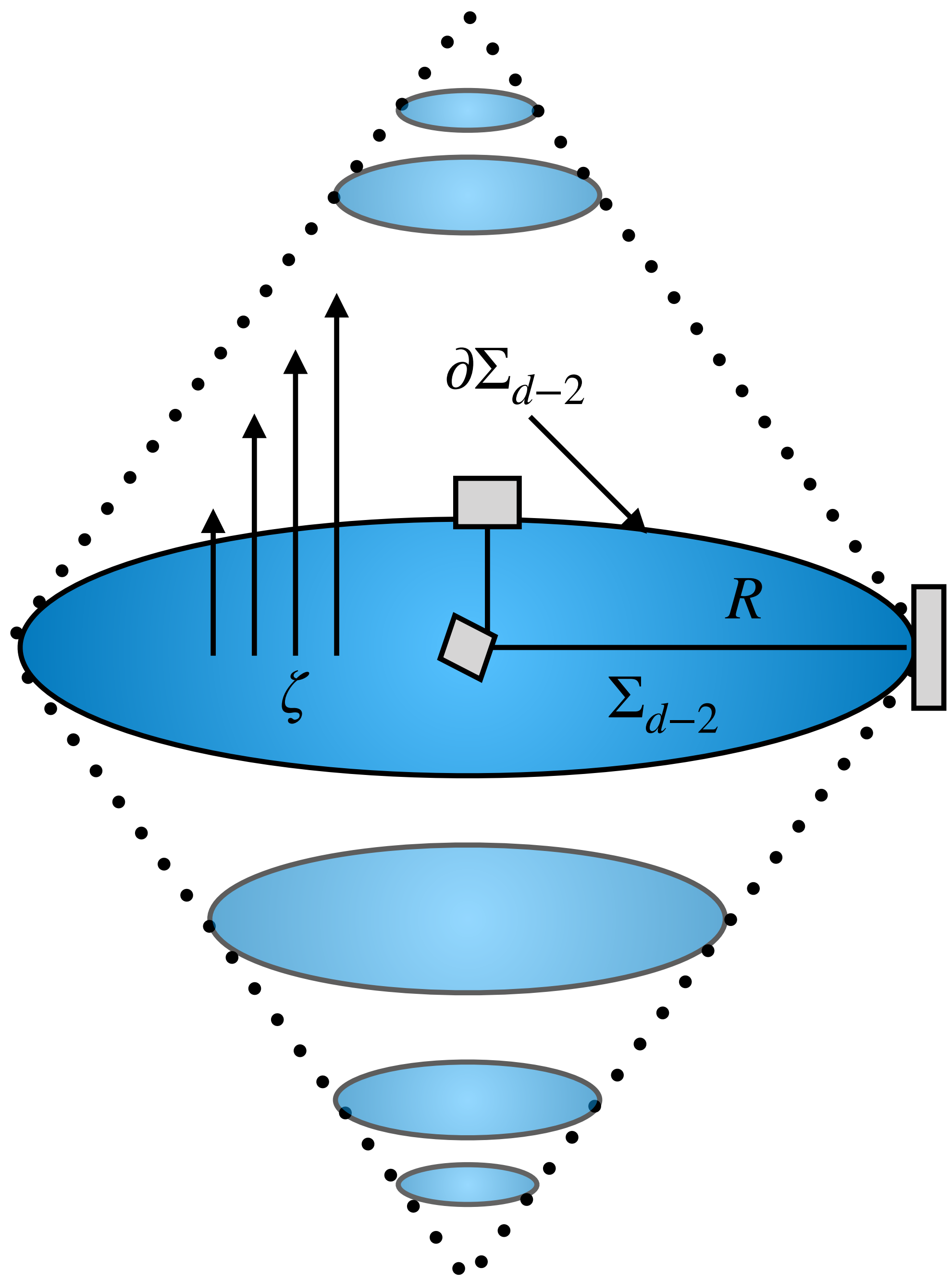
Fluctuations on Horizons

- Brookhaven Nov. 3 2021



$\partial\Sigma_{d-2}$
R
ζ
Σ_{d-2}

TB and K Zurek hep-th 2108.04806 - based on S.Carlip gr-qc 9409052,hep-th 980602, S.Solodukhin hep-th 9812056

- Consider a solution of Einstein's equations and a null surface where the metric is $g_{ab}dy^a dy^b + \rho^2(y) d\Sigma_{d-2}^2$; any d-2 dimensional Riemannian manifold and the two dimensional metric is Rindler - generically true in near horizon limit.

- $ds^2 = \rho^2(y)[\rho^{-2} g_{ab} dy^a dy^b + d\Sigma_{d-2}^2] \equiv \rho^2 \hat{g}_{mn} dx^m dx^n$

- $R_d(\hat{g}) = R_2(\rho^{-2} g_{ab}) + R_{d-2}$

- Plug into d dimensional Einstein action. Geometry of Σ enters only through an overall factor of its volume, and one term in which its integrated curvature multiplies a function of ρ.

