Long Range Non-Perturbative Effects in a t- channel Simplified Dark Matter Model

Dipan Sengupta

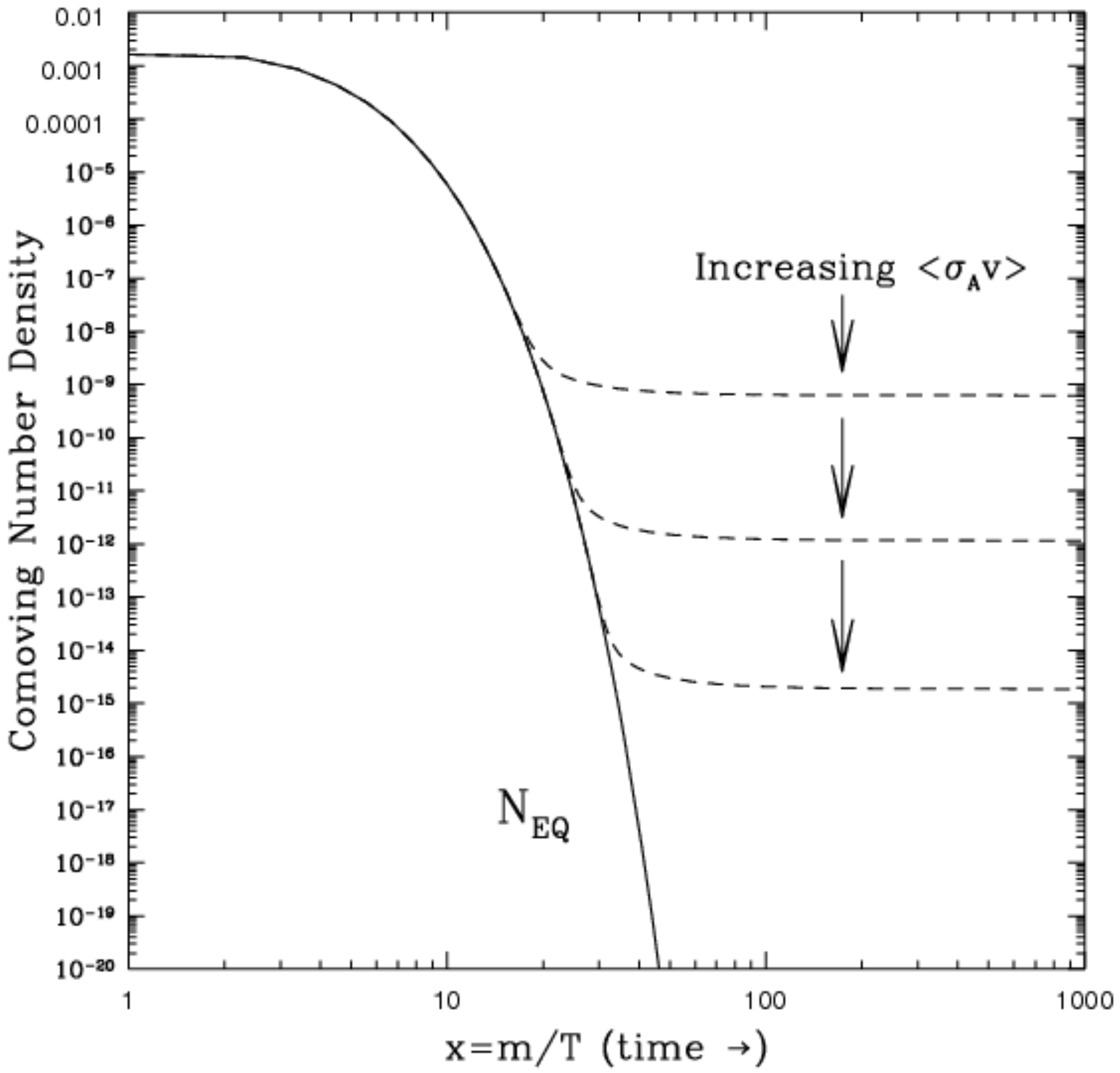
With

Emanuele Copello, Matthias Becker and Julia Harz (T.U. Munich)

as well as

Kirtimaan A. Mohan (M.S.U, East Lansing)

The Thermal Freeze-out scenario



- **Boltzmann Equation :**

$$\frac{dn}{dt} = -3Hn - \langle \sigma v \rangle \left[n^2 - n_{\rm eq}^2 \right]$$

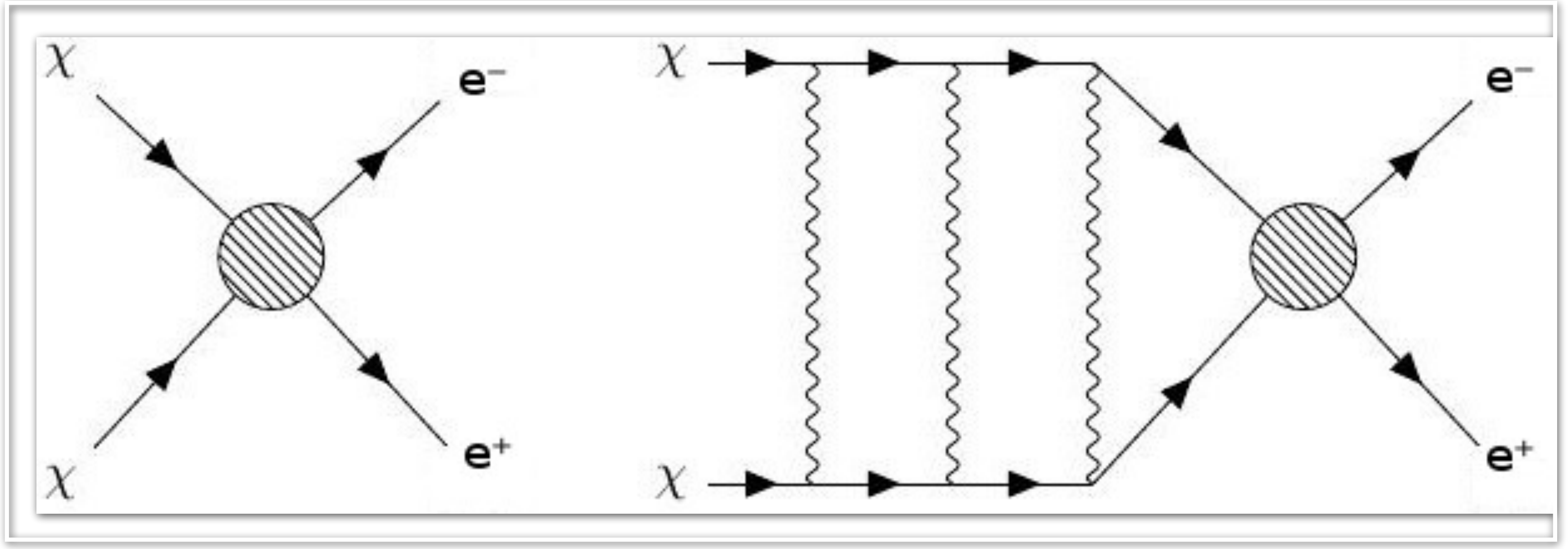
Dilution from expansion $\quad X + X \to SM + SM \quad SM + SM \to X + X$

$$\langle \sigma_{\chi\chi} v \rangle = \langle s_0 + s_1 v^2 + \mathcal{O}(v^4) \rangle$$

How do radiative corrections and non-perturbative effects change the threshold behavior ?

Sommerfeld Enhancements

- *Distortion of two body wave functions by a long range potential*
- *In the low velocity limit : Can be approximated by a Schrodinger equation with a potential*



Resumming an infinite set of ladder diagrams

$$\left[-\frac{1}{2mr^2}\frac{d}{dr}\left(r^2\frac{d}{dr}\right)+\frac{\ell(\ell+1)}{2mr^2}+V(r)-E\right]R_\ell(r;E)=0$$

