An Analytic Treatment of Underdamped Axionic Blue Isocurvature Perturbations

2110.02272

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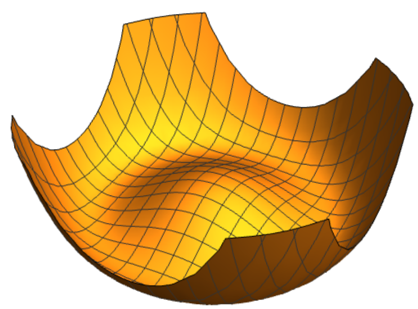
Axionic isocurvature is generic

(# field dof) > 1 → possibility of isocurvature perturbations

The axion solution to the strong CP problem gives an argument for > 1 dynamical degrees of freedom during the quasi-dS era.

After PQ symmetry breaking axion field directions would have remained flat during inflation because of the U(1) symmetry:



Therefore, whenever PQ symmetry is already broken during inflation, and whenever the field is very weakly interacting with the SM, we expect there to be **isocurvature degree of freedom.**

Standard CDM axion isocurvature has been studied well in the past:

Axions as dark matter: Preskill, Wise, Wilczek 83; Abbott, Sikivie 83; Dine, Fischler 83

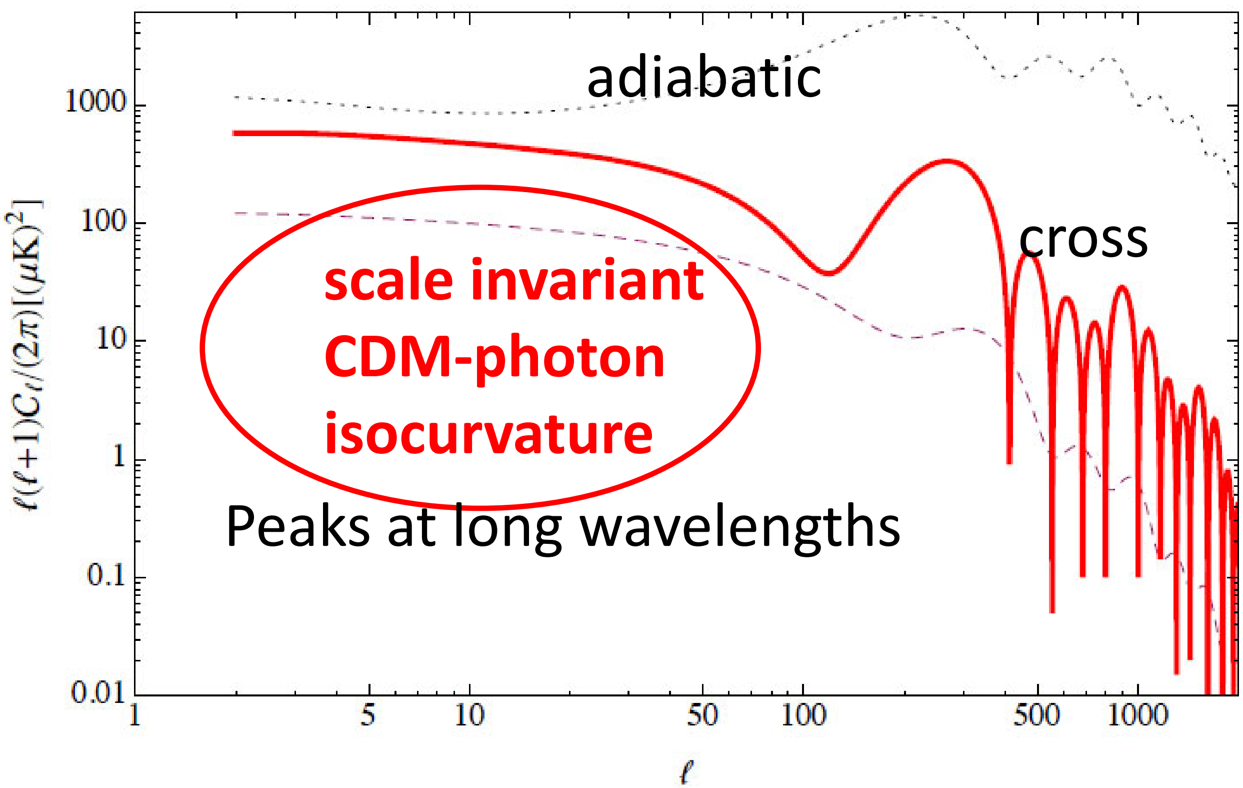
In this talk, I will be restricting to CDM-photon isocurvature

Axion isocurvature sample: Turner, Wilczek, Zee 83; Steinhardt, Turner 83; Axenides, Brandenberger, Turner 83; Linde 84, 85; Seckel, Turner 85; Efstathiou, Bond 86; Hogan, Rees 88; Lyth 90; Linde, Lyth 90; Turner, Wilczek 91; Linde 91; Lyth 92; Kolb, Tkachev 94; Kawasaki, Sugiyama, Yanagida 95; Kasuya, Kawasaki, Yanagida 97; Valviita, Muhonen 03; Ferrer, Rasanen, Valviita 04; Fox, Pierce, Thomas 04; Beltran, Garcia-Belliodo, Lesgourgues, Riazuelo 04; ; Kurki-Suonio, Muhonen, Valviita 05;Beltran, Garcia-Bellido, Lesgourgues, Viel 05; Beltran, Garcia-Bellido, Lesgourgues 06; Keskitalo, Kurki-Suonio, Muhonen, Valviita 07; Hertzberg, Tegmark, Wilczek 08; Holder, Nollett, Engelen 09; Kasuya, Kawasaki 09; Gordon, Pritchard 09; Hamann, Hannestad, Raffelt, Wong 09; Hikage, Koyama, Matsubara, Takahashi, Yamaguchi 09; Gordon, Pritchard 09; Sollom, Challinor, Hobson 09 Grin, Dore, Kamionkowski 11; Kawasaki, Sekiguchi, Takahashi 11; Hikage, Kawasaki, Sekiguchi, Takahashi 12; Dent, Easson, Tashiro 12; ; Grin, Hanson, Holder, Dore, Kamionkowski 13; Marsh, Grin, Hlozek, Ferreira 13; Sekiguchi, Tashiro, Silk, Sugiyama 13; Choi, Jeong, Seo 14; Kadota, Gong, Ichiki, Matsubara 14; Higaki, Jeong, Takahashi 14; Takeuchi, Chongchitnan 13; Nakayama, Takimoto 15; Harigaya, Ibe, Kawasaki, Yanagida 15; He, Grin, Hu 15; Nomura, Rajendran, Sanches 15; Estevez, Santillan 16; Kearney, Orlofsky, Pierce 16; Valiviita 17; Bae, Kost, Shin 18; …

