2021 RHIC/AGS Annual Users' Meeting

Transverse Momentum Distributions at EIC

Chiara Bissolotti
Università degli Studi di Pavia and INFN

with MAP Collaboration
(Multi-dimensional Analyses of Partonic distributions)

EIC^2 EIC Center at Jefferson Lab

3DSPIN MAPPING THE PROTON IN 3D

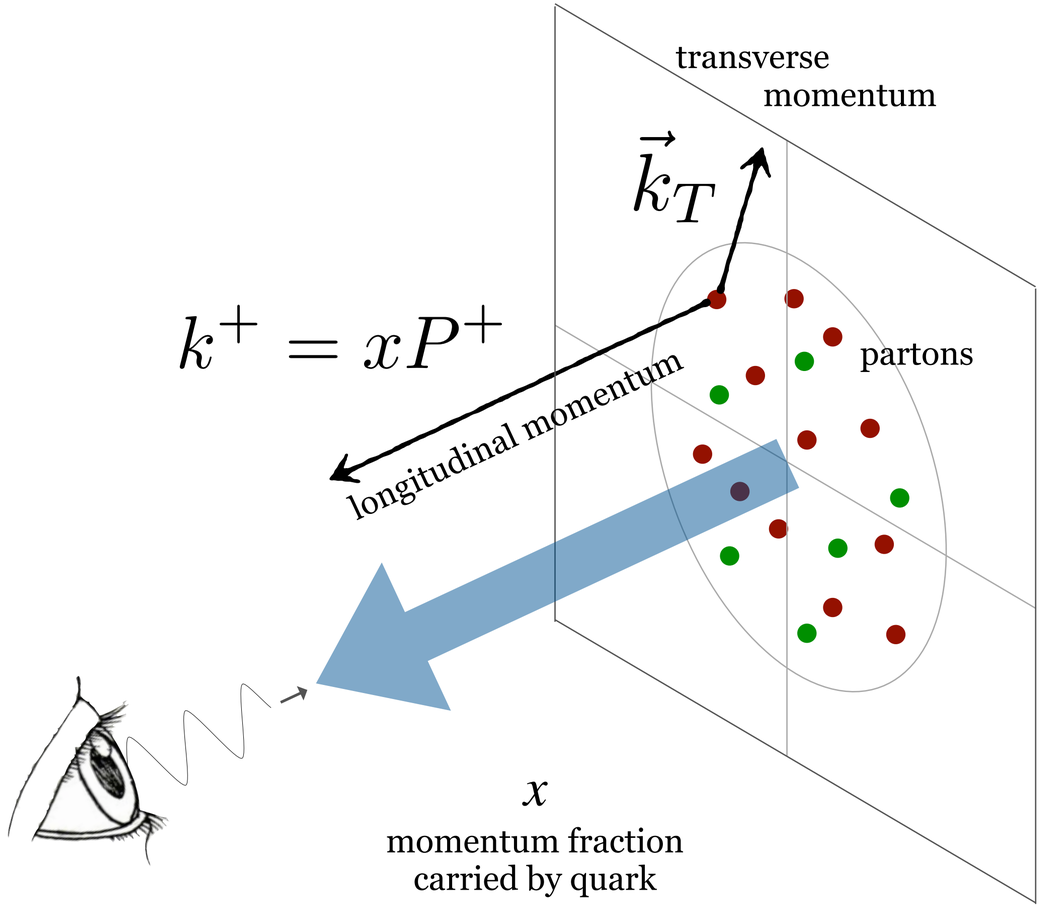
erc European Research Council

INFN Istituto Nazionale di Fisica Nucleare

UNIVERSITÀ DI PAVIA

TMDs: 3D maps

in momentum space



PDFs

Parton Distribution Functions

$$f_1(x)$$

1D maps

$$\int d^2\mathbf{k}_T$$

TMDs

Transverse Momentum Dependent
Parton Distribution Functions

$$f_1(x, k_T^2)$$

3D maps

figure credit: A.Bacchetta

quark TMD PDFs

unpolarized TMD

helicity

quark pol.

nucleon pol.	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

Boer-Mulders

Sivers

Twist-2 TMDs

pretzelosity

worm-gear

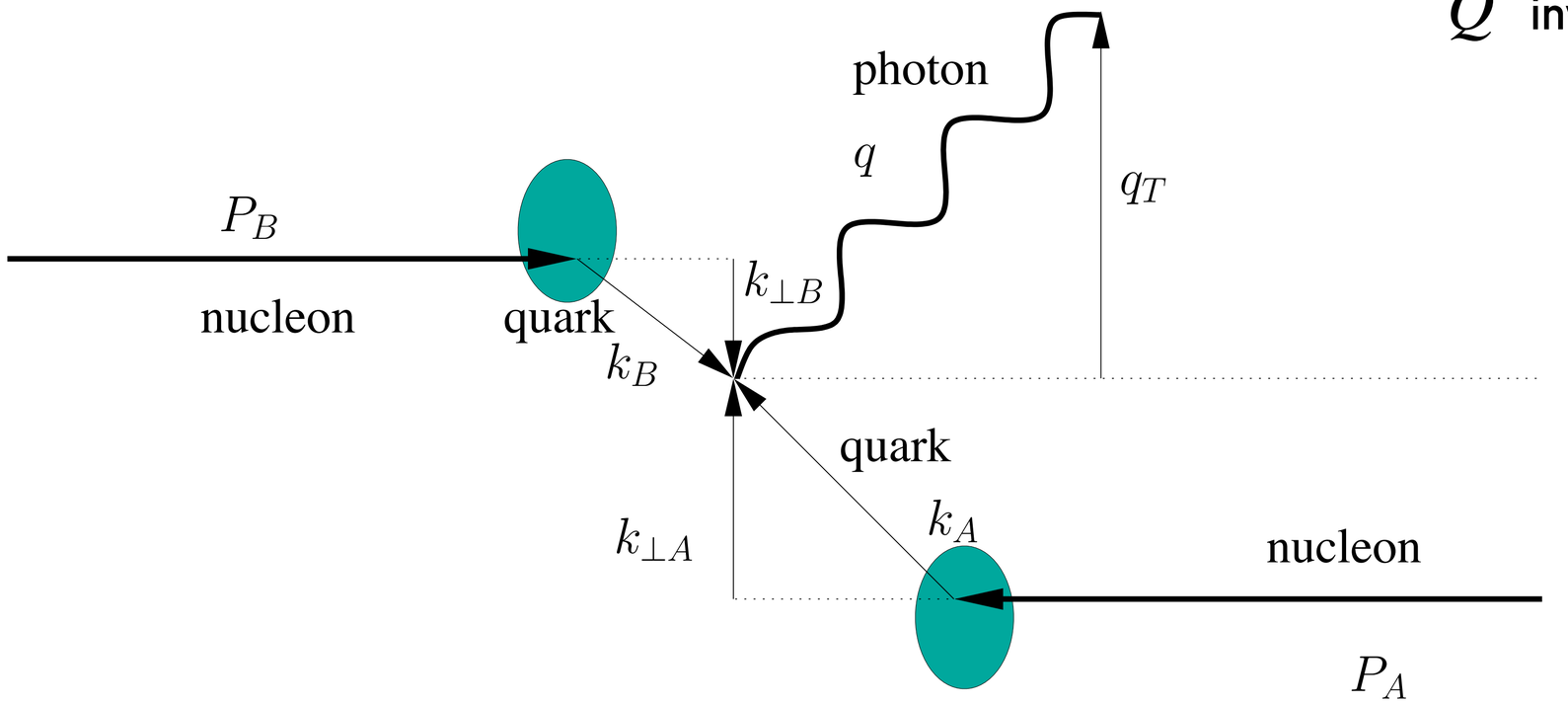
transversity

- TMDs in black survive k_T integration; TMDs in red are T-odd
- similar table for TMD FFs

Drell-Yan

$N(P_A) + N(P_B) \to \gamma^*/Z \to l^+ l^-$

Q invariant mass



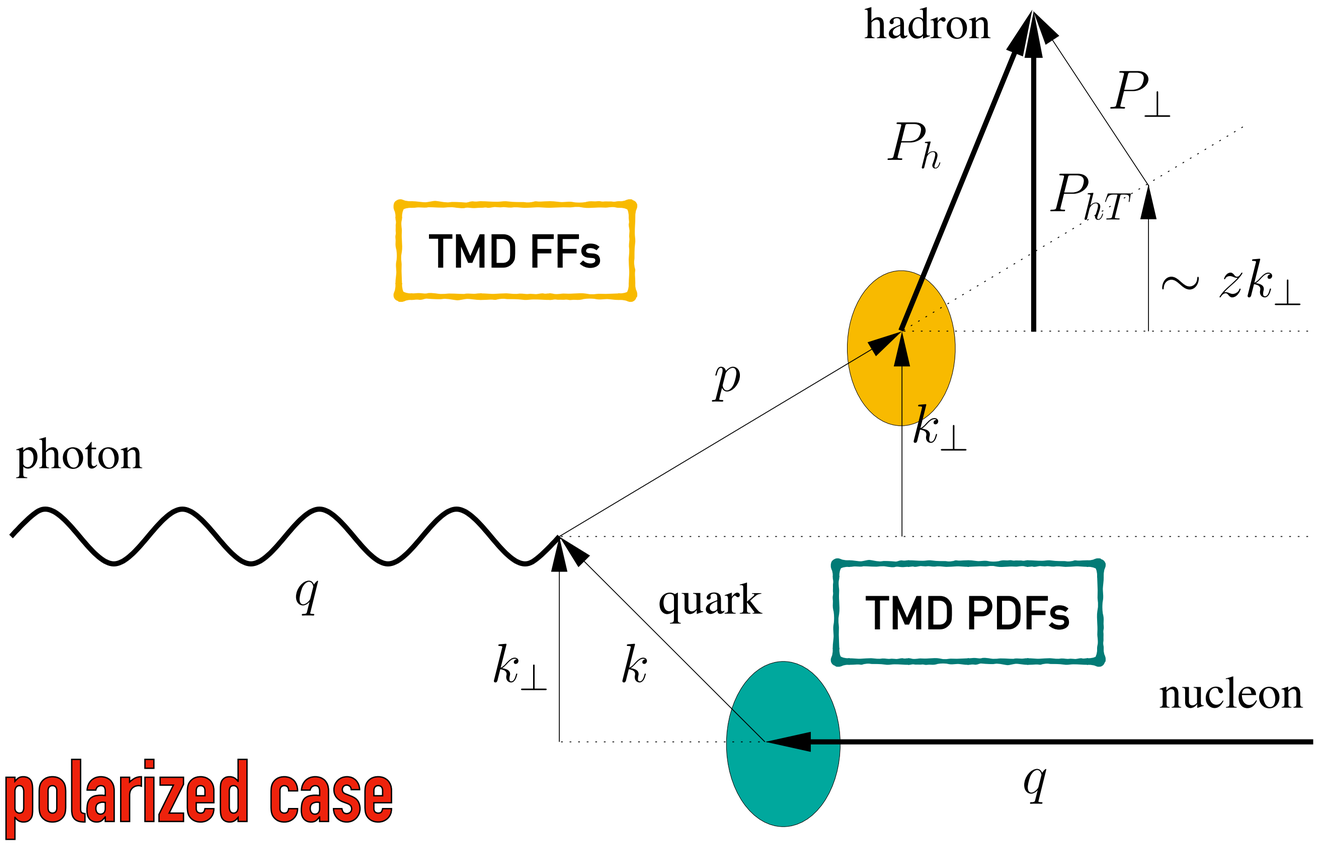
for $q_T \ll Q$

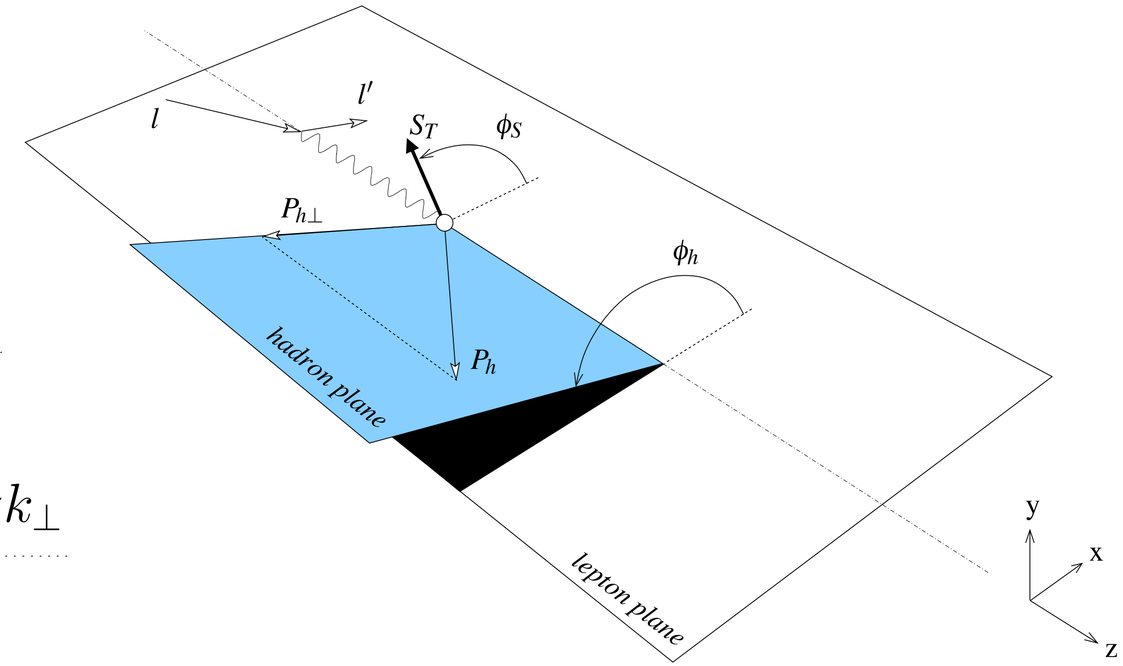
TMD factorization

$$\left(\frac{d\sigma}{dq_T}\right)_{\text{res.}} \propto H(Q,\mu) \int \frac{d^2\mathbf{b}}{4\pi} e^{i\mathbf{b}\cdot\mathbf{q}_T} x_1 \boxed{f_1^q(x_1, \mathbf{b}; \mu, \zeta_1)} x_2 \boxed{f_1^{\bar{q}}(x_2, \mathbf{b}; \mu, \zeta_2)}$$

Semi-Inclusive DIS

$$\ell(l) + N(p) \to \ell(l') + h(P_h) + X$$





$M^2 \ll Q^2 \qquad P_{hT}^2 \ll Q^2$

TMD factorization

unpolarized case

$$\frac{d\sigma}{dq_T} \propto F_{UU,T}(x, z, q_T; Q^2) \propto \int \frac{d^2\mathbf{b}}{4\pi} e^{i\mathbf{b}\cdot\mathbf{q}_T} f_1^q(x, \mathbf{b}; \mu) D_1^{q\to h}(z, \mathbf{b}; \mu)$$

