Interacting dark sector from unimodular gravity: cosmological perturbations with no instability

based on joint work with Ed Wilson-Ewing (UNB, Canada)

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Talk outline

- Energy-momentum (non)conservation in unimodular gravity
    - ✦ Connection with $w = -1$ IDE models
- Cosmological perturbations in unimodular gravity: general framework
- A simple model for energy-momentum transfer
- Solutions and absence of instabilities:
    - ✦ Radiation dominated era
    - ✦ Matter dominated era
- Further generalizations
- Conclusions

Energy-momentum conservation in unimodular gravity?

In general relativity, energy-momentum conservation is a consequence of the field equations:

$$G_{ab} = \kappa T_{ab} \implies \nabla^a T_{ab} = 0$$

In unimodular gravity, the Einstein field equations are replaced by their trace-free counterpart

$$R_{ab} - \frac{1}{4} R\, g_{ab} = T_{ab} - \frac{1}{4} T\, g_{ab}$$

Using the Bianchi identities, we no longer get a conservation law:

$$\kappa \nabla^c T_{ac} = \frac{1}{4} \nabla_a (R + \kappa T) \equiv J_a$$

energy-momentum transfer

In unimodular gravity, energy-momentum conservation for matter fields becomes an independent assumption, and $\nabla^a T_{ab} \neq 0$ in general.

[Ellis, van Elst, Murugan, Uzan; Josset, Perez, Sudarsky]

Effective dark energy from energy-momentum non-conservation

The field equations of UG can be recast in the form of effective Einstein equations

$$R_{ab} - \frac{1}{2} R\, g_{ab} + \left(\Lambda_\infty + \int_\ell J \right) g_{ab} = \kappa T_{ab}$$

To ensure that the geometry only depends on the point (and not on the path ℓ!) the energy-momentum transfer must be integrable

$$dJ = 0 \implies J_a = -\nabla_a Q \longleftarrow \text{energy-momentum transfer potential}$$

Then, the equations of UG read as $G_{ab} = \kappa\,(T_{ab} + \tilde{T}_{ab})$ with $\kappa\,\tilde{T}_{ab} = Q\, g_{ab}$
(dark energy with $w = -1$)

By construction, the total energy-momentum tensor is conserved $\nabla^a (T_{ab} + \tilde{T}_{ab}) = 0$

IDE models with $w = -1$ and integrable transfer are embedded in unimodular gravity

[Perez, Sudarsky, Josset, Wilson-Ewing]

Large scale instabilities?

A large class of interacting dark energy models suffers from well-known large scale non-adiabatic instabilities on super-horizon scales

[Valiviita, Majerotto, Maartens]

Our goal is to show that we can build instability-free IDE models that are embedded in unimodular gravity.

Transfer models

In this context, a model is a specific proposal for the energy-momentum transfer potential Q
This does not follow from the field equations and must be prescribed separately.

There are proposals for the possible microscopic origin of energy-momentum non-conservation due to spacetime discreteness at the Planck scale.

[Josset, Perez, Sudarsky]

Cosmological perturbations in unimodular gravity

We focus on the scalar sector, because that's where the instability found in [Valiviita, Majerotto, Maartens] shows up, and also where modifications introduced by unimodular gravity play a role.

metric perturbations:

$$ds^2 = a(\eta)^2 \left\{ -(1+2\phi)d\eta^2 + 2B_{,i}dx^i d\eta + \left[(1-2\psi)\delta_{ij} + 2E_{,ij}\right] dx^i dx^j \right\}$$

matter perturbations:

$$\delta T^{a}_{A\,b} = (\delta\rho_A + \delta p_A)\bar{u}^a \bar{u}_b + (\bar{\rho}_A + \bar{p}_A)\left(\delta u^a_A \bar{u}_b + \bar{u}^a \delta u^A_b\right) + \delta p_A\, \delta^a_{\ b} + \pi^{a}_{A\,b}$$

dark energy perturbations (recall $\rho_x = -\kappa^{-1}Q$) : $\delta \tilde{T}^a_{\ b} = -\,\delta\rho_x\, \delta^a_{\ b}$

Background evolution

$$\mathscr{H}^2 = \frac{\kappa}{3}a^2\left(\bar{\rho} + \bar{\rho}_x\right) \qquad \bar{\rho}'_A + 3\mathscr{H}(\bar{\rho}_A + \bar{p}_A) = \kappa^{-1}\bar{Q}'_A \qquad \sum_A Q^A = Q$$

$$\bar{\rho}' + 3\mathscr{H}(\bar{\rho} + \bar{p}) = \kappa^{-1}Q' = -\,\bar{\rho}'_x$$

NB: no equation for $\bar{Q}$ at this stage.

