Cold QCD at RHIC: Theory Topics

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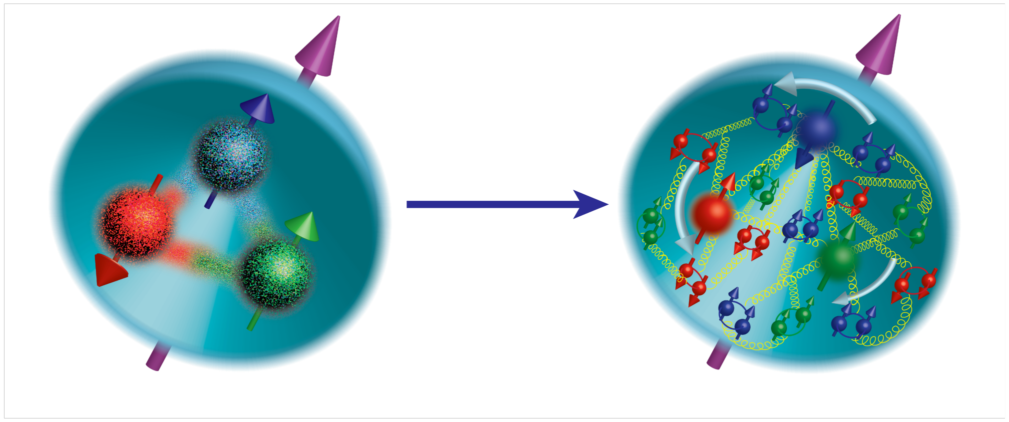
Outline

- Longitudinal spin:
  - proton spin puzzle,
  - gluon spin,
  - spin at small-x and a possible path toward spin puzzle resolution.

- Transverse spin:
  - Sivers function sign reversal,
  - STSA (A_N),
  - transversity.

Longitudinal Spin

Proton Spin

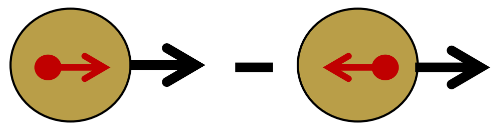


Our understanding of nucleon spin structure has evolved:

- In the 1980's the proton spin was thought of as a sum of constituent quark spins (left panel)
- Currently we believe that the proton spin is a sum of the spins of valence and sea quarks and of gluons, along with the orbital angular momenta of quarks and gluons (right panel)

Helicity Distributions

- To quantify the contributions of quarks and gluons to the proton spin on defines helicity distribution functions: number of quarks/gluons with spin parallel to the proton momentum minus the number of quarks/gluons with the spin opposite to the proton momentum:



- The helicity parton distributions are

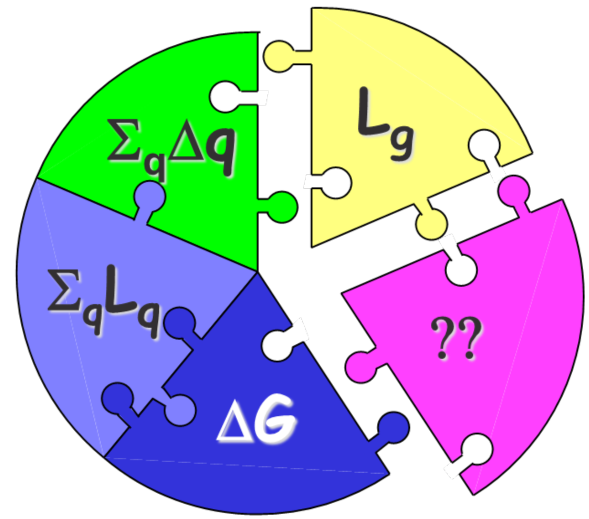
$$\Delta f(x,Q^2) \equiv f^+(x,Q^2) - f^-(x,Q^2)$$

with the net quark helicity distribution

$$\Delta\Sigma \equiv \Delta u + \Delta\bar{u} + \Delta d + \Delta\bar{d} + \Delta s + \Delta\bar{s}$$

and $\Delta G(x,Q^2)$ the gluon helicity distribution.

Proton Helicity Sum Rule



- Helicity sum rule (Jaffe&Manohar, 1989):

$$\frac{1}{2} = S_q + L_q + S_g + L_g$$

with the net quark and gluon spin

$$S_q(Q^2) = \frac{1}{2} \int_0^1 dx\, \Delta\Sigma(x, Q^2) \qquad S_g(Q^2) = \int_0^1 dx\, \Delta G(x, Q^2)$$

- L_q and L_g are the quark and gluon orbital angular momenta (OAM)

Proton Spin Puzzle

- The spin puzzle began when the EMC collaboration measured the proton g_1 structure function ca 1988. Their data resulted in

$$S_q \approx 0.05$$

- It appeared (constituent) quarks do not carry all of the proton spin (which would have corresponded to $S_q = 1/2$).

$$\frac{1}{2} = S_q + L_q + S_g + L_g$$

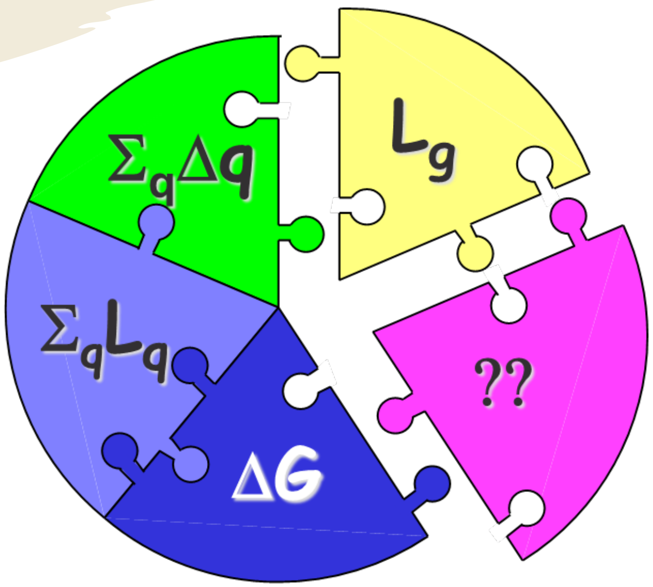
- Missing spin can be
  - Carried by gluons
  - In the orbital angular momenta of quarks and gluons
  - At small x:

$$S_q(Q^2) = \frac{1}{2}\int_0^1 dx\, \Delta\Sigma(x, Q^2) \qquad S_g(Q^2) = \int_0^1 dx\, \Delta G(x, Q^2)$$

Can't integrate down to zero, use x_{min} instead!

  - Or all of the above!

Current Knowledge of Proton Spin



- The proton spin carried by the quarks is estimated to be (for $0.001 < x < 1$)

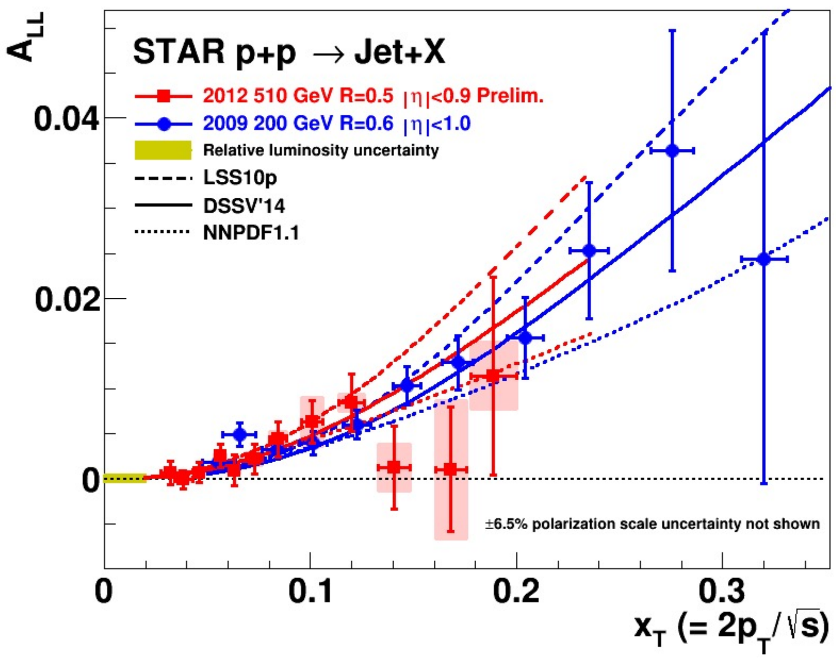
$$S_q(Q^2 = 10\,\mathrm{GeV}^2) \approx 0.15 \div 0.20$$

- The proton spin carried by the gluons is (for $0.05 < x < 1$, STAR+PHENIX+COMPASS +HERMES+… , analyzed by DSSV, JAM, …)

$$S_G(Q^2 = 10\,\mathrm{GeV}^2) \approx 0.13 \div 0.26$$

- Unfortunately, the uncertainties are large. Note also that the x-ranges are limited, with more spin (positive or negative) possible at small x.

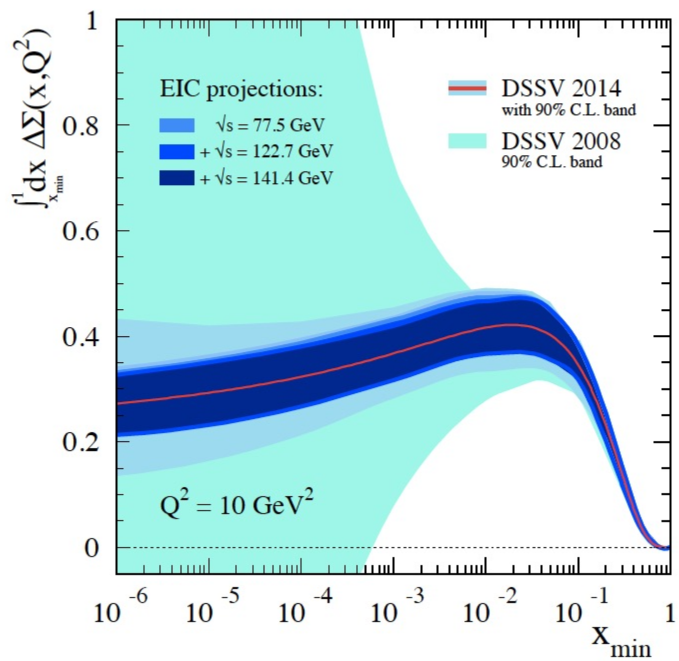
A_{LL} Measurements

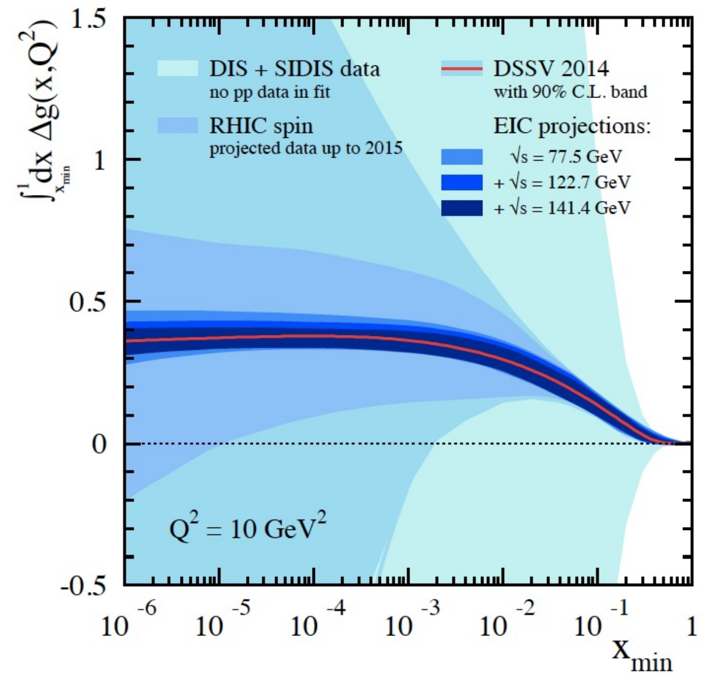


ΔG can be measured in A_{LL} for jets and neutral pions, but may also be measured in A_{LL} for charged pions and direct photons, though with poorer statistics

To me the non-zero ΔG measurement appears as the most important result of RHIC spin program.

How much spin is at small x?





- E. Aschenaur et al, arXiv:1509.06489 **[hep-ph]**
- Uncertainties are very large at small x!

$$\frac{1}{2} = S_q + L_q + S_g + L_g$$

Small-x Spin Challenge

- Can we constrain theoretically the amount of proton spin and OAM coming from small x?

