Fluctuation Measurements and Global Conservation Laws in the BES Program

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2021 RHIC/AGS Annual Users' Meeting

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- Correcting QCD susceptibilities for global B,Q,S conservation
- Proton number cumulants at BES from hydrodynamics

Acknowledgements:

M.I. Gorenstein, V. Koch, R. Poberezhnyuk, O. Savchuk, C. Shen, H. Stoecker

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Alexander von Humboldt Stiftung/Foundation

QCD phase diagram with heavy-ion collisions

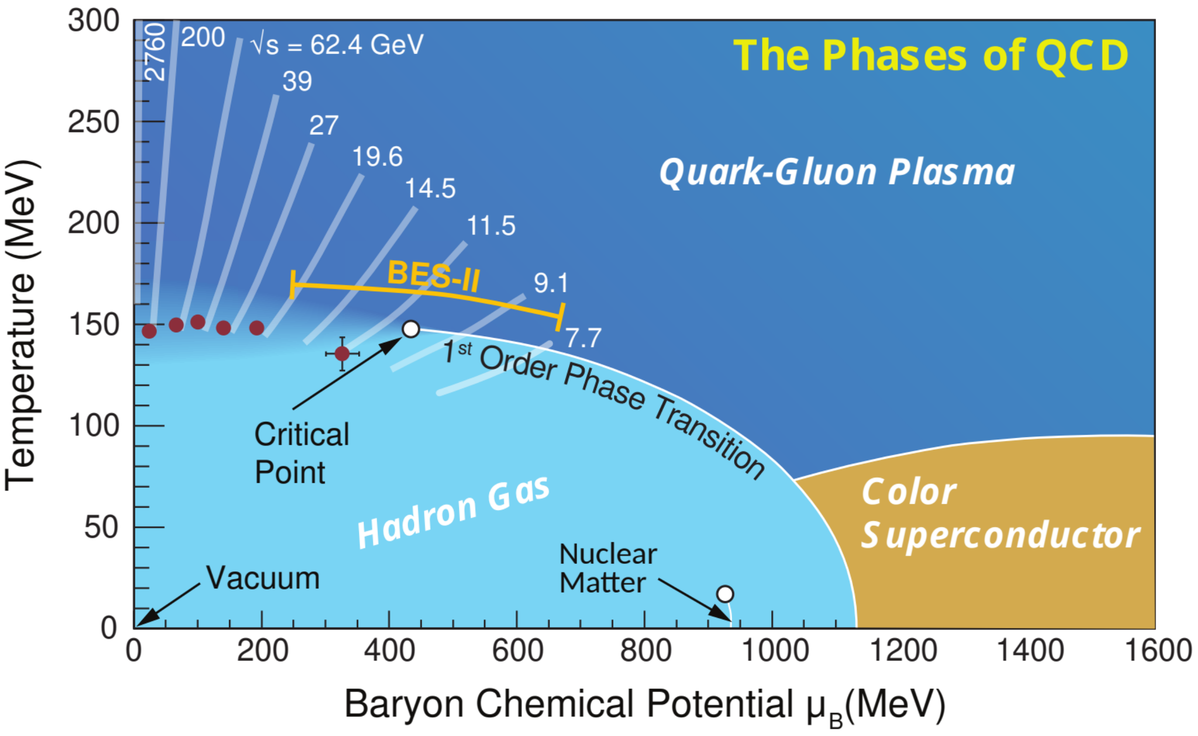
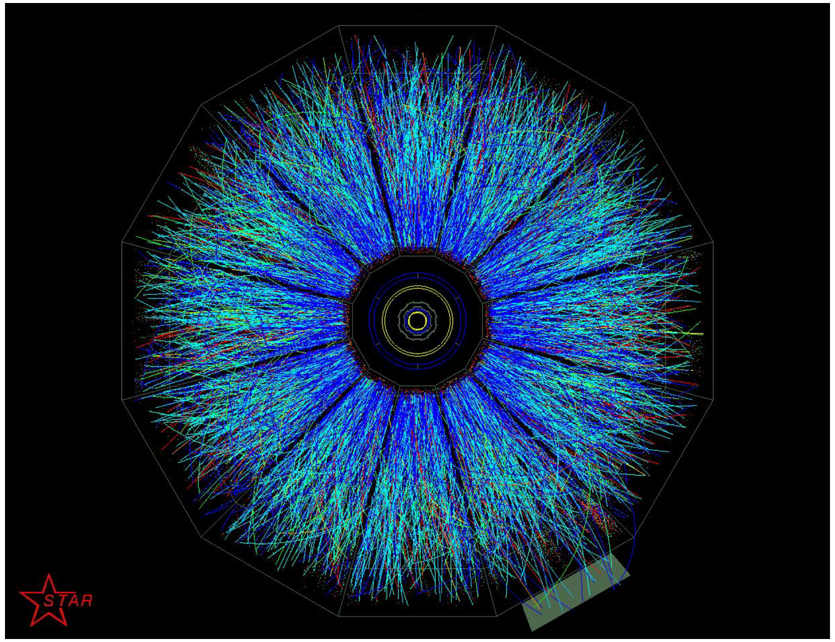


Figure from Bzdak et al., Phys. Rept. '20



STAR event display

Thousands of particles created in relativistic heavy-ion collisions

Apply concepts of statistical mechanics

Event-by-event fluctuations and statistical mechanics

Cumulant generating function

$$K_N(t) = \ln\langle e^{tN}\rangle = \sum_{n=1}^{\infty} \kappa_n \frac{t^n}{n!}$$

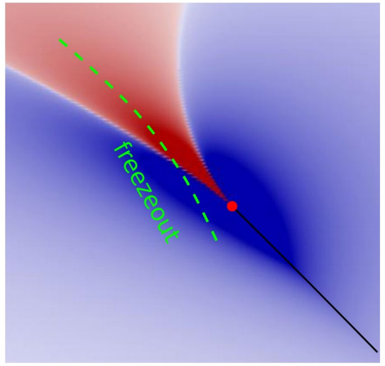
Grand partition function

$$\ln Z^{\text{gce}}(T,V,\mu) = \ln\left[\sum_N e^{\mu N/T} Z^{\text{ce}}(T,V,N)\right]$$

$$\kappa_n \propto \frac{\partial^n (\ln Z^{\text{gce}})}{\partial \mu^n}$$

Cumulants measure chemical potential derivatives of the (QCD) equation of state

- QCD critical point
- Test of (lattice) QCD at $\mu_B \approx 0$
- Freeze-out from fluctuations



M. Stephanov, PRL '09
Energy scans at RHIC (STAR)
and CERN-SPS (NA61/SHINE)

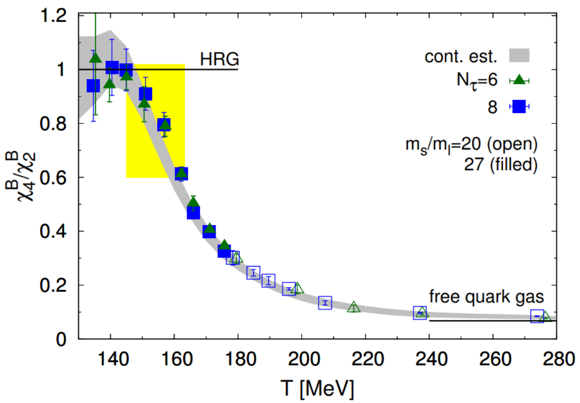
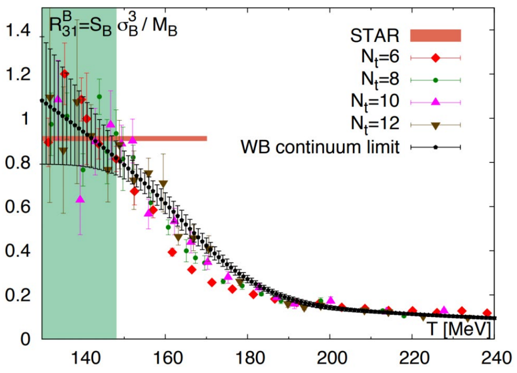


Figure from Bazavov et al. PRD 95, 054504 (2017)
Probed by LHC and top RHIC



Borsanyi et al. PRL 113, 052301 (2014)
Bazavov et al. PRL 109, 192302 (2012)
...

Theory vs experiment: Caveats

- **accuracy of the grand-canonical ensemble (global conservation laws)**
  - **subensemble acceptance method (SAM)**

    VV, Savchuk, Poberezhnyuk, Gorenstein, Koch, PLB 811, 135868 (2020)

- coordinate vs momentum space (thermal smearing)

  Ling, Stephanov, PRC 93, 034915 (2016); Ohnishi, Kitazawa, Asakawa, PRC 94, 044905 (2016)

- proxy observables in experiment (net-proton, net-kaon) vs actual conserved charges in QCD (net-baryon, net-strangeness)

  Kitazawa, Asakawa, PRC 85, 021901 (2012); VV, Jiang, Gorenstein, Stoecker, PRC 98, 024910 (2018)

- volume fluctuations

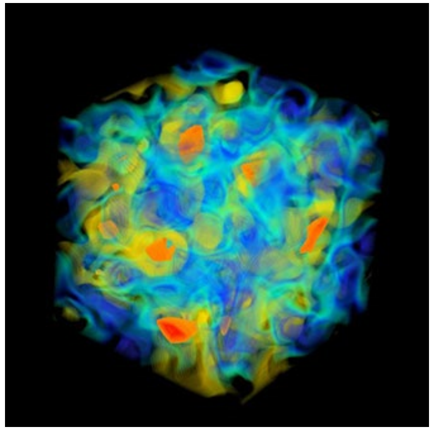
  Gorenstein, Gazdzicki, PRC 84, 014904 (2011); Skokov, Friman, Redlich, PRC 88, 034911 (2013)
  X. Luo, J. Xu, B. Mohanty, JPG 40, 105104 (2013); Braun-Munzinger, Rustamov, Stachel, NPA 960, 114 (2017)

