PRC 103 (2021) 3, 0314901

Finite Density QCD EoS and Net-Proton Cumulants in the BES Program

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Outline

1. Higher-order net-proton cumulants — why do we care?

2. Modeling the critical point and its effects (brief overview and comments)

3. Fourth order baryon number susceptibility in the presence of a QCD critical point

   (DM et al *Phys.Rev.C* 103 (2021) 3, 034901)

4. Conclusions and outlook

Search for the Critical Point – Theory

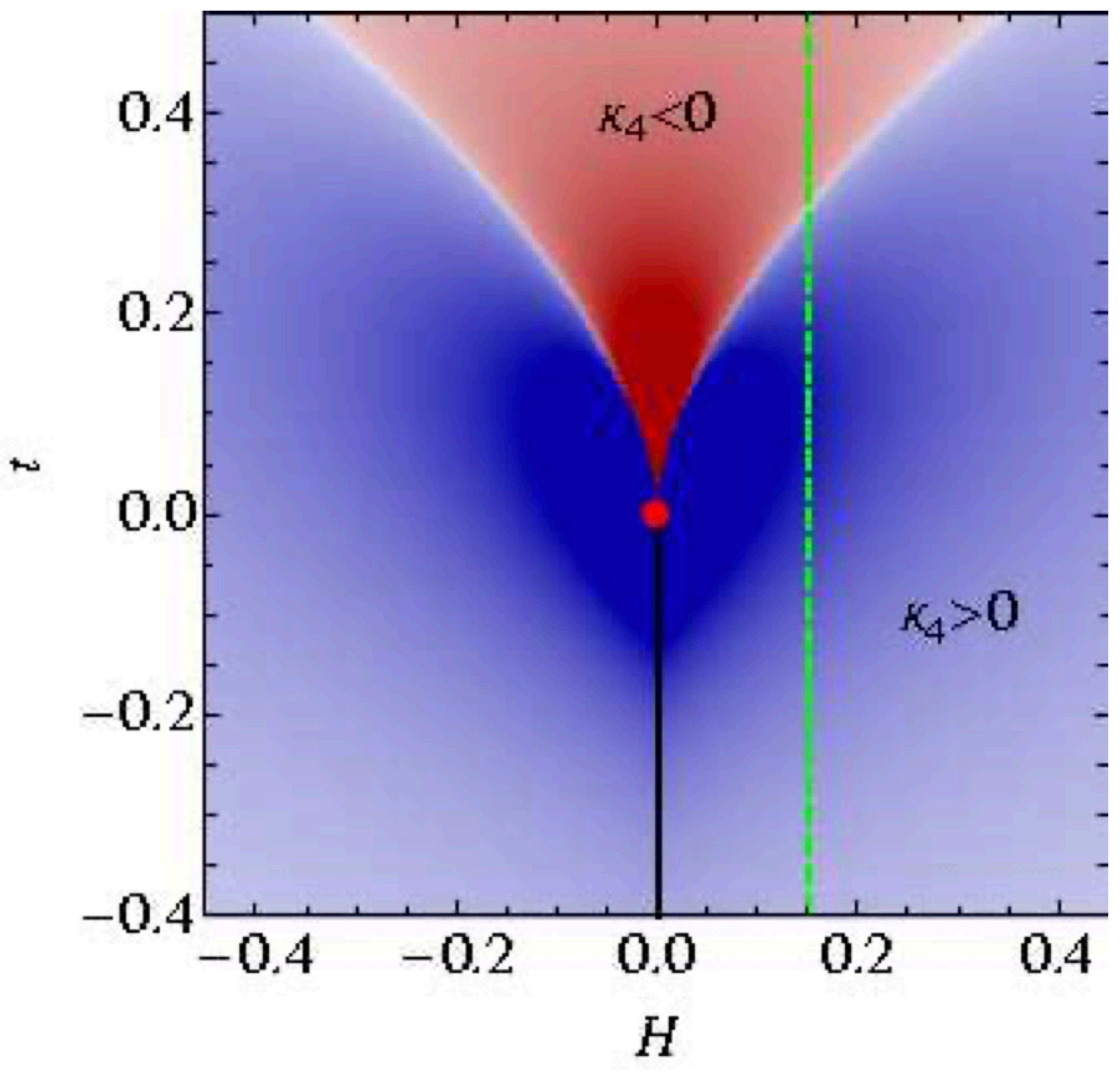
- Beam Energy Scan II at RHIC: looking for a critical point in the hadron-QGP transition.

M. A. Stephanov. PRL 107 (2011)

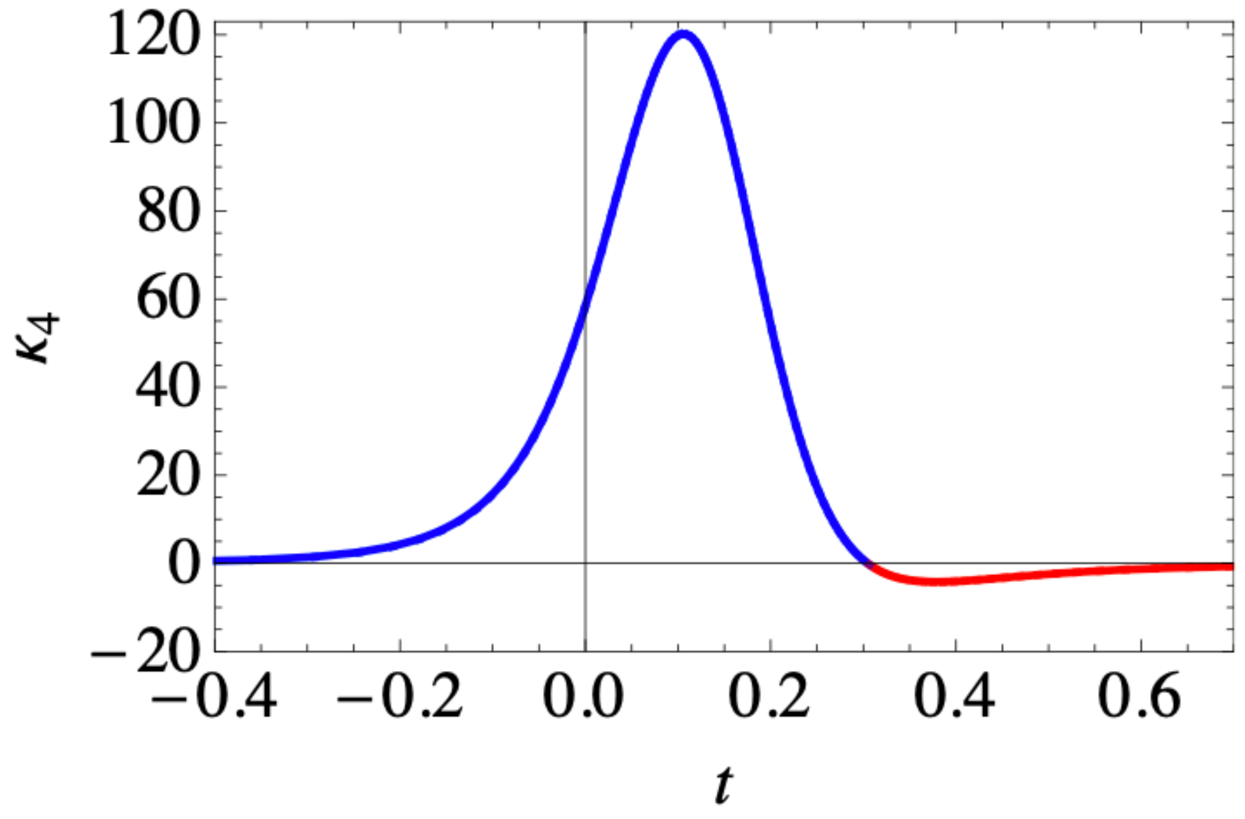
- QCD is in the same universality class as the 3D Ising model.
- Specific non-monotonic behavior of χ_4^B as a function of $\sqrt{s_{NN}}$.

Candidate: **higher order cumulants**

1. Baryon number susceptibilities diverge at the critical point
2. Higher order ~ higher powers of correlation length



$$\chi_4^{\text{Ising}}(r,h) = \left(\frac{\partial^4 G}{\partial h^4}\right)_r = \left(\frac{\partial^3 M}{\partial h^3}\right)_r$$



Search for the Critical Point — Experiment

Experimental proxy for $\chi_n^B \rightarrow$ net-proton moments

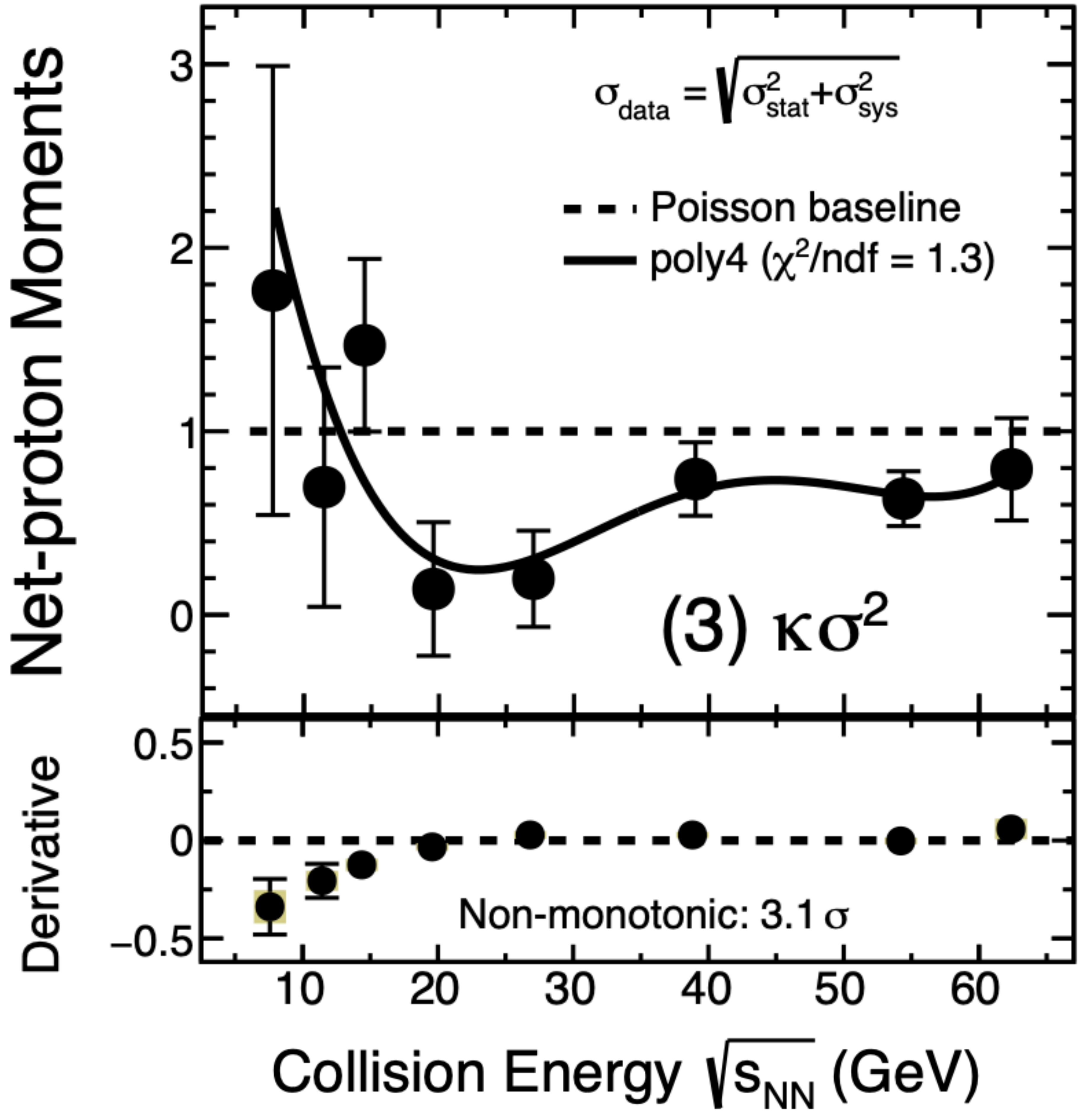
BES-I: Non-monotonic behavior of 3.1σ significance, a.k.a "the dip"

BES-II (ongoing): Improved statistics → smaller error bars

Is the dip followed by a diverging peak?

