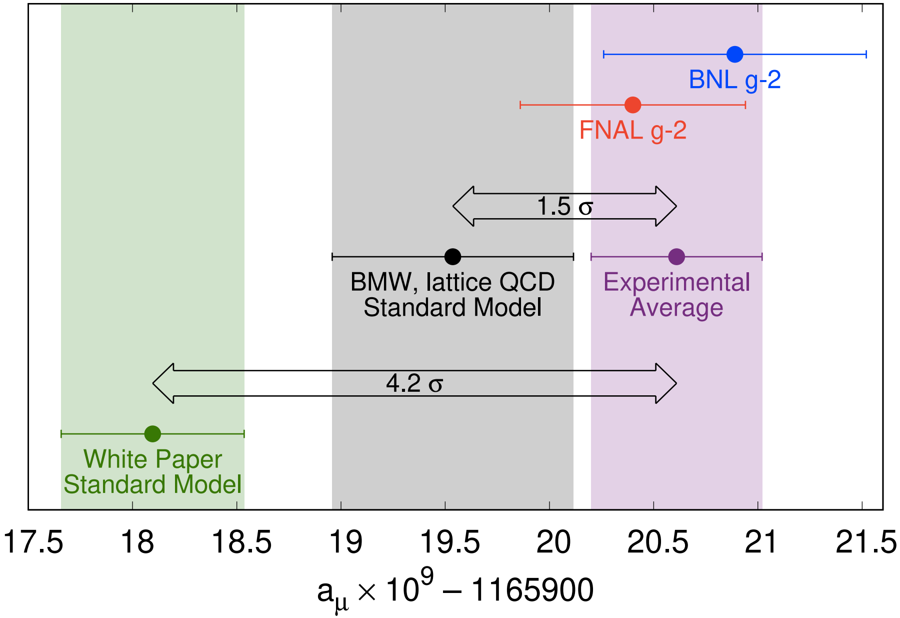
2. Theory.

$$a_\mu^{\text{the}} = 11659181.0(4.3) \times 10^{-10}$$

3. They agree to eight decimal places.
4. They disagree at the ninth decimal place.

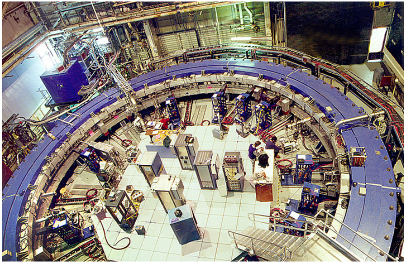
$$a_\mu^{\text{exp}} - a_\mu^{\text{the}} = 25.1(5.9) \times 10^{-10}$$

Outline

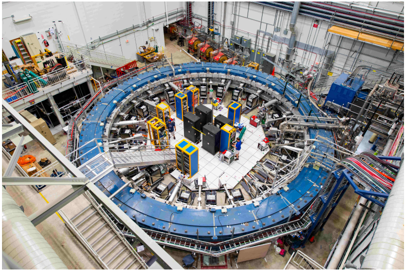


BNL g-2
FNAL g-2
1.5 σ
BMW, lattice QCD
Standard Model
Experimental
Average
4.2 σ
White Paper
Standard Model
17.5
18
18.5
19
19.5
20
20.5
21
21.5
$a_\mu \times 10^9 - 1165900$

Experiments



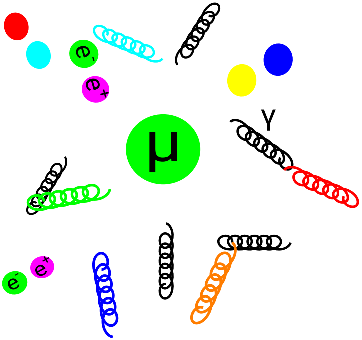
- Brookhaven 2004:
  $a_\mu^{\text{BNL}} = 11659209.1(6.3)$



- Fermilab 2021:
  $a_\mu^{\text{FNAL}} = 11659204.0(5.4)$
- combination:
  $a_\mu^{\text{exp}} = 11659206.1(4.1)$
- target final error (1.6)

- J-PARC >2021

Theory: Standard Model



Sum over all known physics:

1. QED: photons, leptons
2. electroweak: W, Z bosons, neutrinos, Higgs
3. strong: quarks and gluons

- [2006.04822] White Paper of Muon g-2 Theory Initiative

	$a_\mu \times 10^{-10}$
QED	11658471.9(0.0)
electroweak	15.4(0.1)
strong	693.7(4.3)
total	11659181.0(4.3)

Theory: QED

- $\alpha = \frac{e^2}{4\pi} \ll 1$ $\rightarrow$ rapidly converging series, was a key to success of QED

$$\left(\frac{g-2}{2}\right) = \left(\frac{\alpha}{\pi}\right) a^{(1)} + \left(\frac{\alpha}{\pi}\right)^2 a^{(2)} + \left(\frac{\alpha}{\pi}\right)^3 a^{(3)} + \ldots$$

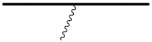
- all contributions with photons and leptons (e, μ, τ)

n-loop	$a_\mu^{\mathrm{QED}} \times 10^{-10}$
1	11614097.330(0.008)
2	41321.762(0.010)
3	3014.190(0.000)
4	38.081(0.030)
5	0.448(0.140)
total	11658471.811(0.160)

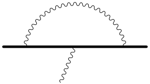
- inputs are m_μ/m_e, m_μ/m_τ and α

Theory: QED

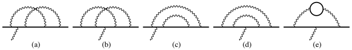
$\alpha^0 \to g = 2$



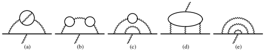
$\alpha^1 \to \alpha/(2\pi)$



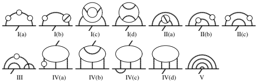
α^2



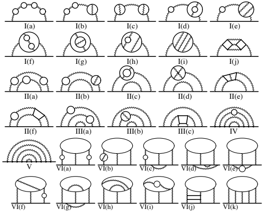
α^3



α^4



α^5

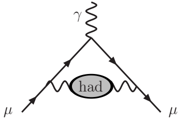


Five loops by [Kinoshita et al ’15]

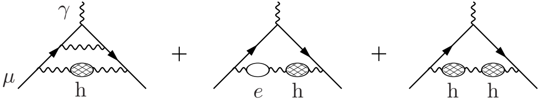
- 12672 diagrams
- automated diagram generation, numerical evaluation of integrals, only some diagrams known analytically
- not yet confirmed by other groups

Strong contributions

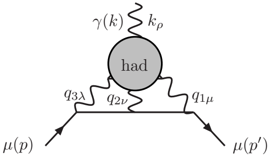
LO hadron vacuum polarization (LO-HVP, $(\frac{\alpha}{\pi})^2$)



NLO hadron vacuum polarization (NLO-HVP, $(\frac{\alpha}{\pi})^3$)



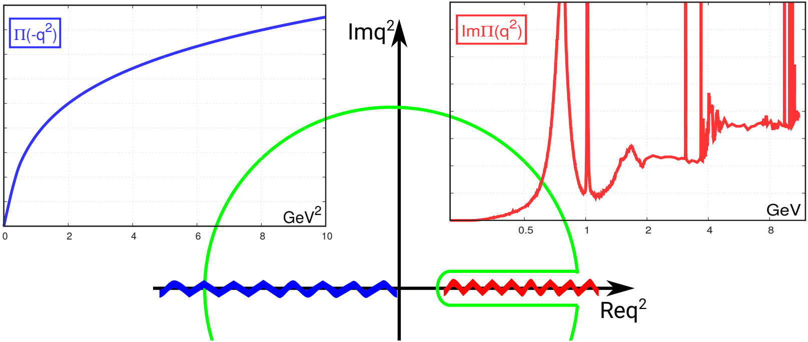
Hadronic light-by-light (HLbL, $(\frac{\alpha}{\pi})^3$)



- pheno $a_\mu^{\text{HLbL}} = 10.5(2.6)$
  [Prades et al '09]
- lattice a_μ^{HLbL}=7.9(3.1)(1.8) or 10.7(1.5)
  [RBC/UKQCD '19] and [Mainz '21]

Hadronic vacuum polarization

- $\Pi_{\mu\nu}(q) = \left(q_\mu q_\nu - g_{\mu\nu} q^2\right) \Pi(q^2)$ analytic + branch-cut