- Any existing and future experiment probes the helicity distributions and OAM down to some x_{min} .

$$S_q(Q^2) = \frac{1}{2} \int\limits_0^1 dx \, \Delta\Sigma(x, Q^2) \qquad S_g(Q^2) = \int\limits_0^1 dx \, \Delta G(x, Q^2)$$

$$L_{q+\bar{q}}(Q^2) = \int\limits_0^1 dx \, L_{q+\bar{q}}(x, Q^2) \qquad L_G(Q^2) = \int\limits_0^1 dx \, L_G(x, Q^2)$$

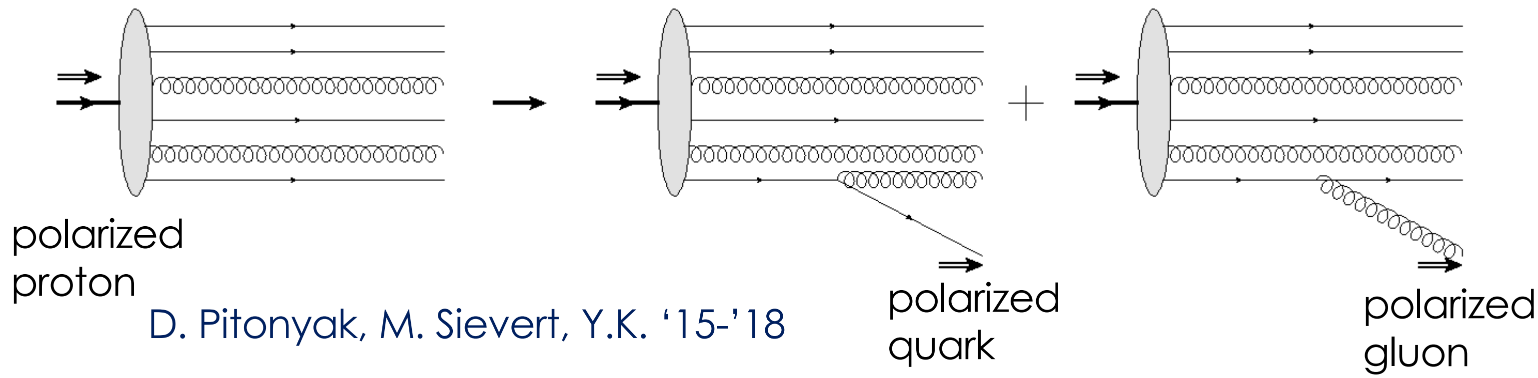
- At very small x (for the proton), saturation sets in: that region likely carries a negligible amount of proton spin. But what happens at larger (but still small) x?

Philosophy of our approach

- DGLAP equation evolves in Q^2, it does not evolve in x.
- Hence, DGLAP-based analyses (DSSV, NNPDF, standard JAM) cannot predict the x-dependence of PDFs.
- If we want to predict helicity PDFs at small x, we need a different evolution equation evolving in x.
- Such equations were constructed by D. Pitonyak, M. Sievert, and YK, 2015-2018 using an approach similar to the BK/JIMWLK evolution.

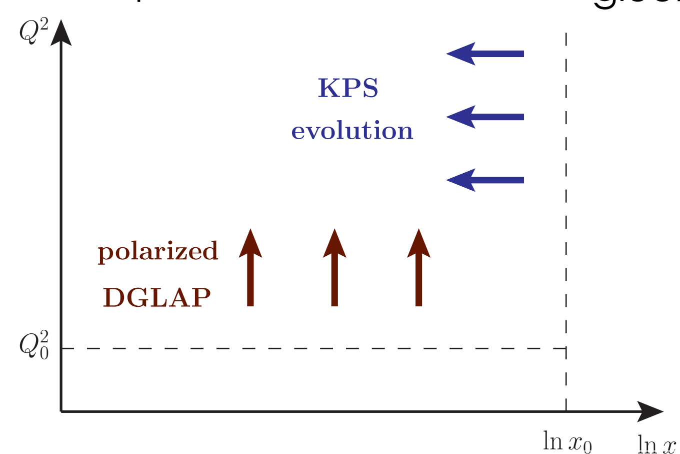
Helicity Evolution at Small x

- To understand how much of the proton's spin is at small x one can construct a helicity analogue of the BFKL equation:



D. Pitonyak, M. Sievert, Y.K. '15-'18

- This new helicity evolution equation is subtle, since it must keep track of both quark and gluon helicities. (BFKL/BK/JIMWLK only have gluons at leading order.)



Large-N_c Evolution

Resummation parameter: $\alpha_s \ln^2 \frac{1}{x}$

Double-logarithmic approximation (DLA)

- In the strict DLA limit and at large N_c, we get (here Γ is an auxiliary function we call the 'neighbor dipole amplitude') (KPS '15)

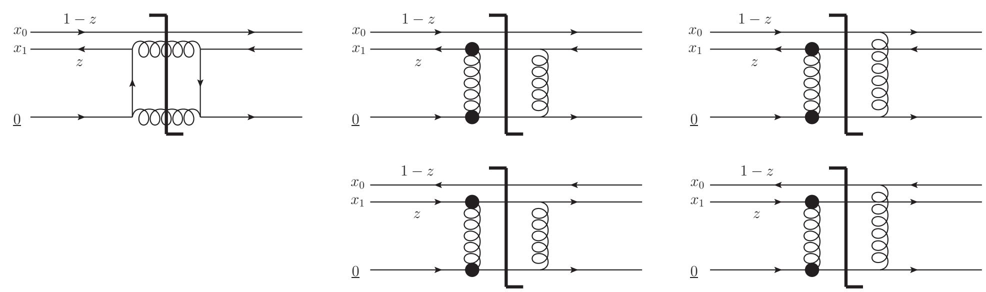
$$G(x_{10}^2, z) = G^{(0)}(x_{10}^2, z) + \frac{\alpha_s N_c}{2\pi} \int\limits_{\frac{1}{x_{10}^2 s}}^{z} \frac{dz'}{z'} \int\limits_{\frac{1}{z' s}}^{x_{10}^2} \frac{dx_{21}^2}{x_{21}^2} \left[\Gamma(x_{10}^2, x_{21}^2, z') + 3\, G(x_{21}^2, z')\right]$$

$$\Gamma(x_{10}^2, x_{21}^2, z') = \Gamma^{(0)}(x_{10}^2, x_{21}^2, z') + \frac{\alpha_s N_c}{2\pi} \int\limits_{\frac{1}{x_{10}^2 s}}^{z'} \frac{dz''}{z''} \int\limits_{\frac{1}{z'' s}}^{\min\left\{x_{10}^2, x_{21}^2 \frac{z'}{z''}\right\}} \frac{dx_{32}^2}{x_{32}^2} \left[\Gamma(x_{10}^2, x_{32}^2, z'') + 3\, G(x_{32}^2, z'')\right]$$

- The initial conditions are given by the Born-level graphs

$$\Gamma^{(0)}(x_{10}^2, x_{21}^2, z) = G^{(0)}(x_{10}^2, z)$$

$$G^{(0)}(x_{10}^2, z) = \frac{\alpha_s^2 C_F}{N_c} \pi \left[C_F \ln \frac{zs}{\Lambda^2} - 2\, \ln(zs\, x_{10}^2)\right]$$



(Adamiak, Melnitchouk, Pitonyak, Sato, Sievert, YK, 2102.06159 [hep-ph] = JAMsmallx)

Small-x Polarized DIS Data

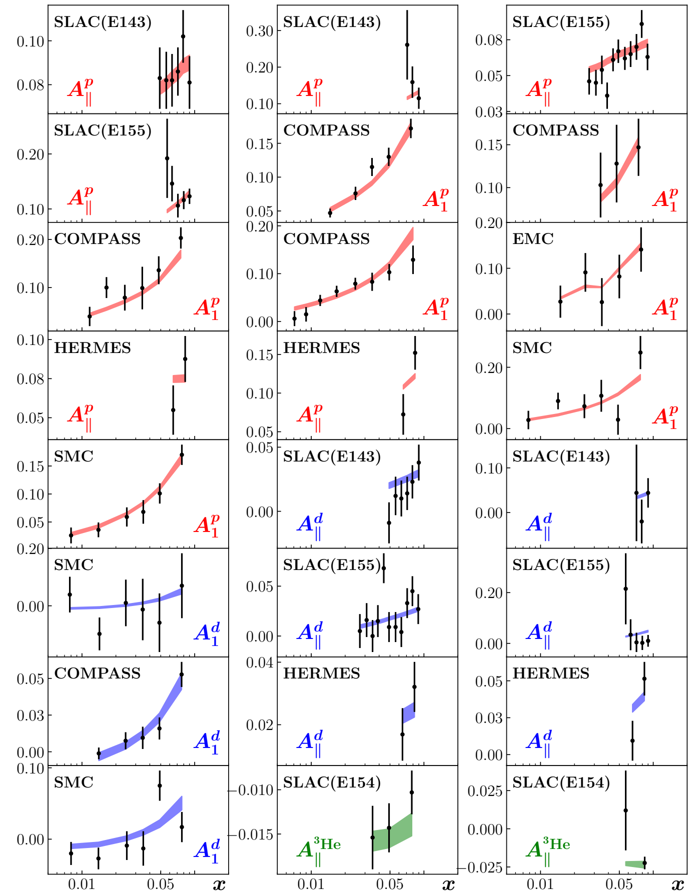
$$A_1 \sim A_{\|} = \frac{\sigma_{+-} - \sigma_{++}}{\sigma_{+-} + \sigma_{++}} \sim \frac{g_1}{F_1}$$

- We have analyzed all existing world polarized DIS data with $x<0.1=x_0$, $Q^2 > m_c^2$ (122 data points) using the large-N_C KPS evolution with the Born-inspired initial conditions (8 parameters for 2 flavors, 11 parameters for 3 flavors).

$$G^{(0)}(x_{10}^2, z) \propto a_q \ln \frac{zs}{\Lambda^2} + b_q \ln \frac{1}{x_{10}^2 \Lambda^2} + c_q$$

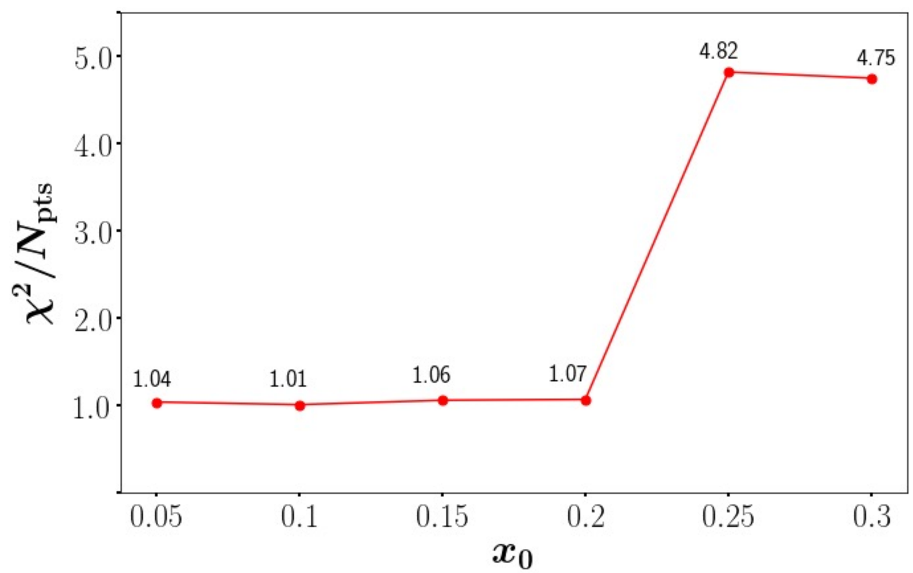
- It worked well, with χ^2/N_{pts} =1.01 (cf. JAM16: χ^2/N_{pts} =1.07)

- Small-x evolution starts at x_0=0.1 ! (cf. x_0=0.01 for unpolarized BK/JIMWLK evolution) Our approach fails at larger x as expected (x_0=0.3 gives χ^2/N_{pts} =4.75).

