

Un-binned Angular Analysis of $B \rightarrow D^* \ell \nu$ and the Right-handed Current

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Motivation

Semileptonic $B \rightarrow D^{(*)} \ell \nu$ decay

- $R(D^{(*)})$ anomalies

$$R(D^{(*)}) = \frac{\mathcal{B}(B \rightarrow D^{(*)} \tau \nu)}{\mathcal{B}(B \rightarrow D^{(*)} \ell \nu)}, \quad \text{with } \ell = \mu, e$$

- V_{cb} puzzle

inclusive decay $B \rightarrow X_c \ell \nu$ ($X_c = D, D^*, D_0^* \dots$)

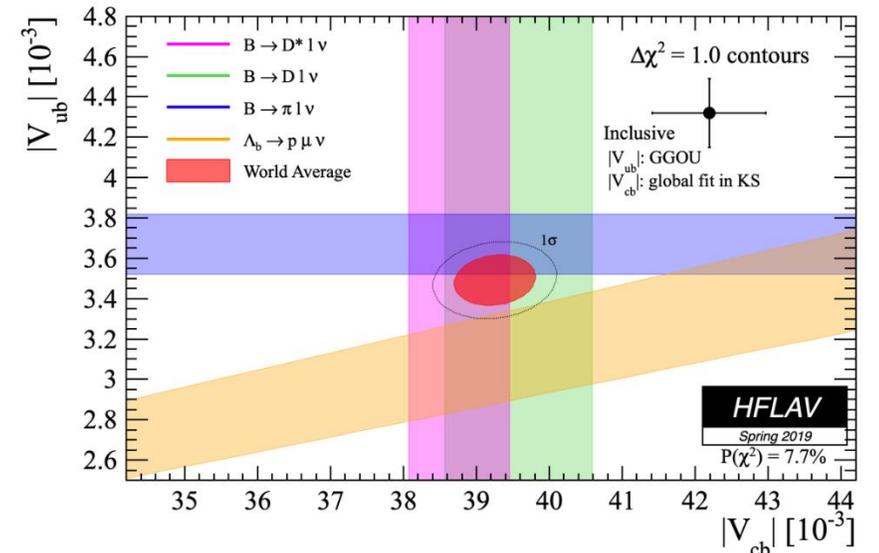
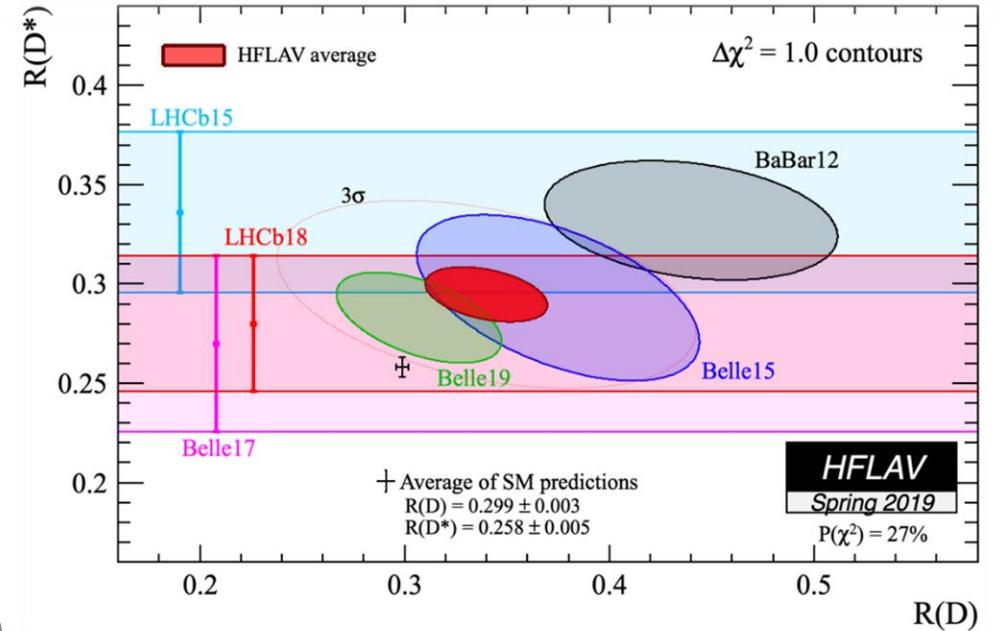
HQE, Optical theorem, OPE

