Open Quantum Systems for Quarkonia

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Review: XY, 2102.01736

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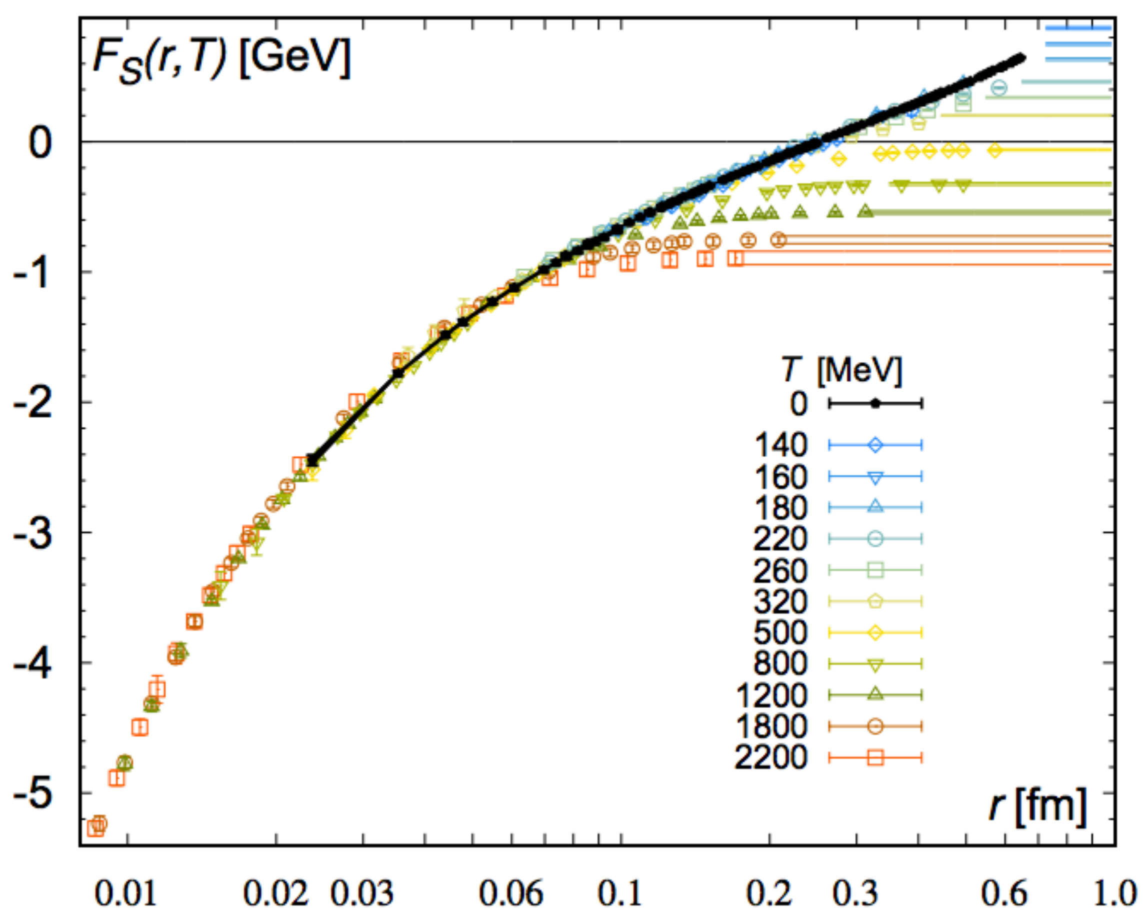
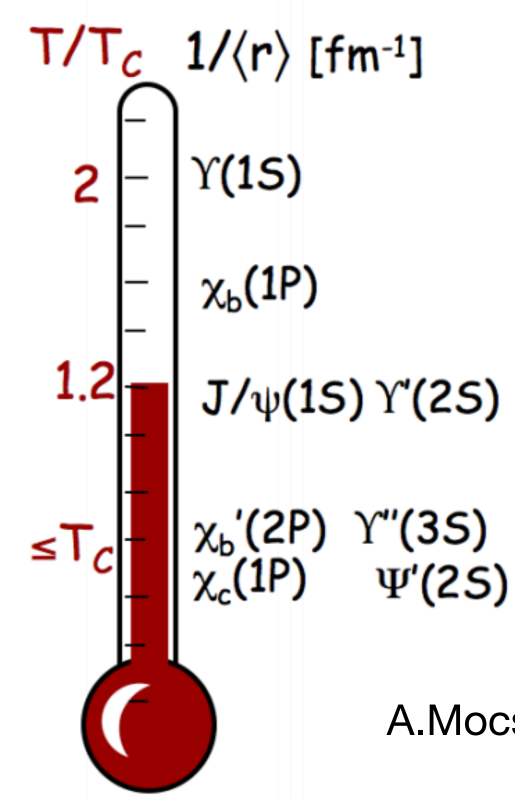
Quarkonium as Probe of Quark-Gluon Plasma

- **Static screening**: suppression of color attraction —> melting at high T —> reduced production —> thermometer

$$T = 0 : V(r) = -\frac{A}{r} + Br \longrightarrow T \neq 0 :$$ Confining part flattened



1804.10600, A.Bazavov, N.Brambilla
P.Petreczky, A.Vairo, J.H.Weber



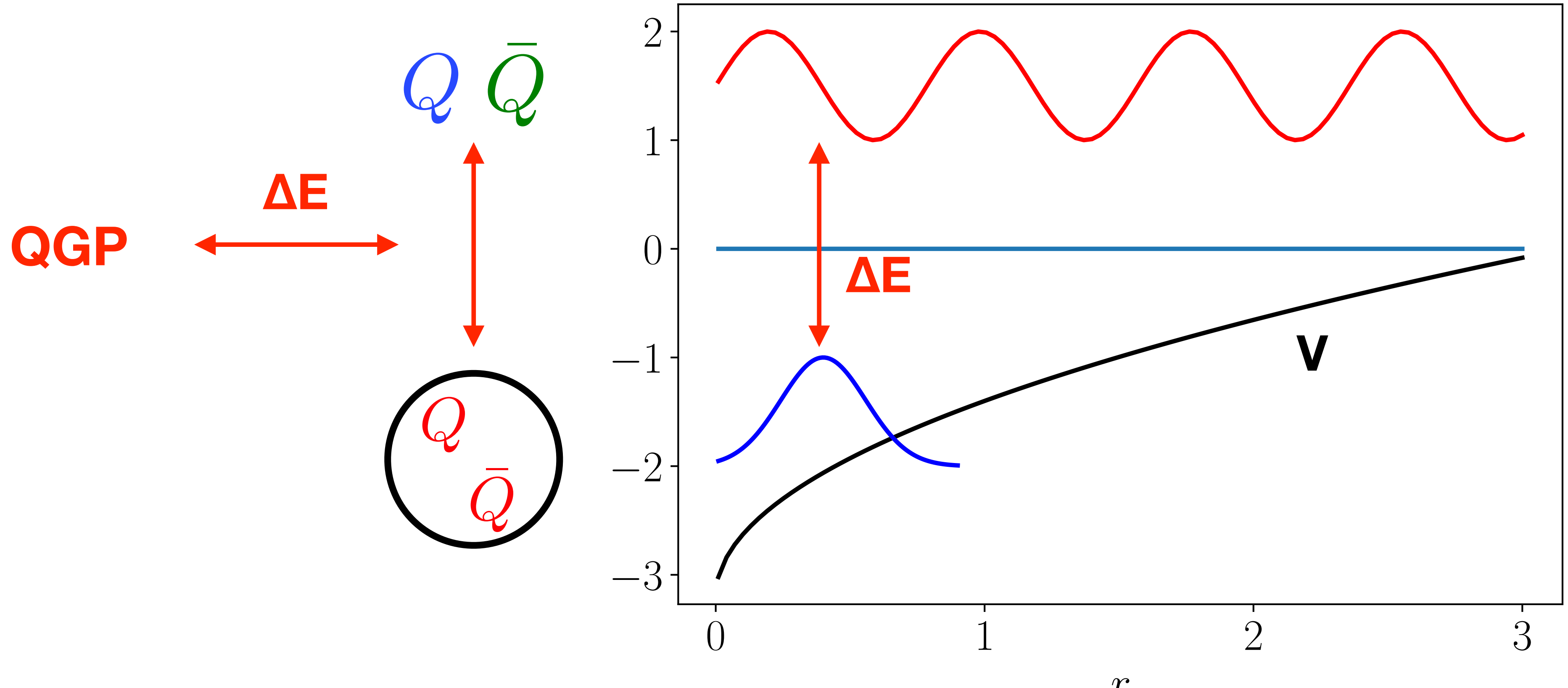
A.Mocsy, 0811.0337

Quarkonium as Probe of Quark-Gluon Plasma

- Static screening: suppression of color attraction —> melting at high T —> reduced production —> thermometer

