Finite Density QCD EoS and Net-Proton Cumulants in the BES Program
Outline

1. Higher-order net-proton cumulants — why do we care?

2. Modeling the critical point and its effects (brief overview and comments)

3. Fourth order baryon number susceptibility in the presence of a QCD critical point

4. Conclusions and outlook
Beam Energy Scan II at RHIC: looking for a critical point in the hadron-QGP transition.

Candidate: higher order cumulants

1. Baryon number susceptibilities diverge at the critical point
2. Higher order $\sim$ higher powers of correlation length

• QCD is in the same universality class as the 3D Ising model.
• Specific non-monotonic behavior of $\chi^B_4$ as a function of $\sqrt{s_{NN}}$.

M. A. Stephanov. PRL 107 (2011)
Experimental proxy for $\chi_n^B \rightarrow$ net-proton moments

BES-I: Non-monotonic behavior of 3.1σ significance, a.k.a “the dip”
BES-II (ongoing): Improved statistics $\rightarrow$ smaller error bars
Is the dip followed by a diverging peak?

Theory: How do we interpret BES results?
Finite Density EoS for QCD with a CP

I) HRG with vdW interactions + quantum statistics

- CP at high densities (liquid-gas)
- Impose charge conservation constraints
- Effect of global conservation laws

Talk: Vovchenko

Figure captions:

- QvdW-HRG, Q/B = 0.4, S = 0
- Along freeze-out

References:


R. V. Poberezhnyuk, V. Vovchenko et al, Phys. Rev. C 100, 054904
II) Black hole holography

- CP at high densities: $T_{\text{CEP}} = 89 \text{ MeV}$, $\mu_B^{\text{CEP}} = 724 \text{ MeV}$
- Large coverage of phase diagram
- Agrees with LQCD

$T_C = 89 \text{ MeV}$, $\mu_C = 724 \text{ MeV}$
Finite Density EoS for QCD with a CP

III) Functional methods

- CP at high densities: $T_{CEP} = 109$ MeV, $\mu_B^{CEP} = 610$ MeV
- Large coverage of phase diagram
- Agrees with LQCD

N$_f = 2+1$

F. Gao, JM. Pawlowski. 2010.13705

$T_C = 109$ MeV, $\mu_C = 610$ MeV

$N_f = 2+1$

W. Fu et al 2101.06035
Finite Density EoS for QCD with a CP

IV) CP from 3D Ising Universality class (BEST EoS)

Up to $\mathcal{O}(\mu_B^4)$:
IV) CP from 3D Ising Universality class (BEST EoS)

Critical Contribution

1. Define parameterization of the 3D Ising Model in the vicinity of the critical point:

\[ M = M_0 R^\beta \theta \]
\[ h = h_0 R^{\delta} \tilde{h}(\theta) \]
\[ r = R(1 - \theta^2) \]

\[ M_0 \simeq 0.605 \quad h_0 \simeq 0.364 \]
\[ \beta \simeq 0.326, \quad \delta \simeq 4.80 \]


2. Map the 3D-Ising phase diagram to QCD variables:

\[ (r, h) \leftrightarrow (T, \mu_B) : \]
\[ \frac{T - T_C}{T_C} = w (r \rho \sin \alpha_1 + h \sin \alpha_2) \]
\[ \frac{\mu_B - \mu_{BC}}{T_C} = w (-r \rho \cos \alpha_1 - h \cos \alpha_2) \]

Parameterized EoS with CP from 3D Ising

IV) CP from 3D Ising Universality class (BEST EoS)

Non-Ising Contribution

3. Impose matching to lattice QCD at $\mu_B = 0$:

$$T^4 c_n^{\text{LAT}}(T) = T^4 c_n^{\text{Non-Ising}}(T) + c_n^{\text{Ising}}(T)$$

4. Reconstruct the full pressure:

$$P(T, \mu_B) = T^4 \sum_{n} c_n^{\text{Non-Ising}}(T) \left( \frac{\mu_B}{T} \right)^n + P_{\text{crit}}^{\text{QCD}}(T, \mu_B)$$

- Reduce number of free parameters by imposing:

$$T = T_0 + \kappa T_0 \left( \frac{\mu_B}{T_0} \right)^2 + O(\mu_B^4), \quad \alpha_1 = \tan^{-1} \left( \frac{2 \kappa}{T_0} \mu_{BC} \right)$$

Up to $\mathcal{O}(\mu_B^4)$:

Size and shape of Ising Contribution

\* Dependence on mapping parameters?