exclusive decay $B \rightarrow D^{(*)} \ell \nu$

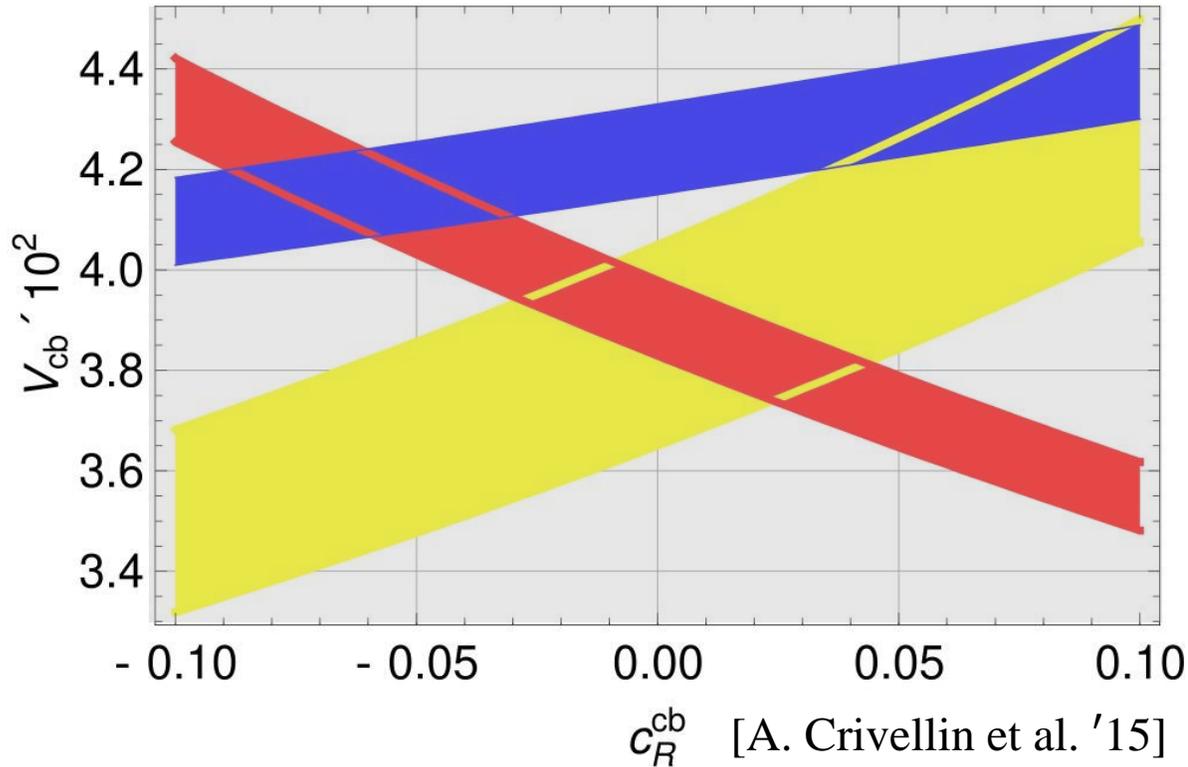
in. 42.16(50) vs ex. 39.70(60)
 $\sim 3\sigma$ deviation

form factor calculation: *lattice, LCSR*
 parametrization: *CLN(-like)/BGL*

[M. Bordone et al. '21] [S. Iguro et al. '20]



Relation between the R.H. vector current and the V_{cb} puzzle



$B \rightarrow D^* \ell \nu$ vs $B \rightarrow X_c \ell \nu$: $C_{V_R} \sim -5\%$

$B \rightarrow D \ell \nu$ vs $B \rightarrow X_c \ell \nu$: $C_{V_R} \sim 5\%$

$$\mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{cb} [C_{V_L} \mathcal{O}_{V_L} + C_{V_R} \mathcal{O}_{V_R}] + \text{h.c.}$$

$$\mathcal{O}_{V_L} = (\bar{c}_L \gamma^\mu b_L)(\bar{\ell}_L \gamma_\mu \nu_L), \quad \mathcal{O}_{V_R} = (\bar{c}_R \gamma^\mu b_R)(\bar{\ell}_L \gamma_\mu \nu_L).$$

$C_{V_L} = 1$ and $C_{V_R} = 0$ in the SM

$C_{V_R} \neq 0$ in the Left-Right symmetric model
from $W_L - W_R$ mixing [E. Kou et al. '13]

Considerable ex. uncertainties.

Theo. uncertainty from lattice QCD input.

More measurements needed!

Theoretical Framework



Differential decay rate ($m_{\mu,e} \rightarrow 0$):

