Gravitational waves from first-order phase transition during inflation

Haipeng An (Tsinghua University)

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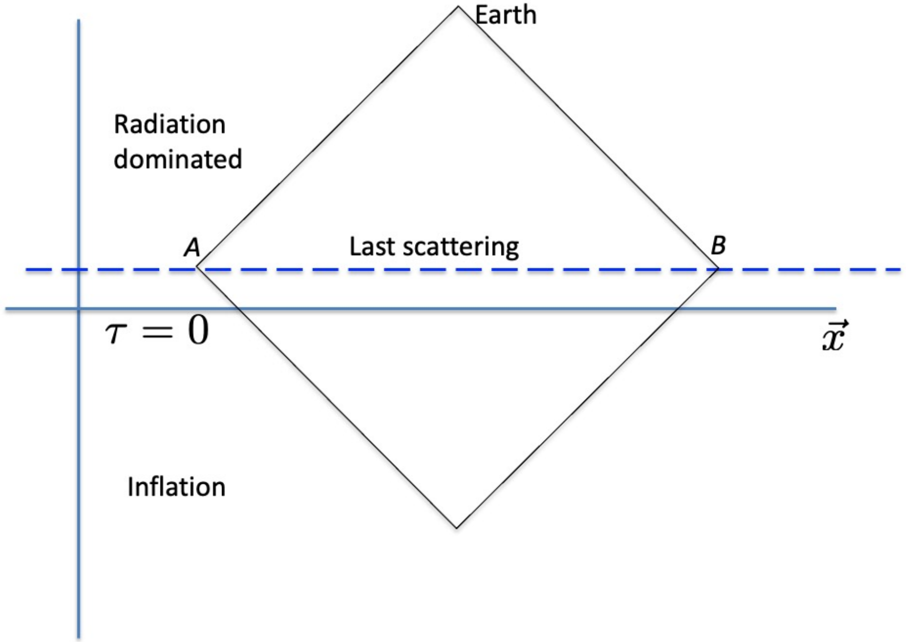
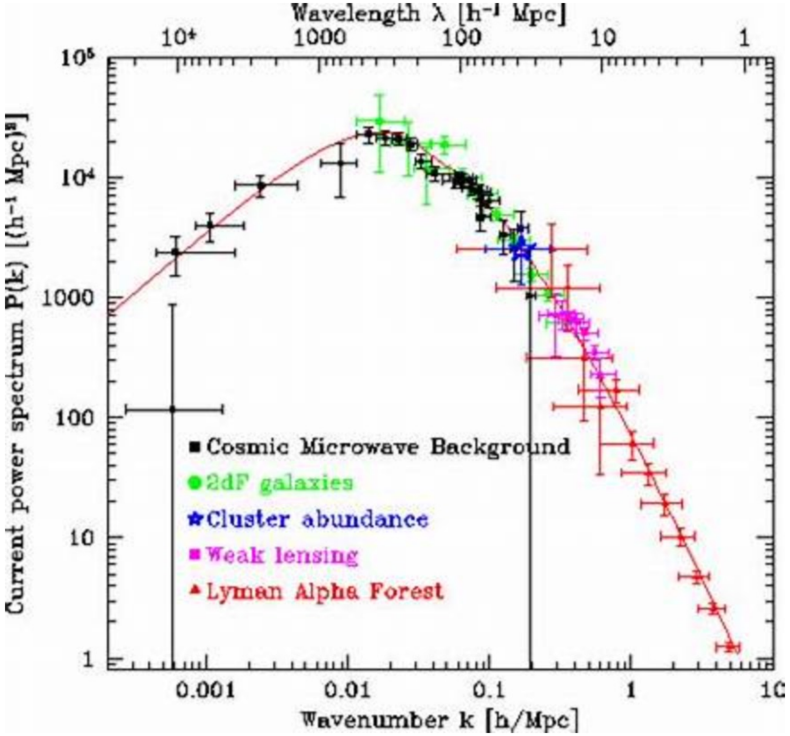
In collaboration with Kun-Feng Lyu, Lian-Tao Wang and Siyi Zhou

2009.12381, 211X.XXXXX

Very brief history of our universe

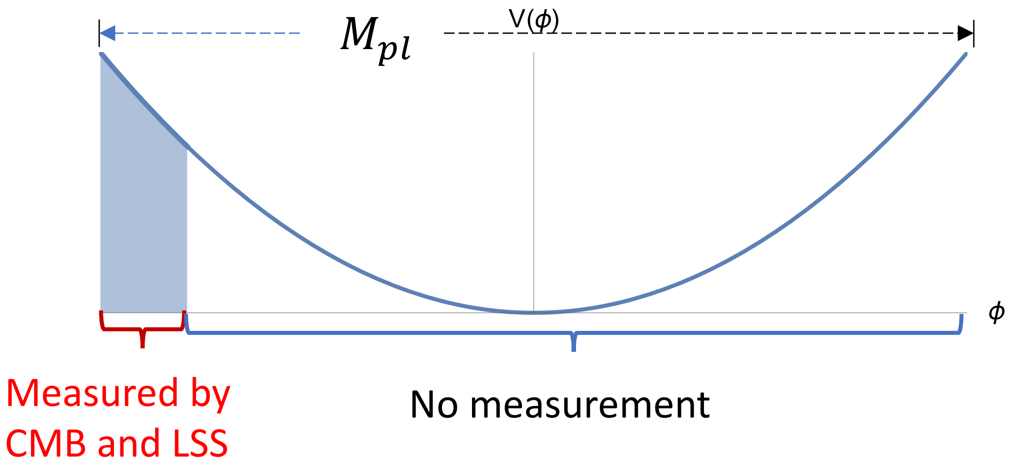




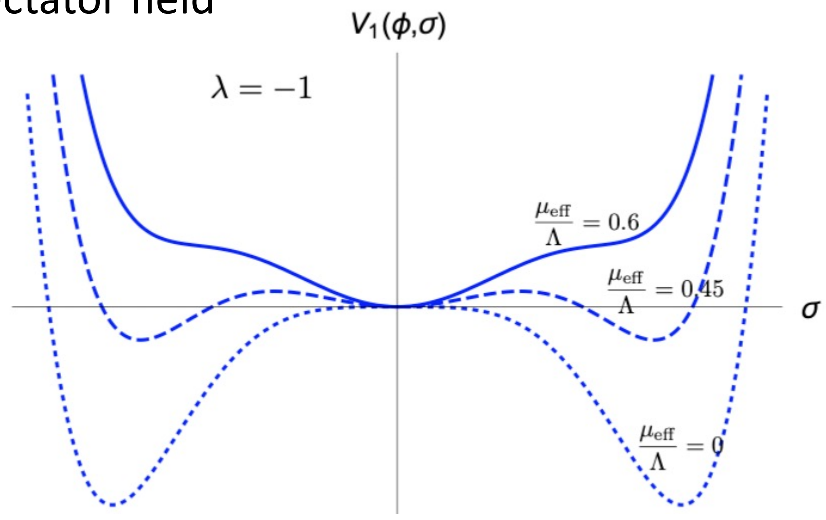
- To solve the problems, 40 to 60 e-folds is required, BUT we can only observe ten!