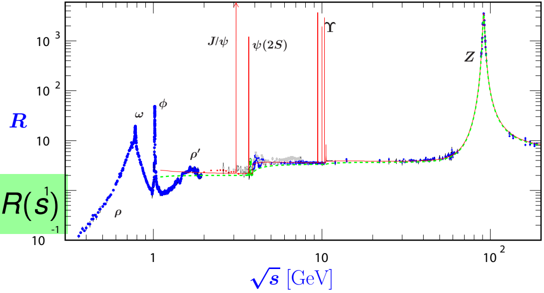
- Minkowski from R-ratio experiments
- Euclidean from lattice QCD or exp. like MUonE
- Minkowski → Euclidean via dispersion relation ($Q^2 = -q^2$)

$$\Pi(Q^2) = \int_{s_{\rm th}}^{\infty} ds \, \frac{Q^2}{s(s+Q^2)} \frac{1}{\pi} \mathrm{Im}\Pi(s)$$

a_μ^{HVP} from R-ratio

Use $e^+e^- \to$ had data of CMD, SND, BES, KLOE, BABAR, ...

$$a_\mu^{\text{LO-HVP}} = \left(\frac{\alpha}{\pi}\right)^2 \int \frac{ds}{s^2} K_\mu(s) R(s)$$

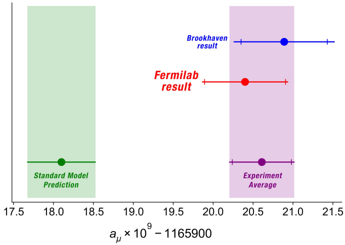


LO	[Jegerlehner '18]	688.1(4.1)	0.60%
LO	[Davier et al '19]	693.9(4.0)	0.58%
LO	[Keshavarzi et al '19]	692.78(2.42)	0.35%
LO	[Hoferichter et al '19]	692.3(3.3)	0.48%
NLO	[Kurz et al '14]	-9.87(0.09)	
NNLO	[Kurz et al '14]	1.24(0.01)	

$$a_\mu^{\text{exp}} - a_\mu^{\text{QED}} - a_\mu^{\text{weak}} - a_\mu^{\text{HLbL}} - a_\mu^{\text{HVP}} = 25.1(5.9)$$

Discrepancy

- $a_\mu^{\text{exp}} - a_\mu^{\text{theory}} = 25.1(5.9)$ around 4.2σ significance



error budget:

$(4.1)_{\text{exp}}(0.2)_{\text{QED}}(0.6)_{\text{weak}}(3.3)_{\text{HVP}}(2.6)_{\text{HLbL}}$

- is about $2\times$ electroweak contribution

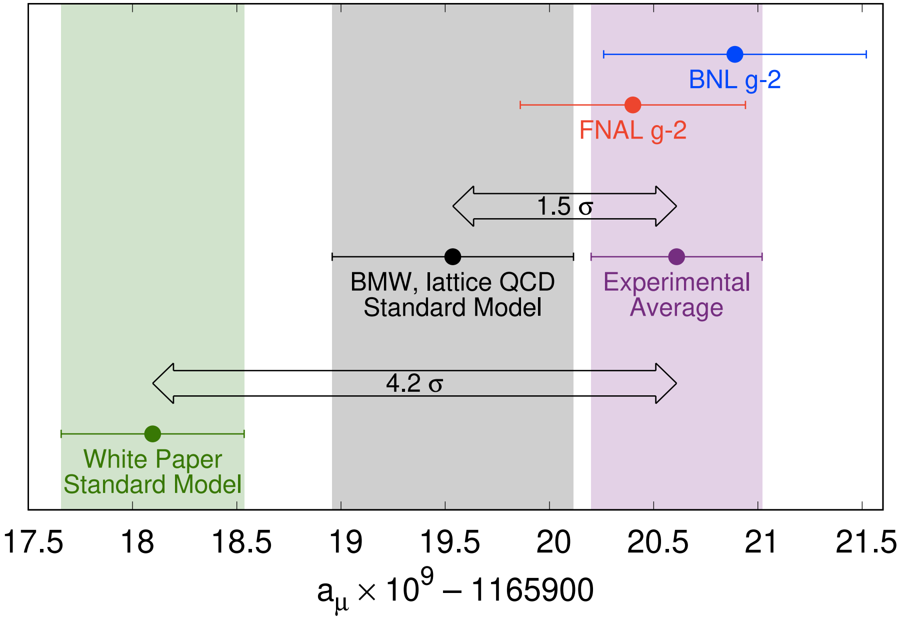
For new physics:

- final FNAL + same theory errors 6σ
- final FNAL + HLbL 10% + HVP 0.2% 11σ

For no new physics:

- 4% larger HVP, $a_\mu^{\text{LO-HVP}} = 720.0(6.8)$
- 360% larger HLbL, $a_\mu^{\text{HLbL}} = 37.9(7.1)$

Outline



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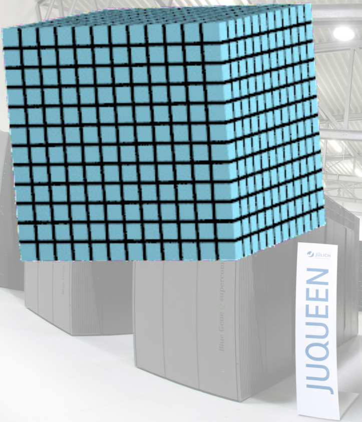
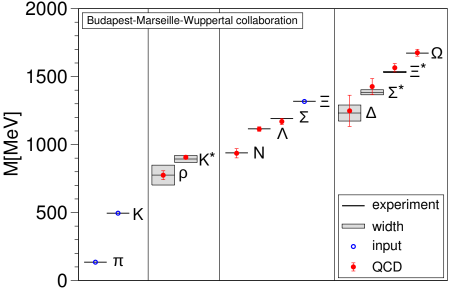
Lattice QCD

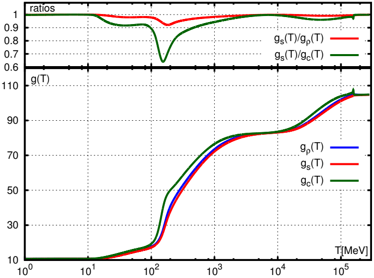


- ab-initio: use equations of QCD and nothing more
- discretize equations on space-time lattice and solve numerically ($\rightarrow$ supercomputer)
- sum over all Feynman-diagrams at once (and beyond)
- only works in imaginary time (no problem for a_μ)

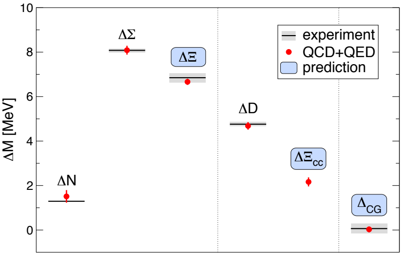
Lattice highlights



hadron spectrum [BMWc'08]



equation of state [WB '16]



neutron-proton difference [BMWc'14]

- isospin symmetry breaking (strong and QED)

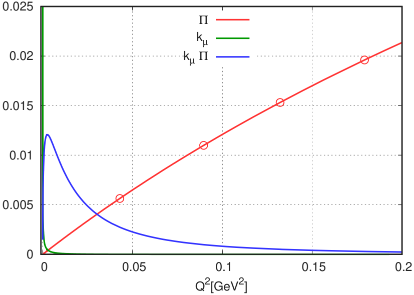
a_μ^{HVP} from lattice QCD

- get Π from Euclidean current-current correlator [Blum '02]

$$\Pi_{\mu\nu} = \int dx\; e^{iQx} \langle J_\mu(x) J_\nu(0) \rangle = \left(Q_\mu Q_\nu - \delta_{\mu\nu} Q^2 \right) \Pi(Q^2)$$

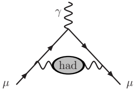
problems: we need $\Pi(Q^2) - \Pi(0)$, but $\Pi(0)$ is not directly accessible; also Q is available at discrete momenta only

- smooth interpolation in Q and prescription for $\Pi(0)$ [Bernecker,Meyer '11], [HPQCD'14], ...