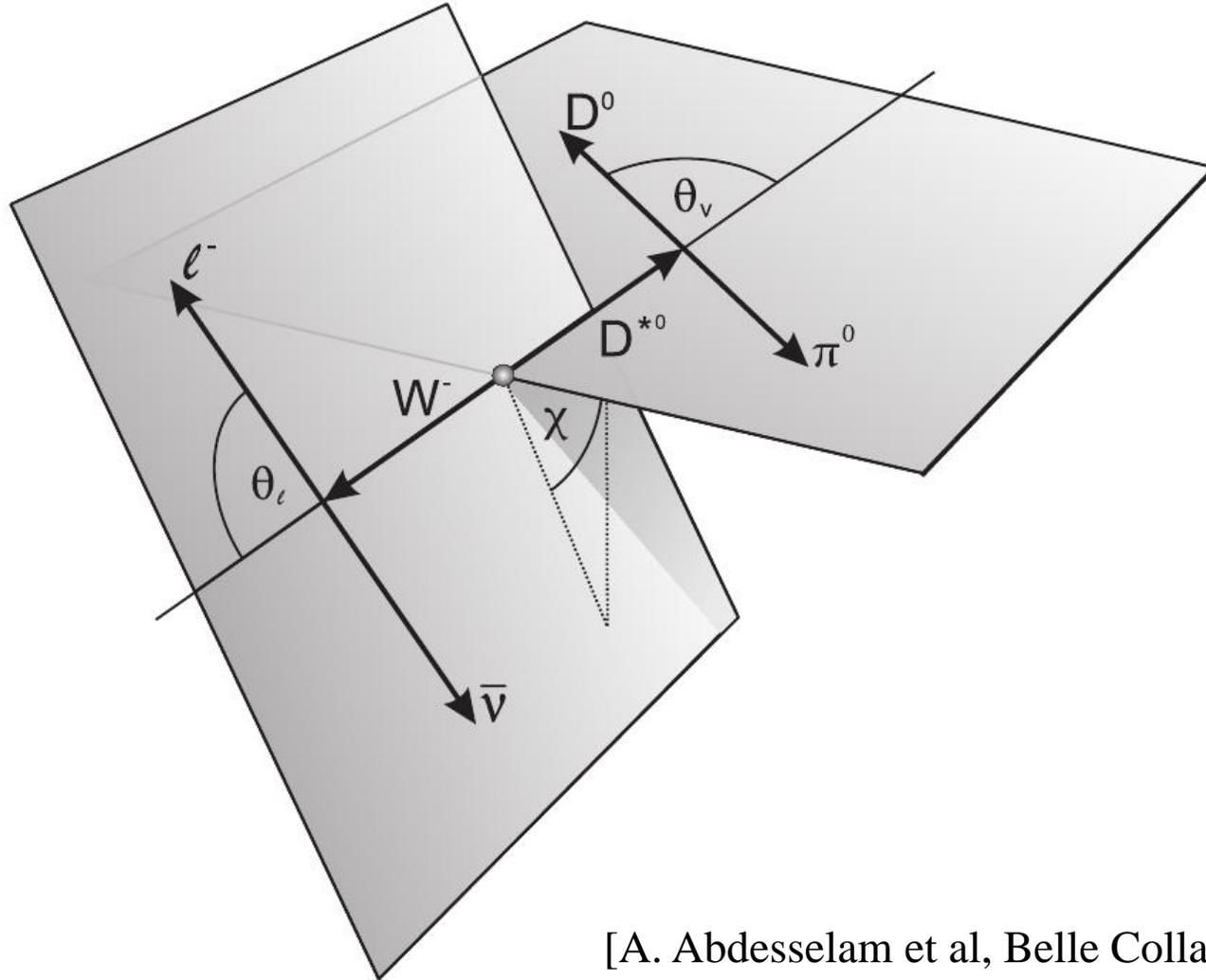
$$\begin{aligned} & \frac{d\Gamma(\bar{B} \rightarrow D^*(\rightarrow D\pi) \ell^- \bar{\nu}_\ell)}{dw d \cos \theta_V d \cos \theta_\ell d\chi} \\ &= \frac{6m_B m_{D^*}^2}{8(4\pi)^4} \sqrt{w^2 - 1} (1 - 2wr + r^2) G_F^2 |V_{cb}|^2 \mathcal{B}(D^* \rightarrow D\pi) \\ & \times \left\{ J_{1s} \sin^2 \theta_V + J_{1c} \cos^2 \theta_V + (J_{2s} \sin^2 \theta_V \right. \\ & \quad + J_{2c} \cos^2 \theta_V) \cos 2\theta_\ell \\ & \quad + J_3 \sin^2 \theta_V \sin^2 \theta_\ell \cos 2\chi \\ & \quad + J_4 \sin 2\theta_V \sin 2\theta_\ell \cos \chi + J_5 \sin 2\theta_V \sin \theta_\ell \cos \chi \\ & \quad + (J_{6s} \sin^2 \theta_V + J_{6c} \cos^2 \theta_V) \cos \theta_\ell \\ & \quad + J_7 \sin 2\theta_V \sin \theta_\ell \sin \chi + J_8 \sin 2\theta_V \sin 2\theta_\ell \sin \chi \\ & \quad \left. + J_9 \sin^2 \theta_V \sin^2 \theta_\ell \sin 2\chi \right\}, \end{aligned}$$

J_i experimentally measurable, includes H_+ , H_- , H_0 , C_{V_L} and C_{V_R} (SM and BSM).