First order phase transition driven by the evolution of the inflaton

- ϕ: inflaton field



σ: spectator field



$$V_1(\phi,\sigma) = -\frac{1}{2}(\mu^2 - c^2\phi^2)\sigma^2 + \frac{\lambda}{4}\sigma^4 + \frac{1}{8\Lambda^2}\sigma^6$$

- It is generic to expect the inflaton to couple to some spectator sector.
- The masses or couples in the spectator sector can be changed drastically due to the evolution of the inflaton field.
- It may induce first order phase transitions in the spectator sector so that classical gravitational waves might be generated.

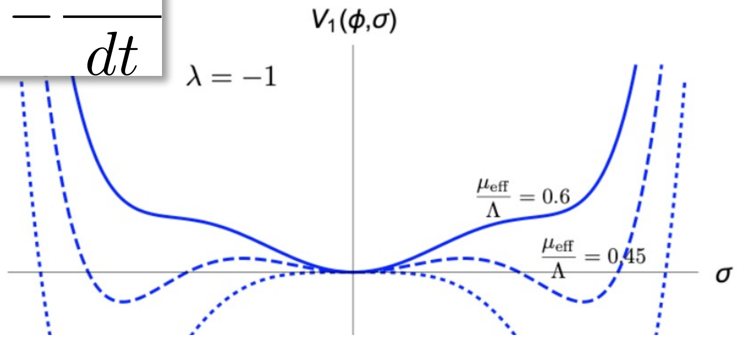
First order phase transition during inflation

- Bubble nucleation rate: $\frac{\Gamma}{V} = I_0 m_\sigma^4 e^{-S_4}$
- Phase transition starts: $\mathcal{O}(1) = \int_{-\infty}^{t} dt' H^{-3} I_0 m_\sigma^4 e^{-S_4(t')}$
- The bounce: $S_4 \sim \log\left(\frac{\phi H}{\dot{\phi}} \frac{m_\sigma^4}{H^4}\right) \sim \log\left(\frac{\phi}{\epsilon^{1/2} M_{\rm pl}} \frac{m_\sigma^4}{H^4}\right)$
- First order phase transiton: $S_4 \gg 1$

$$H^4 \ll m_\sigma^4 \ll 3 M_{\rm pl}^2 H^2$$

First order phase transition during inflation

$$\beta = -\frac{dS_4}{dt}$$

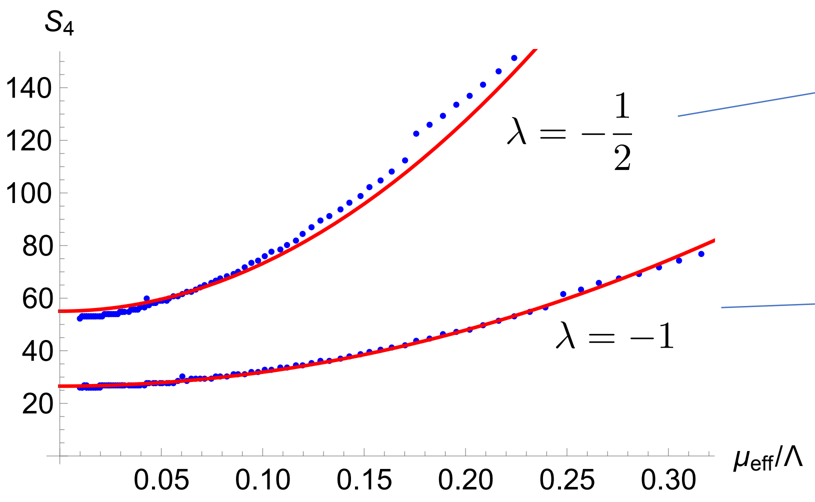


$$V_1(\phi,\sigma) = -\frac{1}{2}(\mu^2 - c^2\phi^2)\sigma^2 + \frac{\lambda}{4}\sigma^4 + \frac{1}{8\Lambda^2}\sigma^6$$

$$\frac{\beta}{H} = \left|\frac{dS_4}{d\log\mu_{\rm eff}^2}\right|(2\epsilon)^{1/2} \times \frac{M_{\rm pl}}{\left|\phi\left(1-\frac{\mu^2}{c^2\phi^2}\right)\right|}$$

$$\sim \mu_{\rm eff}^2/\Lambda^2$$

$$\boxed{\int_{\phi_{\rm end}}^{\phi_{\rm PT}} \frac{d\phi}{\sqrt{2\epsilon}M_{\rm pl}} = N_{\rm e}}$$



$$\frac{\beta}{H} \sim \frac{3800}{N_e}$$

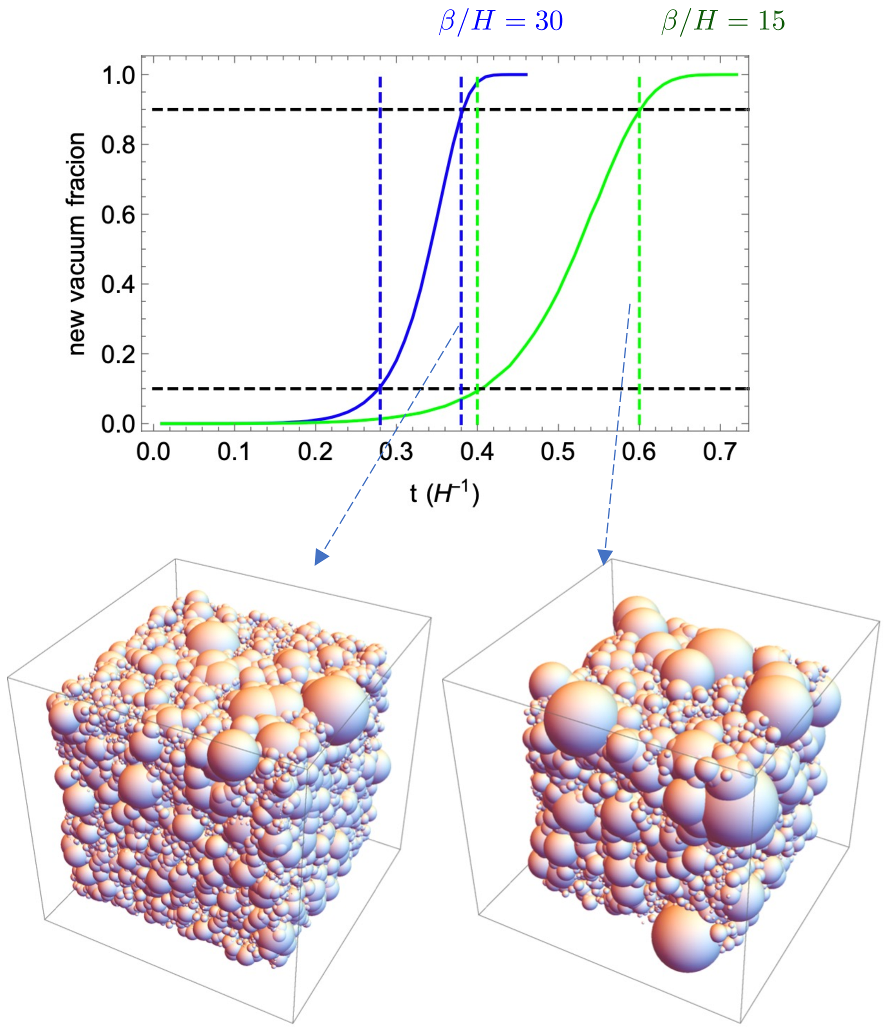
$$\frac{\beta}{H} \sim \frac{500}{N_e}$$

