Open Quantum Systems for Quarkonia

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Quarkonium as Probe of Quark-Gluon Plasma

- **Static screening**: suppression of color attraction $\rightarrow$ melting at high $T$ $\rightarrow$ reduced production $\rightarrow$ thermometer

\[ T = 0 : V(r) = -\frac{A}{r} + Br \quad \rightarrow \quad T \neq 0 : \text{Confining part flattened} \]
Quarkonium as Probe of Quark-Gluon Plasma

- **Static screening**: suppression of color attraction $\rightarrow$ melting at high $T$ $\rightarrow$ reduced production $\rightarrow$ thermometer

- **Dynamical screening**: related to imaginary potential, dissociation induced by dynamical process, lead to suppression even when $T(\text{QGP}) <$ melting $T$

- **Recombination**: unbound heavy quark pair forms quarkonium, can happen below melting $T$, crucial for phenomenology and theory consistency
What QGP properties are we probing by measuring quarkonium?

This talk:

- In certain limit, we are probing chromoelectric correlators of QGP/nuclear medium
- Gauge invariant object, all-order (in coupling) construction
- Tools: open quantum systems + effective field theory (EFT)
Contents

• Introduction: open quantum system

• General procedure: derive semiclassical transport from open quantum system, with effective field theory

• Two temperature regimes:
  
  • **High temperature**: quantum Brownian motion, *Langevin* equations
  
  • **Low temperature**: quantum optical limit, *Boltzmann* equations

• Momentum-dependent & independent chromoelectric correlators of QGP
Open Quantum System

Total system = subsystem + environment: \[ H = H_S + H_E + H_I \]

\[ \rho(t = 0) = \rho_S \otimes \rho_E \]

Subsystem & environment (Heavy quark pairs & QGP)

Unitary evolution

Time reversible

\[ U(t, 0)(\rho_S \otimes \rho_E)U^\dagger(t, 0) \]

Subsystem & environment

Trace out (integrate out) environment

\[ \rho_S(t = 0) \]

Subsystem

Non-unitary

Time irreversible

\[ \text{Tr}_E \left[ U(t, 0)(\rho_S \otimes \rho_E)U^\dagger(t, 0) \right] \]
Subsystem + environment: von Neumann equation

Trace out environment

Subsystem: non-unitary, time-irreversible evolution

Quantum optical limit (low T)

(pre-)Lindblad equation

Markovian (weak-coupling)

Wigner transform
Semiclassical (gradient expansion)

Quantum Brownian motion (high T)

Lindblad equation

Boltzmann

Fokker-Planck/Langevin

Wigner transform
\[ f_{nl}(x, k, t) \equiv \int \frac{d^3 k'}{(2\pi)^3} e^{i k' \cdot x} \langle k + \frac{k'}{2}, nl, 1 | \rho_S(t) | k - \frac{k'}{2}, nl, 1 \rangle \]
Physical Pictures of Two Limits

- Quantum optical limit (low T)
- Quantum Brownian motion (high T)

Transitions between levels:

- 1S
- 2S
- Unbound

Diffusion of heavy Q pair:

- Wavefunction decoherence ➔ dissociation

Resolving power of QGP
Two Limits and Hierarchy of Time Scales

- Quantum optical limit (low T)
  \[ \tau_R \gg \tau_E, \tau_R \gg \tau_S \]

- Quantum Brownian motion (high T)
  \[ \tau_R \gg \tau_E, \tau_S \gg \tau_E \]

- \( \tau_E \): environment correlation time, \( \tau_E \sim \frac{1}{T} \) for QGP at equilibrium

- \( \tau_S \): subsystem intrinsic time scale, \( \tau_S \sim \frac{1}{E_b} \), inverse of quarkonium binding energy

- \( \tau_R \): subsystem relaxation time, depends on coupling strength between subsystem and environment

- \( \tau_R \gg \tau_E \): Markovian dynamics, environment correlation lost during subsystem evolution, generally true in weak coupling limit (between subsystem and environment)
Separation of Scales and NREFT

Separation of scales

\[ M \gg Mv \gg Mv^2, \Lambda_{QCD} \]

- **QCD**
  - **Hard** \( M \) 
    - perturbative matching
        - Heavy quark physics, A.Manohar, M.Wise
          - hep-ph/9407339, G.Bodwin, E.Braaten, G.Lepage
  - **HQET/NRQCD** 
    - perturbative / non-perturbative matching
        - hep-ph/9907240, hep-ph/0410047,
          N.Brambilla A.Pineda, J.Soto, A.Vairo
  - **potential NRQCD** 

- **Ultrasoft** \( Mv^2 \sim 500 \text{ MeV} \)

\[ v^2 \sim 0.3 \quad \text{charmonium} \]
\[ v^2 \sim 0.1 \quad \text{bottomonium} \]