- **Dynamical screening**: related to imaginary potential, **dissociation** induced by dynamical process, lead to suppression even when T(QGP) < melting T

- **Recombination**: unbound heavy quark pair forms quarkonium, can happen below melting T, crucial for phenomenology and theory consistency



Thermometer Picture Breaks Down

What QGP properties are we probing by measuring quarkonium?

This talk:

- **In certain limit, we are probing chromoelectric correlators of QGP/nuclear medium**
- **Gauge invariant object, all-order (in coupling) construction**
- **Tools: open quantum systems + effective field theory (EFT)**

Contents

- Introduction: **open quantum system**
- General procedure: derive semiclassical transport from open quantum system, with **effective field theory**
- Two temperature regimes:
    - **High temperature:** quantum Brownian motion, **Langevin** equations
    - **Low temperature:** quantum optical limit, **Boltzmann** equations
- Momentum-dependent & independent chromoelectric correlators of QGP

Open Quantum System

Total system = subsystem + environment: $H = H_S + H_E + H_I$

$$\rho(t=0) = \rho_S \otimes \rho_E$$

Subsystem & environment

(Heavy quark pairs & QGP)

Unitary evolution

Time reversible

$$U(t,0)(\rho_S \otimes \rho_E)U^\dagger(t,0)$$

Subsystem & environment

Trace out (integrate out) environment

$$\rho_S(t=0)$$

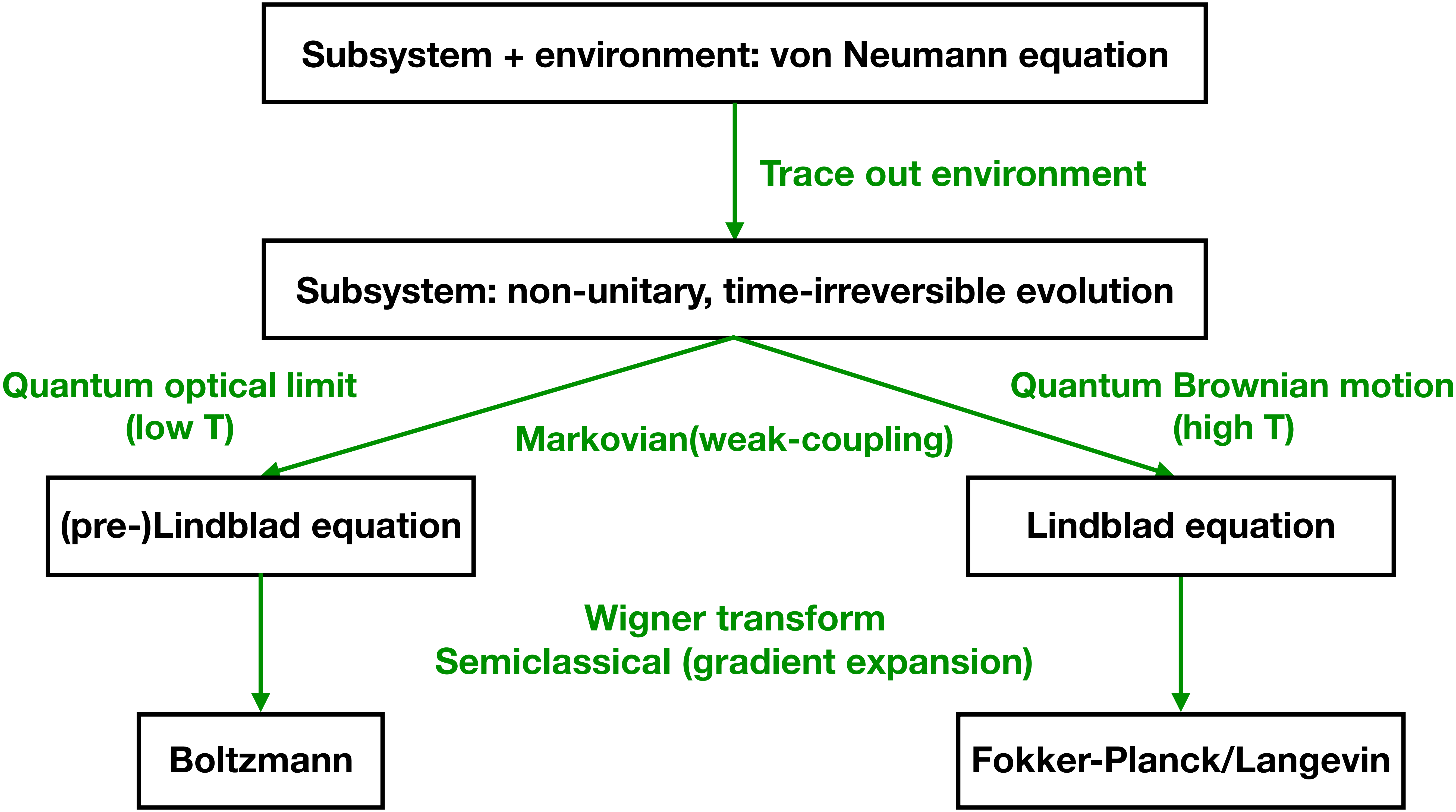
Subsystem

Non-unitary

Time irreversible

$$\mathrm{Tr}_E\Big[U(t,0)(\rho_S \otimes \rho_E)U^\dagger(t,0)\Big]$$

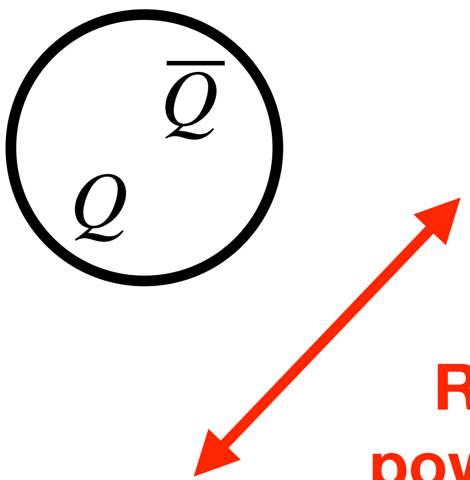
From Open Quantum System to Semiclassical Transport



Wigner transform $$f_{nl}(\boldsymbol{x},\boldsymbol{k},t) \equiv \int \frac{d^3k'}{(2\pi)^3} e^{i\boldsymbol{k}'\cdot\boldsymbol{x}} \left\langle \boldsymbol{k}+\frac{\boldsymbol{k}'}{2}, nl, 1 \middle| \rho_S(t) \middle| \boldsymbol{k}-\frac{\boldsymbol{k}'}{2}, nl, 1 \right\rangle$$

