Gravitational soft theorem from emergent soft gauge symmetries

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Gravitational soft theorem from emergent soft gauge symmetries

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Based on

[2110.02969]

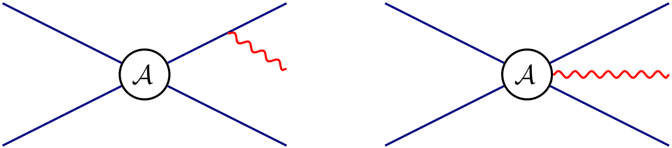
[2111.0XXXX]

in collaboration with

Martin Beneke (TU München), Robert Szafron (BNL)

Soft Theorem

- Consider soft emission of a gluon/graviton from energetic particles



- Low-Burnett-Kroll (LBK) for QED/QCD and Soft Theorem for gravity:
  [Low 1958; Weinberg 1965; Burnett, Kroll 1968; Cachazo, Strominger 1404.4091]

$$\mathcal{A}^{\gamma}_{\rm rad} = -g \sum_i Q_i \left(\frac{\varepsilon_\mu p_i^\mu}{p_i \cdot k} + \frac{k_\nu \varepsilon_\mu J_i^{\mu\nu}}{p_i \cdot k} \right) \mathcal{A}_0$$

$$\mathcal{A}^{h}_{\rm rad} = \frac{\kappa}{2} \sum_i \left(\frac{\varepsilon_{\mu\nu} p_i^\mu p_i^\nu}{p_i \cdot k} + \frac{k_\nu \varepsilon_{\mu\rho} p_i^\rho J_i^{\mu\nu}}{p_i \cdot k} + \frac{1}{2} \frac{k_\rho k_\sigma \varepsilon_{\mu\nu} J^{\mu\rho} J^{\nu\sigma}}{p_i \cdot k} \right) \mathcal{A}_0$$

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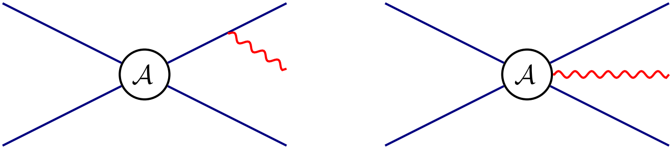

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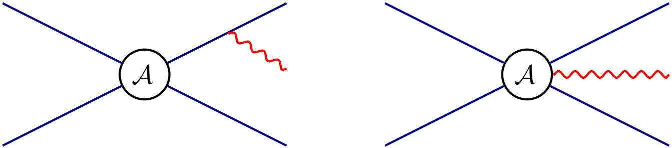

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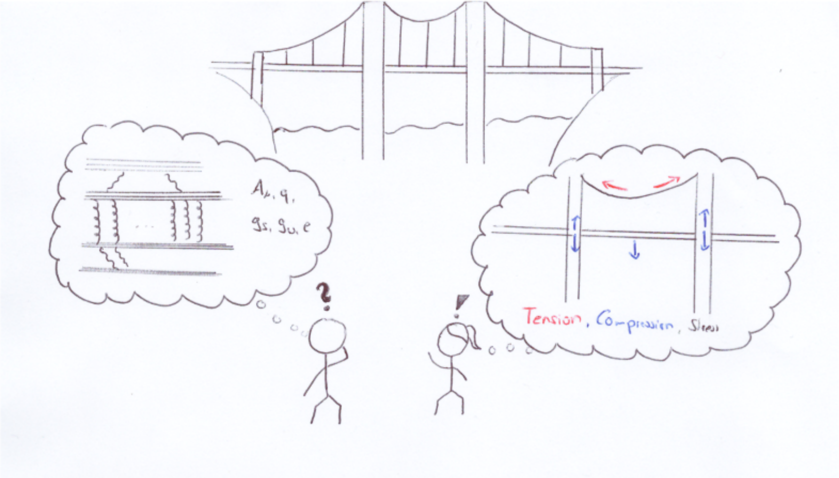
Effective Field Theory

- Description of a physical system in terms of degrees of freedom and interactions that are important for the given length-scale.