Theory: How do we interpret BES results?

Central Au+Au - STAR (2021): 2101.12413



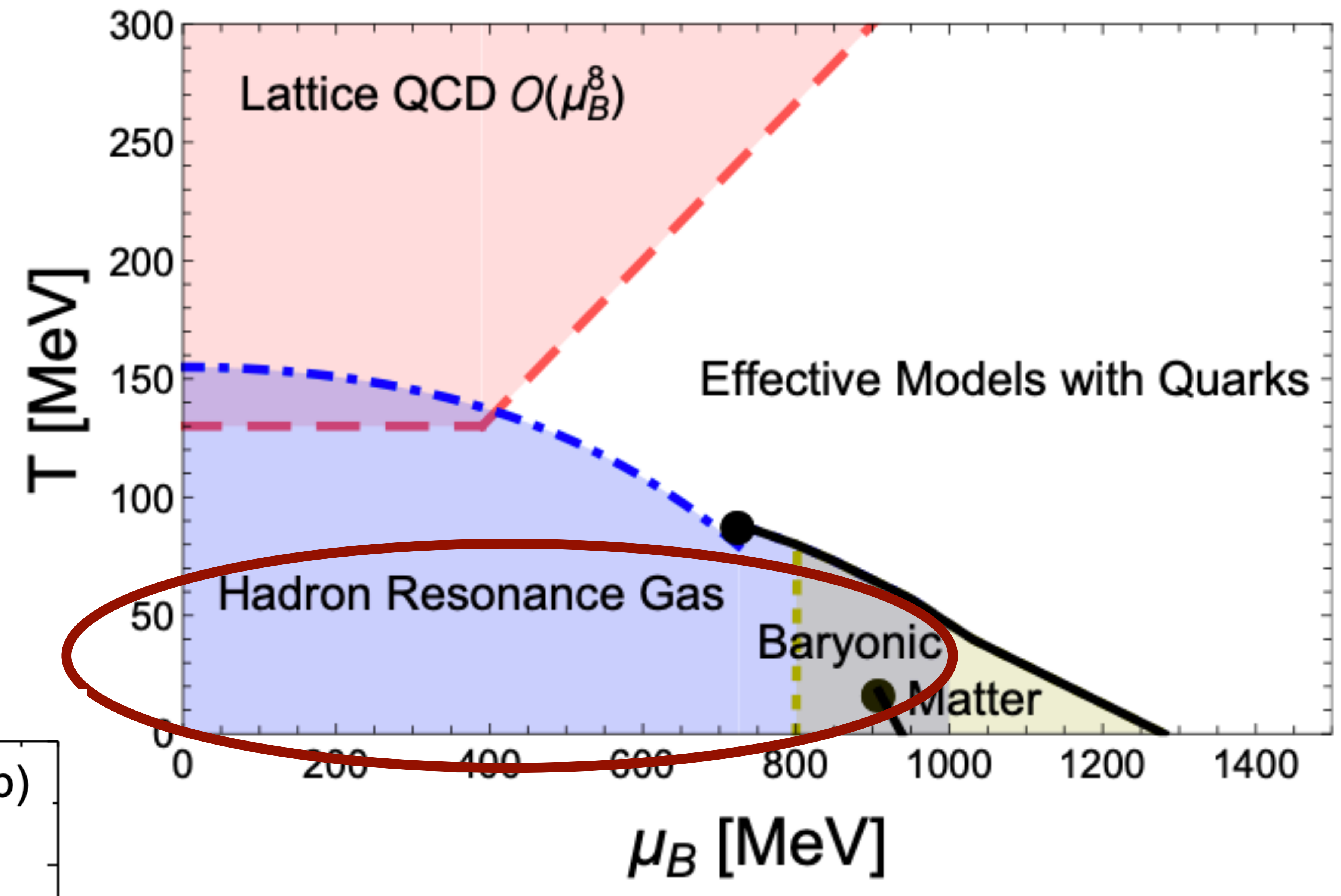
Finite Density EoS for QCD with a CP

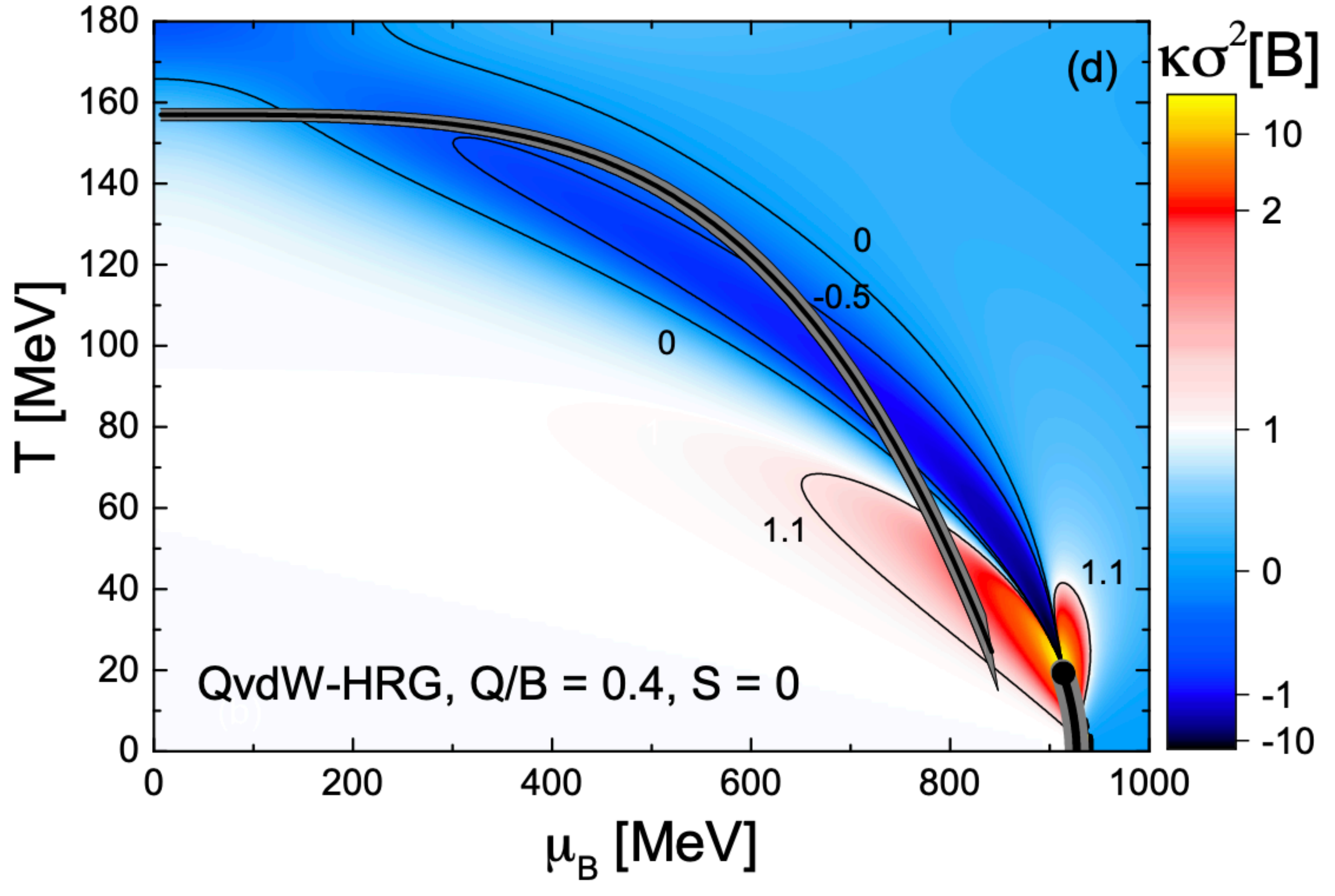
I) HRG with vdW interactions + quantum statistics

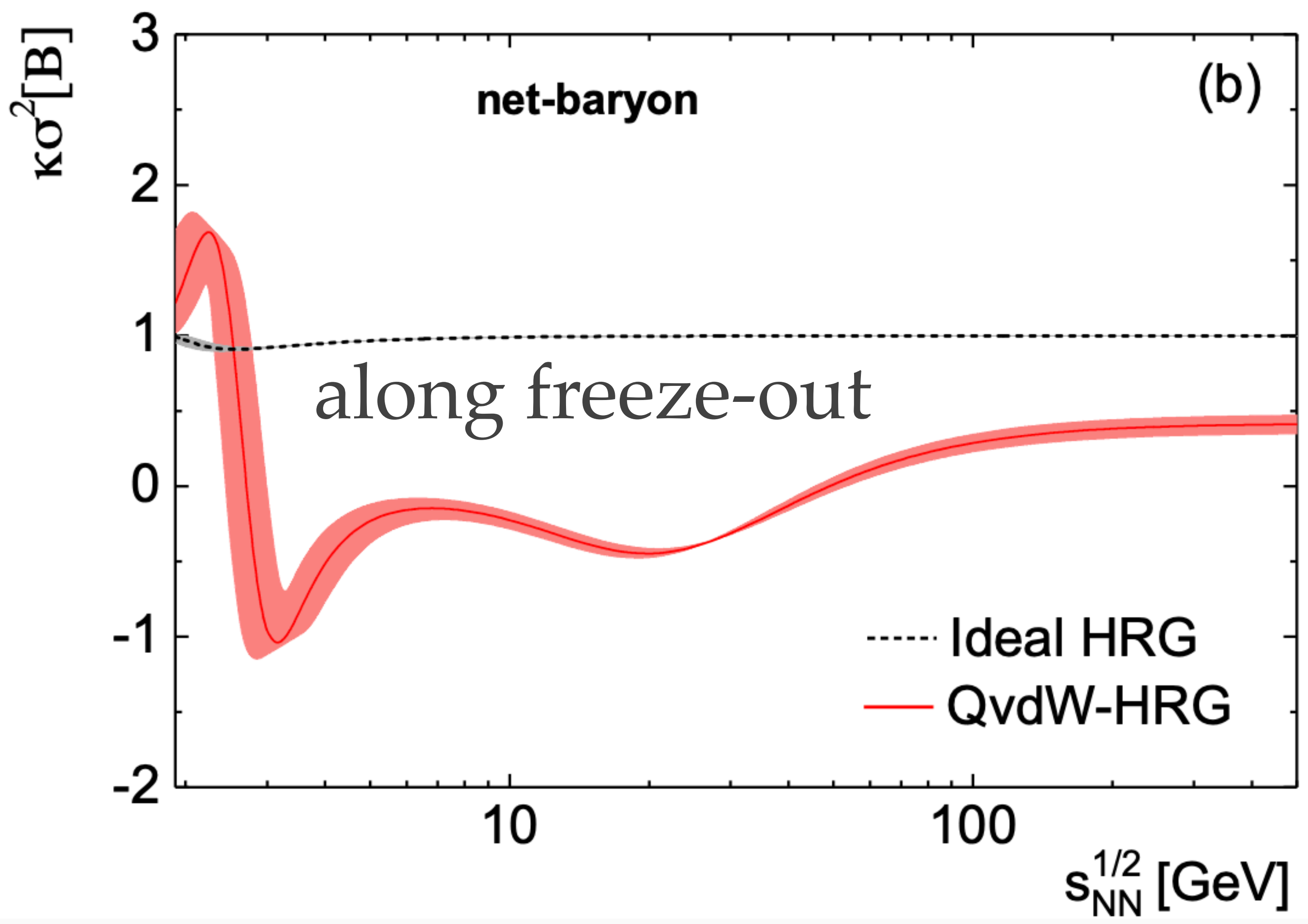
- CP at high densities (liquid-gas)
- Impose charge conservation constraints
- Effect of global conservation laws