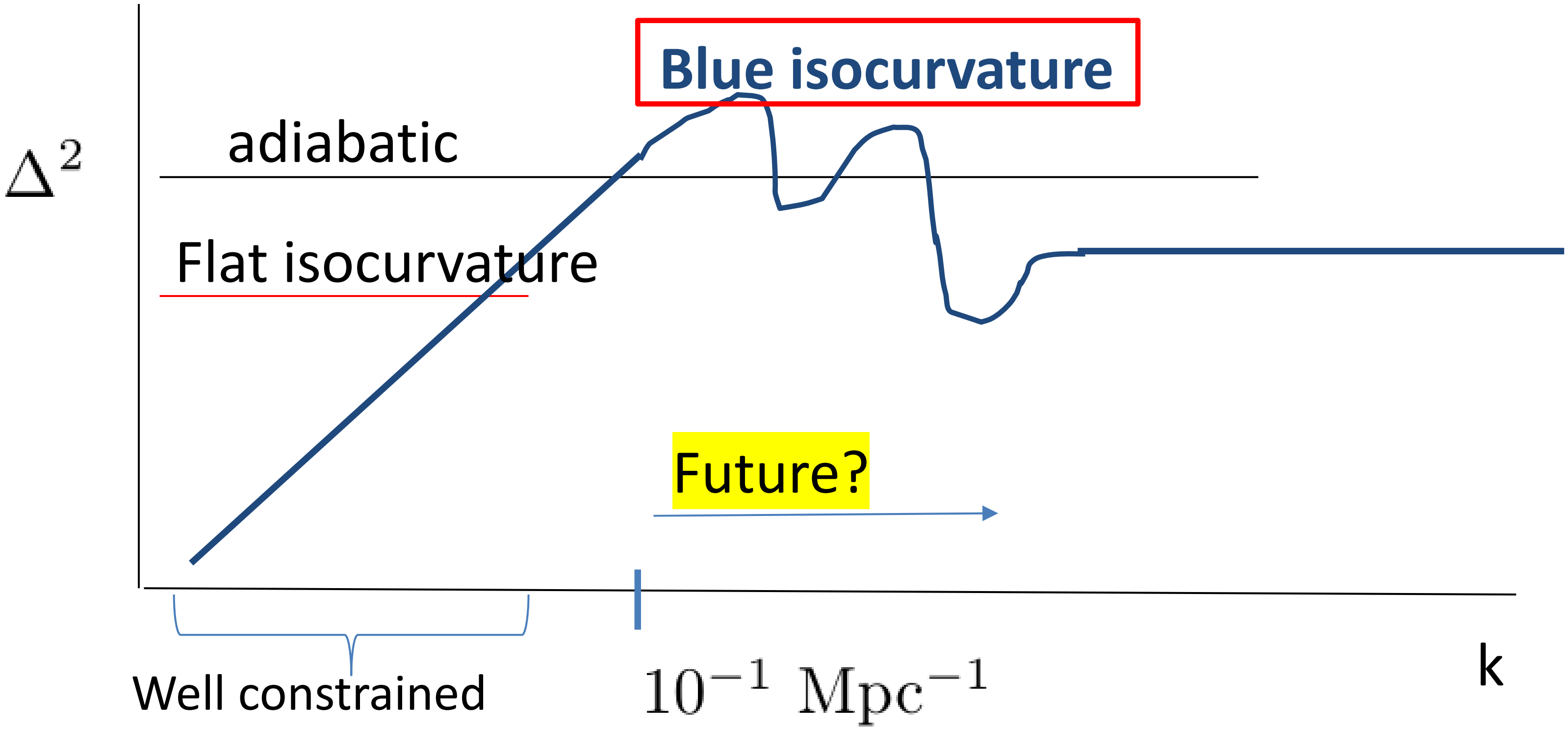
Blue isocurvature may be discoverable in the future

Currently, scale invariant isocurvature perturbations are observationally constrained to be less than about 2% of the two-point function power on large scales. [1807.06211]

Non-ultra-blue isocurvature is small even if somehow correlated:



Blue isocurvature may be discoverable in the future



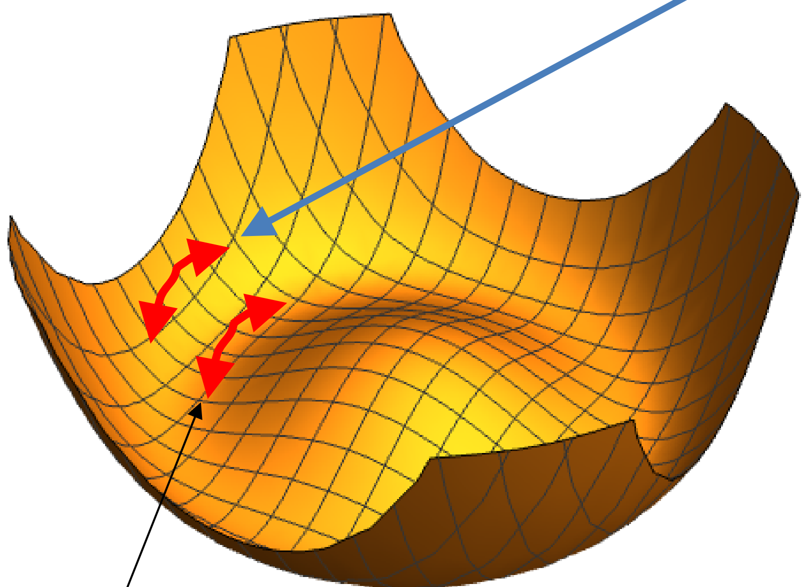
Thus far, there is no statistically significant blue isocurvature in data:

[1711.06736, 1807.06211]

Axions can generate blue isocurvature

[Kasuya, Kawasaki 0904.3800]

Blue isocurvature spectrum



flat isocurvature spectrum

Intuition:

$$\left(\Box - \frac{\Box\varphi_+}{\varphi_+}\right) a = 0$$

Non-equilibrium mass

Cannot be too steep to not decouple the field

Isocurvature computation

Solve background fields $\phi_{\pm}(T)$ then solve for the quantum perturbation mode vector $I(T)$

$$\left(\partial_T^2 + 3\partial_T\right) I + \left(\frac{Ka(0)}{a(T)}\right)^2 I + \tilde{M}^2 I = 0$$

$$\tilde{M}^2(T) \equiv \begin{pmatrix} c_+ & F^2 \\ F^2 & c_- \end{pmatrix} + \begin{pmatrix} \phi_-^2(T) & 0 \\ 0 & \phi_+^2(T) \end{pmatrix}$$

Isocurvature:

$$\Delta_S^2(t,\vec{k}) \approx 4\omega_a^2 \frac{k^3}{2\pi^2} I^\dagger \begin{pmatrix} r_+^2 & 0 \\ 0 & r_-^2 \end{pmatrix} I$$

Can give rise to a complicated Schroedinger potential

Background field dynamics

Realize radial field along a **flat direction**

e.g.