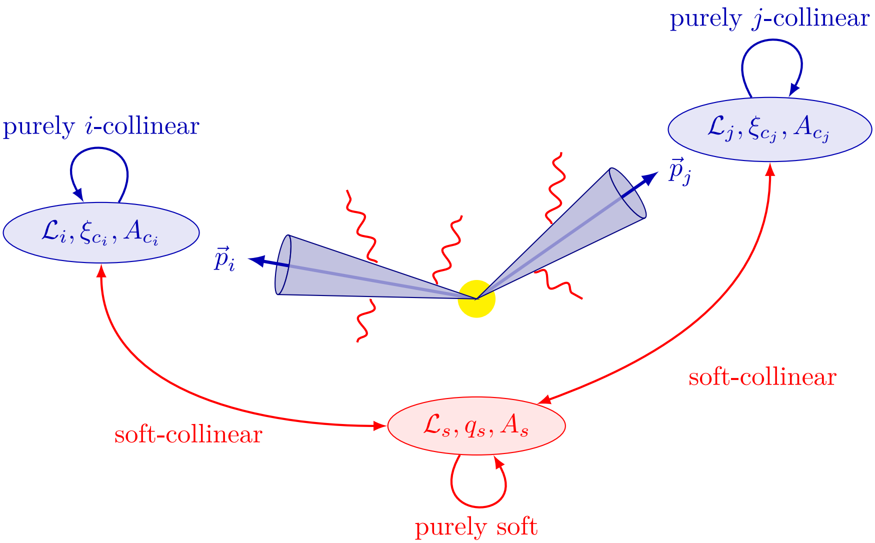
SCET – A modern EFT

- "Conventional" EFT:
  - integrate out heavy fields
  - one-to-one correspondence of light fields in full theory and EFT
  - power-counting related to mass-dimension
  - keep original gauge symmetry
- "Modern" EFT:
  - integrate out certain regions in phase-space
  - full-theory field described by multiple field operators in the EFT
  - power-counting no longer fixed to mass-dimension
  - each region has separate gauge symmetry
- Goal: construct effective theory with homogeneous power-counting that organises the interactions

Soft-collinear Effective Theory

- Collinear modes that describe energetic particles
- Soft fields describe isotropic radiation
- Power-counting parameter $\lambda \sim \frac{p_\perp}{n_+ p} \ll 1$
- Soft fields must be multipole expanded around $x_- = n_+ x \frac{n_-}{2}$
  - Gauge-symmetry must respect multipole expansion
- These modes differ in power-counting and gauge symmetry:
  - Soft fields appear as background fields that live on the classical trajectory x_-
  - Collinear fields are realised as small fluctuations on the soft background

Intuitive Picture



QCD vs Gravity

	QCD	Gravity
Fundamental Degree of Freedom	$A_\mu \sim p_\mu$	$h_{\mu\nu} \sim \frac{p_\mu p_\nu}{\lambda}$
Field-strength / curvature	$F_{\mu\nu} \sim \partial A$	$R^{\mu}{}_{\nu\alpha\beta} \sim \partial^2 h$
Gauge Symmetry	$SU(3)$	Diff(M)
Coupling Dimensionful?	no	yes

Two Sources of Inhomogeneity

- In full theory: gauge charges P^{μ} and coupling κ are inhomogeneous in λ
  - Leads to relations for higher-order terms to form geometric objects
  - This is different from QCD – gauge charges have no scaling in λ
- From multipole expansion: evaluate soft fields at x_{-}.
  - Conceptually the same as in gauge theory
  - Deal with it in similar fashion

SCET Gravity Construction

- Minimally-coupled scalar field

$$\mathcal{L} = \frac{1}{2}\sqrt{-g}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