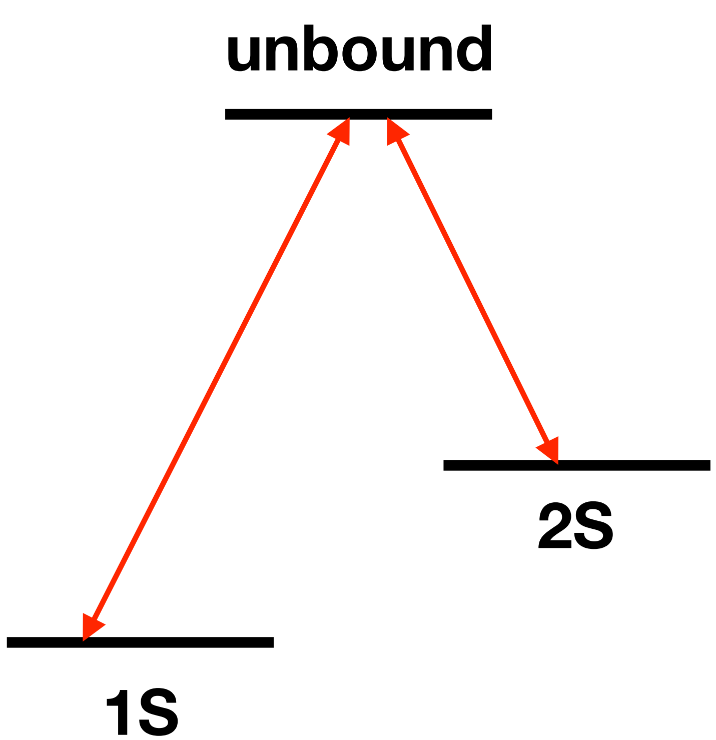
Physical Pictures of Two Limits

- **Quantum optical limit (low T)**

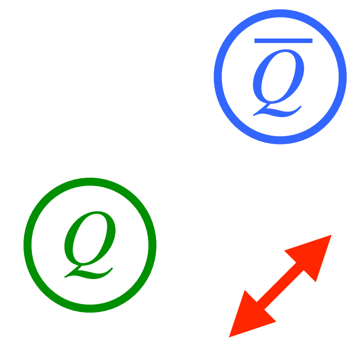


Resolving power of QGP

Transitions between levels

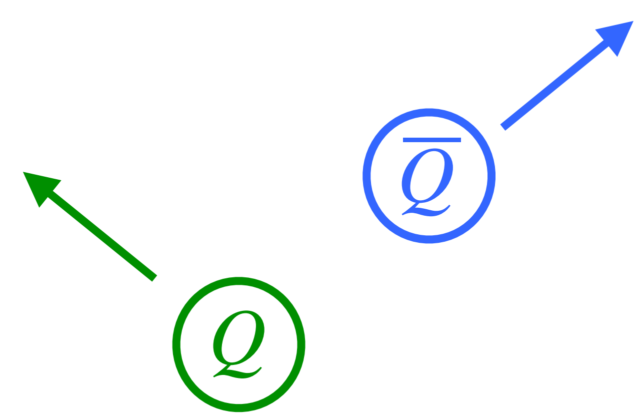


- **Quantum Brownian motion (high T)**



Resolving power of QGP

Diffusion of heavy Q pair



Wavefunction decoherence
—> dissociation

Two Limits and Hierarchy of Time Scales

- **Quantum optical limit (low T)**

$$\tau_R \gg \tau_E,\ \tau_R \gg \tau_S$$

- **Quantum Brownian motion (high T)**

$$\tau_R \gg \tau_E,\ \tau_S \gg \tau_E$$

- τ_E: **environment correlation time**, $\tau_E \sim \dfrac{1}{T}$ for QGP at equilibrium

- τ_S: **subsystem intrinsic time scale**, $\tau_S \sim \dfrac{1}{E_b}$, inverse of quarkonium binding energy

- τ_R: **subsystem relaxation time**, depends on coupling strength between subsystem and environment

- $\tau_R \gg \tau_E$: **Markovian dynamics**, environment correlation lost during subsystem evolution, **generally true in weak coupling limit** (between subsystem and environment)

Separation of Scales and NREFT

Separation of scales

$$M \gg Mv \gg Mv^2,\ \Lambda_{QCD}$$

$v^2 \sim 0.3$ charmonium

$v^2 \sim 0.1$ bottomonium

QCD

Hard M — perturbative matching

HQET/NRQCD

Heavy quark physics, A.Manohar, M.Wise

hep-ph/9407339, G.Bodwin, E.Braaten, G.Lepage

Soft Mv — perturbative / non-perturbative matching

potential NRQCD

hep-ph/9907240, hep-ph/0410047,
N.Brambilla A.Pineda, J.Soto, A.Vairo

Ultrasoft

$Mv^2 \sim 500$ MeV

Different descriptions depending on where T fits into the hierarchy

High Temperature 1: NRQCD $T \gg Mv^2$

Lindblad equation in limit of quantum Brownian motion

$$\frac{d\rho_S(t)}{dt} = -i\big[H_S + \Delta H_S, \rho_S(t)\big] + \frac{1}{N_c^2-1}\int \frac{d^3q}{(2\pi)^3} D^{>}(q_0 = 0, \boldsymbol{q}) \times \Big(\widetilde{O}^a(\boldsymbol{q})\rho_S(t)\widetilde{O}^{a\dagger}(\boldsymbol{q}) - \frac{1}{2}\big\{\widetilde{O}^{a\dagger}(\boldsymbol{q})\widetilde{O}^a(\boldsymbol{q}), \rho_S(t)\big\}\Big)$$

Environment correlator $D^{>ab}(x_1, x_2) = g^2 \mathrm{Tr}_E\big(\rho_E A_0^a(t_1, \boldsymbol{x}_1) A_0^b(t_2, \boldsymbol{x}_2)\big)$

$$\widetilde{O}^a(\boldsymbol{q}) = e^{\frac{i}{2}\boldsymbol{q}\cdot\hat{\boldsymbol{x}}_Q}\Big(1 - \frac{\boldsymbol{q}\cdot\hat{\boldsymbol{p}}_Q}{4MT}\Big)e^{\frac{i}{2}\boldsymbol{q}\cdot\hat{\boldsymbol{x}}_Q}T_F^a - e^{\frac{i}{2}\boldsymbol{q}\cdot\hat{\boldsymbol{x}}_{\bar{Q}}}\Big(1 - \frac{\boldsymbol{q}\cdot\hat{\boldsymbol{p}}_{\bar{Q}}}{4MT}\Big)e^{\frac{i}{2}\boldsymbol{q}\cdot\hat{\boldsymbol{x}}_{\bar{Q}}}T_F^{*a}$$

Dissipation effect, important for thermalization

Approximations:

Stochastic Schrödinger equation with dissipation R.Katz, P.B.Gossiaux, 1504.08087; T.Miura, Y.Akamatsu, M.Asakawa, A.Rothkopf, 1908.06293