- non-equilibrium (memory) effects

  Mukherjee, Venugopalan, Yin, PRC 92, 034912 (2015)

- hadronic phase

  Steinheimer, VV, Aichelin, Bleicher, Stoecker, PLB 776, 32 (2018)



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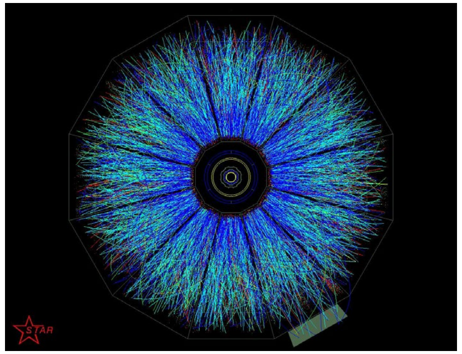


STAR event display

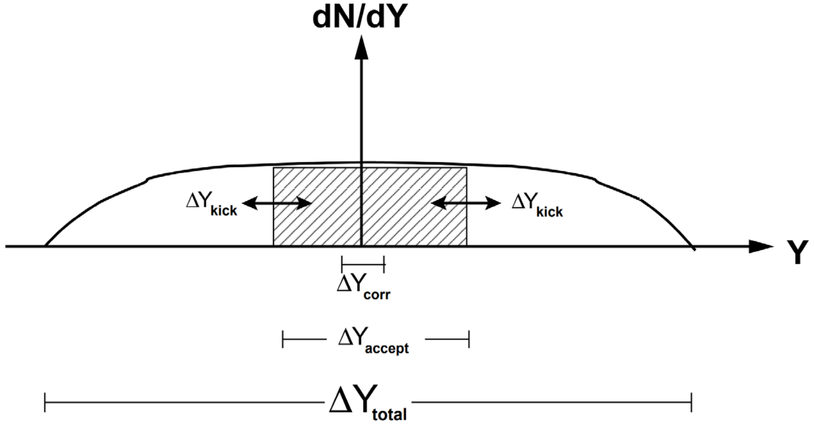
When are the measured fluctuations grand-canonical?

- Consider event-by-event fluctuations of particle number in acceptance ΔY_{accept} around midrapidity

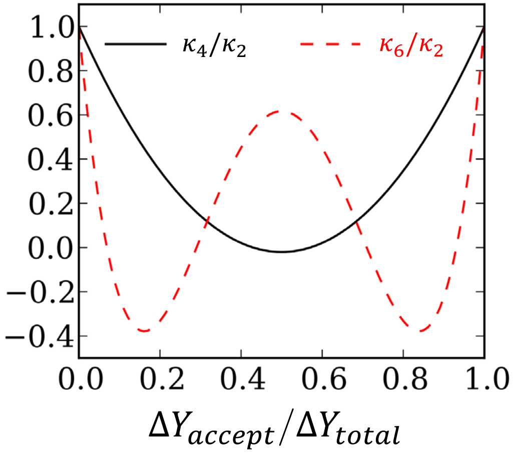
- Scales
  - ΔY_{accept} – acceptance
  - ΔY_{total} – full space
  - ΔY_{corr} – rapidity correlation length (thermal smearing)
  - ΔY_{kick} – diffusion in the hadronic phase

- **GCE applies if $\Delta Y_{total} \gg \Delta Y_{accept} \gg \Delta Y_{kick}, \Delta Y_{corr}$**

- In practice $\Delta Y_{total} \gg \Delta Y_{accept}$ and $\Delta Y_{accept} \gg \Delta Y_{corr}$ are not simultaneously satisfied
  - Corrections from global conservation are large [Bzdak et al., PRC '13]
  - $\Delta Y_{corr} \sim 1 \sim \Delta Y_{accept}$ [Ling, Stephanov, PRC '16]



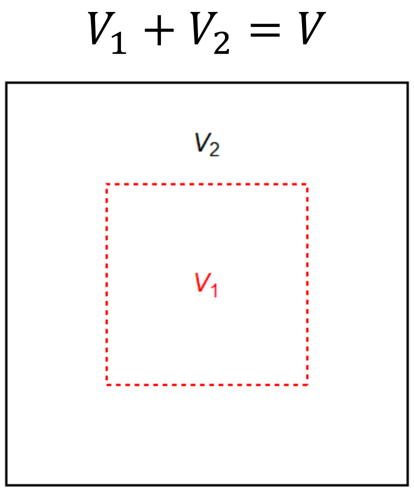
V. Koch, arXiv:0810.2520



Baryon number conservation 2.0: SAM

VV, Savchuk, Poberezhnyuk, Gorenstein, Koch, PLB 811, 135868 (2020)

Subensemble acceptance method (SAM) – method to correct *any* EoS (e.g. *lattice QCD*) for **charge conservation**



Partition a thermal system with a globally conserved charge B *(canonical ensemble)* into two subsystems which can exchange the charge

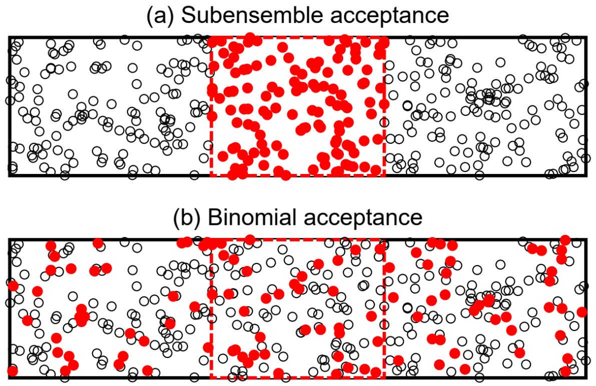
Assume thermodynamic limit:

$$V, V_1, V_2 \to \infty; \quad \frac{V_1}{V} = \alpha = const; \quad \frac{V_2}{V} = (1-\alpha) = const;$$

$$V_1, V_2 \gg \xi^3, \qquad \xi = \text{correlation length}$$

The canonical partition function then reads:

$$Z^{ce}(T,V,B) = \text{Tr}\, e^{-\beta \hat{H}} \approx \sum_{B_1} Z^{ce}(T,V_1,B_1)\, Z^{ce}(T,V-V_1,B-B_1)$$



The probability to have charge B_1 is:

$$P(B_1) \propto Z^{ce}(T,\alpha V,B_1)\, Z^{ce}(T,(1-\alpha)V,B-B_1), \qquad \alpha \equiv V_1/V$$

SAM: Computing the cumulants

VV, Savchuk, Poberezhnyuk, Gorenstein, Koch, PLB 811, 135868 (2020)

Cumulant generating function for B_1:

$$G_{B_1}(t) \equiv \ln\langle e^{t\,B_1}\rangle = \ln\left\{\sum_{B_1} e^{t\,B_1}\,\exp\left[-\frac{\alpha V}{T}\,f(T,\rho_{B_1})\right]\exp\left[-\frac{\beta V}{T}\,f(T,\rho_{B_2})\right]\right\} + \tilde{C} \qquad \beta = 1-\alpha$$

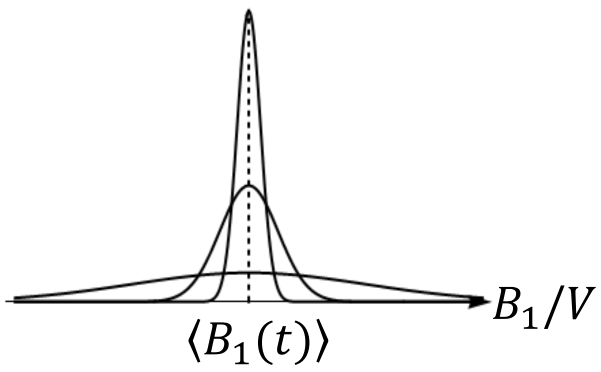
$$\tilde{\kappa}_1[B_1(t)] = \frac{\sum_{B_1} B_1\,\tilde{P}(B_1;t)}{\sum_{B_1}\tilde{P}(B_1;t)} \equiv \langle B_1(t)\rangle \qquad \text{with} \qquad \tilde{P}(B_1;t) = \exp\left\{tB_1 - V\,\frac{\alpha f(T,\rho_{B_1}) + \beta f(T,\rho_{B_2})}{T}\right\}.$$

Thermodynamic limit: $\tilde{P}(B_1;t)$ *highly peaked* at $\langle B_1(t)\rangle$

$\langle B_1(t)\rangle$ is a solution to equation $d\tilde{P}/\mathrm{d}B_1 = 0$:

$$t = \hat{\mu}_B[T,\rho_{B_1}(t)] - \hat{\mu}_B[T,\rho_{B_2}(t)]$$

$$\text{where} \quad \hat{\mu}_B \equiv \mu_B/T, \qquad \mu_B(T,\rho_B) = \partial f(T,\rho_B)/\partial\rho_B$$



$\boldsymbol{t = 0}$**:** $\rho_{B_1} = \rho_{B_2} = B/V$, $B_1 = \alpha B$, i.e. charge uniformly distributed between the subsystems

SAM: Cumulant ratios in terms of GCE susceptibilities

$$\kappa_n[B_1] = \left.\frac{\partial^{n-1}\tilde{\kappa}_1[B_1(t)]}{\partial t^{n-1}}\right|_{t=0} \quad \longleftrightarrow \quad \frac{\partial^n}{\partial t^n}:\ t = \hat{\mu}_B[T,\rho_{B_1}(t)] - \hat{\mu}_B[T,\rho_{B_2}(t)]$$

scaled variance
$$\frac{\kappa_2[B_1]}{\kappa_1[B_1]} = (1-\alpha)\,\frac{\chi_2^B}{\chi_1^B},$$

$$\chi_n^B \equiv \partial^{n-1}(\rho_B/T^3)/\partial(\mu_B/T)^{n-1}$$

skewness
$$\frac{\kappa_3[B_1]}{\kappa_2[B_1]} = (1-2\alpha)\,\frac{\chi_3^B}{\chi_2^B},$$

kurtosis
$$\frac{\kappa_4[B_1]}{\kappa_2[B_1]} = (1-3\alpha\beta)\,\frac{\chi_4^B}{\chi_2^B} - 3\alpha\beta\left(\frac{\chi_3^B}{\chi_2^B}\right)^2.$$

