Towards a model-independent reconstruction approach for late-time Hubble data
(2106.08688 & 2105.12970)

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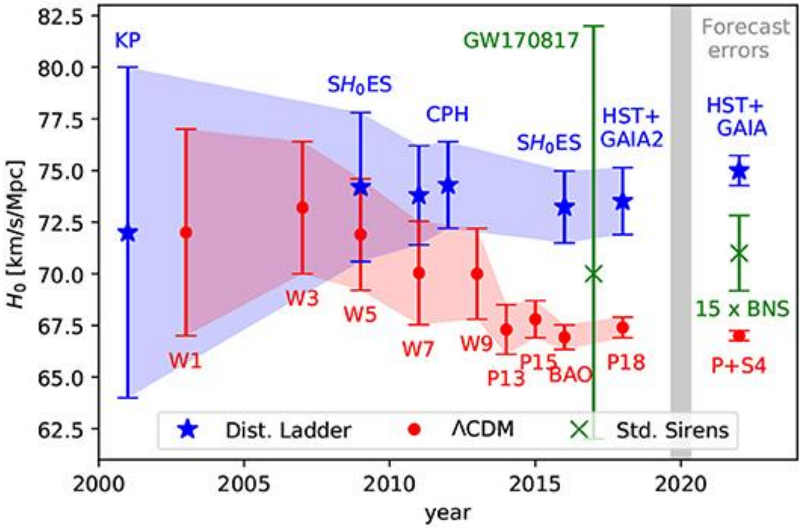
Outline

1. Motivation
2. Gaussian processes, late-time data
   - Basics, perks and quirks of GPs
   - Late-time Hubble data: CC, SNe
3. Kernel selection via evolutionary algorithms
   - 2 hyperparameters: Approximate Bayesian computation
   - 10 hyperparameters: Genetic algorithms
4. Outlook

Background: Modified gravity, The Hubble tension, All that



Hubble Tension. Ezquiaga, J. M., & Zumalacárregui, M. (2018). Dark energy in light of multi-messenger gravitational-wave astronomy. *Frontiers in Astronomy and Space Sciences*, *5*, 44.

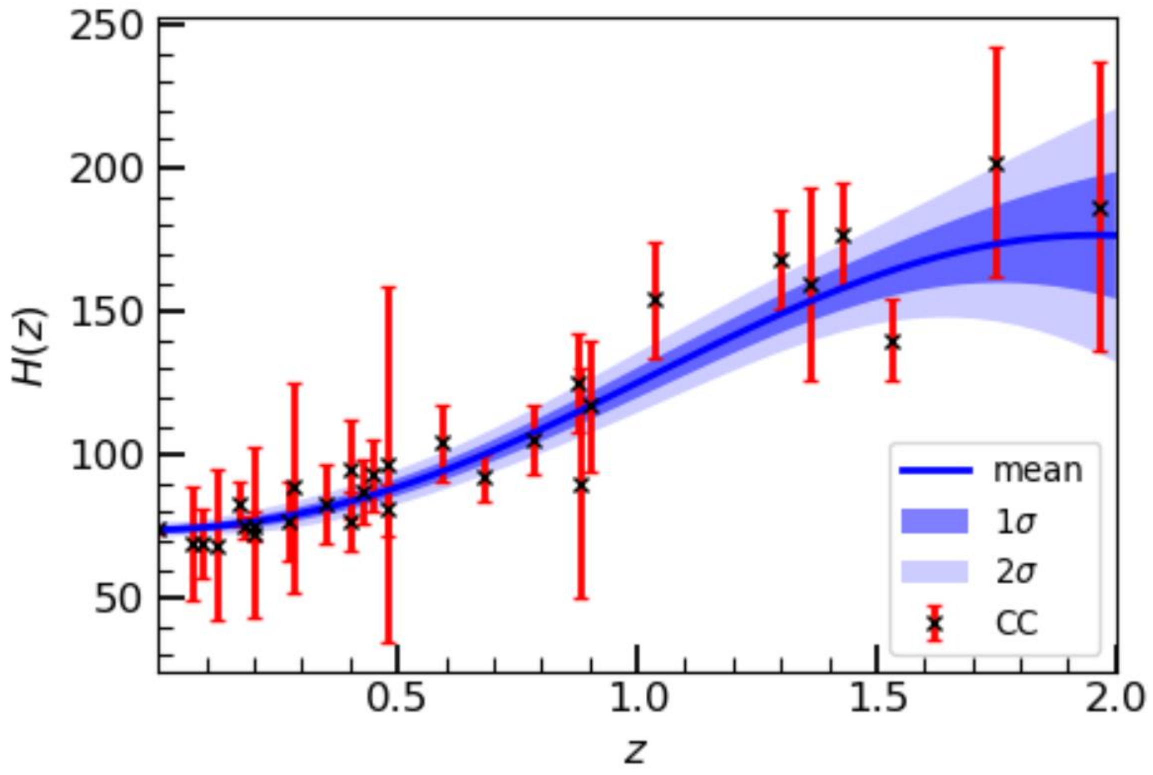
The solution?