- Perform κ expansion in collinear sector $g_{\mu\nu} = g_{s\mu\nu} + \kappa h_{\mu\nu}$
  - Collinear graviton $h_{\mu\nu}$ in presence of soft dynamical background $g_{s\mu\nu}$
  - Duplicate soft and collinear gauge symmetry, not yet homogeneous

- Field content $h_{\mu\nu}$, $g_{s\mu\nu} = \eta_{\mu\nu} + \kappa s_{\mu\nu}$, ϕ_c, ϕ_s

- Introduce collinear light-cone gauge $\mathfrak{h}_{+\mu} = 0$
  - $h_{+\mu}$ only appear in Wilson lines
  - Controls dangerous scaling $h_{++} \sim \lambda^{-1}$, $h_{+\perp} \sim \lambda^0$

    [Beneke, Kirilin 1207.4926; Okui, Yunesi 1710.07685, + Chakraborty 1910.10738]

Soft Multipole-expansion

- Homogeneous gauge symmetry: linear transformations in $(x - x_-)$

$$x^{\mu\prime} = x^\mu + \varepsilon^\mu(x_-) + \omega^\mu{}_\nu(x_-)(x - x_-)^\nu + \mathcal{O}(\varepsilon^2)$$

where $\omega_{\mu\nu} = \frac{1}{2}(\partial_\mu \varepsilon_\nu - \partial_\nu \varepsilon_\mu)$

- Specify a new soft background field $\hat{g}_{s\mu\nu}$
  - Light-cone generalisation of Riemann Normal Coordinates

- Should treat ε^μ and $\omega_{\mu\nu}$ as *independent* parameters

- Two fields appear in covariant derivative

- Systematically achieved by analogue of Wilson lines

Main Takeaway

"Homogeneous" symmetry in Gravity consists of local translations and local Lorentz transformations. This implies a covariant derivative that contains the momentum as well as the Lorentz generators. All other interaction terms are expressed via Riemann tensor and its derivative.

Schematically, the scalar-soft graviton Lagrangian takes the form

$$\mathcal{L}_{\phi\phi s} = \frac{1}{2} n_+\partial\phi n_- D_s \phi + \frac{1}{2}\partial_\perp\phi\partial_\perp\phi - \frac{\kappa}{8} x_\perp^\alpha x_\perp^\beta R_{\alpha-\beta-} n_+\phi n_+\phi + \mathcal{O}(\lambda^3)\,,$$

where

$$n_- D_s = n_-\partial - \underbrace{\frac{\kappa}{2} s_{-\alpha}\partial^\alpha}_{\text{from vierbein}} - \underbrace{\frac{\kappa}{4}(\partial_\alpha s_{\beta-} - \partial_\beta s_{\alpha-})}_{\text{from spin-connection}} J^{\alpha\beta} + \mathcal{O}(s^2)$$

$$J^{\alpha\beta} = (x - x_-)^\alpha \partial^\beta - (x - x_-)^\beta \partial^\alpha$$

This is the transparent form we wanted, similar to QCD.

Observations

- The operator basis contains no soft building blocks up to
  - $F^s_{\mu\nu} \quad \sim \lambda^4 \sim k_s^2$ in QCD
  - $R^s_{\mu\nu\alpha\beta} \sim \lambda^6 \sim k_s^3$ in Gravity

- Any contribution to soft emission up to this order has to stem from the Lagrangian interactions, and is thus universal.

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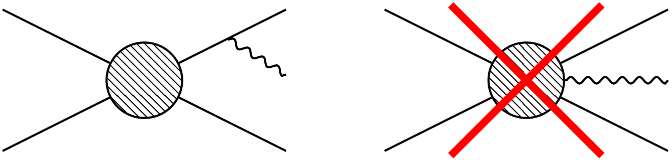

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- This universality implies the soft theorem.

