Precision Calculation of Inflation Correlators at One Loop

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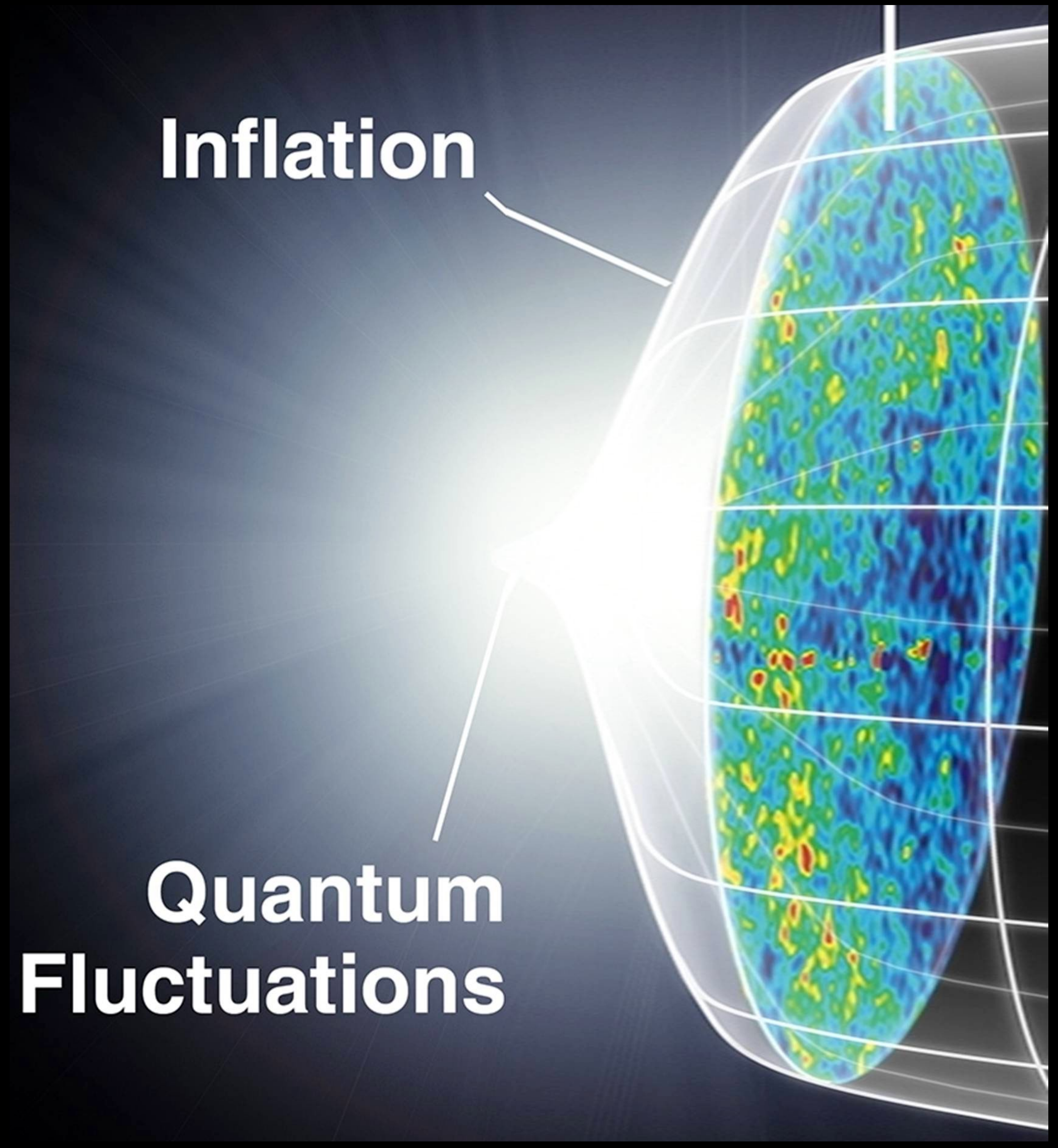
Outline

- Introduction:
  - What is the cosmological collider (CC)?
  - Why we consider 1-loop process?
  - Why we do it numerically?
- Numerical procedure & results
- Summary

Inflation

- The leading paradigm in explaining why the universe is so homogeneous and isotropic.
- Quantum fluctuations got stretched and imprinted at superhorizon scales.
- Predicts a near-scale invariant power spectrum. It is confirmed by CMB data.
- Anything more?



$$\mathcal{P}_\zeta(k) \equiv \frac{k^3}{2\pi^2}\langle \zeta_{\mathbf{k}}\zeta_{-\mathbf{k}}\rangle'$$

$$\sim k^{\text{slow-roll parameter}}$$

Primordial non-Gaussianities

- The simplest single-field slow-roll inflation predicts a **Gaussian** primordial spectra $\Rightarrow$ nothing new in higher-pt correlators

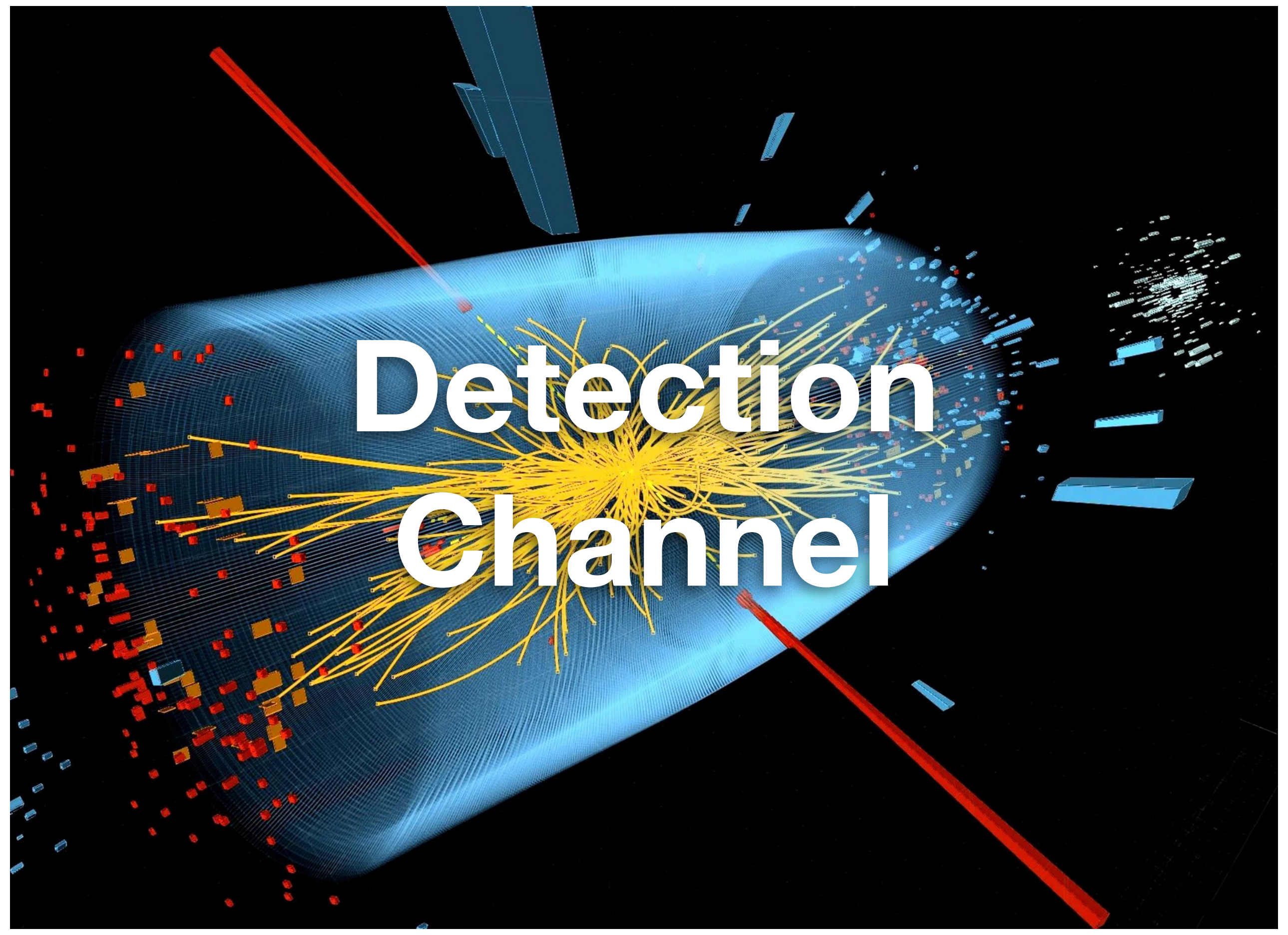
$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle' = ?$$