Talk: Vovchenko

V Dexheimer et al J. Phys. G: Nucl. Part. Phys. 48 (2021)







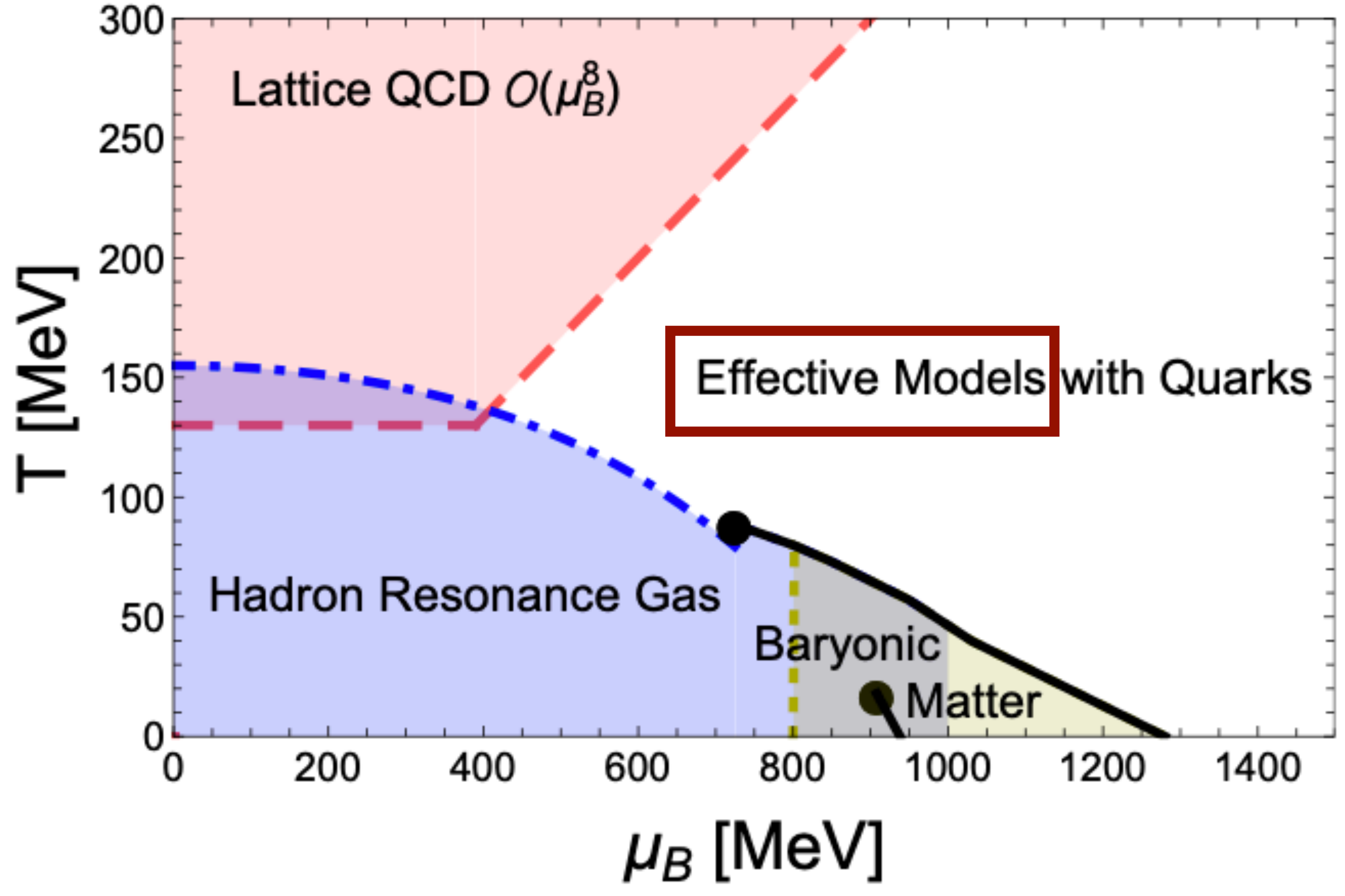
R. V. Poberezhnyuk, V. Vovchenko et al
Phys. Rev. C **100**, 054904

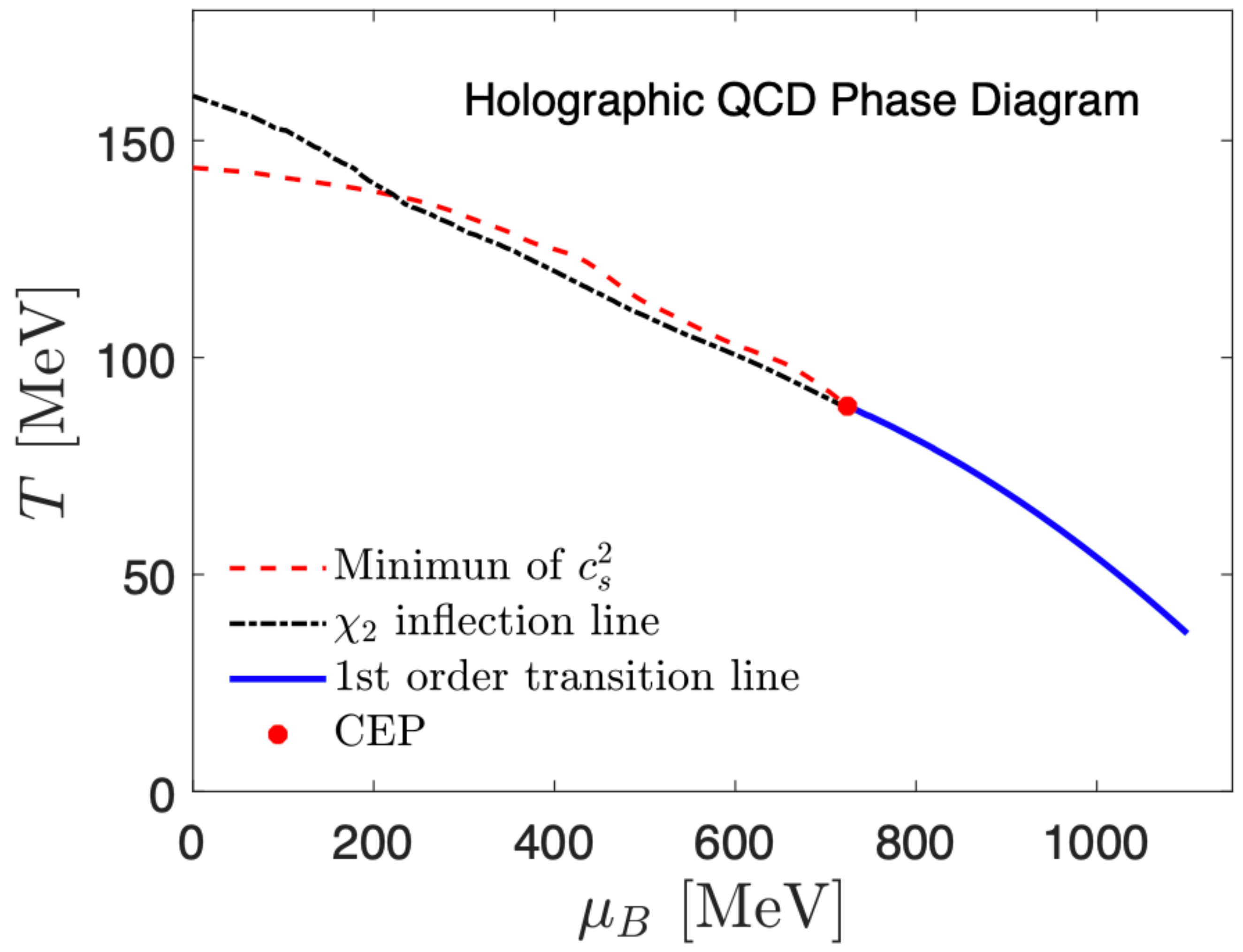
Finite Density EoS for QCD with a CP

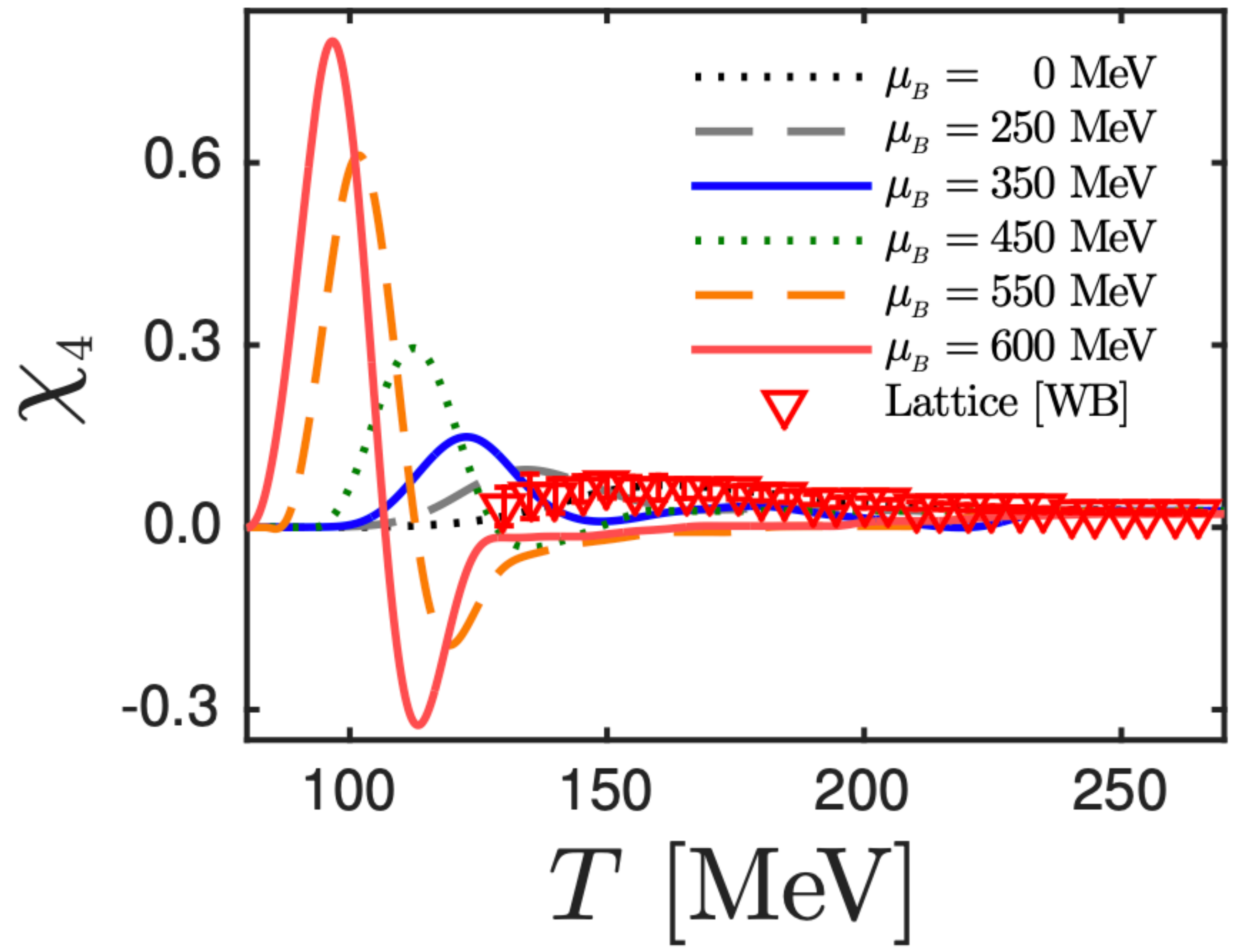
II) Black hole holography

- CP at high densities: $T_{CEP} = 89$ MeV, $\mu_B^{CEP} = 724$ MeV
- Large coverage of phase diagram
- Agrees with LQCD