Semiclassical limit Langevin equations J.-P. Blaizot, M.A.Escobedo, 1711.10812

High Temperature 2: pNRQCD $Mv \gg T \gg Mv^2$

Lindblad equation in limit of quantum Brownian motion

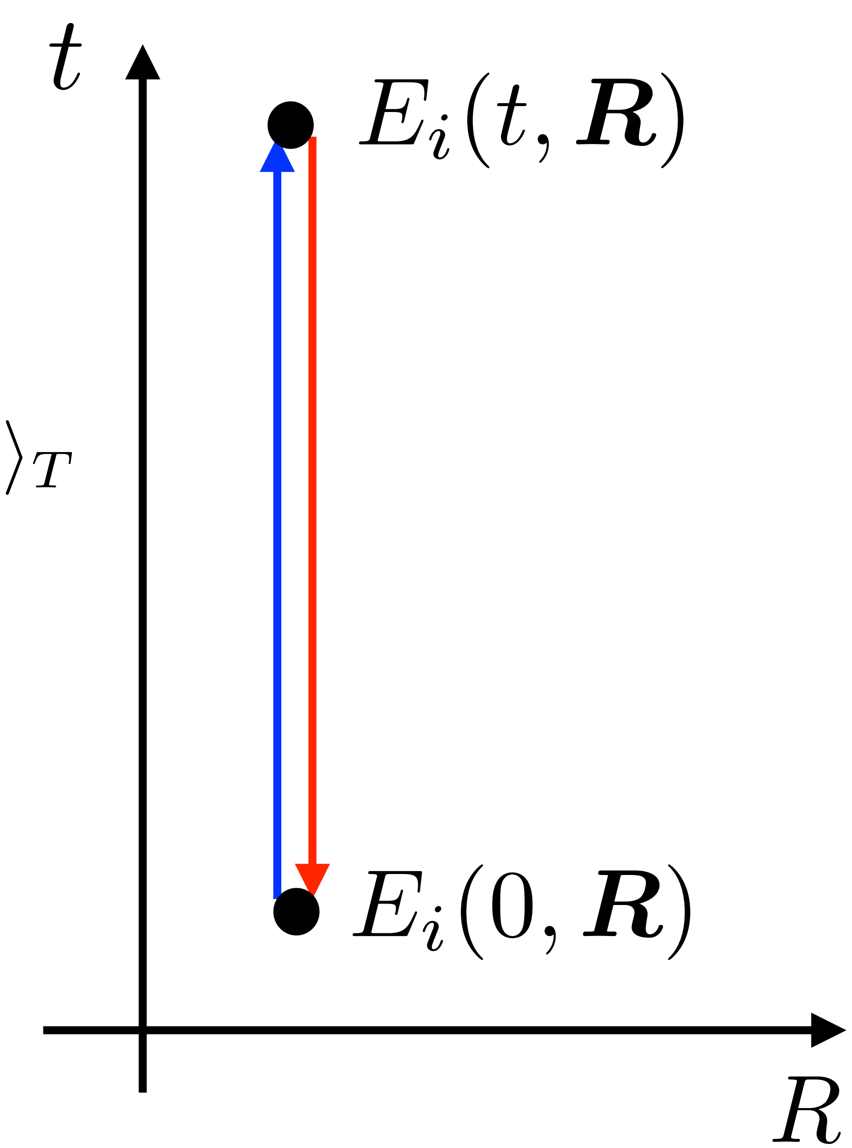
$$\frac{d\rho_S(t)}{dt} = -i\left[H_S + \Delta H_S,\, \rho_S(t)\right] + \frac{D(\omega=0, \boldsymbol{R}=0)}{N_c^2-1}\left(L_{\alpha i}\rho_S(t)L_{\alpha i}^\dagger - \frac{1}{2}\left\{L_{\alpha i}^\dagger L_{\alpha i},\, \rho_S(t)\right\}\right)$$

N.Brambilla, M.A.Escobedo, M.Strickland, A.Vairo, P.V.Griend, J.H.Weber arXiv:2012.01240

Evolution determined by transport coefficients

$$D(\omega=0, \boldsymbol{R}=0) = g^2 \int dt\, \langle E_i(t,\boldsymbol{R})\mathcal{W}_{[t,0]}E_i(0,\boldsymbol{R})\rangle_T$$

$$\Sigma(\omega=0, \boldsymbol{R}=0) = g^2 \mathrm{Im} \int dt\, \langle \mathcal{T} E_i(t,\boldsymbol{R})\mathcal{W}_{[t,0]}E_i(0,\boldsymbol{R})\rangle_T$$



D is just the heavy quark diffusion coefficient

Why HQ diffusion coefficient affects quarkonium?

$T \gg Mv^2$ **binding energy effect is subleading**

Low Temperature: pNRQCD $Mv \gg Mv^2 \gtrsim T$

Quantum optical and semiclassical limits: Boltzmann equation

$$\frac{\partial}{\partial t} f_{nl}(\boldsymbol{x}, \boldsymbol{k}, t) + \frac{\boldsymbol{k}}{2M} \cdot \nabla_{\boldsymbol{x}} f_{nl}(\boldsymbol{x}, \boldsymbol{k}, t) = \mathcal{C}^{+}_{nl}(\boldsymbol{x}, \boldsymbol{k}, t) - \mathcal{C}^{-}_{nl}(\boldsymbol{x}, \boldsymbol{k}, t)$$

T.Mehen, XY: 1811.07027, 2009.02408

Dissociation term

$$\mathcal{C}^{-}_{nl} = \frac{T_F}{N_c} \sum_{i_1, i_2} \int \frac{d^3 p_{\rm cm}}{(2\pi)^3} \frac{d^3 p_{\rm rel}}{(2\pi)^3} \frac{d^4 q}{(2\pi)^4} (2\pi)^4 \delta^3(\boldsymbol{k} - \boldsymbol{p}_{\rm cm} + \boldsymbol{q}) \delta(E_{nl} - E_p + q^0)$$
$$\times \langle \psi_{nl} | r_{i_1} | \Psi_{\boldsymbol{p}_{\rm rel}} \rangle \langle \Psi_{\boldsymbol{p}_{\rm rel}} | r_{i_2} | \psi_{nl} \rangle D_{i_1 i_2}(q^0, \boldsymbol{q}) f_{nl}(\boldsymbol{x}, \boldsymbol{k})$$

Chromoelectric correlator of QGP (gauge invariant, scale independent)

$$D_{i_1 i_2}(q^0, \boldsymbol{q}) = g^2 \int dt\, d^3R\, e^{iq^0(t_1 - t_2) - i\boldsymbol{q}\cdot(\boldsymbol{R}_1 - \boldsymbol{R}_2)} \langle E_{i_1}(t_1, \boldsymbol{R}_1) \mathcal{W} E_{i_2}(t_2, \boldsymbol{R}_2) \rangle_T$$

More general than the previous case:

Binding energy effect matters here: different quarkonium states respond differently

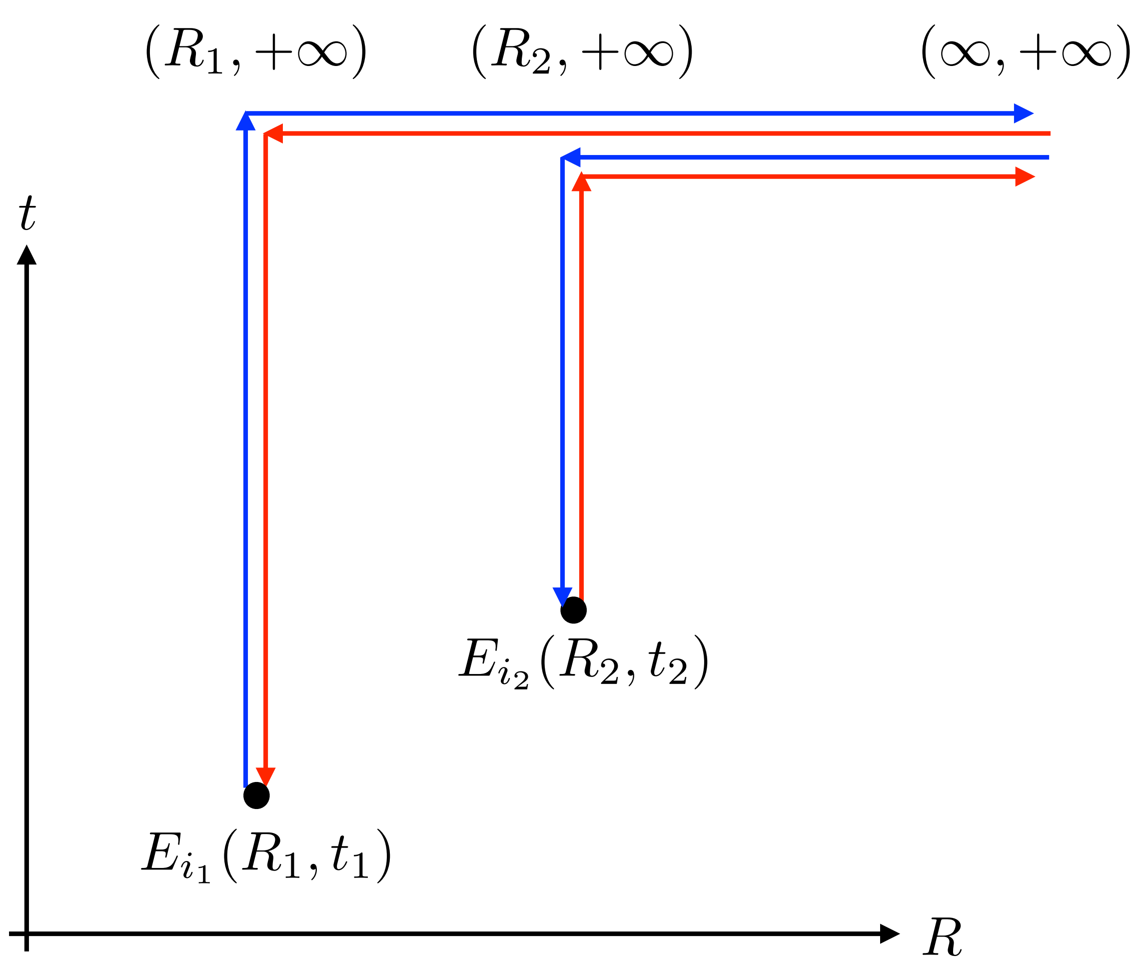
Finite momentum transfer, momentum dependence