R-symmetry generates flat direction

$$W = h(\Phi_+\Phi_- - F_a^2)\Phi_0$$

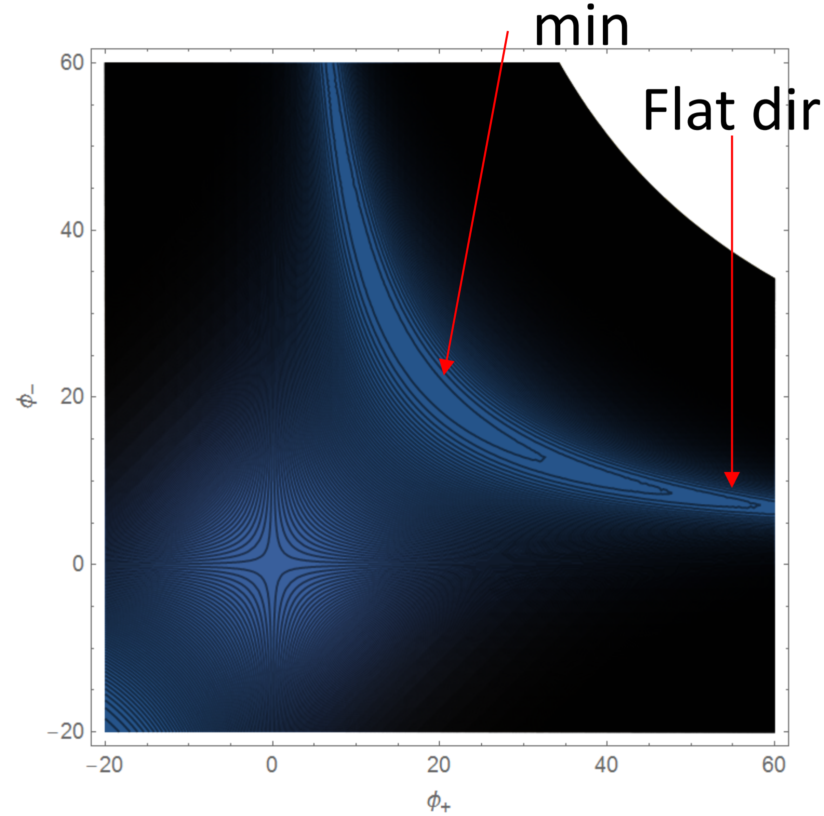
$$V_K = c_+ H^2|\Phi_+|^2 + c_- H^2|\Phi_-|^2 + c_0 H^2|\Phi_0|^2$$

$$\Phi_\pm \equiv \frac{\varphi_\pm}{\sqrt{2}} \exp\left(i\frac{a_\pm}{\sqrt{2}\varphi_\pm}\right)$$

$$a = \frac{\varphi_+}{\sqrt{\varphi_+^2+\varphi_-^2}} a_+ - \frac{\varphi_-}{\sqrt{\varphi_+^2+\varphi_-^2}} a_-$$

$$\phi_\pm \equiv \Phi_\pm \frac{h}{H} \qquad K \equiv \frac{k}{a(0)H}$$

$$F = hF_a/H \qquad a(T) = a(0)\exp(T)$$



$$\ddot{\phi}_+(T) + 3\dot{\phi}_+(T) + c_+\phi_+ + \xi(\phi_+,\phi_-)\phi_- = 0$$

$$\ddot{\phi}_-(T) + 3\dot{\phi}_-(T) + c_-\phi_- + \xi(\phi_+,\phi_-)\phi_+ = 0$$

$$\xi(\phi_+,\phi_-) \equiv \phi_+\phi_- - F^2$$

Key nonlinearity

$$V \approx h^2|\Phi_+\Phi_- - F_a^2|^2 + c_+ H^2|\Phi_+|^2 + c_- H^2|\Phi_-|^2$$

3 Lagrangian parameters

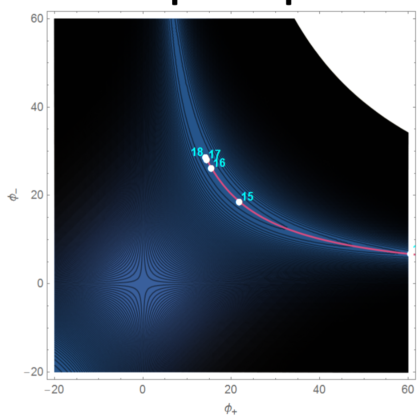
“2” initial condition parameters

$$\phi_+(0) \qquad \varepsilon_0 \equiv \frac{\dot{\phi}_+(0)}{\phi_+(0)}$$

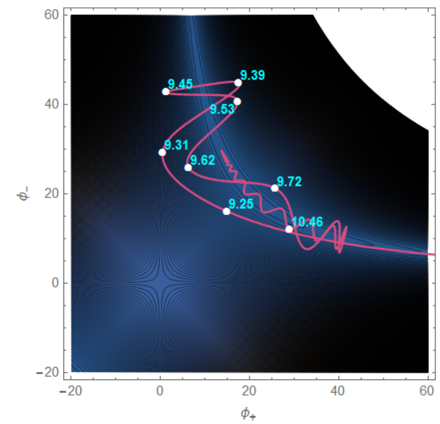
[0904.3800, 1610.04284]
Known from the past