TMD PDF

scales

$$\mu = Q$$

$$\zeta_1 = \zeta_2 = \zeta = Q^2$$

matching to the collinear region

collinear PDFs, longitudinal non perturbative part

$$f_1^q(x, b; \mu, \zeta) = \sum_j \left(C_{q/j} \otimes f^j\right)(x, b_*; \mu_b)$$

unpolarized quark

TMD PDF

$$\times \exp\left\{K(\mu_0) \ln \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}} + \int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F - \gamma_K \ln \frac{\sqrt{\zeta}}{\mu'}\right]\right\}$$

Sudakov form factor: perturbative evolution and resummation of large logs

$$\times f_{\mathrm{NP}}(x, b; \zeta)$$

parametrized and fitted to data non perturbative transverse content

Logarithmic accuracy

$$\left(\frac{d\sigma}{dq_T}\right) \propto H(Q,\mu) \int \frac{d^2\mathbf{b}}{4\pi} e^{i\mathbf{b}\cdot\mathbf{q}_T} x_1 f_1^q(x_1,\mathbf{b};\mu,\zeta_1) x_2 f_1^{\bar{q}}(x_2,\mathbf{b};\mu,\zeta_2)$$

perturbative expansion in $\alpha_s(\mu)$

$$f_1^q(x,b;\mu,\zeta) = \sum_j \left(C_{q/j} \otimes f^j\right)(x,b_*;\mu_b)$$
$$\times \exp\left\{K(\mu_0)\ln\frac{\sqrt{\zeta}}{\sqrt{\zeta_0}} + \int_{\mu_0}^{\mu}\frac{d\mu'}{\mu'}\left[\gamma_F(\alpha_s(\mu')) - \gamma_K(\alpha_s(\mu'))\ln\frac{\sqrt{\zeta}}{\mu'}\right]\right\}$$
$$\times f_{\rm NP}(x,b;\zeta)$$

Accuracy	H and C	K and γ_F	γ_K	PDF and α_s evolution
LL	0	-	1	-
NLL	0	1	2	LO
NLL'	1	1	2	NLO
NNLL	1	2	3	NLO
NNLL'	2	2	3	NNLO
N^3LL	2	3	4	NNLO

Recent TMD fits of unpolarized data

$f_1(x, k_T^2)$

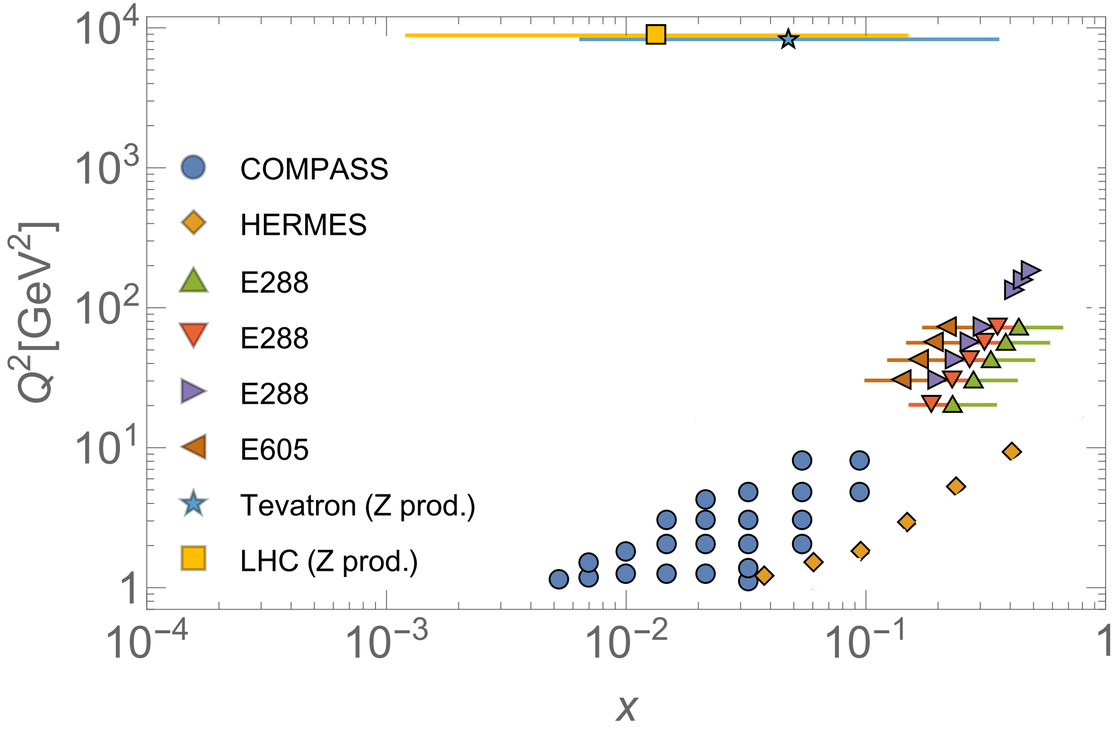
$D_1^h(z, P_T^2)$

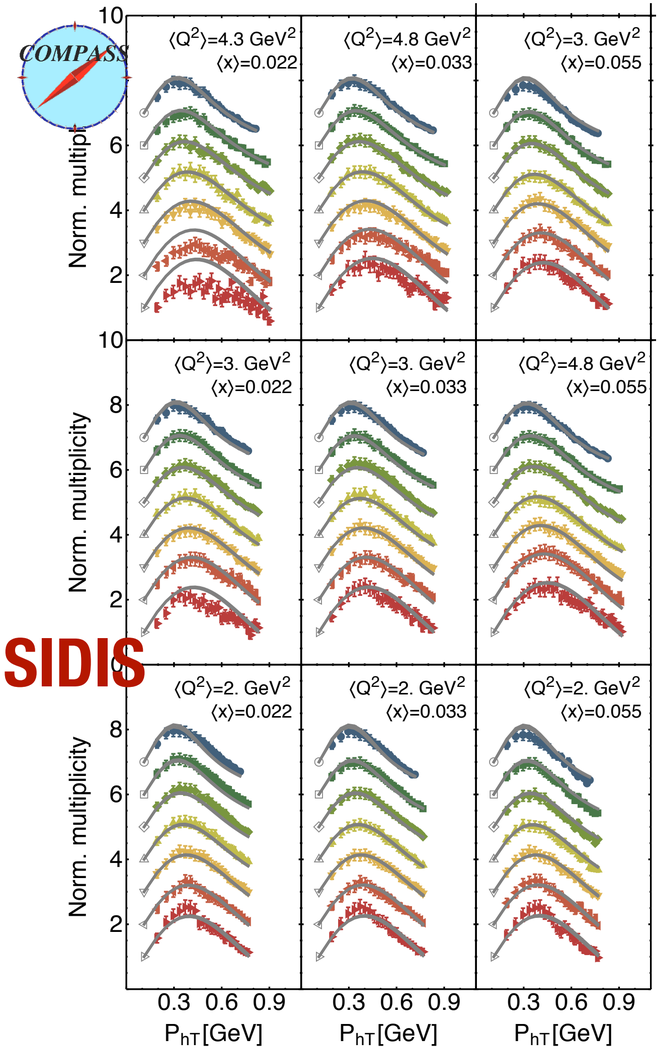
	Framework	HERMES	COMPASS	DY	Z production	N of points	χ^2/N_{points}
Pavia 2017 arXiv:1703.10157	NLL	✔	✔	✔	✔	8059	1.55
SV 2017 arXiv:1706.01473	NNLL′	✘	✘	✔	✔	309	1.23
BSV 2019 arXiv:1902.08474	NNLL′	✘	✘	✔	✔	457	1.17
SV 2019 arXiv:1912.06532	NNLL′	✔	✔	✔	✔	1039	1.06
Pavia 2019 arXiv:1912.07550	N^3LL	✘	✘	✔	✔	353	1.02

PV17

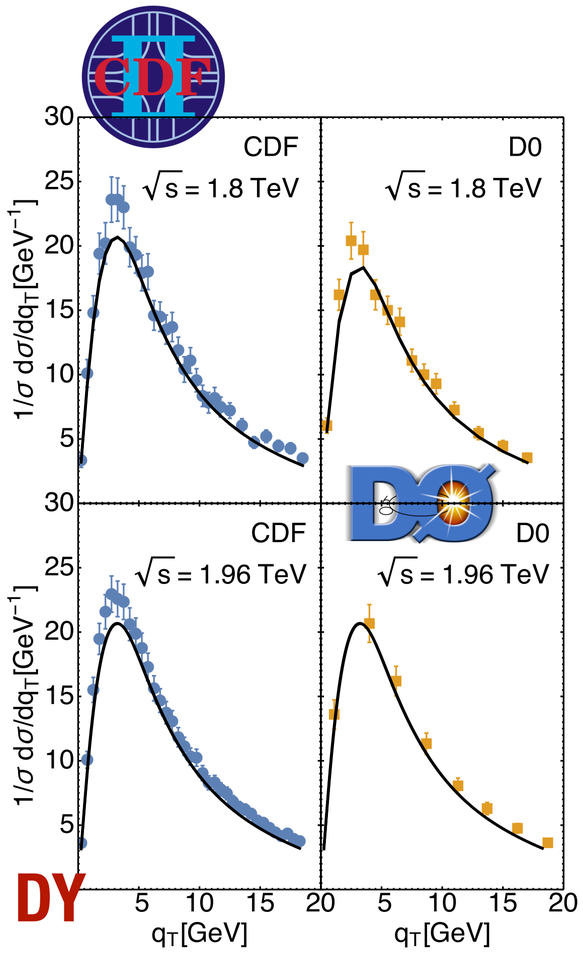
A. Bacchetta, F. Delcarro, C. Pisano, M. Radici, A. Signori
arXiv:1703.10157

8059
data points





SIDIS



DY

with normalization coefficients

NLL

global

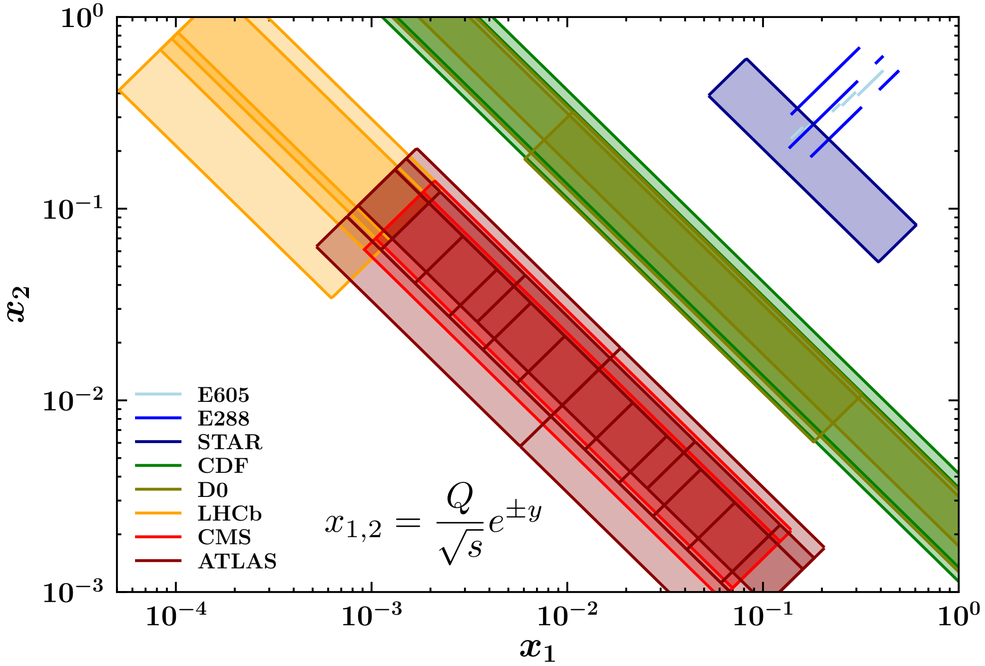
$$\chi^2 = 1.55$$

N3LL Drell-Yan fit

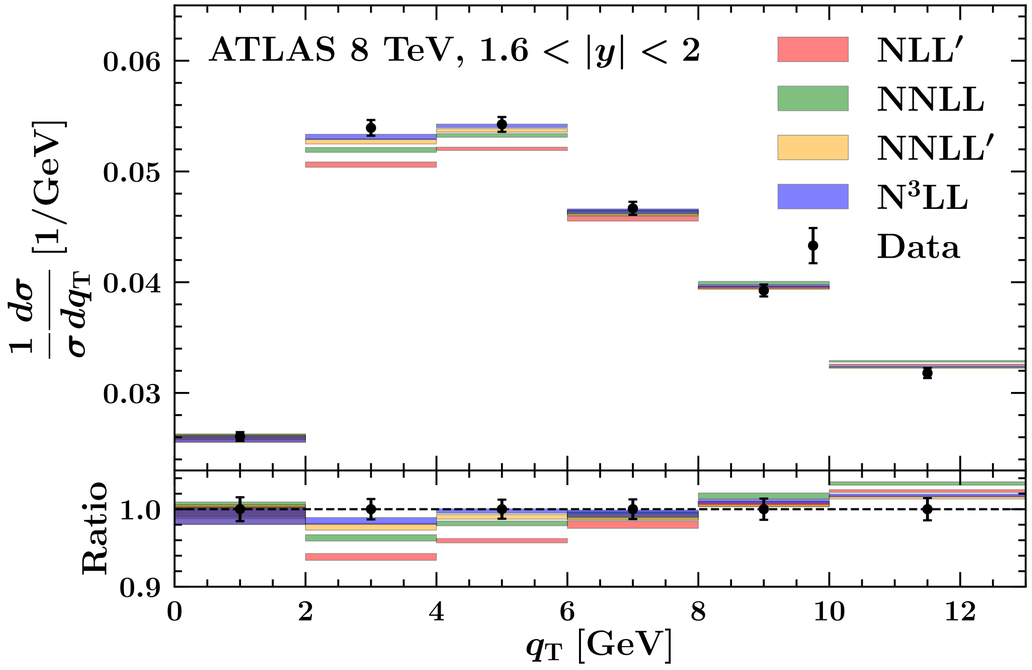
A. Bacchetta, V. Bertone, C. Bissolotti, G. Bozzi, F. Delcarro, F. Piacenza, M. Radici
JHEP 07 (2020) 117 e-Print: 1912.07550