Different descriptions depending on where \( T \) fits into the hierarchy
High Temperature 1: NRQCD \( T \gg M v^2 \)

Lindblad equation in limit of quantum Brownian motion

\[
\frac{d\rho_S(t)}{dt} = -i[H_S + \Delta H_S, \rho_S(t)] + \frac{1}{N_c^2 - 1} \int \frac{d^3q}{(2\pi)^3} D^>(q_0 = 0, q) \\
\times \left( \tilde{O}^a(q)\rho_S(t)\tilde{O}^{a\dagger}(q) - \frac{1}{2}\{\tilde{O}^{a\dagger}(q)\tilde{O}^a(q), \rho_S(t)\} \right)
\]

Environment correlator

\[
D^{>ab}(x_1, x_2) = g^2 Tr_E \left( \rho_E A_0^a(t_1, x_1) A_0^b(t_2, x_2) \right)
\]

\[
\tilde{O}^a(q) = e^{i\frac{q}{2} \cdot \hat{x} \cdot q} \left( 1 - \frac{q \cdot \hat{p} Q}{4MT} \right) e^{i\frac{q}{2} \cdot \hat{x} \cdot q} T_F^a - e^{i\frac{q}{2} \cdot \hat{x} \cdot q} \left( 1 - \frac{q \cdot \hat{p} Q}{4MT} \right) e^{i\frac{q}{2} \cdot \hat{x} \cdot q} T_F^{*a}
\]

Dissipation effect, important for thermalization

Approximations:
R.Katz, P.B.Gossiaux, 1504.08087
T.Miura, Y.Akamatsu, M.Asakawa, A.Rothkopf, 1908.06293

Stochastic Schrödinger equation with dissipation

Semiclassical limit Langevin equations J.-P. Blaizot, M.A.Escobedo, 1711.10812
High Temperature 2: pNRQCD $M_v \gg T \gg M_v^2$

Lindblad equation in limit of quantum Brownian motion

$$\frac{d\rho_S(t)}{dt} = -i[H_S + \Delta H_S, \rho_S(t)] + \frac{D(\omega = 0, R = 0)}{N_c^2 - 1} \left( L_{\alpha i} \rho_S(t) L_{\alpha i}^\dagger - \frac{1}{2} \{ L_{\alpha i}^\dagger L_{\alpha i}, \rho_S(t) \} \right)$$


Evolution determined by transport coefficients

$$D(\omega = 0, R = 0) = g^2 \int dt \langle E_i(t, R) \mathcal{W}_{[t,0]} E_i(0, R) \rangle_T$$

$$\Sigma(\omega = 0, R = 0) = g^2 \text{Im} \int dt \langle \mathcal{T} E_i(t, R) \mathcal{W}_{[t,0]} E_i(0, R) \rangle_T$$

D is just the heavy quark diffusion coefficient

Why HQ diffusion coefficient affects quarkonium?

$T \gg M_v^2$ binding energy effect is subleading
Low Temperature: pNRQCD $M_V \gg M_V^2 \gtrsim T$

Quantum optical and semiclassical limits: Boltzmann equation

$$\frac{\partial}{\partial t} f_{nl}(x, k, t) + \frac{k}{2M} \cdot \nabla_x f_{nl}(x, k, t) = C^+_{nl}(x, k, t) - C^-_{nl}(x, k, t)$$

Dissociation term

$$C^-_{nl} = \frac{T_F}{N_c} \sum_{i_1, i_2} \int \frac{d^3 p_{cm}}{(2\pi)^3} \frac{d^3 p_{rel}}{(2\pi)^3} \frac{d^4 q}{(2\pi)^4} (2\pi)^4 \delta^3(k - p_{cm} + q) \delta(E_{nl} - E_p + q^0)$$

$$\times \langle \psi_{nl} | r_{i_1} | \Psi_{p_{rel}} \rangle \langle \Psi_{p_{rel}} | r_{i_2} | \psi_{nl} \rangle D_{i_1 i_2}(q^0, q) f_{nl}(x, k)$$

Chromoelectric correlator of QGP (gauge invariant, scale independent)

$$D_{i_1 i_2}(q^0, q) = g^2 \int dt \, d^3 R \, e^{i q_0 (t_1 - t_2) - i q \cdot (R_1 - R_2)} \langle E_{i_1}(t_1, R_1) W E_{i_2}(t_2, R_2) \rangle_T$$

More general than the previous case:

Binding energy effect matters here: different quarkonium states respond differently