$$V(r)=\frac{g_Z^2\delta e^{-\delta r}}{1-e^{-\delta r}}$$

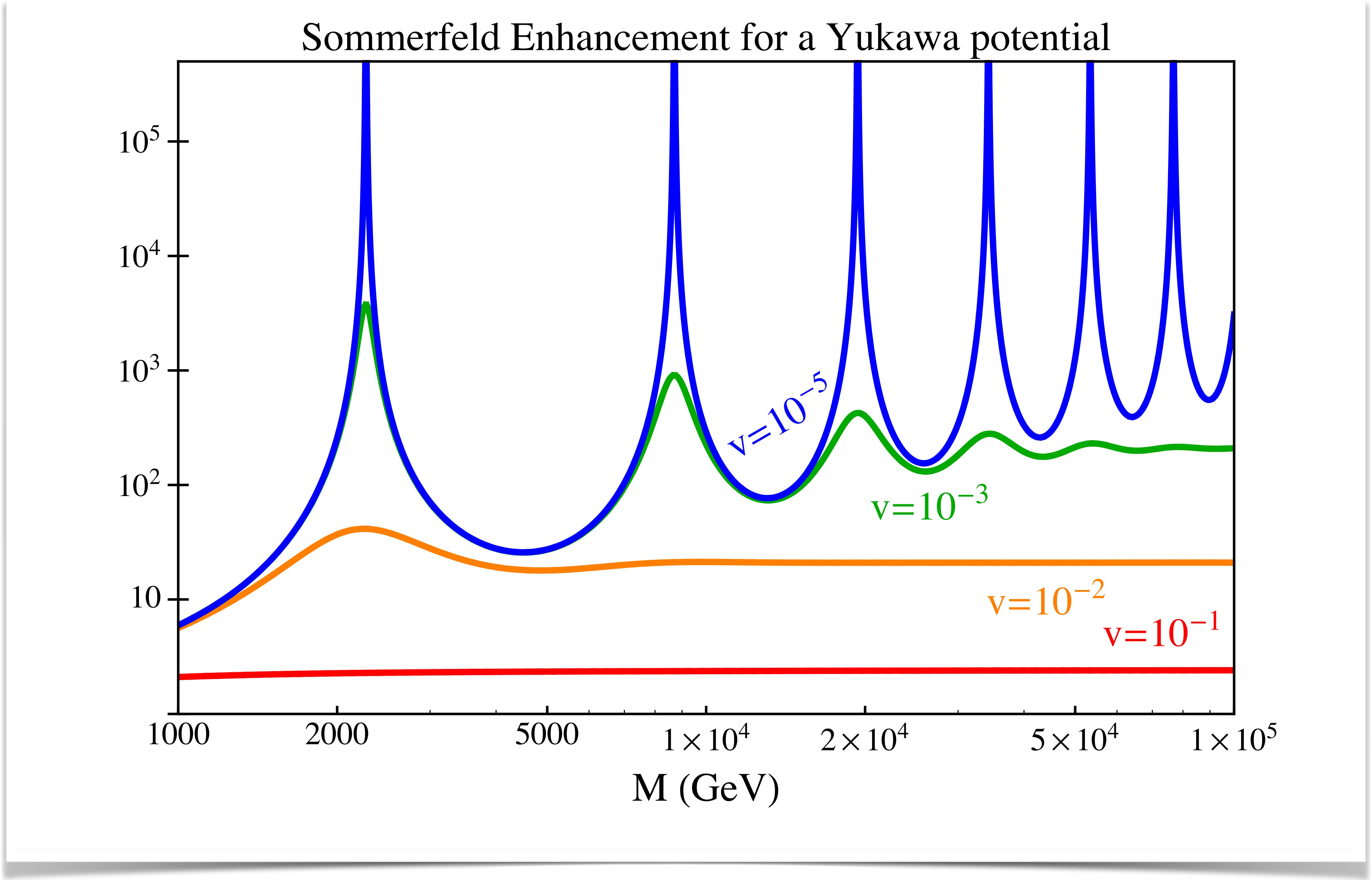
Hulthen Potential

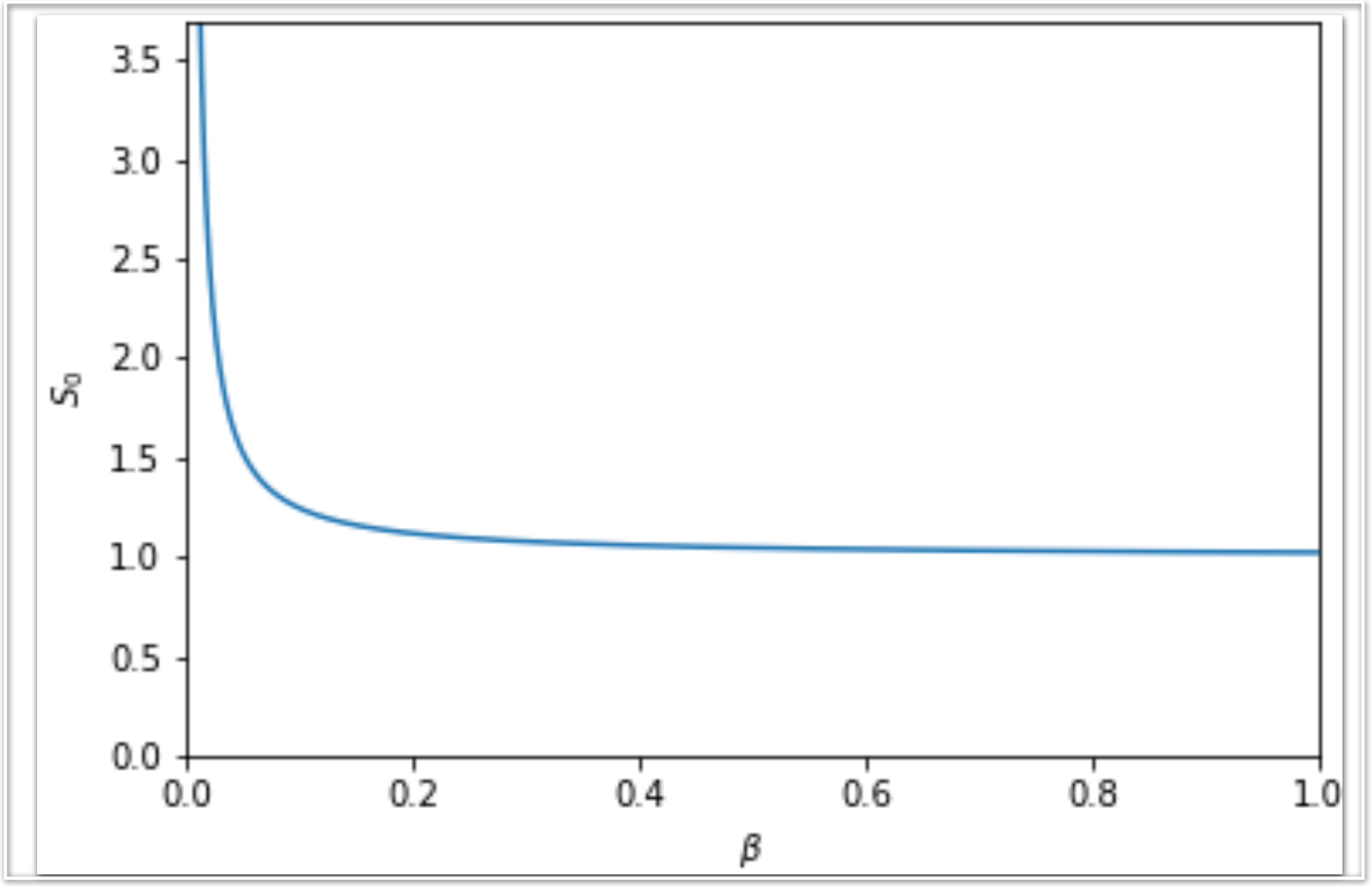
$$\psi_k(\vec{r})=\sum_{\ell=0}^{\infty}A_\ell\,P_\ell(\cos\theta)\,R_\ell(r;E)$$

$$\left|\psi_k(\vec{0})\right|^2=\frac{\pi g_Z^2}{v}\frac{\sinh\frac{2vm_\chi\pi}{\delta}}{\cosh\frac{2vm_\chi\pi}{\delta}-\cos\left(2\pi\sqrt{\frac{g_Z^2m_\chi}{\delta}-\frac{v^2m_\chi^2}{\delta^2}}\right)}$$

