# REVISITING THE CKM PARADIGM

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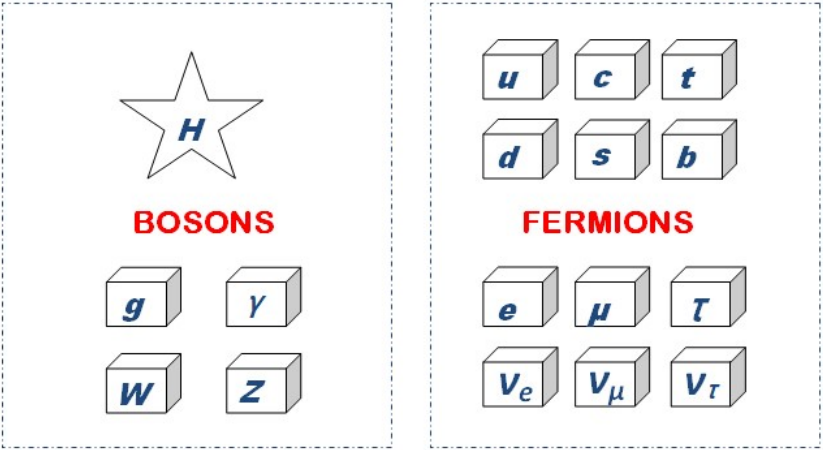
# Standard Model of Particle Physics



# INTRODUCTION

Symmetries are indispensable in the theory of Particle Physics. Once an accepted symmetry is found violated, it is more intriguing as it is a reminder that there exist underlying hidden concepts to cognize. One of the very important symmetries having deep implications is the $CP$ symmetry which is characterized by the product of charge conjugation symmetry($C$) and the spatial inversion symmetry - Parity($P$). $CP$V entails an intrinsic distinction between matter and antimatter, which can help us unravel the mystery of matter dominance in the present day universe.

# CKM Paradigm

In the Standard Model, at present the only way to accommodate $CP$V seems the Cabibbo-Kobayashi-Maskawa (CKM) mechanism, which dictates mixing via CKM matrix. Here, the $d$, $s$ and $b$ quarks are mixed via the CKM matrix to give $d'$, $s'$ and $b'$ eigenstates, respectively, as:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} .$$

The irremovable phase of the $V_{CKM}$ allows $CP$ violation in the SM and the origin of the phase is the complex quark mass matrices.

# Standard Parametrization

Thus, $V_{CKM}$ is the quark mixing matrix connecting mass eigenstates to the weak eigenstates. The $V_{CKM}$ can be expressed in terms of three real angles and one non-trivial phase which is responsible for $CP$ violation in the SM.

# Standard Parametrization

Thus, $V_{CKM}$ is the quark mixing matrix connecting mass eigenstates to the weak eigenstates. The $V_{CKM}$ can be expressed in terms of three real angles and one non-trivial phase which is responsible for $CP$ violation in the SM. The standard parametrization used by Particle Data Group (PDG) for the representation of the $V_{CKM}$ is given by

$$V_{CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}.$$

where $s_{ij} \equiv \sin\theta_{ij}$ and $c_{ij} \equiv \cos\theta_{ij}$

# CPV in CKM Paradigm

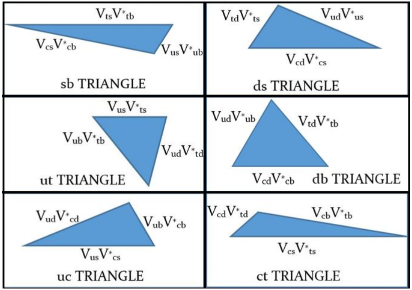
Unitarity of the CKM matrix ($VV^\dagger = I$) implies