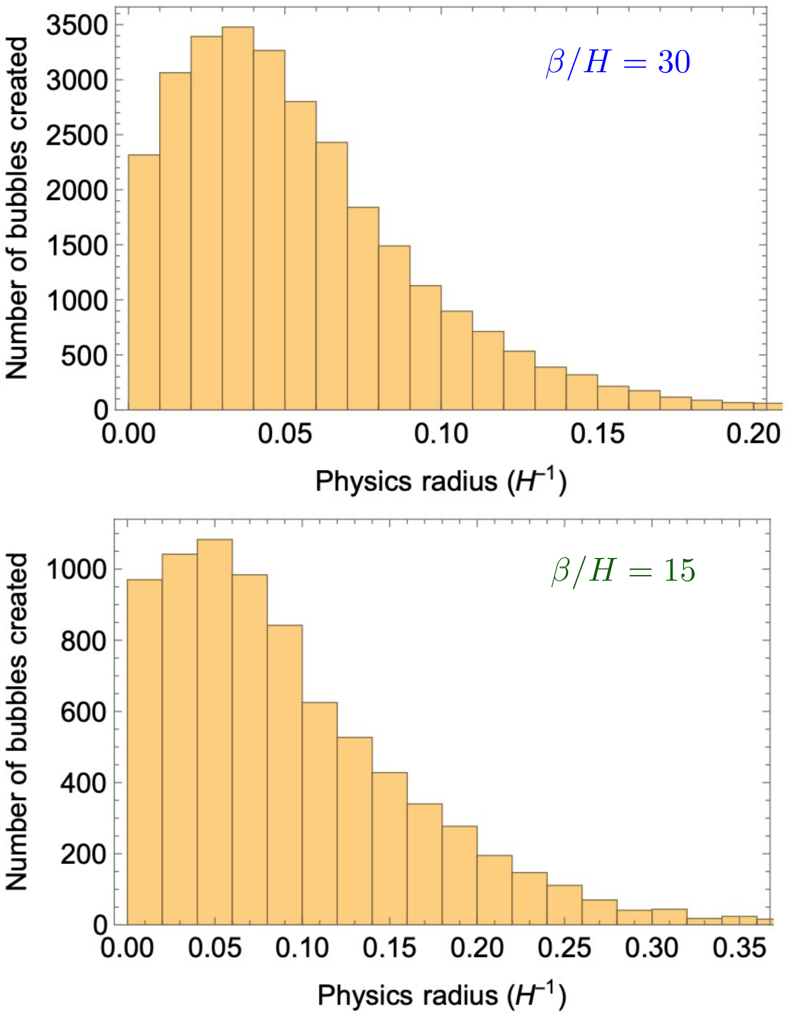
$$\frac{\beta}{H} \sim \mathcal{O}(10) - \mathcal{O}(100)$$

CosmoTransitions

Numerical results

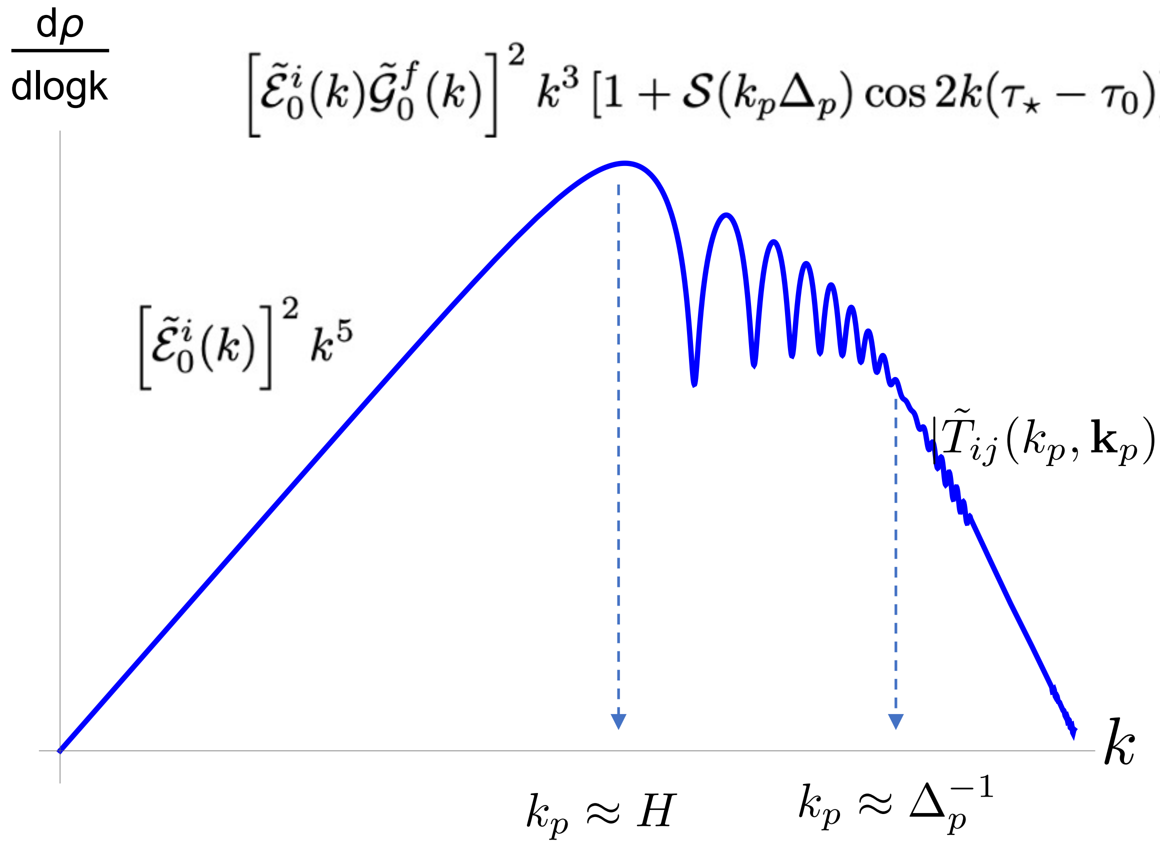
$$R_{\text{bubble}} \approx \beta^{-1} \ll H^{-1}$$





Result

- Generic shape of the GW spectrum produced by first order phase transition during inflation



How to calculate GW?