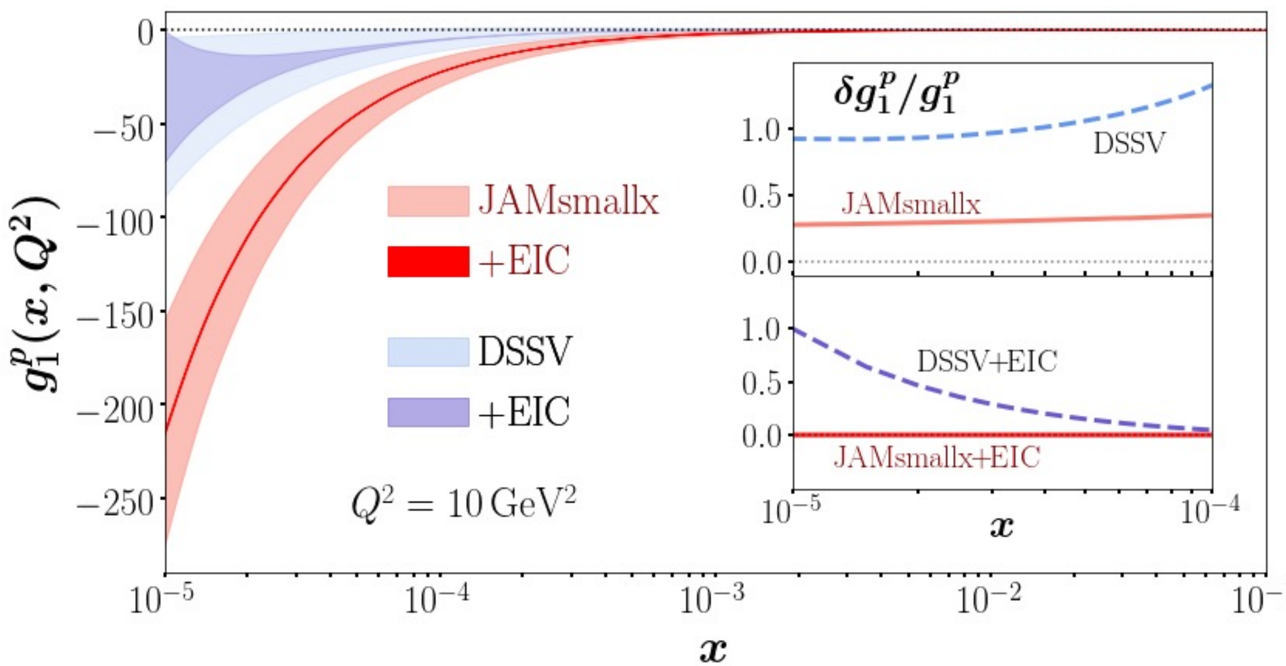


Where to start small-x evolution

- The evolution starts at $x=x_0$, and continues toward smaller x.
- The quality of our fit rapidly deteriorates for $x_0>0.2$, as expected from a small-x approach.
- In unpolarized BK/JIMWLK evolution, typically $x_0=0.01$, so the fact that our fit works up to such a high x_0 is quite remarkable.



Prediction for g_1 structure function

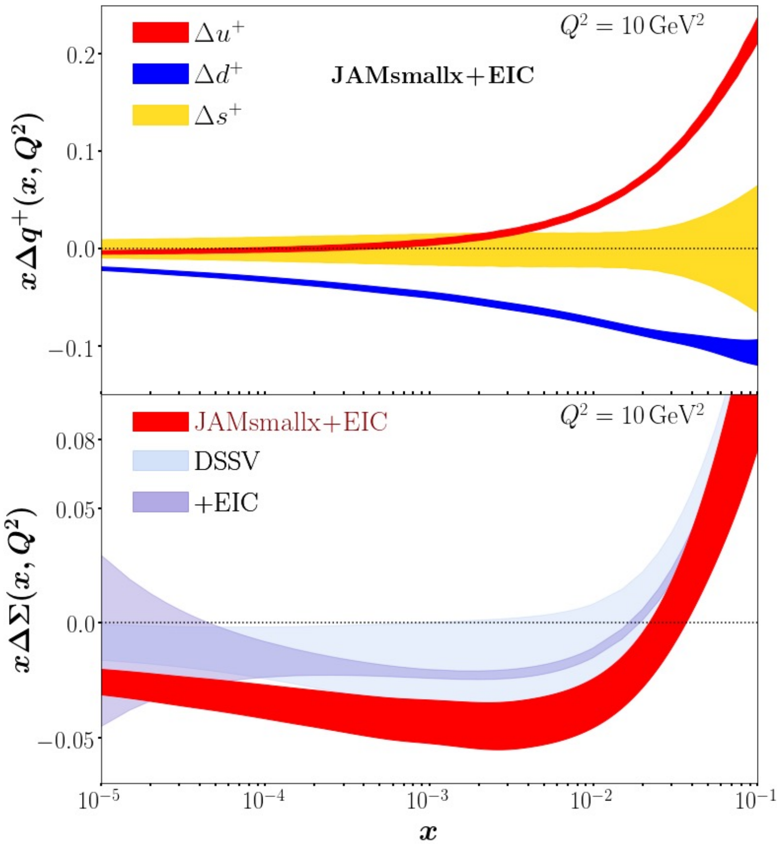


g_1 structure function ≈ spin-dependent part of σ^{e+p}

$$g_1(x,Q^2) = \frac{1}{2}\sum_f e_f^2\left[\Delta q_f + \Delta q_{\bar{f}}\right]$$

Thick band: 1σ CL; thin band: impact of EIC data. With the EIC pseudo-data we have 1096 data points.

Predictions for helicity PDFs



Our (red) error band does not explode in the unmeasured region.

D. Adamiak, W. Melnitchouk, D. Pitonyak, N. Sato, M. Sievert & YK, 2102.06159 [hep-ph], in the JAM Collaboration framework.

$$\Delta q^+ = \Delta q + \Delta \bar{q}$$

$$\Delta\Sigma(x, Q^2) = \sum_f \left[\Delta q_f + \Delta q_{\bar{f}}\right]$$

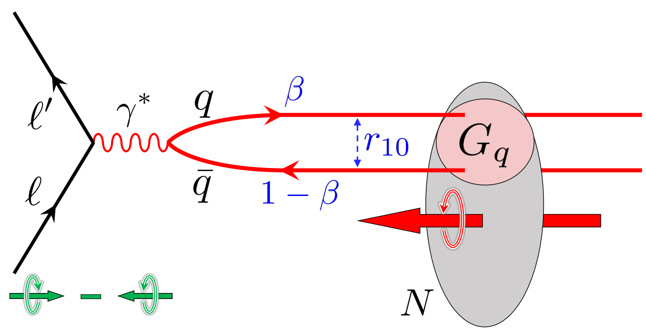
If we plug in $\alpha_S = 0.25$ we get $\alpha_h^q = 0.80$

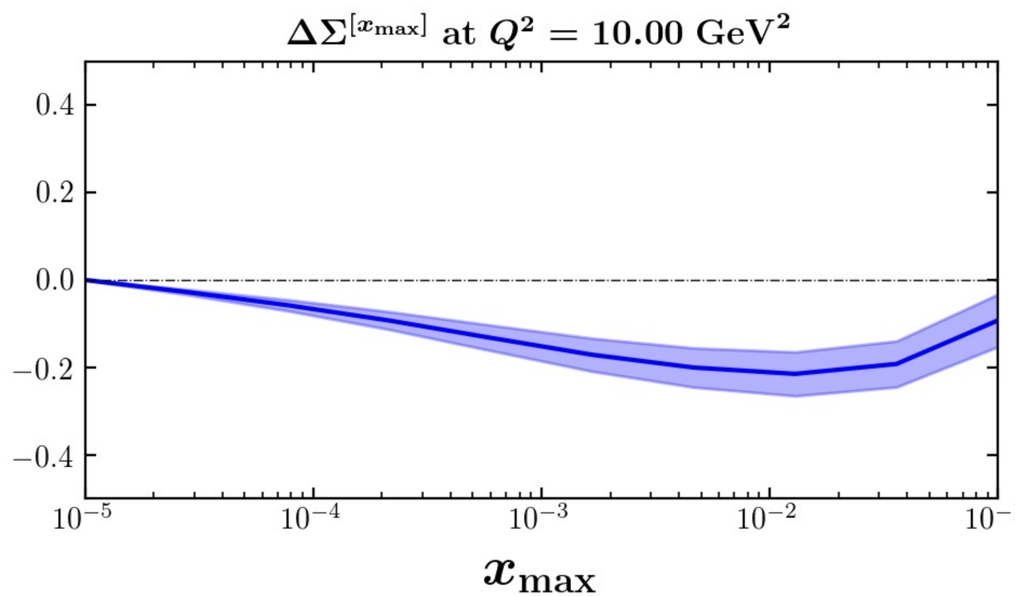
$$\Delta q(x, Q^2) \sim \left(\frac{1}{x}\right)^{\alpha_h^q} \quad \text{with} \quad \alpha_h^q = \frac{4}{\sqrt{3}}\sqrt{\frac{\alpha_s N_c}{2\pi}} \approx 2.31\sqrt{\frac{\alpha_s N_c}{2\pi}}$$

Small-x quarks impact on the proton spin

- Potentially negative 10-20% of the proton spin may be carried by small-x quarks helicity (JAMsmallx, preliminary):

$$\Delta\Sigma^{[x_{max}]}(Q^2) = \int_{10^{-5}}^{x_{max}} dx\, \Delta\Sigma(x, Q^2)$$





Speculation on a path to resolving the spin puzzle

- Above we discussed quark helicity at small x. Let's add the orbital angular momentum (OAM) (Hatta & Yang, '18; YK '19):

$$\frac{1}{2}\,\Delta\Sigma(x,Q^2) + L_{q+\bar{q}}(x,Q^2) = -\frac{1}{2}\,\Delta\Sigma(x,Q^2)$$

- So, the net quark (1/2) helicity+OAM = (-1/2) helicity.
- For x<0.001 we thus expect (preliminary!)

$$\left[\frac{1}{2}\Delta\Sigma + L_{q+\bar{q}}\right]_{Q^2=10\,\mathrm{GeV}^2,\ x<0.001} \approx -\frac{1}{2}\,(-0.2) = 0.1$$

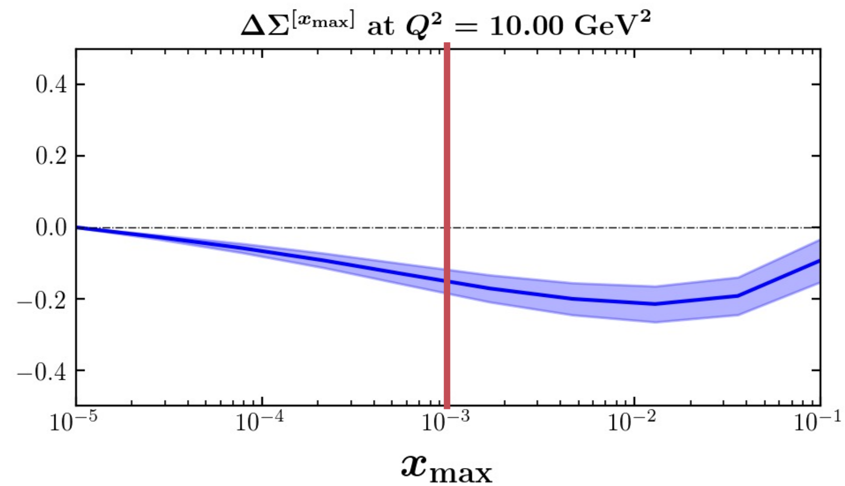
- Add to this the larger-x numbers

$$S_q(Q^2 = 10\,\mathrm{GeV}^2, x > 0.001) \approx 0.18$$
$$S_G(Q^2 = 10\,\mathrm{GeV}^2, x > 0.05) \approx 0.2$$

- We get

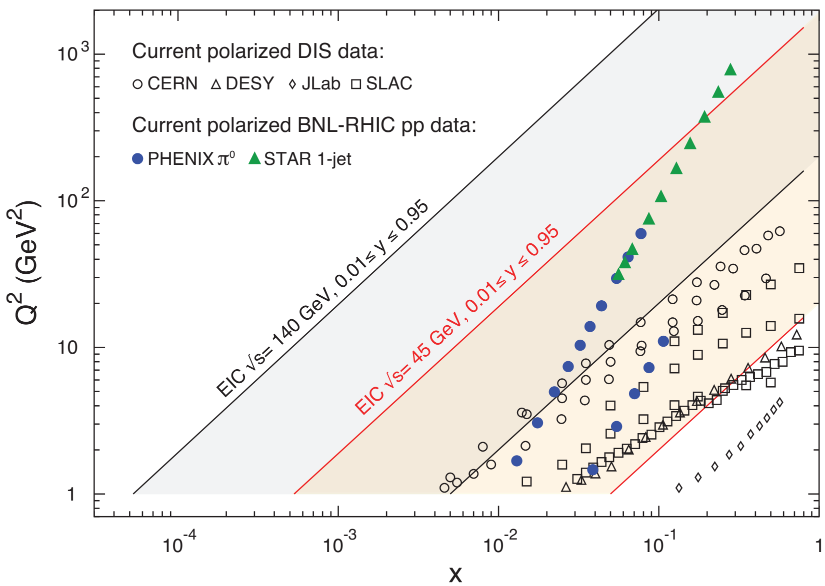
$$0.18 + 0.2 + 0.1 = 0.48$$

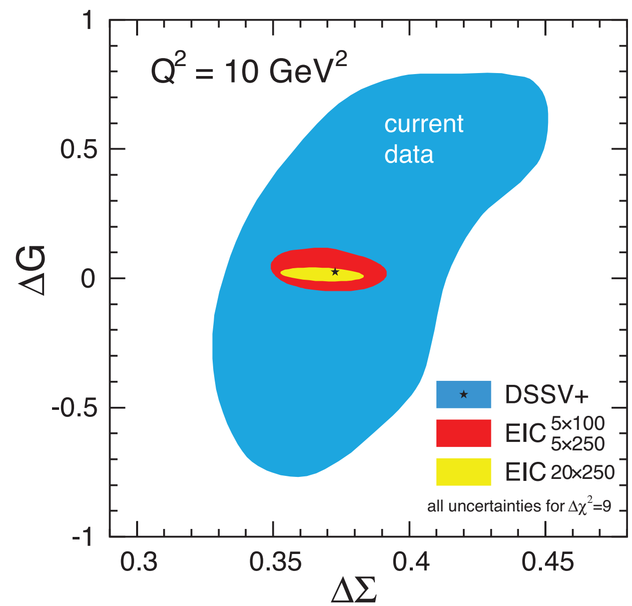
JAMsmallx, preliminary, Adamiak, Melnitchouk, Pitonyak, Sato, Sievert, YK



EIC & Spin Puzzle

- Parton helicity distributions are sensitive to low-x physics.
- EIC would have an unprecedented low-x reach for a polarized DIS experiment, allowing to pinpoint the values of quark and gluon contributions to proton's spin:





- ΔG and $\Delta\Sigma$ are integrated over x in the $0.001 < x < 1$ interval.