PV19

353 data points

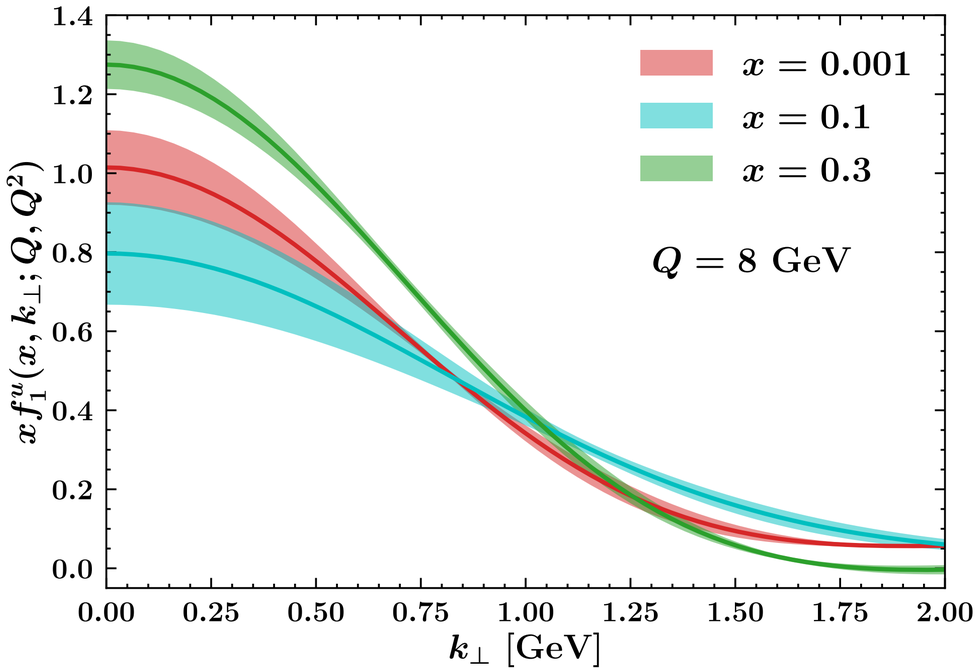


NO normalization coefficients



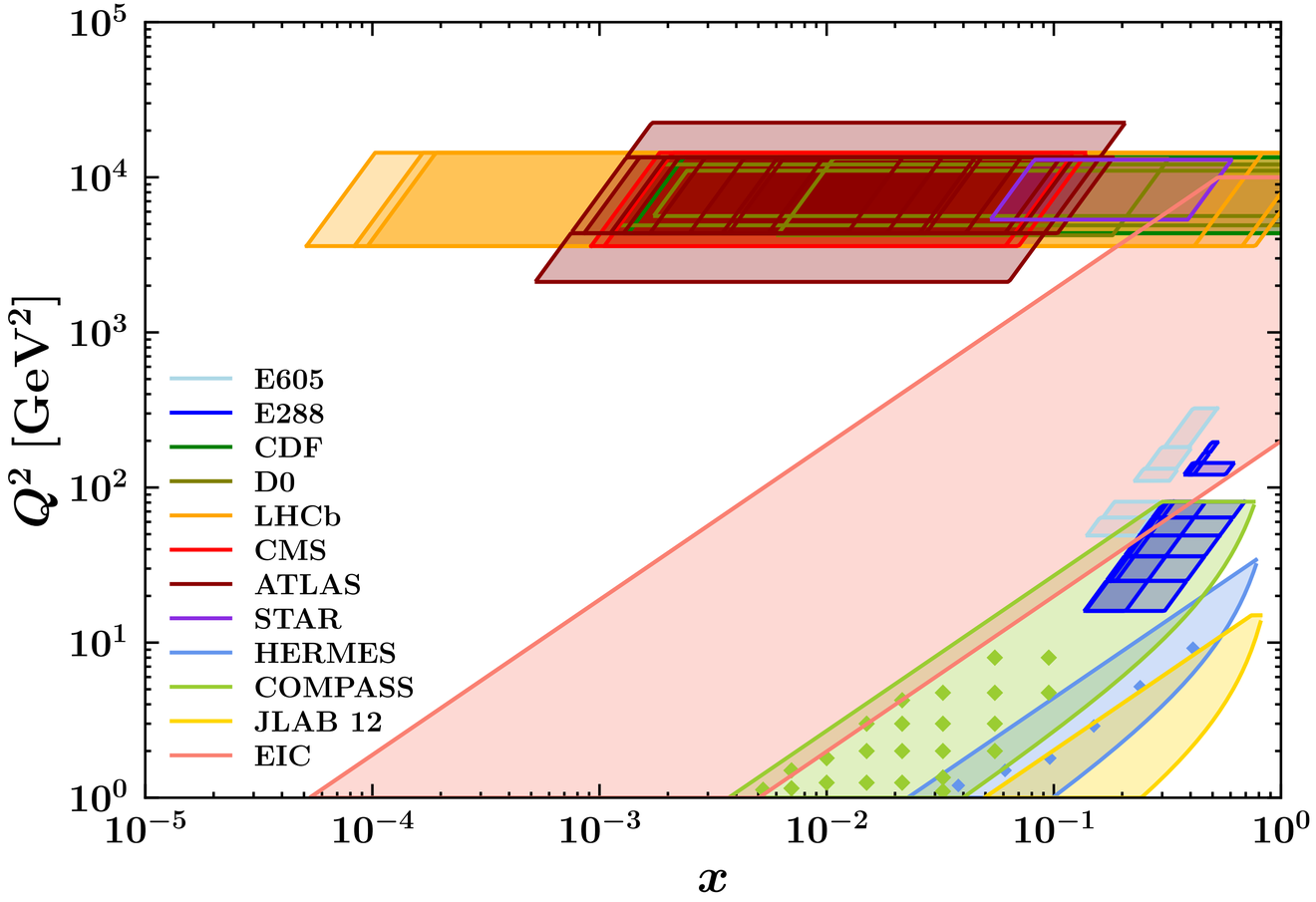
global

$\chi^2 = 1.02$



Electron-Ion Collider

SIDIS as main process to access TMDs

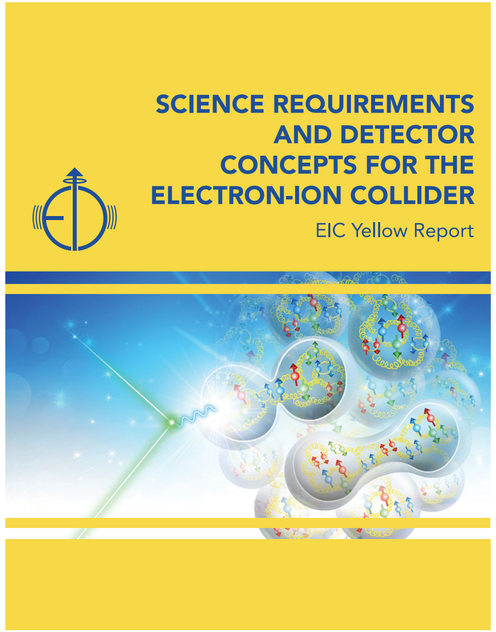


from TMD

extractions

make

predictions



Unpolarized quark TMDs at EIC

EIC pseudodata

generated by Ralf Seidl and available on
https://github.com/VladimirovAlexey/EIC_YR_TMD

5x41 18x275

configurations

5x100 18x100

10x100

π^+

π^-

final state hadrons

K^+

K^-

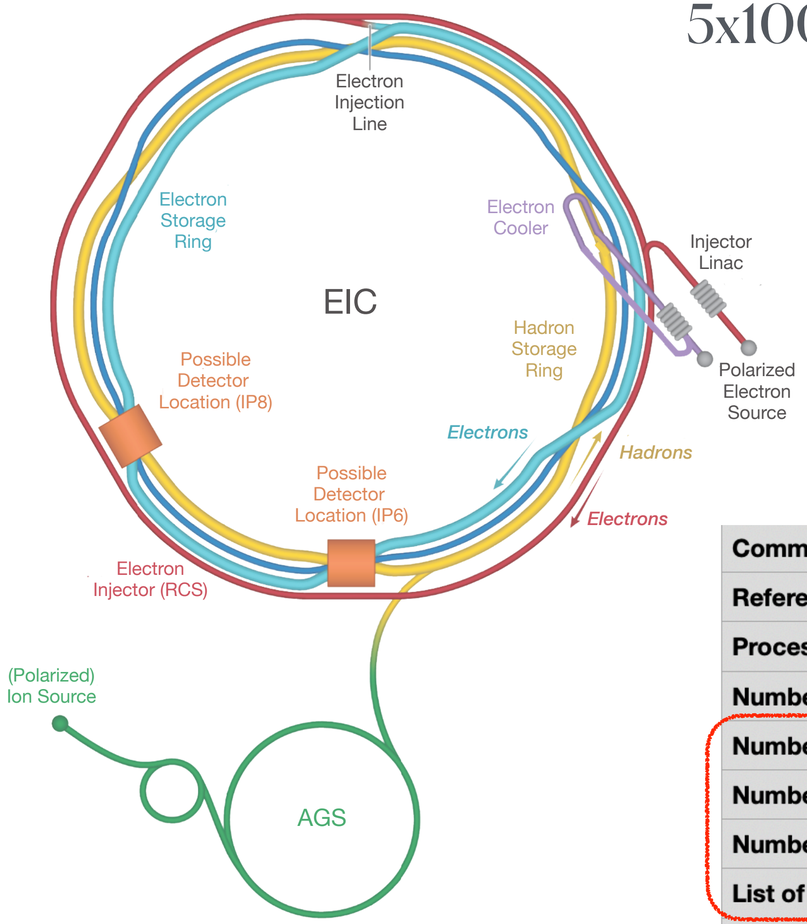


image from EIC YR
arXiv:2103.05419

Comment	Ralf's pseudo data for EIC.
Reference	Ralf
Process type	SIDIS
Number of points	2958
Number of uncorr.errors	2
Number of corr.errors	0
Number of norm.errors	1
List of norm.errors (relative)	0.03
Total cross-section normalized	False

uncertainties

eight options for EIC settings:
we choose option 8

EIC impact studies

sensitivity coefficients

from E. Aschenauer, I. Borsa, G. Lucero, A. S. Nunes, R. Sassot
arXiv:2007.08300

$$F_{UU,T}(x, z, q_T; Q^2)$$

observable

distribution

TMD parameters

$$S[f_i, \mathcal{O}] = \frac{\langle \mathcal{O} \cdot f_i \rangle - \langle \mathcal{O} \rangle \langle f_i \rangle}{\xi \cancel{\Delta \mathcal{O}} \Delta f_i}$$

experimental uncertainty
(from pseudodata)

$$\xi \equiv \frac{\delta \mathcal{O}}{\cancel{\Delta \mathcal{O}}}$$

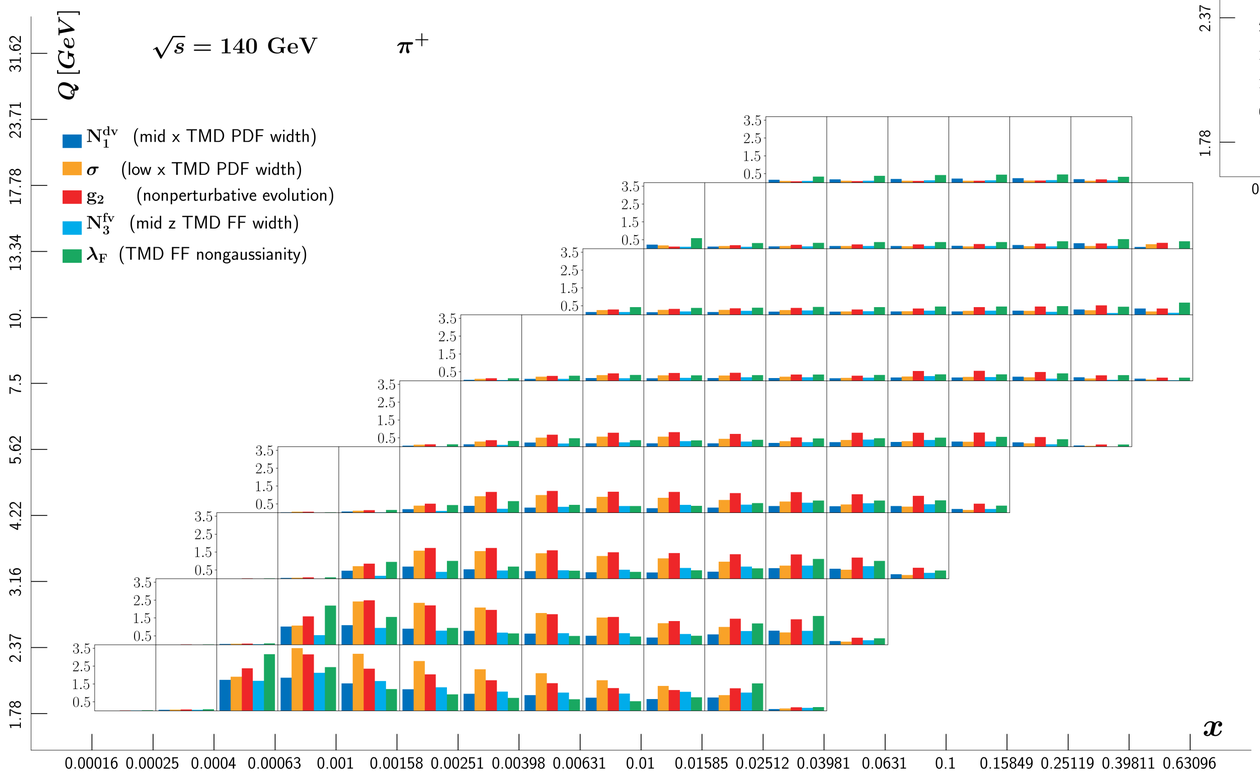
~~theoretical uncertainty~~

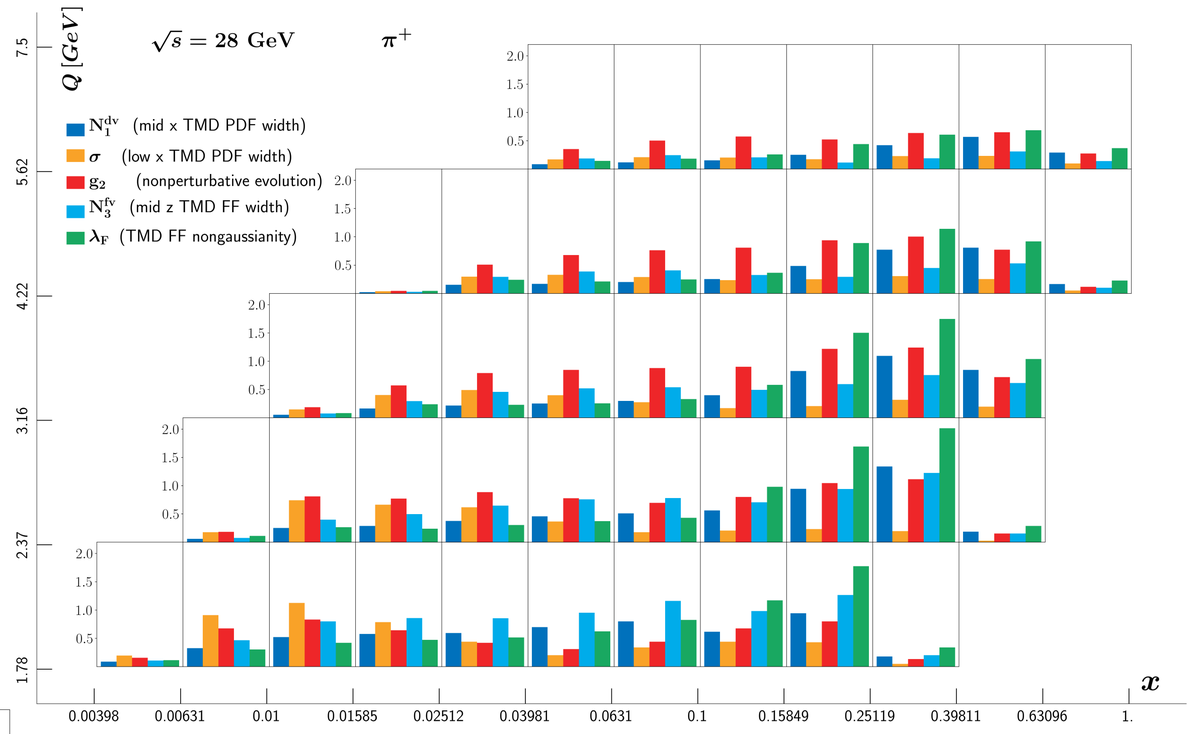
$$\langle \mathcal{O} \rangle = \frac{1}{N} \sum_{k=1}^{N} \mathcal{O}[f_i^{(k)}]$$