$$\sum_{\alpha=d,s,b} V_{i\alpha} V^*_{j\alpha} = \delta_{ij}$$

$$\sum_{i=u,c,t} V_{i\alpha} V^*_{i\beta} = \delta_{\alpha\beta}$$

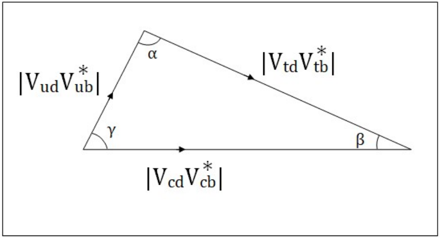
$$\begin{aligned}
sb \quad & V_{us}V^*_{ub} + V_{cs}V^*_{cb} + V_{ts}V^*_{tb} = 0, \\
ds \quad & V_{ud}V^*_{us} + V_{cd}V^*_{cs} + V_{td}V^*_{ts} = 0, \\
ut \quad & V_{ud}V^*_{td} + V_{us}V^*_{ts} + V_{ub}V^*_{tb} = 0, \\
db \quad & V_{ud}V^*_{ub} + V_{cd}V^*_{cb} + V_{td}V^*_{tb} = 0, \\
uc \quad & V_{ud}V^*_{cd} + V_{us}V^*_{cs} + V_{ub}V^*_{cb} = 0, \\
ct \quad & V_{cd}V^*_{td} + V_{cs}V^*_{ts} + V_{cb}V^*_{tb} = 0.
\end{aligned}$$

which represents six unitarity triangles in the complex plane



# The reference triangle

$$V_{ud}V_{ub}^{*} + V_{cd}V_{cb}^{*} + V_{td}V_{tb}^{*} = 0.$$



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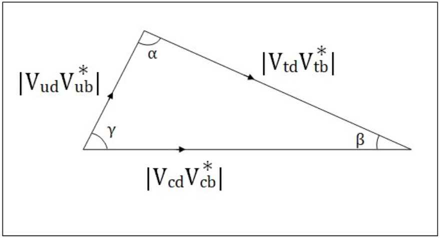


$$\alpha \equiv arg\left[-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right], \ \beta \equiv arg\left[-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right], \ \gamma \equiv arg\left[-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right].$$

# Unitarity based analysis

$$\begin{aligned}
\beta &\equiv arg\left[-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right] \\
= \quad arg \quad & \left[\frac{(s_{12}c_{23}+c_{12}s_{23}s_{13}e^{i\delta})(s_{23}c_{13})}{(s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta})(c_{23}c_{13})}\right] \\
= \quad arg \quad & [\frac{s_{23}(s_{12}^2c_{23}s_{23}-c_{12}^2s_{23}c_{23}s_{13}^2-s_{12}c_{12}s_{13}(c_{23}^2-s_{23}^2)\cos\delta)}{c_{23}(s_{12}^2s_{23}^2+c_{12}^2c_{23}^2s_{13}^2-2s_{12}s_{23}c_{12}c_{23}s_{13}\cos\delta)} \\
& +\frac{i\ s_{23}(s_{12}c_{12}s_{13}\sin\delta)}{c_{23}(s_{12}^2s_{23}^2+c_{12}^2c_{23}^2s_{13}^2-2s_{12}s_{23}c_{12}c_{23}s_{13}\cos\delta)}] \\
= \quad \tan^{-1} \quad & [\frac{s_{12}c_{12}s_{13}\sin\delta}{s_{23}c_{23}(s_{12}^2-c_{12}^2s_{13}^2)-c_{12}s_{12}s_{13}(c_{23}^2-s_{23}^2)\cos\delta}]
\end{aligned}$$

# Unitarity based analysis(cont.)

$$\tan\beta = \frac{s_{12}c_{12}s_{13}\sin(\alpha+\beta)}{s_{23}c_{23}(s_{12}^2 - c_{12}^2 s_{13}^2) + c_{12}s_{12}s_{13}(c_{23}^2 - s_{23}^2)\cos(\alpha+\beta)}$$

## Approximation

$s_{12}^2 >> c_{12}^2 s_{13}^2$ and $s_{23}^2 << c_{23}^2$

$$\tan\beta = \frac{s_{12}c_{12}s_{13}\sin(\alpha+\beta)}{s_{23}c_{23}(s_{12}^2) + c_{12}s_{12}s_{13}(c_{23}^2)\cos(\alpha+\beta)}$$

# Similar analysis with $\gamma$

$$\tan\gamma = \frac{s_{12}c_{23}(2\tan\frac{\delta}{2})}{s_{12}c_{23}(1-\tan^2\frac{\delta}{2}) + c_{12}s_{23}s_{13}(1+\tan^2\frac{\delta}{2})}$$

$$\delta = \gamma + \sin^{-1}\left(\sin\gamma\frac{c_{12}s_{23}s_{13}}{s_{12}c_{23}}\right)$$

$$\delta \simeq \gamma$$

# Input

$$
\begin{aligned}
V_{us} &= 0.2245 \pm 0.0008 \quad [PDG], \\
\alpha &= (84.9 \pm 5.1)^\circ \qquad [PDG], \\
\beta &= (22.2 \pm 0.7)^\circ \qquad [HFLAV], \\
V_{cb} &= (42.2 \pm 0.8) \times 10^{-3} \; [PDG].
\end{aligned}
$$