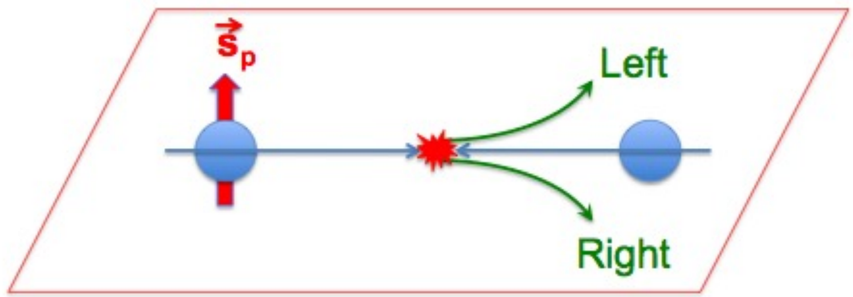
Transverse Spin

Single Transverse Spin Asymmetry & Sign Reversal

Single Transverse Spin Asymmetry

- Consider transversely polarized proton scattering on an unpolarized proton or nucleus.

$$p(\vec{s}_\perp) + p \to h(\pi^\pm, \pi^0, ...) + X$$



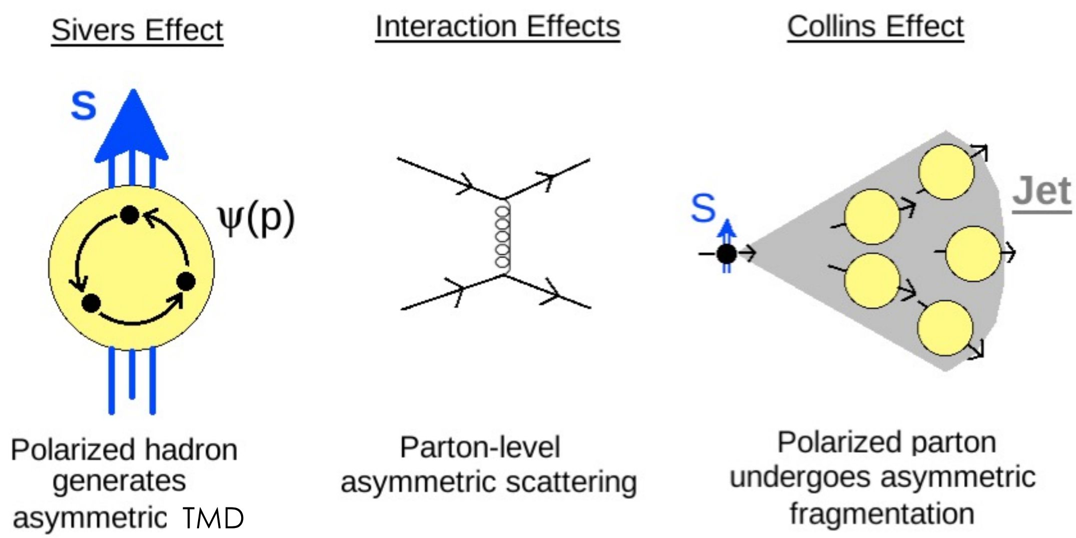
- Single Transverse Spin Asymmetry (STSA) is defined by

$$A_N(\mathbf{k}) \equiv \frac{\frac{d\sigma^\uparrow}{d^2k\,dy} - \frac{d\sigma^\downarrow}{d^2k\,dy}}{\frac{d\sigma^\uparrow}{d^2k\,dy} + \frac{d\sigma^\downarrow}{d^2k\,dy}} = \frac{\frac{d\sigma^\uparrow}{d^2k\,dy}(\mathbf{k}) - \frac{\mathbf{d}\sigma^\uparrow}{\mathbf{d^2k\,dy}}(-\mathbf{k})}{\frac{d\sigma^\uparrow}{d^2k\,dy}(\mathbf{k}) + \frac{\mathbf{d}\sigma^\uparrow}{\mathbf{d^2k\,dy}}(-\mathbf{k})} \equiv \frac{d(\Delta\sigma)}{2\,d\sigma_{unp}}$$

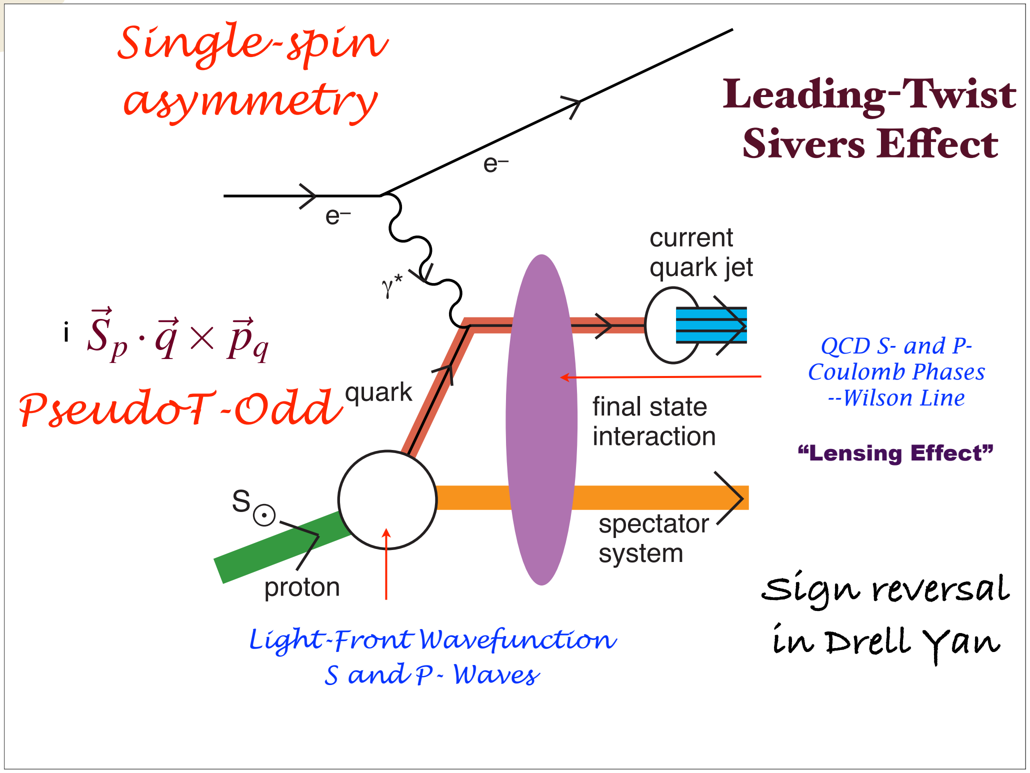
Theoretical Explanations (TMD factorization framework)

The origin of STSA is in

- Transverse spin-dependent TMD (Sivers effect)
- Transverse spin-dependent fragmentation (Collins effect)
- Hard scattering (small effect)



STSA IN SIDIS

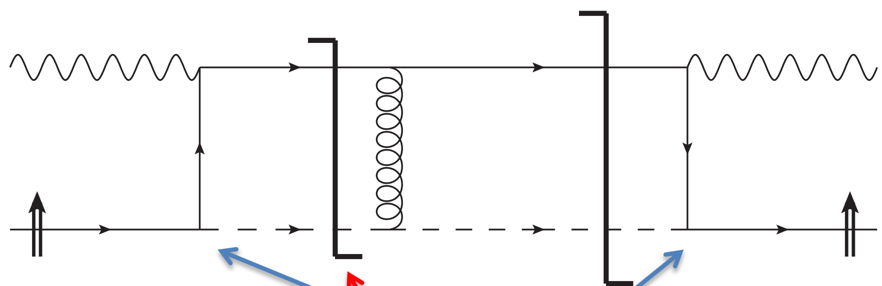


Brodsky, Hwang, Schmidt '02; Collins '02

- To generate STSA need a final state interaction (the blob above) -- lensing.
- In TMD factorization this is usually absorbed into the polarized proton TMD and is referred to as the initial-state effect, and hence identified with the Sivers function.

STSA in SIDIS

- STSA arises from the interference diagrams between Born-level and the one-rescattering graphs:



- Spin-dependence comes from the vertex. This brings in an “i”. Need another “i” to get a real cross-section. Hence, need a phase.

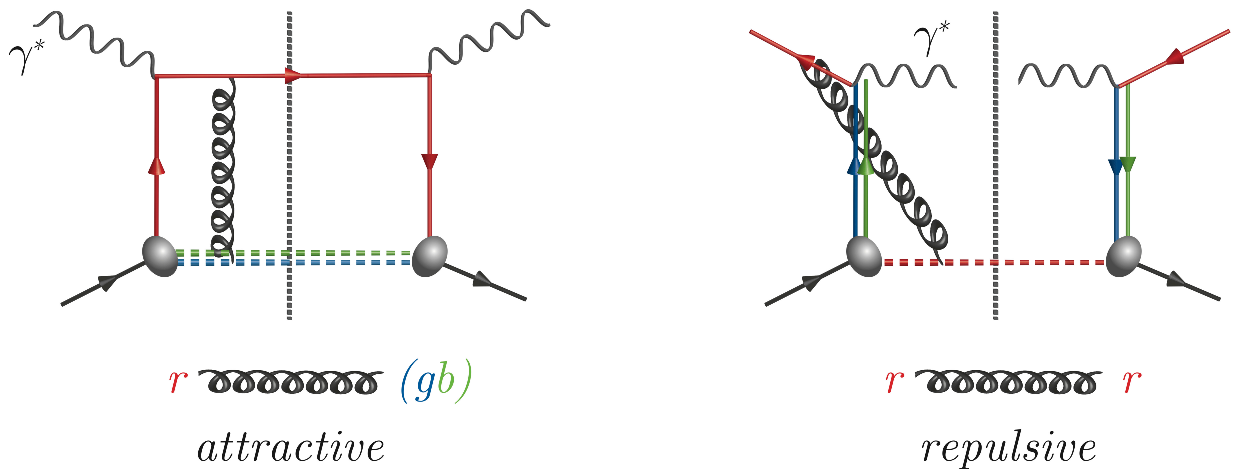
- The phase is generated by an extra rescattering, which gives the amplitude an Im part represented by the second “cut”.

Brodsky, Hwang, Schmidt ’02;
(see also Brodsky, Hwang, YK, Schmidt, Sievert ’13)

Sivers effect sign reversal

A fundamental prediction of TMD factorization:

$$f_{1T}^{\perp\,SIDIS}(x,k_T^2) = -f_{1T}^{\perp\,DY}(x,k_T^2)$$



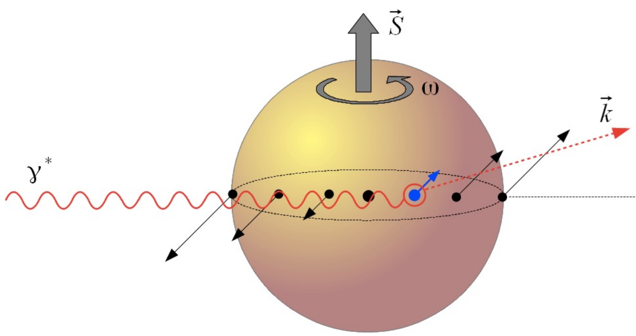
Collins '02; Brodsky, Hwang, Schmidt '02;
Brodsky, Hwang, YK, Schmidt, Sievert '13 (redo).

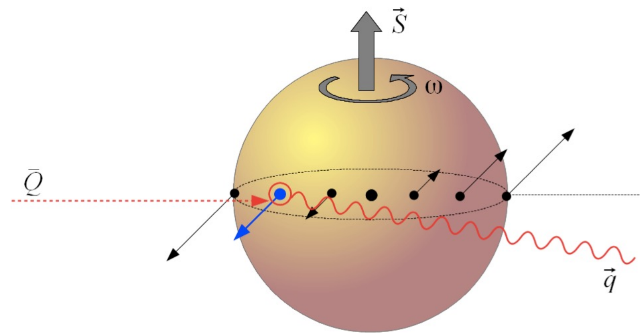
Origin of Sign Reversal

$$f_{1T}^{\perp A}(x, k_T)\Big|_{SIDIS} = -f_{1T}^{\perp A}(x, k_T)\Big|_{DY}$$

- In the transversity (Sivers density) channel the origin of the sign reversal is simple: the (LO) Sivers function of the nucleon changes sign, and multiple rescatterings do not affect this.