$$S=\frac{\sigma}{\sigma^0}=\frac{|\psi(0)|^2}{|\psi^0(0)|^2}$$

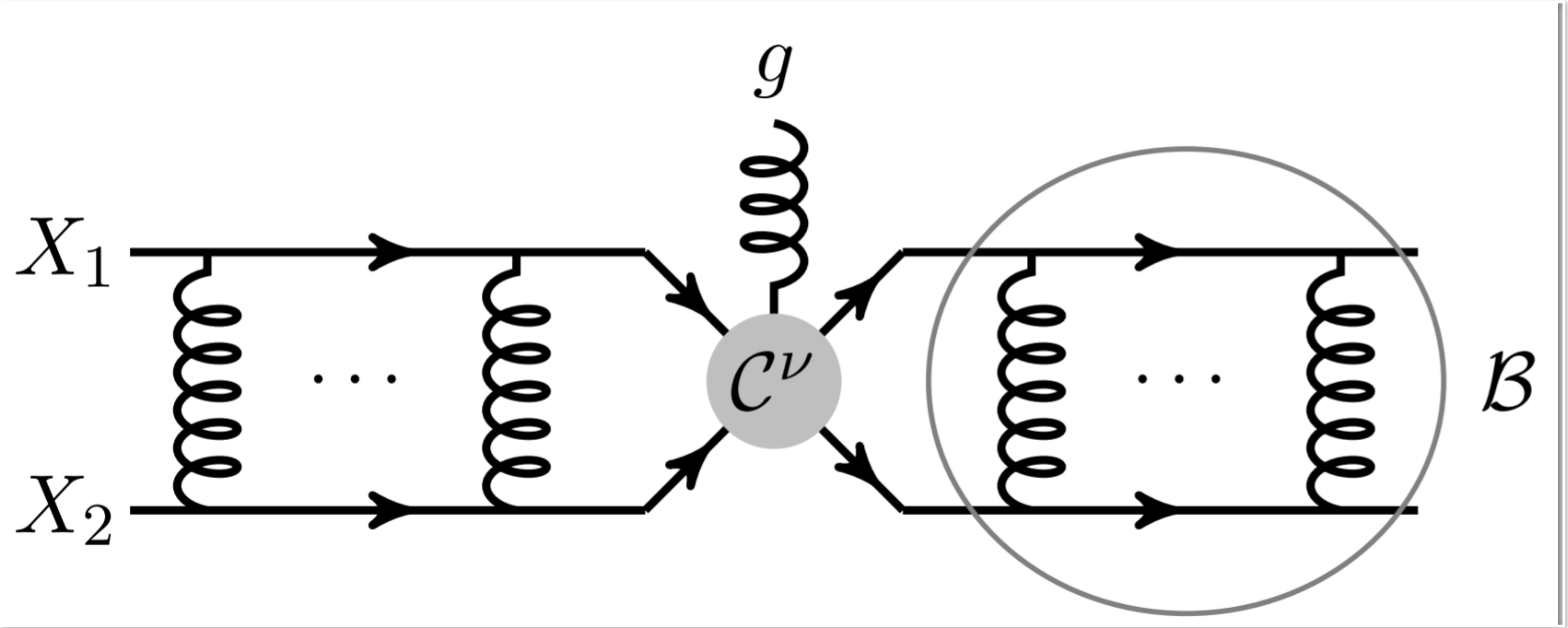
Sommerfeld Enhancements





arXiv:1603.03797 [hep-ph] ***1705.01987***

Bound State Formations



Radiative Capture into a bound state

$$X_1 + X_2 \to \mathcal{B}(X_1 X_2) + g.$$

A Simplified Model

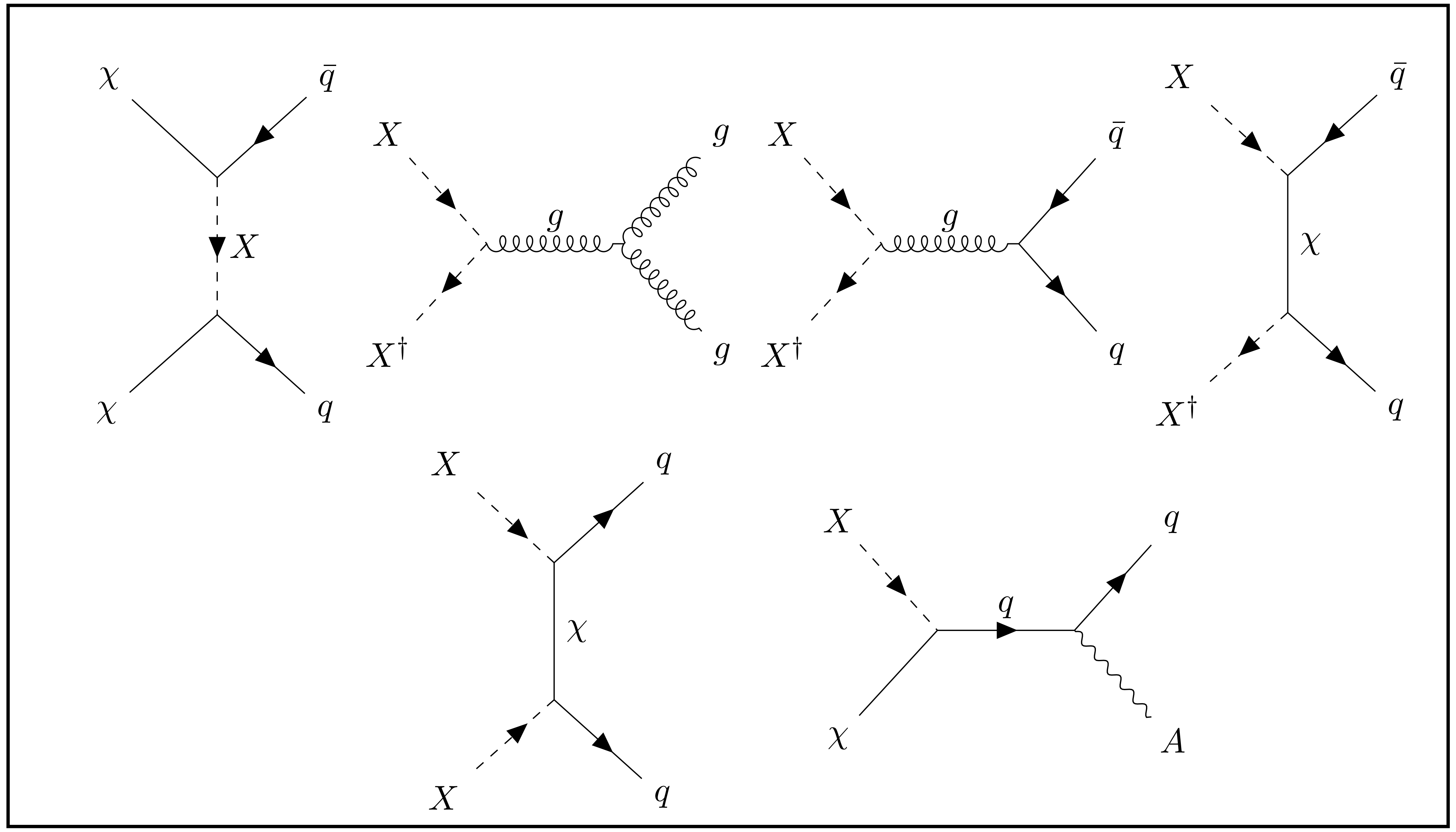
$$\mathcal{L}_{u_R} = \sum_u \left[(D_\mu \tilde{u})^*(D^\mu \tilde{u}) - M_{\tilde{u}}^2 \; \tilde{u}^*\tilde{u} + g_{DM} \; \tilde{u}^* \; \bar{\chi} P_R u + g_{DM}^* \; \tilde{u} \; \bar{u} P_L \chi\right] \quad \boxed{(3,1)_{2/3}}$$

$$\mathcal{L}_{d_R} = \sum_d \left[(D_\mu \tilde{d})^*(D^\mu \tilde{d}) - M_{\tilde{d}}^2 \; \tilde{d}^*\tilde{d} + g_{DM} \; \tilde{d}^* \; \bar{\chi} P_R d + g_{DM}^* \; \tilde{d} \; \bar{d} P_L \chi\right] \quad \boxed{(3,1)_{-1/3}}$$

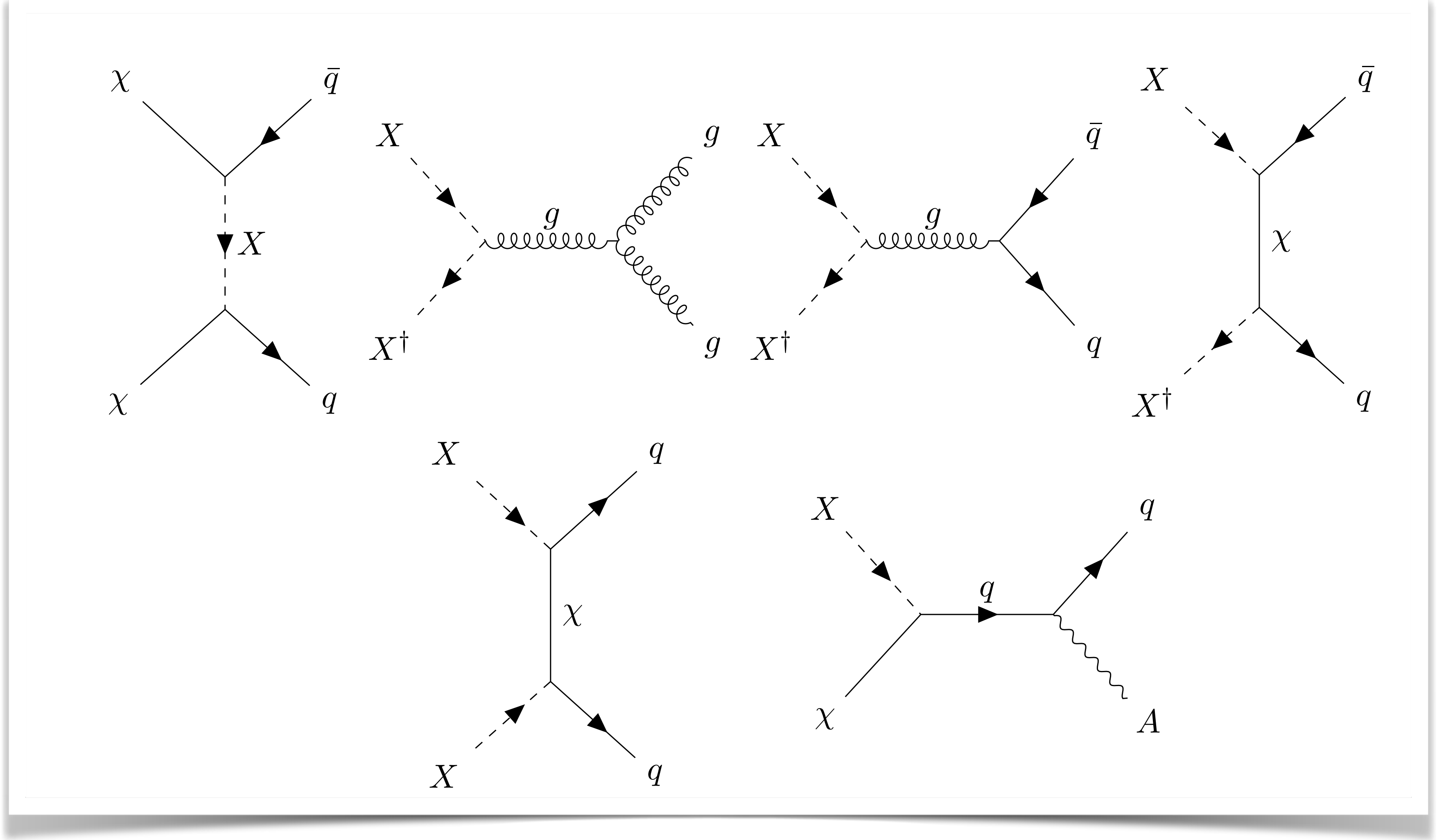
$$\mathcal{L}_{q_L} = \sum \left[(D_\mu \tilde{q})^*(D^\mu \tilde{q}) - M_{\tilde{q}}^2 \; \tilde{q}^*\tilde{q} + g_{DM} \; \tilde{q}^* \; \bar{\chi} P_L q + g_{DM}^* \; \tilde{q} \; \bar{q} P_R \chi)\right] \quad \boxed{(3,2)_{-1/6}}$$

$$\mathcal{L}_\chi = \frac{1}{2}\left(i\bar{\chi}\partial\!\!\!/\chi - M_\chi \bar{\chi}\chi\right)$$