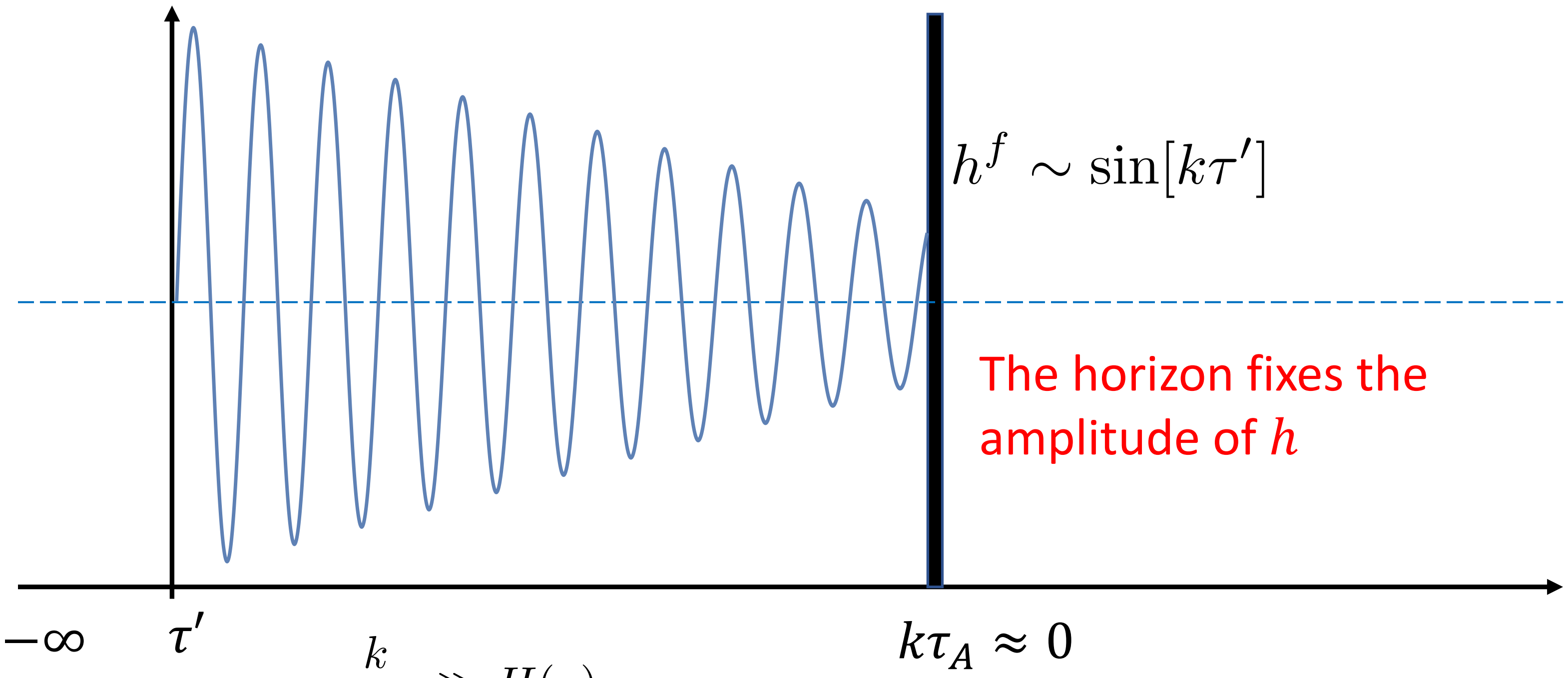
- In E&M: $\partial_\mu F^{\mu\nu} = J^\nu$
  - We solve the Green's function first.
  - We convolute the Green's function with the source.

- In GR: $G_{\mu\nu} = 8\pi G T_{\mu\nu}$ $\quad h''_{ij} + \frac{2a'}{a} h'_{ij} - \nabla^2 h_{ij} = 16\pi^2 G_N a^2 \sigma_{ij}$
  - Green's function in Fourier space

$$h''(\tau, \mathbf{k}) + \frac{2a'}{a} h'(\tau, \mathbf{k}) + k^2 h(\tau, \mathbf{k}) = 16\pi G_N a^{-1} T \delta(\tau - \tau')$$

GW from instantaneous and local sources (qualitative study)

- $$h''(\tau,\mathbf{k}) + \frac{2a'}{a}h'(\tau,\mathbf{k}) + k^2 h(\tau,\mathbf{k}) = 16\pi G_N a^{-1} T\delta(\tau-\tau')$$



$$\frac{k}{a(\tau)} \gg H(\tau)$$

$$h \sim \frac{\sin k(\tau-\tau')}{a(\tau)}$$

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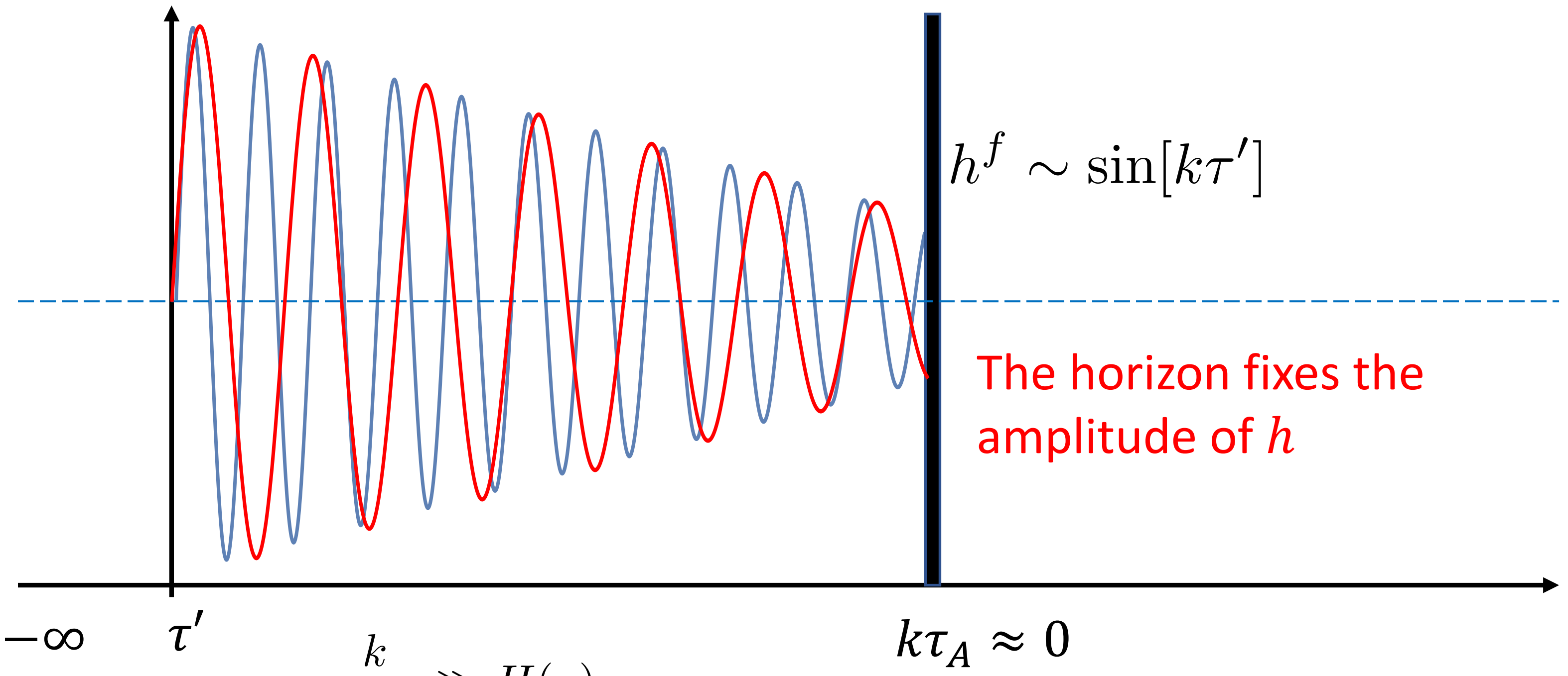
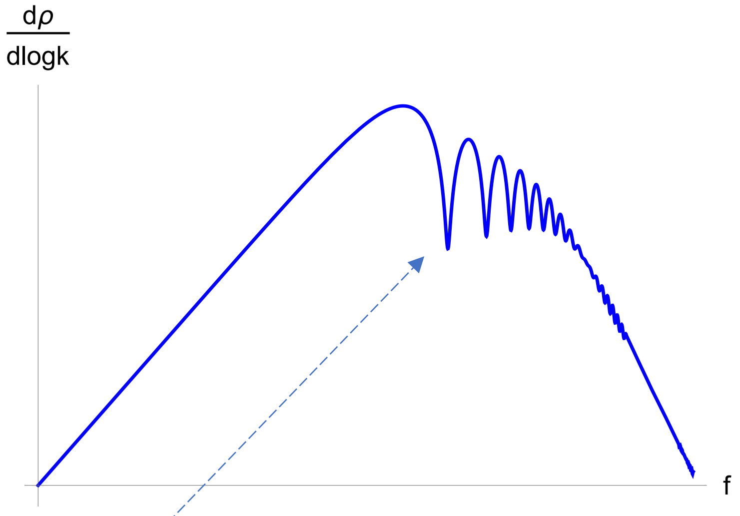