V Dexheimer *et al* 2021 *J. Phys. G: Nucl. Part. Phys.* **48** 073001







J. Grefa et al. 2102.12042

R. Critelli et al Phys. Rev. D **96**, 096026

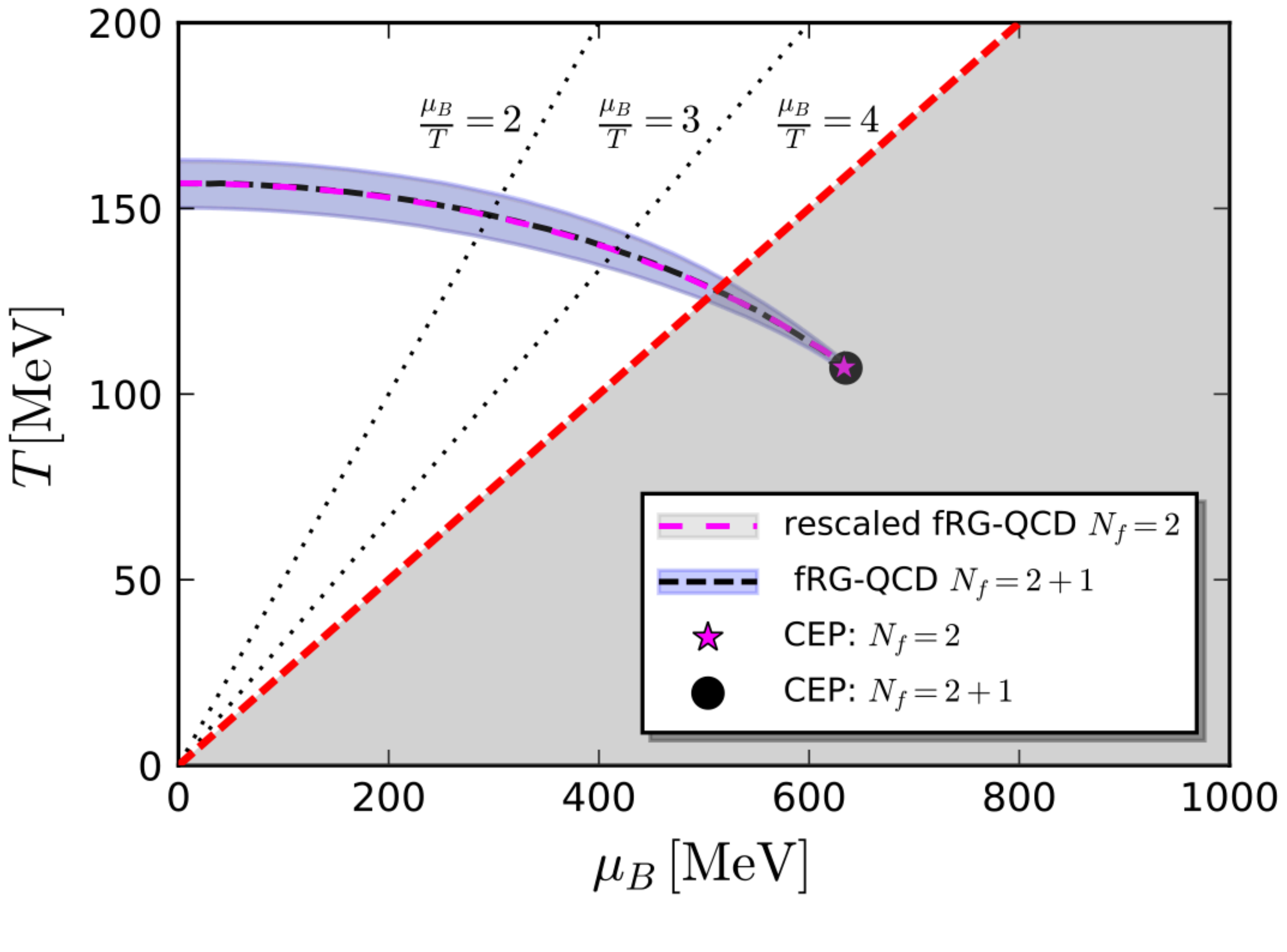
Finite Density EoS for QCD with a CP

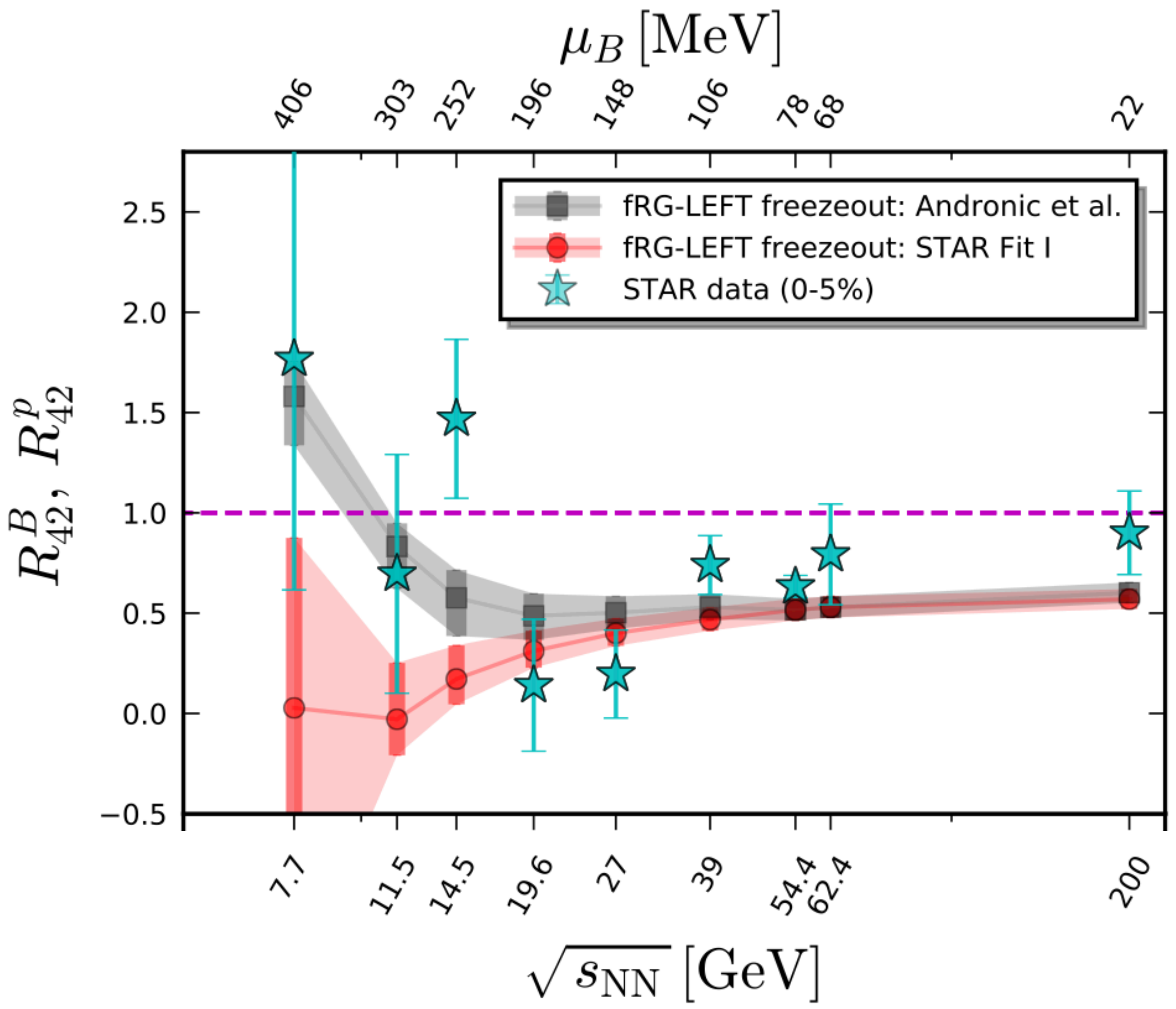
III) Functional methods

$N_f = 2+1$

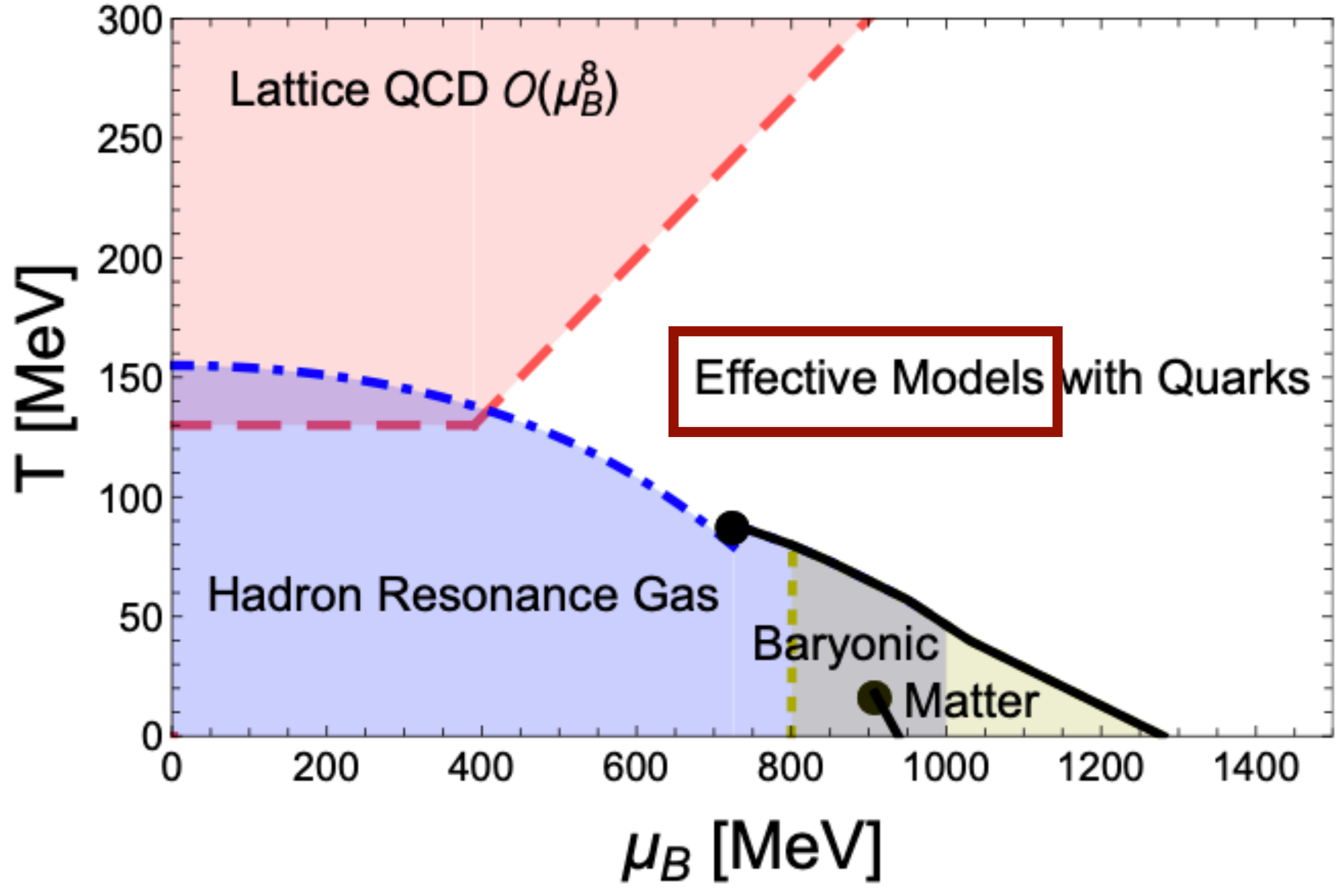
F. Gao, JM. Pawlowski. 2010.13705