- In the OAM channel the reversal ultimately happens for the simple reason pictured here:

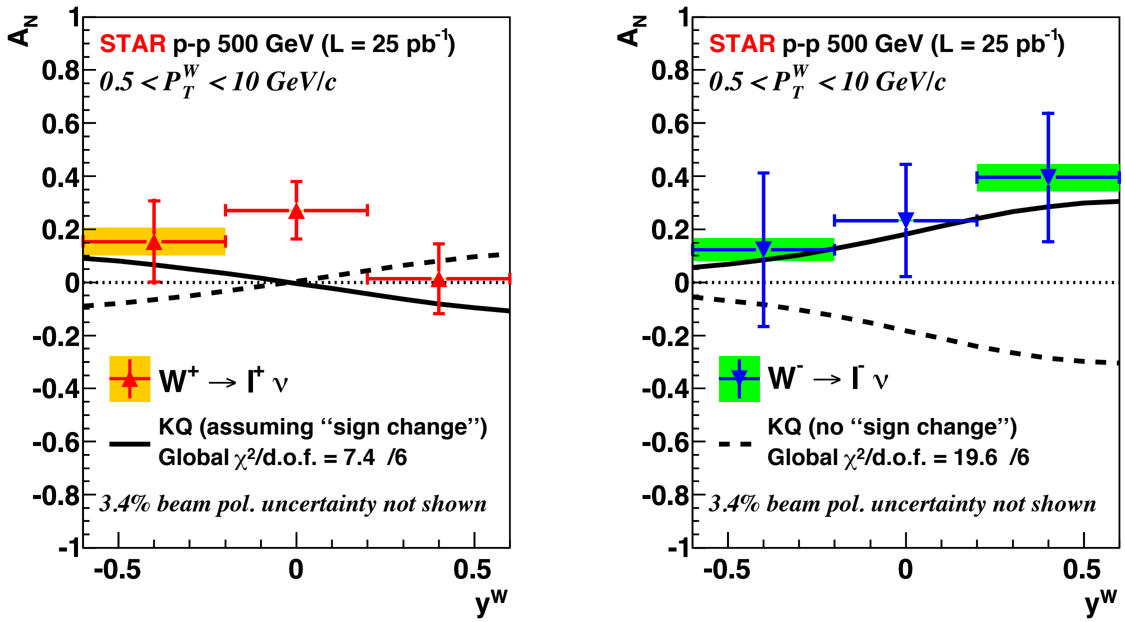




YK, Sievert, '13

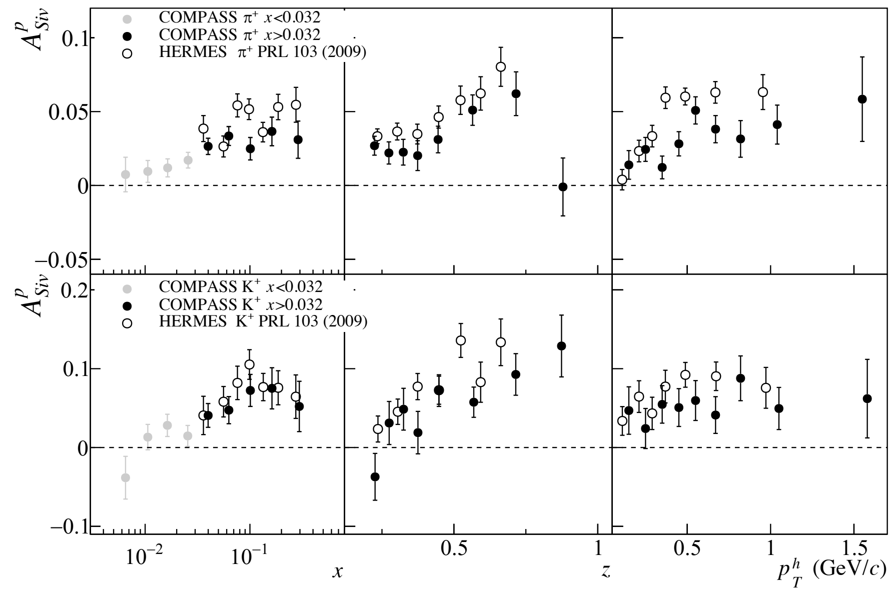
SIDIS vs DY Data

- A_N in W/Z producton is positive, consistent with the sign reversal with respect to SIDIS scenario.

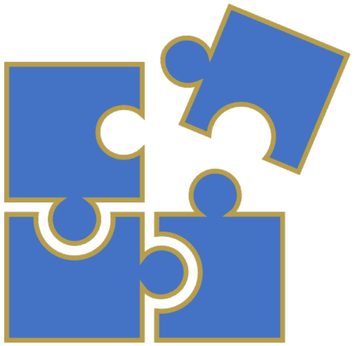


SIDIS vs DY Data

- In SIDIS, here's a compilation of HERMES and COMPASS data:



- Where is the sign reversal? Well, π^+ are sensitive to u, W^- are sensitive to d, u's and d's may have opposite-sign Sivers functions… So, it maybe OK.

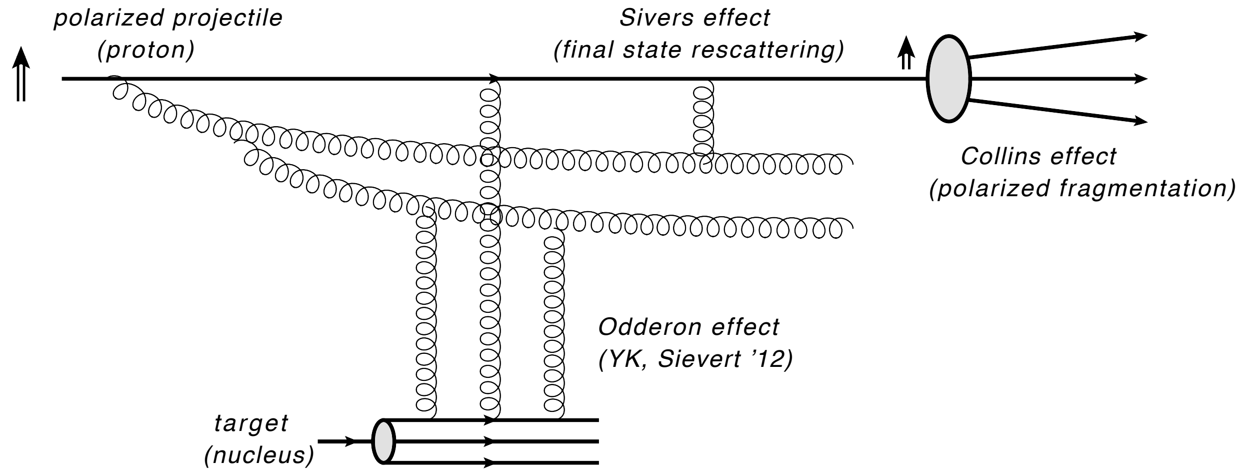


A_N Puzzles

Possible Origins of A_N

The single transverse spin asymmetry in $p^{\uparrow}$+p or $p^{\uparrow}$+A can be generated via the following mechanisms:

- Sivers effect – multiple rescattering in $p^{\uparrow}$+p – This work.
- Collins effect – fragmentation of a polarized quark
- Odderon – multiple rescattering in the unpolarized target



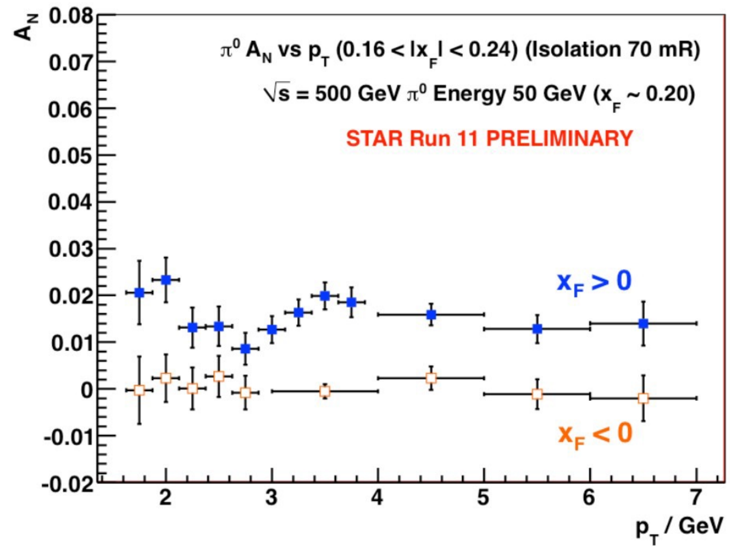
p_T Dependence of the A_N

- A_N does not seem to fall off with p_T!

- Naively all the multiple interactions are "higher twists": hence, A_N should fall off with p_T. This is also a prediction of the lensing mechanism described above.

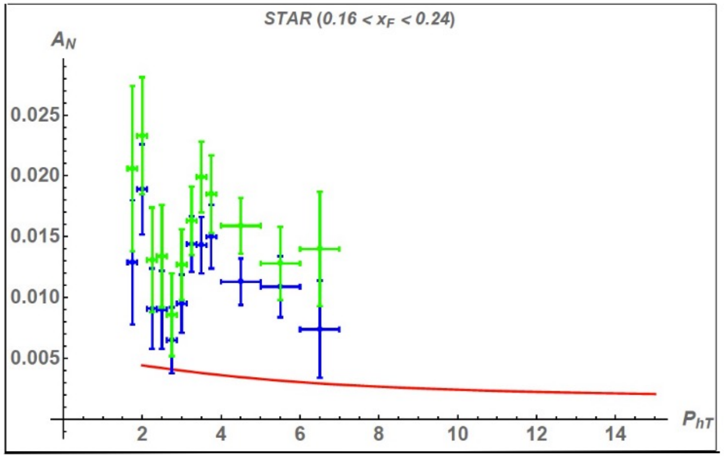
- Explanations?

- Metz, Pitonyak '12; Kanazawa, Koike, Metz, Pitonyak '14: higher twist effects in the fragmentation function for SIDIS. (see also Benic&Hatta '18)



p_T Dependence of the A_N

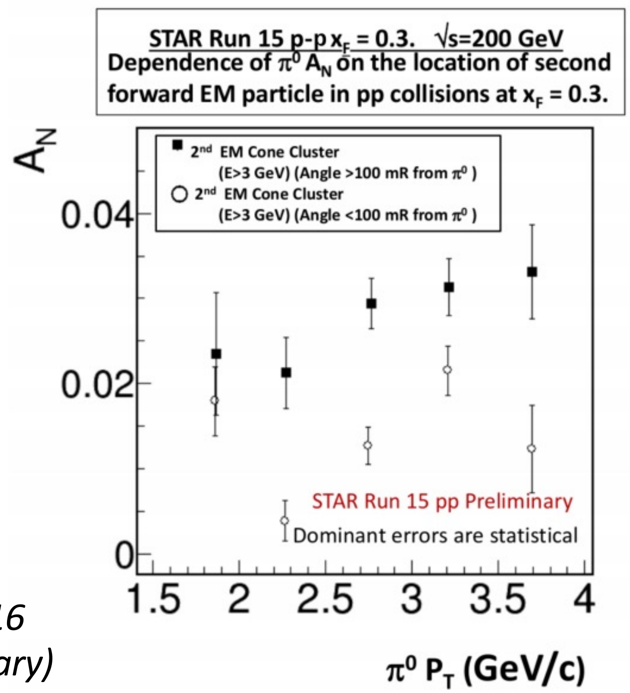
- For $p^{\uparrow}+p \to \pi+X$, the higher-twist approach was generalized by Gamberg, Kang, Pitonyak and Prokudin '17.
- Why no (or little) fall-off with p_T? Even if this is a higher-twist effect?