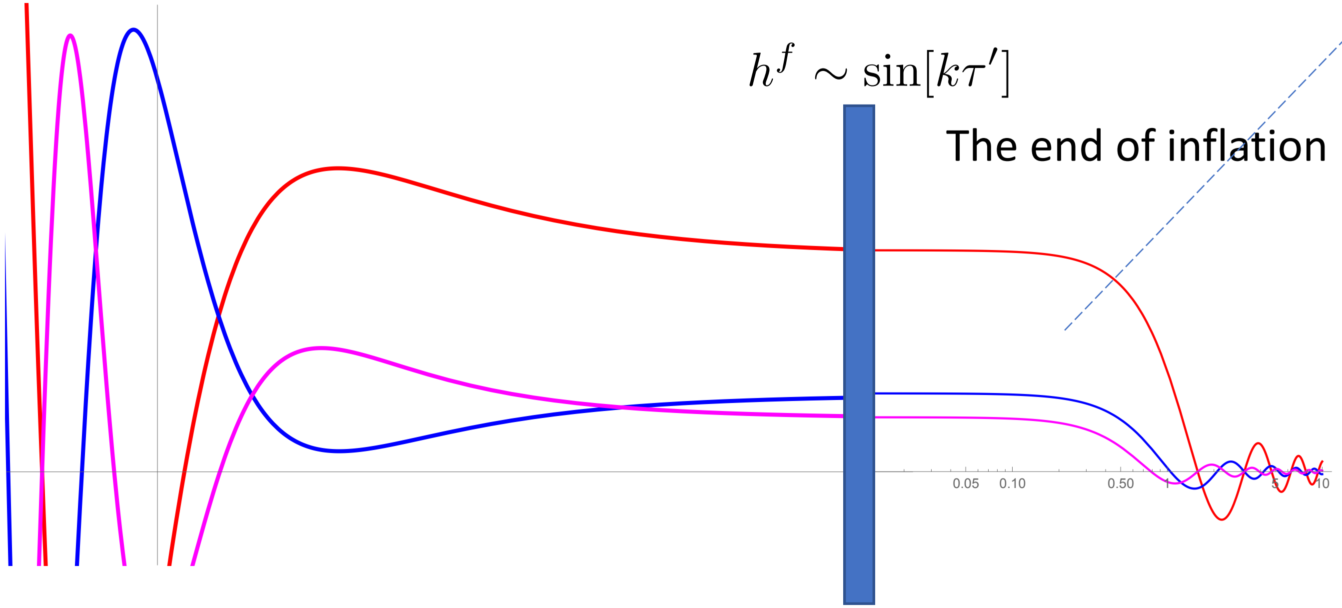
$$\frac{k}{a(\tau)} \gg H(\tau)$$

$$h \sim \frac{\sin k(\tau-\tau')}{a(\tau)}$$

After inflation

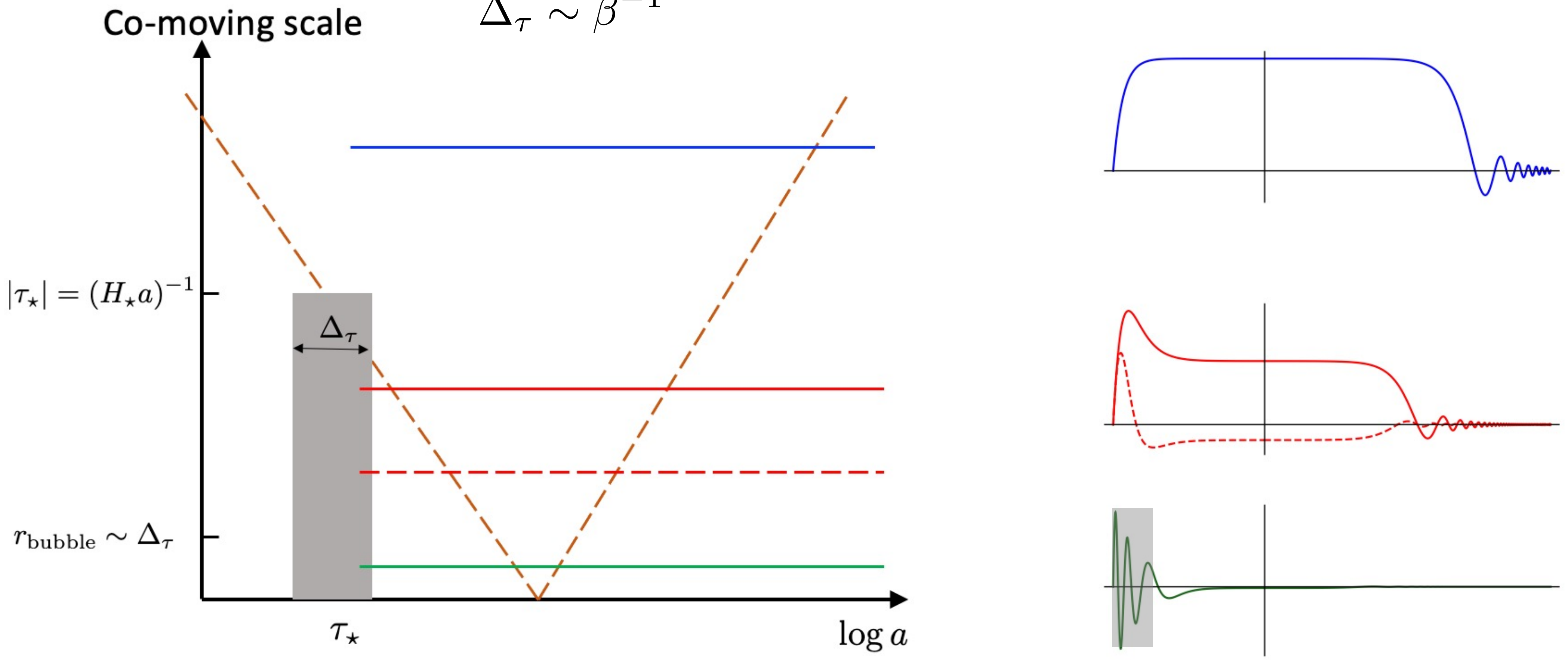
- $h^f(k)$ is the initial amplitude for the GW oscillation after inflation.
- All the modes start to oscillate with the same phase.
- Example, in RD, the oscillation is $\sin k\tau\,/k\tau$