Chromoelectric Correlator of QGP

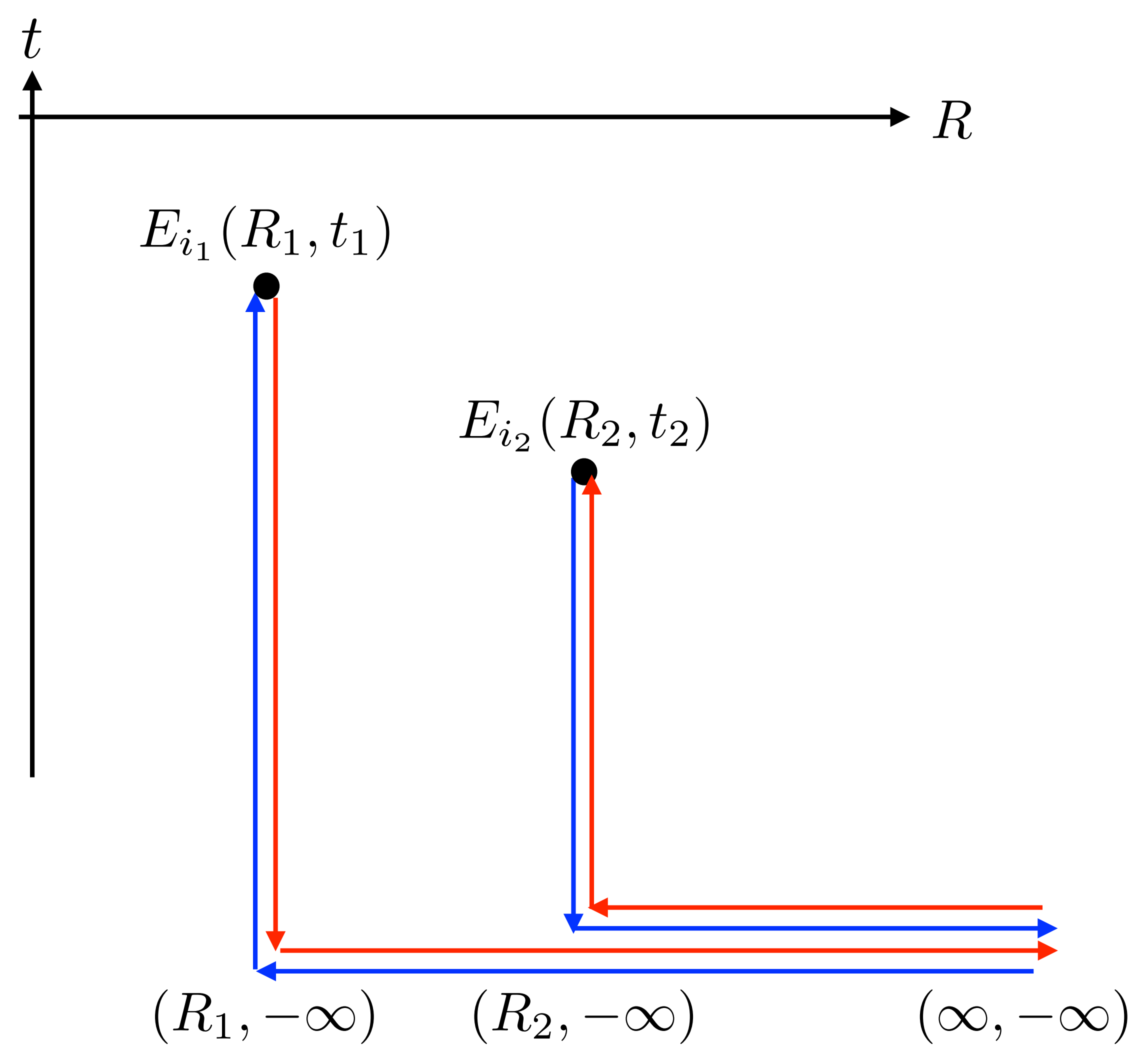
Staple shaped Wilson lines

$$D_{i_1 i_2}(q^0, \boldsymbol{q}) = g^2 \int dt\, d^3R\, e^{iq^0(t_1-t_2)-i\boldsymbol{q}\cdot(\boldsymbol{R}_1-\boldsymbol{R}_2)} \langle E_{i_1}(t_1, \boldsymbol{R}_1) \mathcal{W} E_{i_2}(t_2, \boldsymbol{R}_2)\rangle_T$$

For dissociation: final-state interaction



For recombination: initial-state interaction



Inclusive v.s. Differential Reaction Rates

Take dissociation rate as example

$$R^-_{nl} = \sum_{i_1,i_2} \int \frac{d^3p_{\rm cm}}{(2\pi)^3}\frac{d^3p_{\rm rel}}{(2\pi)^3}\frac{d^4q}{(2\pi)^4}(2\pi)^4\delta^3(\boldsymbol{k}-\boldsymbol{p}_{\rm cm}+\boldsymbol{q})\delta(E_{nl}-E_p+q^0)d^{nl}_{i_1i_2}(\boldsymbol{p}_{\rm rel})D_{i_1i_2}(q^0,\boldsymbol{q})$$

$$d^{nl}_{i_1i_2}(\boldsymbol{p}_{\rm rel}) = \frac{T_F}{N_c}\langle\psi_{nl}|r_{i_1}|\Psi_{\boldsymbol{p}_{\rm rel}}\rangle\langle\Psi_{\boldsymbol{p}_{\rm rel}}|r_{i_2}|\psi_{nl}\rangle$$

Inclusive rate

$$R^-_{nl} = \int \frac{d^3p_{\rm rel}}{(2\pi)^3}\,\bar{d}^{nl}(\boldsymbol{p}_{\rm rel})D\Big(\frac{p^2_{\rm rel}}{M}-E_{nl},\boldsymbol{R}=0\Big)$$

$$D(q^0,\boldsymbol{R}=0) = g^2\int dt\, e^{iq^0t}\langle E_i(t,\boldsymbol{R})\mathcal{W}_{[t,0]}E_i(0,\boldsymbol{R})\rangle_T$$

Momentum independent distribution

Zero frequency limit = HQ diffusion coefficient, appear in quantum Brownian motion