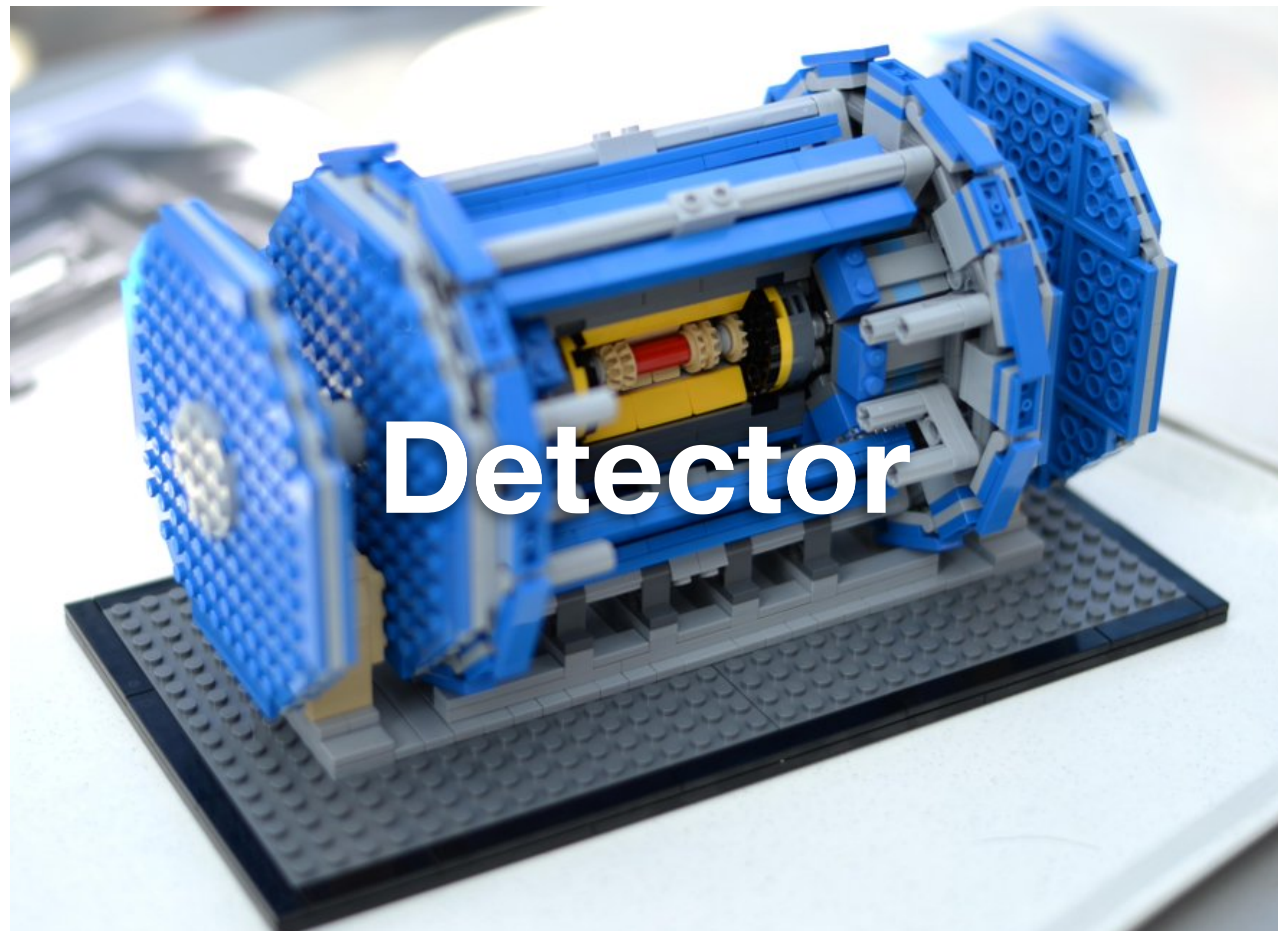
- Many inflation models predict non-trivial higher-pt correlators

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \zeta_{\mathbf{k}_4} \rangle' = ?$$

- Non-Gaussianities $\Rightarrow$ break the degeneracies among inflation models

……





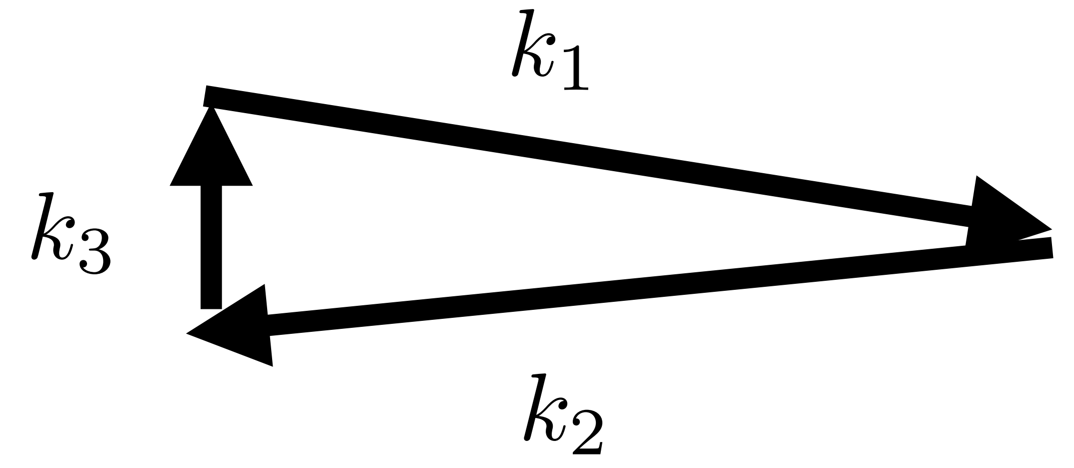
Inflation may access energy scale up to 10^{14} GeV ⇨ Imprint features in the primordial spectra ⇨ Measured by CMB, LSS surveys, & future 21 cm

Chen & Wang '09, '09, '12, Pi et al '12, Gong et al, '13, Arkani-Hamed & Maldacena '15……