- CP at high densities: $T_{CEP} = 109$ MeV, $\mu_B^{CEP} = 610$ MeV
- Large coverage of phase diagram
- Agrees with LQCD





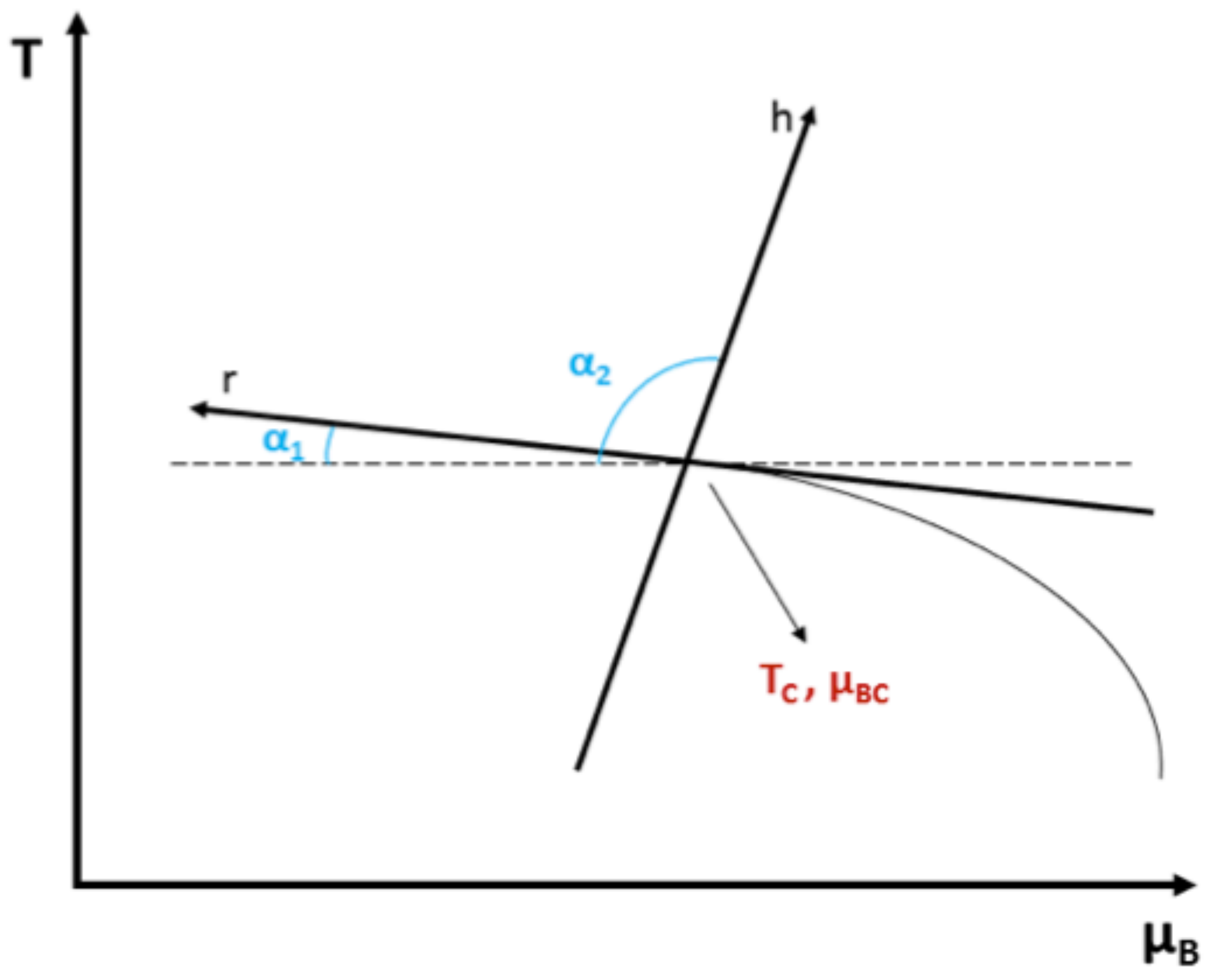
V Dexheimer *et al* 2021 *J. Phys. G: Nucl. Part. Phys.* **48** 073001



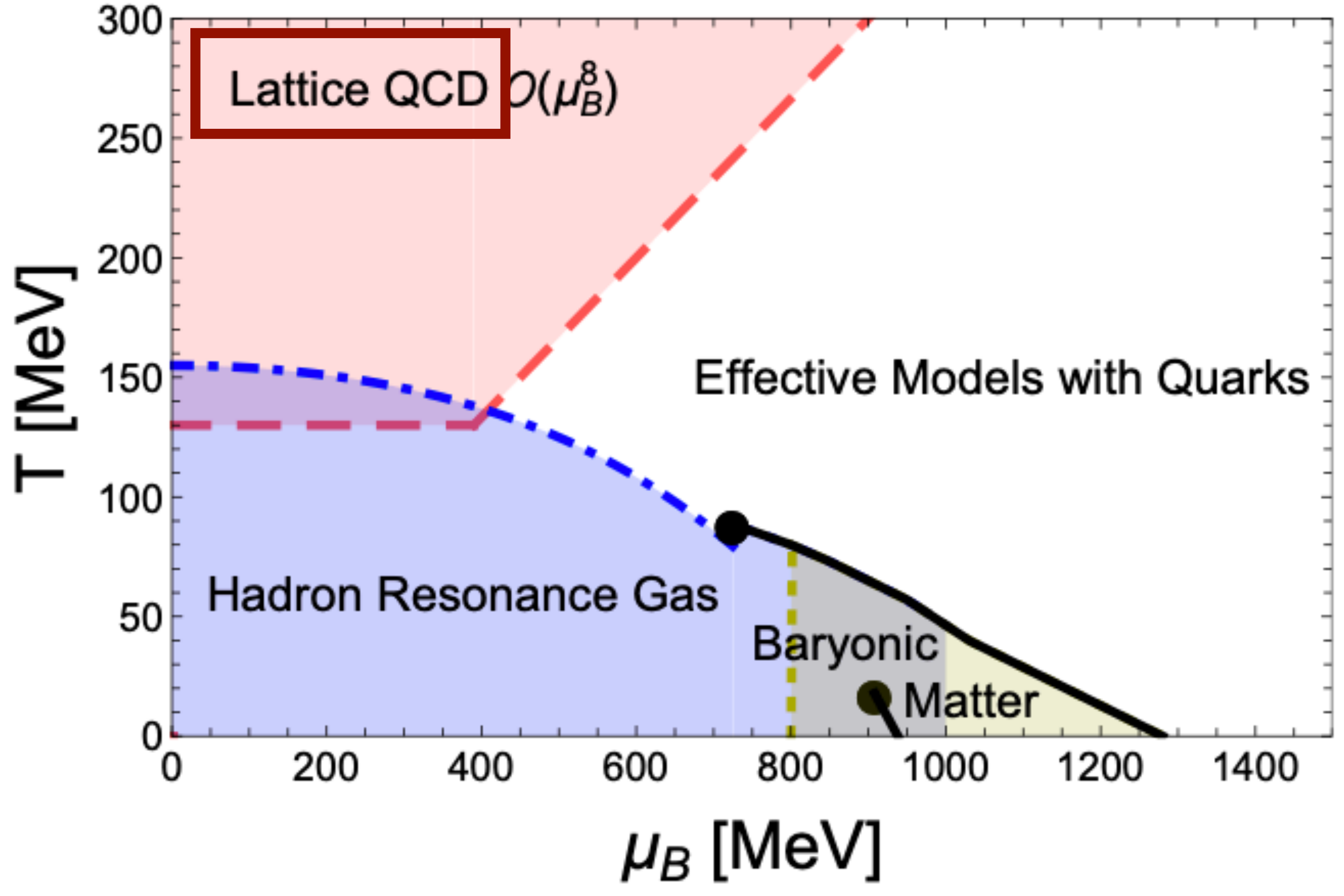
W. Fu et al 2101.06035

Finite Density EoS for QCD with a CP

IV) CP from 3D Ising Universality class (BEST EoS)