J_i functions:

$$\begin{aligned} J_{1s} &= \frac{3}{2}(H_+^2 + H_-^2)(|C_{V_L}|^2 + |C_{V_R}|^2) - 6H_+H_- \text{Re}[C_{V_L}C_{V_R}^*] \\ J_{1c} &= 2H_0^2(|C_{V_L}|^2 + |C_{V_R}|^2 - 2\text{Re}[C_{V_L}C_{V_R}^*]) \\ J_{1c} &= 2H_0^2(|C_{V_L}|^2 + |C_{V_R}|^2 - 2\text{Re}[C_{V_L}C_{V_R}^*]) \\ J_{2s} &= \frac{1}{2}(H_+^2 + H_-^2)(|C_{V_L}|^2 + |C_{V_R}|^2) - 2H_+H_- \text{Re}[C_{V_L}C_{V_R}^*] \\ J_{2c} &= -2H_0^2(|C_{V_L}|^2 + |C_{V_R}|^2 - 2\text{Re}[C_{V_L}C_{V_R}^*]) \\ J_3 &= -2H_+H_- (|C_{V_L}|^2 + |C_{V_R}|^2) + 2(H_+^2 + H_-^2) \text{Re}[C_{V_L}C_{V_R}^*] \\ J_4 &= (H_+H_0 + H_-H_0)(|C_{V_L}|^2 + |C_{V_R}|^2 - 2\text{Re}[C_{V_L}C_{V_R}^*]) \\ J_5 &= -2(H_+H_0 - H_-H_0)(|C_{V_L}|^2 - |C_{V_R}|^2) \\ J_{6s} &= -2(H_+^2 - H_-^2)(|C_{V_L}|^2 - |C_{V_R}|^2) \\ J_{6c} &= 0 \\ J_7 &= 0 \\ J_8 &= 2(H_+H_0 - H_-H_0) \text{Im}[C_{V_L}C_{V_R}^*] \\ J_9 &= -2(H_+^2 - H_-^2) \text{Im}[C_{V_L}C_{V_R}^*] \end{aligned}$$

Kinematic variables in $B \rightarrow D^* (\rightarrow D\pi)\ell\nu$



θ_ℓ the angle between the lepton and the direction opposite the B-meson in the virtual W-boson rest frame;

θ_ν the angle between the D meson and the direction opposite the B meson in the D^* rest frame;

χ the tilting angle between the two decay planes spanned by the W and D systems in the B meson rest frame;

W the dimensionless four-momentum transfer.

[A. Abdesselam et al, Belle Collaboration '17]

Helicity amplitudes in CLN and BGL parametrizations

$$H_{\pm}(w) = m_B \sqrt{r} (w+1) h_{A_1}(w) \times \left[1 \mp \sqrt{\frac{w-1}{w+1}} R_1(w) \right]$$

$$H_0(w) = m_B^2 \sqrt{r} (w+1) \frac{1-r}{\sqrt{q^2}} h_{A_1}(w) \times \left[1 + \frac{w-1}{1-r} (1 - R_2(w)) \right]$$

CLN parametrization (HQE based)

$$h_{A_1}(w) = h_{A_1}(1) (1 - 8\rho_{D^*}^2 z + (53\rho_{D^*}^2 - 15)z^2 - (231\rho_{D^*}^2 - 91)z^3)$$

$$R_1(w) = R_1(1) - 0.12(w-1) + 0.05(w-1)^2$$

$$R_2(w) = R_2(1) + 0.11(w-1) - 0.06(w-1)^2$$

$$H_{\pm}(w) = f(w) \mp m_B |\mathbf{p}_{D^*}| g(w)$$

$$H_0(w) = \frac{\mathcal{F}_1(w)}{\sqrt{q^2}}$$

BGL parametrization (analyticity based)

$$g(z) = \frac{1}{P_g(z)\phi_g(z)} \sum_{n=0}^N a_n^g z^n$$

$$f(z) = \frac{1}{P_f(z)\phi_f(z)} \sum_{n=0}^N a_n^f z^n$$

$$\mathcal{F}_1(z) = \frac{1}{P_{\mathcal{F}_1}(z)\phi_{\mathcal{F}_1}(z)} \sum_{n=0}^N a_n^{\mathcal{F}_1} z^n$$

Blaschke factors:

P_g, P_f, P_{F_1}

outer functions:

$\phi_g, \phi_f, \phi_{F_1}$

Un-binned Angular Analysis



Normalised PDF:

$$\hat{f}_{\langle \vec{g} \rangle}(\cos \theta_V, \cos \theta_\ell, \chi) = \frac{9}{8\pi} \times \left\{ \frac{1}{6}(1 - 3\langle g_{1c} \rangle + 2\langle g_{2s} \rangle + \langle g_{2c} \rangle) \sin^2 \theta_V + \langle g_{1c} \rangle \cos^2 \theta_V + (\langle g_{2s} \rangle \sin^2 \theta_V + \langle g_{2c} \rangle \cos^2 \theta_V) \cos 2\theta_\ell + \langle g_3 \rangle \sin^2 \theta_V \sin^2 \theta_\ell \cos 2\chi + \langle g_4 \rangle \sin 2\theta_V \sin 2\theta_\ell \cos \chi + \langle g_5 \rangle \sin 2\theta_V \sin \theta_\ell \cos \chi + (\langle g_{6s} \rangle \sin^2 \theta_V + \langle g_{6c} \rangle \cos^2 \theta_V) \cos \theta_\ell + \langle g_7 \rangle \sin 2\theta_V \sin \theta_\ell \sin \chi + \langle g_8 \rangle \sin 2\theta_V \sin 2\theta_\ell \sin \chi + \langle g_9 \rangle \sin^2 \theta_V \sin^2 \theta_\ell \sin 2\chi \right\},$$