- Global conservation (α) and equation of state (χ_n^B) effects factorize in cumulants up to the 3rd order, starting from κ_4 not anymore
- $\alpha \to 0$ – GCE limit*, $\alpha \to 1$ – CE limit

*As long as $V_1 \gg \xi^3$ holds

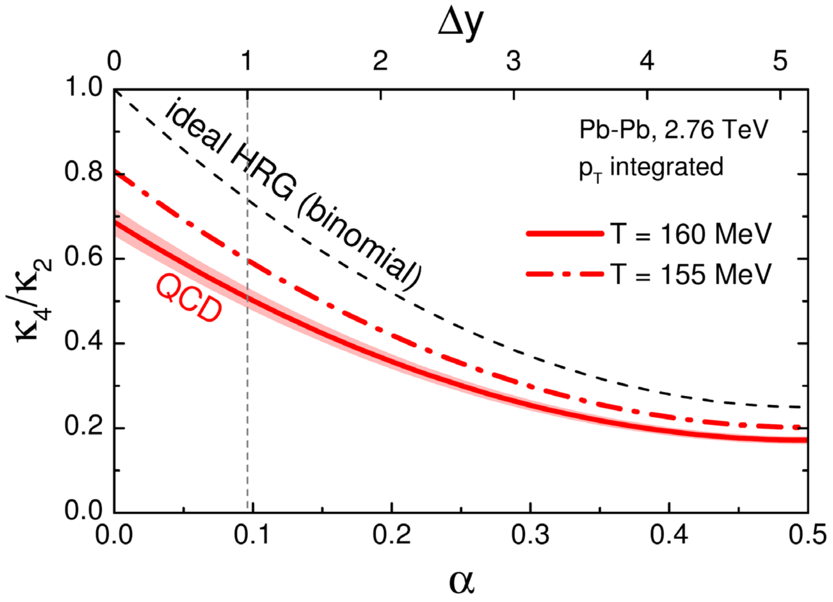
For *multiple conserved charges* (joint B,Q,S cumulants up to 6th order)

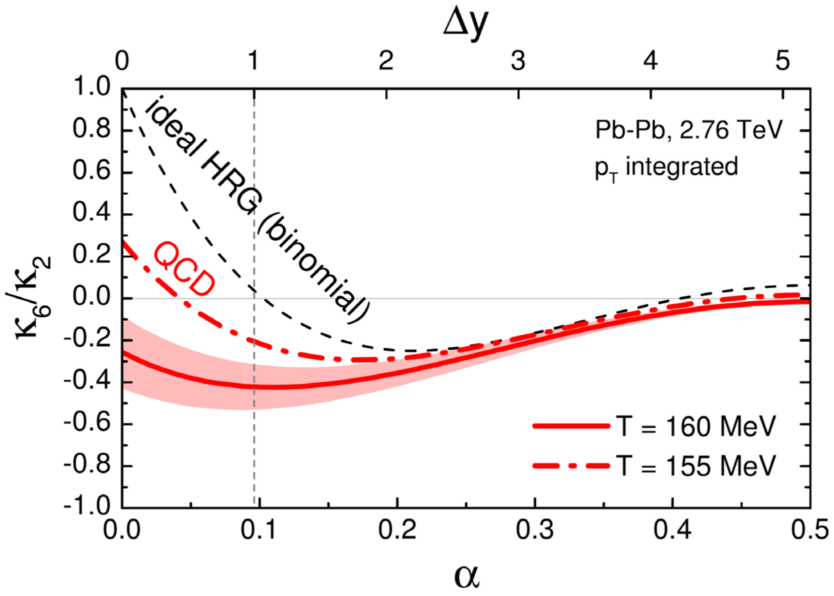
see VV, Poberezhnyuk, Koch, JHEP 10, 089 (2020)

Net baryon fluctuations at LHC ($\mu_B = 0$)

VV, Savchuk, Poberezhnyuk, Gorenstein, Koch, PLB 811, 135868 (2020)

$$\left(\frac{\kappa_4}{\kappa_2}\right)_{LHC} = (1-3\alpha\beta)\,\frac{\chi_4^B}{\chi_2^B} \qquad \left(\frac{\kappa_6}{\kappa_2}\right)_{LHC} = [1-5\alpha\beta(1-\alpha\beta)]\,\frac{\chi_6^B}{\chi_2^B} - 10\alpha(1-2\alpha)^2\beta\left(\frac{\chi_4^B}{\chi_2^B}\right)^2$$





Lattice data for χ_4^B/χ_2^B and χ_6^B/χ_2^B from Borsanyi et al., 1805.04445

Theory: negative χ_6^B/χ_2^B is a possible signal of chiral criticality [Friman, Karsch, Redlich, Skokov, EPJC '11]

Experiment: $\alpha \approx \frac{N_{ch}(\Delta y)}{N_{ch}(\infty)} \approx \mathrm{erf}\left(\frac{\Delta y}{2\sqrt{2}\sigma_y}\right)$, for $\Delta y \approx 1$ the κ_6/κ_2 is mainly sensitive to the EoS

$N_{ch}(\Delta y)$ measurement: ALICE Collaboration, PLB 726 (2013) 610-622

SAM-2.0

VV, in preparation

SAM limitations: uniform thermal system and **coordinate** space

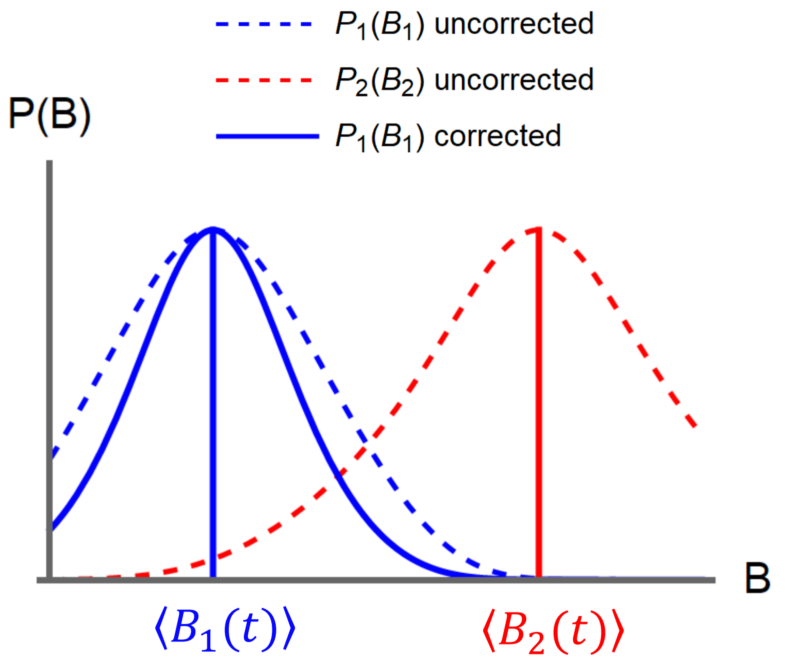
SAM-2.0: apply the correction for *arbitrary* distributions inside and outside the acceptance that are peaked at the mean

- Spatially inhomogeneous systems (e.g. RHIC)
- Momentum space
- Non-conserved quantities (e.g. proton number)
- Map "grand-canonical" cumulants inside and outside the acceptance to the "canonical" cumulants inside the acceptance

$$\kappa_{p,B}^{\text{in,ce}} = \text{SAM}\left[\kappa_{p,B}^{\text{in,gce}}, \kappa_{p,B}^{\text{out,gce}}\right]$$

↓

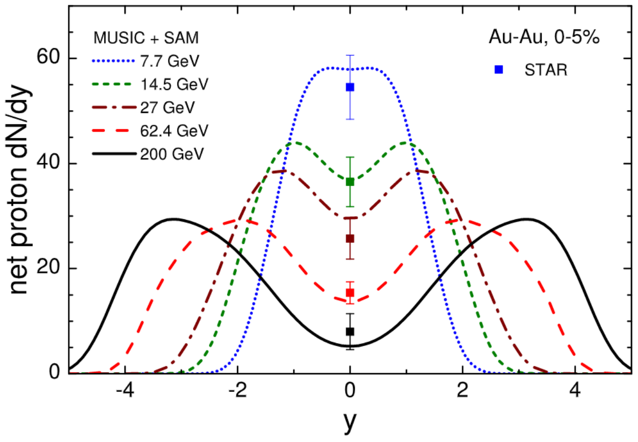
*Applicable at **RHIC-BES***



Proton cumulants at RHIC-BES from hydro

VV, C. Shen, V. Koch, in preparation

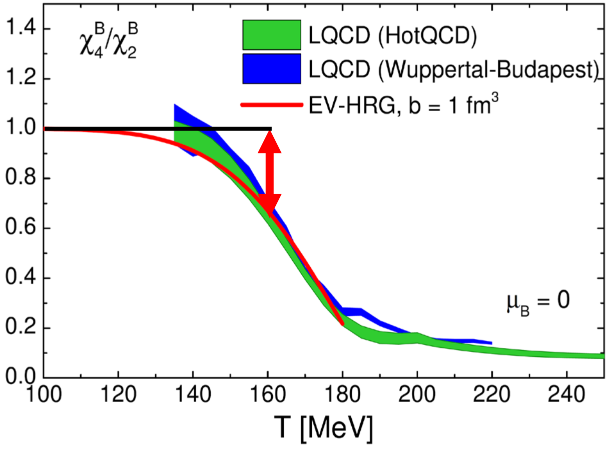
- Collision geometry based 3D initial state [Shen, Alzhrani, PRC '20]
  - Constrained to net proton distributions