V Dexheimer *et al* 2021 *J. Phys. G: Nucl. Part. Phys.* **48** 073001

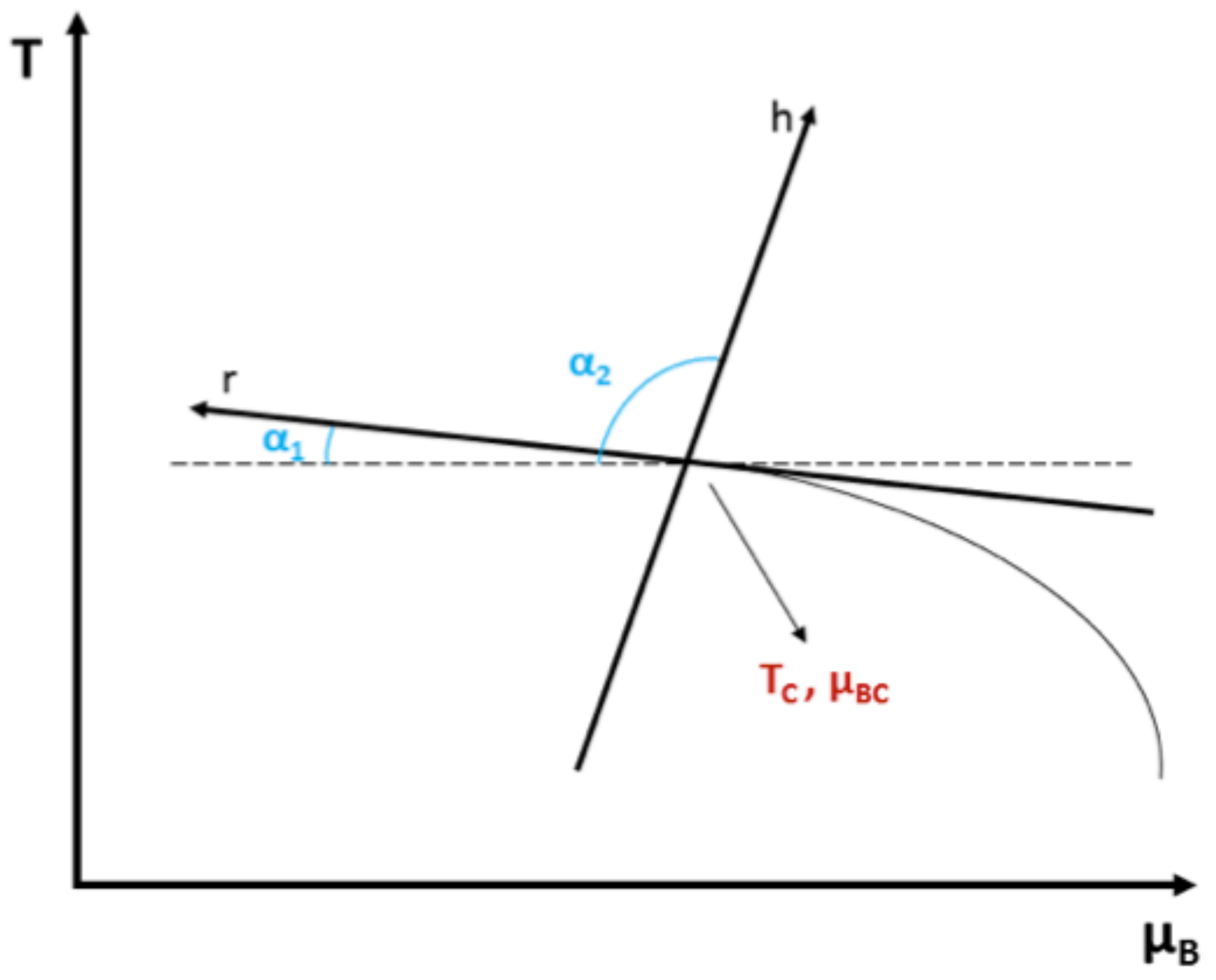


Up to $\mathcal{O}(\mu_B{}^4)$:

P. Parotto, DM, et al Phys. Rev C (2020)

Parameterized EoS with CP from 3D Ising

IV) CP from 3D Ising Universality class (BEST EoS)



Up to $\mathcal{O}(\mu_B^4)$:

P. Parotto, DM, et al Phys. Rev C (2020)

Critical Contribution

1. Define parameterization of the 3D Ising Model in the vicinity of the critical point:

$$M = M_0 R^\beta \theta \qquad M_0 \simeq 0.605 \quad h_0 \simeq 0.364$$
$$h = h_0 R^{\beta\delta} \tilde{h}(\theta) \qquad \tilde{h}(\theta) = \theta(1 + a\theta^2 + b\theta^4)$$
$$r = R(1-\theta^2) \qquad \beta \simeq 0.326, \quad \delta \simeq 4.80$$

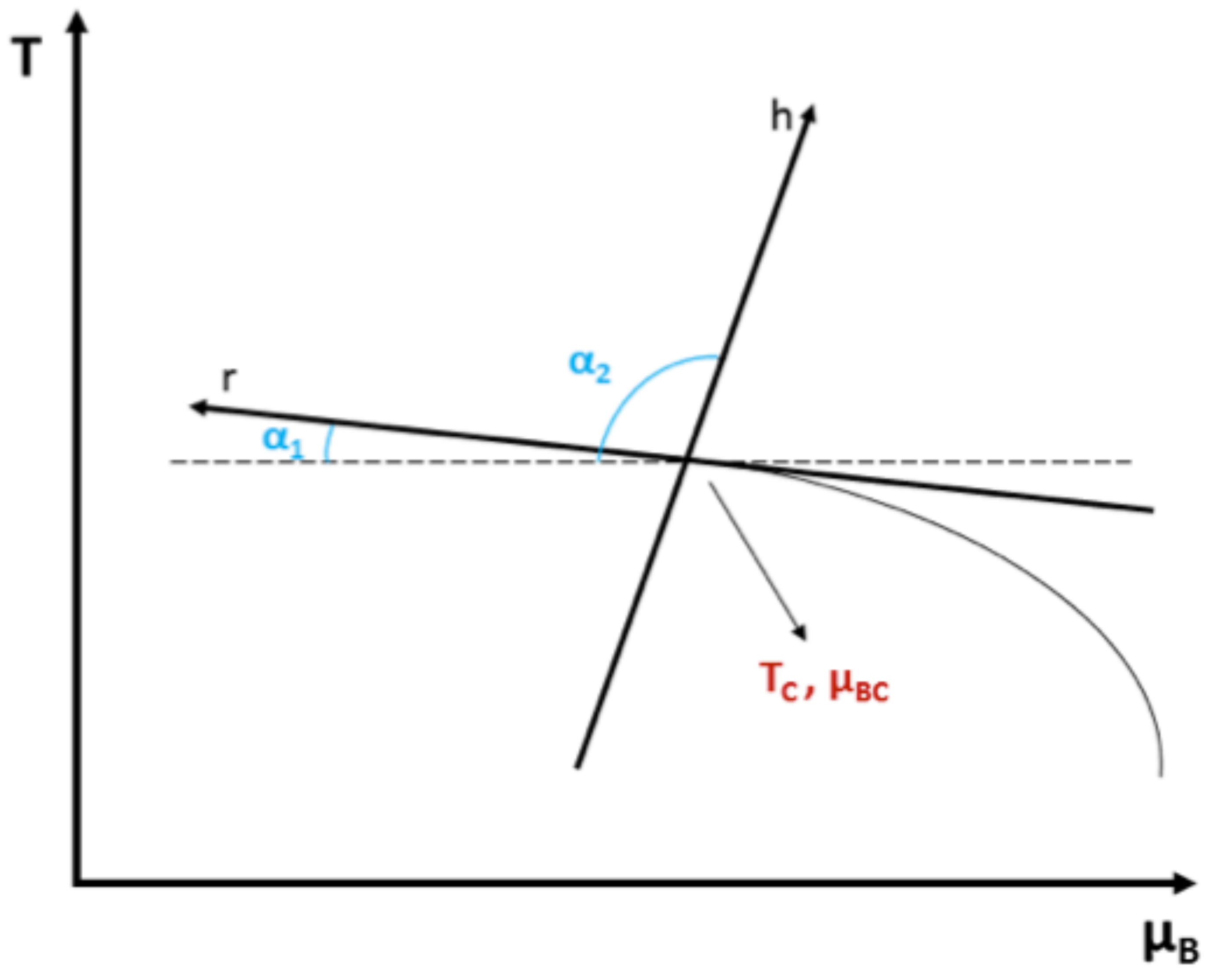
C. Nonaka, M. Asakawa, Phys. Rev C (2005)

2. Map the 3D-Ising phase diagram to QCD variables:

$$(\mathbf{r}, \mathbf{h}) \longleftrightarrow (\mathbf{T}, \mu_\mathbf{B}) : \quad \frac{T - \mathbf{T_C}}{\mathbf{T_C}} = \mathbf{w}\,(r\rho \,\sin\alpha_1 + h\,\sin\alpha_2)$$
$$\frac{\mu_B - \mu_{\mathbf{BC}}}{\mathbf{T_C}} = \mathbf{w}\,(-r\rho\,\cos\alpha_1 - h\,\cos\alpha_2)$$

Parameterized EoS with CP from 3D Ising

IV) CP from 3D Ising Universality class (BEST EoS)



Up to $\mathcal{O}(\mu_B{}^4)$:

P. Parotto, DM, et al Phys. Rev C (2020)

Non-Ising Contribution

3. Impose matching to lattice QCD at $\mu_B = 0$:

$$T^4 c_n^{\mathrm{LAT}}(T) = T^4 c_n^{\mathrm{Non-Ising}}(T) + c_n^{\mathrm{Ising}}(T)$$

4. Reconstruct the full pressure:

$$P(T,\mu_B) = T^4 \sum_n c_n^{\mathrm{Non-Ising}}(T) \left(\frac{\mu_B}{T}\right)^n + P_{\mathrm{crit}}^{\mathrm{QCD}}(T,\mu_B)$$

- Reduce number of free parameters by imposing:

$$T = T_0 + \kappa T_0 \left(\frac{\mu_B}{T_0}\right)^2 + O(\mu_B^4), \qquad \alpha_1 = \tan^{-1}\left(2\frac{\kappa}{T_0}\mu_{BC}\right)$$

Size and shape of Ising Contribution

- Dependence on mapping parameters?

We can estimate the size of the critical region along the crossover, h=0:

$$\chi_4^{\text{Ising}} \sim AG_{\mu\mu\mu\mu}(r,0) \sim AG_{hhhh}(r,0)h_\mu^4$$

Along the critical chemical potential line, r ~ 0:

$$\chi_4^{\text{Ising}} \sim AG_{\mu\mu\mu\mu}(0,h) \sim AG_{hhhh}(0,h)h_\mu^4$$

Comparing to the regular contribution $\chi_4^{\text{reg}} \sim 1$, we find

$$\Delta\mu_B \sim T_C \rho w \cos\alpha_1 \left(\frac{A^{1/4}}{T_C} \frac{sin\alpha_1}{wT_C \sin\alpha_1 - \alpha_2} \right)^{\frac{4}{\beta(3\delta-1)}}$$

$$\Delta T \sim T_C \left(\frac{A}{T_C^4} \right)^{\frac{\delta}{3\delta-1}} \frac{\sin\alpha_1}{\cos\alpha_1} \left(\frac{\sin\alpha_1}{w \sin\alpha_1 - \alpha_2} \right)^{\frac{\delta+1}{3\delta-1}}$$

Weak dependence on w, while smaller α_1 - α_2 yields a larger critical region for the same w, ρ:

$$\Delta\mu_B \sim w^{1/7}, \quad \Delta T \sim w^{-3/7}$$

$$\Delta\mu_B \sim \sin\alpha_1 - \alpha_2{}^{-6/7}, \quad \Delta T \sim \sin\alpha_1 - \alpha_2{}^{-3/7}$$

Parameter choices

- Two sets of parameters

	μ_{BC}	T_C	α_1	$\alpha_2 - \alpha_1$	w	ρ
I.	420 MeV	138 MeV	4.6°	90°	$0.5, 1, 2$	$0.5, 1, 2$
II.	420 MeV	138 MeV	4.6°	-3°	$0.5, 1, 2$	$0.5, 1, 2$

Common choice in literature. Orthogonal axes.

Motivated by M.S. Pradeep, M. Stephanov, Phys. Rev. D 100 (2019)

- Large μ_{BC} to allow for maximum freedom in parameter choice within the range of the Taylor expansion.
- T_C and α_1 not free → follow from choice of μ_{BC}.
- Compute critical contribution to χ_4^B for each choice of parameters.