New work with **Sai Tadepalli**:
2110.02272

Overdamped param region



Underdamped param region



$$\tilde{M}^2(T) \equiv \begin{pmatrix} c_+ & F^2 \\ F^2 & c_- \end{pmatrix} + \begin{pmatrix} \phi_-^2(T) & 0 \\ 0 & \phi_+^2(T) \end{pmatrix}$$

$$(\partial_T^2 + 3\partial_T) I + \left(\frac{Ka(0)}{a(T)}\right)^2 I + \tilde{M}^2 I = 0$$

$$\Delta_S^2(t,\vec{k}) \approx 4\omega_a^2 \frac{k^3}{2\pi^2} I^\dagger \begin{pmatrix} r_+^2 & 0 \\ 0 & r_-^2 \end{pmatrix} I$$

Analytic approximation accurate to 30-50% in the nontrivial k-region

https://pages.physics.wisc.edu/~stadepalli/Blue-Axion-IsoCurvSpec-Underdamped.nb

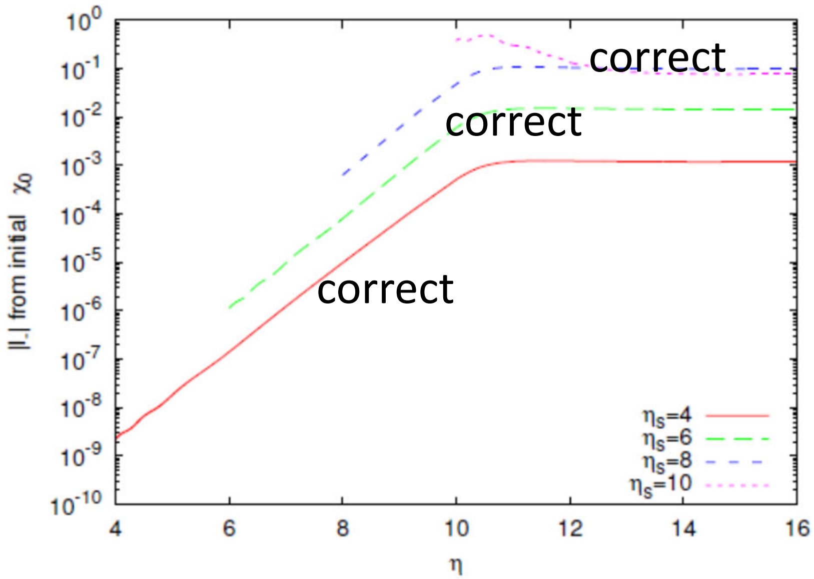
Numerical fitting function accurate to 10-20% in the nontrivial k region

Why calculate analytically?

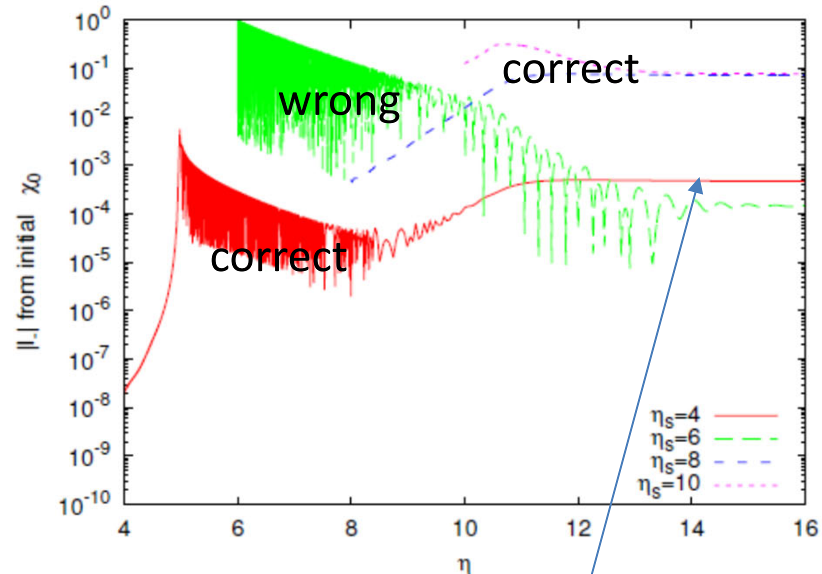
1) Gain a human understanding of the physics

2) Purely numerical situation had confusion

model 1



model 2



3) Easier for data fitting since pure numerical solutions take a long time to compute on an ODE solver