Differential rate

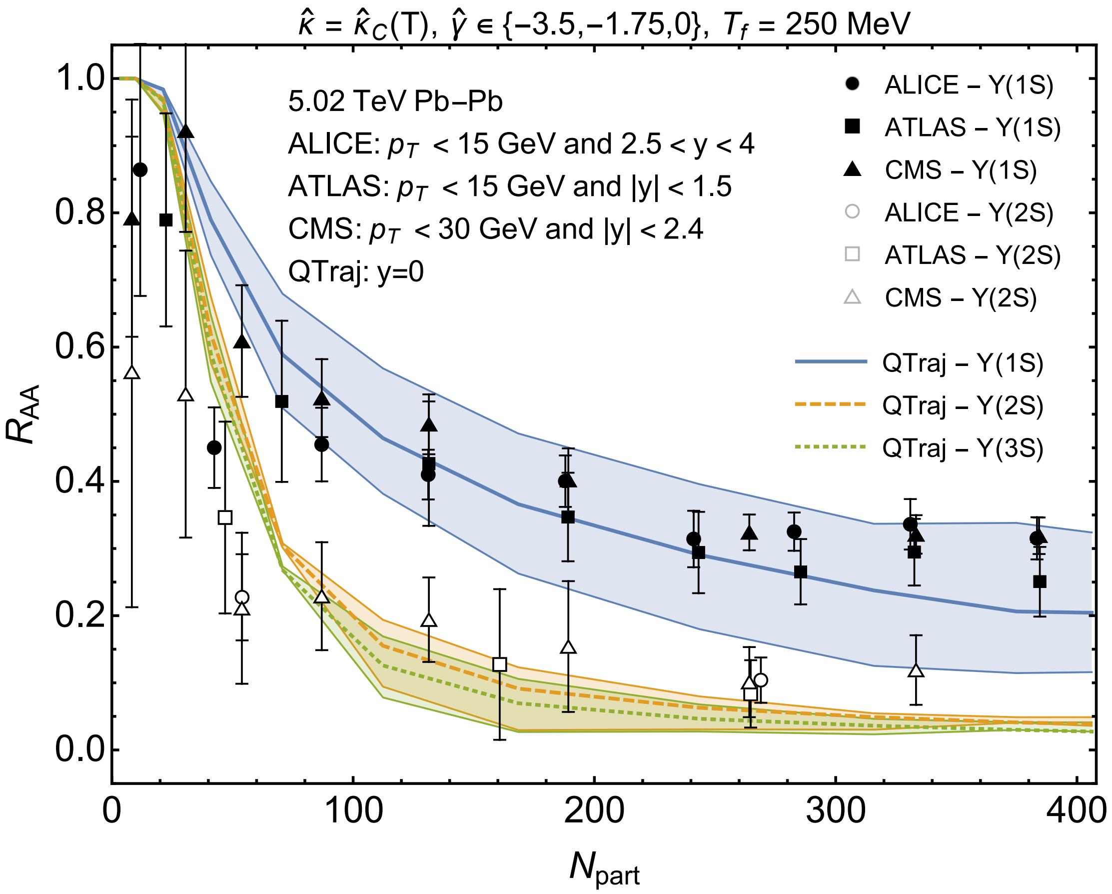
$$(2\pi)^3\frac{dR^-_{nl}}{d^3p_{\rm cm}} = \int \frac{d^3p_{\rm rel}}{(2\pi)^3}\,\bar{d}^{nl}(\boldsymbol{p}_{\rm rel})D\Big(\frac{p^2_{\rm rel}}{M}-E_{nl},\boldsymbol{p}_{\rm cm}-\boldsymbol{k}\Big)$$

Momentum dependent distribution

Similar to PDF v.s. TMDPDF, though different in time axis

Phenomenological Results for Bottomonia

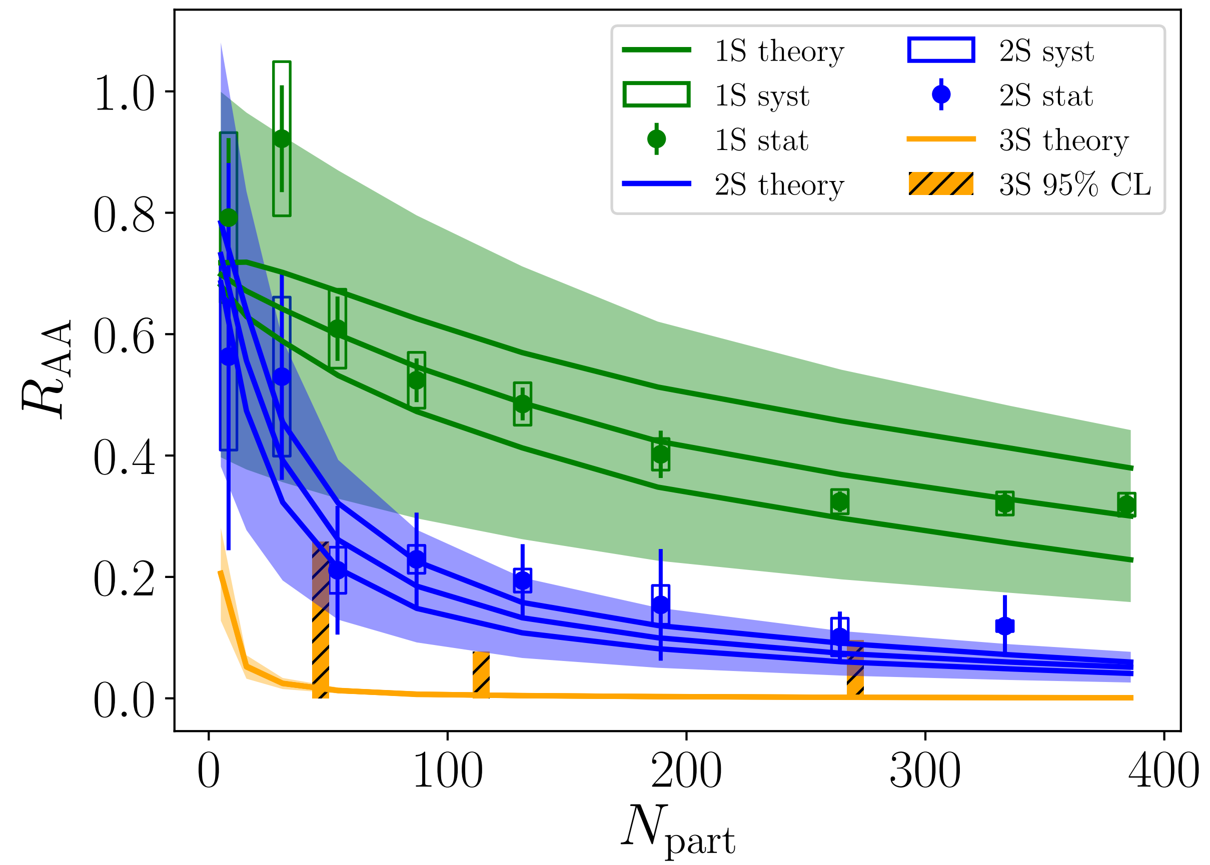
Lindblad equation for quantum Brownian motion



N.Brambilla, M.A.Escobedo, M.Strickland, A.Vairo, P.V.Griend, J.H.Weber arXiv:2012.01240

No nPDF effects

Coupled Boltzmann equation for quantum optical limit



XY, W.Ke, Y.Xu, S.A.Bass, B.Müller, 2004.06746

Uncertainty of nPDF dominates

Summary

- What are we probing by measuring quarkonium?
  Chromoelectric correlator of QGP

- **Open quantum + EFT**: derive quantum and semiclassical transport equations

  - High temperature: Langevin equations, dynamics governed by heavy quark diffusion coefficient & another transport coefficient

  - Low temperature: Boltzmann equations, dynamics governed by energy and momentum dependent chromoelectric correlator

Backup: $Mv \gg T \gg Mv^2$

Lindblad equation in the limit of quantum Brownian motion

$$\frac{d\rho_S(t)}{dt} = -i\big[H_S + \Delta H_S,\, \rho_S(t)\big] + \frac{D(\omega=0,\boldsymbol{R}=0)}{N_c^2-1}\Big(L_{\alpha i}\rho_S(t)L_{\alpha i}^\dagger - \frac{1}{2}\big\{L_{\alpha i}^\dagger L_{\alpha i},\, \rho_S(t)\big\}\Big)$$

$$\Delta H_S = \frac{\Sigma(\omega=0,\boldsymbol{R}=0)}{2(N_c^2-1)} r^2 \begin{pmatrix} C_F & 0 \\ 0 & \frac{N_c^2-2}{4N_c} \end{pmatrix}$$

$$L_{1i} = \sqrt{C_F}\Big(r_i + \frac{1}{2MT}\nabla_i - \frac{N_c}{8T}\frac{\alpha_s r_i}{r}\Big)\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$L_{2i} = \sqrt{\frac{T_F}{N_c}}\Big(r_i + \frac{1}{2MT}\nabla_i + \frac{N_c}{8T}\frac{\alpha_s r_i}{r}\Big)\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$L_{3i} = \sqrt{\frac{N_c^2-4}{4N_c}}\Big(r_i + \frac{1}{2MT}\nabla_i\Big)\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