Existing binned analysis
(projected χ^2 fit):
Belle '17 '18; BaBar '19

$$\langle g_i \rangle \equiv \frac{\langle J'_i \rangle}{6\langle J'_{1s} \rangle + 3\langle J'_{1c} \rangle - 2\langle J'_{2s} \rangle - \langle J'_{2c} \rangle}$$

$$J'_i \equiv J_i \sqrt{w^2 - 1} (1 - 2wr + r^2)$$

The experimental determination of $\langle g_i \rangle$ can be pursued by the *maximum likelihood method*:

$$\mathcal{L}(\langle \vec{g}_i \rangle) = \sum_{i=1}^N \ln \hat{f}_{\langle \vec{g}_i \rangle}(e_i)$$

Angular observables allow to determine C_{V_R} without the intervention of the V_{cb} puzzle!

Pseudo data generation

Pseudo data generated using CLN parameters fitted by Belle [E. Waheed et al, '18]

$$N_{event} = (5306, 8934, 10525, 11241, 11392, 11132, 10555, 9726, 8693, 7497)$$

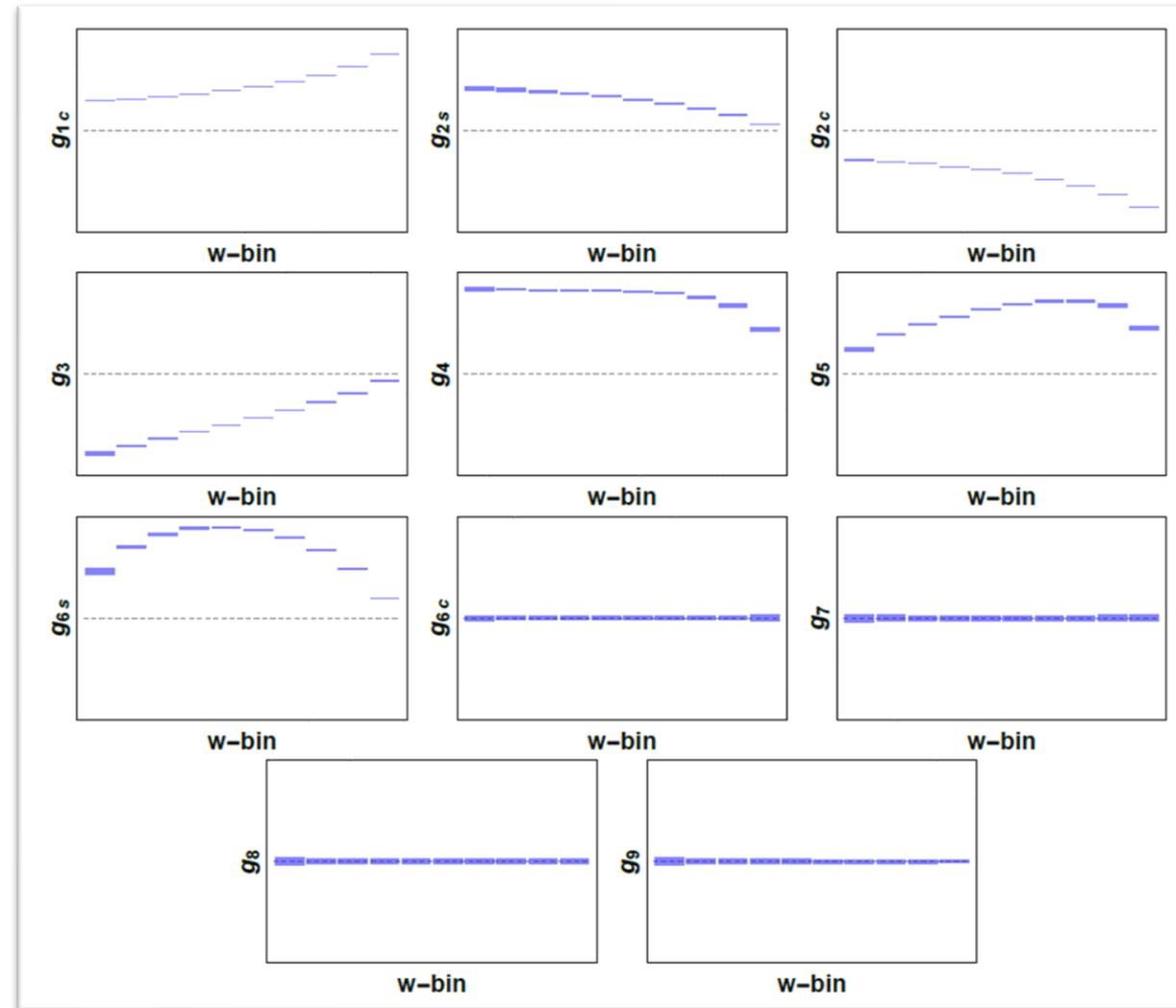
Pseudo data generated using BGL parameters fitted by Belle [E. Waheed et al, '18]