each color represents 1 value of k in Δ_S^2

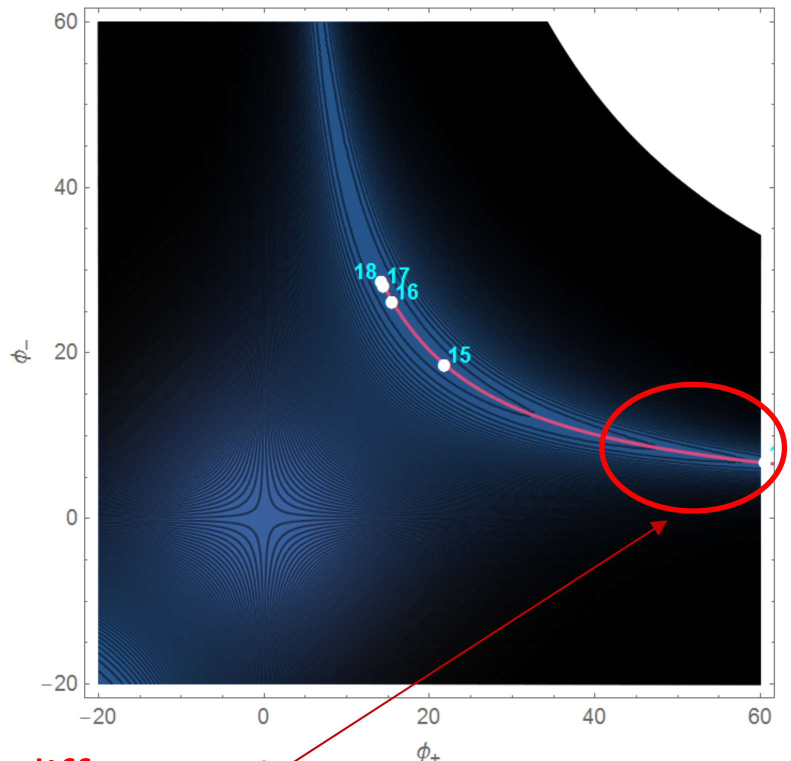
overdamped

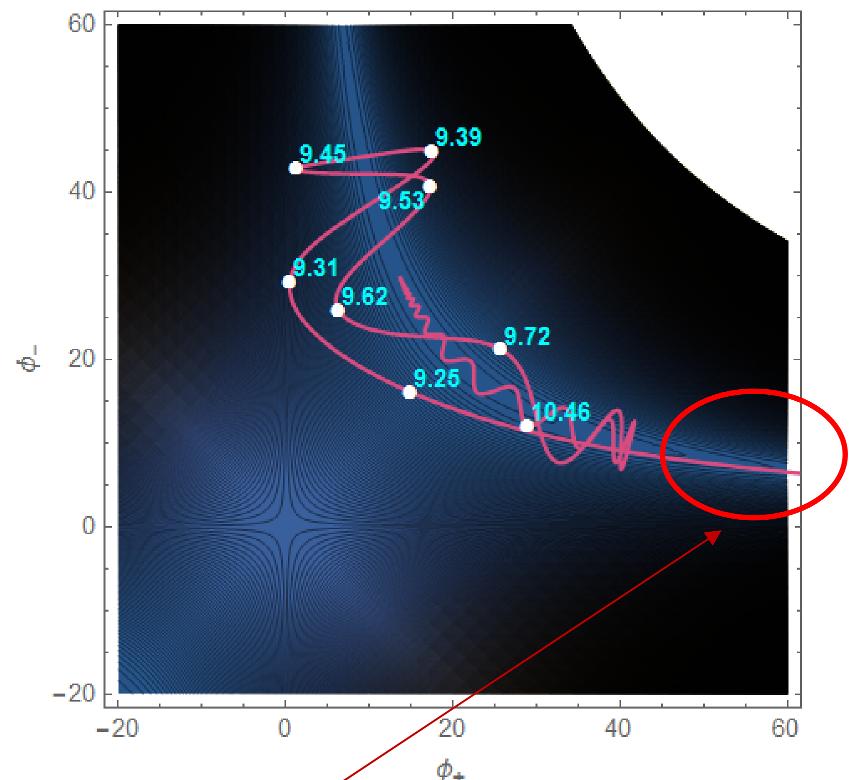
underdamped

Tiny parametric difference

$$c_+ = \frac{9}{4} - 0.1$$

$$c_+ = \frac{9}{4} + 0.1$$





Key difference 1:

Famous dynamical battle outcome: gravity "friction" vs. potential force

$$\phi_+(T) \sim \exp\left(\left[-1+\sqrt{1-\tfrac{4}{9}\tfrac{m^2_{\text{descent}}}{H^2}}\right]\tfrac{3}{2}T\right)$$

$$\phi_-(T) \sim \frac{F^2}{\phi_+}$$

$$\phi_+(T) \sim \exp\left(-\frac{3}{2}T\right)\cos\left(\sqrt{\frac{m^2_{\text{descent}}}{H^2}-\frac{9}{4}}\,T-\varphi\right)$$

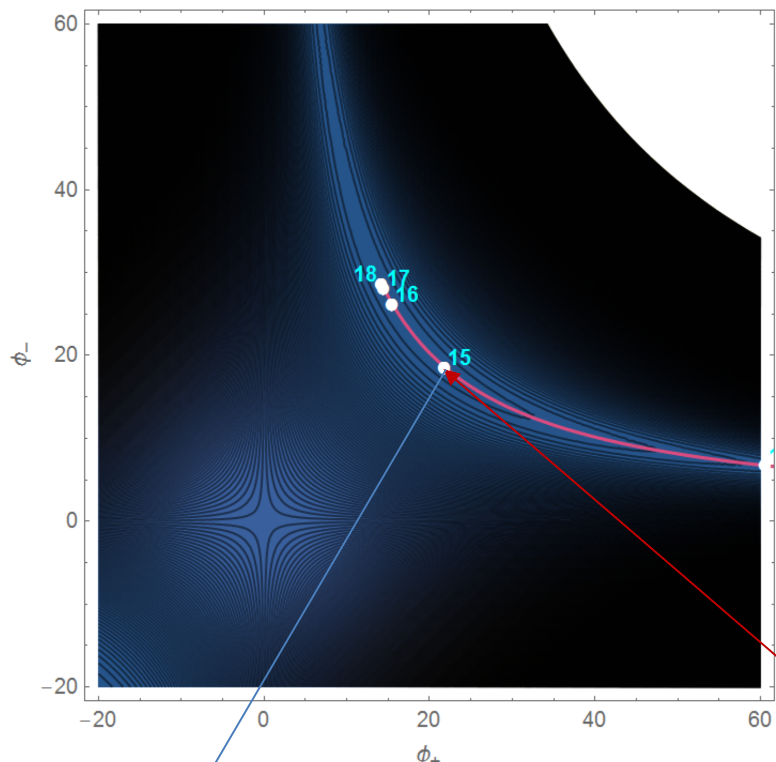
$$\phi_-(T) \sim \frac{F^2}{\phi_+}$$

Possibility of large kinetic energy

This class of scenarios have a transition

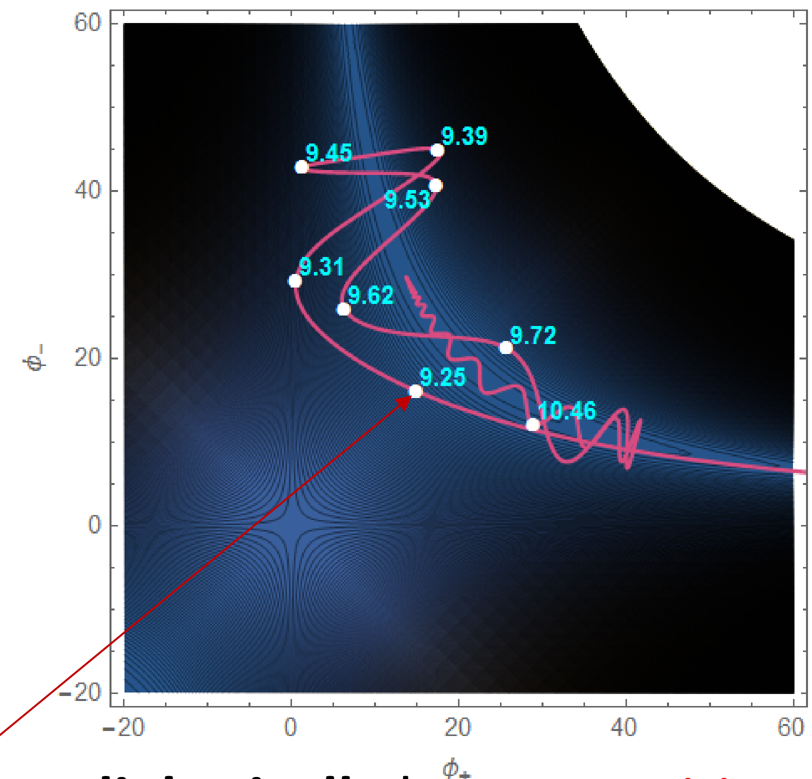
overdamped

$$c_+ = \frac{9}{4} - 0.1$$

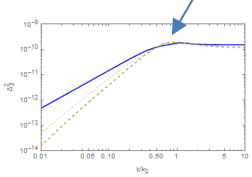


underdamped

$$c_+ = \frac{9}{4} + 0.1$$



$\partial_{\phi_i}\partial_{\phi_j} V$ eigenvector (ϕ_+, ϕ_-) rotates **nonadiabatically** here: transition