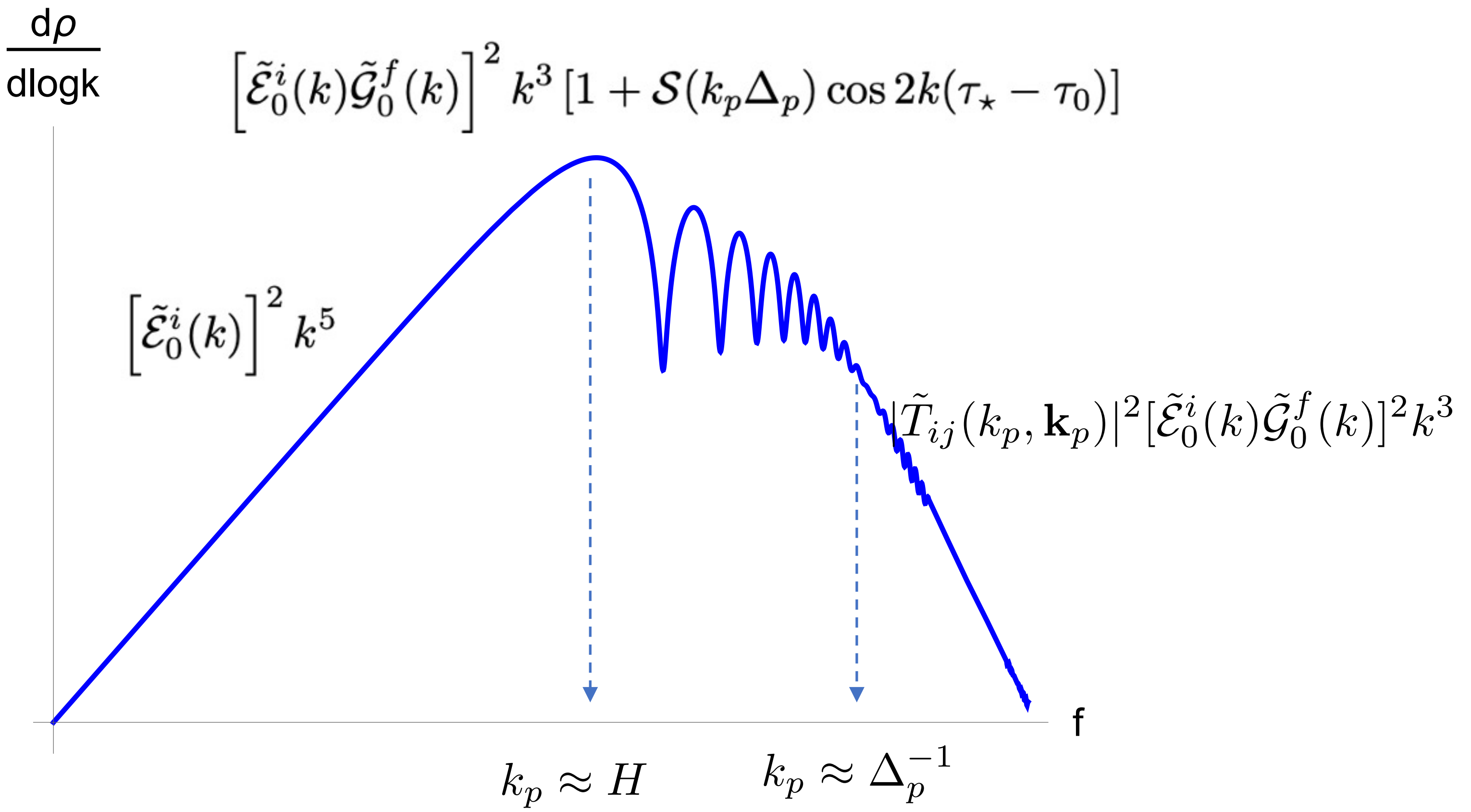
GW spectrum for a real source

$$\Delta_\tau \sim \beta^{-1}$$

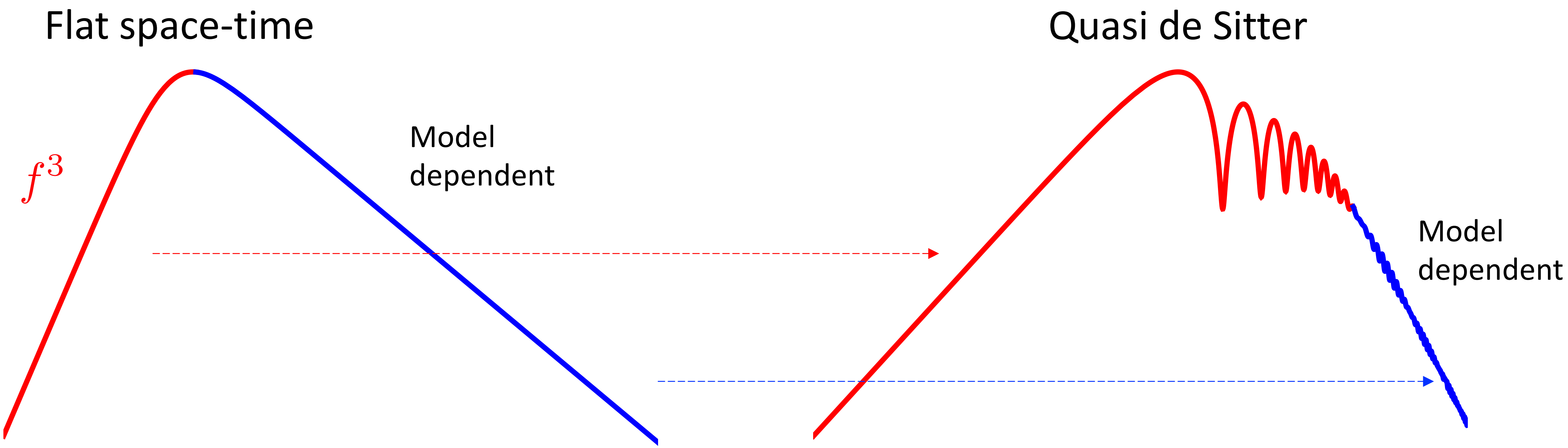


Generic features of GW spectrum

- Inflation models
  - de Sitter inflation $\tilde{\mathcal{G}}_0^f \sim \frac{1}{k}$
  - t^p inflation $\tilde{\mathcal{G}}_0^f \sim k^{\frac{p}{1-p}}$
- Evolution after inflation
  - In RD, $\tilde{\mathcal{E}}_0^i \sim k^{-1}$
  - In MD, $\tilde{\mathcal{E}}_0^i \sim k^{-2}$
  - In $t^{\tilde{p}}$, $\tilde{\mathcal{E}}_0^i \sim k^{\tilde{p}/(\tilde{p}-1)}$