Relic Annihilations + Co-annihilations



Relic denity without Sommerfeld Enhancements and BSF



Total Relic includes co-annhilations

$$\boxed{\chi - X,\ \chi - X^\dagger,\ X - X^\dagger \text{ and } X - X}$$

$$\boxed{\tilde{Y} = Y_\chi + \sum_{i=u,c,t} Y_{X_i} + Y^\dagger_{X_i} = Y_\chi + 2\sum_{i=u,c,t} Y_{X_i}}$$

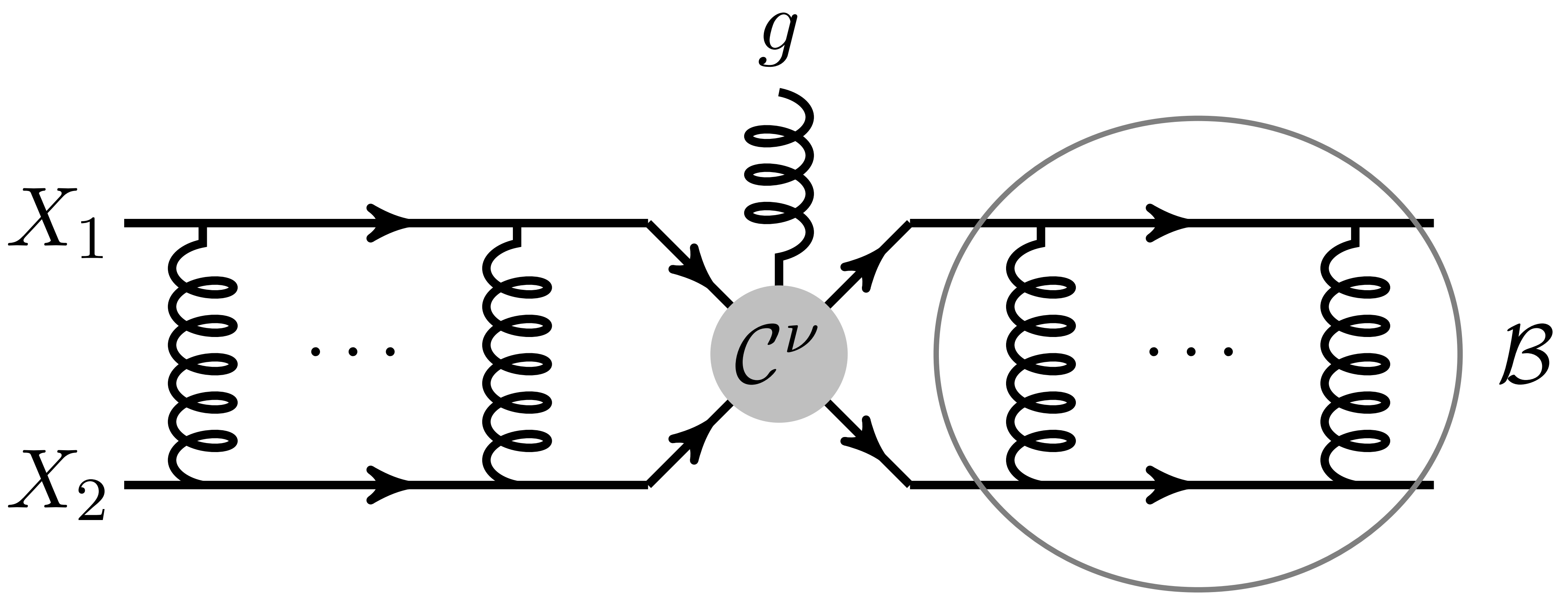
$$\boxed{\frac{\mathrm{d}\tilde{Y}}{\mathrm{d}x} = -c\, g^{1/2}_{*,\mathrm{eff}} \frac{\langle \sigma_{\mathrm{eff}} v_{\mathrm{rel}} \rangle}{x^2} \left(\tilde{Y}^2 - \tilde{Y}^2_{\mathrm{eq}} \right)}$$

$$\langle \sigma_{\mathrm{eff}} v_{\mathrm{rel}} \rangle = \langle \sigma_{XX^\dagger} v_{\mathrm{rel}} \rangle \left(\frac{2 g_X^2 (1+\delta)^3 e^{-2\delta x}}{[g_\chi + 2\sum_i g_{X_i} (1+\delta)^{3/2} e^{-\delta x}]^2} \right)$$

$$\boxed{\delta \equiv \frac{m_X - m_\chi}{m_\chi} \equiv \frac{\Delta}{m_\chi}, \qquad \Delta = m_X - m_\chi}$$

In the degenerate limit $\langle \sigma_{\mathrm{eff}} v_{\mathrm{rel}} \rangle \simeq \langle \sigma_{XX^\dagger} v_{\mathrm{rel}} \rangle \dfrac{2 g_X^2}{(g_\chi + 2\sum_i g_{X_i})^2}$

Sommerfeld Effect and Bound State Formation



n-gluon exchanges contribute with $\left(\frac{\alpha}{v}\right)^n$ for $\alpha \sim v$

Sommerfeld effect

$$\sigma(X_1 X_2 \to SM\,SM) = S\left(\frac{\alpha}{v}\right)\sigma_{\text{pert.}}$$

Bound State Formation (BSF)

$$\sigma(X_1 X_2 \to \mathcal{B}\,(X_1 X_2)\ g) = \sigma_{\text{BSF}}$$

Bound state as an additional particle in the thermal bath.

Figure from [Harz,Petraki (2018)]

Sommerfeld Enhancements and Bound States

Color Potential

$$V(r) = -\frac{\alpha_s}{2r}\left[C_2(\mathbf{R_1}) + C_2(\mathbf{R_2}) - C_2(\mathbf{R})\right]$$

Color Configurations

$$\mathbf{3} \times \bar{\mathbf{3}} = \mathbf{1} + \mathbf{8}$$

$$\mathbf{3} \times \mathbf{3} = \bar{\mathbf{3}} + \mathbf{6}$$

$$\mathbf{R_1} \otimes \mathbf{R_2} = \bigoplus_{\hat{\mathbf{R}}} \hat{\mathbf{R}}$$

$$V_{[\hat{\mathbf{R}}]}(r) = -\alpha_g^{[\hat{\mathbf{R}}]}/r$$

$$\boxed{\alpha_g^{[\hat{\mathbf{R}}]} = \alpha_s \times \frac{1}{2}[C_2(\mathbf{R_1}) + C_2(\mathbf{R_1}) - C_2(\hat{\mathbf{R}})] \equiv \alpha_s \times k_{[\hat{\mathbf{R}}]}}$$

$$V(r)_{\mathbf{3}\otimes\bar{\mathbf{3}}} = \begin{cases} -\dfrac{4}{3}\dfrac{\alpha_s}{r} & [\mathbf{1}] \\ +\dfrac{1}{6}\dfrac{\alpha_s}{r} & [\mathbf{8}] \end{cases} \quad ; \quad V(r)_{\mathbf{3}\otimes\mathbf{3}} = \begin{cases} -\dfrac{2}{3}\dfrac{\alpha_s}{r} & [\bar{\mathbf{3}}] \\ +\dfrac{1}{3}\dfrac{\alpha_s}{r} & [\mathbf{6}] \end{cases}$$

Sommerfeld Enhancements and Bound States

Sommerfeld Enhancement

$$\sigma_{\rm SE} = S_{0,[\mathbf{R}]}\, \sigma_0$$

$$S_{0,[\mathbf{R}]} = S_0\left(k_{[\mathbf{R}]}\frac{\alpha_s}{v_{\rm rel}}\right)$$