- Viscous hydrodynamics evolution – MUSIC-3.0
  - NEOS-BSQ equation of state [Monnai, Schenke, Shen, PRC '19]
  - Shear viscosity via IS-type equation

- Cooper-Frye particlization at $\epsilon_{sw} = 0.26$ GeV/fm^3

$$\omega_p \frac{dN_j}{d^3p} = \int_{\sigma(x)} d\sigma_\mu(x)\, p^\mu \, \frac{d_j\, \lambda_j^{\rm ev}(x)}{(2\pi)^3} \, \exp\left[\frac{\mu_j(x) - u^\mu(x) p_\mu}{T(x)}\right].$$



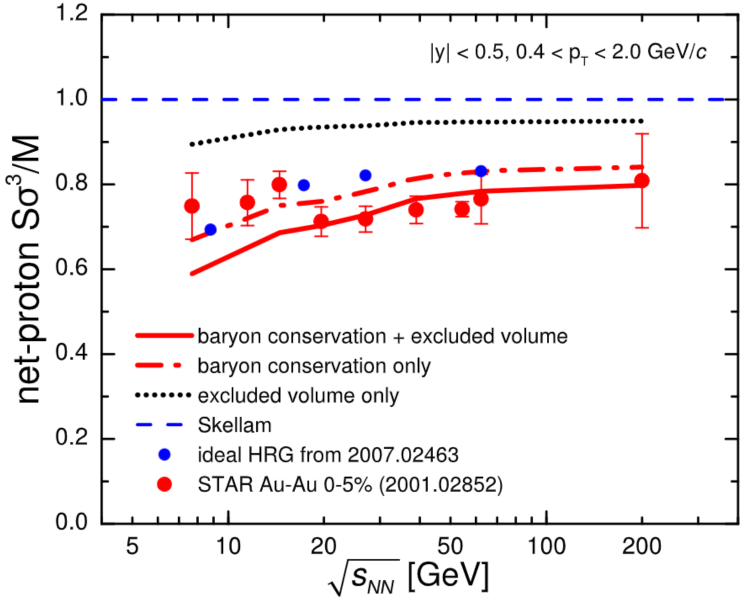
- Particlization includes QCD-based baryon number distribution
  - Here incorporated via baryon excluded volume

details in [VV, Koch, PRC 103, 044903 (2021)]

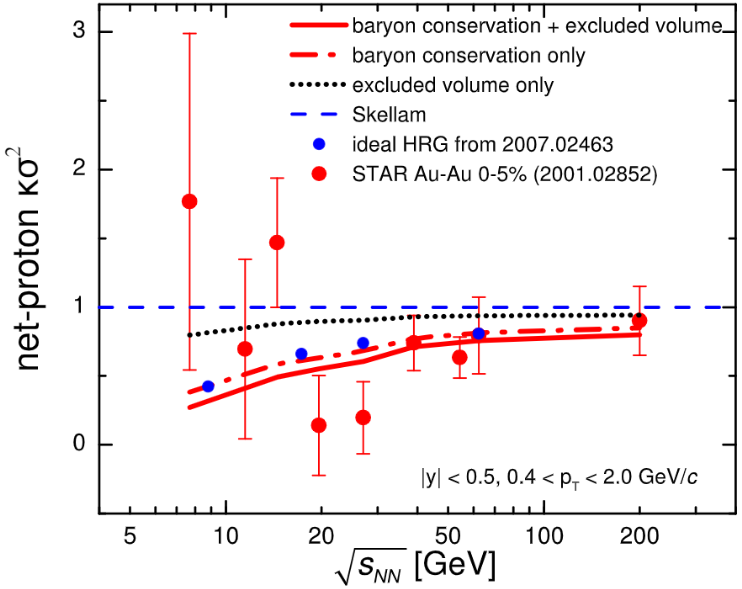
- Correction for global baryon conservation via SAM-2.0

Net proton cumulant ratios

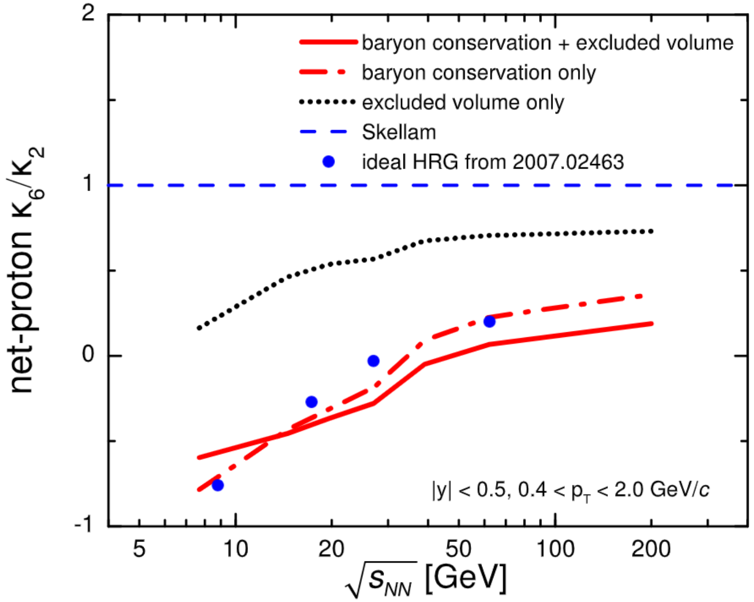
κ_3/κ_1



κ_4/κ_2



κ_6/κ_2



- Both the baryon conservation and repulsion needed to describe data at $\sqrt{s_{NN}} \geq 20$ GeV quantitatively
- Effect from baryon conservation is larger than from repulsion
- Canonical ideal HRG limit is consistent with the data-driven study of [Braun-Munzinger et al., 2007.02463]
- κ_6/κ_2 turns negative at $\sqrt{s_{NN}}$~50 GeV

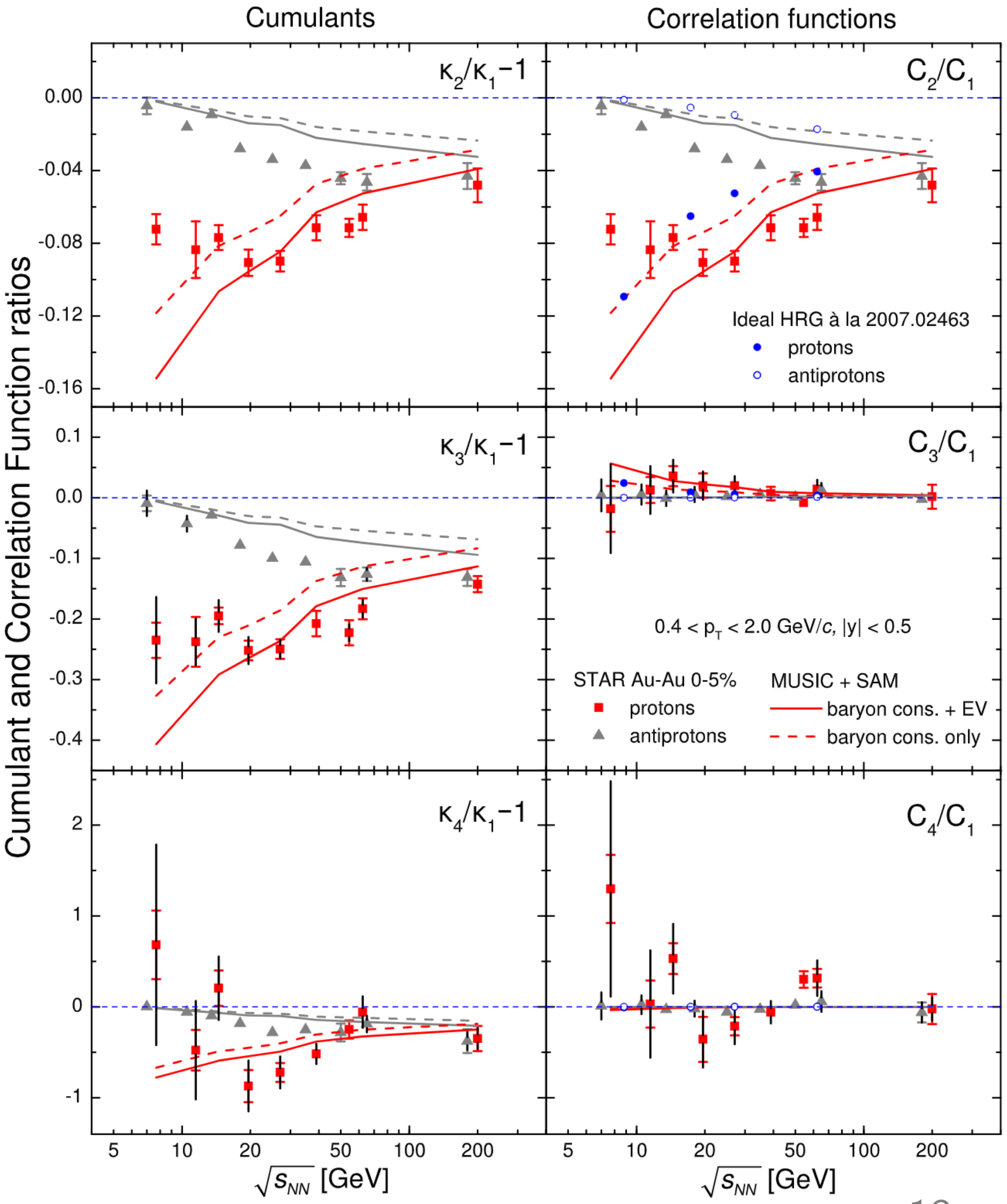
Cumulants vs Correlation Functions

- Analyze genuine multi-particle correlations via **factorial cumulants** [Bzdak, Koch, Strodthoff, PRC '17]

$$\hat{C}_1 = \kappa_1, \qquad \hat{C}_3 = 2\kappa_1 - 3\kappa_2 + \kappa_3,$$
$$\hat{C}_2 = -\kappa_1 + \kappa_2, \quad \hat{C}_4 = -6\kappa_1 + 11\kappa_2 - 6\kappa_3 + \kappa_4.$$