- $f(R)$, Galileon ghost gondensate, generalized Galileons, Teleparallel Gravity, etc.
- Relaxed Cosmological Principle
- Interacting dark energy models
- ??
- ???

2103.01183, Di Valentino, *et al.*, In the Realm of the Hubble tension…

Gaussian processes + Late-time data

- *Nonparameteric* method for reconstructing observational data
- *Integrates* well with *Late-time Hubble data* (e.g., CC, SNe)
- Advantages:
  - (Cosmology) model-independent
  - Bayesian -> Mean + Uncertainty
  - Easy to implement
- Quirks:
  - Overfitting, underestimates unc.
  - **Kernel selection (2106.08688)**



GP Reconstruction of Cosmic Chronometers data set. Hubble function $H(z)$ as a function of the redshift z.

main

notebooks / supp_ntbks_arxiv.2106.08688 / gp_abcsmc_reconstruction.ipynb

reggiebernardo added 1702.00418 to CC references

Latest con

1 contributor

1016 lines (1016 sloc) | 523 KB

Gaussian processes with approximate Bayesian computation and sequential Monte for the reconstruction of late-time Hubble data

Investigation of kernel selection for the Gaussian process using various cosmological datasets (2106.08688). This notebook (https://pyabc.readthedocs.io/en/latest/) for approximate Bayesian computation with sequential Monte-Carlo for model selection. The analysis is s first one using cosmic chronometers and the second one using the compressed Pantheon samples.

References to the data can be found at the end of the notebook.

In [1]:
```
%matplotlib inline
import numpy as np
import matplotlib.pyplot as plt
import os
import tempfile
import scipy.stats as st
```

Kernel selection

- Radial basis function (RBF)

$$K(r) = A^2 e^{-r^2/2l^2}$$

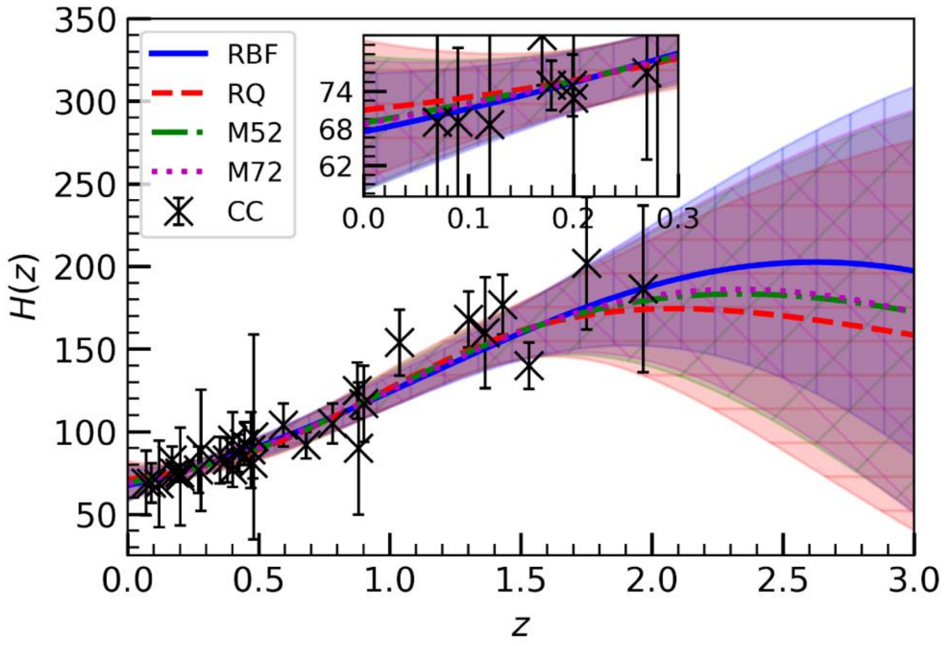
- Rational quadratic (RQ)

$$K(r) = A^2 \left(1 + \frac{r^2}{2\alpha l^2}\right)^{\alpha}$$

- Matern function (Mν)

$$K(r) = A^2 \frac{2^{1-\nu}}{\Gamma(\nu)} \left(\frac{\sqrt{2\nu r^2}}{l}\right)^{\nu} K_\nu\left(\frac{\sqrt{2\nu r^2}}{l}\right)$$