Finite momentum transfer, momentum dependence
Chromoelectric Correlator of QGP

Staple shaped Wilson lines

\[ D_{i_1 i_2}(q^0, q) = g^2 \int dt \, d^3 R \, e^{iq^0(t_1-t_2)-i\mathbf{q} \cdot (\mathbf{R}_1-\mathbf{R}_2)} \langle E_{i_1}(t_1, \mathbf{R}_1) \mathcal{W} E_{i_2}(t_2, \mathbf{R}_2) \rangle_T \]

For dissociation: final-state interaction

For recombination: initial-state interaction
Inclusive v.s. Differential Reaction Rates

Take dissociation rate as example

\[ R_{nl}^- = \sum_{i_1, i_2} \int \frac{d^3 p_{cm}}{(2\pi)^3} \frac{d^3 p_{rel}}{(2\pi)^3} \frac{d^4 q}{(2\pi)^4} (2\pi)^4 \delta^3 (k - p_{cm} + q) \delta (E_{nl} - E_p + q^0) d_{i_1 i_2}^{nl} (p_{rel}) D_{i_1 i_2} (q^0, q) \]

Inclusive rate

\[ R_{nl}^- = \int \frac{d^3 p_{rel}}{(2\pi)^3} \bar{d}^{nl} (p_{rel}) D \left( \frac{p_{rel}^2}{M} - E_{nl}, R = 0 \right) \]

\[ D(q^0, R = 0) = g^2 \int dt e^{iq^0 t} \langle E_i (t, R) \mathcal{W}[t, 0] E_i (0, R) \rangle_T \]

Momentum independent distribution

Zero frequency limit = HQ diffusion coefficient, appear in quantum Brownian motion

Differential rate

\[ (2\pi)^3 \frac{dR_{nl}^-}{d^3 p_{cm}} = \int \frac{d^3 p_{rel}}{(2\pi)^3} \bar{d}^{nl} (p_{rel}) D \left( \frac{p_{rel}^2}{M} - E_{nl}, p_{cm} - k \right) \]

Momentum dependent distribution

Similar to PDF v.s. TMDPDF, though different in time axis
Phenomenological Results for Bottomonia

Lindblad equation for quantum Brownian motion

\[
\dot{\rho} = \rho_{C}(T), \quad \gamma \in (-3.5, -1.75, 0), \quad T_f = 250 \text{ MeV}
\]

5.02 TeV Pb–Pb
ALICE: \( p_T < 15 \text{ GeV} \) and \( 2.5 < y < 4 \)
ATLAS: \( p_T < 15 \text{ GeV} \) and \( |y| < 1.5 \)
CMS: \( p_T < 30 \text{ GeV} \) and \( |y| < 2.4 \)
QTraj: \( y=0 \)

\[
\begin{align*}
&\text{ALICE – Y(1S)} \\
&\text{ATLAS – Y(1S)} \\
&\text{CMS – Y(1S)} \\
&\text{ALICE – Y(2S)} \\
&\text{ATLAS – Y(2S)} \\
&\text{CMS – Y(2S)} \\
&\text{QTraj – Y(1S)} \\
&\text{QTraj – Y(2S)} \\
&\text{QTraj – Y(3S)}
\end{align*}
\]


No nPDF effects

Coupled Boltzmann equation for quantum optical limit

XY, W.Ke, Y.Xu, S.A.Bass, B.Müller, 2004.06746

Uncertainty of nPDF dominates
Summary

• What are we probing by measuring quarkonium? **Chromoelectric correlator of QGP**

• **Open quantum + EFT**: derive quantum and semiclassical transport equations

  • High temperature: Langevin equations, dynamics governed by heavy quark diffusion coefficient & another transport coefficient

  • Low temperature: Boltzmann equations, dynamics governed by energy and momentum dependent chromoelectric correlator
Lindblad equation in the limit of quantum Brownian motion

\[
\frac{d\rho_S(t)}{dt} = -i[H_S + \Delta H_S, \rho_S(t)] + \frac{D(\omega = 0, R = 0)}{N_c^2 - 1} \left( L_{\alpha i} \rho_S(t) L_{\alpha i}^\dagger - \frac{1}{2} \{ L_{\alpha i} L_{\alpha i}, \rho_S(t) \} \right)
\]

\[
\Delta H_S = \sum(\omega = 0, R = 0)r^2 \left( \begin{array}{cc} C_F & 0 \\ 0 & \frac{N_c^2 - 2}{4N_c} \end{array} \right)
\]

\[
L_{1i} = \sqrt{C_F} \left( r_i + \frac{1}{2MT} \nabla_i - \frac{N_c \alpha_s r_i}{8T} \right) \left( \begin{array}{cc} 0 & 0 \\ 1 & 0 \end{array} \right)
\]

\[
L_{2i} = \sqrt{\frac{T_F}{N_c}} \left( r_i + \frac{1}{2MT} \nabla_i + \frac{N_c \alpha_s r_i}{8T} \right) \left( \begin{array}{cc} 0 & 1 \\ 0 & 0 \end{array} \right)
\]

\[
L_{3i} = \sqrt{\frac{N_c^2 - 4}{4N_c}} \left( r_i + \frac{1}{2MT} \nabla_i \right) \left( \begin{array}{cc} 0 & 0 \\ 0 & 1 \end{array} \right)
\]
Backup: Leading Power