EIC impact studies

sensitivity coefficients

$$S[f_i, \mathcal{O}] = \frac{\langle \mathcal{O} \cdot f_i \rangle - \langle \mathcal{O} \rangle \langle f_i \rangle}{\delta \mathcal{O} \Delta f_i}$$





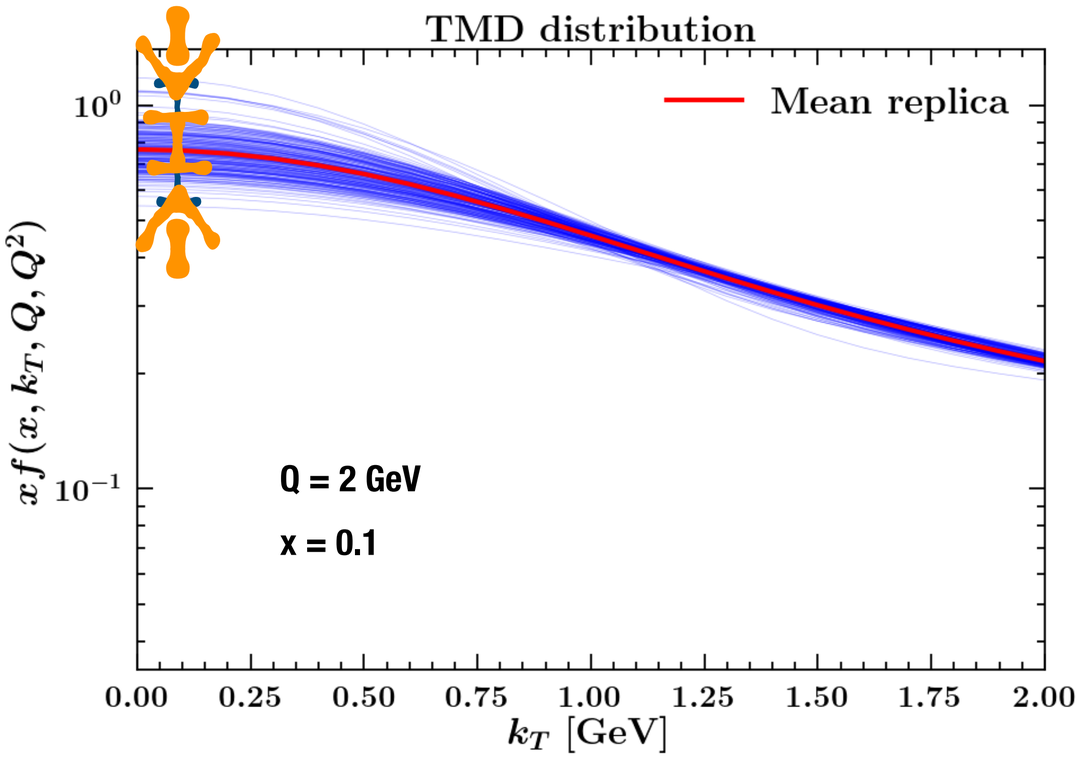
higher impact for low $\sqrt{s}$

attempts at reweighing

200 replicas are compared with pseudodata

$$\chi^2_k = \chi^2_{k,\mathrm{EIC}} + \chi^2_{k,\mathrm{PV17}}$$

'original' χ^2 with respect to PV17 data



weights

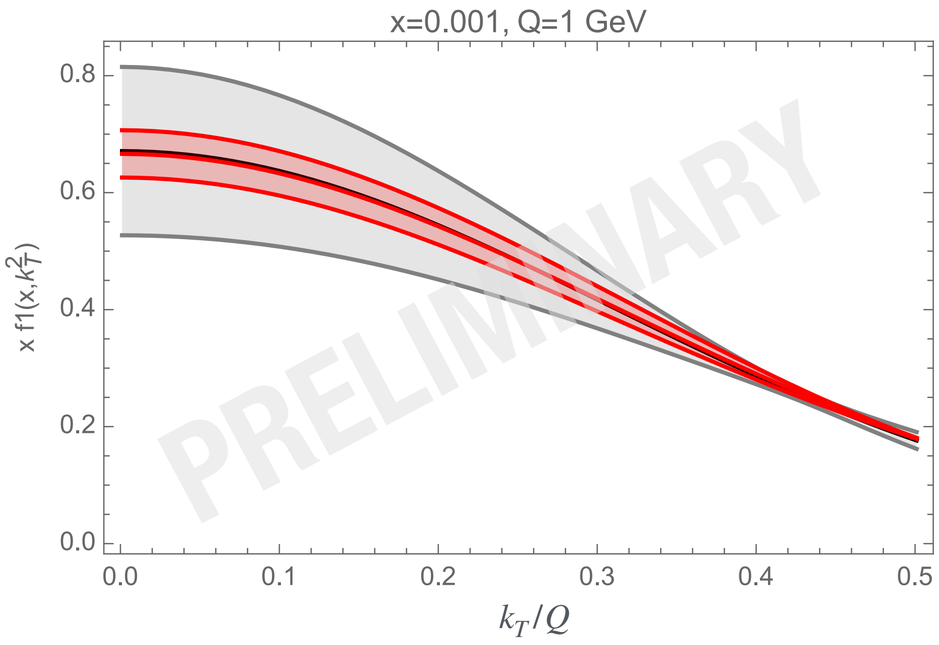
$$w_k \propto \mathcal{P}(f_k|\chi_k) \propto \chi_k^{n-1} e^{-\frac{1}{2}\chi_k^2}$$

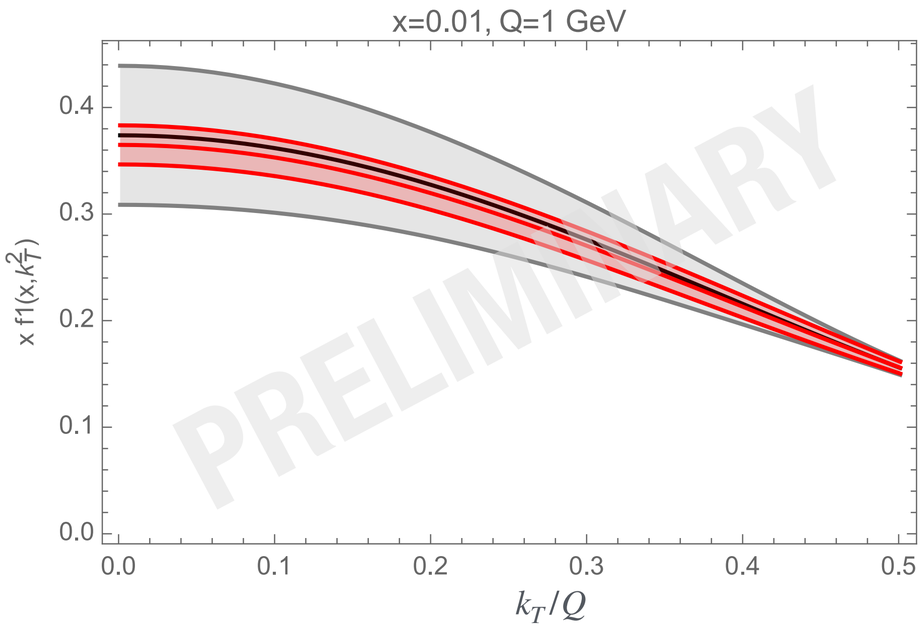
used to select replicas

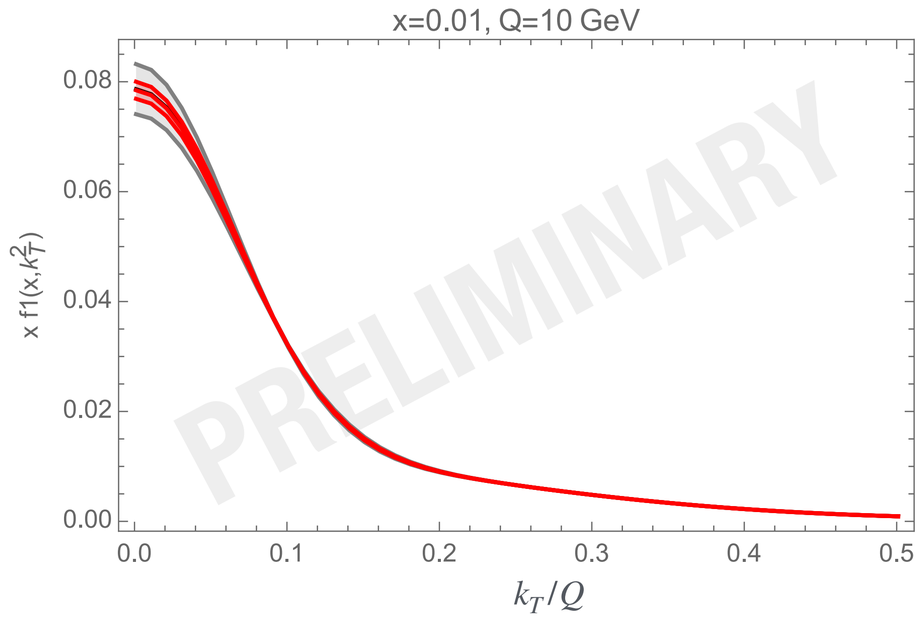
reflect the impact of EIC data on extracted TMDs

from NNPDF Collaboration
arXiv:1108.1758

attempts at reweighing







we combined pseudodata of different configurations

5x41, 5x100, 10x100, 18x275

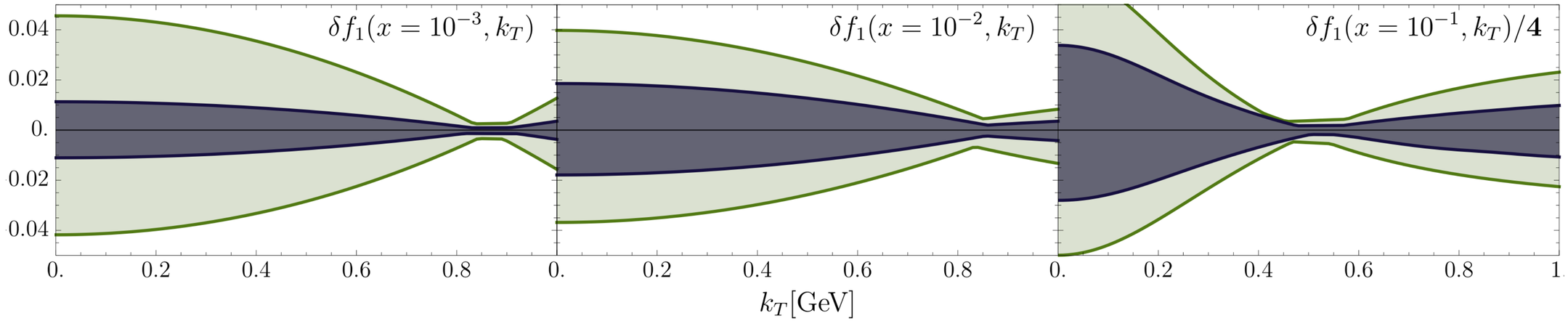
EIC impact - fit of pseudodata

on up quark unpolarized TMD PDF f_1

green band is the SV19 extraction

I. Scimemi, A. Vladimirov

JHEP 06 (2020) 137, arXiv: 1912.06532



dark grey is a fit of SV19 + EIC pseudodata

from EIC Yellow Report

Sivers quark TMDs

quark Sivers TMDs

A. Bacchetta, F. Delcarro, C. Pisano, M. Radici
arXiv:2004.14278

NLL analysis

$$\hat{f}_{1T}^{\perp(1)a}(x, b_T^2; Q^2) = e^{S(\mu_b^2, Q^2)}\ e^{g_K(b_T)\ln(Q^2/Q_0^2)} f_{1T}^{\perp(1)a}(x; \mu_b^2)\ \hat{f}_{1T\mathrm{NP}}^{\perp(1)a}(x, b_T^2)$$