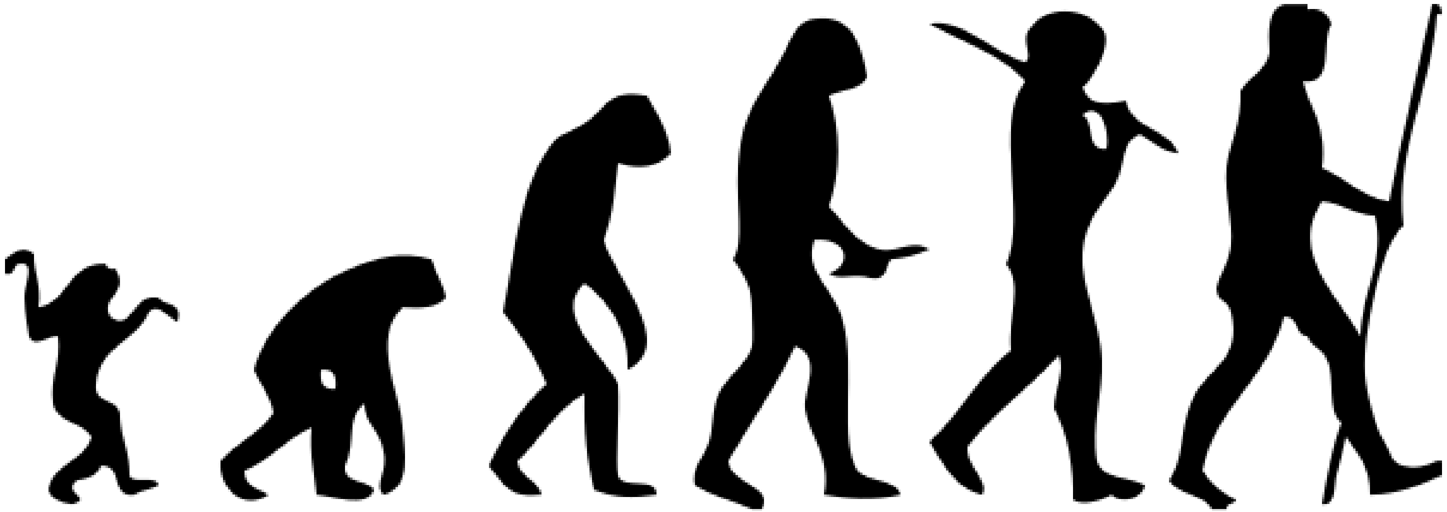
Kernel	H_0 $\left[\mathrm{km\ s^{-1} Mpc^{-1}}\right]$
RBF	67.4 ± 4.7
RQ	71.0 ± 5.6
M52	68.9 ± 5.4
M72	68.7 ± 5.2



How do we choose the kernel?

Based on: 2106.08688 & 2105.12970

Evolution (Accessed 04 Nov. 2021): https://upload.wikimedia.org/wikipedia/commons/thumb/c/c2/Human_evolution_scheme.svg/1212px-Human_evolution_scheme.svg.png