- Most general 2d dilaton gravity Lagrangian

- $\mathscr{L} = \sqrt{-g}[cSR + F(S)(\nabla S)^2 - V(S)]$

- In our case $S \propto \rho^{d-2}$ and goes to a constant near the horizon. One can also do an S dependent Weyl transformation on g_{ab} to change the kinetic term. The curvature of Σ enters only in the potential

- Near the horizon, the first two terms are well approximated by
- $\mathscr{L}_{Liouville} = q\sqrt{S_h}\phi\sqrt{-g}R + \frac{1}{2}\sqrt{-g}(\nabla\phi)^2$
- This Lagrangian is invariant under classical conformal transformations $(t + x) \to f(t + x)$, the symmetry of 2d Rindler space with ingoing wave boundary conditions on the past horizon. The potential terms scale to zero under this transformation.
- $L_n = \int (z_+)^n T_{++}(z_+)$ Virasoro algebra with c = $6q^2S_h$
- Note, the L_0 generator of this algebra is the modular Hamiltonian of the diamond (Casini, Huerta, Myers)

Liouville Theory

- For ANY 2d CFT, the correlators of the stress tensor T_{++} are completely determined by the Virasoro Ward identities and the central charge. Liouville theory, with the right value of the central charge will have the same stress tensor correlators.

- Cardy's Formula: Consider a CFT on a torus and the partition function $Z = \mathrm{Tr}\ q^{L_0}\bar{q}^{\bar{L}_0} \quad q = e^{2\pi i\tau}$

- High temperature behavior of Z the same as behavior of partition function at fixed temperature and volume going to zero. But the latter is dominated by LOWEST eigenvalue of L_0

- Cardy: $n(L_0) = e^{2\pi\sqrt{cL_0}/6}$ IF lowest lying state has $L_0 = 0$. Liouville theory does not have this property. However Carlip and Solodukhin showed, independently, that if we take the L_0 value of the classical solution of Liouville, satisfying the boundary conditions, then this formula reproduces BH entropy for ALL black holes.

- Liouville is a classical state counting device. It does not have the quantum states predicted by Cardy: but c.f. Strominger's reinterpretation of Brown-Henneaux Virasoro algebra on the boundary for classical 3d gravity.

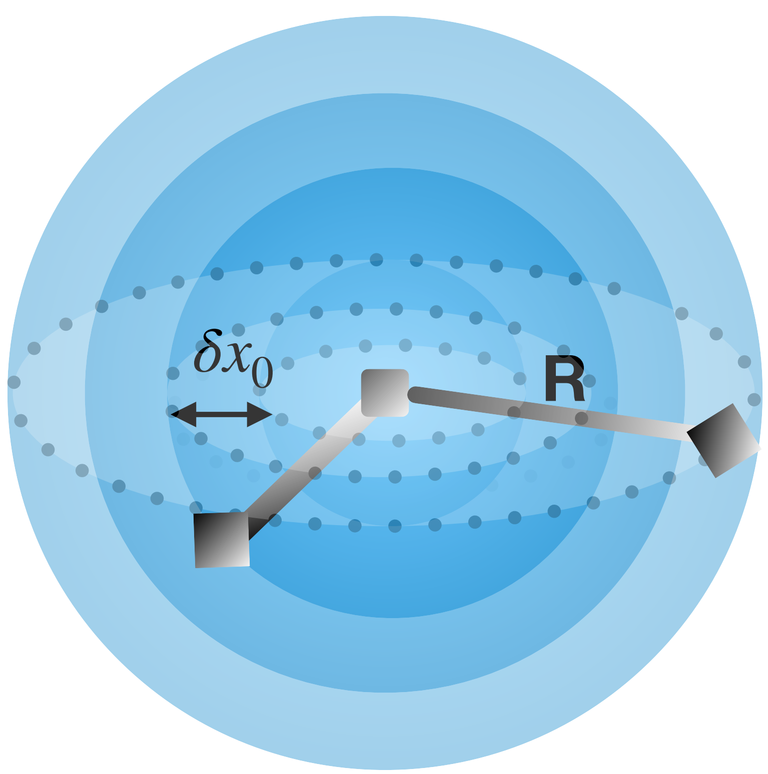
- TB and Zurek: Carlip/Solodukhin conjecture implies

- $(\Delta K)^2 = S$ for ALL black holes. Locally on the horizon the generic model of QG, when EH is a good approximation to classical physics, is a $1+1$ CFT

- Implies Planck length fluctuation $\Delta L = L_p$ in each causal diamond traversed by beam in interferometer.

- Fluctuations independent in each diamond because of large increase in entropy

- Implies $\Delta L_{interferometer} \sim \sqrt{L_{interferometer} L_P}$


δx_0
R