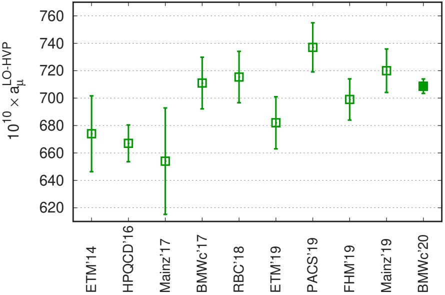


$$a_\mu^{\mathrm{HVP}} = \frac{\alpha^2}{\pi^2} \int dQ^2\; k_\mu(Q^2)\, \Pi(Q^2)$$

$k_\mu(Q^2)$ describes the leptonic part of diagram



a_μ^{HVP} from lattice QCD



several ongoing lattice efforts

BMWc'17 $\rightarrow$ BMWc'20

711.0(18.9) $\rightarrow$ 707.5(5.5)

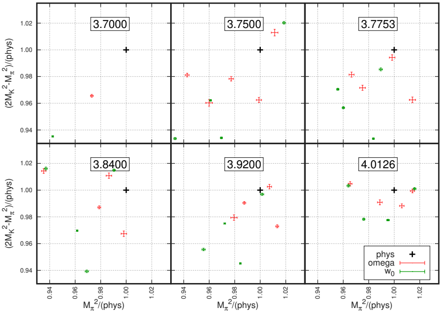
statistical (7.5) $\rightarrow$ (2.3)

finite-size (13.5) $\rightarrow$ (2.5)

cont.extrap (8.0) $\rightarrow$ (4.1)

isospin-breaking (5.1) $\rightarrow$ (1.4)

Simulations [2002.12347]



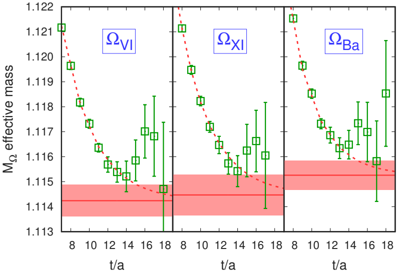
- $N_f = 2 + 1 + 1$ staggered
- 27 high statistics simulations
- bracketing physical point
- 6 lattice spacings
- $L \sim 6$ fm, $T \sim 9$ fm
- dedicated finite-size study
- overlap crosscheck of cont.limit

Techniques

conserved current - EigCG - Low Mode Averaging [Giusti et al '04]- All Mode Averaging [Blum et al '13] - Solver Truncation [Bali et al '09] - close to 10M/40M conn/disc measurements

Setting the lattice spacing a

Q^2 in a_μ^{HVP} has to be converted to physical units $\to$ need a



- use Omega mass to set the scale $a = (aM_\Omega)^{\mathrm{latt.}} / M_\Omega^{\mathrm{exp}}$
- use multi-exponential fits and/or GEVP
- reach permil level accuracy

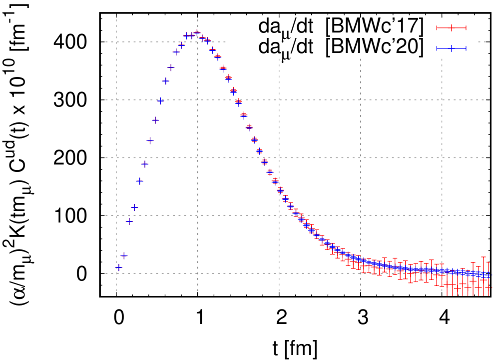
Determine an intermediate scale w_0 with four permil:

$$w_0 = 0.17236(29)(63)[70] \text{ fm}$$

- use w_0 to separate isospin breaking effects
- consistent with $w_0 = 0.1715(9)$ fm [HPQCD'13]

Noise reduction

noise/signal in $C(t) = \langle J(t)J(0)\rangle$ grows for large distances



- Low Mode Averaging: use exact (all2all) quark propagator in IR and stochastic in UV
- decrease noise by replacing $C(t)$ by upper/lower bounds above t_c

$$0 \leq C(t) \leq C(t_c)e^{-E_{2\pi}(t-t_c)}$$

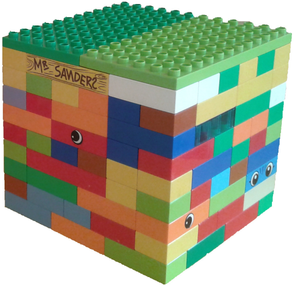
$\rightarrow$ few permil level accuracy on each ensemble

Finite-size effects

Typical runs use $L < 6$ fm, earlier model estimates gave $O(2)\%$.