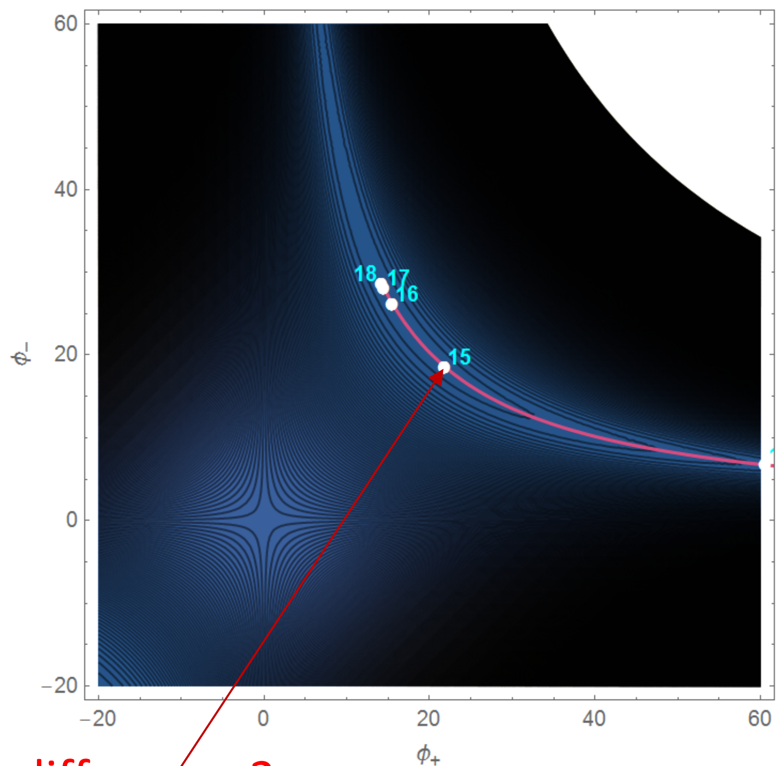
Comparable at transition

$$\tilde{M}^2(T) \equiv \begin{pmatrix} c_+ & F^2 \\ F^2 & c_- \end{pmatrix} + \begin{pmatrix} \phi_-^2(T) & 0 \\ 0 & \phi_+^2(T) \end{pmatrix}$$

Large kinetic energy makes a difference at the transition

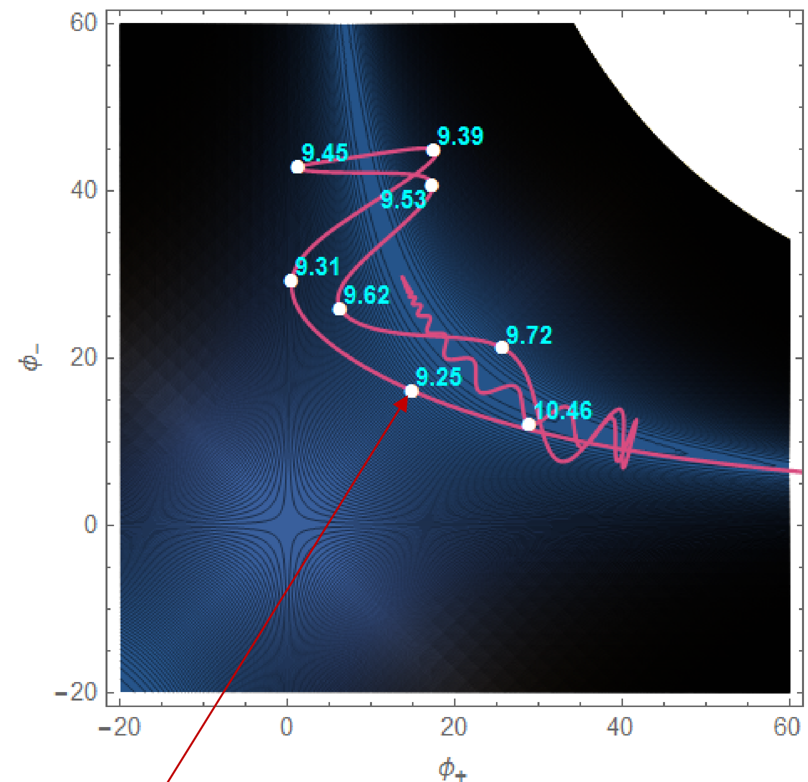
overdamped

$$c_+ = \frac{9}{4} - 0.1$$



underdamped

$$c_+ = \frac{9}{4} + 0.1$$



Key difference 2:

Transition has negligible effects from

Transition has effects from overshooting deviations from the flat direction

$$V \approx h^2|\Phi_+\Phi_- - F_a^2|^2 + c_+H^2|\Phi_+|^2 + c_-H^2|\Phi_-|^2$$

Key difference 2 → definition of resonant transition

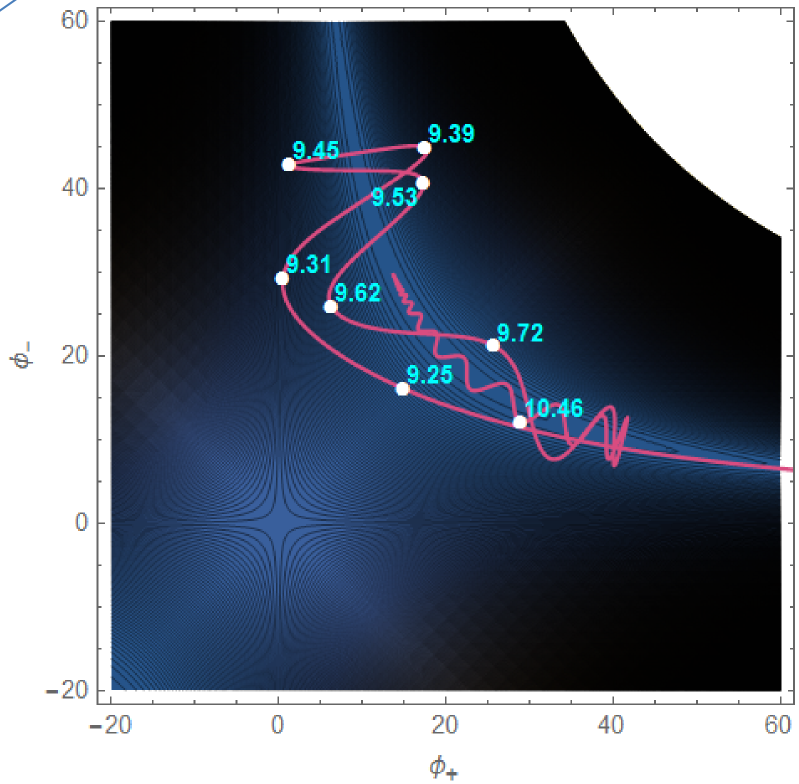
Transition can be resonant: i.e. timing of the $\phi_-(T)$ is such that the it reaches F about the same time ϕ_+ reaches F.

$\longrightarrow$ (ϕ_+, ϕ_-) can deviate significantly from flat direction

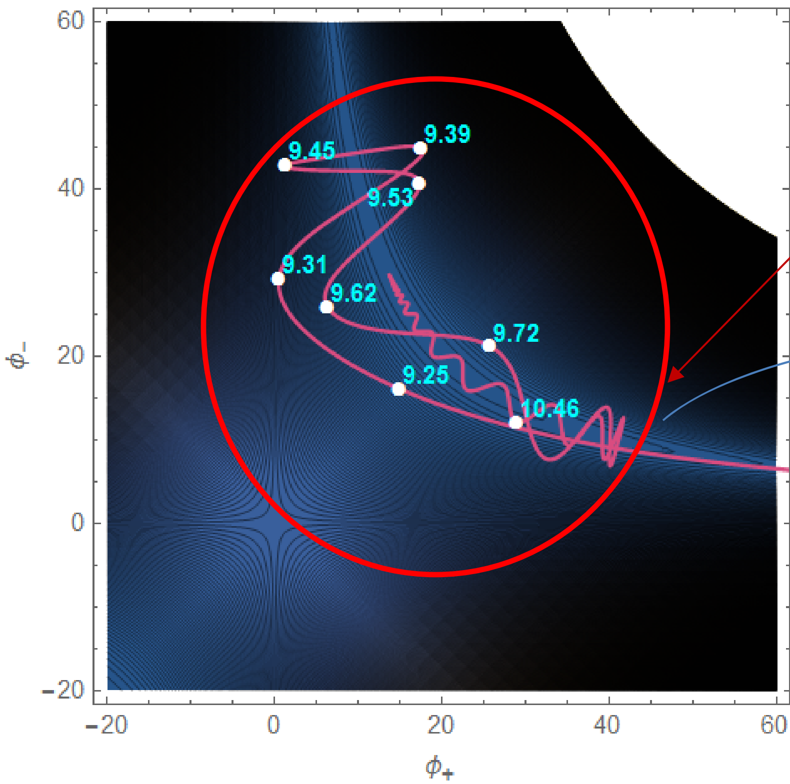
$$\text{nonlinear force on } \phi_+ = -(\phi_+\phi_- - F^2)\phi_-$$

$\|$

Deviation from flat direction



This can transfer large kinetic energy back and forth between ϕ_+ and ϕ_-



Complexity invites approximations for analytic solvability

$$\tilde{M}^2(T) \equiv \begin{pmatrix} c_+ & F^2 \\ F^2 & c_- \end{pmatrix} + \begin{pmatrix} \phi_-^2(T) & 0 \\ 0 & \phi_+^2(T) \end{pmatrix}$$

1) Decoupling heavy mass modes by Lagrangian parameter restriction
2) Calculate effective potential through integrating out fast time modes
3) Piecewise model with square wells and exponentials
    - Perturbation theory
    - Polynomial analytic fits
    - Exponential potential time averaging
    - Nonlinear field redefinition

V. Novicenko, J. Ruseckas and E. Anisimovas, *Quantum dynamics in potentials with fast spatial oscillations*, *Phys. Rev. A* **99** (Apr, 2019) 043608. [1]

grungy but nontrivial;
needed because PT breaks down;
good to only about 20-40%

Final results are intricate as expected (but stories can be told through them):

For resonant, not too energetic case:

$$\Delta_S^2(K) \approx |f_{\text{correction}}(K)|^2 \times \begin{cases} C_1 K^3 \left| 1 - i\frac{\Gamma(i\omega)\Gamma(i\omega+1)}{\pi(1+i\cot(i\omega\pi))} e^{-2i\omega \ln\left(\frac{-K\tau_1}{2}\right)} \right|^2 & -K\tau_c \ll 1 \\ C_2 \mathfrak{D}^2 \left| H^1_{i\omega}(-K\tau_1) \right|^2 (-K\tau_2) \Big(\sin(-K\tau_2) \\ +(3/2+b\tanh[-b\Delta T]) \left(\frac{\cos[-K\tau_2]}{-K\tau_2} - \frac{\sin[-K\tau_2]}{(-K\tau_2)^2} \right) \Big)^2 & 0.5 \lesssim -K\tau_c < 3 \\ C_3 \mathfrak{D}^2 \cosh^2[b\Delta T] \times \\ \Big| \left(-ie^{iK\tau_2}\right) + \tanh[-b\Delta T] \times \\ \left(\left(\frac{b}{-K\tau_2} \right) \cos[-K\tau_2] + \left(i\frac{-K\tau_2}{b} \right) \sin[-K\tau_2] \right) \Big|^2 & 3 \lesssim -K\tau_c < K_2 \\ C_4 \times 1 & K > K_P \end{cases}$$

$$\omega \equiv \sqrt{c_+ - 9/4}$$

$$C_1 \approx C\mathfrak{D}^2 \frac{\pi}{8} e^{-\omega\pi} \cosh^2[b\Delta T] \frac{e^{-3T_2}}{3} \left(\frac{3}{2} - b\tanh[-b\Delta T] \right)^2 \left| \frac{1+i\cot(i\omega\pi)}{\Gamma(i\omega+1)} \right|^2$$

$$C_2 \approx C\frac{\pi}{8} e^{-\omega\pi} \cosh^2[b\Delta T]$$

$$C_3 \approx C\frac{1}{4}$$

$$C_4 \approx \omega_a^2 \frac{h^2}{2\pi^2 \theta_+^2 F^2} \left(\frac{r}{1+r^2} \right)$$

$$C = \omega_a^2 \frac{4}{\pi^2} \frac{r(1+r^4)}{(1+r^2)^3} \frac{h^2}{\theta_+^2 F^2}$$