- Three- and four-particle correlations are small
  - Higher-order cumulants are driven by two-particle correlations
  - Small positive $\hat{C}_3/\hat{C}_1$ in the data is explained by baryon conservation + excluded volume
  - Strong multi-particle correlations would be expected near the critical point [Ling, Stephanov, 1512.09125]

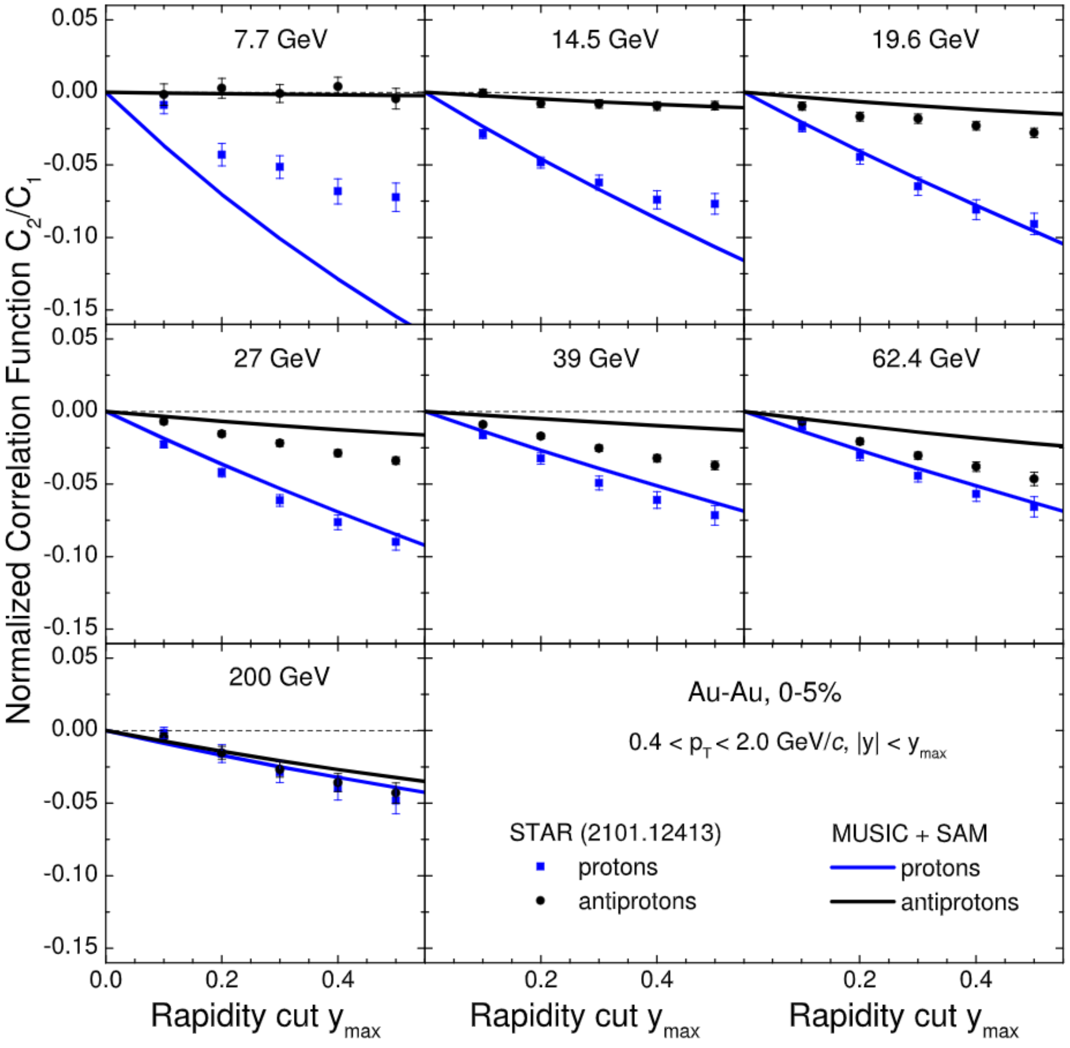
- Two-particle correlations are negative
  - Protons at $\sqrt{s_{NN}} \leq 14.5$ GeV overestimated
  - Antiprotons at $19.6 \leq \sqrt{s_{NN}} \leq 62.4$ GeV underestimated



*We use the notation for (factorial) cumulants from Bzdak et al., Phys. Rept. '20. This is different from STAR's 2101.12413 where it is reversed

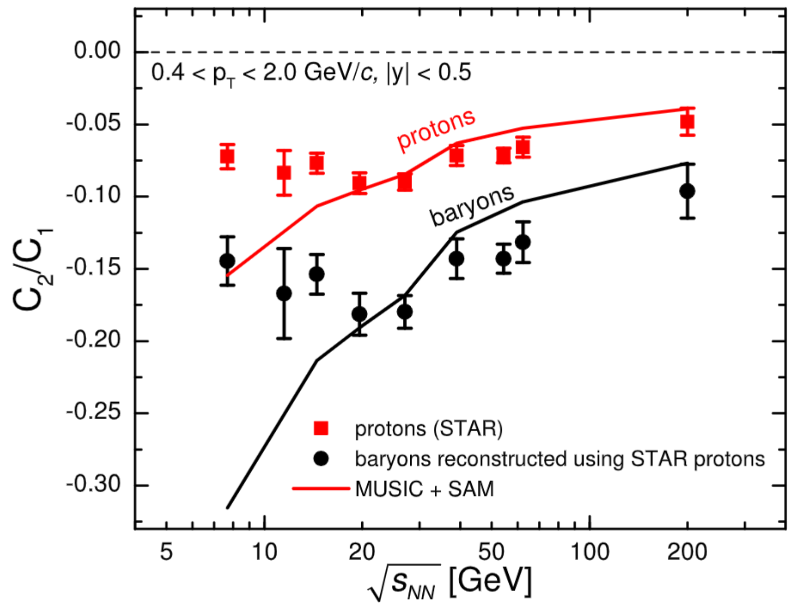
Acceptance dependence of two-particle correlations

- Qualitative agreement with the STAR data
- Data indicate a changing y_{max} slope at $\sqrt{s_{NN}} \leq 14.5$ GeV
- Volume fluctuations? [Skokov, Friman, Redlich, PRC '13]
  - Can improve low energies but spoil high energies?
- Exact electric charge conservation?
  - Worsens the agreement at $\sqrt{s_{NN}} \leq 14.5$, higher energies virtually unaffected (see backup)
- Attractive interactions?
  - Could work if baryon repulsion switches to attraction in the high-μ_B regime
  - **Critical point?**