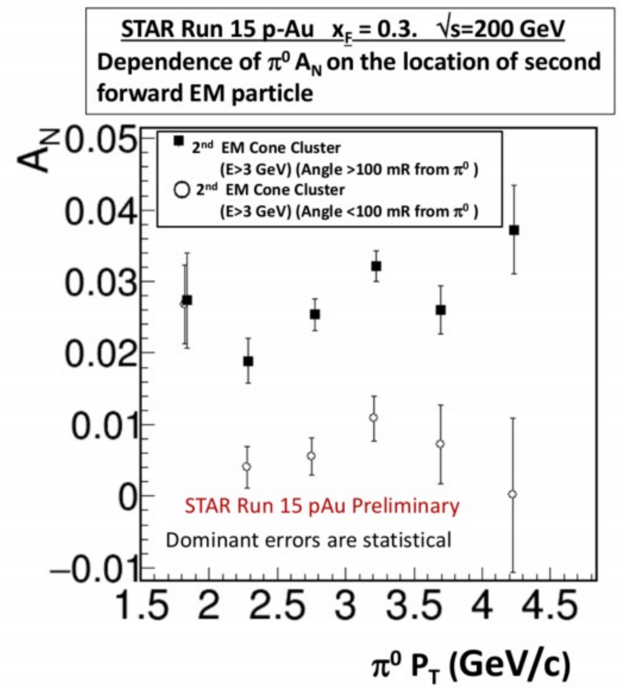


- Their explanation is that the hard function contains a positive power of p_T^2 : this makes the fit work, but is complicated theoretically, probably indicates factorization breakdown.

A_N in Elastic Processes

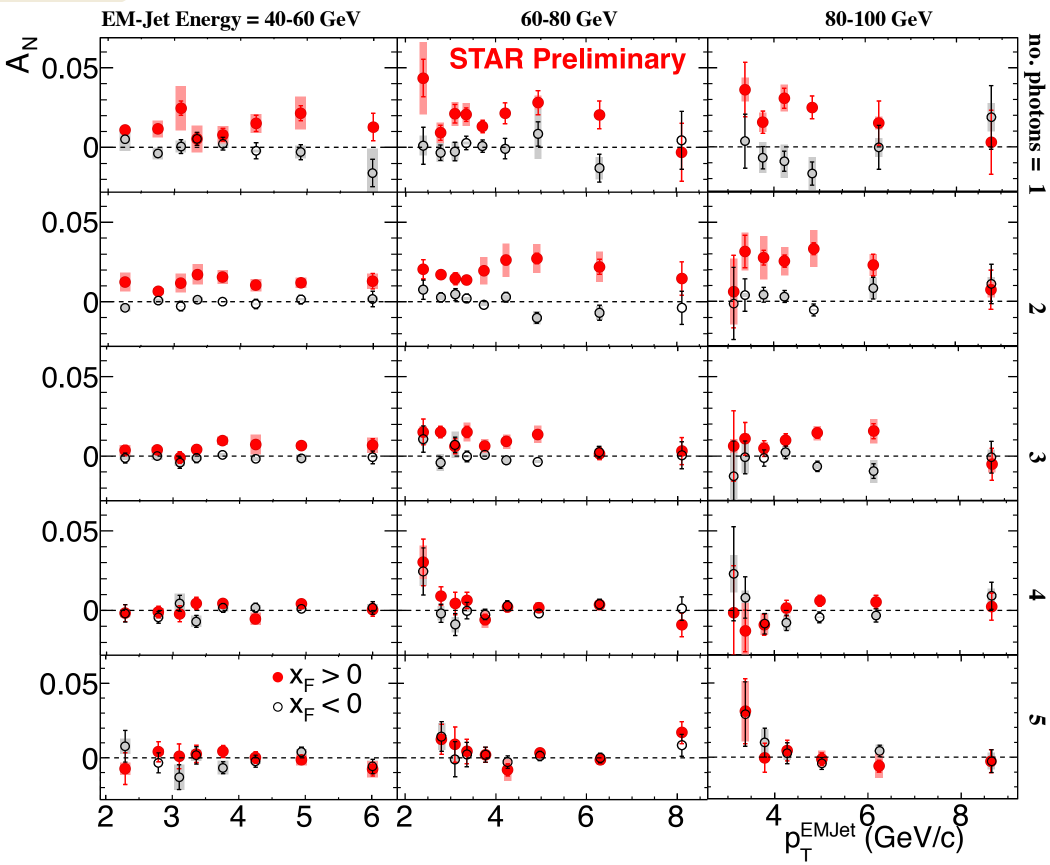
- The asymmetry is larger for elastic processes than for inelastic processes





STAR, 2016 (preliminary)

STSA is stronger in diffractive events

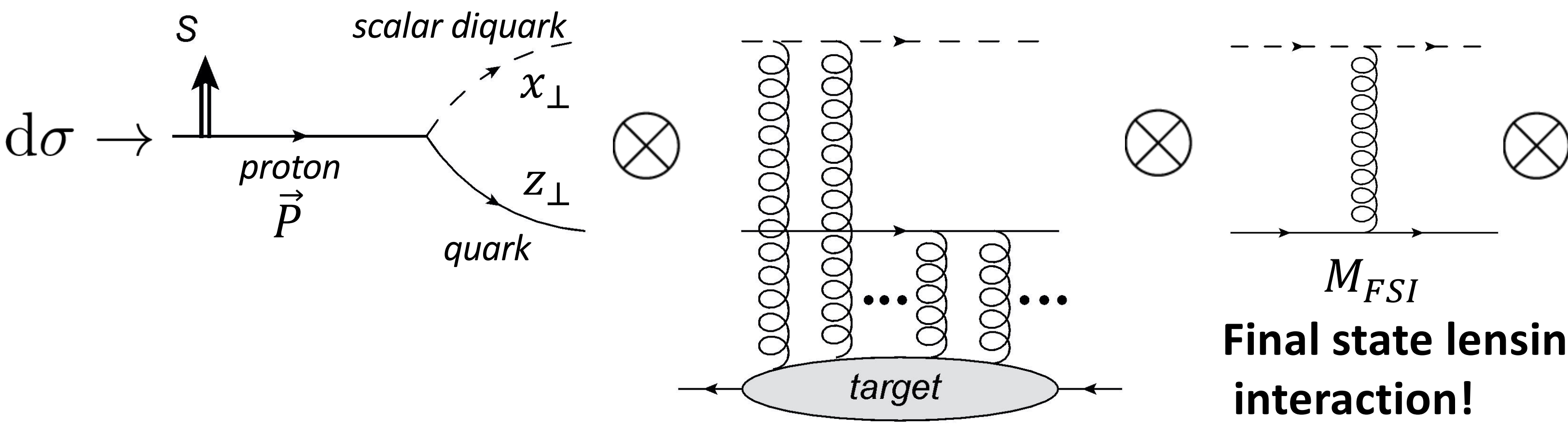


Does the standard TMD factorization have anything to say about this? I think not.
One would have to introduce diffractive TMDs (cf. diffractive PDFs) -- more ad hoc parameters.

Small-x Lensing Mechanism

- The phase needed for A_N may come from a final-state interaction:

M.G. Santiago, YK, 2003.12650 [hep-ph]

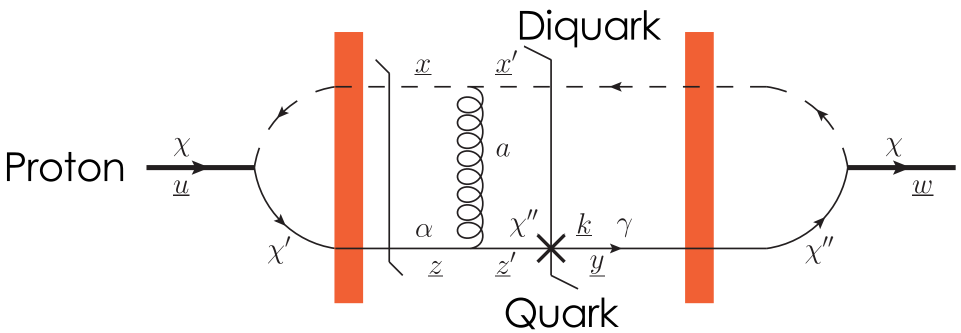


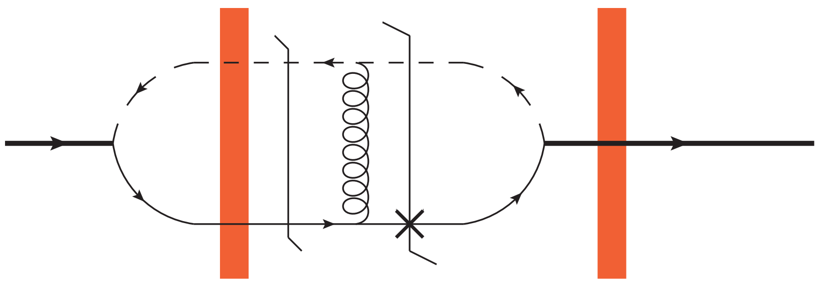
conjugate terms without M_{FSI}

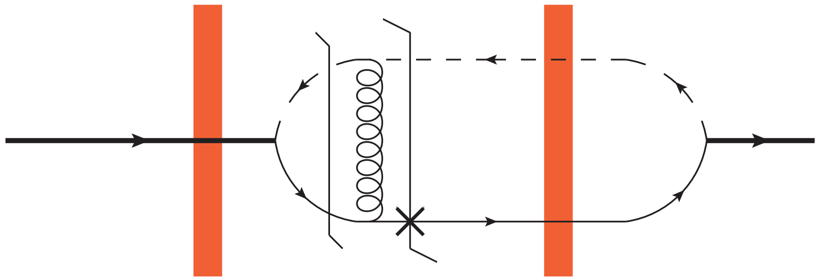
Final state lensing interaction!

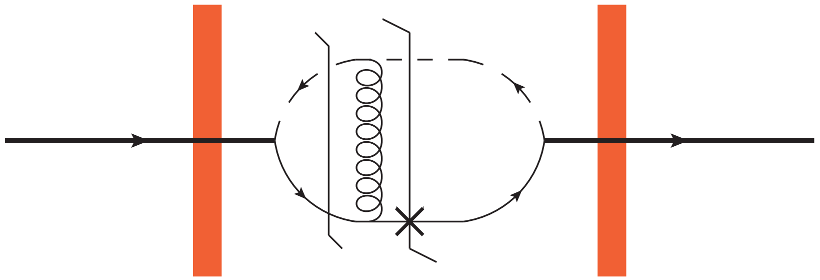
$$\psi(P,S) \otimes V_z V_x^\dagger \otimes M_{FSI} \quad V_x V_y^\dagger \psi^*(P,S)$$

Production process with lensing:







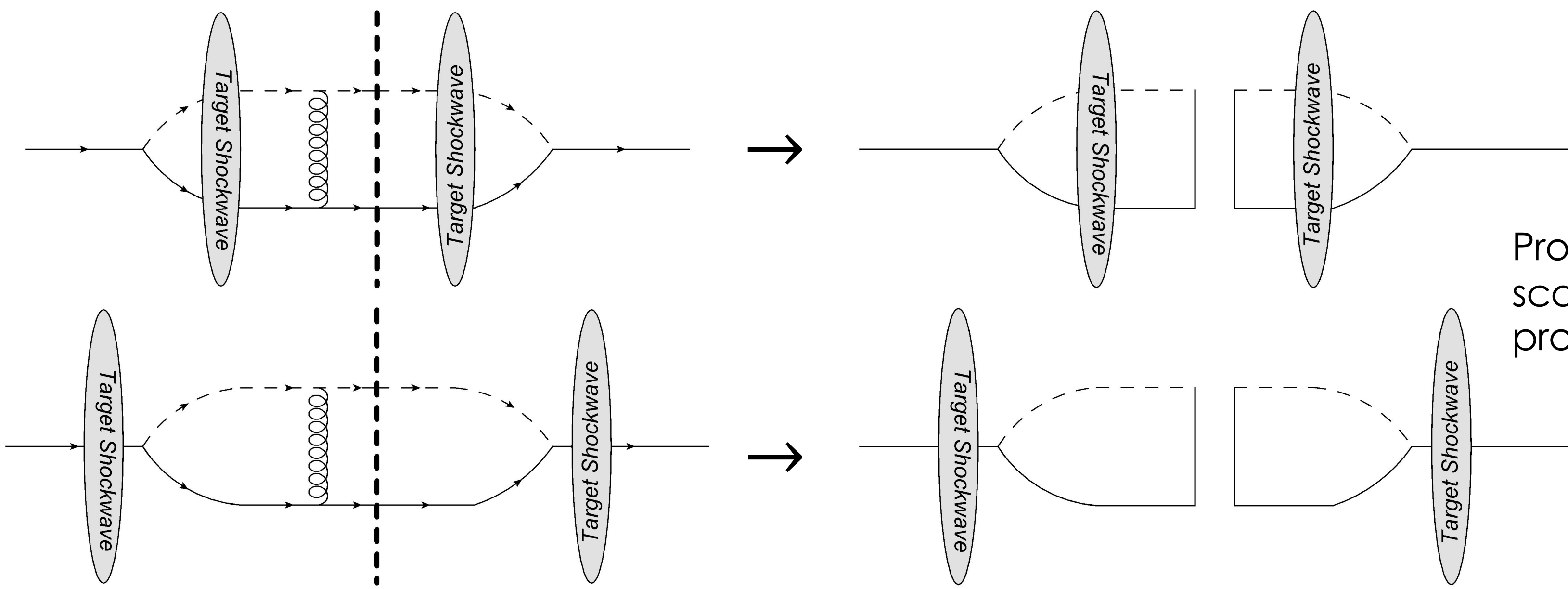


M.G. Santiago, YK, 2003.12650 [hep-ph]

Large-N_c Limit → Elastic Dominance

M.G. Santiago, YK, 2003.12650 [hep-ph]

- At large-N_c the gluon line becomes a double quark line



Product of elastic scattering processes!