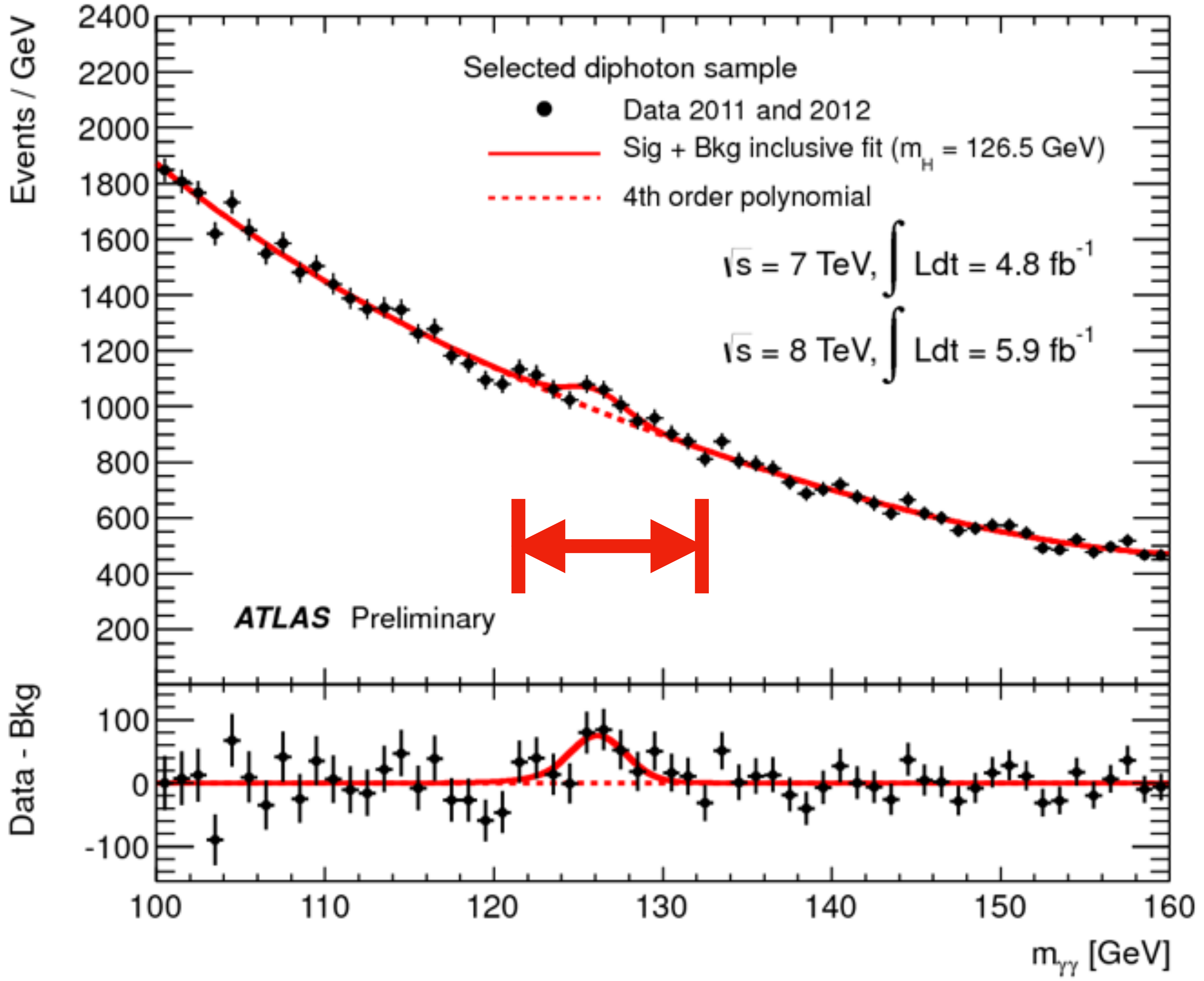
Probe New Physics at very high energy scales