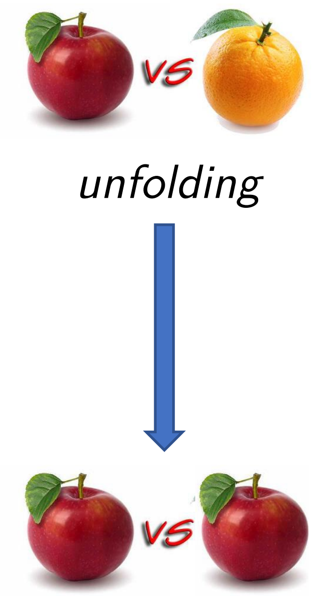


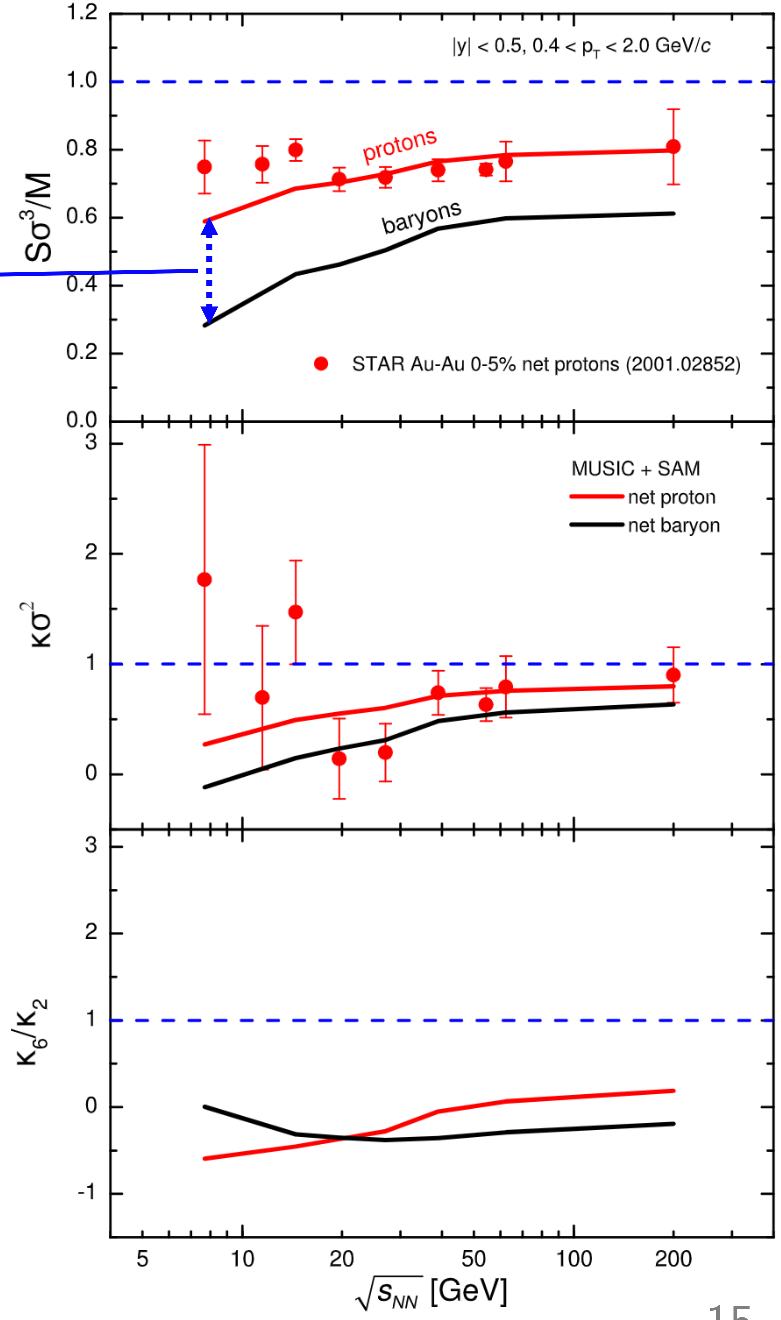
Net baryon vs net proton

- **net baryon** $\neq$ **net proton**
- Baryon cumulants can be reconstructed from proton cumulants via binomial (un)folding based on isospin randomization [Kitazawa, Asakawa, Phys. Rev. C 85 (2012) 021901]
  - Requires the use of joint factorial moments, only experiment can do it model-independently



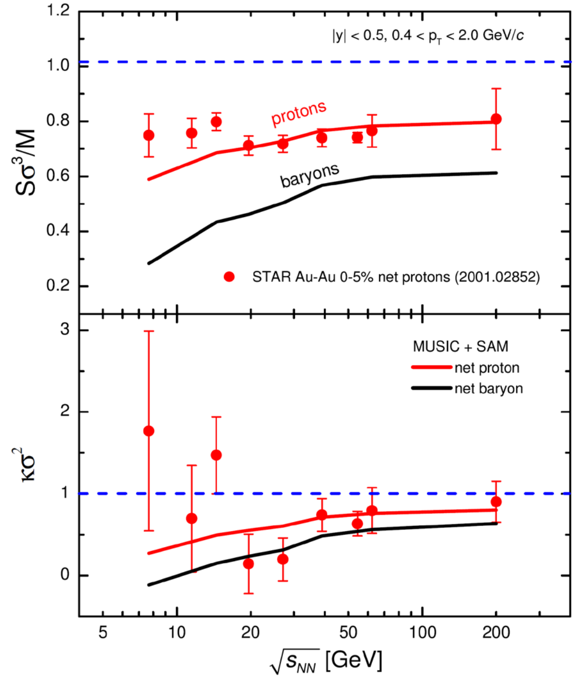
$$\frac{\hat{C}_2^B}{\hat{C}_1^B} \approx 2\,\frac{\hat{C}_2^p}{\hat{C}_1^p}$$

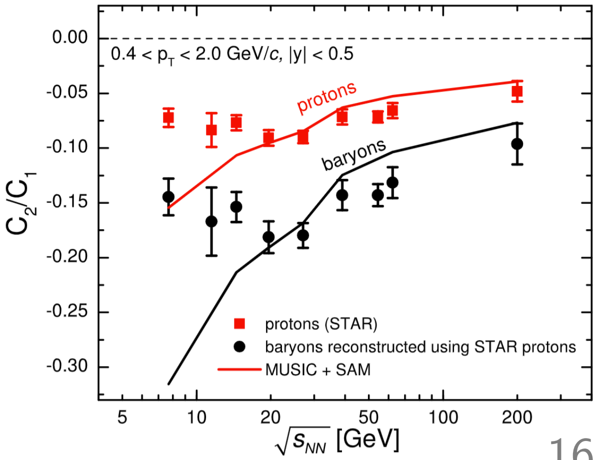




Summary

- Fluctuations are a powerful tool to explore the QCD phase diagram
- SAM corrects QCD cumulants in heavy-ion collisions for global conservation of (multiple) charges
  - important link between theory and experiment
- Quantitative analysis of proton cumulants at $\sqrt{s_{NN}}$=7.7-200 GeV
  - Data at $\sqrt{s_{NN}} > 20$ GeV consistent with baryon conserv. + excluded volume
  - Possible evidence for attractive proton interactions at $\sqrt{s_{NN}} \leq 14.5$ GeV
  - Need to unfold baryon cumulants from measured protons
  - Small three- and four-particle correlations in absence of critical point effects, ordinary cumulants driven by two-particle correlations





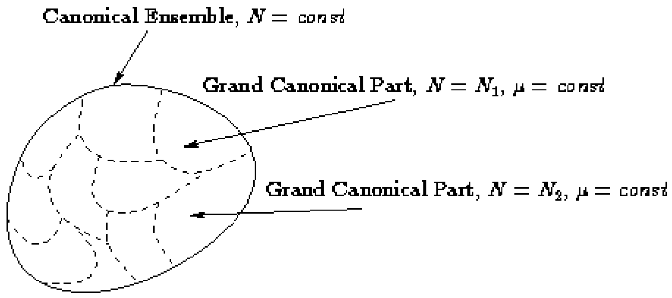
Thanks for your attention!

Backup slides

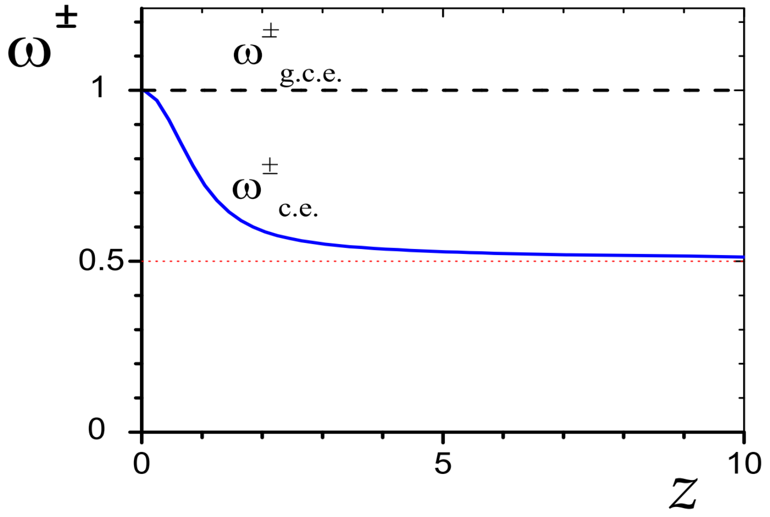
Canonical vs grand-canonical

Grand-canonical ensemble: the system exchanges conserved charges with a heat bath

Canonical ensemble: conserved charges fixed to a same set of values in all microstates



Thermodynamic equivalence: in the limit $V \to \infty$ all statistical ensembles are equivalent wrt to all average quantities, e.g. $\langle N \rangle_{GCE} = N_{CE}$



Begun, Gorenstein, Gazdzicki, Zozulya, PRC '04

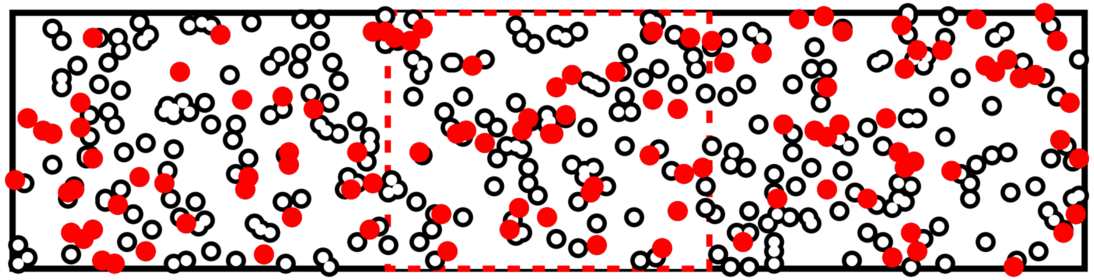
Thermodynamic equivalence does *not* extend to fluctuations. The results are ensemble-dependent in the limit $V \to \infty$

So what ensemble should one use?

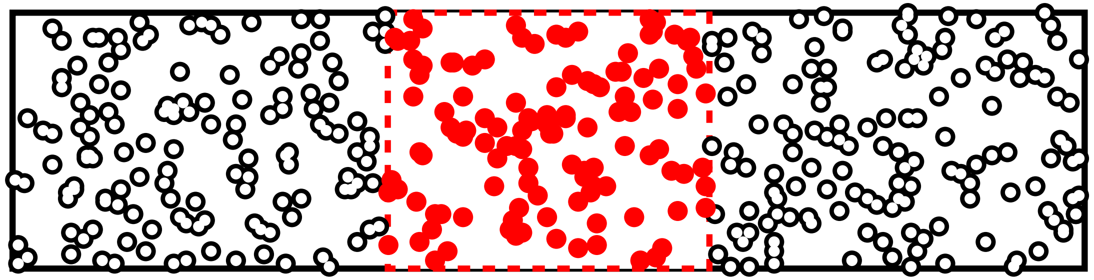
Canonical? Grand-canonical? Something else?