Sivers

$$A_{UT}^{\sin(\phi_h - \phi_S)}(x, z, \boldsymbol{P}_{hT}^2, Q^2) \approx \frac{F_{UT,T}^{\sin(\phi_h - \phi_S)}}{F_{UU,T}}$$

quark Sivers TMDs

A. Bacchetta, F. Delcarro, C. Pisano, M. Radici
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Sivers

$$A_{UT}^{\sin(\phi_h-\phi_S)}(x, z, \boldsymbol{P}_{hT}^2, Q^2) \approx \frac{F_{UT,T}^{\sin(\phi_h-\phi_S)}}{F_{UU,T}}$$

this ingredient is the same and
must be fixed by unpolarized TMDs

consistency with unpolarized TMDs

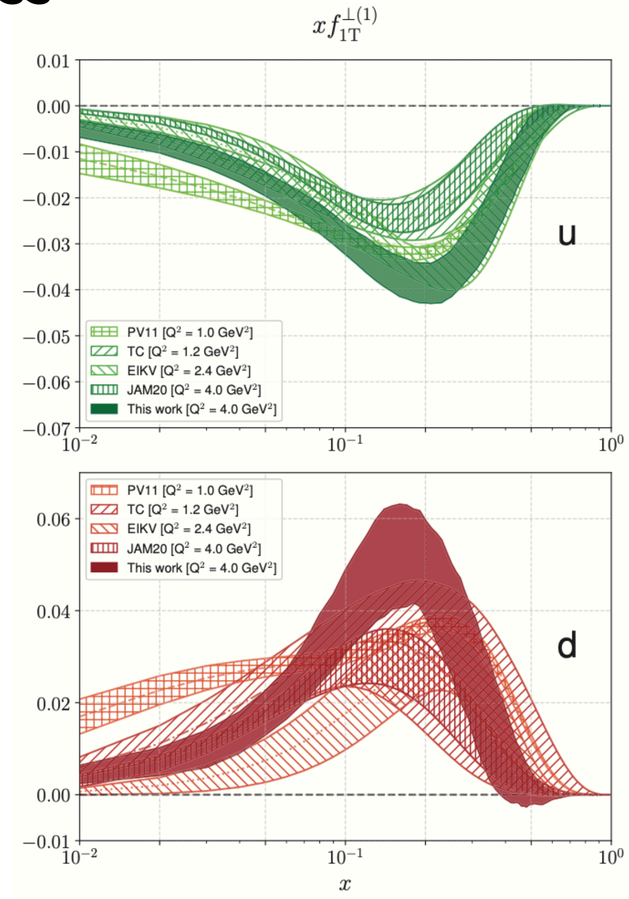
$$\hat{f}_1^a(x, b_T^2; Q^2) = e^{S(\mu_b^2, Q^2)}\; e^{g_K(b_T)\ln(Q^2/Q_0^2)}\; f_1^a(x; \mu_b^2)\; \hat{f}_{1\mathrm{NP}}^a(x, b_T^2)$$

unpolarized TMD PDF

quark Sivers TMD extractions

from SIDIS data

JLab, HERMES and COMPASS



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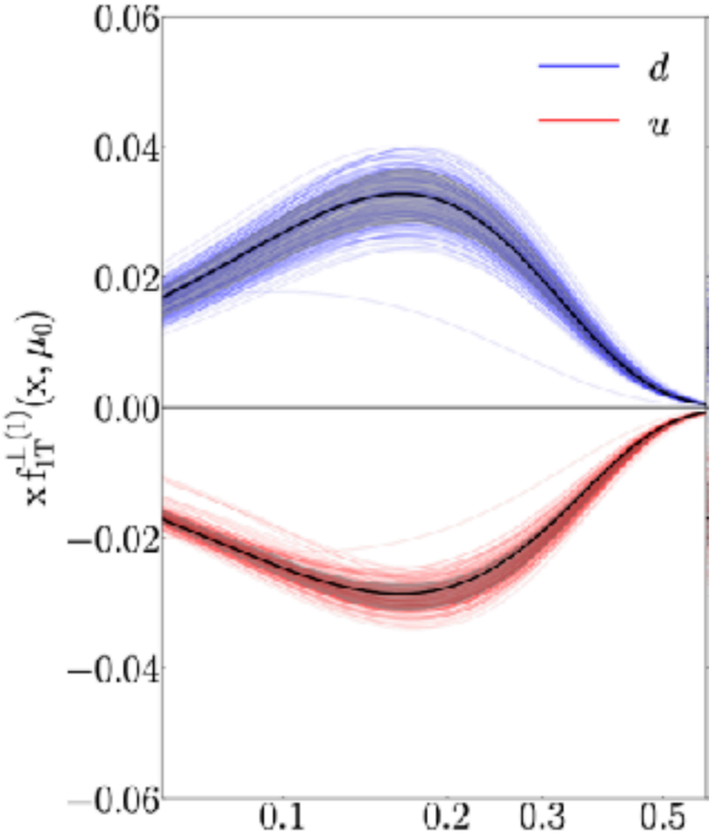
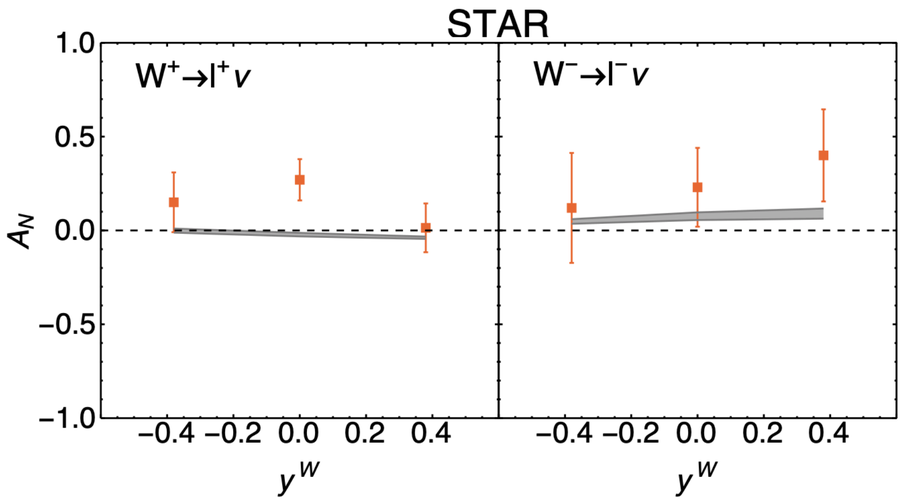


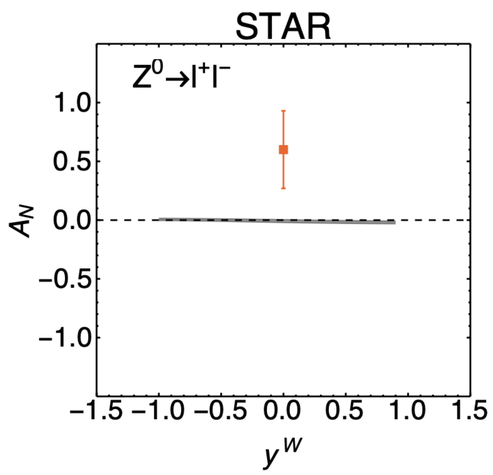
M. Echevarria, Z. Kang, J. Terry
arXiv:2009.10710

Impact of RHIC

Drell-Yan data

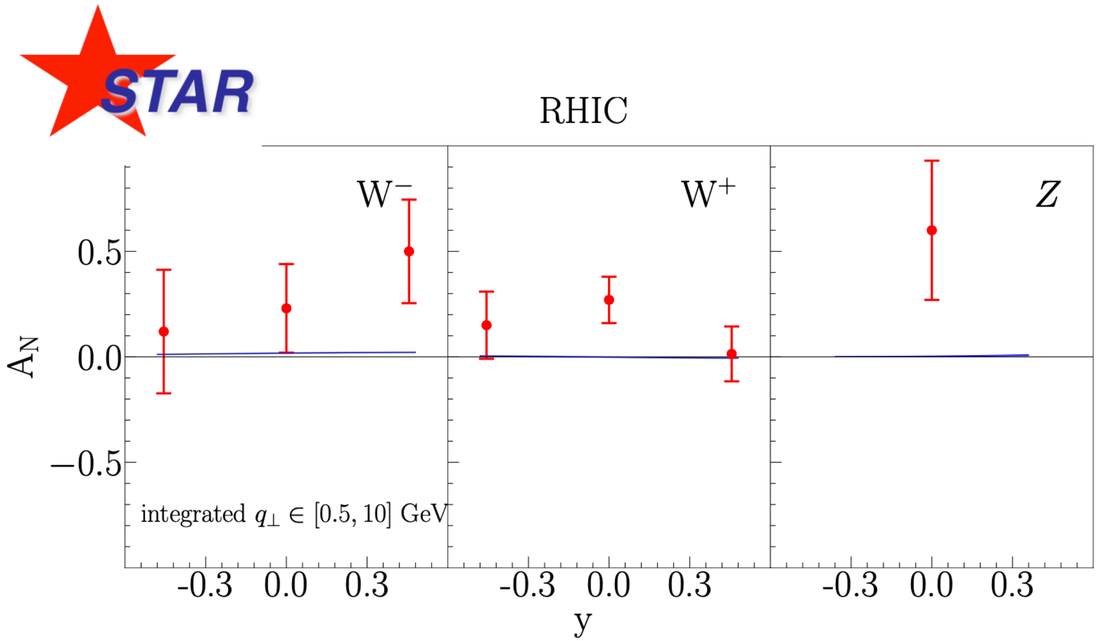
$$f_{1T}^{\perp\,\mathrm{DY}}(x, k_\perp^2; \mu, \zeta) = -f_{1T}^{\perp\,\mathrm{SIDIS}}(x, k_\perp^2; \mu, \zeta)$$





A. Bacchetta, F. Delcarro, C. Pisano, M. Radici
arXiv:2004.14278

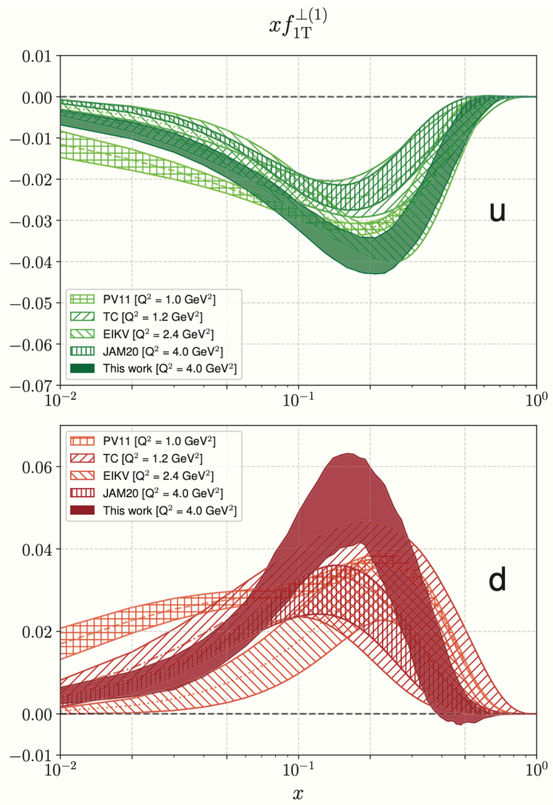
predictions



M. Echevarria, Z. Kang, J. Terry
arXiv:2009.10710

Impact of RHIC data

fit without RHIC W & Z data



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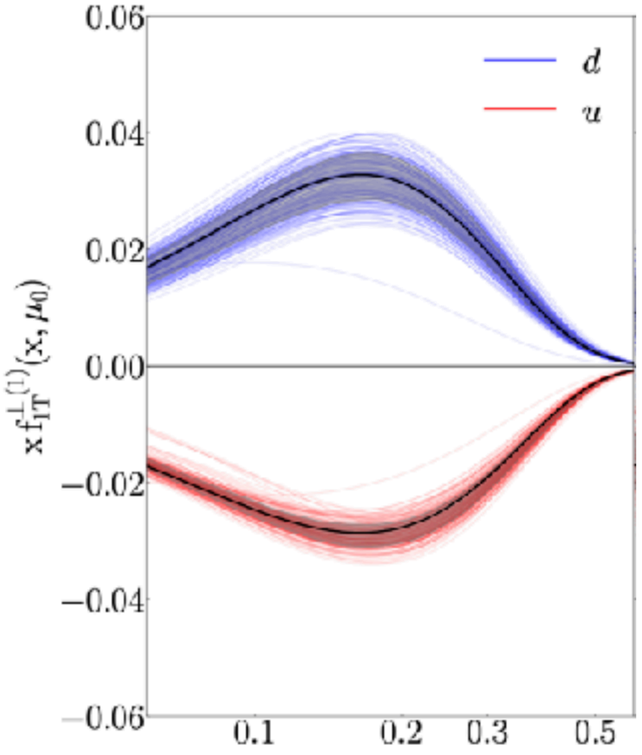
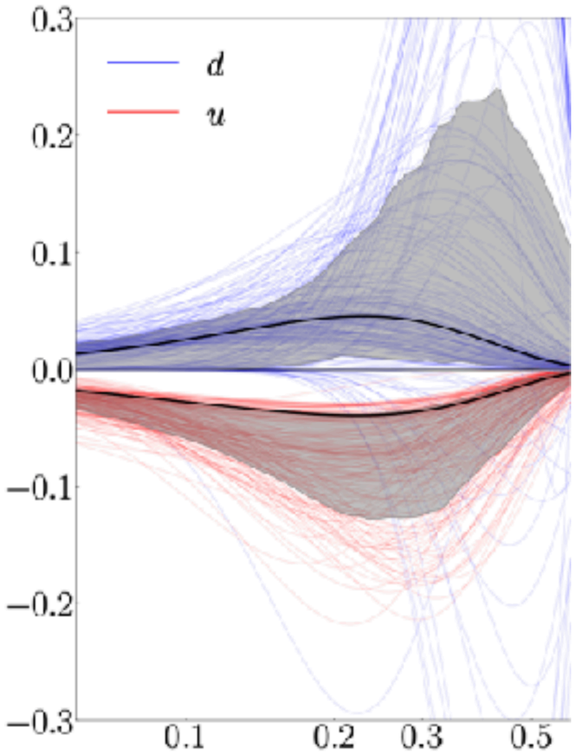
arXiv:2004.14278