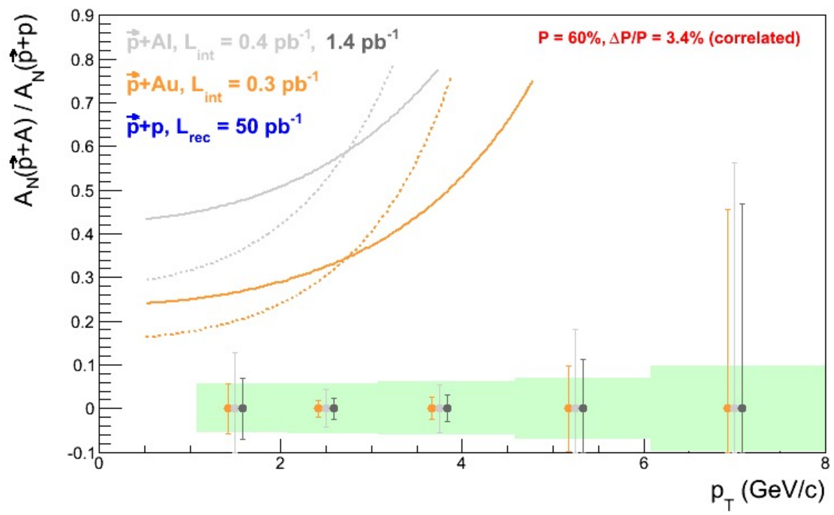
$$3 \otimes \bar{3} = 1 \oplus 8 \quad \Rightarrow \quad N_c \otimes \bar{N}_c = 1 \oplus (N_c^2 - 1) \approx N_c^2 - 1$$

A-dependence of A_N: Theory Predictions

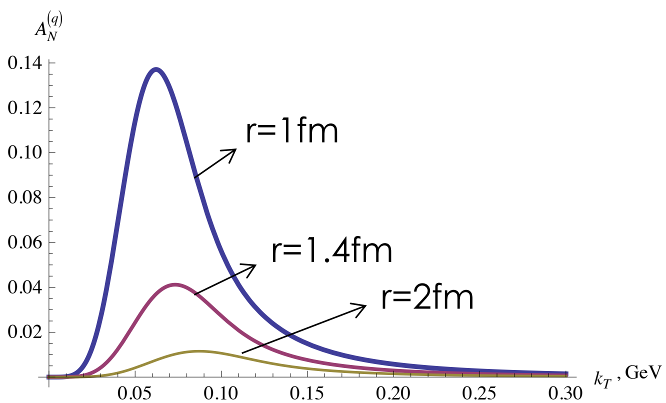
- Kang and Yuan '11: multiple rescattering in the nucleus ``wash out'' the asymmetry,

$$A_N^{p^\uparrow + A} \sim f_{1T}^{\perp}(x, k_T^2) \otimes e^{-k_T^2/Q_s^2}$$

p_T broadening in the nucleus



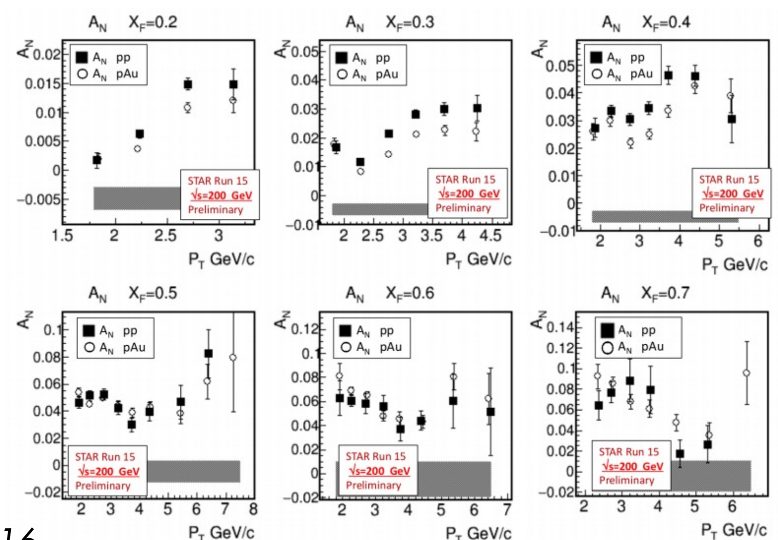
strong suppression of odderon STSA in nuclei.



- YK & Sievert '12: in the odderon exchange mechanism, multiple rescatterings in a nucleus should also quench the asymmetry.

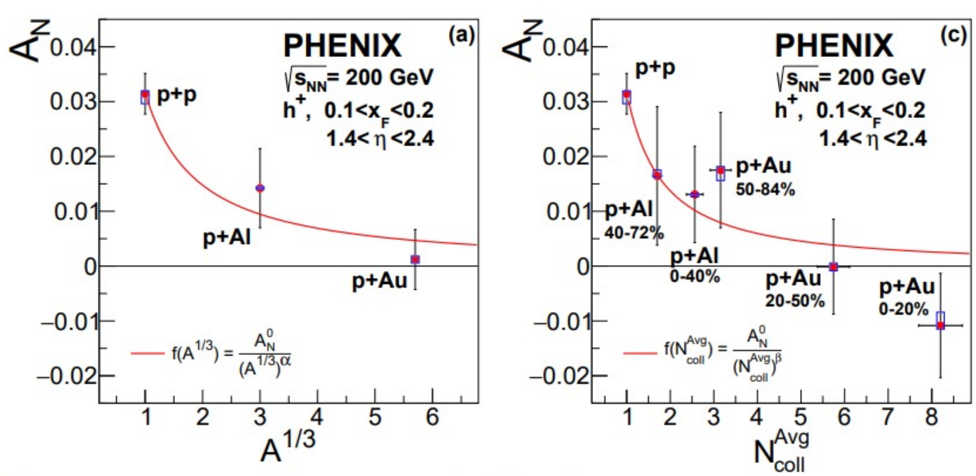
A-Dependence Puzzles

- The asymmetry has been observed to have different dependence on the target's atomic number A in different ranges of x_F



STAR, 2016 (preliminary)

STAR has observed an asymmetry which is roughly independent of A

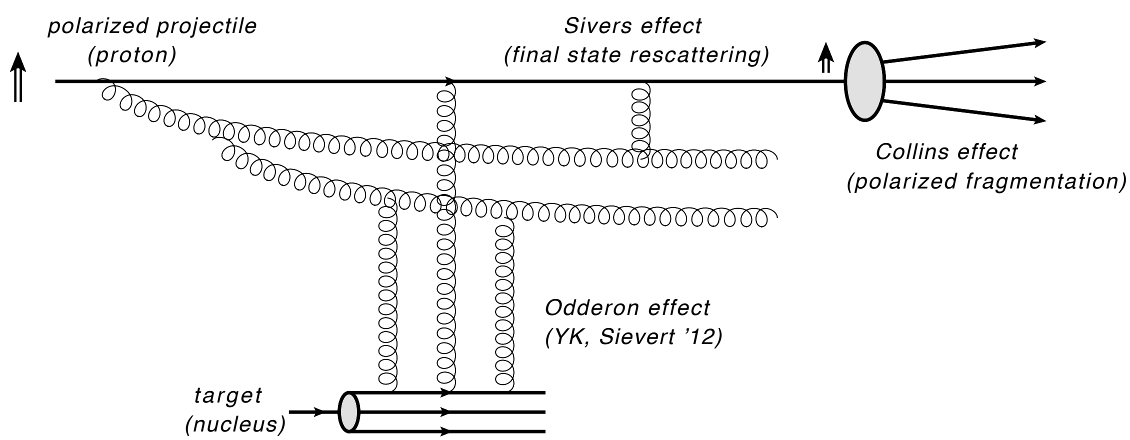


PHENIX, 2019

PHENIX has observed an asymmetry which is suppressed by increasing A

More Theory Predictions

- For a hybrid formalism calculation (PDF or TMD+saturation) see Hatta et al, '16. They have terms in A_N which do not exhibit any A-suppression and terms that do. (See also Boer et al '06, Kang&Xiao '12 , Schafer&Zhou '14, Zhou '17.)



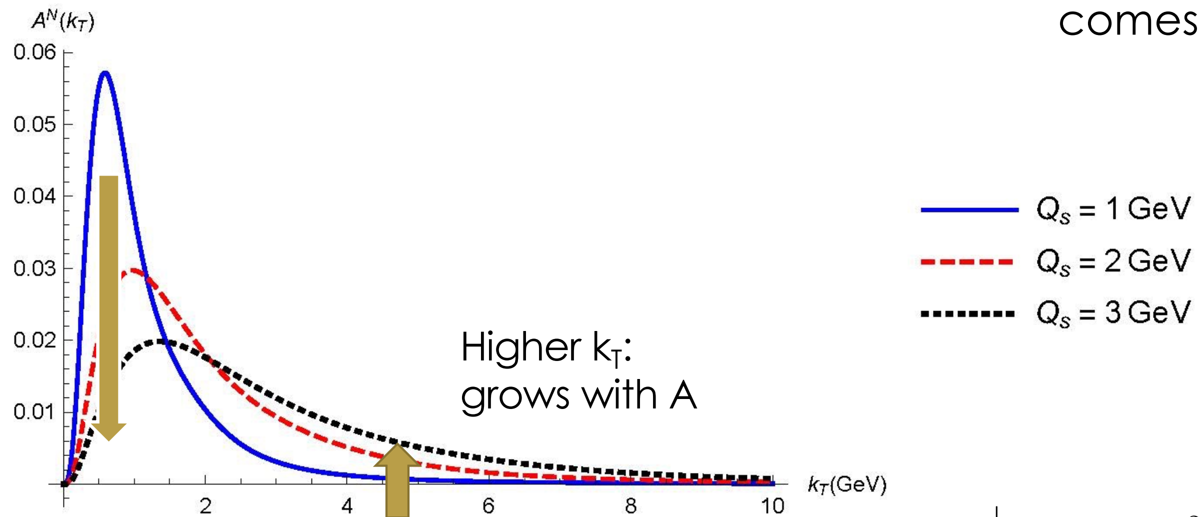
Large-N_C Approximation

M.G. Santiago, YK, 2003.12650 [hep-ph]

- The dipole amplitudes can be evaluated in the MV model
- We plot quark A_N for strong coupling $\alpha_s = 0.3$

A –dependence comes from $Q_s \sim A^{1/6}$

Low k_T: falls off with A



Higher k_T: grows with A

$$A_N(k_T, Q_s)\Big|_{M_P \ll k_T \ll Q_s} \sim \frac{k_T\, M_P}{Q_s^2}\, \ln \frac{Q_s}{M_p}$$

$$A_N(k_T, Q_s)\Big|_{k_T \gg Q_s} \sim \frac{Q_s^2\, M_P}{k_T^3}$$

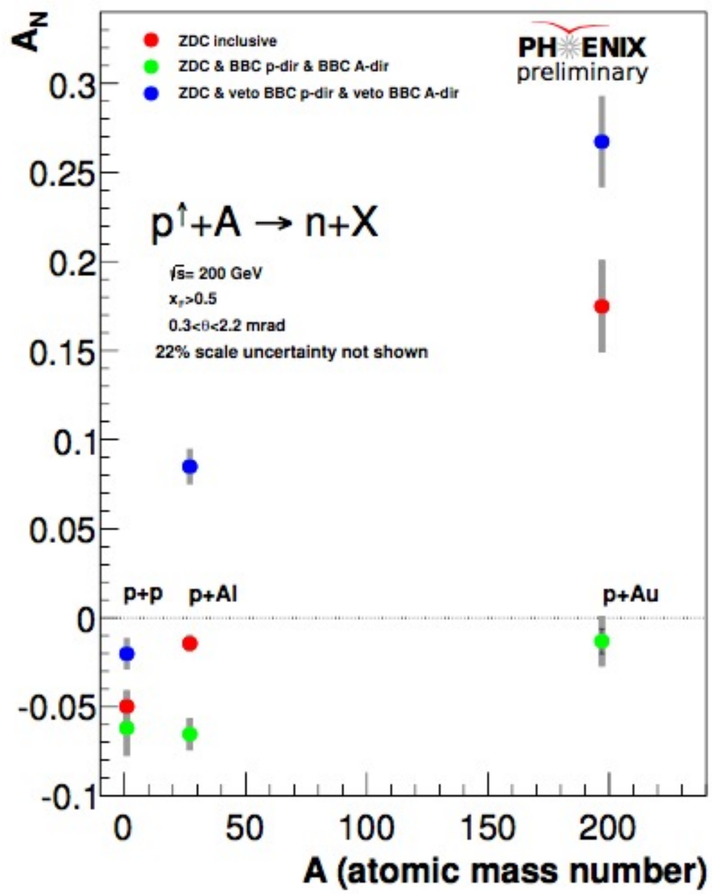
A_N for Neutrons

PHENIX '17 PRL:

A large A_N that grows with A, changing its sign in the process?