$$N_{event} = (5306, 8934, 10525, 11241, 11392, 11132, 10555, 9726, 8693, 7497)$$

$\langle g_i \rangle$ generated in 10 bins with covariance matrices by toy Monte-Carlo method

Total event number: 95k as in Belle analysis

Using pseudo data we fit theoretical formula including C_{V_R} (on top of form factors). **Note** V_{cb} is not possible to fit any more because it cancels in g_i !



$\langle g_i \rangle$ generated in 10 w-bins

χ^2 utilized in the CLN/BGL fit

$$\chi^2(\vec{v}) = \chi_{\text{angle}}^2(\vec{v}) + \chi_{\text{lattice}}^2(\vec{v})$$

$$\chi_{\text{angle}}^2(\vec{v}) = \sum_{w\text{-bin}=1}^{10} \left[\sum_{ij} N_{\text{event}} \hat{V}_{ij}^{-1} (\langle g_i \rangle^{\text{exp}} - \langle g_i^{\text{th}}(\vec{v}) \rangle) (\langle g_j \rangle^{\text{exp}} - \langle g_j^{\text{th}}(\vec{v}) \rangle) \right]_{w\text{-bin}}$$

We include the lattice input by introducing

$$\chi_{\text{lattice}}^2(v_i) = \left(\frac{v_i^{\text{lattice}} - v_i}{\sigma_{v_i}^{\text{lattice}}} \right)^2$$

with $h_{A_1}(1) = 0.906 \pm 0.013$

by Fermilab/MILC

[J.A. Bailey et al, '14]

Notes:

1.) C_{V_R} and V_{cb} are correlated in the fit using only w-dependence as the changes in both parameters directly impact $Br(B \rightarrow D^* \ell \nu)$

2.) the angular fit does not converge as C_{V_R} is not independent of the vector form factor

Lattice input of the vector form factor is crucial for determining C_{V_R} !

$R_1(1) \sim 4\%$ error $h_V(1) \sim 7\%$ error

[T. Kaneko et al, '19]

Fit of C_{V_R}



CLN fit:

$$R_1(1) = \frac{h_V(1)}{h_{A_1}(1)}$$

$$\vec{v} = (\rho_{D^*}^2, R_1(1), R_2(1), C_{V_R})$$

$$= (1.106, 1.229, 0.852, 0)$$

$$\sigma_{\vec{v}} = (3.177, 0.049, 0.018, 0.021)$$

$$\rho_{\vec{v}} = \begin{pmatrix} 1. & -0.016 & -0.763 & 0.095 \\ -0.016 & 1. & 0.006 & -0.973 \\ -0.763 & 0.006 & 1. & -0.117 \\ 0.095 & -0.973 & -0.117 & 1. \end{pmatrix}$$

C_{V_R} can be determined to a precision of ~ 2 (4)% in CLN (BGL) parametrization.

BGL fit:

$$h_V(1) = \frac{m_B \sqrt{r}}{P_g(0) \phi_g(0)} a_0^g$$

$$\vec{v} = (a_0^f, a_1^f, a_1^{\mathcal{F}_1}, a_2^{\mathcal{F}_1}, a_0^g, C_{V_R})$$