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Special case :

$$\alpha = 90^\circ?$$

# Implications on $V_{ub}$

$$|V_{ub}| = \frac{s_{12}s_{23}\sin\beta}{c_{12}\cos\alpha}$$

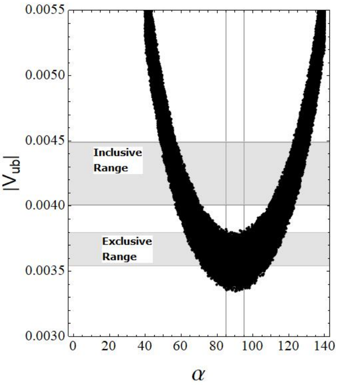
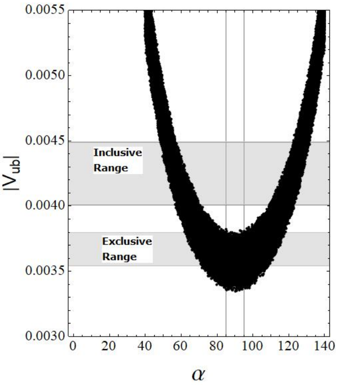



Figure: $V_{ub}$ versus $\alpha$

# Implications on $V_{ub}$

$$|V_{ub}| = \frac{s_{12}s_{23}\sin\beta}{c_{12}\cos\alpha}$$

$$|V_{ub}| = 0.00358 \pm 0.00016.$$

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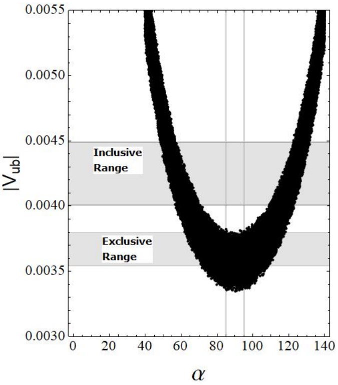
EXCLUSIVE DECAYS

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EXCLUSIVE DECAYS

$$\left|\frac{V_{ub}}{V_{cb}}\right| = 0.087 \pm 0.003$$

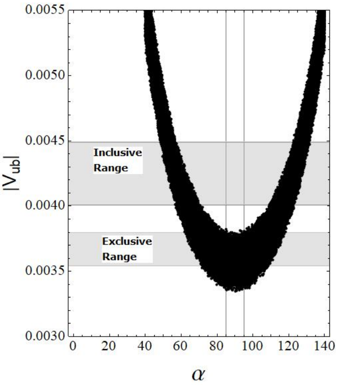


Figure: $V_{ub}$ versus $\alpha$

# Completing the CKM picture

$$\begin{aligned}\gamma &= \pi - \alpha - \beta = (72.9 \pm 5.1)^\circ, \quad \gamma(SC) = (67.8 \pm 0.7)^\circ, \\ \gamma(PDG) &= (72.1^{+4.1}_{-4.5})^\circ\end{aligned}$$

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$$V_{CKM} = \begin{pmatrix} 0.97451(11) & 0.2243(5) & 0.00367(13) \\ 0.22415(49) & 0.97364(11) & 0.0422(8) \\ 0.00876(14) & 0.04144(78) & 0.99910(3) \end{pmatrix}$$

$$V_{CKM}(SC) = \begin{pmatrix} 0.97447(18) & 0.2245(8) & 0.00358(16) \\ 0.22435(80) & 0.97364(19) & 0.0410(14) \\ 0.00883(42) & 0.0402(14) & 0.99915(6) \end{pmatrix}$$

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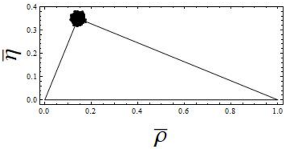
\end{aligned}
$$