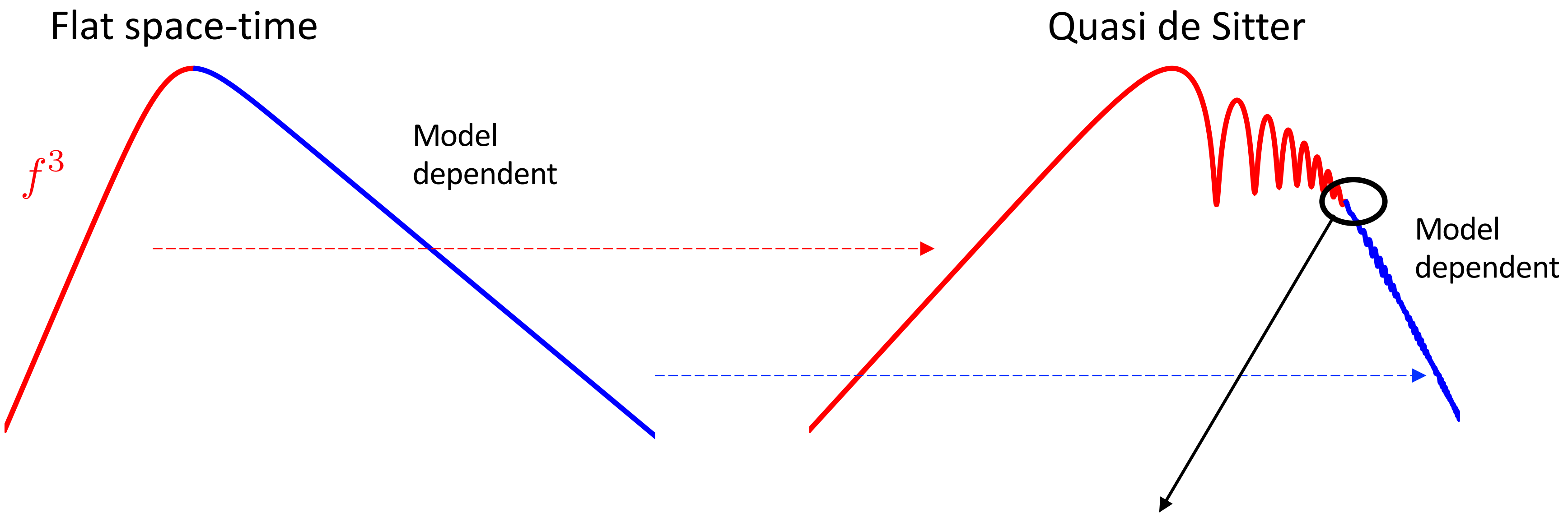


Spectrum distortion



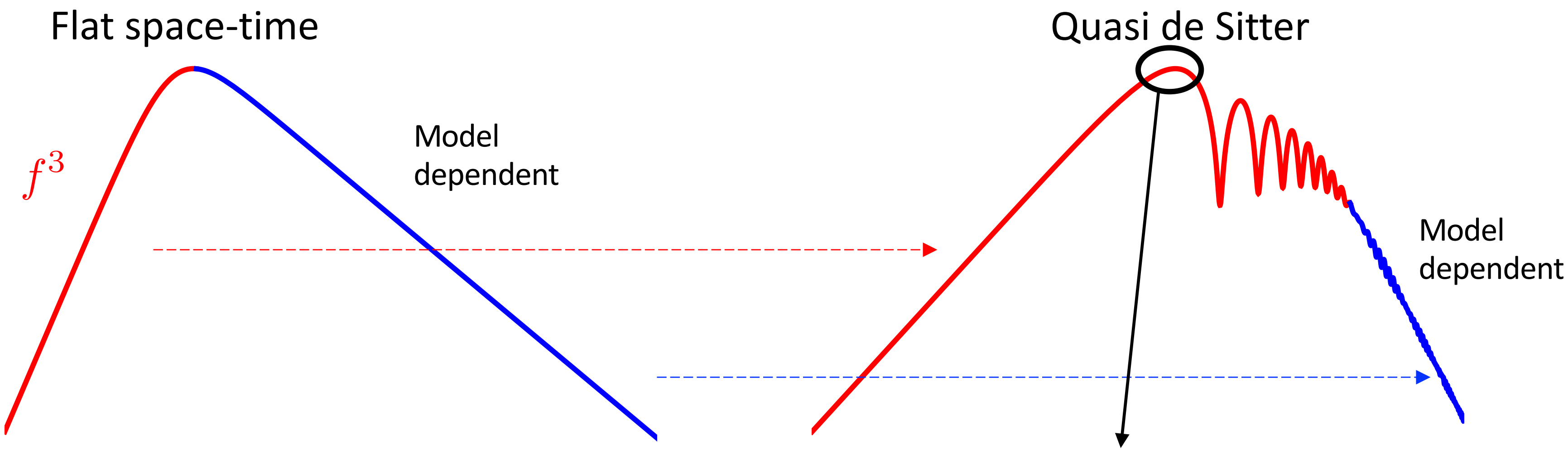
Cai, Pi, Sasaki, 1909.13728

Spectrum distortion by inflation



$$\Omega_{\rm GW} \approx \Omega_R \left(\frac{H_{\rm inf}}{\beta}\right)^6 \frac{\Delta\rho_{\rm vac}}{\rho_{\rm inf}}$$

Spectrum distortion by inflation



$$\Omega_{\rm GW} \approx \Omega_R \left(\frac{H_{\rm inf}}{\beta}\right)^5 \frac{\Delta\rho_{\rm vac}}{\rho_{\rm inf}}$$

$$\approx 10^{-11} \times \left(\frac{H_{\rm inf}}{0.1\beta}\right)^5 \frac{\Delta\rho_{\rm vac}}{0.1\rho_{\rm inf}}$$

$$\approx 10^{-16} \times \left(\frac{H_{\rm inf}}{0.01\beta}\right)^5 \frac{\Delta\rho_{\rm vac}}{0.1\rho_{\rm inf}}$$

GW Power spectrum

- Assume quasi-dS inflation, RD re-entering and fast reheating

$$\Omega_{\rm GW}(k_{\rm today}) = \Omega_R \frac{H_{\rm inf}^4}{k_p^4}\left[\frac{1}{2} + \mathcal{S}(k_p\beta^{-1})\cos\left(\frac{2k_p}{H_{\rm inf}}\right)\right]\frac{\Delta\rho_{\rm vac}}{\rho_{\rm inf}}\frac{d\rho_{\rm GW}^{\rm flat}}{\Delta\rho_{\rm vac}d\log k_p}$$

Dilution factor

Smearing

Suppressed by the energy faction

Flat space spectrum

Redshift

$$\frac{f_{\rm today}}{f_\star} = \frac{a(\tau_\star)}{a_1}\left(\frac{g_{*S}^{(0)}}{g_{*S}^{(R)}}\right)^{1/3}\frac{T_{\rm CMB}}{\left[\left(\frac{30}{g_*^{(R)}\pi^2}\right)\left(\frac{3H_{\rm inf}^2}{8\pi G_N}\right)\right]^{1/4}}$$