fit with RHIC W & Z data

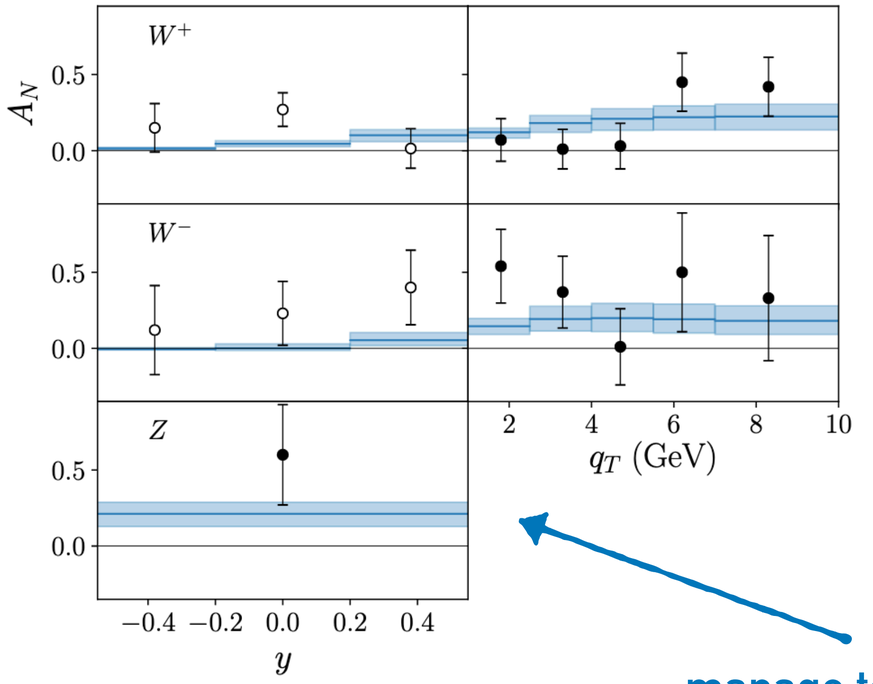


M. Echevarria, Z. Kang, J. Terry
arXiv:2009.10710

Impact of RHIC data

M. Bury, A. Prokudin, A. Vladimirov
JHEP 05 (2021) 151, arXiv: 2103.03270

fit with RHIC W & Z data



manage to raise predictions

CAVEAT: positivity bounds

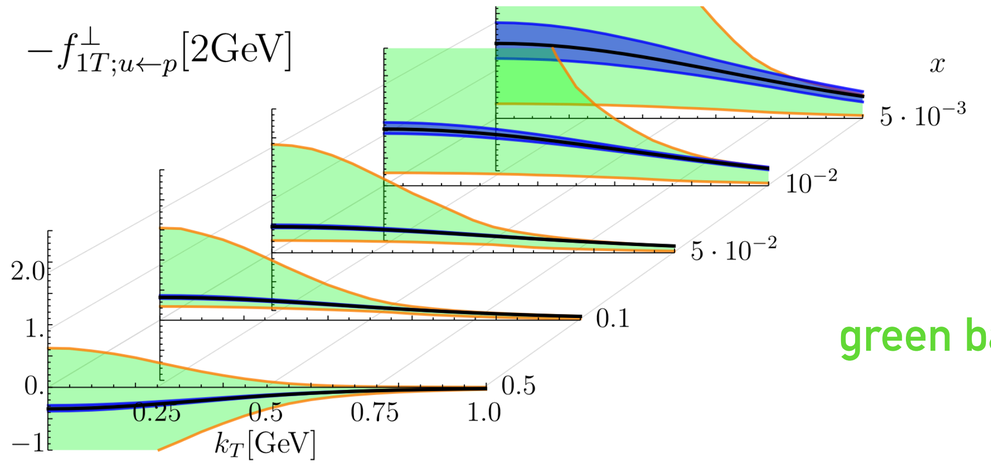
are violated in some regions!

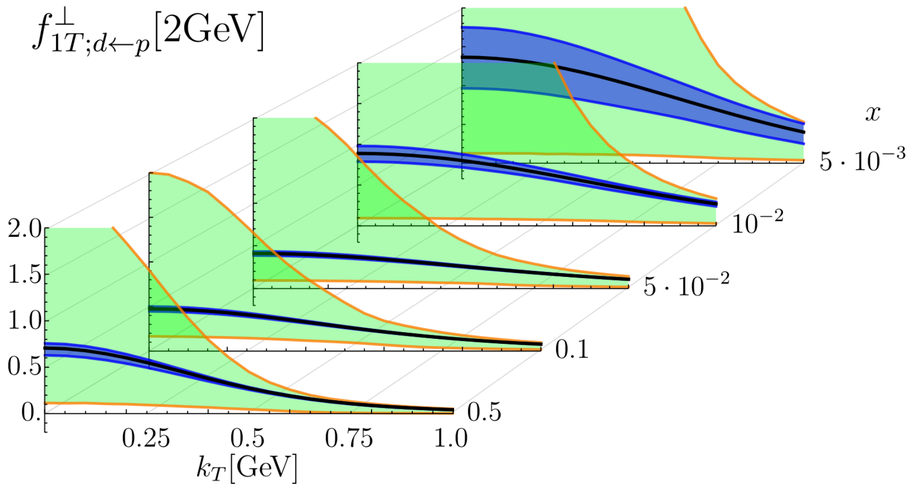
$$\frac{k_T}{M}\left|f_{1T}^{\perp}(x,k_T^2)\right| \leq f_1(x,k_T^2)$$

must hold for any value of x and k_T

Impact of EIC: reweighing

EIC Yellow Report
arXiv: 2103.05419





EIC pseudodata set has about
two orders of magnitude
more points than current data

M. Bury, A. Prokudin, A. Vladimirov
JHEP 05 (2021) 151, arXiv: 2103.03270

green bands represent the current uncertainty

blue bands after reweighing

CAVEAT:

are **positivity bounds** violated?

$$\frac{k_T}{M}\left|f_{1T}^{\perp}(x,k_T^2)\right| \leq f_1(x,k_T^2)$$

must hold for any value of x and k_T

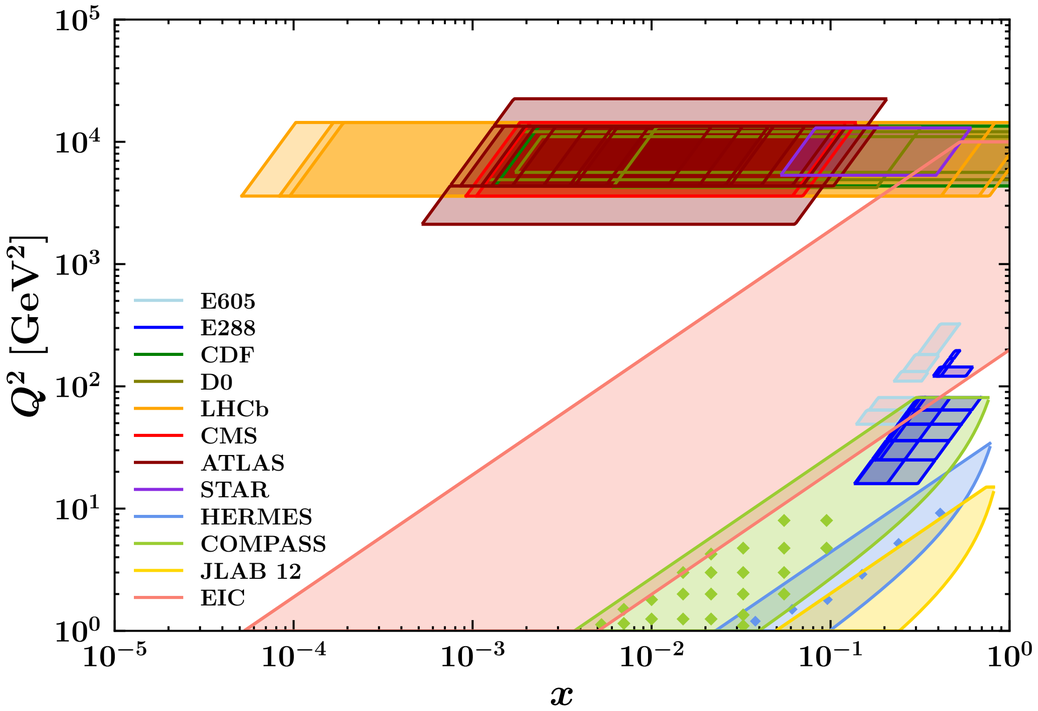
Conclusions

state of the art accuracy of **unpolarized quark TMD PDF** extractions is **N3LL**

- needed to describe present and future very precise data

EIC will have an enormous **impact** on **TMDs**

- largest impact will be in the regions that are not covered by present data
- better determination of f_1 influences the accuracy of polarized TMDs as well
  - **Sivers**
    present extractions have difficulties in describing RHIC data



Backup

TMDs: 3D maps

in momentum space

Wigner distributions

$$\rho\left(x, \mathbf{k}_T, \mathbf{b}_T\right)$$

$$\int d^2\mathbf{k}_T$$

Generalized Parton Distributions

GPDs

$$\int d^2\mathbf{b}_T$$

Transverse Momentum Distributions

TMDs

$$\int d^2\mathbf{k}_T$$

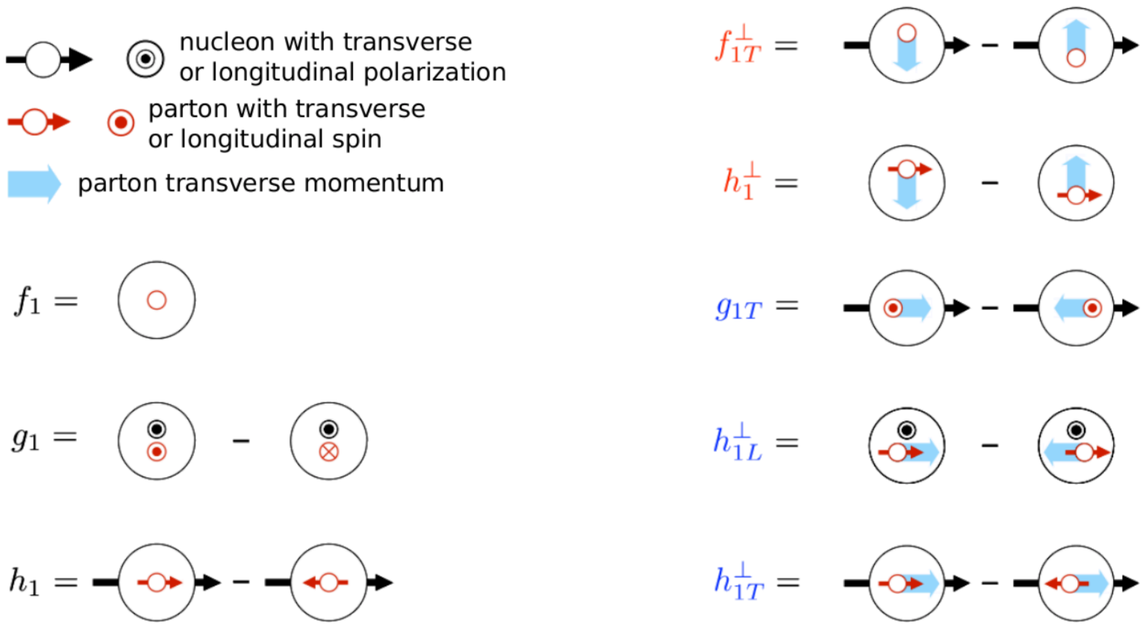
Parton Distribution Functions

PDFs

quark TMDs

TMD PDFs

		Quark polarization		
		U	L	T
Nucleon polarization	U	f_1		$h_1^\perp$
	L		g_{1L}	$h_{1L}^\perp$
	T	$f_{1T}^\perp$	g_{1T}	$h_1, h_1^\perp$



quark TMDs

TMD PDFs

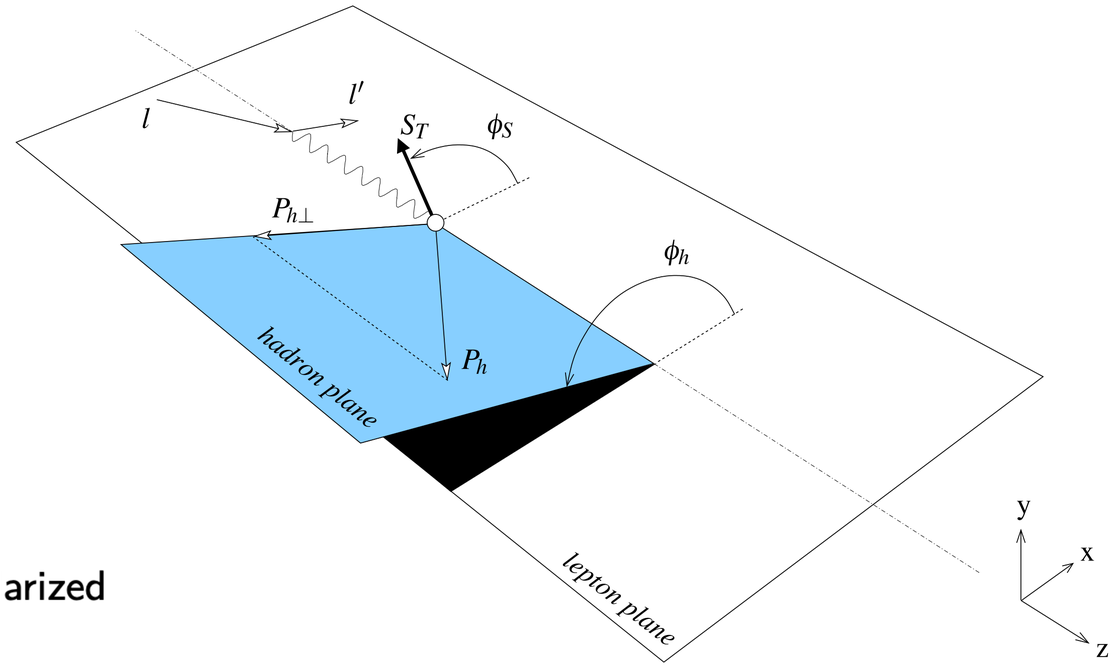
		Quark polarization		
		U	L	T
Nucleon polarization	U	f_1		$h_1^\perp$
	L		g_{1L}	$h_{1L}^\perp$
	T	$f_{1T}^\perp$	g_{1T}	$h_1, h_1^\perp$

TMD FFs

		Quark polarization		
		U	L	T
Hadron polarization	U	D_1		$H_1^\perp$
	L		G_{1L}	$H_{1L}^\perp$
	T	$D_{1T}^\perp$	G_{1T}	$H_{1T}, H_{1T}^\perp$

Semi-Inclusive DIS

$$\ell(l) + N(p) \to \ell(l') + h(P_h) + X$$



$$\frac{d\sigma}{dx\,dy\,dz\,d\psi\,d\phi_h\,dP_{h\perp}^2} = \frac{\alpha^2}{xyQ^2}\,\frac{y^2}{2(1-\epsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Big\{$$

$F_{UU,T}(x, z, P_{h\perp}^2, Q^2)$ → 4D quantities

$+\,\epsilon\,F_{UU,L} + \sqrt{2\epsilon(1+\epsilon)}\cos(\phi_h)\,F_{UU}^{\cos\phi_h} + \epsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}$ → unpolarized

$+\,\lambda_e\,\Big[\ldots 1\,\text{SF}\ldots\Big]$ → polarized terms

$+\,S_{||}\,\Big[\ldots 2\,\text{SFs}\ldots\Big] +$

$+\,S_{||}\,\lambda_e\,\Big[\ldots 2\,\text{SFs}\ldots\Big]$

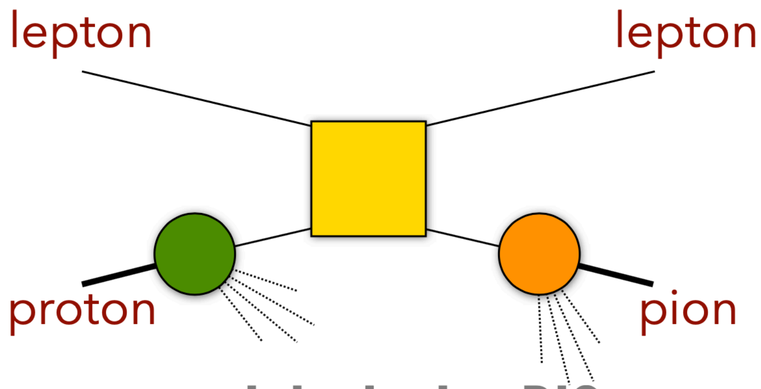
$+\,S_{\perp}\,\Big[\ldots 6\,\text{SFs}\ldots\Big]$