# COMPLETING THE CKM PICTURE

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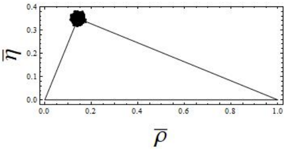


| Expression | Our value | Special case | PDG value |
|---|---|---|---|
| $J$ | $(3.07 \pm 0.25) \times 10^{-5}$ | $(3.13 \pm 0.14) \times 10^{-5}$ | $(3.18 \pm 0.15) \times 10^{-5}$ |
| $\lvert\epsilon_K\rvert$ | $0.00215 \pm .00035$ | $0.00223 \pm .00025$ | $0.002228 \pm .000011$ |
| $\left\lvert\frac{\Delta m_d}{\Delta m_s}\right\rvert$ | $0.0321 \pm 0.0025$ | $0.0297 \pm 0.0009$ | $0.0285 \pm 0.0001$ |

# Concluding Remark

- We have done a unitarity based analysis and also considered the possibility of a right angled unitarity triangle,
  1. $|V_{ub}|$ – exclusive
  2. Value of the $\delta$, J, $V_{CKM}$, $|\epsilon_K|$, $\left|\frac{\Delta m_d}{\Delta m_s}\right|$
- The CKM paradigm evaluated is very well consistent with the present experimental values by PDG(2020), meanwhile increasing the precision to a great extent, in case the UT angle $\alpha$ is exactly right.

# THANK YOU

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# Extra Slides–1

references where RUTs are discussed
Z. Xing, Phys.Lett.B **679**, 111 (2009).
Z. Xing, Phys. Rev. D **86**, 113006 (2012).
S. Antusch, C. Hohl, C. K. Khosa and V. Susič, J. High Energ. Phys. 25 (2018). S. Antusch, M. Holthausen, M.A. Schmidt and M. Spinrath, Nucl. Phys. B **877**, 752 (2013).
Z. Xing and J. Zhu, Nucl. Phys. B **908**, 302 (2016).
Y. Mimura, arXiv:1805.07773v2.
P.F. Harrison, D.R.J. Roythorne and W.G. Scott, arXiv:0904.3014v1 [hep-ph]

# Extra Slides–2

| | |
|---|---|
| $\beta \equiv \phi_1$ | $(22.2 \pm 0.7)^\circ$ |
| $\alpha \equiv \phi_2$ | $(84.9\,^{+5.1}_{-4.5})^\circ$ |
| $\gamma \equiv \phi_3$ | $(71.1\,^{+4.6}_{-5.3})^\circ$ |

FIGURE: hflav data

| | |
|---|---|
| α [deg] | 91.7 [+1.7 -1.1] |
| α [deg] (meas. not in the fit) | 91.8 [+2.7 -1.0] |
| α [deg] (dir. meas.) | 86.4 [+4.5 -4.3] \|\| -1.8 [+4.3 -5.1] |
| β [deg] | 22.56 [+0.47 -0.40] |
| β [deg] (meas. not in the fit) | 23.7 [+1.3 -1.2] |
| β [deg] (dir. meas.) | 22.14 [+0.69 -0.67] |
| γ [deg] | 65.80 [+0.94 -1.29] |
| γ [deg] (meas. not in the fit) | 65.66 [+0.90 -2.65] |
| γ [deg] (dir. meas.) | 72.1 [+5.4 -5.7] |

FIGURE: ckmfitter data

| | |
|---|---|
| $\alpha[^\circ]$ | $93.3 \pm 5.6$ and $166.6 \pm 0.6$ |
| $\beta[^\circ]$ | – |
| $\gamma[^\circ]$ | $-109.9 \pm 4.2$ and $70.0 \pm 4.2$ |

FIGURE: utfit data

# Extra Slides–3

Exclusive decays:
select a particular charmless state.
provides a better background rejection
$X \to a + - - -$

Inclusive decay:
Integrate over all charmless states.
provides higher signal efficiency.
$X \to a + b + c$

Buras- 0.0864(25)
HFLAV- 0.080(4)(4)
LHCb- 0.083(4)(4).

# Extra Slides–4

$$\left|\epsilon_K\right| = \frac{\kappa_\epsilon G_f^2 m_W^2 m_K f_k^2 b_k x_{SD}}{(12\sqrt{2}\pi^2 \Delta m_k)}$$

where

$x_{SD} = (\eta_{tt} f[x_t] Im[(V_{ts} V_{td}^*)^2] + 2\eta_{ct} f[x_c, x_t] Im[V_{cs} V_{cd}^* V_{ts} V_{td}^*] + \eta_{cc} x_c Im[(V_{cs} V_{cd}^*)^2])$

$\left|\frac{\Delta m_d}{\Delta m_s}\right| = \frac{m_{B_d} f_b^2 |V_{td}|^2}{(m_{B_s} f_s^2 |V_{ts}|^2)}$