Coulomb Potential $\longrightarrow$

$$S_0(\zeta_s) = \frac{2\pi\zeta_s}{1-e^{-2\pi\zeta_s}}$$

$$\zeta_s = \alpha_{g,[\mathbf{R}]}/v_{\rm rel} = k_{[\mathbf{R}]}\,\alpha_s/v_{\rm rel}$$

$$S_{0,[\mathbf{1}]} = S_0\left(\frac{4\alpha_s^S}{3v_{\rm rel}}\right),\ S_{0,[\mathbf{8}]} = S_0\left(\frac{-\alpha_s^S}{6v_{\rm rel}}\right),\ S_{0,[\bar{\mathbf{3}}]} = S_0\left(\frac{2\alpha_s^S}{3v_{\rm rel}}\right),\ S_{0,[\mathbf{6}]} = S_0\left(\frac{-\alpha_s^S}{3v_{\rm rel}}\right)$$

Bound States : Capture Processes

Particle-Anti-Particle

$$(X+X^\dagger)_{[\mathbf{8}]} \to \mathcal{B}(XX^\dagger)_{[\mathbf{1}]} + g,$$
$$(X+X^\dagger)_{[\mathbf{1}]} \to \{\mathcal{B}(XX^\dagger)_{[\mathbf{8}]} + g\}_{[\mathbf{1}_S]},$$
$$(X+X^\dagger)_{[\mathbf{8}]} \to \{\mathcal{B}(XX^\dagger)_{[\mathbf{8}]} + g\}_{[\mathbf{8}_S]\ \text{or}\ [\mathbf{8}]_A}$$

Particle-Particle

$$(X+X)_{[\bar{\mathbf{3}}]} \to \{\mathcal{B}(XX)_{[\mathbf{6}]} + g\}_{[\bar{\mathbf{3}}]}$$
$$(X+X)_{[\bar{\mathbf{3}}]} \to \{\mathcal{B}(XX)_{[\bar{\mathbf{3}}]} + g\}_{[\bar{\mathbf{3}}]}$$
$$(X+X)_{[\mathbf{6}]} \to \{\mathcal{B}(XX)_{[\bar{\mathbf{3}}]} + g\}_{[\mathbf{6}]}$$
$$(X+X)_{[\mathbf{6}]} \to \{\mathcal{B}(XX)_{[\mathbf{6}]} + g\}_{[\mathbf{6}]}$$

Sommerfeld Enhancements and Bound States

Process	Contribution to $\langle \sigma v \rangle$	$v_{\rm rel}$	Color Structure	BSF
$\chi\chi \to q_i \bar{q}_i$	$g_{\rm DM}^4$	$v_{\rm rel}^2$ $(m_q = 0)$ const. $(m_q \neq 0)$	none	✗
$X_i X_j^\dagger \to gg$	$g_s^4 e^{-2x\delta}$	const.	$\lvert\mathcal{M}\rvert^2 \sim \frac{2}{7}[\mathbf{1}] + \frac{5}{7}[\mathbf{8}]$	✓
$X_i X_j \to q_i q_j$	$g_{\rm DM}^4 e^{-2x\delta}$	$v_{\rm rel}^2$	$\lvert\mathcal{M}\rvert^2 \sim \frac{1}{3}[\bar{\mathbf{3}}] + \frac{2}{3}[\mathbf{6}]$	(✓)
$X_i X_i \to q_i q_i$	$g_{\rm DM}^4 e^{-2x\delta}$	$v_{\rm rel}^2$	$\lvert\mathcal{M}\rvert^2 \sim [\mathbf{6}]$	(✓)
$X_i X_j^\dagger \to q_i \bar{q}_j$	$(\alpha g_{\rm DM}^2 + \beta g_s^2)^2 e^{-2x\delta}$	$v_{\rm rel}^2$	$\lvert\mathcal{M}\rvert^2 \sim f_1\,(g_{\rm DM}, g_s)\,[\mathbf{1}]$ $+ f_8\,(g_{\rm DM}, g_s)\,[\mathbf{8}]$	✓
$X_i \chi \to q_i A$	$g_{\rm DM}^2 g_{\rm gauge}^2 e^{-x\delta}$	const.	none	✗

We take only singlet states for this work

Capture into ground state

$(nlm) = (100)$ at a given representation $\mathbf{R}$

$$\sigma_{\mathbf{k}\to\{100\}} v_{\rm rel} = \frac{\pi \alpha_s^{\rm BSF} \alpha_g^B}{\mu^2} \frac{2^7 C_2(\mathbf{R})}{3\, d_{\mathbf{R}}^2} \; f_c \; S_{\rm BSF}(\zeta_S, \zeta_B)$$

$$f_c = d_{\mathbf{R}}^2(\eta_1^2 + \eta_2^2) - 2 + C_2(\mathbf{adj}) \left[d_{\mathbf{R}} C_2(\mathbf{R}) - \frac{C_2(\mathbf{adj})}{2} \right] \left(\frac{\alpha_s^{\rm NA}}{\alpha_g^B} \right)^2$$

$$S_{\rm BSF}(\zeta_S, \zeta_B) = \left(\frac{2\pi\zeta_S}{1 - e^{-2\pi\zeta_S}} \right) (1 + \zeta_S^2) \frac{\zeta_B^4 e^{-4\zeta_S \,{\rm arccot}(\zeta_B)}}{(1 + \zeta_B^2)^3}$$

For attractive potentials and small velocities

$$S_{\rm BSF} \sim v_{\rm rel}^{-1}$$

Sommerfeld Enhancements and Bound States

- X-X* capture into a singlet Bound State

$$\sigma_{\rm BSF}^{[\mathbf{8}]\to[\mathbf{1}]} v_{\rm rel} = \frac{2^7\, 17^2}{3^5} \frac{\pi \alpha_s^{\rm BSF} \alpha_{s,[\mathbf{1}]}^B}{m_X^2} S_{\rm BSF}(\zeta_S, \zeta_B)$$

- ***Thermal Average***

$$\langle \sigma_{\rm BSF} v_{\rm rel} \rangle = \left(\frac{\mu}{2\pi T}\right)^{3/2} \int {\rm d}^3 v_{\rm rel} \exp\left(-\mu v_{\rm rel}^2/2T\right) \underbrace{[1 + f_g(\omega)]}_{\textit{Bose Enhancement}} \sigma_{\rm BSF} v_{\rm rel}$$

$$\underbrace{\omega = \mu/2 \left[(\alpha_g^B)^2 + v_{\rm rel}^2\right]}_{\text{Energy emitted by radiated gluon}}$$