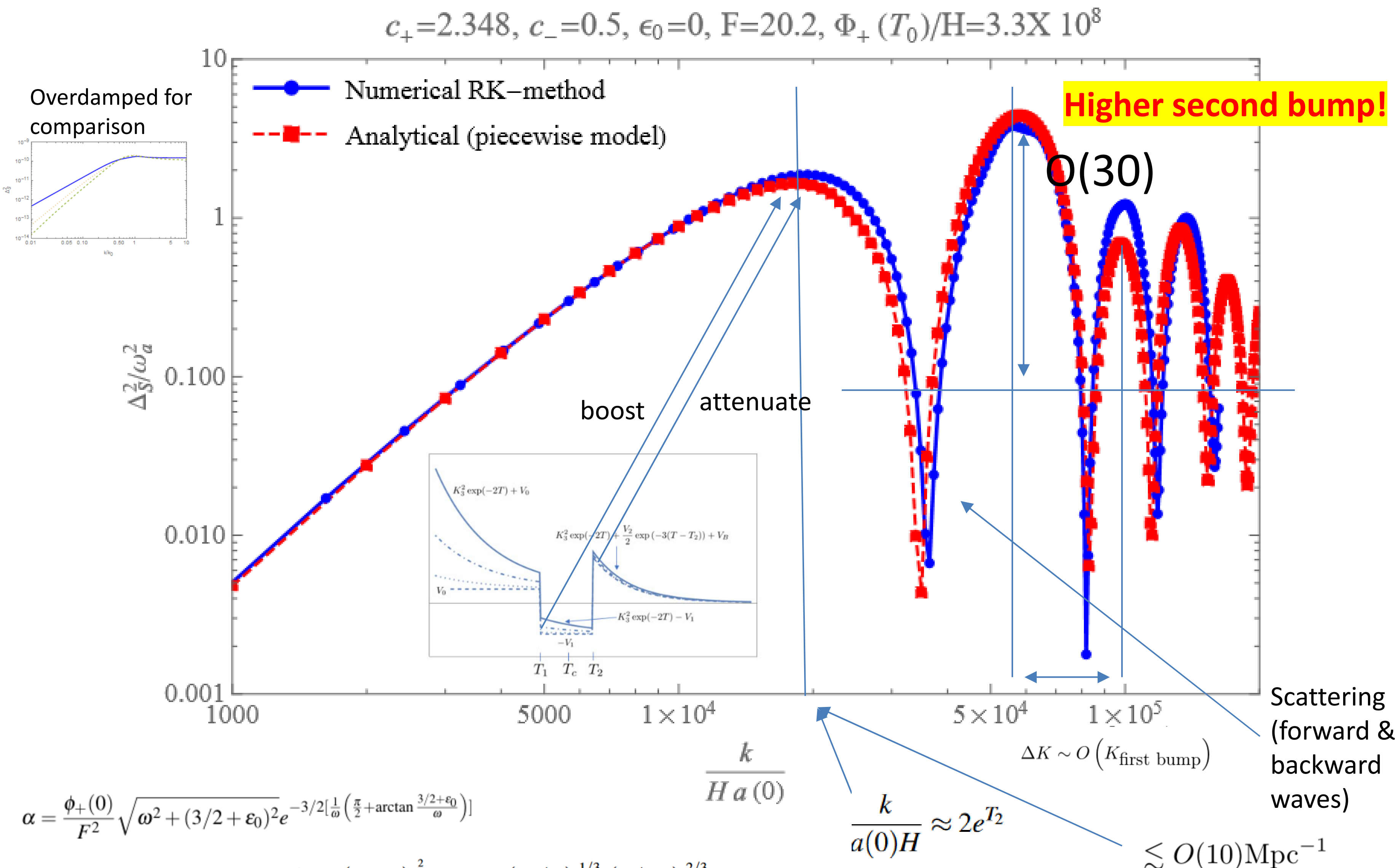
$$r = \sqrt{c_+/c_-}$$

$$V_B \approx c_- + \frac{1}{(T_L - T_2)} \left(\frac{1063}{3072} + \frac{106793 c_-}{393216 c_+} \right)$$

$$\mathfrak{D} \approx \exp\left(\left(-\frac{3}{2} + \sqrt{\frac{9}{4} - V_B} \right) (T_B - \tilde{T}) \right)$$

... more ... → https://pages.physics.wisc.edu/~stadepalli/Blue-Axion-IsoCurvSpec-Underdamped.nb

An interesting example plot of the analytic result:



$$\alpha = \frac{\phi_+(0)}{F^2}\sqrt{\omega^2+(3/2+\varepsilon_0)^2}\,e^{-3/2\left[\frac{1}{\omega}\left(\frac{\pi}{2}+\arctan\frac{3/2+\varepsilon_0}{\omega}\right)\right]}$$

$$\frac{k_\star}{a_0} \sim \left(\frac{\varphi_{\rm init}}{0.3M_p}\right)^{\frac{2}{3}} e^{-(N_e-50)}\left(\frac{T_{\rm rh}/H}{10^{-1}}\right)^{1/3}\left(\frac{H/\varphi_{\rm fin}}{10^{-3}}\right)^{2/3}(10\,\mathrm{Mpc}^{-1})$$

Summary

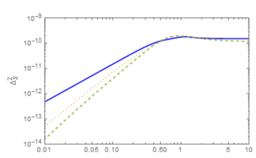
- In 2110.02272, the underdamped blue axionic isocurvature spectrum has been computed analytically to about 30-40% accuracy in the resonant region. Easy to use mathematica package can be found at

https://pages.physics.wisc.edu/~stadepalli/Blue-Axion-IsoCurvSpec-Underdamped.nb

- The dominant physics is the **nonlinear** interaction of the classical background field and sharp-edged time-space potentials. This in turn leads to scattering of quantum waves that generate the spectrum.

- Enhancements and interesting features exist whose physics and parametric dependences can be found in the paper.

overdamped



vs.

underdamped