Binomial acceptance vs the actual acceptance

Binomial acceptance: accept each particle (charge) with a probability α independently from all other particles



SAM:



SAM for multiple conserved charges (B,Q,S)

VV, Poberezhnyuk, Koch, JHEP 10, 089 (2020)

Key findings:

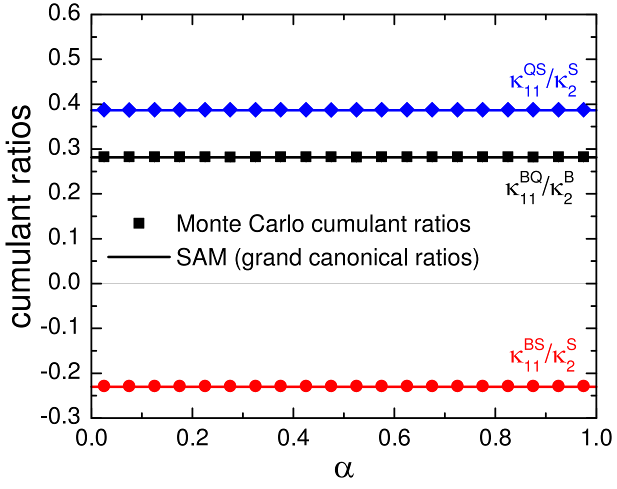
- Cumulants up to 3rd order factorize into product of binomial and grand-canonical cumulants

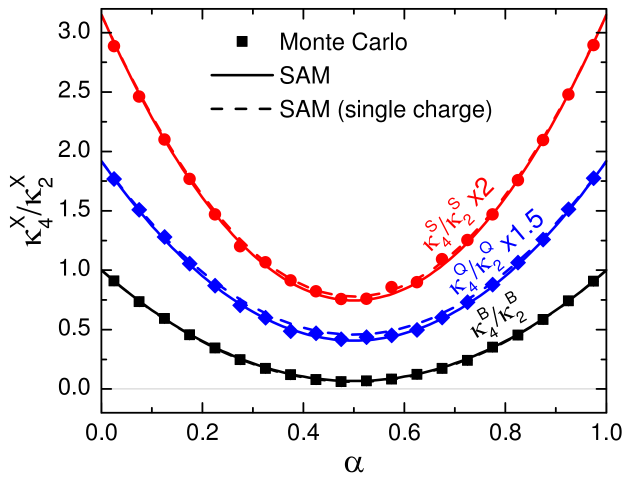
$$\kappa_{l,m,n} = \kappa^{\text{bino}}_{l+m+n}(\alpha) \times \kappa^{\text{gce}}_{l,m,n}\,, \qquad l+m+n \leq 3$$

  - Ratios of second and third order cumulants are NOT sensitive to charge conservation
  - Also true for the measurable ratios of covariances involving one non-conserved charge, such as κ_{pQ}/κ_{kQ}

- For order $n > 3$ charge cumulants "mix". Effect in HRG is tiny

$$\kappa_4^B = \kappa_4^{B,\text{gce}}\,\beta\left[(1-3\alpha\beta)\,\chi_4^B - 3\,\alpha\,\beta\,\frac{(\chi_3^B)^2\chi_2^Q - 2\chi_{21}^{BQ}\chi_{11}^{BQ}\chi_3^B + (\chi_{21}^{BQ})^2\chi_2^B}{\chi_2^B\chi_2^Q - (\chi_{11}^{BQ})^2}\right]$$





Experiment: Measurements of the off-diagonal cumulants are in progress, e.g. [STAR Collaboration, arXiv:1903.05370]

Mathematica notebook to express any B,Q,S-cumulant of order $n \leq 6$ in terms of grand-canonical susceptibilities available at https://github.com/vlvovch/SAM

Calculating cumulants at particlization

- Strategy:
  1. Calculate proton cumulants in experimental acceptance in the grand-canonical limit*
  2. Apply correction for exact baryon number conservation

First step:

- Sum contributions from each fluid element x_i
  - Cumulants of joint (anti)proton/(anti)baryon distribution
  - Assumes small correlation length $\xi \to 0$

$$\kappa_{n,m}^{B^\pm,p^\pm,\mathrm{gce}}(\Delta p_{\mathrm{acc}}) = \sum_{i \in \sigma} \delta\kappa_{n,m}^{B^\pm,p^\pm,\mathrm{gce}}(x_i; \Delta p_{\mathrm{acc}})$$

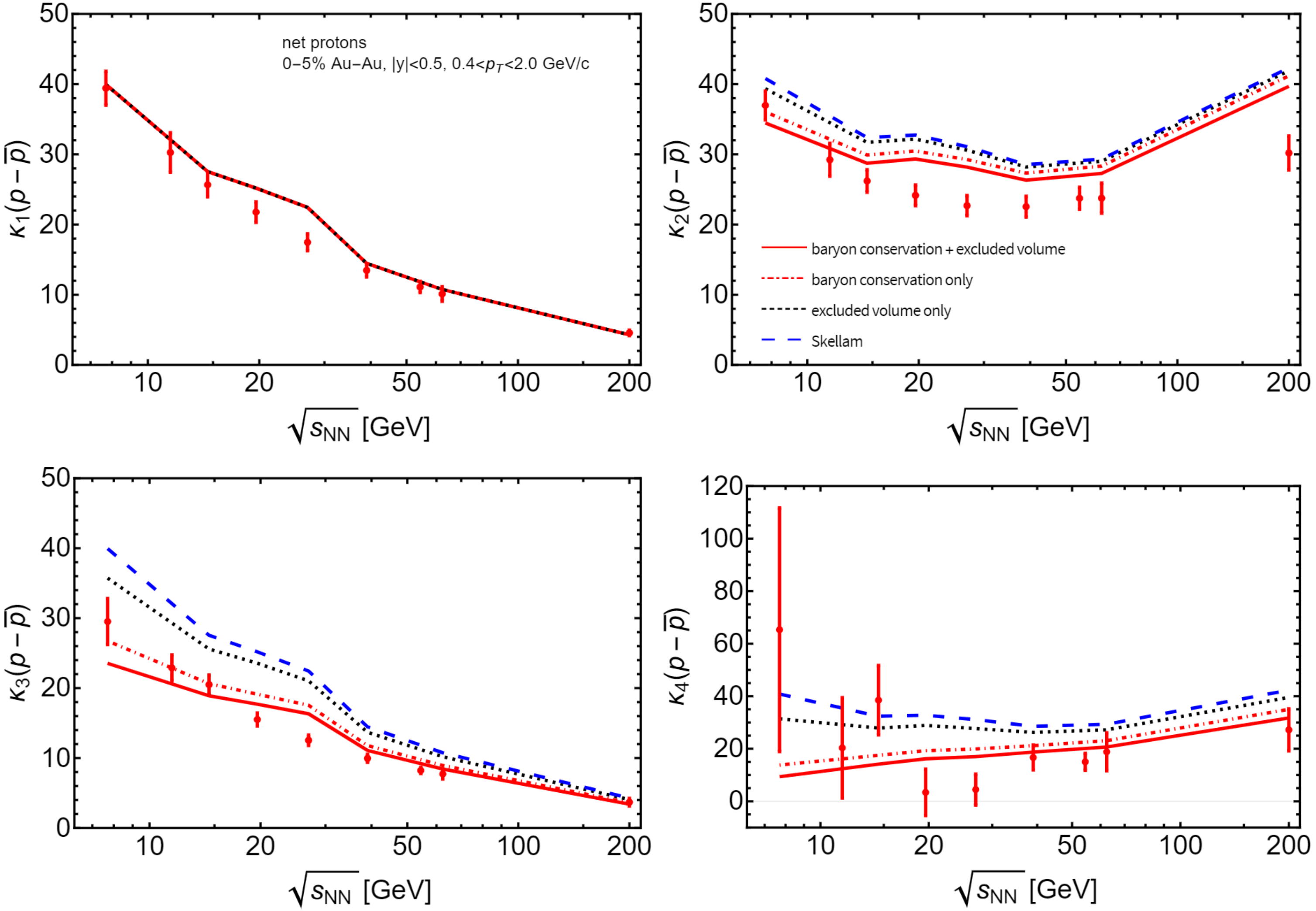
- To compute each contribution
  - Grand-canonical susceptibilities $\chi^{B^\pm}(x_i)$ of (anti)baryon number
  - Each baryon ends up in acceptance Δp_{acc} with binomial probability
  - Each baryon is a proton with probability $q(x_i) = \langle N_p(x_i) \rangle / \langle N_B(x_i) \rangle$

[Kitazawa, Asakawa, Phys. Rev. C 85 (2012) 021901]