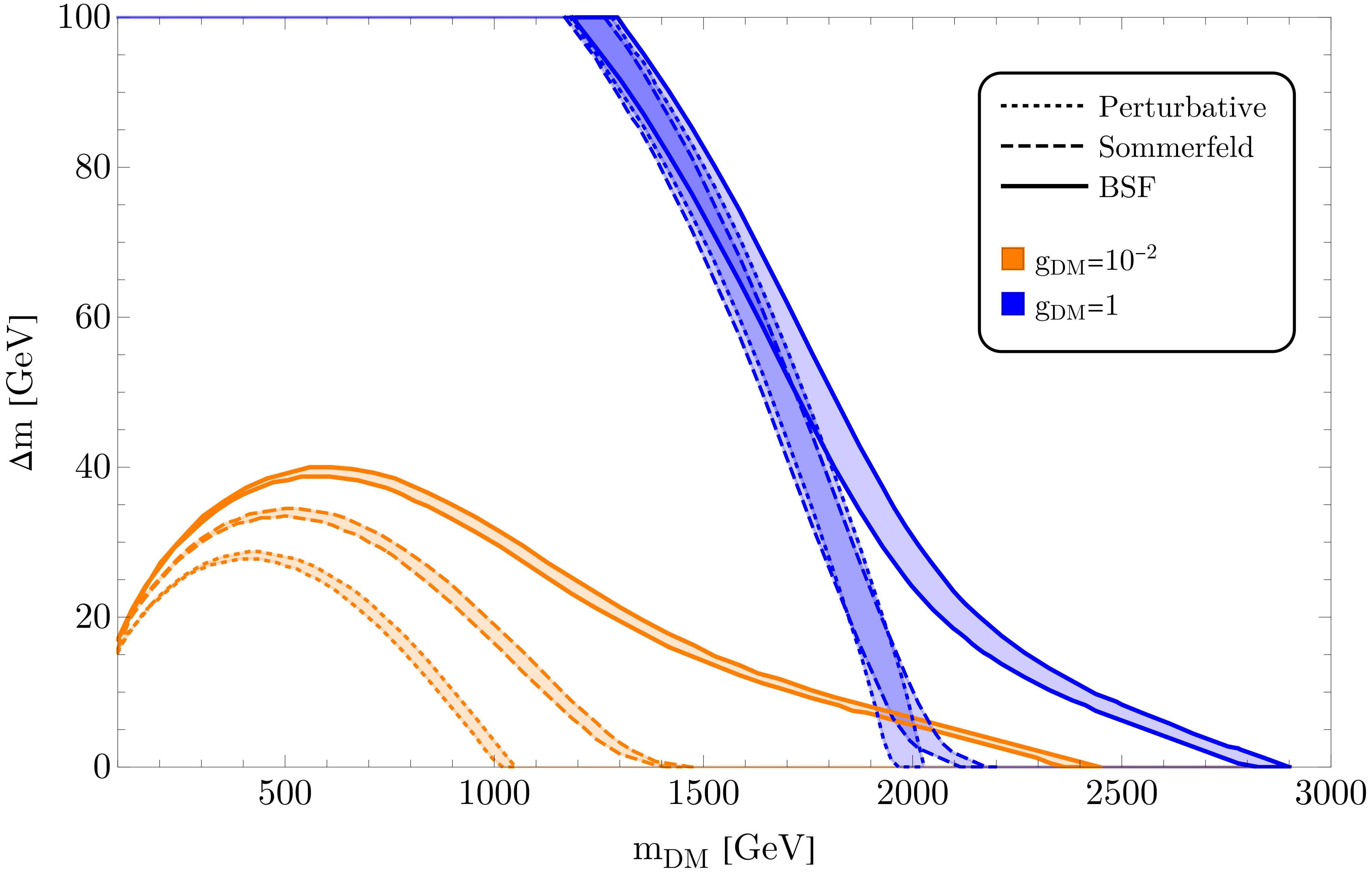
Bounds states once formed can

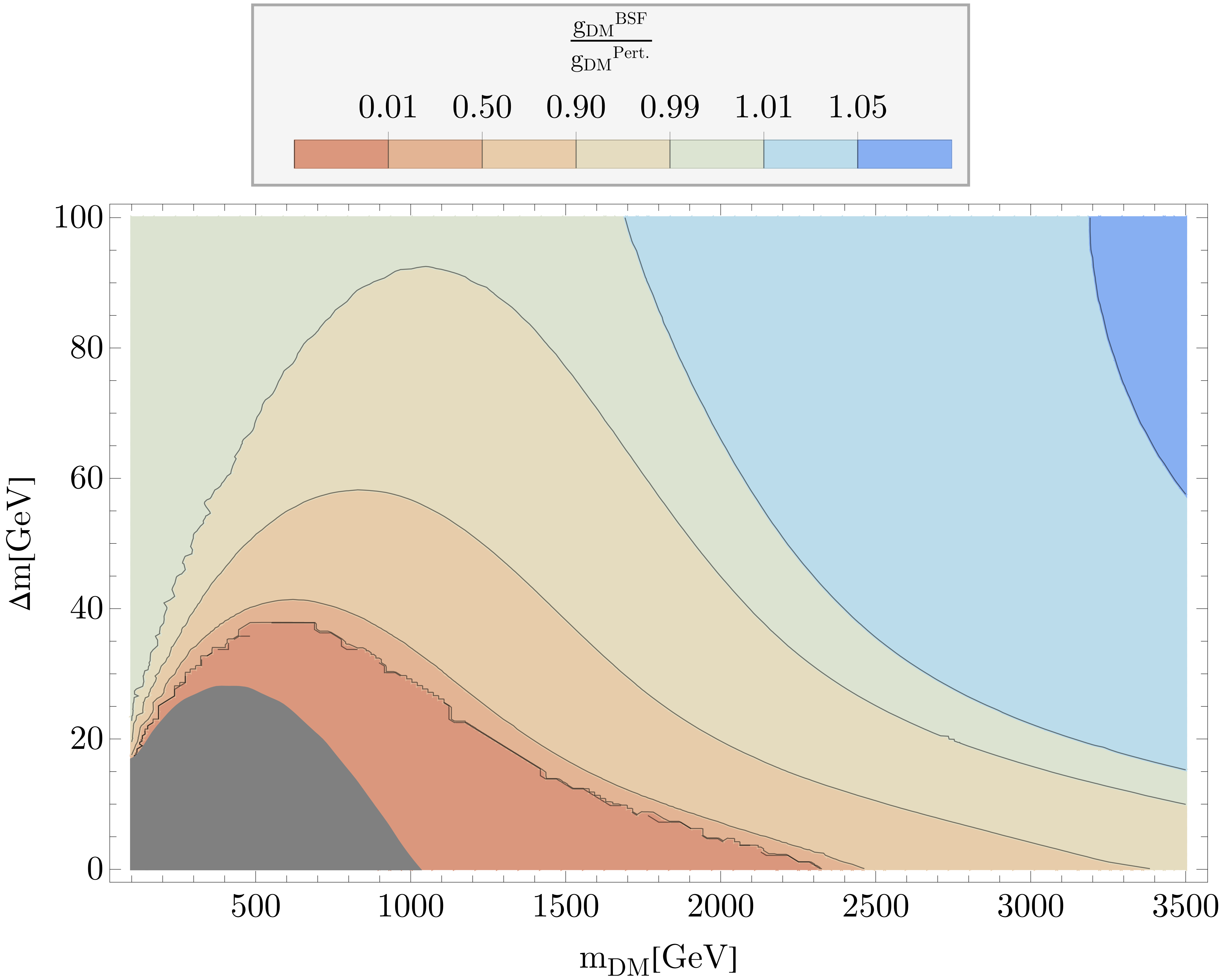
1. Can Ionize and dissolve into their constituents by energetic gluons the plasma. Net number of constituent particles and BSP is a constant.
2. Directly decay into radiation. Binding particles eventually decay into radiation

$$\langle \sigma_{\rm BSF} v_{\rm rel} \rangle_{\rm eff} \equiv \langle \sigma_{\rm BSF}^{[\mathbf{8}]\to[\mathbf{1}]} v_{\rm rel} \rangle \frac{\Gamma_{{\rm dec}[\mathbf{1}]}}{\Gamma_{{\rm dec}[\mathbf{1}]} + \Gamma_{{\rm ion},[\mathbf{1}]}} \longrightarrow$$

Large temperatures dominated by Ionisation
Low Temperatures dominated by decay

Sommerfeld Enhancements and Bound States





Calculation of the Relic Density

We adjusted micrOMEGAs 5.2.7 such that

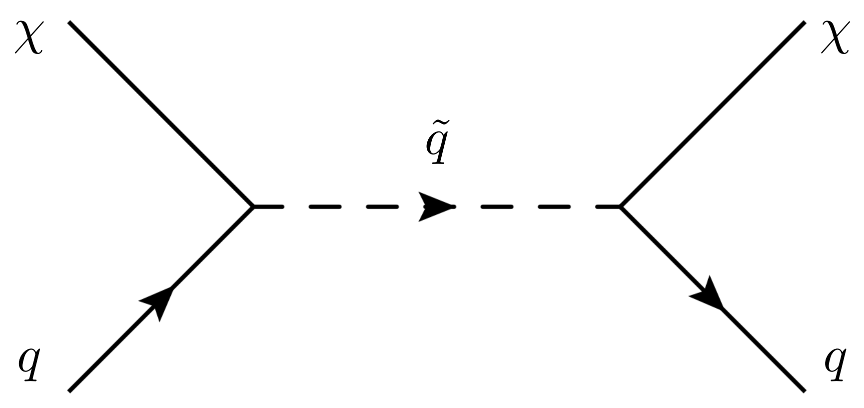
- the Sommerfeld effect is included for colored scalars up to the adjoint representation
- Bound state effects are included for colored scalars up to the adjoint representation

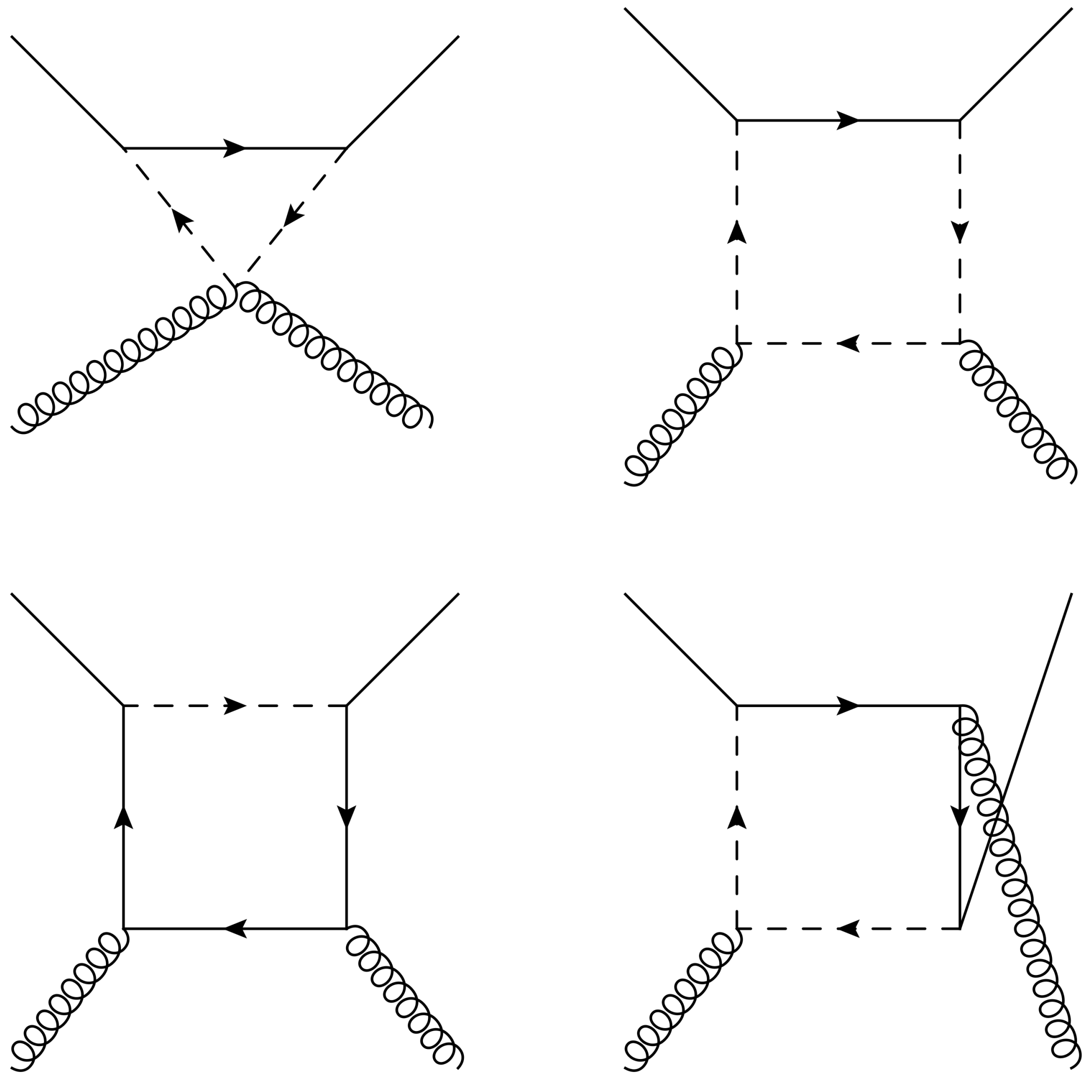
Determine $g_{DM,0}$ for each data point (M_χ, Δ) such that DM is *not* overproduced.