$+\,S_{\perp}\,\lambda_e\,\Big[\ldots 3\,\text{SFs}\ldots\Big]\Big\}$

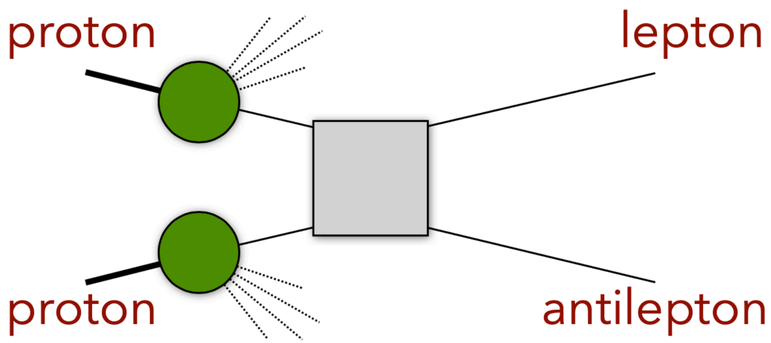
$M^2 \ll Q^2 \qquad P_{hT}^2 \ll Q^2$

TMD factorization

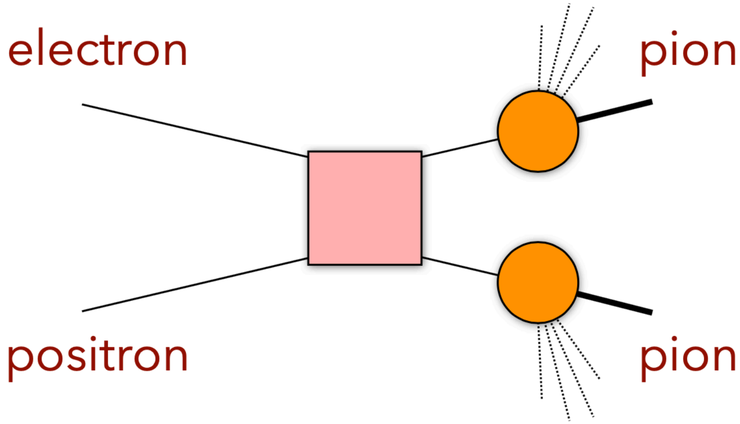
Factorization



semi-inclusive DIS



Drell-Yan



e^-e^+ to pions

TMD UNIVERSALITY

in these cases, TMD factorization is well understood

see, e.g., Ji, Ma, Yuan, PRD 71
Collins, "Foundations of Perturbative QCD"
Rogers, Aybat, PRD 83
Echevarria, Idilbi, Scimemi JHEP 1207

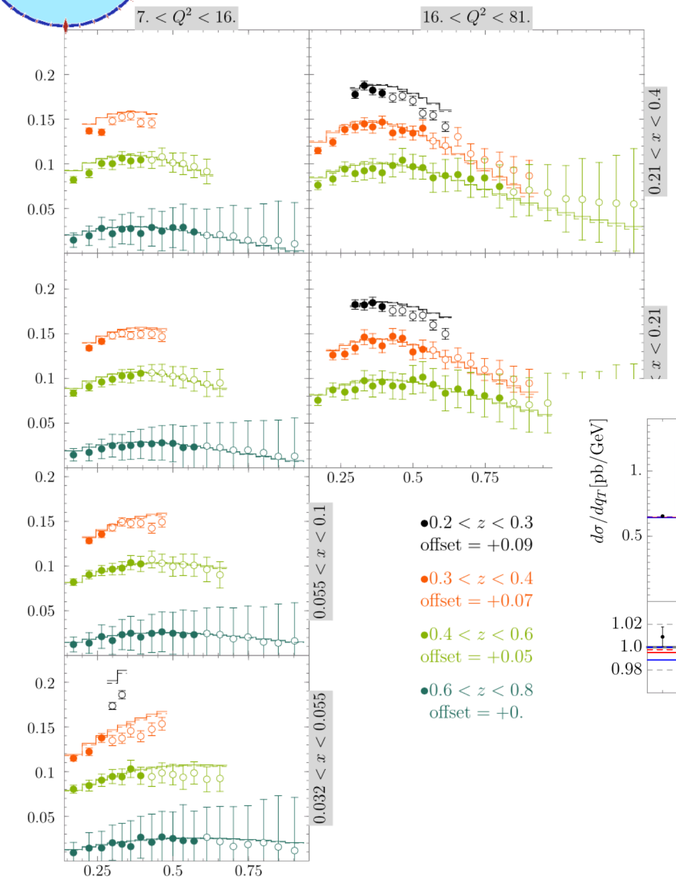
SV19

I. Scimemi, A. Vladimirov, arXiv:1912.06532

COMPASS

$z^2 \times M(z, p_T)$

$d \to h^+$

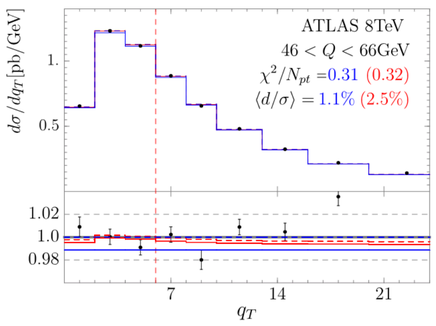


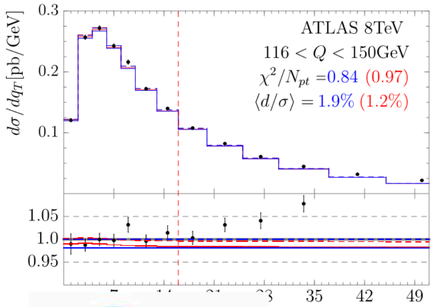
NNLL'

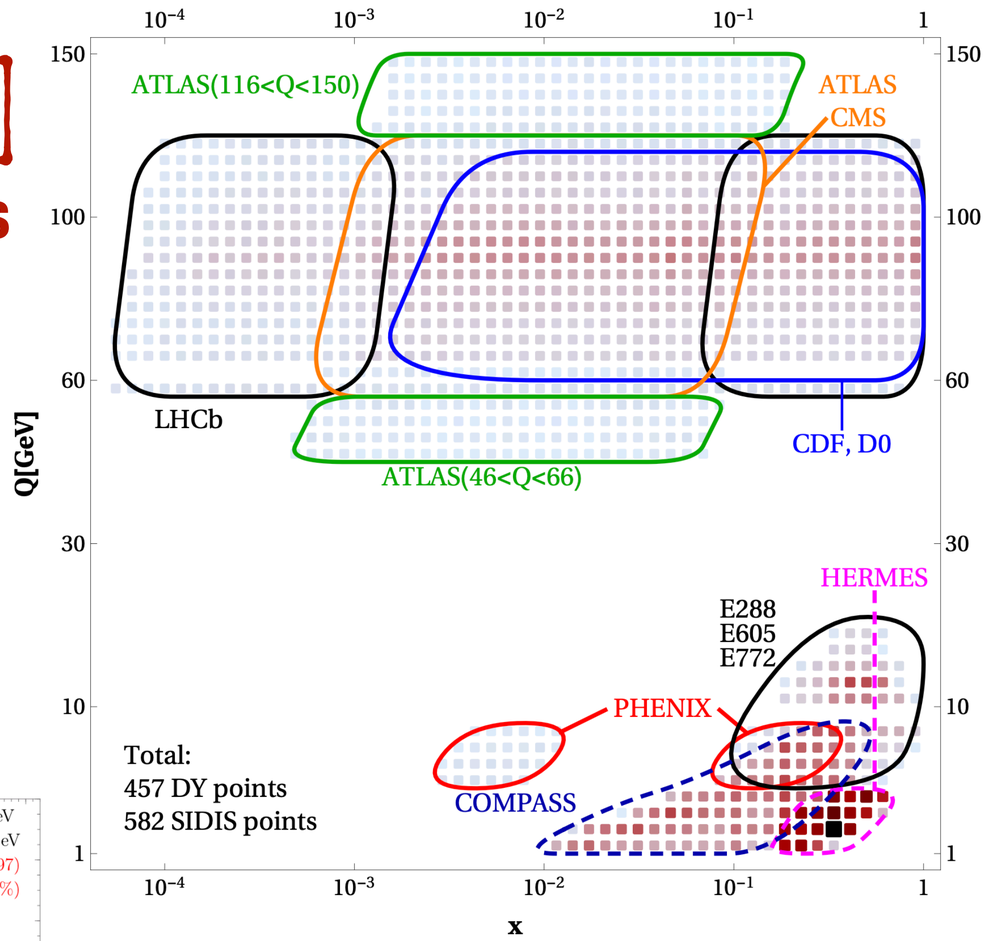
1039 data points

global

$$\chi^2 = 1.06$$







PV17 non perturbative functions

A. Bacchetta, F. Delcarro, C. Pisano, M. Radici, A. Signori
arXiv:1703.10157

$$f^a_{1\mathrm{NP}}(x,\mathbf{k}_\perp^2) = \frac{1}{\pi}\frac{(1+\lambda\mathbf{k}_\perp^2)}{g_{1a}+\lambda g_{1a}^2}e^{-\frac{\mathbf{k}_\perp^2}{g_{1a}}}$$

$$D^{a\to h}_{1\mathrm{NP}}(z,\mathbf{P}_\perp^2) = \frac{1}{\pi}\frac{1}{g_{3a\to h}+(\lambda_F/z^2)g^2_{4a\to h}}\left(e^{-\frac{\mathbf{P}_\perp^2}{g_{3a\to h}}}+\lambda_F\frac{\mathbf{P}_\perp^2}{z^2}\,e^{-\frac{\mathbf{P}_\perp^2}{g_{4a\to h}}}\right)$$

x-dependence

$$g_1(x) = N_1\frac{(1-x)^\alpha\,x^\sigma}{(1-\hat{x})^\alpha\,\hat{x}^\sigma}$$

$$g_{3,4}(z) = N_{3,4}\,\frac{(z^\beta+\delta)\,(1-z)^\gamma}{(\hat{z}^\beta+\delta)\,(1-\hat{z})^\gamma}$$

non-perturbative Sudakov factor

$$g_K(b_T) = -g_2 b_T^2/2$$

total of **11 parameters**

PV19 non perturbative function

Q-Gaussian

Gaussian

$$f_{\rm NP}(x,b,\zeta) = \left[\frac{1-\lambda}{1+g_1(x)\frac{b^2}{4}} + \lambda \exp\left(-g_{1,B}(x)\frac{b^2}{4}\right)\right]$$

non perturbative contribution to evolution

$$\times \exp\left[-\left(g_2 + g_{2B} b^2\right)\log\left(\frac{\zeta}{Q_0^2}\right)\frac{b^2}{4}\right]$$