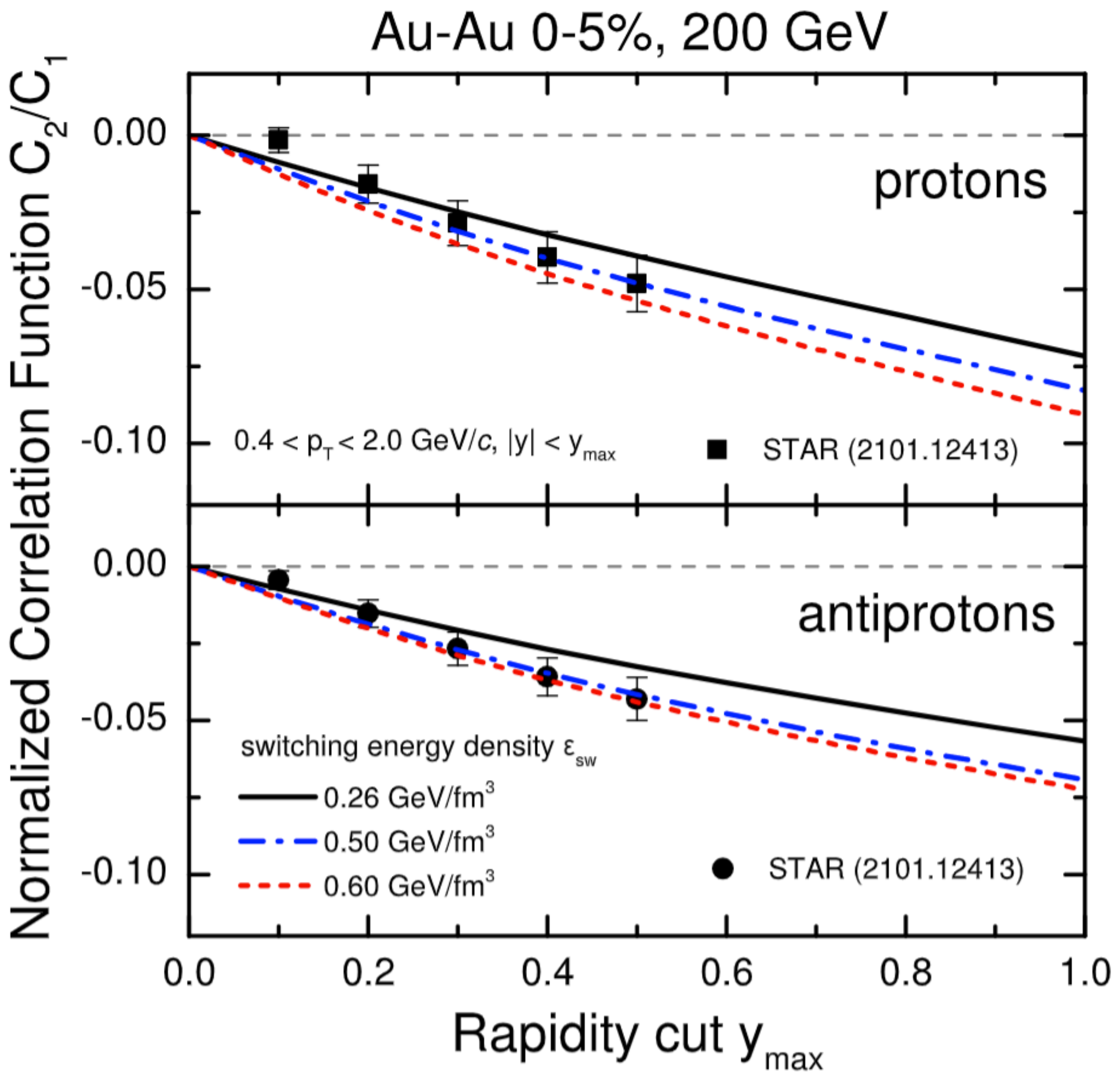
$$p_{\mathrm{acc}}(x_i; \Delta p_{\mathrm{acc}}) = \frac{\int_{p \in \Delta p_{\mathrm{acc}}} \frac{d^3p}{\omega_p} \delta\sigma_\mu(x_i)\, p^\mu\, f[u^\mu(x_i) p_\mu; T(x_i), \mu_j(x_i)]}{\int \frac{d^3p}{\omega_p} \delta\sigma_\mu(x_i)\, p^\mu\, f[u^\mu(x_i) p_\mu; T(x_i), \mu_j(x_i)]}.$$

*For similar calculations of critical fluctuations see Ling, Stephanov, 1512.09125 and Jiang, Li, Song, 1512.06164

Net proton cumulants at RHIC

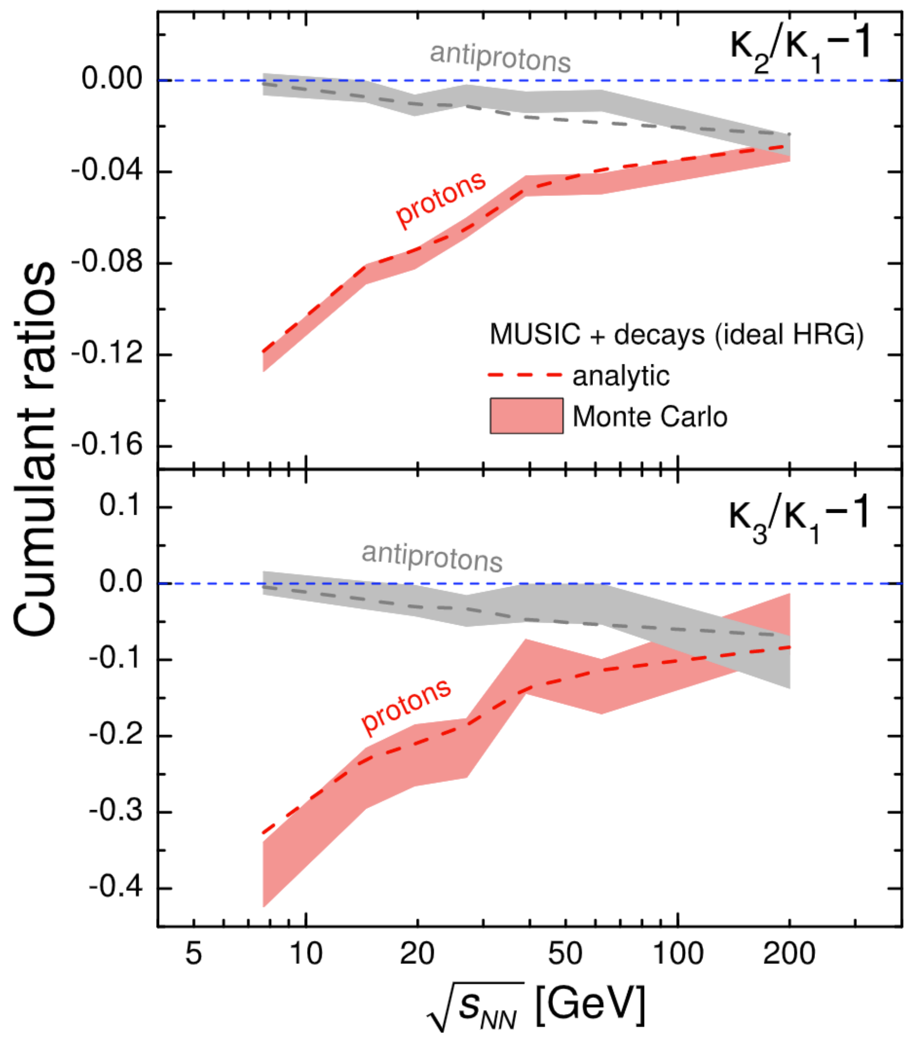


Dependence on the switching energy density



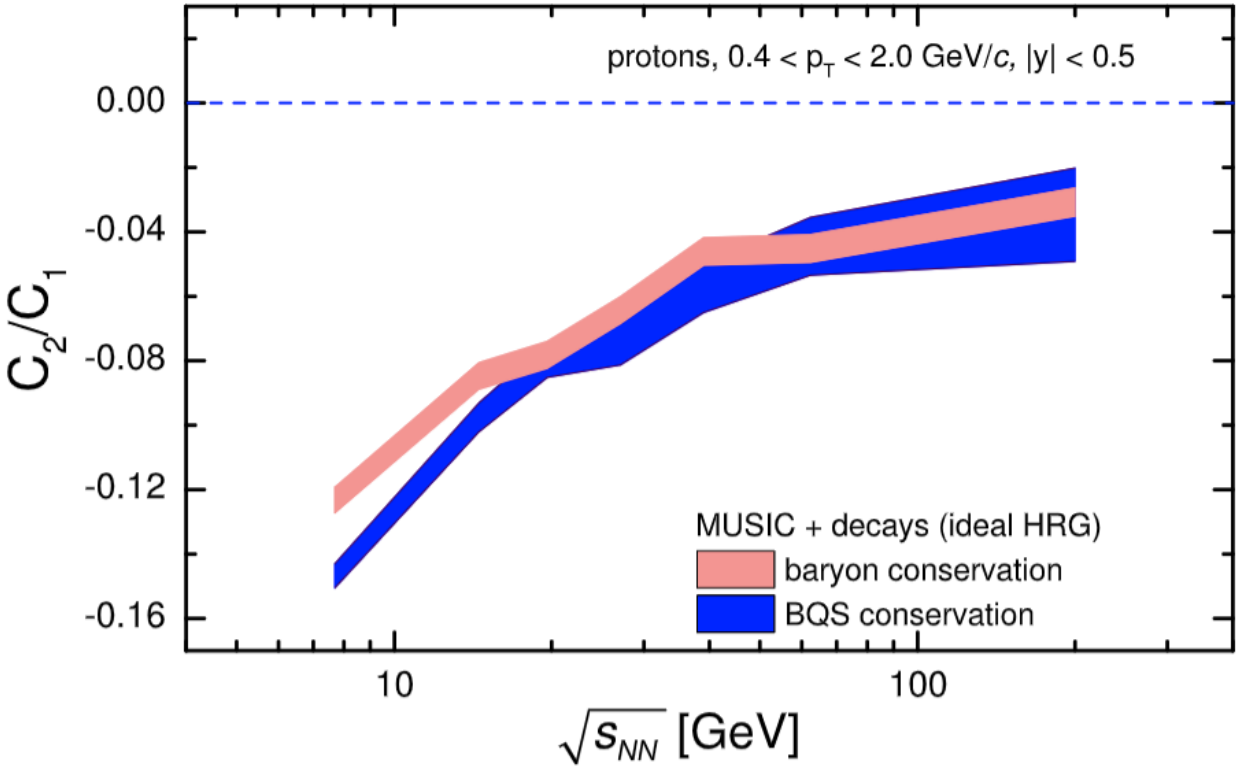
Cross-checking the cumulants with Monte Carlo

- Sample canonical ideal HRG model at particlization with Thermal-FIST
- Analytic results agree with Monte Carlo within errors



Exact conservation of electric charge

- Sample ideal HRG model at particlization with exact conservation of baryon number, electric charge, and strangeness using Thermal-FIST
- Protons are affected by electric charge conservation at $\sqrt{s_{NN}} \leq 14.5$



Effect of the hadronic phase

Sample ideal HRG model at particlization with exact conservation of baryon number using Thermal-FIST and run through hadronic afterburner UrQMD