- perform continuum limit in $L = 6$ fm, reference box
- on a coarse lattice compute finite size effect with $L = 11$ fm

"ref" $\rightarrow$



$\leftarrow$ "big"

- compare "big" - "ref" on the lattice to various models

	lattice	NNLO	MLLGS	HP	RHO
"big" - "ref"	18.1(2.0)(1.4)	15.3	18.3	16.3	15.2

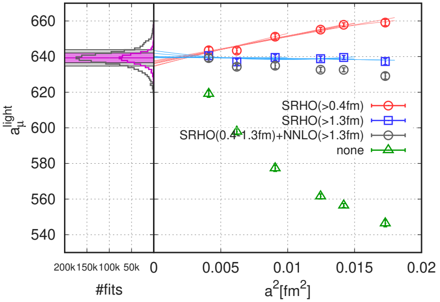
- use models for remnant finite-size effect of "big" $\sim 0.1\%$

Continuum limit

Improve continuum extrapolations by models ([HPQCD'16])

$$a_\mu(a) \to a_\mu(a) + \left[a_\mu^{\mathrm{RHO}} - a_\mu^{\mathrm{SRHO}}(a)\right]$$

$\to$ reduced sensitivity to β-cuts; improved slope and curvature



Systematic error:

- skip coarse lattices
- consider different models (SMLLGS/SRHO/SXPT)
- change the point, where we start applying the improvement
- use $a^2\alpha^\Gamma[1/a]$ instead of a^2
- use linear or quadratic extrapolations

Isospin breaking

include all e^2 and $\delta m = m_d - m_u$ effects; not only for $\langle JJ \rangle$ but also for masses, scale setting

compute the derivatives

$$m_l \frac{\partial X}{\partial \delta m} \qquad \frac{\partial X}{\partial e} \qquad \frac{1}{2}\frac{\partial^2 X}{\partial e^2}$$

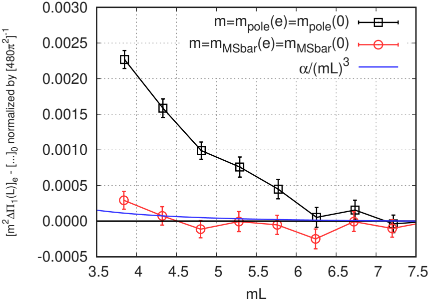
also distinguish between sea and valence electric charges

$$\frac{1}{2}\frac{\partial^2 X}{\partial e_v^2} \qquad \frac{\partial^2 X}{\partial e_v \partial e_s} \qquad \frac{1}{2}\frac{\partial^2 X}{\partial e_s^2}$$

Use two approaches from the literature:

1. operator insertion [DeDivitiis et al '13] $\rightarrow$ sea effects
2. direct approach [Eichten et al '97] $\rightarrow$ valence effects
   $\partial_e^2 X \approx \frac{1}{e^2}[X_{+e} + X_{-e} - 2X_0]$

Isospin breaking



QED finite size effects

- masses $\to$ universal α/L [BMWc'14,Davoudi-Savage'14]
- HVP $\to$ tiny α/L^3 [Bijnens et al'19] verified in scalar QED

Isospin decomposition

$\langle O\rangle_{\text{iso}}$ $\langle O\rangle_{\text{qed}}$ $\langle O\rangle_{\text{sib}}$

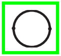
use a hadronic scheme based on w_0 and M_{uu}, M_{dd}, M_{ss} masses

Comparison with others

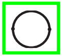
good agreement w/ [RBC'18] [ETMC'19] [HPQCD'18]

Overview of contributions

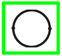
Isospin symmetric



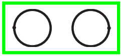
connected light
633.7(2.1)(4.2)



connected strange
53.393(89)(68)



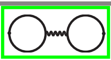
connected charm
14.6(0)(1)

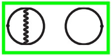


disconnected
-13.36(1.18)(1.36)