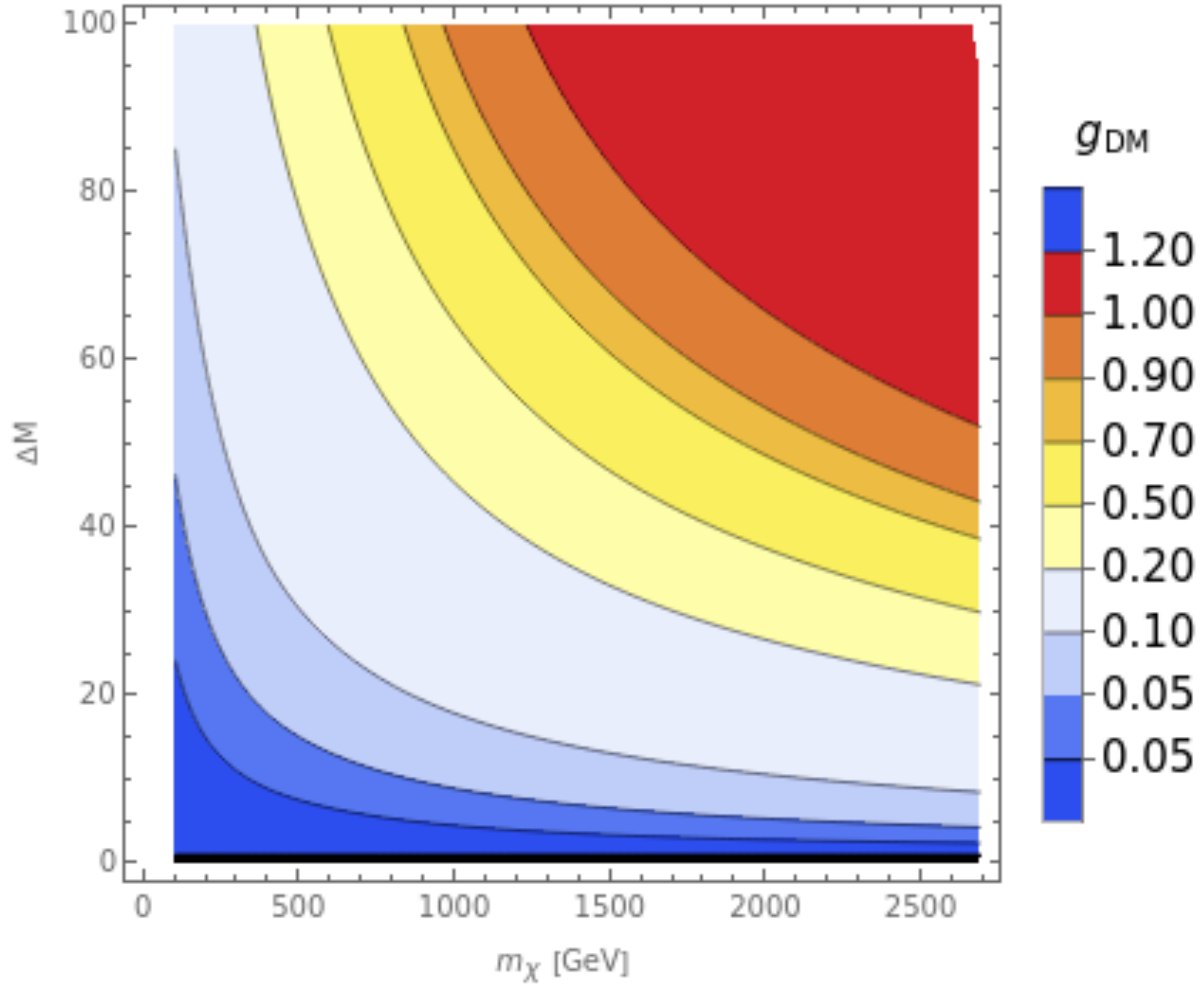
For instance, we find $g_{DM,0}\left(M^{\text{u.b.}}, 0\right) = \sqrt{4\pi}$ for

Constraints on the Model : Direct Detection

Direct Detection :





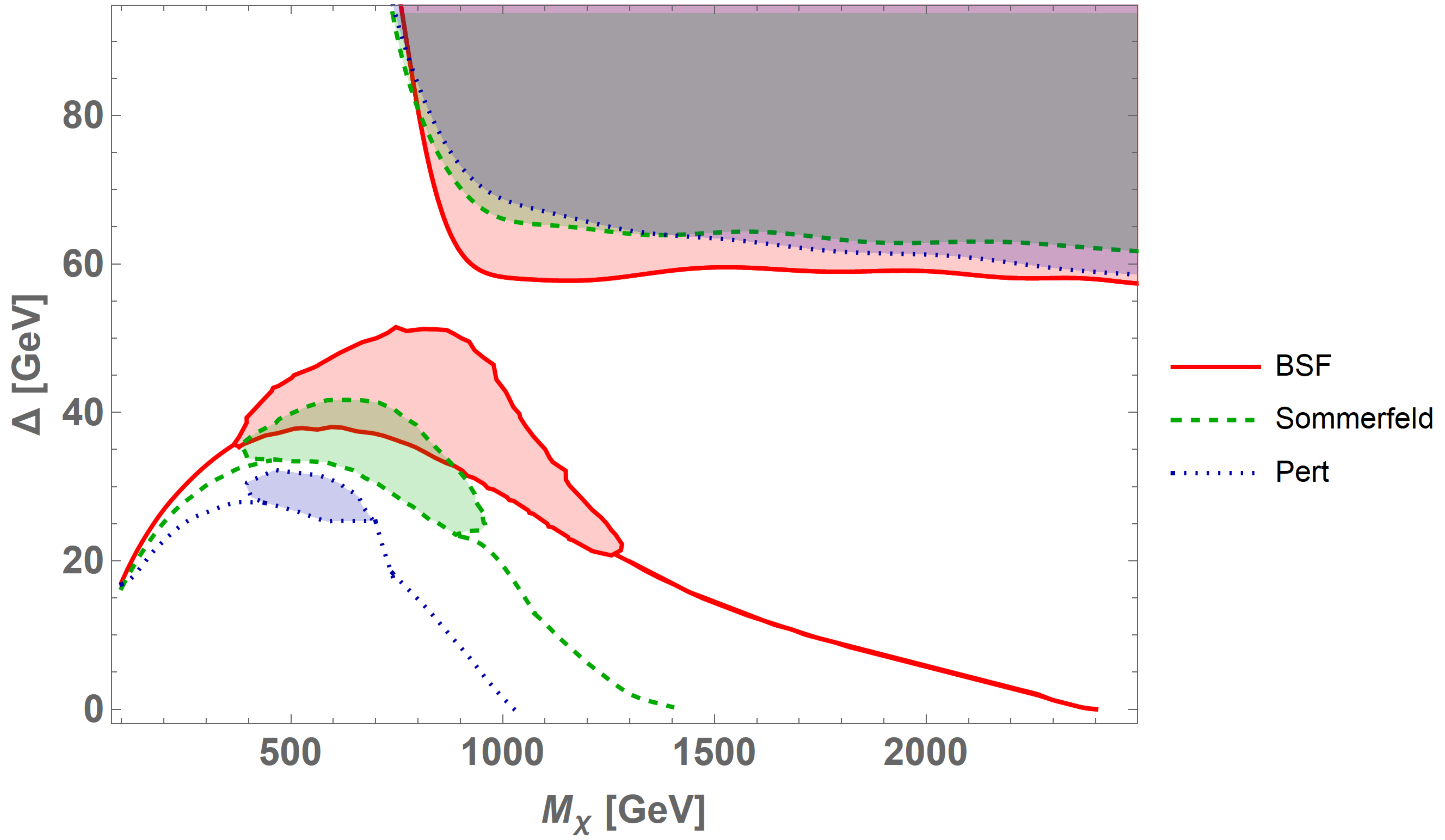


- Wilson coefficients of the effective *DM-q*/*DM-g* interaction are RGE evolved from $\mu \sim M_X$ to $\mu \sim \mathrm{GeV} \rightarrow$ factor ~ 2 on amplitude level. [Mohan et. al (2019)]
- 1-loop or velocity suppressed SI contribution typically more constraining than spin-dependent limits (for coannihilating regions).