Nonrelativistic & multipole expansions: $v$ & $r$

$$\mathcal{L}_{\text{pNRQCD}} = \int d^3r \text{Tr}\left(S^\dagger (i\partial_0 - H_s)S + O^\dagger (iD_0 - H_o)O + V_A (O^\dagger r \cdot gE_S + \text{h.c.}) + \frac{V_B}{2} O^\dagger \{r \cdot gE, O\} + \cdots\right)$$

Dipole interaction

Boltzmann equation at leading-power in $v$ & $r$, leading-order in $g$

Dissociation and recombination rates depend on QGP via

$$\text{Tr}_E \left(\rho_E E_i(t_1, x_1) E_i(t_2, x_2)\right) = \langle E_i(t_1, x_1) E_i(t_2, x_2)\rangle_T$$

Not gauge invariant!
Backup: All-Order Construction: Sum A0
Interactions

\[ \mathcal{L}_{\text{pNRQCD}} = \int d^3r \text{Tr} \left( S^\dagger (i\partial_0 - H_s)S + O^\dagger (iD_0 - H_o)O + V_A (O^\dagger r \cdot gE_S + \text{h.c.}) + \frac{V_B}{2} O^\dagger \{r \cdot gE, O\} + \cdots \right) \]

Octet—A0 interaction not suppressed by \( v \) or \( r \)

Need sum A0 to all orders at leading power

Field redefinition:

\[
O(R, r, t) = \mathcal{W}_{[(R,t),(R,t_0)]} \tilde{O}(R, r, t)
\]

\[
\tilde{E}_i(R, t) = \mathcal{W}_{[(R,t_0),(R,t)]} E_i(R, t)
\]

\[
\mathcal{W}_{[(R,t_f),(R,t_i)]} = \mathcal{P} \exp \left( ig \int_{t_i}^{t_f} ds A_0(R, s) \right)
\]

New form of dipole interaction:

\[
g \int d^3r \text{Tr} \left( \tilde{O}^\dagger r_i \tilde{E}_i S + S^\dagger r_i \tilde{E}_i^\dagger \tilde{O} \right)
\]
Backup: Chromoelectric Correlator of QGP

\[ g_{i_1i_2}^{E++}(t_1, t_2, R_1, R_2) = \left\langle E_{i_1}(R_1, t_1) W_{[(R_1, t_1), (R_1, +\infty)]} W_{[(R_2, +\infty), (R_2, t_2)]} E_{i_2}(R_2, t_2) \right\rangle_T \]

Wilsons not connected at infinite time!

For gauge invariance, need spatial gauge link
Backup: Wilson Lines at Infinite Time

\[ \mathcal{L}_{\text{NRQCD}} = \int d^3r \text{Tr} \left( S^\dagger (i\partial_0 - H_s) S + O^\dagger (iD_0 - H_o) O + V_A (O^\dagger r \cdot gE S + \text{h.c.}) + \frac{V_B}{2} O^\dagger \{r \cdot gE, O\} + \cdots \right) \]

Coulomb interaction between octet heavy quark pair included in potential

But Coulomb between octet center-of-mass motion and medium not considered

For Coulomb modes

\[ p^\mu_c \sim A^\mu_c \sim M (v^2, v, v, v) \]

\[ \int d^3r \text{Tr} \left( O^\dagger (R, r, t) \left( iD_0 + \frac{D^2_R}{4M} + \frac{\nabla^2_r}{M} - V_o(r) + \cdots \right) O(R, r, t) \right) \]

C.m. kinetic term same order as D0, so leading power in v

Write out c.m. kinetic term

\[ \int d^3r \text{Tr} \left( O^\dagger (R, r, t) \frac{\nabla^2_R}{4M} O(R, r, t) - \frac{ig}{4M} O^\dagger (R, r, t) \left( A(R, t) \cdot \nabla_R + \nabla_R \cdot A(R, t) \right) O(R, r, t) - \frac{g^2}{4M} O^\dagger (R, r, t) A^2(R, t) O(R, r, t) \right) \]
Backup: Wilson Lines at Infinite Time: Resum Coulomb

Single Coulomb attachment

Double Coulomb attachment

Calculations have some similarity with A.V. Belitsky, X. Ji, F. Yuan hep-ph/0208038