QED isospin-breaking: valence

connected -1.23(40)(31)

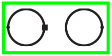




disconnected -0.55(15)(10)

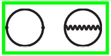
Strong isospin-breaking

connected
6.60(63)(53)

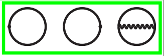


disconnected
-4.67(54)(69)

QED isospin-breaking: sea



connected 0.37(21)(24)



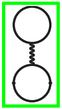
disconnected -0.040(33)(21)

Etc.

bottom; higher order; perturbative

0.11(4)

QED isospin-breaking: mixed



connected -0.0093(86)(95)

disconnected 0.011(24)(14)

Finite-size effects

isospin-symmetric
18.7(2.5)

isospin-breaking
0.0(0.1)

$$10^{10}\times a_\mu^{\text{LO-HVP}} = 707.5(2.3)_{\text{stat}}(5.0)_{\text{sys}}[5.5]_{\text{tot}}$$

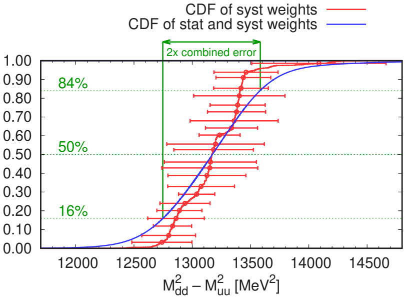
Global analysis

expand observable Y around physical point:

$$Y = A + BX_l + CX_s + DX_{\delta m} + Ee_v^2 + Fe_v e_s + Ge_s^2$$

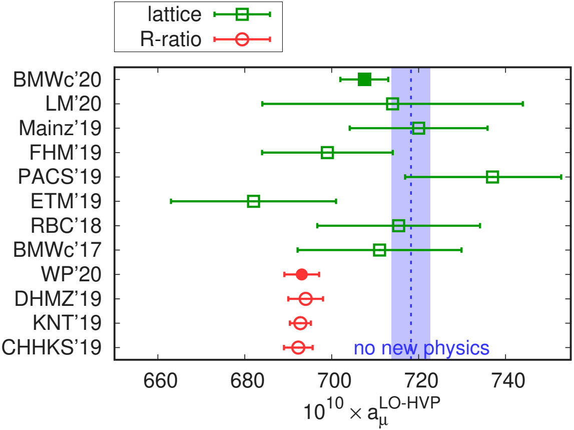
B,C: light/strange quark dependence; D: strong isospin breaking; E,F,G: sea/valence electric charge

- determine $A, B, \ldots$ from χ^2-fit to all ensembles
- isospin breaking derivatives can be incorporated easily



Many thousands of fits depending on measurement procedure, functional form of $A, B, \ldots$ and so on. Build weighted histogram and read off central value and error (stat and syst).

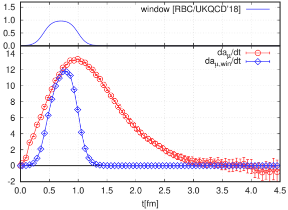
Final result

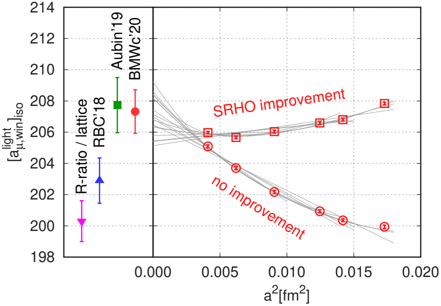


- $a_\mu^{\text{LO-HVP}} = 707.5(2.3)(5.0)[5.5]$ with 0.8% accuracy
- consistent with FNAL and 1.5σ away from BNL+FNAL
- 2.0σ larger than [DHMZ19], 2.5σ than [KNT19]

Crosscheck - window

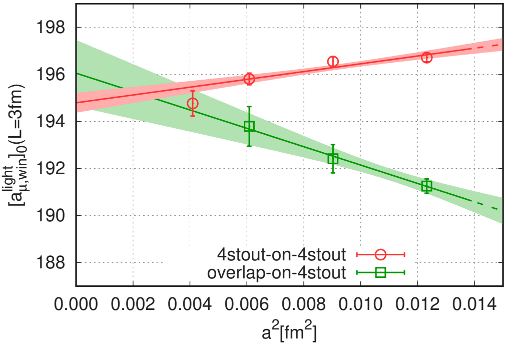
- restrict correlator to window
  0.4 – 1.0 fm [RBC/UKQCD'18]
- fewer difficulties