Backup: Leading Power

Nonrelativistic & multipole expansions: v & r

$$\mathcal{L}_{\mathrm{pNRQCD}} = \int d^3r\, \mathrm{Tr}\Big(\mathrm{S}^\dagger(i\partial_0 - H_s)\mathrm{S} + \mathrm{O}^\dagger(iD_0 - H_o)\mathrm{O} + \boxed{V_A(\mathrm{O}^\dagger \boldsymbol{r}\cdot g\boldsymbol{E}\mathrm{S} + \mathrm{h.c.})} + \frac{V_B}{2}\mathrm{O}^\dagger\{\boldsymbol{r}\cdot g\boldsymbol{E}, \mathrm{O}\} + \cdots\Big)$$

Dipole interaction

Boltzmann equation at leading-power in v & r, leading-order in g

Dissociation and recombination rates depend on QGP via

XY, T.Mehen 1811.07027

$$\mathrm{Tr}_E\big(\rho_E E_i(t_1,\boldsymbol{x}_1)E_i(t_2,\boldsymbol{x}_2)\big) = \langle E_i(t_1,\boldsymbol{x}_1)E_i(t_2,\boldsymbol{x}_2)\rangle_T$$

Not gauge invariant !

Backup: All-Order Construction: Sum A0 Interactions

$$\mathcal{L}_{\rm pNRQCD} = \int d^3r \, {\rm Tr}\Big({\rm S}^\dagger (i\partial_0 - H_s){\rm S} + \boxed{{\rm O}^\dagger (iD_0 - H_o){\rm O}} + V_A({\rm O}^\dagger \boldsymbol{r}\cdot g\boldsymbol{E}{\rm S} + {\rm h.c.}) + \frac{V_B}{2}{\rm O}^\dagger\{\boldsymbol{r}\cdot g\boldsymbol{E}, {\rm O}\} + \cdots \Big)$$

Octet—A0 interaction not suppressed by v or r

Need sum A0 to all orders at leading power

Field redefinition:

$$\begin{aligned} {\rm O}(\boldsymbol{R},\boldsymbol{r},t) &= \mathcal{W}_{[(\boldsymbol{R},t),(\boldsymbol{R},t_0)]}\widetilde{\rm O}(\boldsymbol{R},\boldsymbol{r},t) \\ \widetilde{E}_i(\boldsymbol{R},t) &= \mathcal{W}_{[(\boldsymbol{R},t_0),(\boldsymbol{R},t)]}E_i(\boldsymbol{R},t) \\ \mathcal{W}_{[(\boldsymbol{R},t_f),(\boldsymbol{R},t_i)]} &= \mathcal{P}\exp\left(ig\int_{t_i}^{t_f} ds\mathcal{A}_0(\boldsymbol{R},s)\right) \end{aligned}$$

New form of dipole interaction:

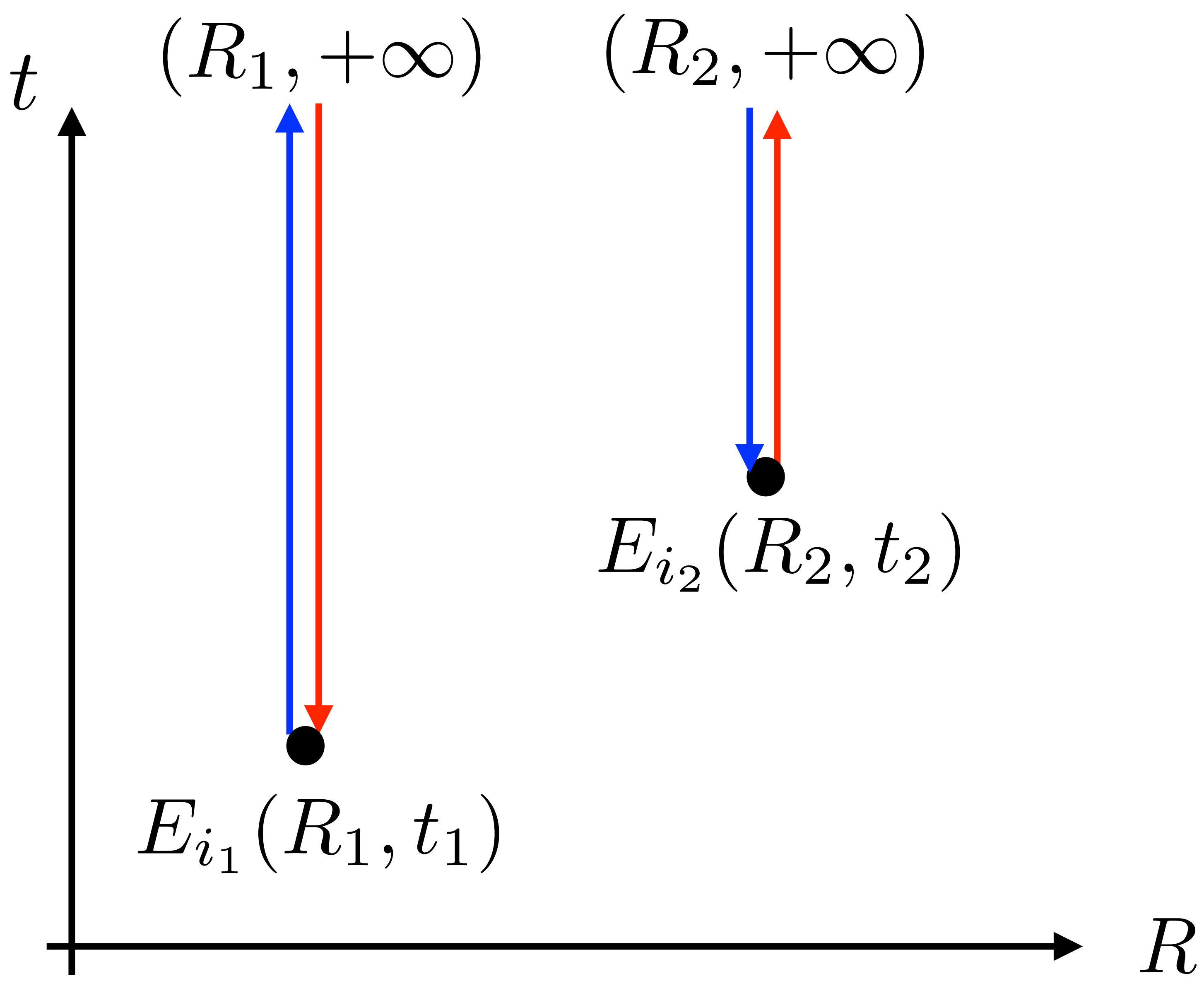
$$g\int d^3r {\rm Tr}\Big(\widetilde{\rm O}^\dagger r_i \widetilde{E}_i {\rm S} + {\rm S}^\dagger r_i \widetilde{E}_i^\dagger \widetilde{\rm O}\Big)$$

Backup: Chromoelectric Correlator of QGP

$$g^{E++}_{i_1 i_2}(t_1, t_2, \boldsymbol{R}_1, \boldsymbol{R}_2) = \Big\langle E_{i_1}(\boldsymbol{R}_1, t_1)\mathcal{W}_{[(\boldsymbol{R}_1,t_1),(\boldsymbol{R}_1,+\infty)]}\mathcal{W}_{[(\boldsymbol{R}_2,+\infty),(\boldsymbol{R}_2,t_2)]}E_{i_2}(\boldsymbol{R}_2, t_2)\Big\rangle_T$$

Wilsons not connected at infinite time!