Strong constraints for small mass splittings.

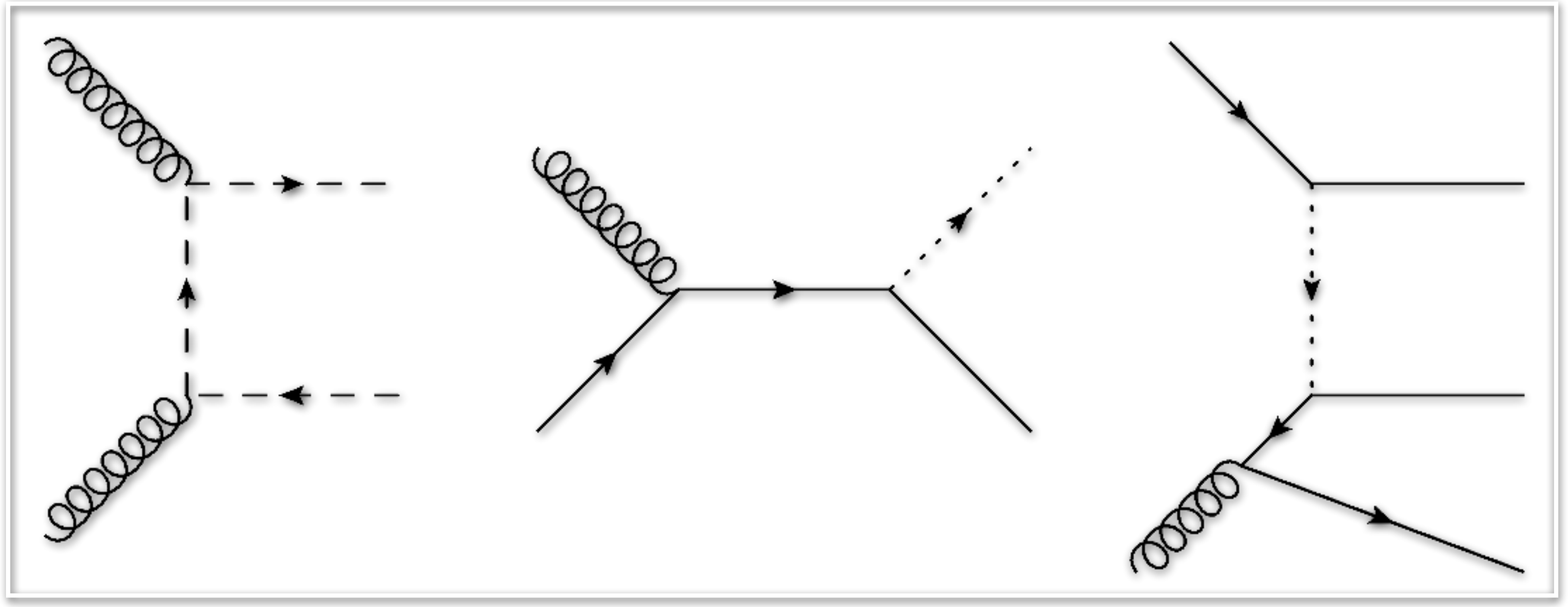
Constraints on the Model

Exclusion Limits from Direct Detection



- BSF has a sizable impact for small mass splittings.
- Shift in largest allowed mass from $M_\chi \sim 1\,\mathrm{TeV}$ to $M_\chi \sim 2.5\,\mathrm{TeV}$ for $\Delta \to 0$.
- Viable region for $\Delta \sim 100\,\mathrm{GeV}$ slightly enlarged.

Constraints on the Model : Collider Constraints



- pair production of the colored mediators ($\tilde{q}$), followed by their decay into dark matter (χ) plus a quark;
- associated production of $\tilde{q}$ with χ; and
- pair production of the dark matter in association with a jet from initial state radiation, $pp \to \chi\chi j$.

Prompt Searches :

- mono-jet + $\not{E}_T$ [Atlas (2021)]
- multi-jet + $\not{E}_T$ [Atlas (2020)]

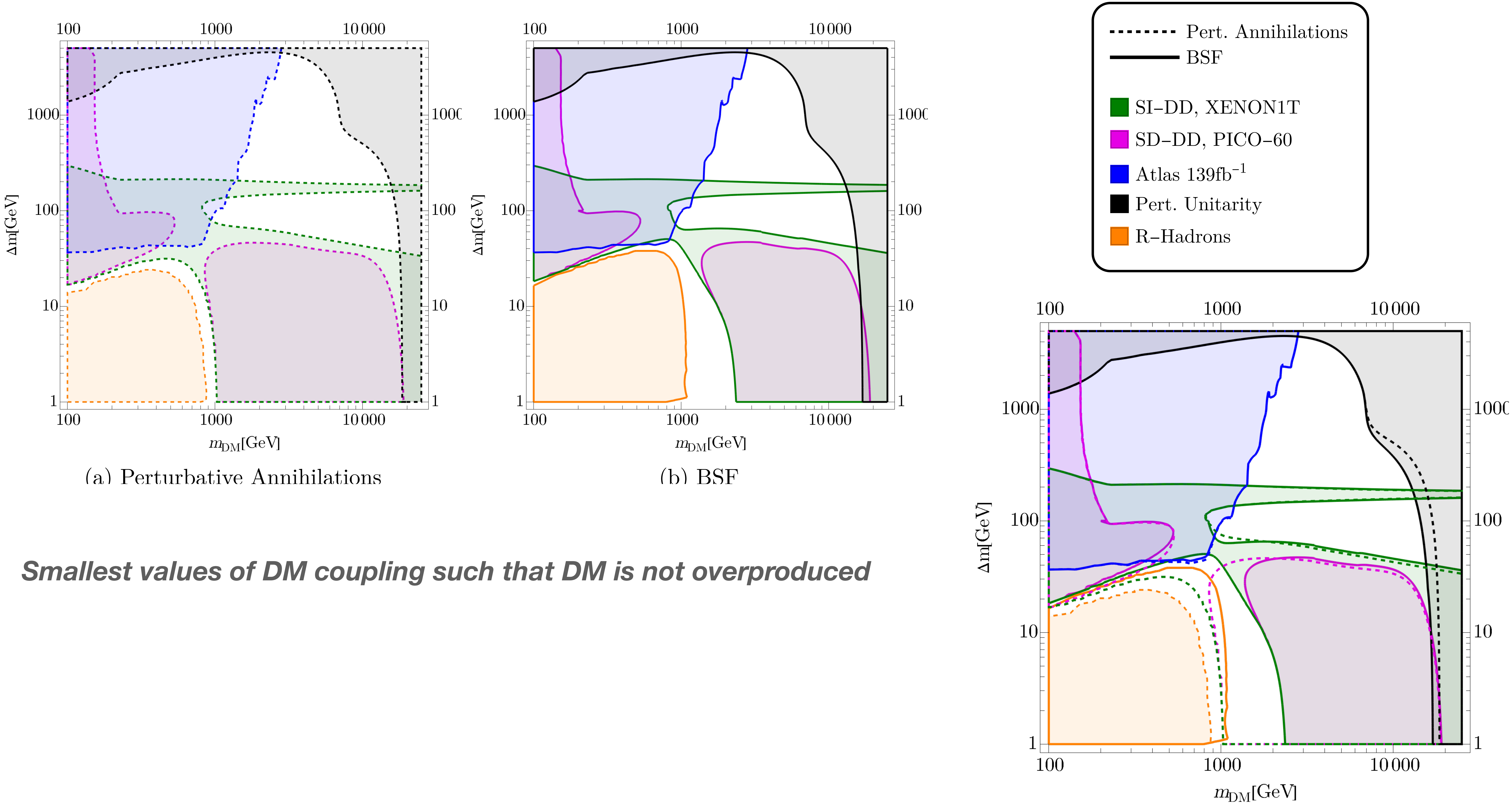
→ *Recasted and Reinterpreted with* **MadAnalysis5**

LLP Searches :

- Heavy Stable Charged Particle (HSCP) searches
- Displaced Vertices

→ *Re-interpreted from CMS searches*

Summary of the constraints



(a) Perturbative Annihilations

(b) BSF

Smallest values of DM coupling such that DM is not overproduced

Conclusions

- The Sommerfeld effect and bound state formation (BSF) arise for long range interactions in a dark sector
- BSF and subsequent bound state decay into SM particles efficiently provides an additional DM annihilation channel
- We have analyzed a model of colored coannihilation (Simplified t-channel: S3M-uR) using a modified micrOMEGAs version.
- The coannihilating region strongly impacted by non-perturbative effects. Viable paramter space involving tiny couplings of DM to the SM is shifted from 1 TeV to 2.5 TeV

- Direct Detection constrains a large part of the co-annihilating DM.
- Prompt searches at the LHC constrain large mass gaps between parent particle and DM.
- For small mass gaps, long lived particle searches constrain a significant part of the parameter space