Observables

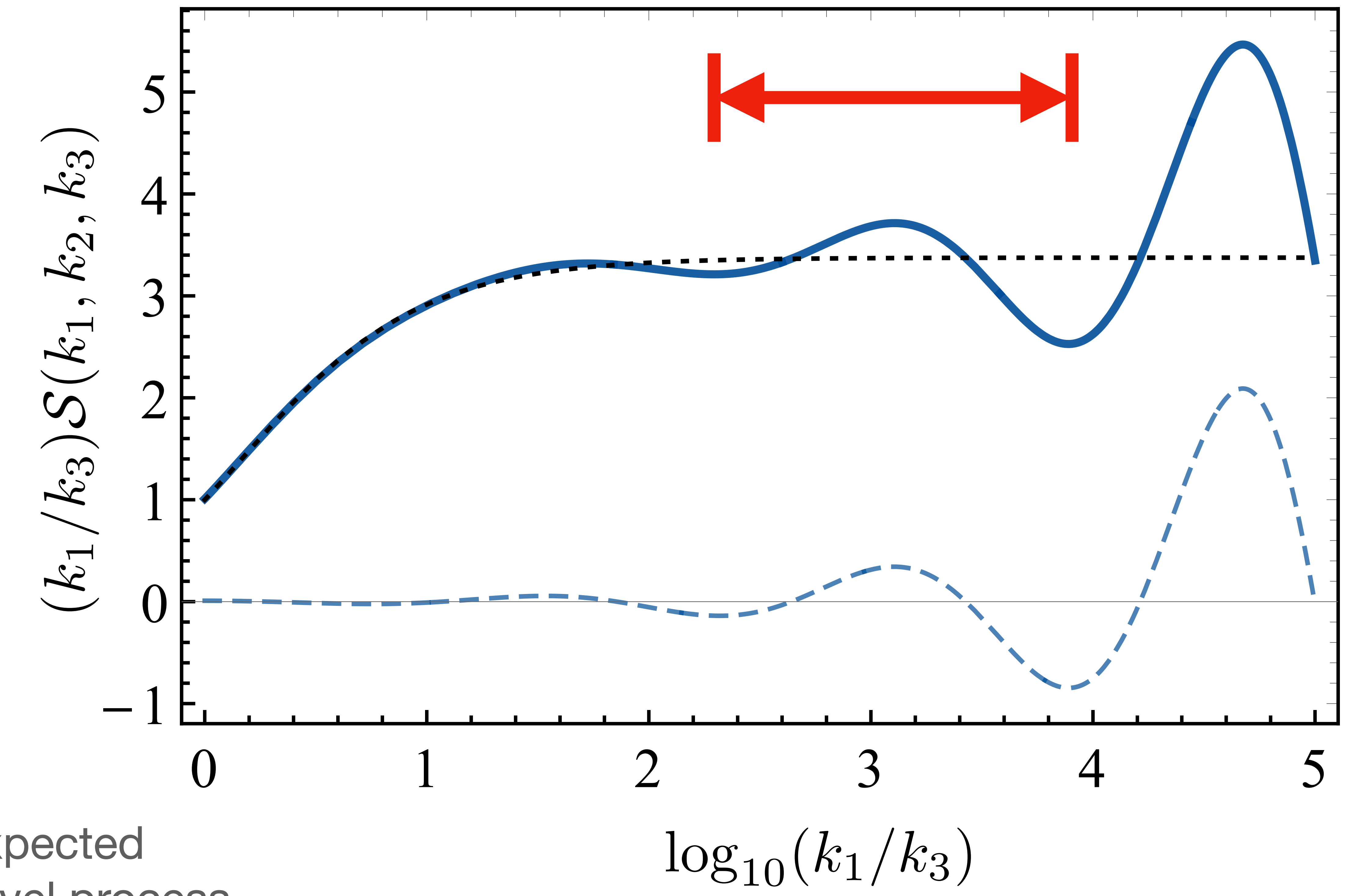
squeezed shape



Invariant mass for di-photons



Shape function of 3-pt correlator

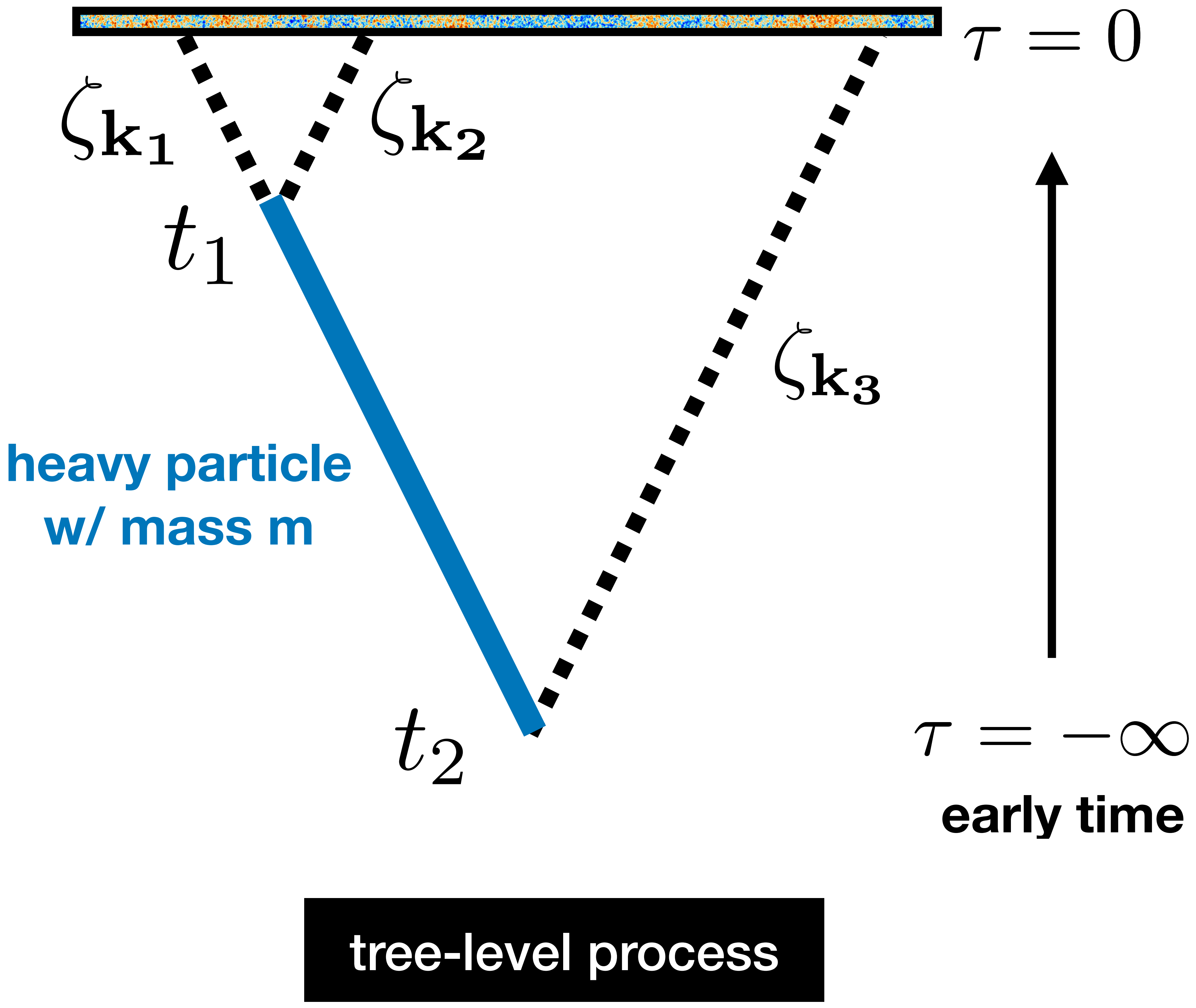


expected
tree-level process

Bump ⇒ Mass

Oscillations in log(*k* ratio) ⇒ Mass

Oscillations in log(*k* ratio)



$$t = \frac{1}{H}\log a$$

$$m \sim k/a$$

$$\sim e^{im(t_2 - t_1)} \sim \sin\left(\frac{m}{H}\log\frac{k_1}{k_3}\right)$$

$$\sqrt{\left(\frac{m}{H}\right)^2 - \#}$$

Why we need to go to 1-loop?

- For many cosmo-collider processes, the amplitude for the log-oscillation are Boltzmann-suppressed.

$$\exp\left(-\pi\frac{m}{H}\right)\sin\left(\tilde{\omega}\log\frac{k_1}{k_3}\right)$$

- Look for models that boost the amplitude

- Chemical-potential-enhanced models

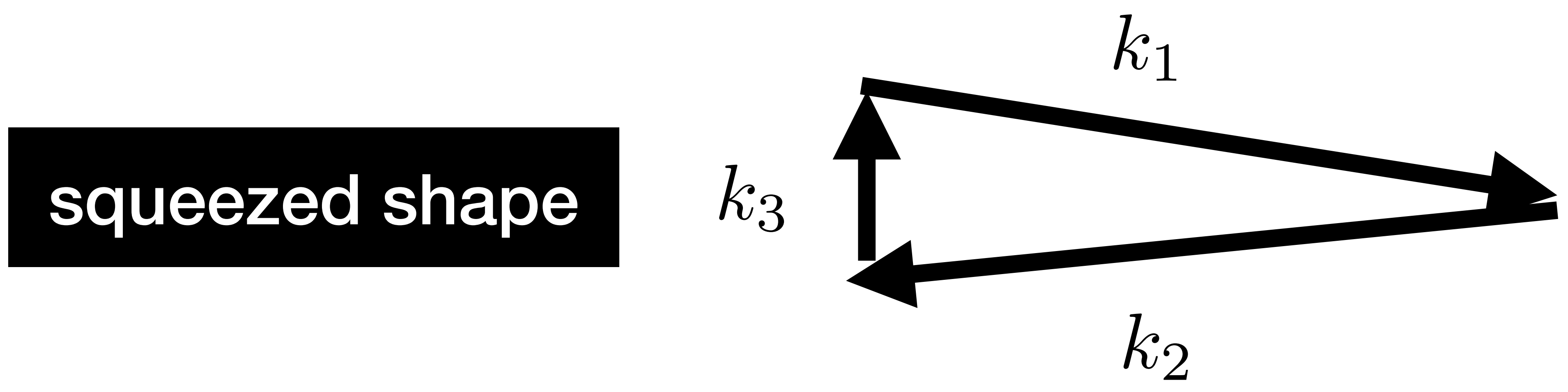
Wang & Xianyu '19, '20

chemical poentential

$$\exp\left(2\pi\frac{\mu-m}{H}\right)\sin\left(\tilde{\omega}'\log\frac{k_1}{k_3}\right)$$