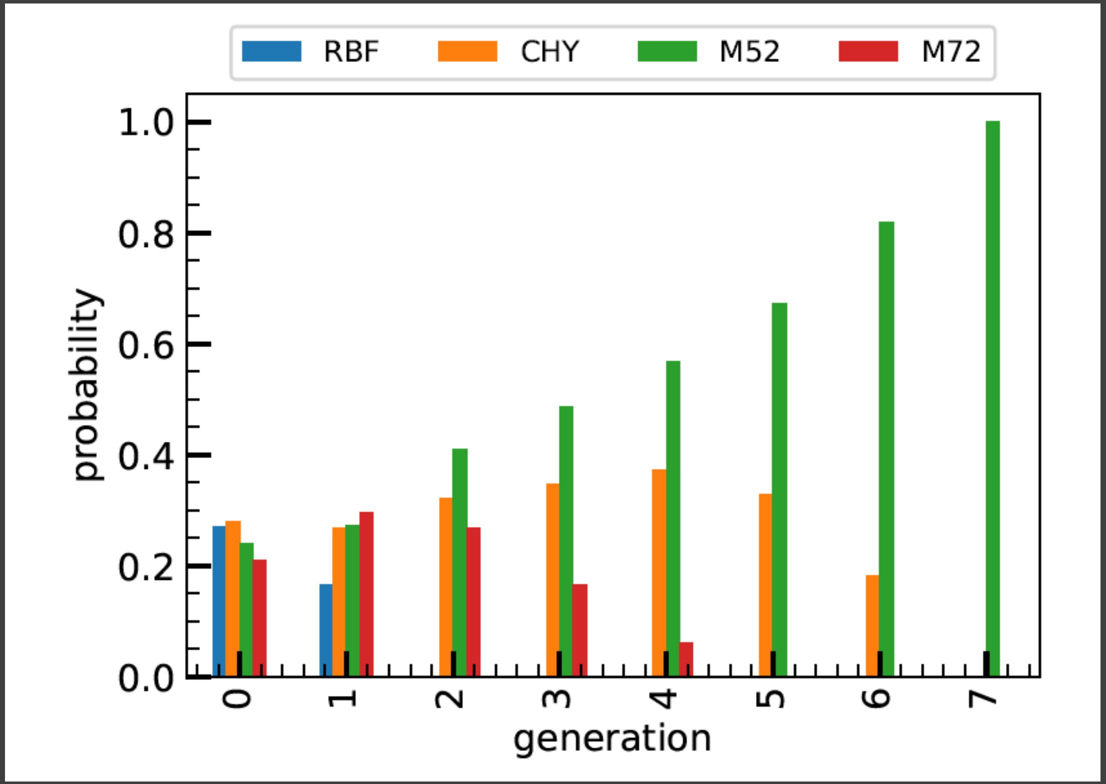


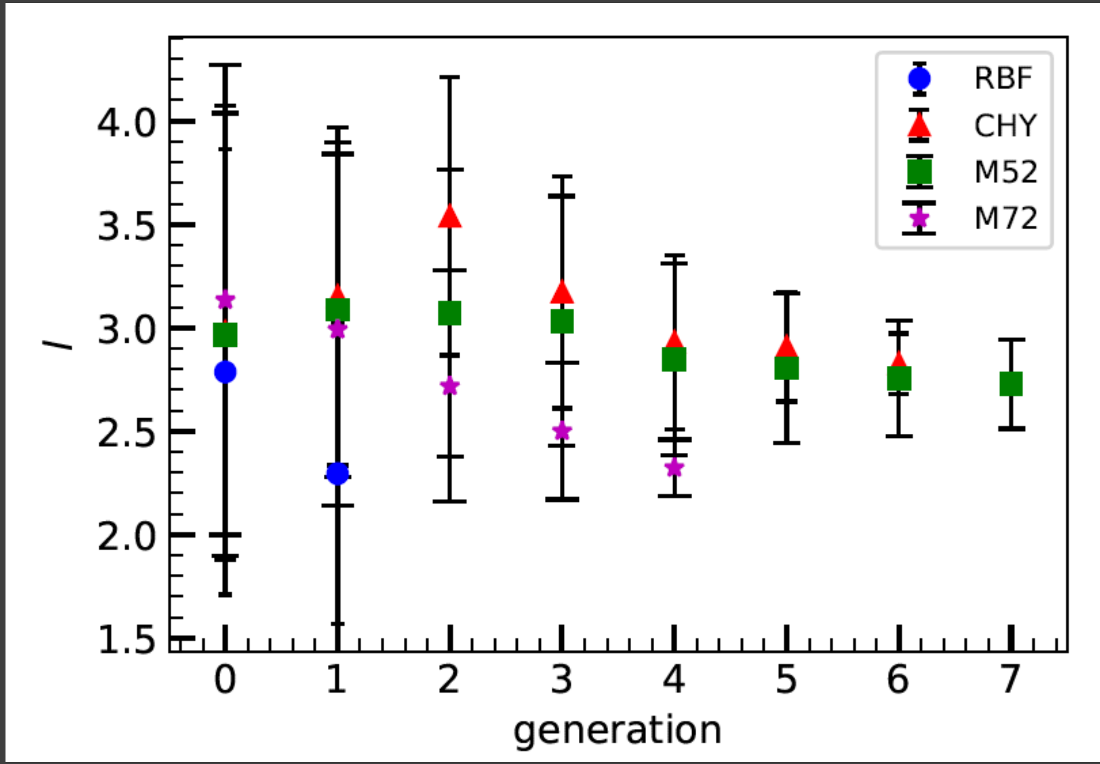
Approximate Bayesian computation

- The goal:
$$P(\theta|D,M) \propto L(D|\theta,M)$$
- *particle* $(M,\theta) \in$ *population*
- *distance function* $\Delta(R)$

if $\Delta(R) < \varepsilon$, pass

else, shall not pass





Kernel evolution via ABC-SMC

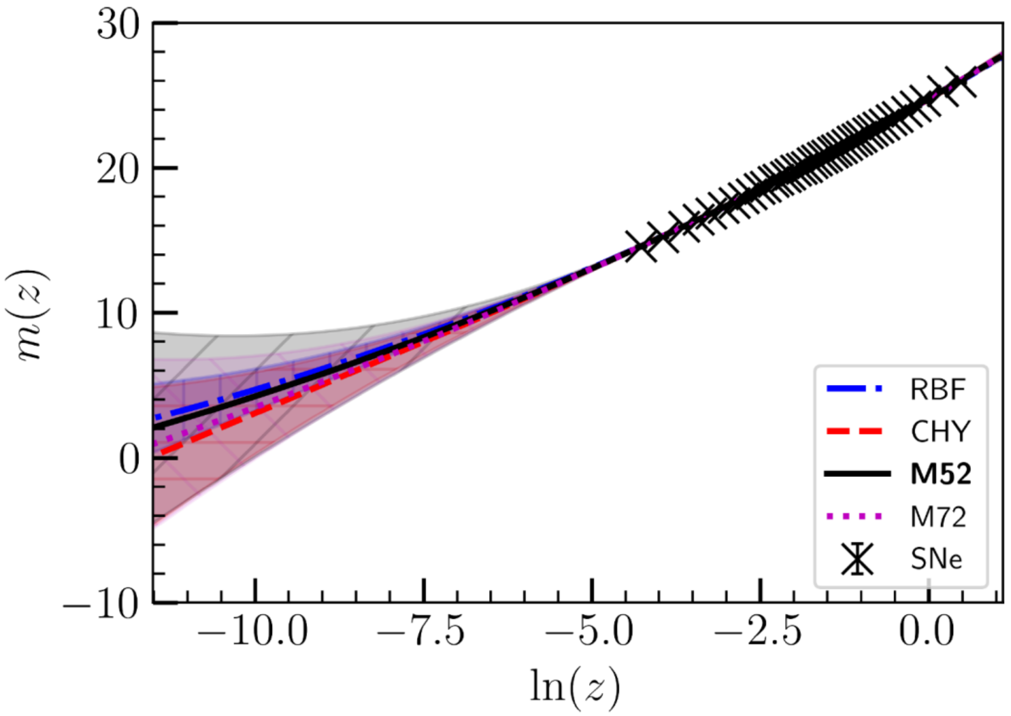
(**left**) *Kernel posteriors* in a joint kernel space per generation obtained using Approximate Bayesian Computation and