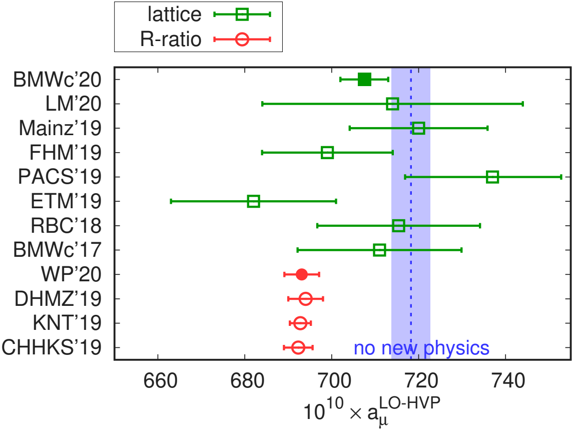
good agreement w/ other lattice, but 3.7σ tension with R-ratio

Crosscheck - overlap



- compute $a_{\mu,\text{win}}$ with overlap valence
- local current instead of conserved $\rightarrow$ had to compute Z_V
- cont.limit in $L = 3$ fm box consistent w/ staggered valence

Final result



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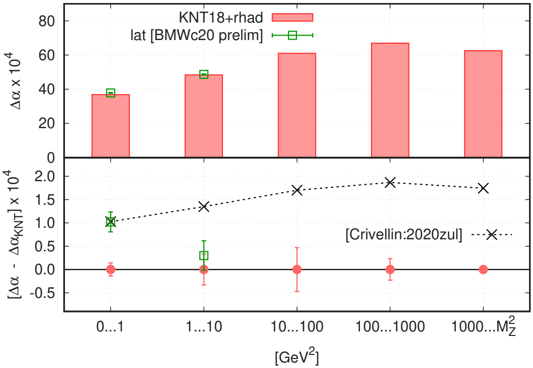
How about EWP?



- **1st bin:** same relative deviation (2.8%) as in a_μ or $a_{\mu,\mathrm{win}}$
- [Crivellin et al'20] assumes 2.8% dev. in all(!) bins $\rightarrow 4.2\sigma$ discrepancy with EWP
- **2nd bin:** already consistent with R-ratio

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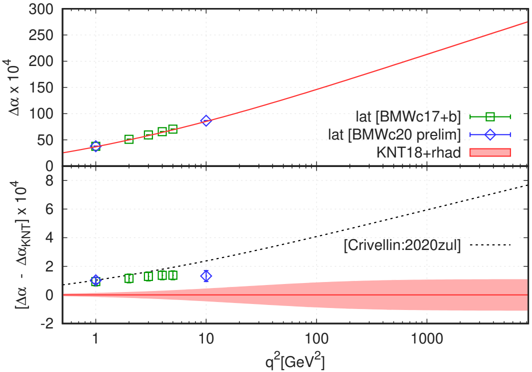

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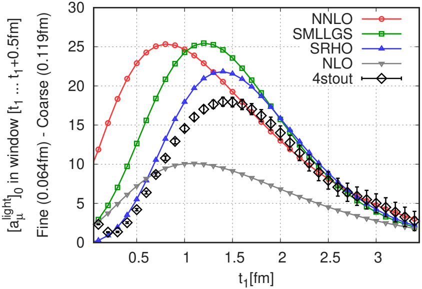
Continuum limit

large lattice artefacts due to staggered taste violations, can be described by

1. NNLO SXPT based on [Lee,Sharpe,VandeWater,Bailey $\geq$'98] $\rightarrow$ depends only on 1 LEC (l_6) and masses of pion tastes
2. SRHO [Jegerlehner,Szafron'11;HPQCD'16] $\rightarrow$ depends on the mass and decay parameters of rho
3. SMLLGS [Meyer,Lellouch,Luscher,Gounaris,Sakurai...]



for $t \gtrsim 2.0$ fm reproduce discretization effects well (not fits)