For gauge invariance, need spatial gauge link



Backup: Wilson Lines at Infinite Time

$$\mathcal{L}_{\text{pNRQCD}} = \int d^3r\, \mathrm{Tr}\Big(\mathrm{S}^\dagger(i\partial_0 - H_s)\mathrm{S} + \mathrm{O}^\dagger(iD_0 - H_o)\mathrm{O} + V_A(\mathrm{O}^\dagger \boldsymbol{r}\cdot g\boldsymbol{E}\mathrm{S} + \mathrm{h.c.}) + \frac{V_B}{2}\mathrm{O}^\dagger\{\boldsymbol{r}\cdot g\boldsymbol{E}, \mathrm{O}\} + \cdots\Big)$$

Coulomb interaction between octet heavy quark pair included in potential

But Coulomb between octet center-of-mass motion and medium not considered

For Coulomb modes $p_c^\mu \sim A_c^\mu \sim M(v^2, v, v, v)$

$$\int \mathrm{d}^3r\, \mathrm{Tr}\bigg(\mathrm{O}^\dagger(\boldsymbol{R},\boldsymbol{r},t)\Big(iD_0 + \boxed{\frac{\boldsymbol{D}_{\boldsymbol{R}}^2}{4M}} + \frac{\nabla_{\boldsymbol{r}}^2}{M} - V_o(\boldsymbol{r}) + \cdots\Big)\mathrm{O}(\boldsymbol{R},\boldsymbol{r},t)\bigg)$$

C.m. kinetic term same order as D0, so leading power in v

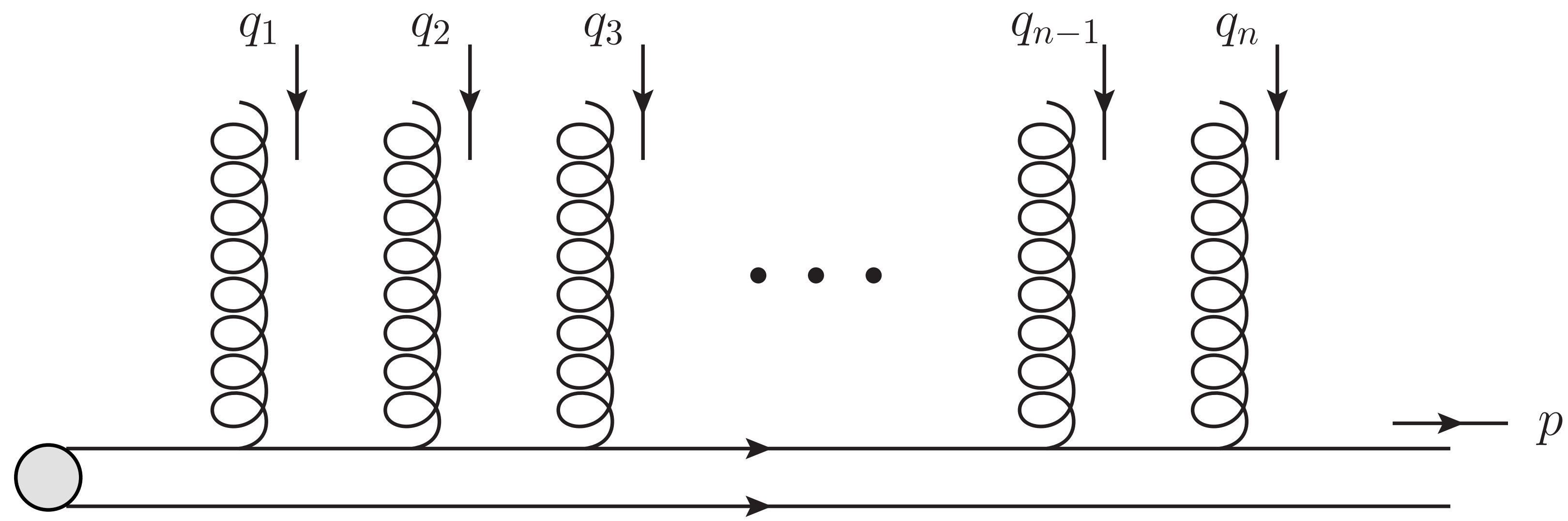
Write out c.m. kinetic term

$$\int \mathrm{d}^3r\, \mathrm{Tr}\bigg(\mathrm{O}^\dagger(\boldsymbol{R},\boldsymbol{r},t)\frac{\nabla_{\boldsymbol{R}}^2}{4M}\mathrm{O}(\boldsymbol{R},\boldsymbol{r},t) - \frac{ig}{4M}\mathrm{O}^\dagger(\boldsymbol{R},\boldsymbol{r},t)\Big(\boldsymbol{A}(\boldsymbol{R},t)\cdot\nabla_{\boldsymbol{R}}$$

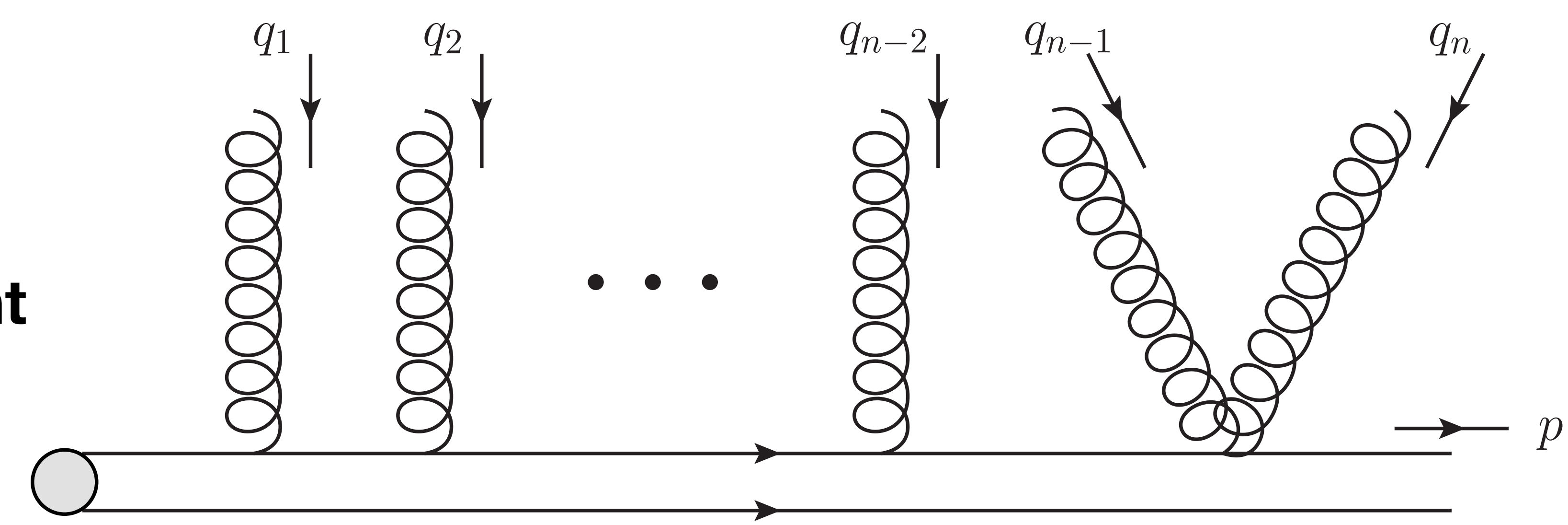
$$+ \nabla_{\boldsymbol{R}}\cdot\boldsymbol{A}(\boldsymbol{R},t)\Big)\mathrm{O}(\boldsymbol{R},\boldsymbol{r},t) - \frac{g^2}{4M}\mathrm{O}^\dagger(\boldsymbol{R},\boldsymbol{r},t)\boldsymbol{A}^2(\boldsymbol{R},t)\mathrm{O}(\boldsymbol{R},\boldsymbol{r},t)\bigg).$$

Backup: Wilson Lines at Infinite Time: Resum Coulomb

Single Coulomb attachment



Double Coulomb attachment



Calculations have some similarity with A.V. Belitsky, X. Ji, F. Yuan hep-ph/0208038