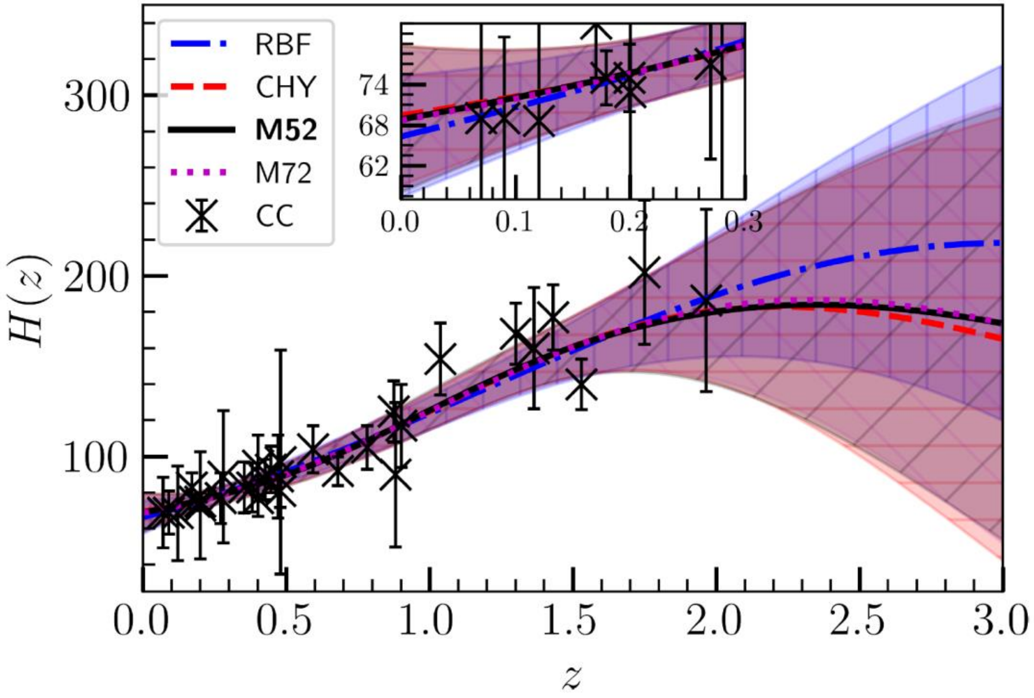
(**right**) the corresponding evolution of the hyperperameters per kernel.

GP reconstruction with ABC-SMC

(**left**) *Kernel posteriors* in a joint kernel space per generation obtained using Approximate Bayesian Computation and

(**right**) the corresponding evolution of the hyperperameters per kernel.





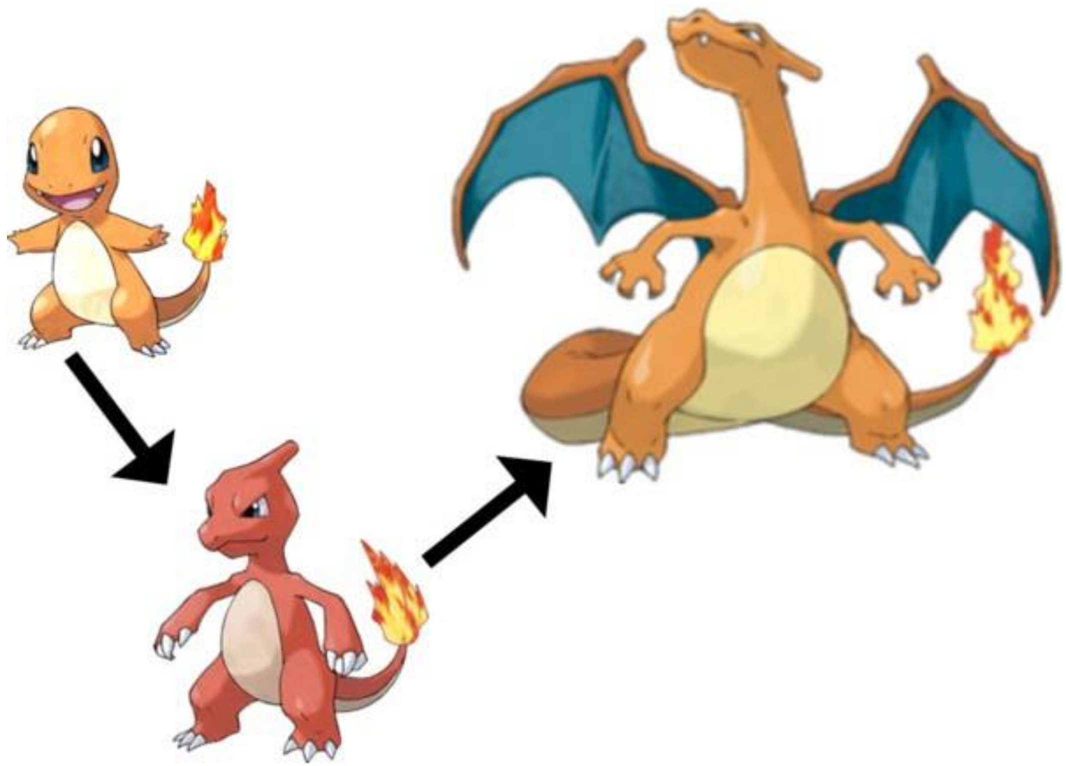
Kernel mutations with Genetic algorithms