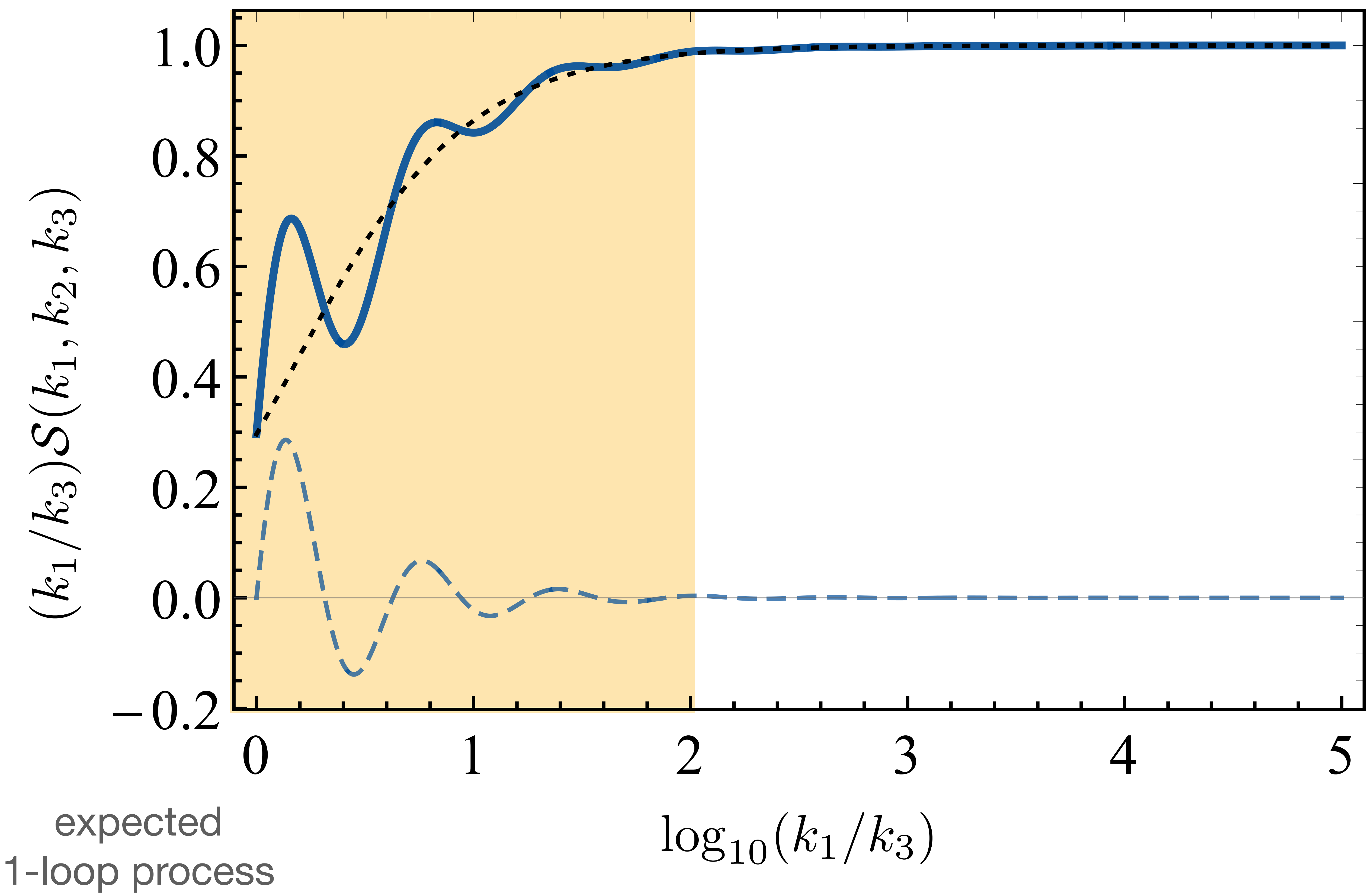
- Firstly appear at **1-loop order** (instead of tree-level) for the 3-pt correlator

Why we do it numerically?



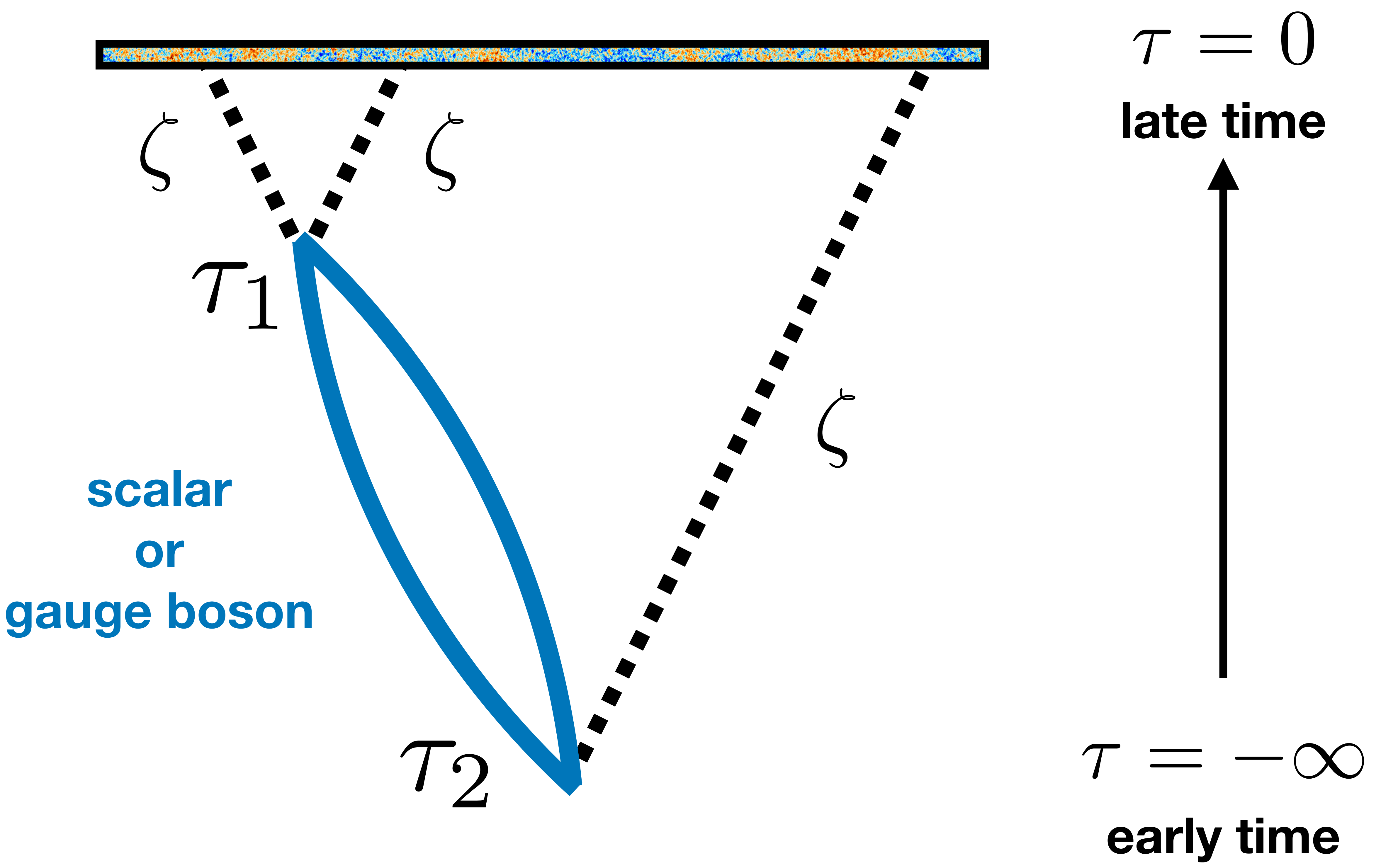
Shape function of 3-pt correlator

- Hard to do analytically (lack of symmetries)
- Hard to estimate the smooth “background” piece of the signal
- Hard to estimate the oscillations at first a few *k*-ratio, where oscillations are the most significant.



The target process

$$\mathcal{S}(k_1, k_2, k_3) =$$



use real-time Schwinger-Keldysh formalism

Challenges

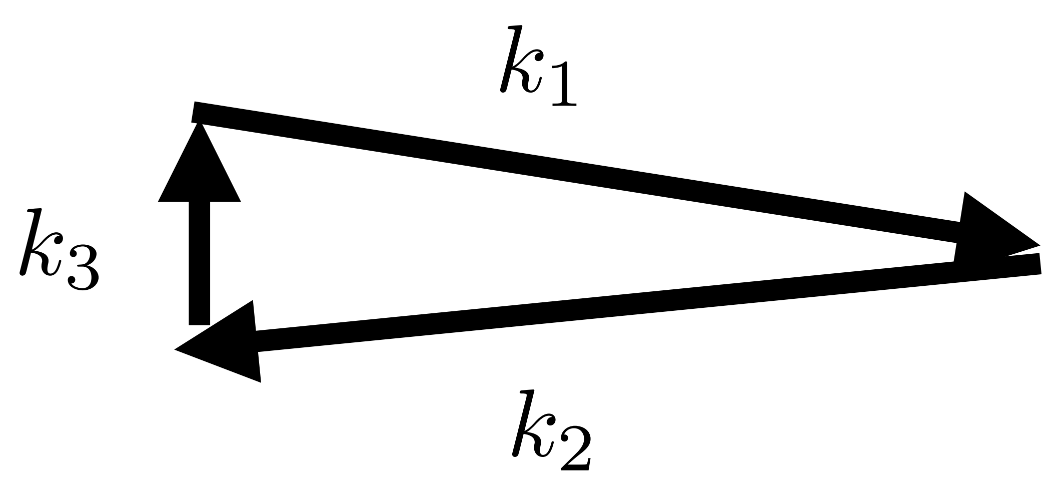
- Like the flat-space QFT: multi-dim integral, UV regulator is needed
- Unlike the flat-space QFT: no Monte-Carlo, more diagrams, propagators are very oscillatory special functions (Whittaker W functions).
- Optimize at multiple levels in the numeral procedure.
- Each process takes O(0.1 M) CPU hours.