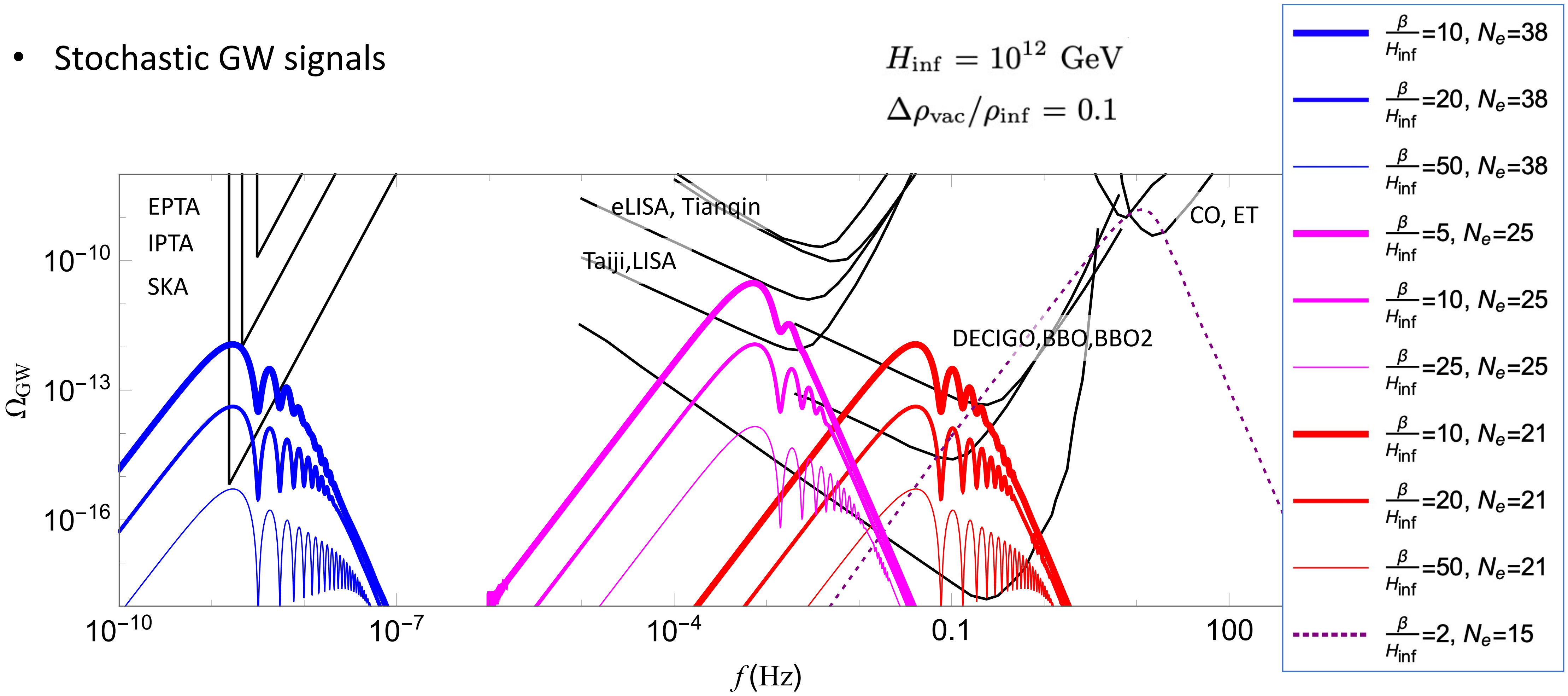
$$e^{-N_e}$$

N_e: e-folds before the end of inflation

First order phase transition during inflation

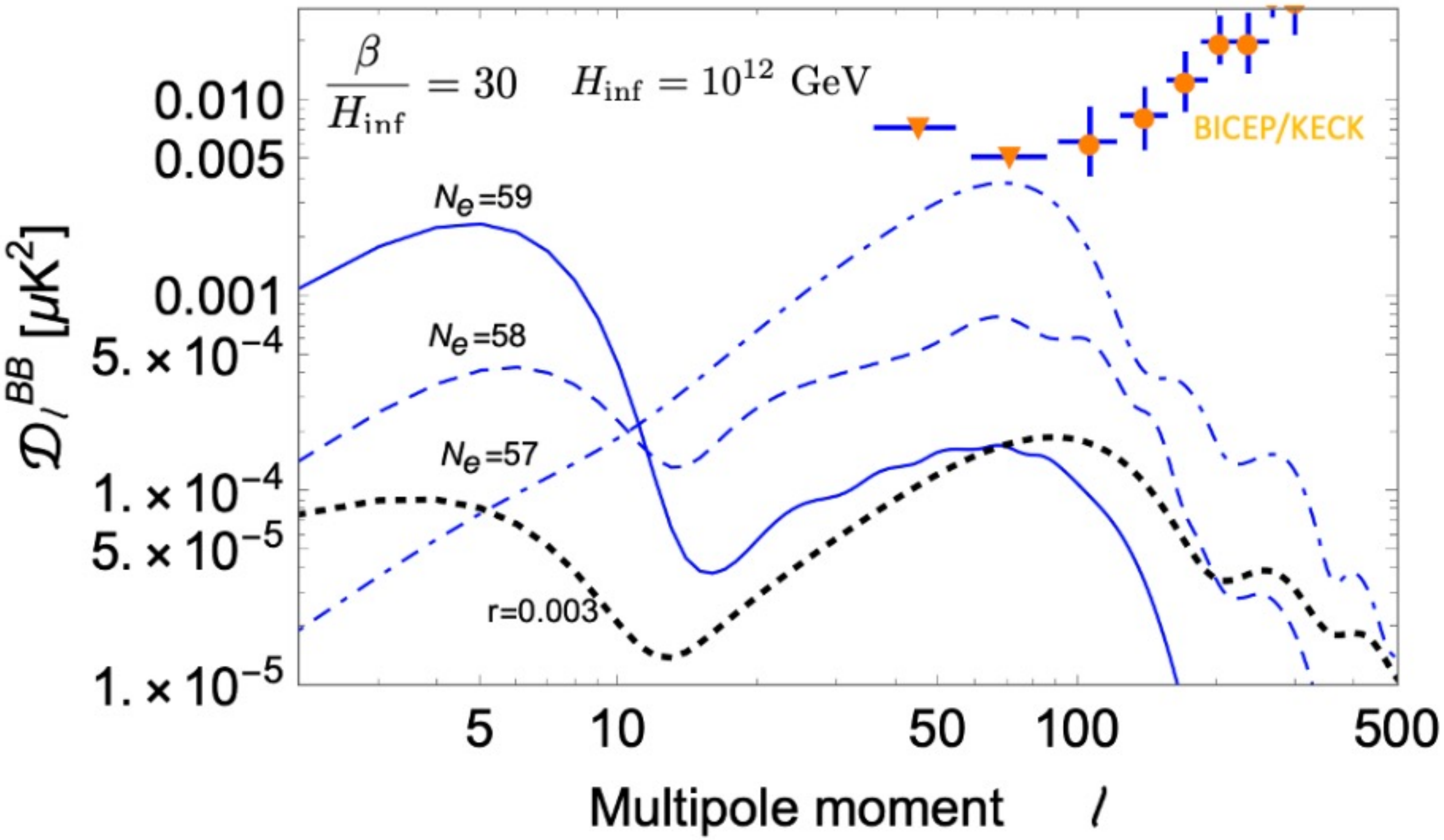
- Stochastic GW signals

$$H_{\rm inf} = 10^{12}\ \mathrm{GeV}$$
$$\Delta\rho_{\rm vac}/\rho_{\rm inf} = 0.1$$



First order phase transition during inflation

- CMB B modes



Summary

- We show that there is an oscillatory feature in the spectrum.
- The slopes of the spectrum can tell us information about the inflation model and evolution of the universe when the modes re-enter the horizon.
- First order phase transition during inflation can be realized with simple models.
- If we are lucky enough, such a signal can be detected by future GW detectors.