Key terms:

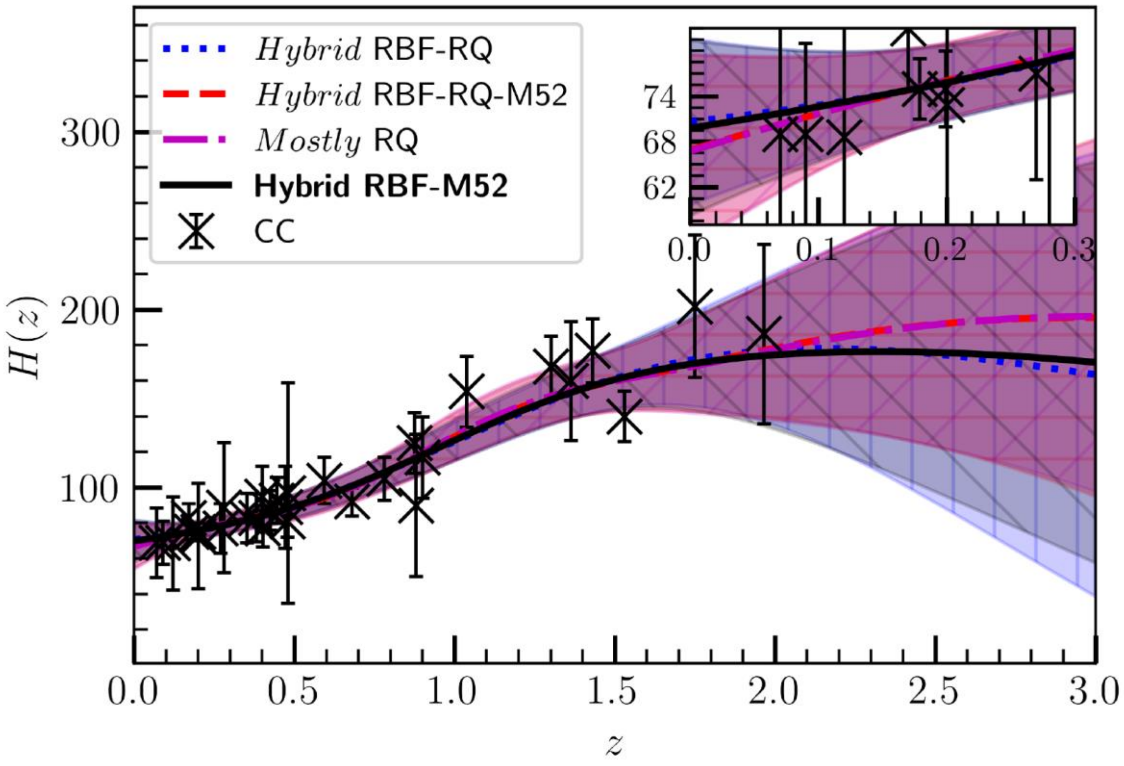
- An *individual*:

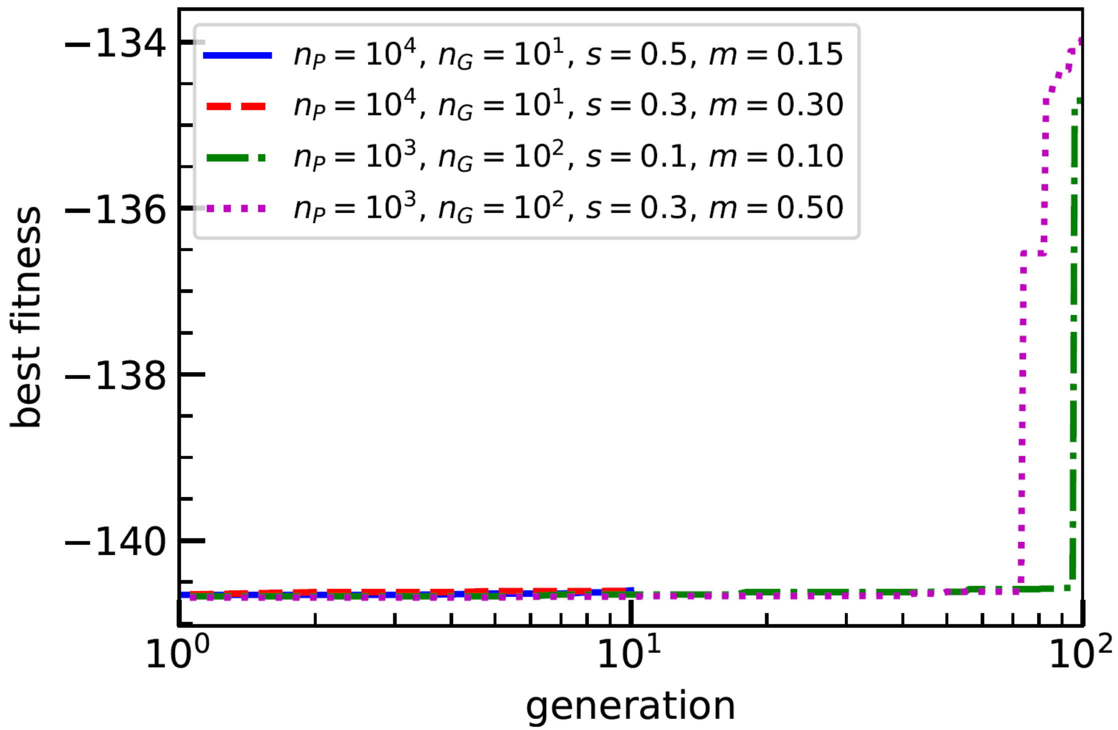
$$K(r|\theta) = C_{RBF}^2 K_{RBF}(r|l_{RBF})^{n_{RBF}} + C_{RQ} K_{RQ}\left(r|l_{RQ}, \alpha_{RQ}\right)^{n_{RBF}} + \cdots$$

