Lepton Anomalous Magnetic Moment with Singlet-Doublet Fermion Dark Matter in Scotogenic $L_\mu - L_\tau$ Model

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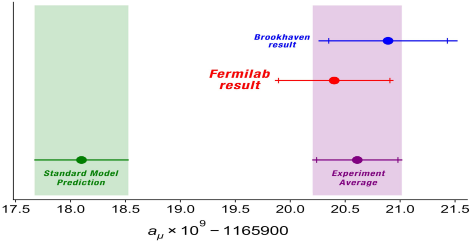
Anomalous Magnetic Moments

$$\overrightarrow{\mu_\ell} = g_\ell \left(\frac{q}{2m}\right) \overrightarrow{S}\ , \qquad g_\ell = 2.$$

$$a_\ell = \frac{1}{2}(g_\ell - 2)$$

Anomalous Muon Magnetic Moment



arXiv:2104.03281[E989-FERMI Lab]

The recent measurement of a_μ, by the E989 experiment at Fermilab shows a discrepancy with respect to (SM)

$$a_\mu^{\mathrm{FNAL}} = 116592040(54) \times 10^{-11}$$
$$a_\mu^{\mathrm{SM}} = 116591810(43) \times 10^{-11}$$

which when combined with the previous Brookhaven determination

$$a_\mu^{\mathrm{BNL}} = 116592089(63) \times 10^{-11}$$

$$\Delta a_\mu = 251(59) \times 10^{-11}.$$

Anomalous Electron Magnetic Moment

$$\Delta a_e = a_e^{\mathrm{exp}} - a_e^{\mathrm{SM}} = (-87 \pm 36) \times 10^{-14}$$

Science, 360, 191–195(2018).

From Precision measurement of the fine structure constant using Caesium atoms.

$U(1)_{L_\mu - L_\tau}$ Extension

$$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_{L_\mu - L_\tau}$$

- **Well Motivated** :
  Anomaly free.
  Interesting phenomenology related to neutrino mass, DM.
  Can explain Muon anomalous magnetic moment $(g-2)_\mu$.
- Better prospects of detection $\Longrightarrow$ Muonic Probes
- Kinetic mixing term between $U(1)_Y$ and $U(1)_{L_\mu - L_\tau}$:

$$\frac{\epsilon}{2} B^{\alpha\beta} Y_{\alpha\beta}$$

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Anomalous Muon Magnetic Moment

- Any radiative correction, which couples the muon spin to the virtual fields, contributes to its magnetic moment:

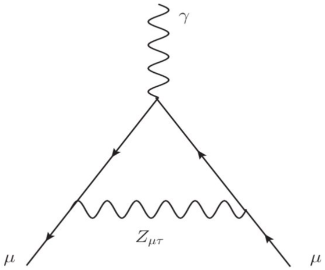
Anomalous Muon Magnetic Moment

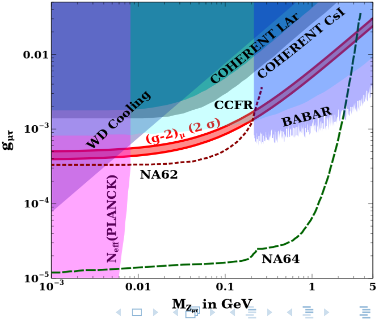
- Any radiative correction, which couples the muon spin to the virtual fields, contributes to its magnetic moment:

One loop diagram mediated by $Z_{\mu\tau}$ boson.

$$\Delta a_\mu = \frac{\alpha'}{2\pi}\int_0^1 dx \frac{2m_\mu^2 x^2(1-x)}{x^2 m_\mu^2 + (1-x)M_{Z_{\mu\tau}}^2} \approx \frac{\alpha'}{2\pi}\frac{2m_\mu^2}{3M_{Z_{\mu\tau}}^2}$$

where $\alpha' = g_{\mu\tau}^2/(4\pi)$.