Critical contribution to χ_4^B

$(\alpha_2 - \alpha_1 = 90^{\circ})$

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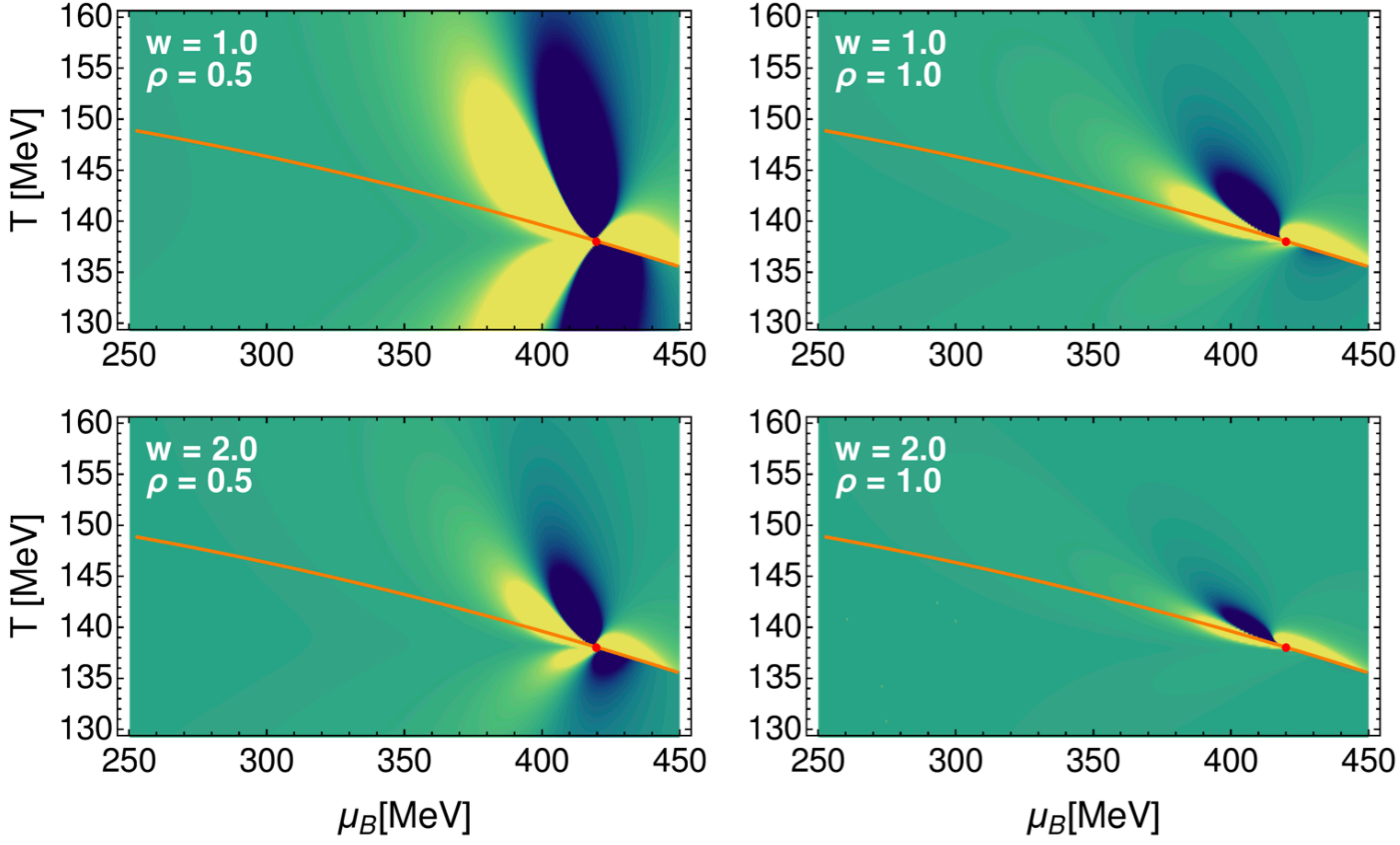
green → yellow: positive values, blue: negative.
orange curve: parameterized transition line.



- Smaller w yields larger critical region.
- ρ stretches critical region along μ_B
- **Approach to critical point characterized by peak rather than dip, except for immediate vicinity.**

Critical contribution to χ_4^B

$(\alpha_2 - \alpha_1 = -3^\circ)$

green → yellow: positive values, blue: negative.
orange curve: parameterized transition line.

- **Negative angle difference bends negative lobe downwards.**
- **Agrees with leading singularity prediction.**
- Note: this choice is **not thermodynamically stable** under current mapping.

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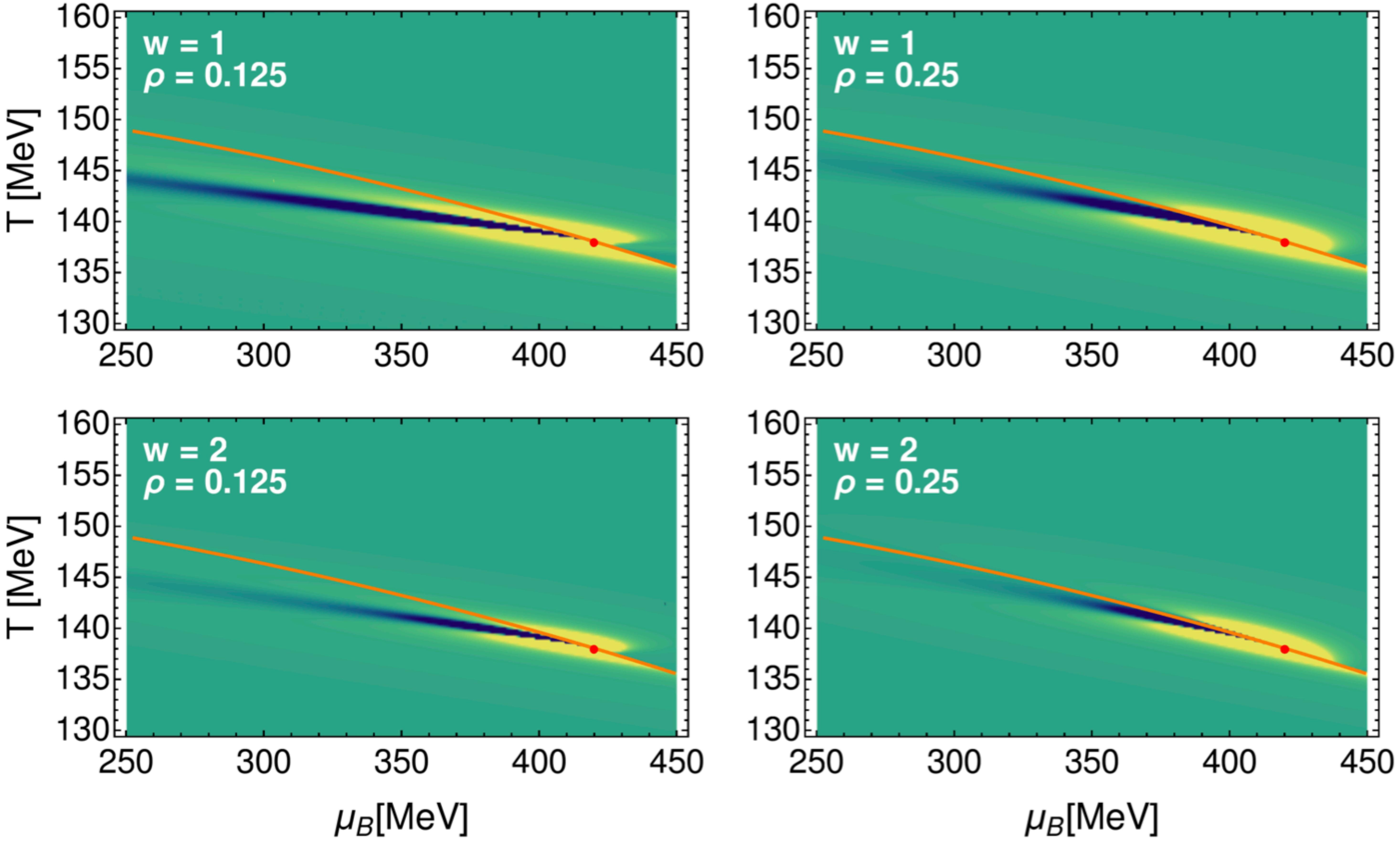

Peak sign of CP, not the dip

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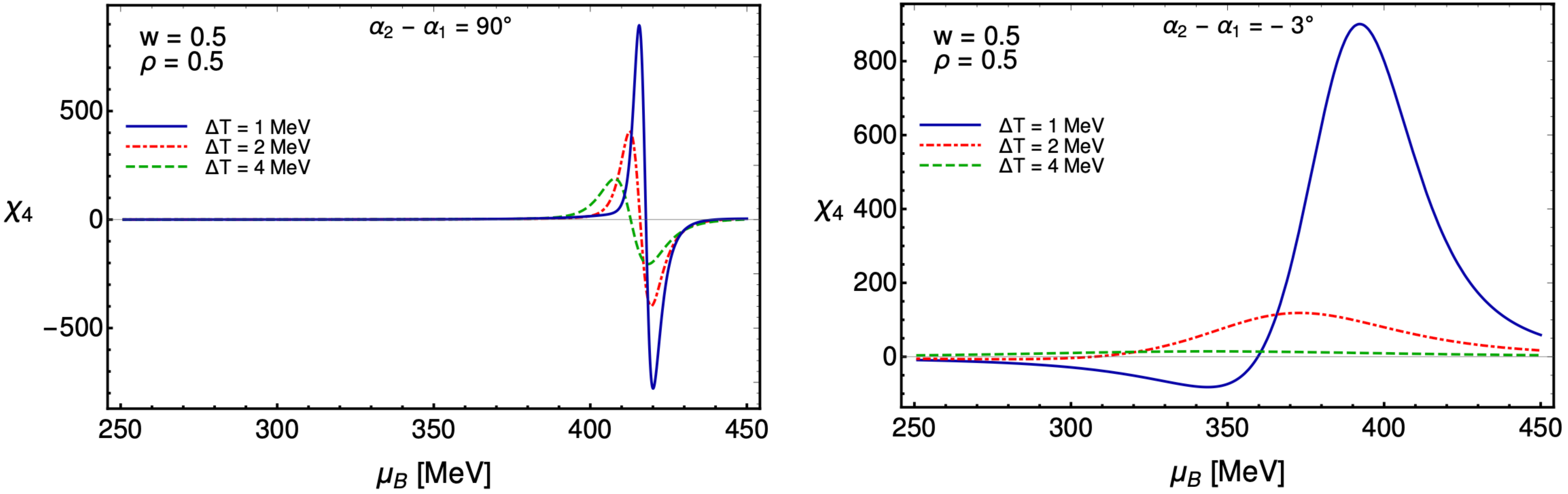


Dip only appears for one choice*, close to the vicinity of the transition line.