- *population* = many individuals
- *selection, crossover,* ***mutation***



Pokemon Evolution (Accessed 04 Nov. 2021): http://i.imgur.com/Vmv2qmX.jpg





Kernel selection via GAs

- (left) Reconstructed Hubble function
- (right) Fitness evolution per GA parameters

Know-your-meme (Accessed 04 Nov. 2021): 4fd.jpg (1265×1241) (kym-cdn.com)

Outlook

- Kernel selection via ABC-SMC and GAs
- GP + Late-time Data -> Constraints on Modified Gravity potentials (2105.12970)
- How about overfitting and underestimating the uncertainties?

References

[1] 2105.12970, **RCB**, Levi Said (2021), *A data-driven Reconstruction of Horndeski gravity via the Gaussian processes*.

[2] 2106.08688, **RCB**, Levi Said (2021), *Towards a model-independent reconstruction approach for late-time Hubble data*.

[3] 2103.01183, Di Valentino, *et al*. (2021), *In the Realm of the Hubble tension - a Review of Solutions*.

[4] 1807.09241, Ezquiaga & Zumalacárregui (2018), *Dark energy in light of multi-messenger gravitational-wave astronomy*.

[5] hep-ph/9712331], Tsamis & Woodard (1998), *Nonperturbative models for the quantum gravitational backreaction on inflation*.

Extra Slides

Horndeski gravity

The Action (def. $X = -(\partial\phi)^2/2$)

$$\boxed{S_g[g,\phi] = \int d^4x\sqrt{-g}\Big(F(\phi)R + K(\phi,X) - G(\phi,X)\partial^2\phi + \cdots\Big)} \quad (1)$$

- Discovery: derived in the 1970s, gained popularity in the late 2000s
- Most general scalar-tensor theory with second-order field equations
- Phenomenologically rich
- Subclasses: $f(R)$, Brans-Dicke theory, Galileons, Fab Four, etc.

Quintessence Reconstruction

- Quintessence dark energy: $K(\phi, X) \to X - V(\phi)$
- Field equations:

$$3H^2 = \rho_\phi + \rho_m \quad (2)$$

$$2\dot{H} + 3H^2 = P_\phi + P_m \quad (3)$$

$$\rho_\phi = \left(\dot{\phi}^2/2\right) + V(\phi) \quad (4)$$

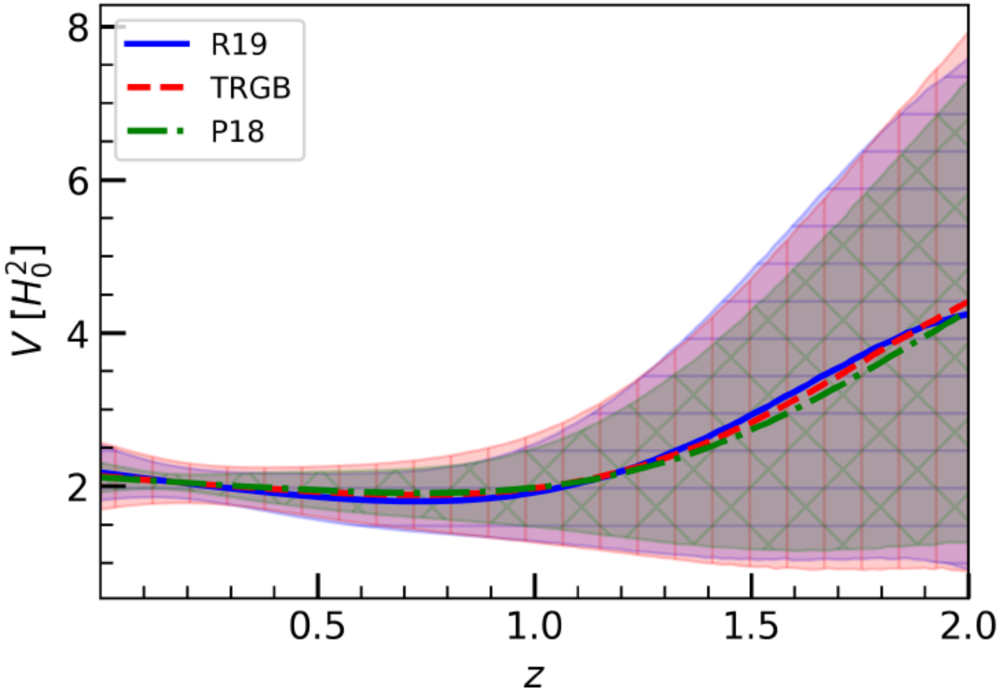
$$P_\phi = \left(\dot{\phi}^2/2\right) - V(\phi) \quad (5)$$