Cosmological perturbations in unimodular gravity

perturbed field equations (longitudinal gauge):

$$-\triangle\psi + 3\mathscr{H}(\psi' + \mathscr{H}\phi) = -\frac{\kappa}{2}a^2(\delta\rho + \delta\rho_x)\ , \quad \mathscr{H}\phi + \psi' = -\frac{\kappa}{2}a^2(\bar{\rho} + \bar{p})v\ , \quad \psi - \phi = \kappa\, a^2\pi\ ,$$

$$\psi'' + \mathscr{H}(\phi' + 2\psi') + \left(2\frac{a''}{a} - \mathscr{H}^2\right)\phi = \frac{\kappa}{2}a^2\left(\delta p - \delta\rho_x + \frac{2}{3}\triangle\pi\right)$$

continuity equations: $\theta_A \equiv \triangle(v_A + B)\ , \ \ \delta_A \equiv \dfrac{\delta\rho_A}{\bar{\rho}_A}$

$$\delta_A' + \left(3\mathscr{H}(c_{s\,A}^2 - w_A) + \kappa^{-1}\frac{\bar{Q}_A'}{\bar{\rho}_A}\right)\delta_A + (w_A + 1)\theta_A - 3(w_A + 1)\psi' + 3\mathscr{H}(c_{s\,A}^2 - c_{a\,A}^2)\left[3\mathscr{H}(1 + w_A) - \kappa^{-1}\frac{\bar{Q}_A'}{\bar{\rho}_A}\right]\frac{\theta_A}{k^2} = \kappa^{-1}\frac{\delta Q_A'}{\bar{\rho}_A}\ ,$$

$$\theta_A' + \mathscr{H}(1 - 3c_{s\,A}^2)\theta_A - k^2\phi - \frac{k^2}{1 + w_A}c_{s\,A}^2\delta_A + \frac{2k^4}{3(1 + w_A)\bar{\rho}_A}\pi_A = \kappa^{-1}\left[\frac{k^2}{\bar{\rho}_A(1 + w_A)}\delta Q_A - \left(\frac{1 + c_{s\,A}^2}{1 + w_A}\right)\frac{\bar{Q}_A'}{\bar{\rho}_A}\theta_A\right]$$

Note that there is no additional equation for $\delta_x = \delta Q/\bar{Q}$

θ_x is not defined for a fluid with $w = -1$.
We can identify it with the velocity perturbation for the total fluid $\theta_x \equiv \theta$.

A simple transfer model

In order to solve the equations, we also need to model the energy-momentum transfer.

We choose a model where the violation of energy-momentum conservation is only due to dark matter and the transfer potential is

$$Q = -\Lambda_\infty + \epsilon\,\kappa\,\rho_c$$

cosmological constant
at future infinity
(integration constant)

energy density of CDM

With this, dark energy evolves adiabatically w.r.t. CDM $\zeta_x = \zeta_c \implies S_{xc} = 0$

The CDM (non)conservation equations then read as:

background: $(1-\epsilon)\bar{\rho}_c' + 3\mathscr{H}\bar{\rho}_c = 0$

perturbations: $(1-\epsilon)\delta_c' + \theta_c - 3\psi' = 0\ , \quad \theta_c' + \left(\frac{1-4\epsilon}{1-\epsilon}\right)\mathscr{H}\theta_c - k^2\phi - \epsilon\,k^2\delta_c = 0$

CDM effectively behaves as a fluid with $c_{s,\mathrm{eff}}^2 = w_{\mathrm{eff}} = \dfrac{\epsilon}{1-\epsilon}$

Effective sound speed of DM

CDM effectively behaves as a fluid with $c^2_{s,\mathrm{eff}} = w_{\mathrm{eff}} = \dfrac{\epsilon}{1-\epsilon}$

To avoid gradient instabilities, we shall require $\epsilon \geq 0$.

This condition also tells us that energy flows from dark matter to dark energy (the effective cosmological 'constant' must be increasing)

The characterisation of DM with parameters $c^2_{s,\mathrm{eff}}$, w_{eff} resembles the *generalized dark matter* phenomenological model (in the inviscid case)

[Hu] [Kopp, Skordis, Thomas]

Radiation dominated era

We solve the gravity+matter equations during RDE at tight coupling for super-horizon modes, taking into account baryons, photons, neutrinos in addition to DE and CDM

Constant mode. $\psi = \left(1 + \frac{2}{5}\Omega_\nu\right)\phi + \frac{4}{15}S_{\nu\gamma}\Omega_\nu(1 - \Omega_\nu)$

Decaying mode. $\psi \approx (k\eta)^n \qquad n \approx -3 + \frac{8}{5}\Omega_\nu \qquad \phi = (1 - \frac{8}{5}\Omega_\nu)\,\psi$

Matter perturbations θ_A , δ_A , σ_ν are also well behaved in both cases

DE density perturbations are also bounded and decreasing in magnitude:

$$\delta_x = -3\frac{\epsilon}{1-\epsilon}\left(\frac{\bar{\rho}_c}{\bar{\rho}_x}\right)\psi$$

No instability

Matter dominated era

Assume only interacting CDM and DE as matter fields.