$$= (0.0132, 0.0169, 0.0070, -0.0852, 0.0241, 0.0024)$$

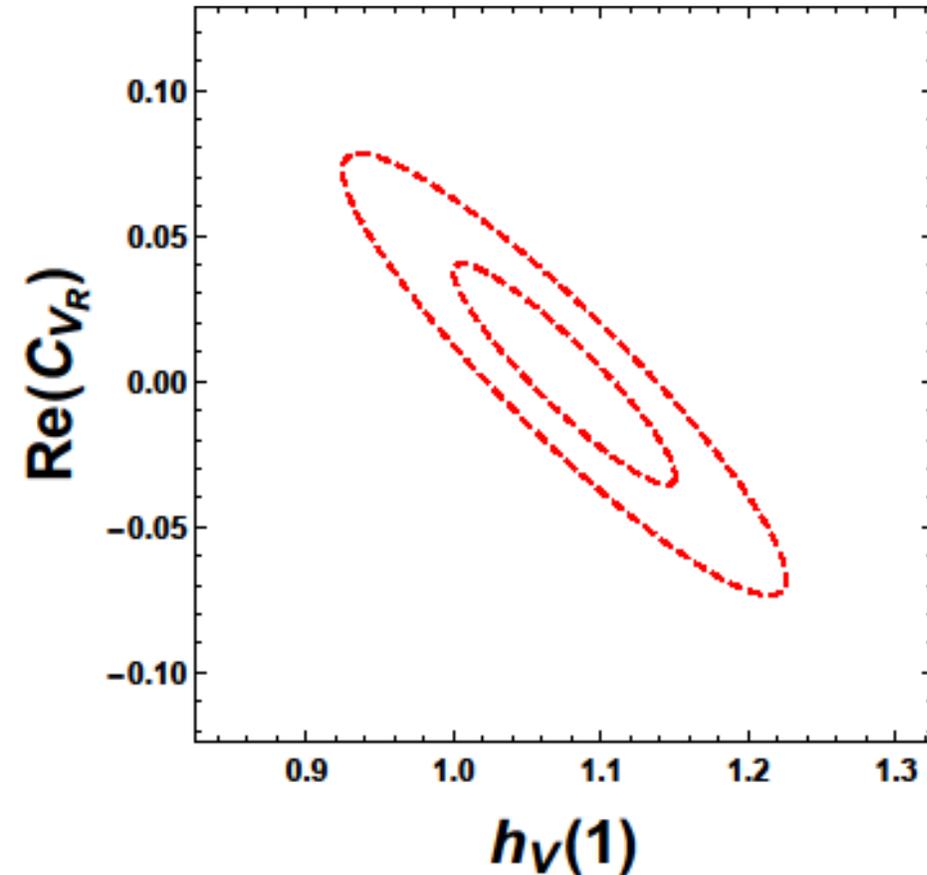
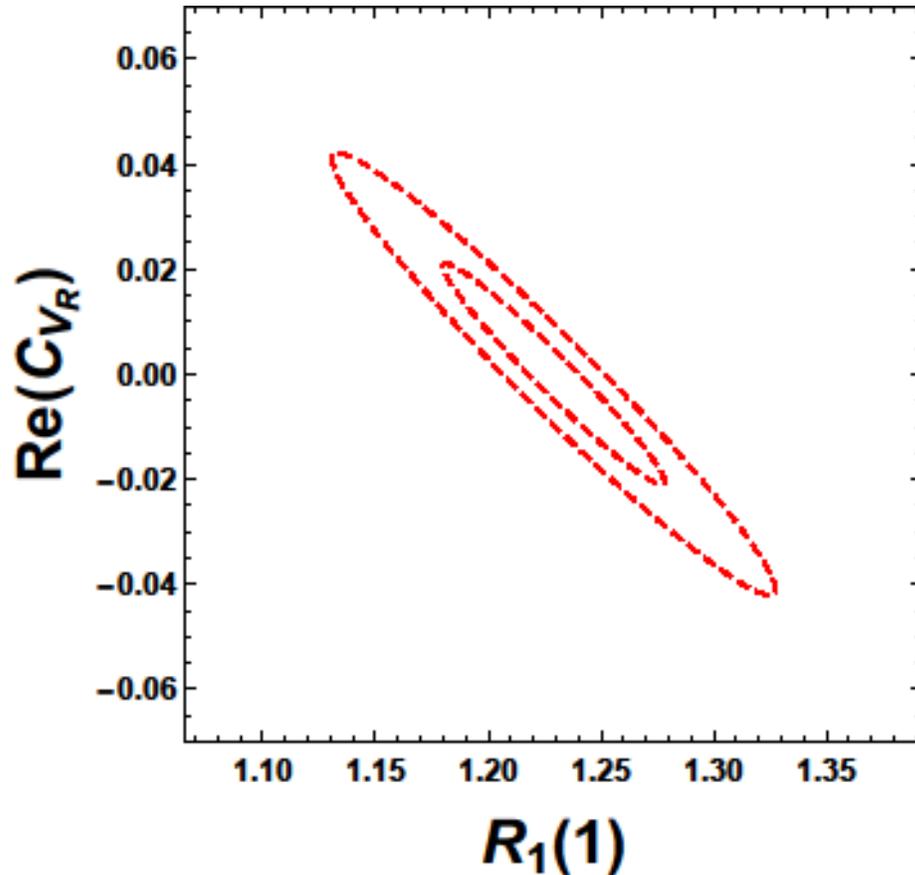
$$\sigma_{\vec{v}} = (0.0002, 0.0109, 0.0026, 0.0352, 0.0017, 0.0379)$$

$$\rho_{\vec{v}} = \begin{pmatrix} 1. & 0.022 & 0.039 & -0.035 & 0.000 & 0.189 \\ 0.022 & 1. & 0.860 & -0.351 & 0.000 & 0.316 \\ 0.039 & 0.860 & 1. & -0.762 & 0.000 & 0.283 \\ -0.035 & -0.351 & -0.762 & 1. & 0.000 & -0.119 \\ 0.000 & 0.000 & 0.000 & 0.000 & 1. & -0.923 \\ 0.189 & 0.316 & 0.283 & -0.119 & -0.923 & 1. \end{pmatrix}$$

Central value of C_{V_R} is ~ 0 because of Belle measurement.