Scotogenic $U(1)_{L_\mu - L_\tau}$ Model

Gauge Group	Fermion Fields			Scalar Field		
	N_e	N_μ	N_τ	Φ_1	Φ_2	η
$SU(2)_L$	1	1	1	1	1	2
$U(1)_Y$	0	0	0	0	0	$\frac{1}{2}$
$U(1)_{L_\mu - L_\tau}$	0	1	-1	1	2	0
Z_2	-1	-1	-1	$+1$	$+1$	-1

$$\mathcal{L} \supseteq \overline{N_\mu} i\gamma^\mu \mathfrak{D}_\mu N_\mu - M_{\mu\tau} N_\mu N_\tau + \overline{N_\tau} i\gamma^\mu \mathfrak{D}_\mu N_\tau - \frac{M_{ee}}{2} N_e N_e - Y_{e\mu}\Phi_1^\dagger N_e N_\mu$$
$$- Y_{e\tau}\Phi_1 N_e N_\tau - Y_\mu \Phi_2^\dagger N_\mu N_\mu - Y_{De}\bar{L_e}\tilde{\eta} N_e - Y_{D\mu}\bar{L_\mu}\tilde{\eta} N_\mu - Y_{D\tau}\bar{L_\tau}\tilde{\eta} N_\tau$$
$$- Y_\tau \Phi_2 N_\tau N_\tau - Y_{le}\overline{L_e} H e_R - Y_{l\mu}\overline{L_\mu} H \mu_R - Y_{l\tau}\overline{L_\tau} H \tau_R + \text{h.c.}$$

$$V(H,\Phi_i,\eta) = -\mu_H^2\left(H^\dagger H\right) + \lambda_H\left(H^\dagger H\right)^2 - \mu_{\Phi_i}^2\left({\Phi_i}^\dagger \Phi_i\right) + \lambda_{\Phi_i}\left({\Phi_i}^\dagger\Phi_i\right)^2$$
$$+ \lambda_{H\Phi_i}(H^\dagger H)\left({\Phi_i}^\dagger\Phi_i\right) + m_\eta^2(\eta^\dagger\eta) + \lambda_2(\eta^\dagger\eta)^2 + \lambda_3(\eta^\dagger\eta)(H^\dagger H)$$
$$+ \lambda_4(\eta^\dagger H)(H^\dagger\eta) + \frac{\lambda_5}{2}[(H^\dagger\eta)^2 + (\eta^\dagger H)^2].$$

The right handed neutrino mass matrix, Dirac neutrino Yukawa and charged lepton mass matrix are given by

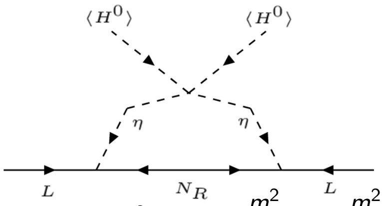
$$M_R = \begin{pmatrix} M_{ee} & Y_{e\mu} v_1 & Y_{e\tau} v_1 \\ Y_{e\mu} v_1 & Y_{\mu} v_2 & M_{\mu\tau} \\ Y_{e\tau} v_1 & M_{\mu\tau} & Y_{\tau} v_2 \end{pmatrix}$$

$$Y_D = \begin{pmatrix} Y_{De} & 0 & 0 \\ 0 & Y_{D\mu} & 0 \\ 0 & 0 & Y_{D\tau} \end{pmatrix}, \; M_\ell = \frac{1}{\sqrt{2}} \begin{pmatrix} Y_e v & 0 & 0 \\ 0 & Y_\mu v & 0 \\ 0 & 0 & Y_\tau v \end{pmatrix}$$

Here $v/\sqrt{2}$ is the VEV of neutral component of SM Higgs doublet H.

Neutrino Mass

- Z_2 symmetry under which RHNs and η are odd.



- Neutrino Mass:

$$(M_\nu)_{ij} = \sum_k \frac{h_{ik} h_{jk} M_k}{32\pi^2} \left[L_k(m^2_{\eta_R}) - L_k(m^2_{\eta_I}) \right] ; \quad L_k(m^2) = \frac{m^2}{m^2 - M_k^2} \ln \frac{m^2}{M_k^2}$$

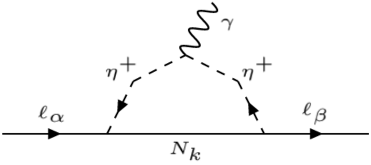
- Casas-Ibarra parametrisation:

$$h_{\alpha i} = \left(U D_\nu^{1/2} R^\dagger \Lambda^{1/2} \right)_{\alpha i}$$