- The covariant derivative explains why there are three terms for Gravity.
  - First two are from $n_- D$
  - Third term is from $R_{\mu\nu\alpha\beta}$

Comparison of Soft Theorems

- Recall LBK and Soft Theorem

$$\mathcal{A}^{\gamma}_{\text{rad}} = -g \sum_i Q_i \left(\varepsilon_\mu \frac{p_i^\mu}{p_i \cdot k} + \frac{k_\nu \varepsilon_\mu J_i^{\mu\nu}}{p_i \cdot k} \right) \mathcal{A}_0$$

$$\mathcal{A}^{h}_{\text{rad}} = \frac{\kappa}{2} \sum_i \left(\left(p_i^\nu \varepsilon_{\nu\mu} + J_i^{\nu\rho} k_\rho \varepsilon_{\nu\mu} \right) \frac{p_i^\mu}{p_i \cdot k} + J^{\mu\rho} \frac{1}{2} \frac{k_\rho k_\sigma \varepsilon_{\mu\nu} J^{\nu\sigma}}{p_i \cdot k} \right) \mathcal{A}_0$$

- First terms explained by covariant derivative

$$(n_- D_s)_{\mathsf{QCD}} = n_- \partial - i g n_- A_s^a Q^a$$

$$(n_- D_s)_{\mathsf{grav}} = n_- \partial - \frac{\kappa}{2} s_{-\nu} \partial^\nu - \frac{\kappa}{4} \omega_{\nu\rho} J^{\nu\rho}$$

- Soft decoupling in Gravity analogous to QCD
  [Bauer, Pirjol, Stewart hep-ph/0109045]

Conclusion and Outlook

- Derived rigorously SCET for Gravity to subleading order
- Transparent structure of two-fold gauge symmetry of soft Gravity
  local translations and local Lorentz symmetry shows form of the soft theorem
- No soft graviton building blocks up to $\mathcal{O}(\lambda^6)$ in the operator basis implies universality of soft theorem
- Beautiful interpretation of the soft theorem based on gauge symmetry
- Similarity of LBK and soft theorem can be understood due to universal part of SCET – multipole expansion and homogeneous gauge symmetry
- Use these principles to extend the EFT to non-relativistic sources and describe radiation

Auxiliary Slides

Full Lagrangian

$$\mathcal{L}^{(0)} = \frac{1}{2}\sqrt{-\hat{g}_s}\hat{g}_s^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\,,$$

$$\mathcal{L}^{(1)} = \frac{1}{2}\sqrt{-\hat{g}_s}\left(-\hat{g}_s^{\mu\alpha}\hat{g}_s^{\nu\beta}h_{\alpha\beta} + \frac{1}{2}\hat{g}_s^{\alpha\beta}h_{\alpha\beta}\hat{g}_s^{\mu\nu}\right)\partial_\mu\phi\partial_\nu\phi\,,$$

$$\mathcal{L}^{(2)} = \frac{1}{2}\sqrt{-\hat{g}_s}\left(\hat{g}_s^{\mu\alpha}\hat{g}_s^{\nu\beta}\hat{g}_s^{\rho\sigma}h_{\alpha\rho}h_{\beta\sigma} - \frac{1}{2}\hat{g}_s^{\alpha\beta}h_{\alpha\beta}\hat{g}_s^{\mu\rho}\hat{g}_s^{\nu\sigma}h_{\rho\sigma} + \frac{1}{8}(\hat{g}_s^{\alpha\beta}h_{\alpha\beta})^2 \right.$$
$$\left. - \frac{1}{4}\hat{g}_s^{\mu\alpha}g_s^{\nu\beta}h_{\mu\nu}h_{\alpha\beta}\right)\partial_\mu\phi\partial_\nu\phi\,,$$

$$\mathcal{L}_R^{(2)} = \frac{1}{2}\left[\partial_\alpha W^{-1}\phi\right]\left[\partial_\beta W^{-1}\phi\right]\Big(\det\left(R^\mu{}_\alpha\right)\left[R^{-1}\sqrt{-g_s}\right]R_\mu{}^\alpha R_\nu{}^\beta\left[R^{-1}g_s^{\mu\nu}(x)\right]$$
$$-\sqrt{-\hat{g}_s}\hat{g}_s^{\alpha\beta}\Big)$$