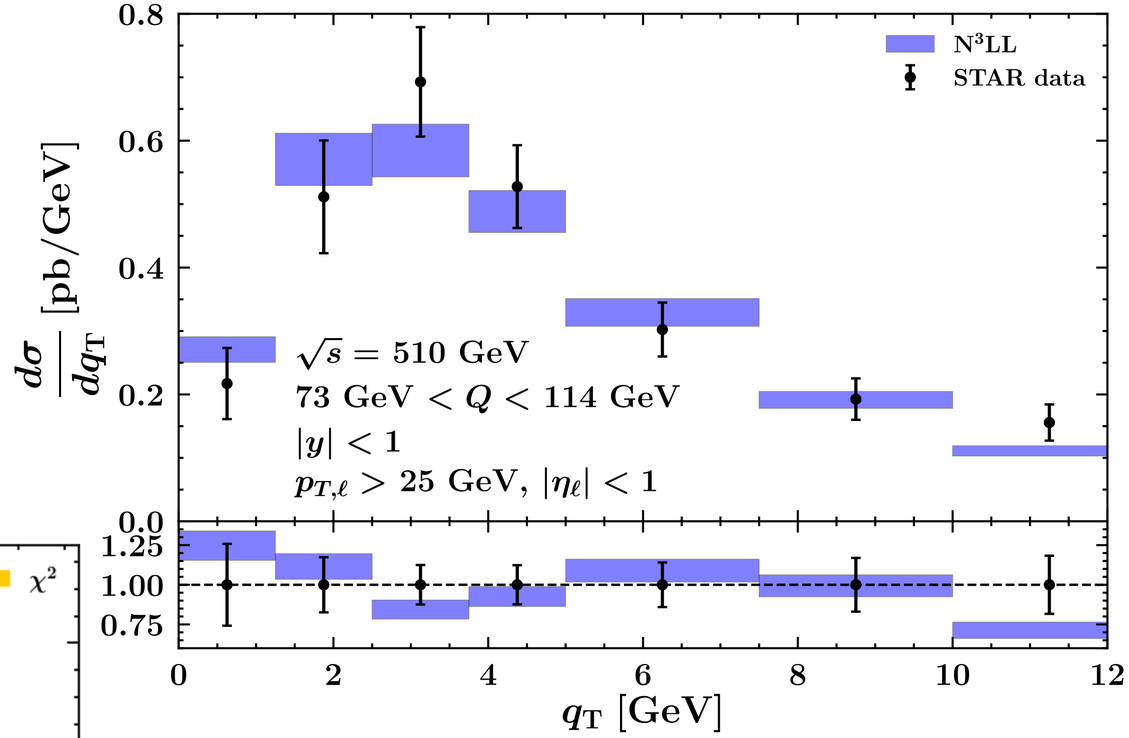
x-dependence

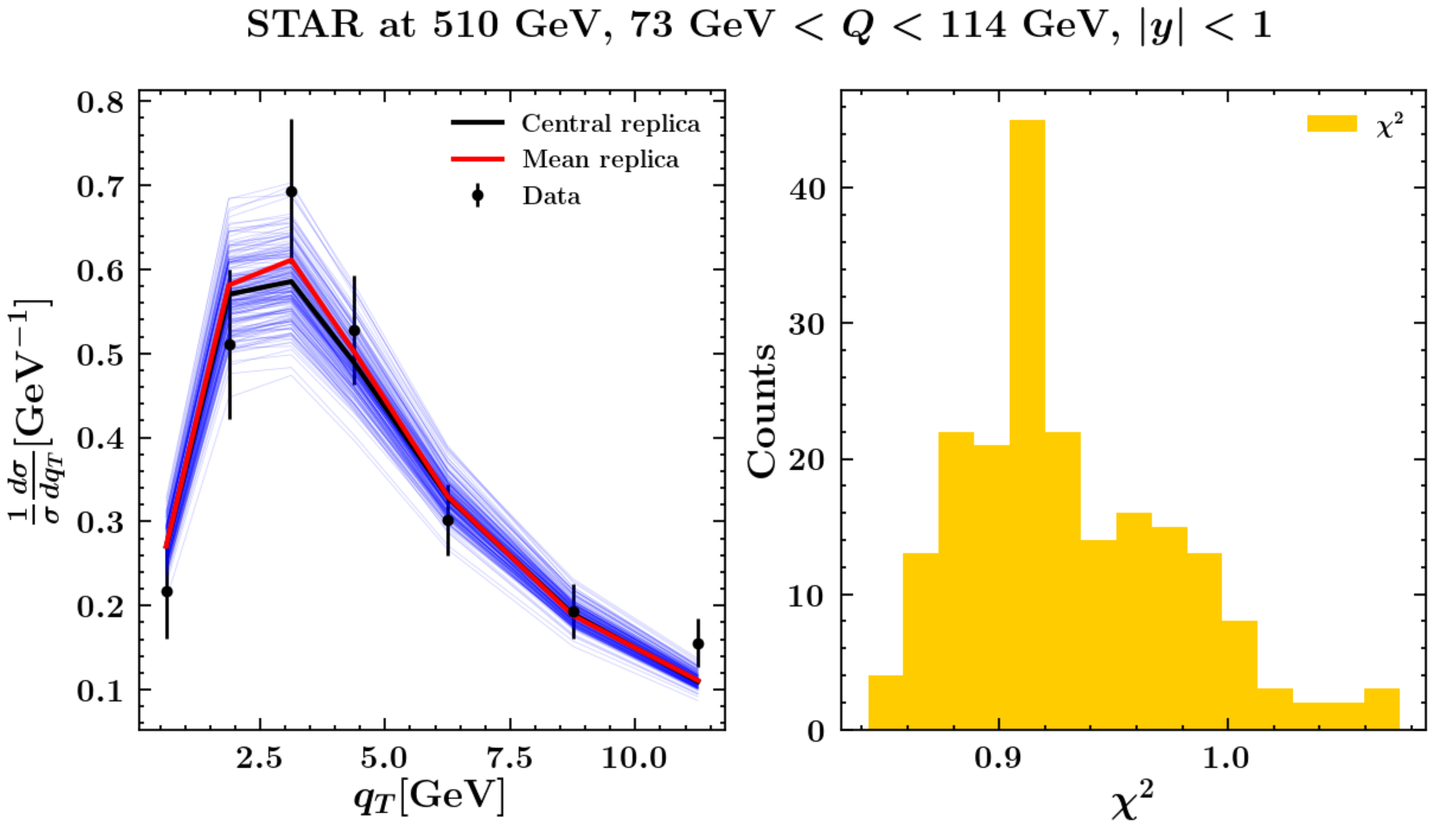
$$g_1(x) = \frac{N_1}{x\sigma}\exp\left[-\frac{1}{2\sigma^2}\ln^2\left(\frac{x}{\alpha}\right)\right]$$

$$g_{1,B}(x) = \frac{N_{1B}}{x\sigma_B}\exp\left[-\frac{1}{2\sigma_B^2}\ln^2\left(\frac{x}{\alpha_B}\right)\right]$$

total of **9 parameters**

PV19 and STAR data





Logarithmic accuracy

perturbative expansion in $\alpha_s(\mu)$

matching to the collinear region

$$\left(\frac{d\sigma}{dq_T}\right) \propto H(Q,\mu) \int \frac{d^2\mathbf{b}}{4\pi} e^{i\mathbf{b}\cdot\mathbf{q}_T} x_1 f_1^q(x_1,\mathbf{b};\mu,\zeta_1) x_2 f_1^{\bar{q}}(x_2,\mathbf{b};\mu,\zeta_2)$$

hard factor

collinear PDFs, longitudinal non perturbative part

$$f_1^q(x,b;\mu,\zeta) = \sum_j \left(C_{q/j} \otimes f^j\right)(x,b_*;\mu_b)$$

unpolarized quark

TMD PDF

$$\times \exp\left\{K(\mu_0)\ln\frac{\sqrt{\zeta}}{\sqrt{\zeta_0}} + \int_{\mu_0}^{\mu}\frac{d\mu'}{\mu'}\left[\gamma_F(\alpha_s(\mu')) - \gamma_K(\alpha_s(\mu'))\ln\frac{\sqrt{\zeta}}{\mu'}\right]\right\}$$

$$\times f_{\rm NP}(x,b;\zeta)$$

non perturbative transverse content

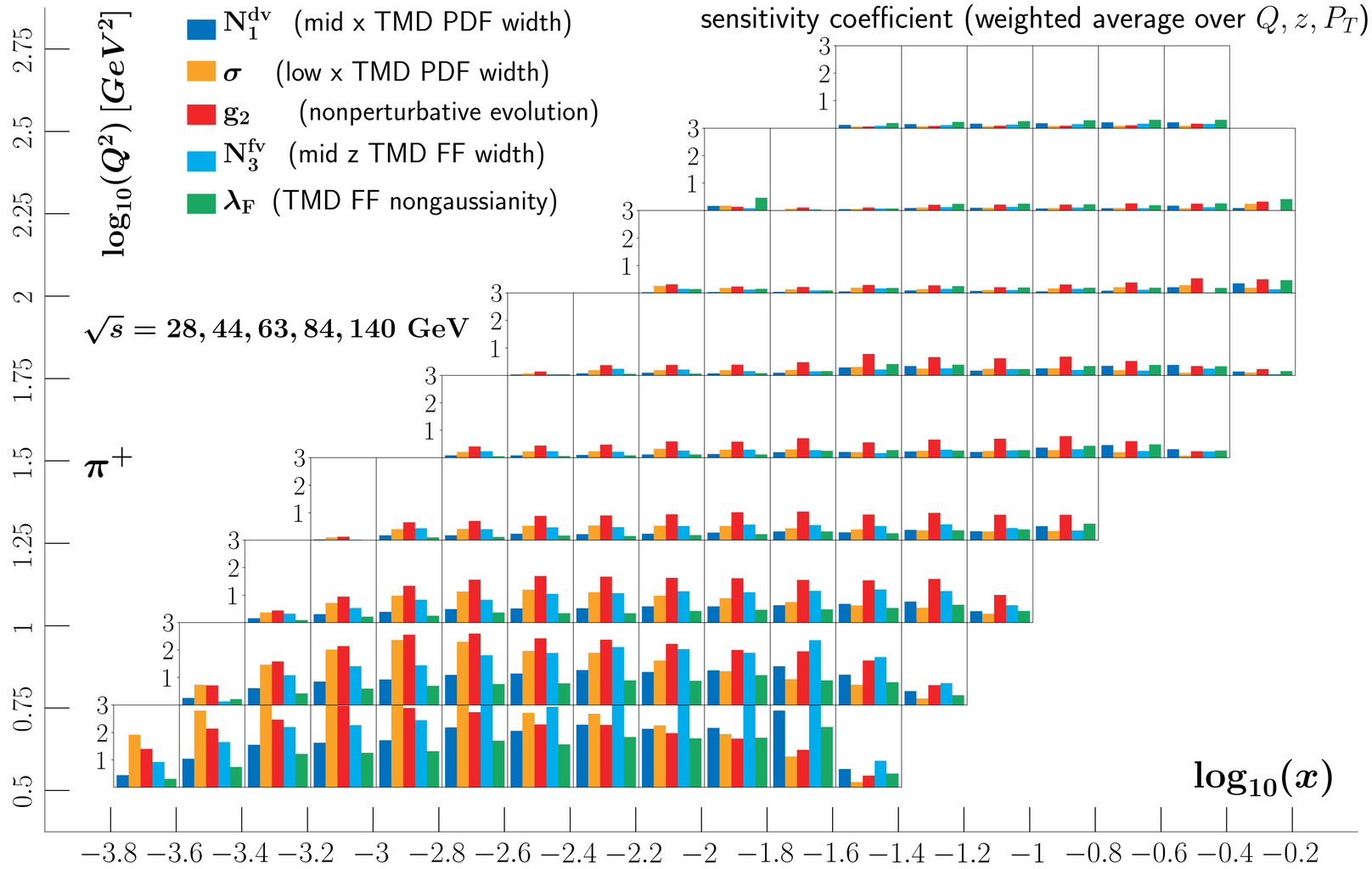
Sudakov form factor: perturbative evolution, resummation of large logs

Accuracy	H and C	K and γ_F	γ_K	PDF and α_s evolution
LL	0	-	1	-
NLL	0	1	2	LO
NLL'	1	1	2	NLO
NNLL	1	2	3	NLO
NNLL'	2	2	3	NNLO
N^3LL	2	3	4	NNLO

EIC impact studies

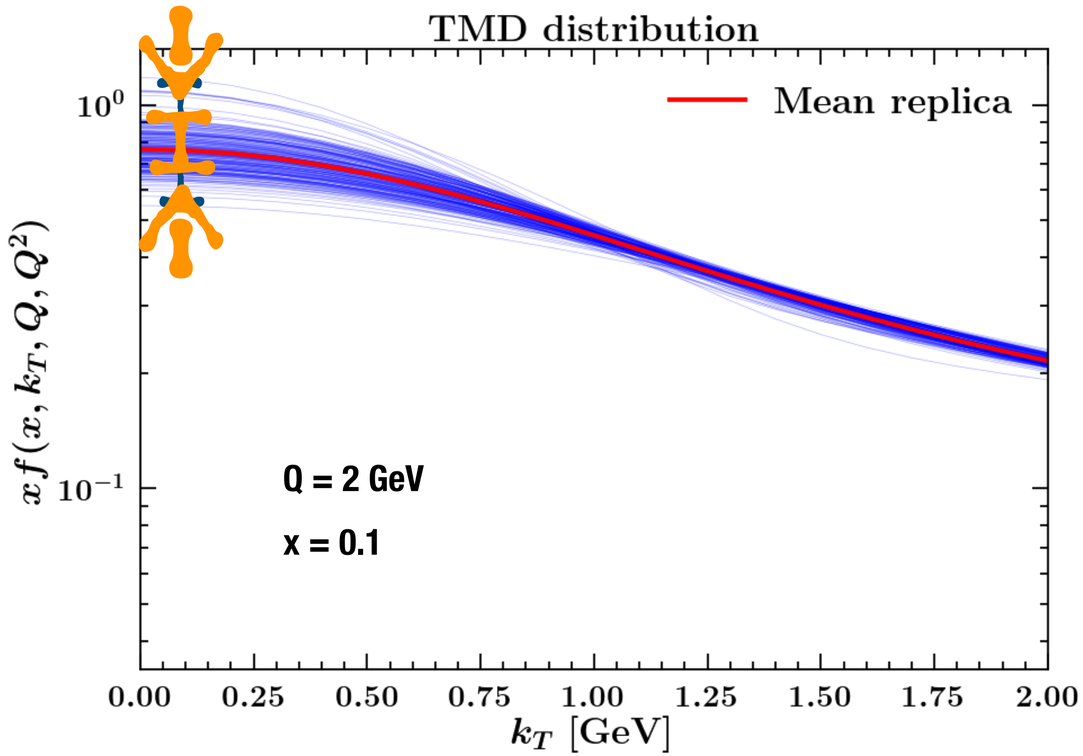
sensitivity coefficients

$$S[f_i, \mathcal{O}] = \frac{\langle \mathcal{O} \cdot f_i \rangle - \langle \mathcal{O} \rangle \langle f_i \rangle}{\xi \Delta \mathcal{O} \Delta f_i}$$



attempts at reweighing

200 replicas compared with pseudodata



different mathematical formulas to compute the weights

N. Sato, J. Owens, H. Prosper, PRD 89 (2014) 114020;
H. Paukkunen, P. Zurita, JHEP 12 (2014) 100

$$w_k \propto \mathcal{P}(f_k|\chi_k) \propto e^{-\frac{1}{2}\chi_k^2}$$

- selects replicas with very low χ^2

NNPDF Collaboration
arXiv:1108.1758

total number of points

$$w_k \propto \mathcal{P}(f_k|\chi_k) \propto \chi_k^{n-1} e^{-\frac{1}{2}\chi_k^2}$$

- suppresses replicas with very high AND very low χ^2

quark Sivers TMDs: parameterization

A. Bacchetta, F. Delcarro, C. Pisano, M. Radici
arXiv:2004.14278

NLL analysis

$$f_{1T\mathrm{NP}}^{\perp}(x, k_T^2) = \frac{(1 + \lambda_S\, k_T^2)\, e^{-k_T^2/M_1^2}}{K\pi\,(M_1^2 + \lambda_S M_1^4)}\, f_{1\mathrm{NP}}(x, k_T^2)$$

$$f_{1T}^{\perp(1)a}(x; Q_0^2) = \frac{N_{\mathrm{Siv}}^a}{G_{\mathrm{max}}^a}\, x^{\alpha_a}(1-x)^{\beta_a}\big[1 + A_a\, T_1(x) + B_a\, T_2(x)\big]\, f_1^a(x; Q_0^2)$$

5 parameters for up, down, sea +2
= 17 free parameters

Sivers results

A. Bacchetta, F. Delcarro, C. Pisano, M. Radici
arXiv:2004.14278

NLL analysis

Slightly updated results w.r.t. arXiv (due to the inclusion of correlated errors)

Total number of data points: 118
Total χ^2/dof = 1.08

	M_1	λ_S	α_d	α_u	α_s	
All replicas	0.81 ± 0.35	-0.50 ± 0.73	1.13 ± 0.98	0.16 ± 0.16	1.61 ± 1.52	
Replica 105	0.78	−0.42	0.42	0.12	0.32	
	β_d	β_u	β_s	A_d	A_u	A_s
All replicas	5.64 ± 4.32	1.47 ± 1.41	4.64 ± 4.55	0.79 ± 8.63	-0.98 ± 3.10	-0.68 ± 6.83
Replica 105	9.70	0.86	0.20	−0.88	−0.08	−1.52
	B_d	B_u	B_s	$N^d_{\rm Siv}$	$N^u_{\rm Siv}$	$N^s_{\rm Siv}$
All replicas	2.05 ± 5.01	2.35 ± 4.57	0.29 ± 3.32	$-4.89 \times 10^{-6} \pm 1.00$	-0.07 ± 0.50	0.02 ± 0.64
Replica 105	0.98	1.49	0.89	−1.00	0.29	0.44