*shown in this work

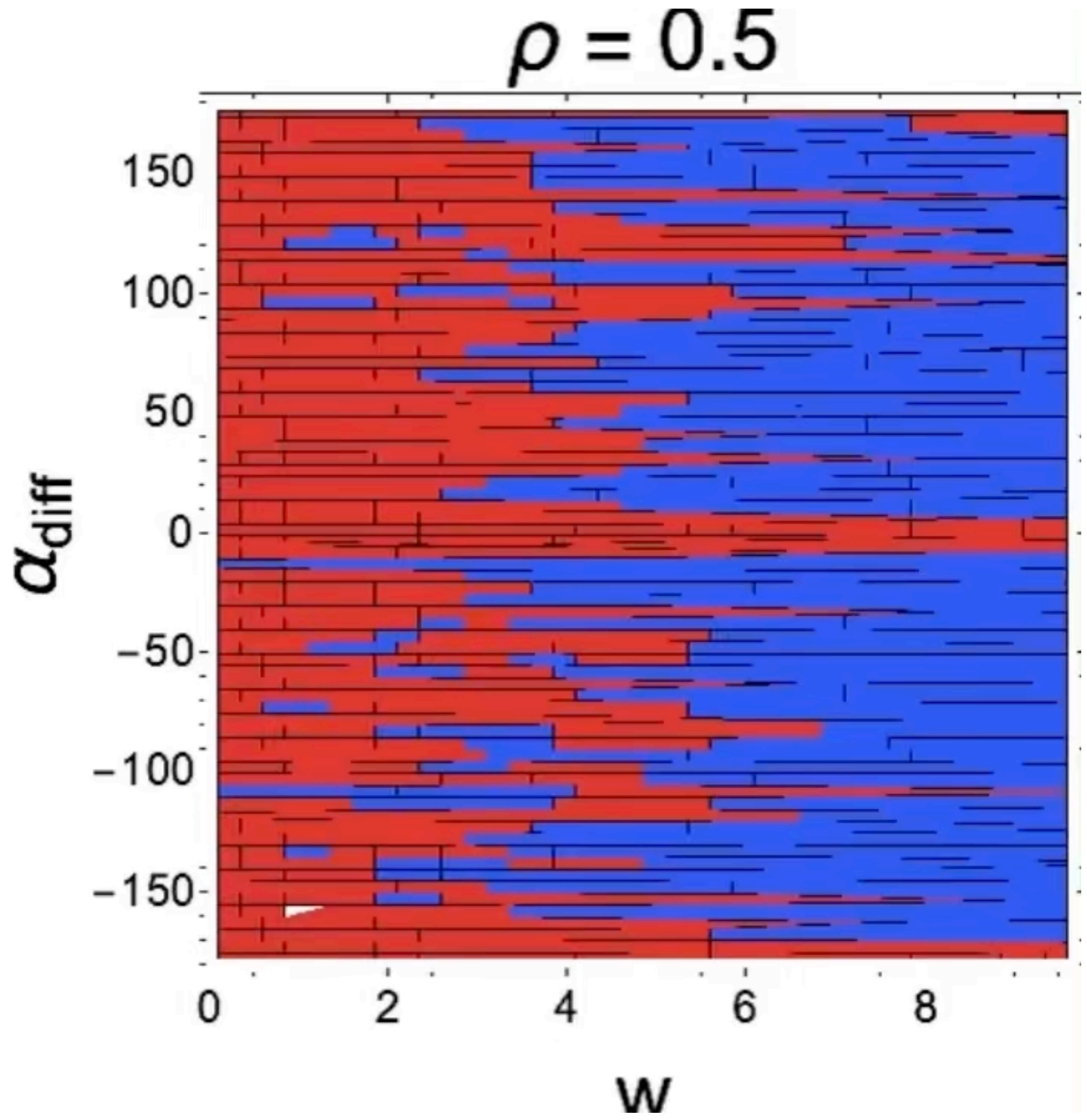
Why the difference?

- Ising to QCD map introduces mixing between r, h:

$$\partial_{\mu_B} = \frac{1}{w\rho T_C \sin\alpha_1 - \alpha_2}(\sin\alpha_1\partial_h + \sin\alpha_2\partial_r)$$

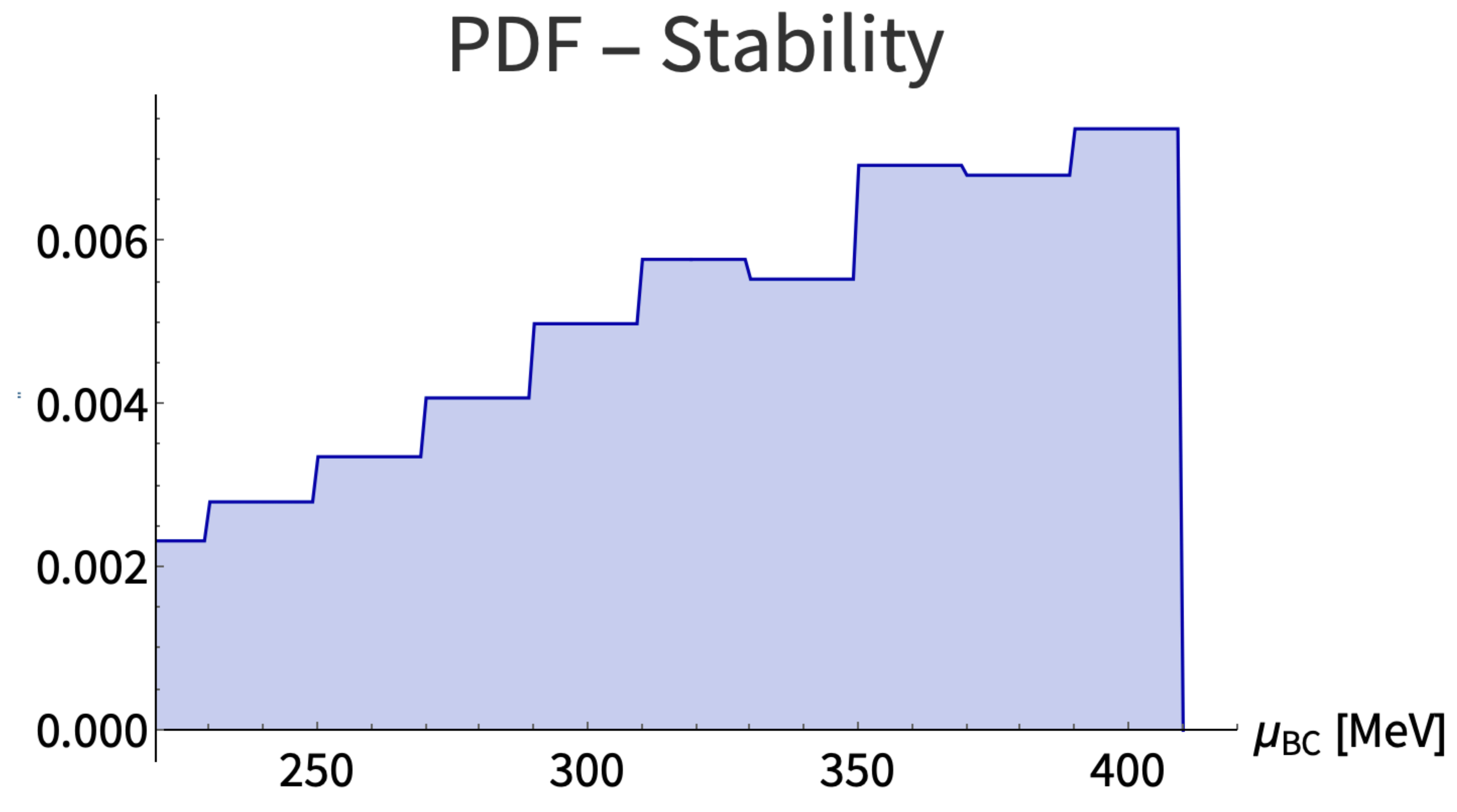
- h has larger scaling dimension: dominant contribution close to the critical point.
- Since α_1 is small, when is α_2 not small → h contribution becomes suppressed (orthogonal case).
- Taking most divergent terms corresponds to $\partial_{\mu_B} \sim \partial_h$
- **but** subleading terms may dominate if leading term is sufficiently suppressed.

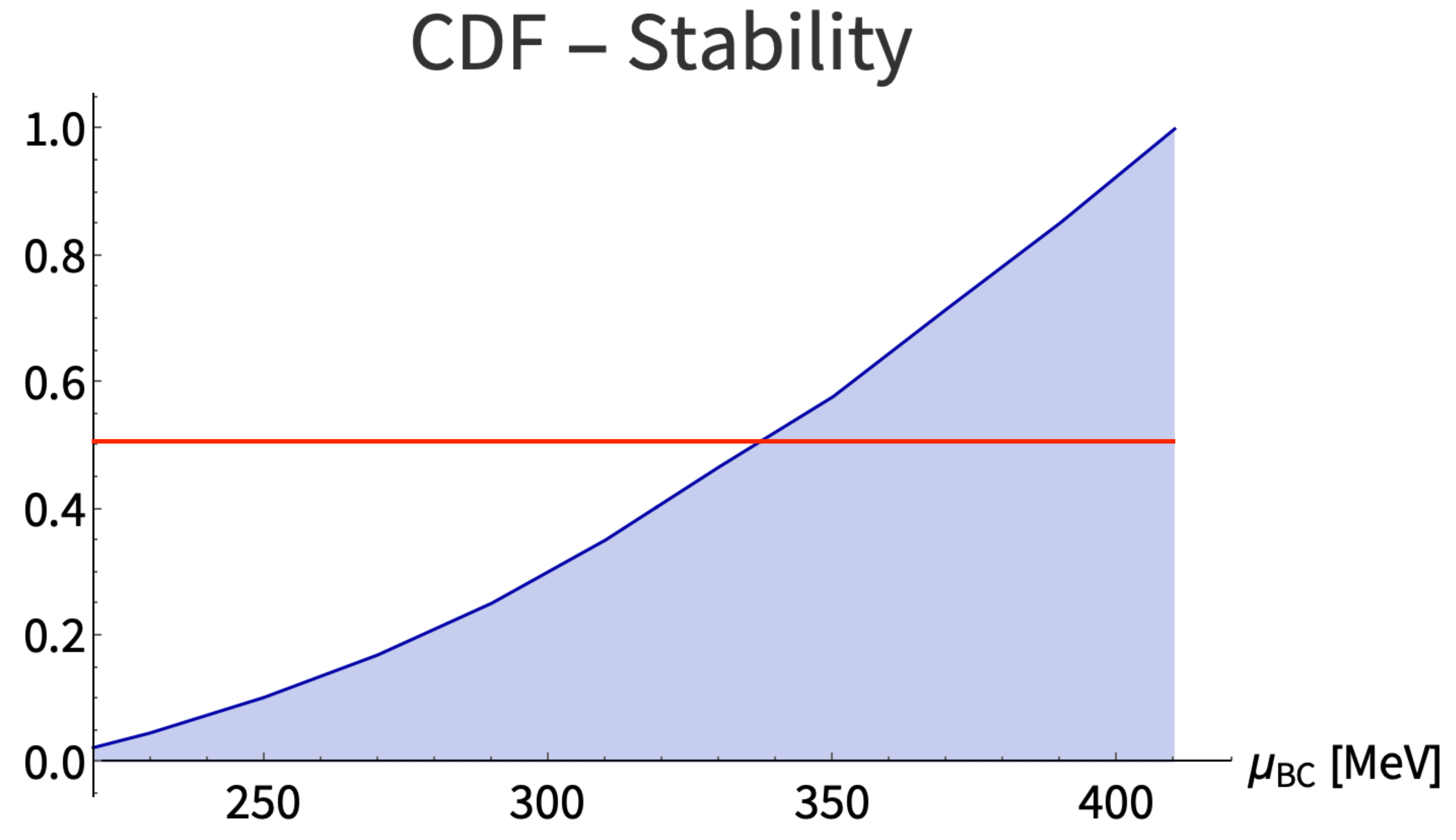
Parameter space scan (preliminary)



- Introduction of free-parameters with Ising → QCD map allows for thermodynamically unstable realizations.
- ML classifier assisted classification of entire parameter space.

Preliminary results show:
i) Larger μ_{BC} is preferred
ii) Hard upper limit on ρ
iii) Small angles strongly disfavored





Conclusions, considerations, outlook

- Investigated behavior of χ_4^B in the presence of the critical point in the 3D Ising model universality class.
- χ_4^B can be affected by sub-leading terms.
- **Diverging peak is a robust signature** of the critical point.
- Dip only present in **very few, thermodynamically unstable** cases under current mapping.
- Current study: **equilibrium** properties of QCD EoS. HIC are dynamical systems, **need EbE relativistic viscous BSQ hydro evolution + critical fluctuations + hadronic transport.**
- Temperature difference between hadronization and freeze-out?
- Machine-learning-assisted study coming soon!