There is no anisotropic stress, so $\phi = \psi$.

$$\mathscr{H} = \left(\frac{1-\epsilon}{1+2\epsilon}\right)\frac{2}{\eta} \qquad\qquad c^2_{s,\mathrm{eff}} = \frac{\epsilon}{1-\epsilon}$$

$$\psi'' + 3\mathscr{H}(1+c^2_{s,\mathrm{eff}})\psi' + \left(2\mathscr{H}' + \mathscr{H}^2(1+3c^2_{s,\mathrm{eff}})\right)\psi + c^2_{s,\mathrm{eff}}k^2\psi = 0$$

Super-horizon modes ($c_{s,\mathrm{eff}}\, k\eta \ll 1$)

$$\psi = \psi_0 + C\,\eta^{-\left(\frac{5+3c^2_{s,\mathrm{eff}}}{1+3c^2_{s,\mathrm{eff}}}\right)} \qquad\qquad \delta_c \sim a^{1+3c^2_{s,\mathrm{eff}}}$$

same as in Hu's GDM

Also in this case there are no instabilities.
The ΛCDM limit $\epsilon \to 0$ is continuous on these scales.

Matter dominated era

Sub-horizon modes ($c_{s,\text{eff}}\, k\eta \gg 1$)

on these scales, the potential and the density contrast oscillate,
the latter with increasing amplitude

$$\psi \sim \cos(c_{s,\text{eff}} k\eta + \varphi_k) \qquad\qquad \delta_c \sim a^{\frac{1}{2}(1+3c_{s,\text{eff}}^2)} \sin(c_{s,\text{eff}} k\eta + \varphi_k)$$

different power compared to
super-horizon modes

in ΛCDM, the sound speed of CDM is exactly zero and $\delta_c \sim a$ on all scales

Therefore, $\epsilon \to 0$ is a singular limit. For any finite non-zero values of ϵ (no matter how small), the behaviour of sub-horizon modes is qualitatively different from the $\epsilon = 0$ case.

This offers an opportunity to test the model and constrain the coupling ϵ.

Further generalizations

We could choose a more general model for the transfer potential. Some examples are:

$$J_a = -\epsilon\,\alpha\,\nabla_a \theta_{\text{CDM}}$$

expansion scalar of CDM four-velocity

$$J_a = -\epsilon\,\nabla_a (\theta_{\text{CDM}})^2$$

$$J_a = -\epsilon\,\kappa\,\nabla_a \Big(F(T^{\text{CDM}}_{uu}, \theta_{\text{CDM}}) \Big)$$

energy density of CDM

One could also include in the functional dependence of F further geometric invariants, or additional interactions with other matter species.

In any case, we would still have $w = -1$, but such modifications may introduce further features to the effective fluid description of DM.

Conclusions

- IDE models with $w = -1$ and integrable transfer are embedded in unimodular gravity (trace-free Einstein equations)
- For this class of models, we derive the general equations for scalar cosmological perturbations. These are a special case of [Valiviita, Majerotto, Maartens]
- We assume a simple model where $Q = -\Lambda_\infty + \epsilon\,\kappa\,\rho_c$ and examine in detail the evolution of perturbations
  - ✦ CDM effectively behaves as a fluid with $c^2_{s,\text{eff}} = w_\text{eff} = \epsilon/(1-\epsilon)$
  - ✦ $\epsilon \geq 0$ ensures that there is no gradient instability. Energy flows from CDM to DE.
  - ✦ We solved the equations analytically for super-horizon modes during RDE, and for both super-horizon and sub-horizon modes during MDE. No large scale instabilities.
  - ✦ The limit $\epsilon \to 0$ is singular for sub-horizon modes. There may be significant differences from ΛCDM on these scales, which gives us an opportunity to test the model and constrain ϵ.
  - ✦ Several generalisations of the model are possible within this framework. Can we identify all models within this class that are free from instabilities?
- Future work should focus on a detailed analysis of the impact of this kind of dark sector interactions on the CMB and structure formation.