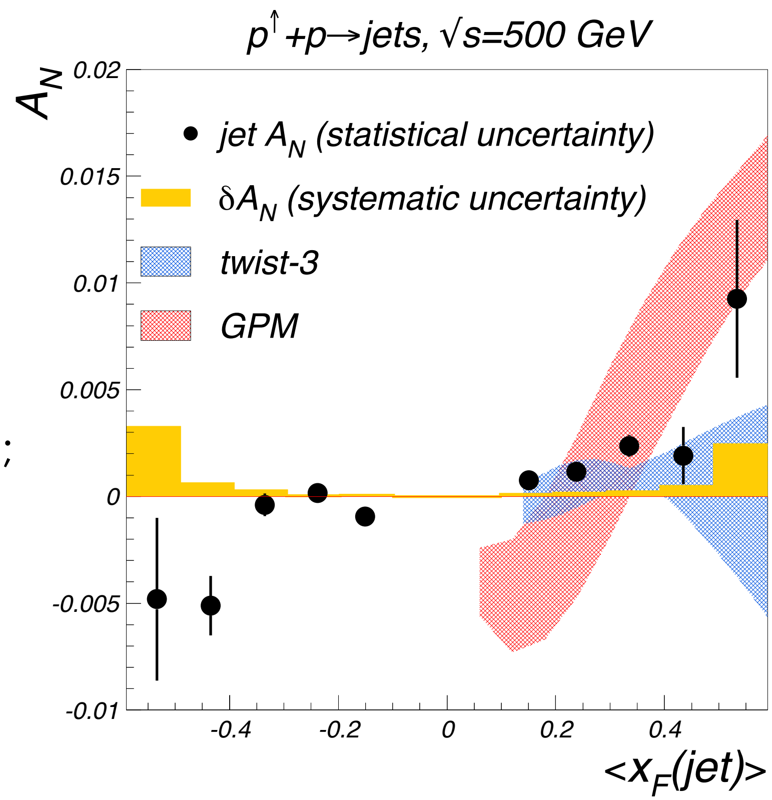
Typical p_T of those neutrons is not large, so probably not a perturbative mechanism.

Still, this is really puzzling…



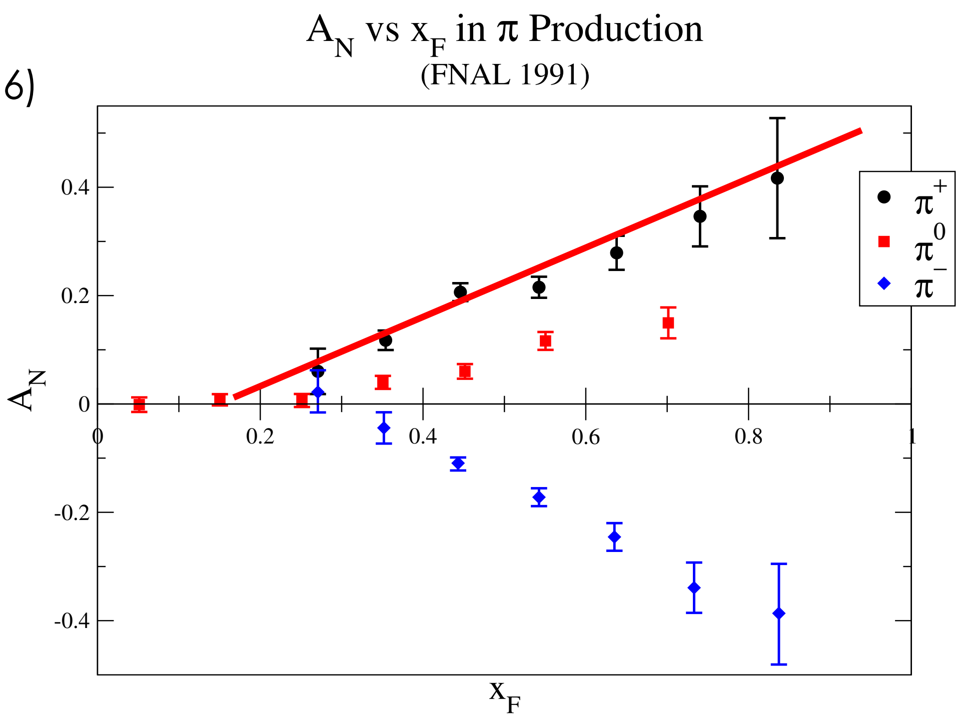
AnDY Experiment

- Non-zero transverse spin asymmetry in the backward direction?
- Spin-dependent odderon? (Gluon Sivers: Szymanowski & Zhou, '16; Boer et al, '15; Quark Sivers: Dong et al, '19; Santiago & YK, in preparation)
- A cross-check of this data would be useful.



x-Dependence of A_N

- The spin-dependent odderon (Zhou et al, '15-'16) predicts A_N=const(x).
- The data indicates more like A_N~x.
- Where is the spin-dependent odderon? What is behind the x-dependence of the data?



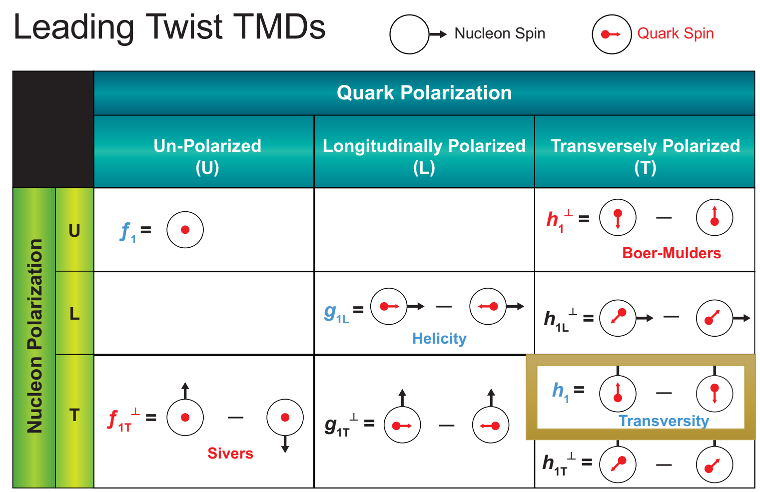
Transversity

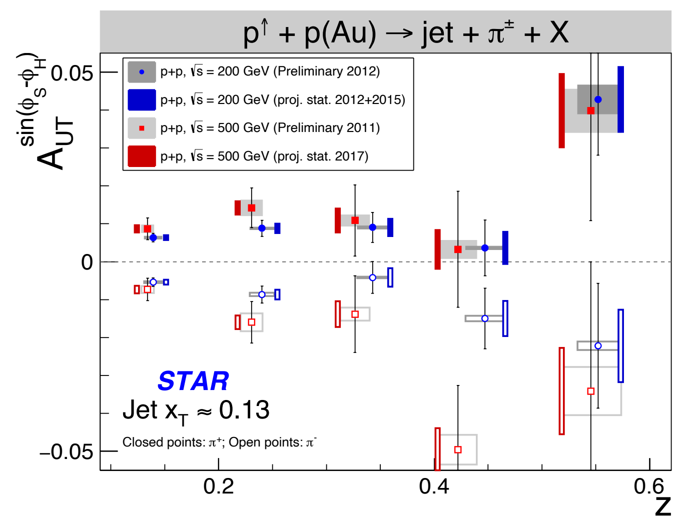
Transversity

- Another fundamental object, but C-odd, hence hard to measure.
- Transversity can be extracted from A_{UT} due to the Collins effect,

  $A_{UT} \sim$ transversity $\times$ interference fragmentation function.

- This is hard. But perhaps doable?





Small-x Asymptotics of Quark Transversity

YK, Sievert, '18

- Solution of the transversity evolution equation is straightforward.
- The resulting small-x asymptotics is (cf. Kirschner et al, 1996)

$$h_{1T}^{NS}(x, k_T^2) \sim h_{1T}^{\perp\, NS}(x, k_T^2) \sim \left(\frac{1}{x}\right)^{\alpha_t^q} \quad \text{with} \quad \alpha_t^q = -1 + 2\sqrt{\frac{\alpha_s\, C_F}{\pi}}$$

- Note the suppression by x^2 compared to the unpolarized quark TMDs.
- For $\alpha_s = 0.3$ we get

$$h_{1T}^{NS}(x, k_T^2) \sim h_{1T}^{\perp\, NS}(x, k_T^2) \sim x^{0.243}$$

- This certainly satisfies the Soffer bound, but is not likely to produce much tensor charge from small x.

$$\delta q(Q^2) = \int\limits_0^1 dx\, h_1(x, Q^2)$$

Conclusions

- RHIC Spin produced important measurements of A_{LL} leading to the first-ever non-zero ΔG extraction. This is a major step towards solving the proton spin puzzle.
- A decent portion of the remaining spin of the proton may be at small x, with theory and phenomenology being developed. EIC will help resolve the issue!
- STSA in $p^{\uparrow}+p$ allows us to test various pQCD predictions, and, so far, resulted in many puzzles with few resolutions.
- x-Dependence of transversity and Sivers function can be investigated, theoretically and experimentally, at RHIC and at EIC.

Backup Slides

Asymptotics of helicity PDFs and OAM

- At large N_c we have obtained the following small-x asymptotics:

$$\Delta q(x, Q^2) \sim \left(\frac{1}{x}\right)^{\alpha_h^q} \quad \text{with} \quad \alpha_h^q = \frac{4}{\sqrt{3}} \sqrt{\frac{\alpha_s N_c}{2\pi}} \approx 2.31 \sqrt{\frac{\alpha_s N_c}{2\pi}}$$

$$\Delta G(x, Q^2) \sim \left(\frac{1}{x}\right)^{\alpha_h^G} \quad \text{with} \quad \alpha_h^G = \frac{13}{4\sqrt{3}} \sqrt{\frac{\alpha_s N_c}{2\pi}} \approx 1.88 \sqrt{\frac{\alpha_s N_c}{2\pi}}$$

$$L_{q+\bar{q}}(x, Q^2) = -\Delta\Sigma(x, Q^2) \sim \left(\frac{1}{x}\right)^{\frac{4}{\sqrt{3}} \sqrt{\frac{\alpha_s N_c}{2\pi}}},$$

KPS ‘17, YK ‘19

$$L_G(x, Q^2) \sim \Delta G(x, Q^2) \sim \left(\frac{1}{x}\right)^{\frac{13}{4\sqrt{3}} \sqrt{\frac{\alpha_s N_c}{2\pi}}}$$

- At large N_c & N_f we have obtained

$$\Delta\Sigma(x, Q^2)\Big|_{\text{large-}N_c\&N_f} \sim \left(\frac{1}{x}\right)^{\alpha_h^q} \cos\left[\omega_q \ln\frac{1}{x} + \varphi_q\right]$$

with

$$\omega_q \approx \frac{0.22 N_f}{1 + 0.1265 N_f} \sqrt{\frac{\alpha_s N_c}{2\pi}}$$

Y. Tawabutr, YK ‘20

Large-N_C Limit → Elastic Dominance

- One can rewrite the color factor in terms of color dipole and color quadrupole scattering amplitudes

$$\frac{N_C^2}{2}\Big\langle S_{zx}S_{yx'} - \frac{1}{N_C^2}Q_{zxyx'} + (S_{zx}+S_{yx'})\Big(\frac{1}{N_C^2}-1\Big) + 1 - \frac{1}{N_C^2}\Big\rangle$$

$$S_{zx} = \frac{1}{N_C}Tr[V_z V_x^\dagger], \quad Q_{zxyx'} = \frac{1}{N_C}Tr[V_z V_x^\dagger V_y V_{x'}^\dagger]$$

- In the large-N_C limit the averaged products of dipole amplitudes factorize into products of averaged dipole amplitudes, and we get the leading term to be purely a product of dipole scattering terms

$$\lim_{N_C\to\infty}\frac{N_C^2}{2}\Big\langle(1-S_{zx})(1-S_{yx'})\Big\rangle \to \frac{N_C^2}{2}\Big\langle 1-S_{zx}\Big\rangle\Big\langle 1-S_{yx'}\Big\rangle$$

Elastic process!

Glauber-Mueller/MV model: $S_{xy} \approx e^{-\frac{1}{4}(x_\perp - y_\perp)^2 Q_s^2}$

Elastic Dominance

- We see that elastic processes dominate in A_N in this lensing+saturation calculation as N_C^2:1, that is, 9:1.
- At high p_T the inelastic contribution may dominate over the elastic one (see M.G. Santiago, YK, 2003.12650 [hep-ph]).
- Higher-order (in the coupling) corrections may also affect this elastic dominance at the leading order, though perhaps they will not destroy it completely.