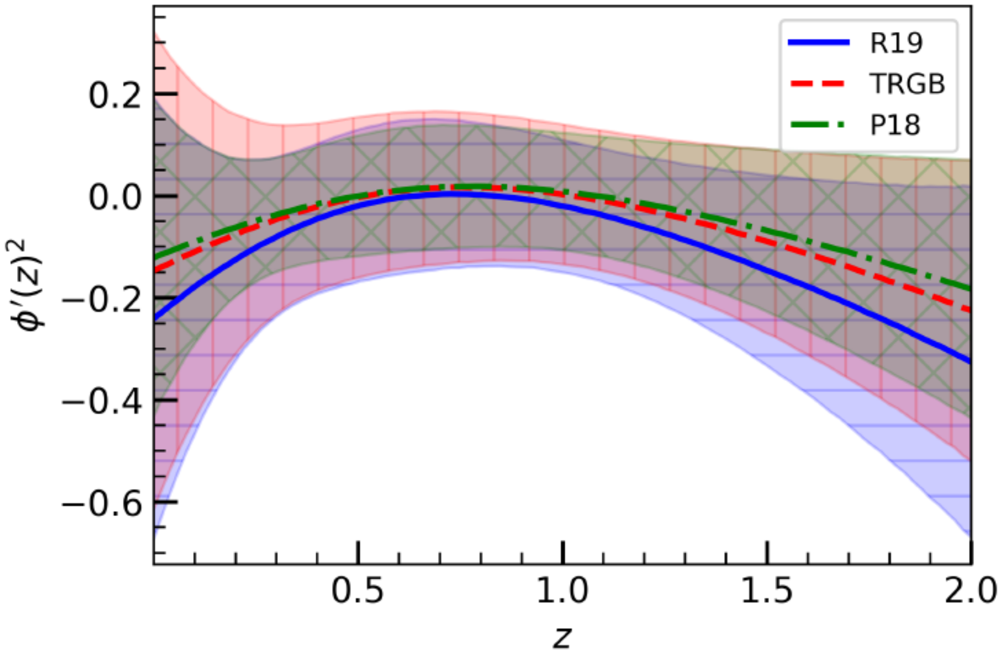
- Add and Subtract Eqs. 2 and 3:

$$\boxed{X[H, \rho_m, P_m], V[H, \rho_m, P_m]} \quad (6)$$

Quintessence Reconstruction

$$
\begin{aligned}
H_0^{\mathrm{P18}} &= \mathbf{67.4 \pm 0.5}\ \mathrm{km\ s^{-1}\ Mpc^{-1}} \\
H_0^{\mathrm{TRGB}} &= \mathbf{69.8 \pm 1.9}\ \mathrm{km\ s^{-1}\ Mpc^{-1}} \\
H_0^{\mathrm{R19}} &= \mathbf{74.03 \pm 1.42}\ \mathrm{km\ s^{-1}\ Mpc^{-1}}
\end{aligned}
$$





(**left**) Reconstructed quintessence potential $V(z)$ and (**right**) $\phi'(z)^2$ for varying H_0 prior. The filled-hatched regions show the 2σ confidence intervals. Hatches: (“--”: R19), (“|”: TRGB), (“×”: P18).

Constraints on the DE EoS

Constraints on the DE EoS in Horndeski cosmology. The columns R19, TRGB, and P18 stand for the GP analysis using the corresponding H_0 priors. P18 priors for Ωs. For designer Horndeski, $c_0 = H_0^{n+2}$, $n = 1$, and $J = H_0$ were assumed.

	$w_{\mathrm{DE}}\,(z=0)$		
Theory + parameters	H_0^{R19}	H_0^{TRGB}	H_0^{P18}
Quintessence + (Ω_{m0})	-1.1 ± 0.1	-1.1 ± 0.1	-1.06 ± 0.08
Designer Horndeski + $(\Omega_{m0}, \Omega_\Lambda, c_0, n, \mathcal{J})$	-0.8 ± 0.2	-0.9 ± 0.3	-0.9 ± 0.1
Tailoring Horndeski + $(\Omega_{m0}, \Omega_\Lambda)$	-1.1 ± 0.1	-1.1 ± 0.1	-1.06 ± 0.08
ΛCDM	-1		
w_0CDM (Planck + SNe + BAO)	-1.03 ± 0.03		
$w_0 w_a$CDM (Planck + SNe + BAO)	-0.96 ± 0.08		