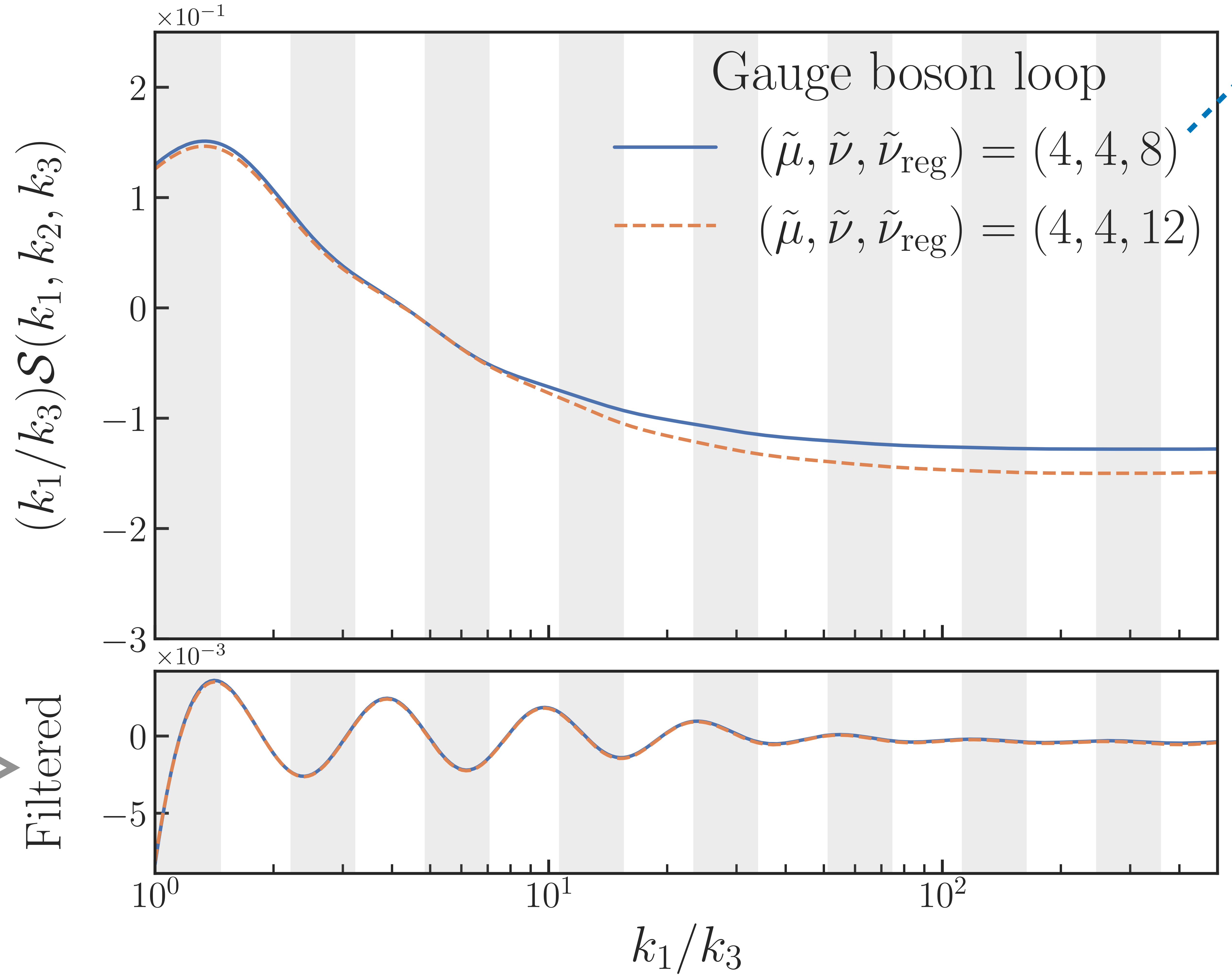
Example of the results

$$\tilde{\mu} = \mu/H$$

$$\tilde{\nu} = \sqrt{\left(\frac{m}{H}\right)^2 - \frac{1}{4}}$$

$$\tilde{\nu}_{\rm reg} = \sqrt{\left(\frac{M}{H}\right)^2 - \frac{1}{4}}$$



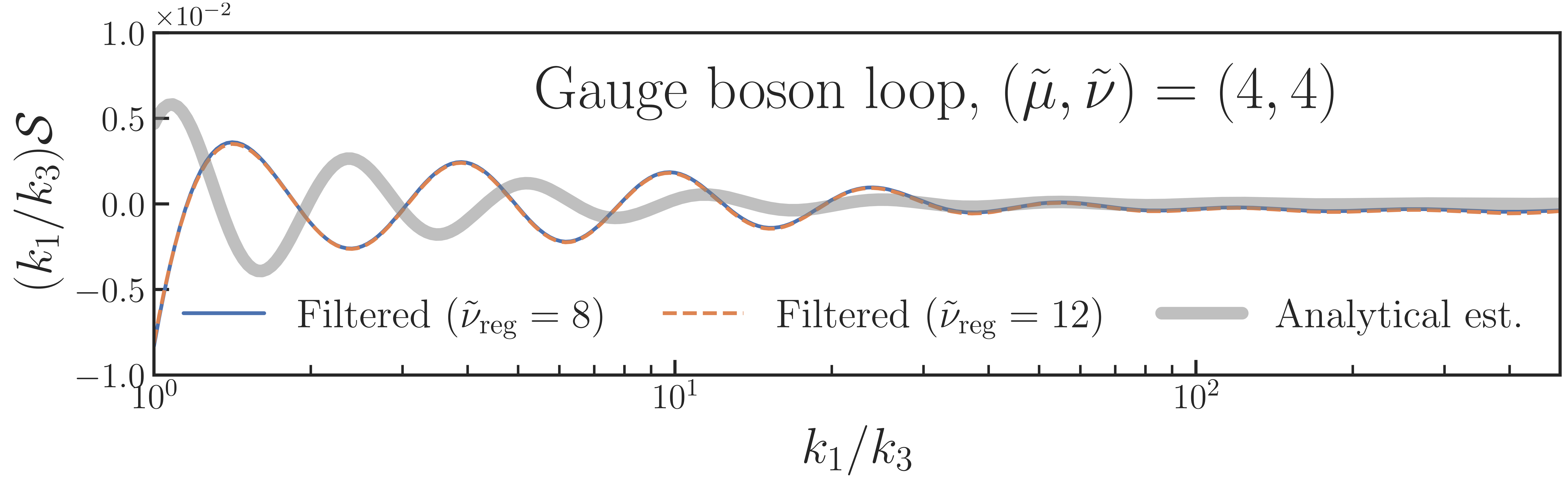


mod out
trivial pre-factors

passing through
a high-pass filter

$$\omega \sim 2\tilde{\nu}$$

Compare with analytic estimates



- Good agreement in frequencies at $k_1/k_3 \gtrsim 20$
- A few present-level disagreement in frequencies at lower k_1/k_3

Summary

- Primordial non-Gaussianities can provide information of physics at very high energy scales.
- Large CC signals may first appear in the 1-loop process.
- The 1-loop 3-pt correlators are difficult to compute. Nevertheless it is possible. We presented the first full numerical results for such process for scalar/gauge boson.
- Methods & techniques we developed can be extend to other 1-loop process.