Metric in FLNC

- Fixed-line normal coordinates

$$x'^{\mu} = x^{\mu} + (E^{\mu}{}_{A} - \delta^{\mu}_{A})(x-x_{-})^{A} - \frac{1}{2}(x-x_{-})^{A}(x-x_{-})^{B}E^{\alpha}{}_{A}E^{\beta}{}_{B}\Gamma^{\mu}{}_{\alpha\beta}$$
$$+ \frac{1}{6}(x-x_{-})^{A}(x-x_{-})^{B}(x-x_{-})^{C}E^{\alpha}{}_{A}E^{\beta}{}_{B}E^{\nu}{}_{C}(2\Gamma^{\mu}{}_{\alpha\lambda}\Gamma^{\lambda}{}_{\beta\nu} - [\partial_{\nu}\Gamma^{\mu}{}_{\alpha\beta}])$$
$$+ \mathcal{O}(x^{3}),$$

- Dressed metric field

$$\tilde{g}_{ab}(x) = \frac{\partial x'^{\mu}}{\partial x^{a}}\frac{\partial x'^{\nu}}{\partial x^{b}}\left(1 + \theta^{\alpha}\partial_{\alpha} + \frac{1}{2}\theta^{\alpha}\theta^{\beta}\partial_{\alpha}\partial_{\beta} + \dots\right)g_{\mu\nu}(x)$$

- Background Metric field

$$\hat{g}_{ab}(x) = \eta_{ab},$$
$$\hat{g}_{a-}(x) = e_{a-} - y^{A}[\omega_{-}]_{Aa},$$
$$\hat{g}_{--}(x) = (e_{-}{}^{A} - y^{R}\omega_{-R}{}^{A})(e_{-}{}^{B} - y^{S}\omega_{-S}{}^{B})\eta_{AB}.$$

Simplified Lagrangian

$$\mathcal{L}^{(0)} = \frac{1}{2}\partial_+\phi D_-\phi + \frac{1}{2}\partial_{\alpha_\perp}\phi\partial^{\alpha_\perp}\phi$$

$$\mathcal{L}_h^{(1)} = -\frac{1}{2}\mathfrak{h}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \frac{1}{4}\mathfrak{h}^{\alpha_\perp}{}_{\alpha_\perp}\left(\partial_+\phi D_-\phi + \partial_{\alpha_\perp}\phi\partial^{\alpha_\perp}\phi\right)$$

$$\mathcal{L}^{(2)} = -\frac{1}{8}x_\perp^\alpha x_\perp^\beta R_{\alpha-\beta-}(\partial_+\phi)^2 + \frac{1}{8}s_{+-}\left(\partial_+\phi D_-\phi + \partial_{\alpha_\perp}\phi\partial^{\alpha_\perp}\phi\right)$$

$$\mathcal{L}_h^{(2)} = \frac{1}{2}\mathfrak{h}^{\mu\alpha}\mathfrak{h}^\nu_\alpha\partial_\mu\phi\partial_\nu\phi + \frac{1}{16}\left((\mathfrak{h}^{\alpha_\perp}_{\alpha_\perp})^2 - 2\mathfrak{h}^{\alpha\beta}\mathfrak{h}_{\alpha\beta}\right)\left(\partial_+\phi D_-\phi + \partial_{\mu_\perp}\phi\partial^{\mu_\perp}\phi\right)$$
$$+ \frac{1}{4}\mathfrak{h}^{\mu\alpha}s_{\alpha-}\partial_+\phi\partial_\mu\phi$$