C_{V_R} and the vector form factor are highly correlated!

Contour Plots



If new lattice results turn out to be different from the experimental fitted value (assuming SM), non-zero C_{V_R} can be hinted.

Fit of C_{V_R} using forward-backward asymmetry (FBA) only

$FBA \sim \langle g_{6s} \rangle$

Advantage: one angle measurement

$$\begin{aligned} \langle \mathcal{A}_{FB} \rangle &\equiv \frac{\int_0^1 \frac{d\Gamma}{d\cos\theta_\ell} d\cos\theta_\ell - \int_{-1}^0 \frac{d\Gamma}{d\cos\theta_\ell} d\cos\theta_\ell}{\int_0^1 \frac{d\Gamma}{d\cos\theta_\ell} d\cos\theta_\ell + \int_{-1}^0 \frac{d\Gamma}{d\cos\theta_\ell} d\cos\theta_\ell} \\ &= 3\langle g_{6s} \rangle \end{aligned}$$

$$\begin{aligned} \vec{v} &= (\rho_{D^*}^2, R_1(1), R_2(1), C_{V_R}) \\ &= (1.106, 1.229, 0.852, 0.000) \end{aligned}$$

$$\sigma_{\vec{v}} = (2.200, 0.049, 0.031, \mathbf{0.022})$$

$$\rho_{\vec{v}} = \begin{pmatrix} 1. & 0.008 & -0.873 & 0.262 \\ 0.008 & 1. & -0.040 & \underline{-0.931} \\ -0.873 & -0.040 & 1. & -0.296 \\ 0.262 & -0.931 & -0.296 & 1. \end{pmatrix}$$

C_{V_R} can be determined at a precision of 2.2% using FBA alone! Almost as good as the full set of $\langle g_i \rangle$!

$Im(C_{V_R})$ can also be determined at precision of 0.7% for both CLN and BGL!

Summary & Conclusions



- The normalized angular observables $\langle g_i \rangle$ for $B \rightarrow D^*(D\pi)\ell\nu$ determined in the **un-binned analysis** are useful for the precision measurement of C_{V_R} by circumventing the V_{cb} puzzle.
- C_{V_R} is highly dependent on the vector form factor, thus it can only be determined with the vector form factor calculated by lattice.
- The real (imaginary) part of C_{V_R} can be determined at precision of 2-4 (1) % using the full set of $\langle g_i \rangle$.
- FBA ($\langle g_{6s} \rangle$) can determine C_{V_R} at almost equally good precision, thus it is highly proposed to be measured in the near future.

Thank you!

Thank you!

Backup

SM fit including V_{cb}

$$\chi^2(\vec{v}) = \chi_{\text{angle}}^2(\vec{v}) + \chi_{\text{lattice}}^2(\vec{v}) + \chi_{w\text{-bin}}^2(\vec{v})$$

w dependence in χ^2 :

$$\chi_{w\text{-bin}}^2(\vec{v}) = \sum_{w\text{-bin}=1}^{10} \frac{([N]_{w\text{-bin}} - \alpha \langle \Gamma \rangle_{w\text{-bin}})^2}{[N]_{w\text{-bin}}}$$

The factor α is a constant, which relates the number of events and the decay rate:

$$\alpha \equiv \frac{4N_{B\bar{B}}}{1 + f_{+0}} \tau_{B^0} \times \epsilon \mathcal{B}(D^0 \rightarrow K^- \pi^+)$$

→ number of $B\bar{B}$ pairs produced from $\Upsilon(4S)$
→ B^0 lifetime
→ B^+ / B^0 production ratio at Belle

SM fit results in CLN parametrization

$$\begin{aligned} \vec{v} &= (h_{A_1}(1), \rho_{D^*}^2, R_1(1), R_2(1), V_{cb}) \\ &= (0.906, 1.106, 1.229, 0.852, 0.0387) \end{aligned}$$

$$\sigma_{\vec{v}} = (0.013, 0.019, 0.011, 0.011, 0.0006)$$

SM fit results in BGL parametrization

$$\begin{aligned} \vec{v} &= (a_0^f, a_1^f, a_1^{\mathcal{F}_1}, a_2^{\mathcal{F}_1}, a_0^g, V_{cb}) \\ &= (0.0132, 0.0169, 0.0070, -0.0853, 0.0242, 0.0384) \end{aligned}$$

$$\sigma_{\vec{v}} = (0.0002, 0.0028, 0.0011, 0.0199, 0.0004, 0.0006)$$

$\alpha = 6.616(6.613) \times 10^{18}$ in CLN (BGL) parametrization
 Experimental efficiency: $\epsilon = \sim 4.8 \times 10^{-2}$