Backups

Signals in the squeezed limit

- More generally, the shape function under squeezed limit is given by

analytic piece / "background part"

$$\mathcal{S} \approx \boxed{A\left(\frac{k_1}{k_3}\right)^{-N}} + \boxed{B\left(\frac{k_1}{k_3}\right)^{-L} \sin\left(\omega \log\frac{k_1}{k_3} + \varphi\right)}$$

nonanalytic piece / "signal part"

		B	L	ω
$s=0,\ m>\frac{3}{2},\ \mu=0$ [7]	tree	$e^{-\pi m}$	$\frac{1}{2}$	$\sqrt{m^2-\frac{9}{4}}$
$s=0,\ 0<m<\frac{3}{2},\ \mu=0$ [7]	tree	–	$\frac{1}{2}-\sqrt{\frac{9}{4}-m^2}$	0
$s>0,\ m>s-\frac{1}{2},\ \mu=0$ [13]	tree	$e^{-\pi m}$	$\frac{1}{2}$	$\sqrt{m^2-(s-\frac{1}{2})^2}$
$s>0,\ 0<m<s-\frac{1}{2},\ \mu=0$ [13]	tree	–	$\frac{1}{2}-\sqrt{(s-\frac{1}{2})^2-m^2}$	0
$s=0,\ m>\frac{3}{2},\ \mu=0$ [7]	1-loop	$e^{-2\pi m}$	2	$2\sqrt{m^2-\frac{9}{4}}$
Dirac fermion, $m>0,\ \mu=0$ [8]	1-loop	$e^{-2\pi m}$	3	$2m$

Arkani-Hamed & Maldacena '15
Chen et al '18
Lee et al '16

c.f. Table 1, H =1

Chemical-potential-enhanced signals

Wang & Xianyu '19, '20

- Production of particles via inflaton rolling

$$\omega^2 = k^2 + m^2 + \cdots$$

$$(\omega \pm \mu)^2 = k^2 + m^2 + \cdots \qquad\qquad \omega^2 = (k \pm \mu)^2 + m^2 + \cdots$$

chemical potential

- Naturally realized for fermions and gauge boson

$$\frac{1}{\Lambda}\partial_\mu \phi\, \psi^\dagger \bar{\sigma}^\mu \psi \qquad \frac{1}{\Lambda}\phi F \tilde{F} \qquad \Rightarrow \quad \mu = \frac{\dot{\phi}_0}{\Lambda} \rightarrow \dot{\phi}^{\frac{1}{2}} \approx 60H$$

Chemical-potential-enhanced signals

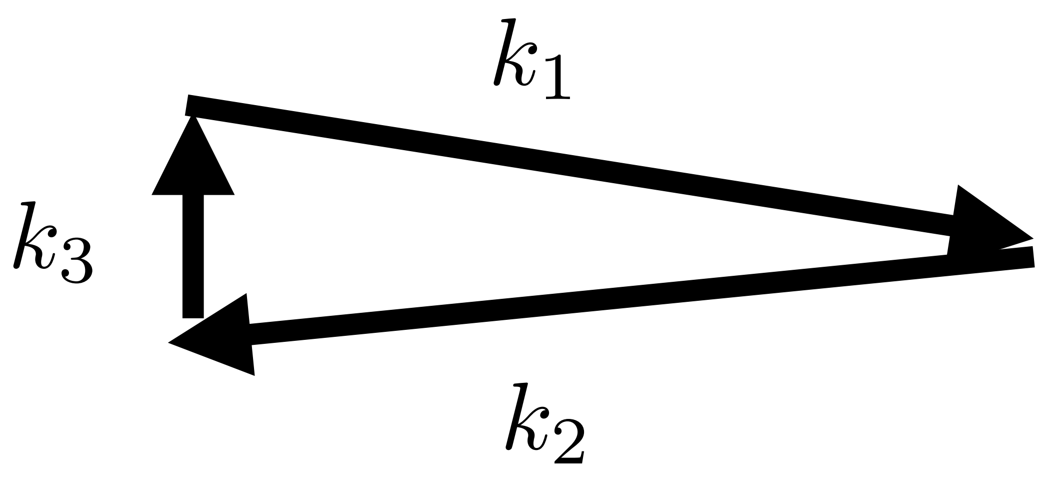
- Firstly appear at **1-loop order** (instead of tree-level) for bispectrum
    - fermions
    - gauge bosons: only one transverse polarization gets enhanced
- Signals are enhanced by $\sim \exp(2\pi\mu/H)$, can overcome the Boltzmann suppression

Chen et al ’18
Wang & Xianyu ‘20

		B	L	ω
Dirac fermion, $m > 0$, $\mu > 0$ [8]	1-loop	$e^{2\pi\mu - 2\pi\sqrt{m^2+\mu^2}}$	2	$2\sqrt{m^2+\mu^2}$
$s = 1$, $m > \frac{1}{2}$, $\mu \geq 0$ [10]	1-loop	$e^{2\pi\mu - 2\pi m}$	2	$2\sqrt{m^2 - \frac{1}{4}}$