We can estimate the size of the critical region along the crossover, \( h=0 \):

\[
\chi_4^{\text{Ising}} \sim AG_{\mu\mu\mu}(r,0) \sim AG_{h\bar{h}h\bar{h}}(r,0) h_\mu^4
\]

Comparing to the regular contribution \( \chi_4^{\text{reg}} \sim 1 \), we find

\[
\Delta \mu_B \sim T_C \rho w \cos \alpha_1 \left( \frac{A^{1/4}}{T_C} \frac{\sin \alpha_1}{w T_C \sin \alpha_1 - \alpha_2} \right)^{\frac{4}{\delta (\delta - 1)}}
\]

\[
\Delta T \sim T_C \left( \frac{A}{T_C^4} \right)^{\frac{\delta}{\delta - 1}} \frac{\sin \alpha_1}{\cos \alpha_1} \left( \frac{\sin \alpha_1}{w \sin \alpha_1 - \alpha_2} \right)^{\frac{\delta + 1}{\delta - 1}}
\]

**Weak dependence on \( w \), while smaller \( \alpha_1 - \alpha_2 \) yields a larger critical region for the same \( w, \rho \):**

\[
\Delta \mu_B \sim w^{1/7}, \quad \Delta T \sim w^{-3/7} \quad \Delta \mu_B \sim \sin \alpha_1 - \alpha_2^{-6/7}, \quad \Delta T \sim \sin \alpha_1 - \alpha_2^{-3/7}
\]

Along the critical chemical potential line, \( r \sim 0 \):

\[
\chi_4^{\text{Ising}} \sim AG_{\mu\mu\mu}(0,h) \sim AG_{h\bar{h}h\bar{h}}(0,h) h_\mu^4
\]
Parameter choices

❖ Two sets of parameters

<table>
<thead>
<tr>
<th></th>
<th>$\mu_{BC}$</th>
<th>$T_C$</th>
<th>$\alpha_1$</th>
<th>$\alpha_2 - \alpha_1$</th>
<th>$w$</th>
<th>$\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I.</td>
<td>420 MeV</td>
<td>138 MeV</td>
<td>4.6°</td>
<td>90°</td>
<td>0.5, 1, 2</td>
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</tr>
</tbody>
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❖ Large $\mu_{BC}$ to allow for maximum freedom in parameter choice within the range of the Taylor expansion.

❖ $T_C$ and $\alpha_1$ not free $\rightarrow$ follow from choice of $\mu_{BC}$.

❖ Compute critical contribution to $\chi_4^B$ for each choice of parameters.
Critical contribution to $\chi_4^B$

$(\alpha_2 - \alpha_1 = 90^\circ)$

green $\rightarrow$ yellow: positive values, blue: negative.
orange curve: parameterized transition line.

- Smaller $w$ yields larger critical region.
- $\rho$ stretches critical region along $\mu_B$
- Approach to critical point characterized by peak rather than dip, except for immediate vicinity.
Critical contribution to $\chi^B_4$

$(\alpha_2 - \alpha_1 = -3^\circ)$

green → yellow: positive values, blue: negative.
orange curve: parameterized transition line.

❖ Negative angle difference bends negative lobe downwards.

❖ Agrees with leading singularity prediction.

❖ Note: this choice is not thermodynamically stable under current mapping.
Peak sign of CP, not the dip

Dip only appears for one choice*, close to the vicinity of the transition line.

*shown in this work
Why the difference?

- Ising to QCD map introduces mixing between r, h:

\[ \partial_{\mu_B} = \frac{1}{\omega \rho T_C \sin \alpha_1 - \alpha_2} (\sin \alpha_1 \partial_h + \sin \alpha_2 \partial_r) \]

- h has larger scaling dimension: dominant contribution close to the critical point.
- Since \( \alpha_1 \) is small, when is \( \alpha_2 \) not small \( \rightarrow \) h contribution becomes suppressed (orthogonal case).
- Taking most divergent terms corresponds to \( \partial_{\mu_B} \sim \partial_h \)
- **but** subleading terms may dominate if leading term is sufficiently suppressed.
Parameter space scan (preliminary)

- Introduction of free-parameters with Ising QCD map allows for thermodynamically unstable realizations.
- ML classifier assisted classification of entire parameter space.

Preliminary results show:

i) Larger $\mu_{BC}$ is preferred
ii) Hard upper limit on $\rho$
iii) Small angles strongly disfavored
Conclusions, considerations, outlook

- Investigated behavior of $\chi_4^B$ in the presence of the critical point in the 3D Ising model universality class.
- $\chi_4^B$ can be affected by sub-leading terms.
- **Diverging peak is a robust signature** of the critical point.
- Dip only present in **very few, thermodynamically unstable** cases under current mapping.
- Current study: equilibrium properties of QCD EoS. HIC are dynamical systems, need EbE relativistic viscous BSQ hydro evolution + critical fluctuations + hadronic transport.
- Temperature difference between hadronization and freeze-out?
- Machine-learning-assisted study coming soon!