c.f. Table 1, H =1

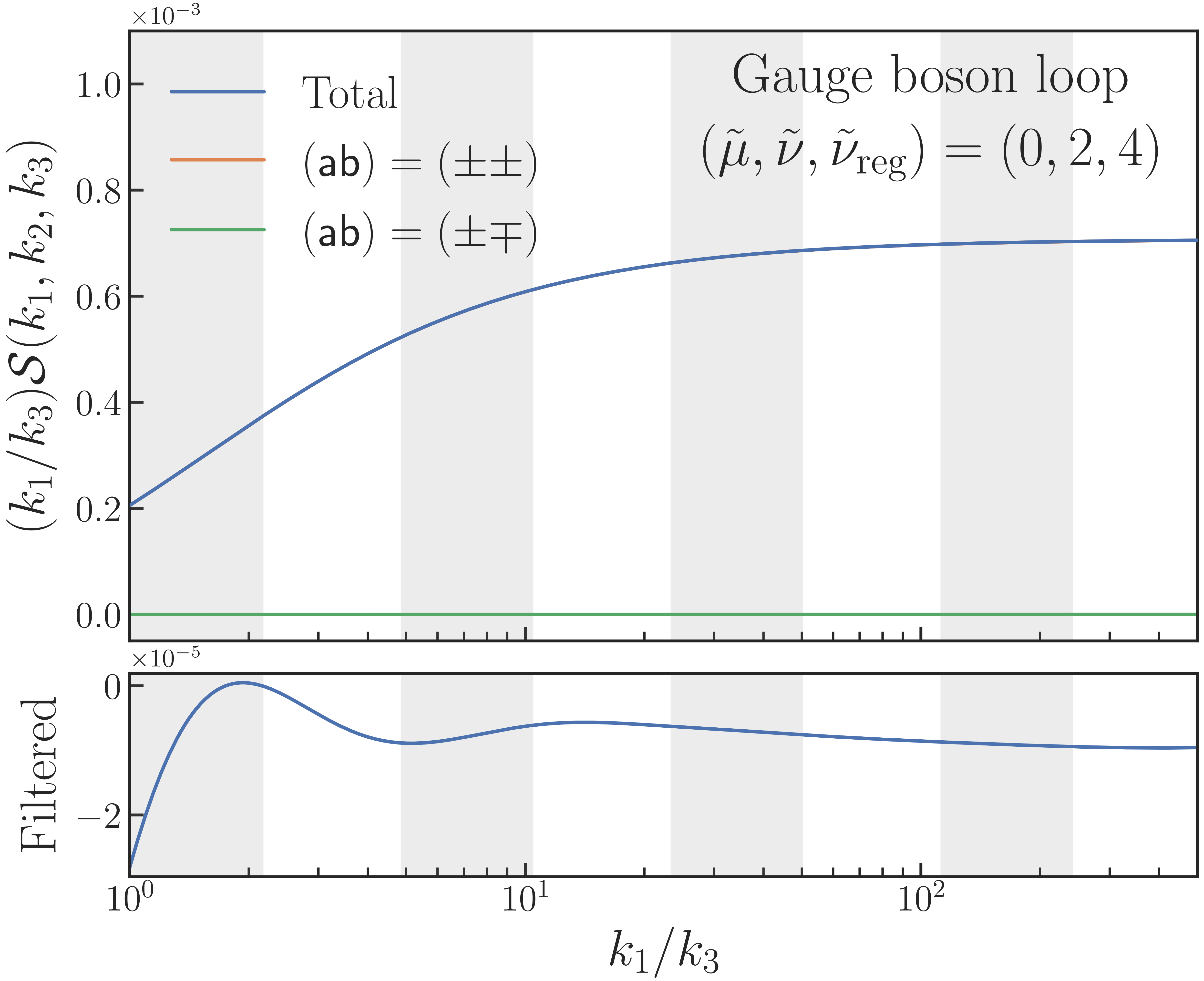
Without chemical potential



$$\tilde{\mu} = \mu/H$$

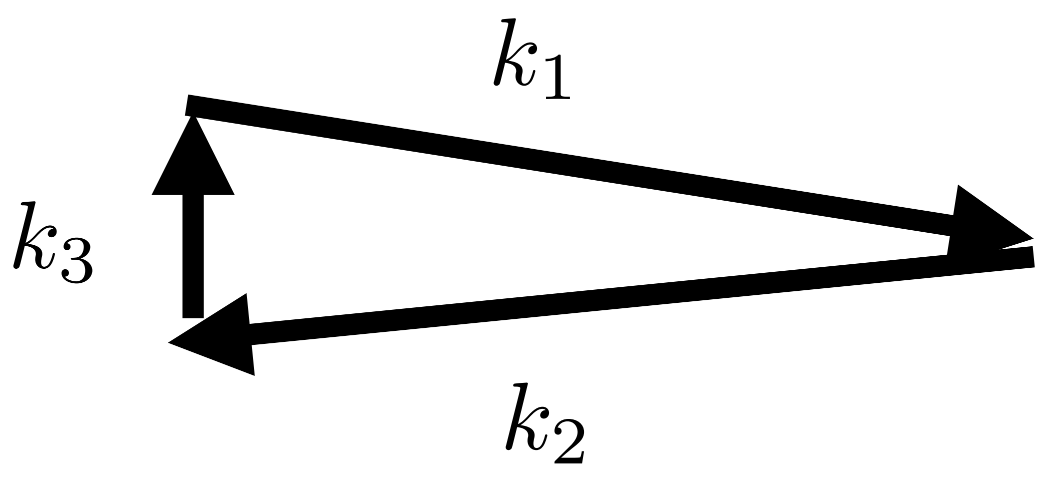
$$\tilde{\nu} = \sqrt{\left(\frac{m}{H}\right)^2 - \frac{1}{4}}$$

$$\tilde{\nu}_{\text{reg}} = \sqrt{\left(\frac{M}{H}\right)^2 - \frac{1}{4}}$$



Oscillation does not show up after filtering

Large chemical potential



$\tilde{\mu} = \mu/H$

$\tilde{\nu} = \sqrt{\left(\frac{m}{H}\right)^2 - \frac{1}{4}}$

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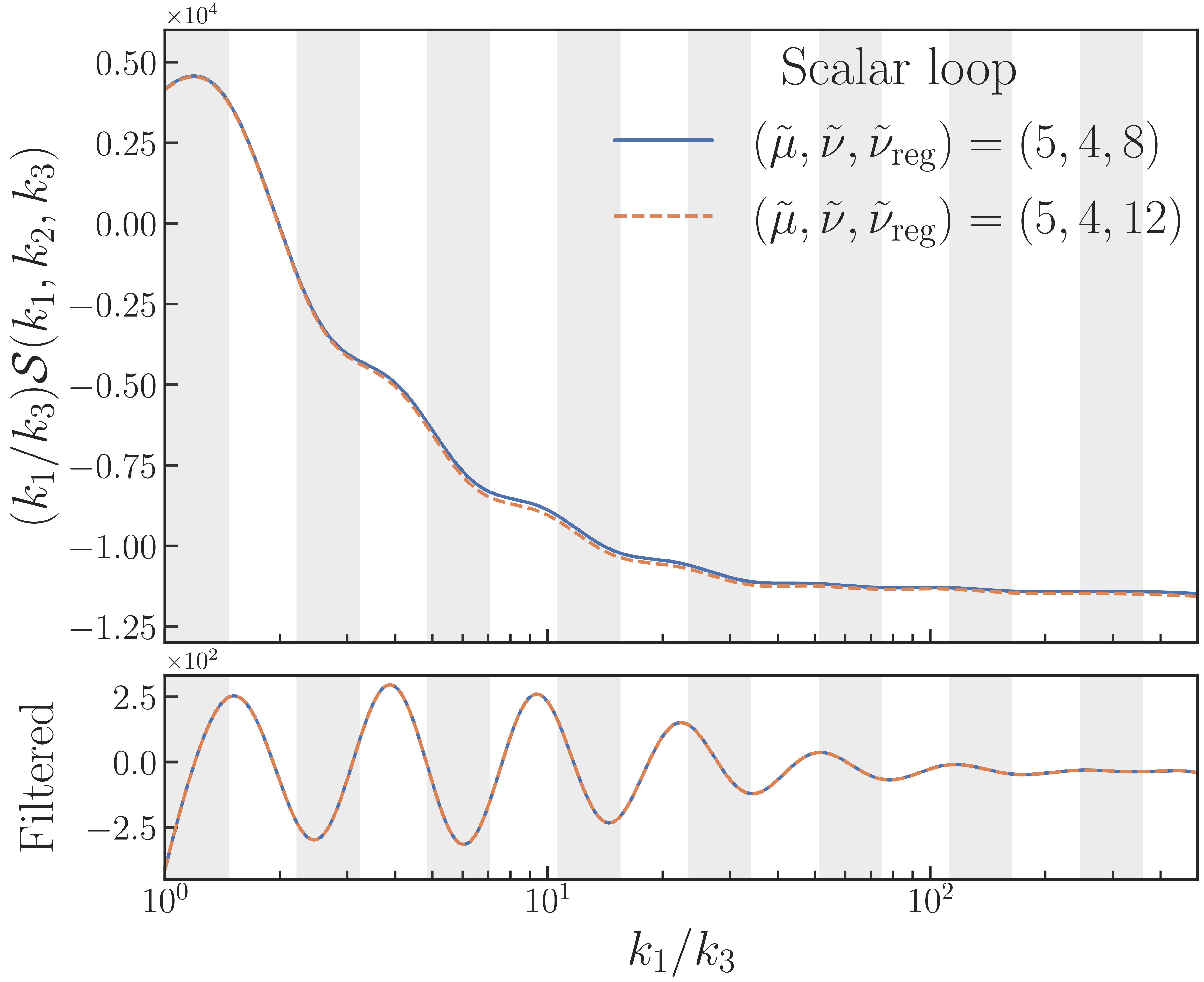

Other shape functions