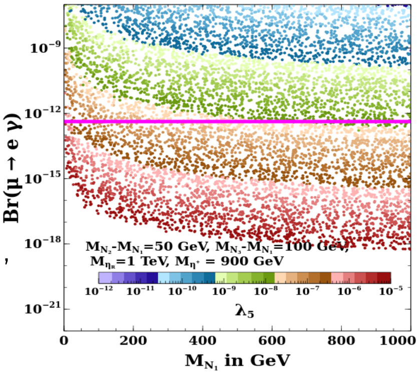
$$\Lambda_k = \frac{2\pi^2}{\lambda_5} \zeta_k \frac{2 M_k}{v^2}, \qquad \zeta_k = \left(\frac{M_k^2}{8(m^2_{\eta_R} - m^2_{\eta_I})} \left[L_k(m^2_{\eta_R}) - L_k(m^2_{\eta_I}) \right] \right)^{-1}$$

Lepton flavour violation

$$\mathrm{Br}(\mu \to e\gamma) = \frac{3(4\pi)^3\alpha}{4G_F^2}|A_D|^2 \mathrm{Br}(\mu \to e\nu_\mu\overline{\nu_e})$$



$$A_D = \sum_k \frac{h^*_{ke}h_{k\mu}}{16\pi^2}\frac{1}{M^2_{\eta^+}}f(t_k)$$
$$\text{where} \quad t_k = m^2_{N_k}/M^2_{\eta^+}$$



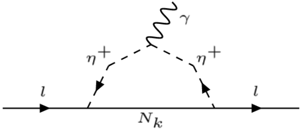
MEG Constraint :
$\mathrm{Br}(\mu \to e\gamma) = 4.2 \times 10^{-13}$

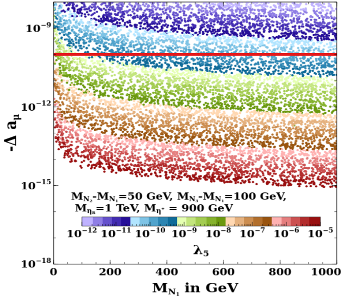
Parameter: $M_1, M_2, M_3 \in [1, 1000]$ GeV ,
$M_{\eta^+} \in [100, 1000]$ GeV
and $\lambda_5 \in [10^{-10}, 10^{-3}]$

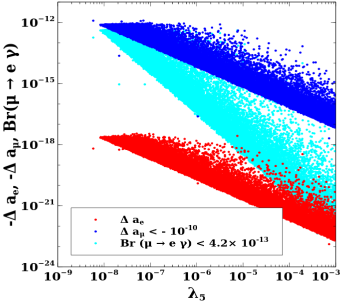
Δa_ℓ in Scotogenic Scenario

$$\Delta a_l = \sum_k -\frac{m_l^2}{8\pi^2 M_{\eta^+}^2}|h_{lk}|^2 f(M_k^2/M_{\eta^+}^2)$$

$$f(x) = \frac{1-6x+3x^2+2x^3-6x^2\log x}{12(1-x)^4}$$







In spite of the possibility of having positive and negative contributions to $(g-2)$ from vector boson and charged scalar loops respectively, the minimal scotogenic $L_\mu - L_\tau$ model can not explain muon and electron $(g-2)$ simultaneously.

VLFD Extension of scotogenic $U(1)_{L_\mu - L_\tau}$ model

Vector like fermion doublet $\Psi^T = (\psi^0, \psi^-) \sim (1, 2, -\frac{1}{2}, 0)$ Z_2 odd.

$$\mathcal{L} = \overline{\Psi}\,(i\gamma^\mu D_\mu - M)\,\Psi - Y_\psi \overline{\Psi}\widetilde{H}\big(N_e + (N_e)^c\big) - Y_{\psi e}\overline{\Psi_L}\eta e_R + \mathrm{h.c.}$$

$$-\mathcal{L}_{\mathrm{mass}} = M\overline{\psi^0_L}\psi^0_R + \frac{1}{2}M_{ee}\overline{N}_e(N_e)^c + m'_D(\overline{\psi^0_L}N_e + \overline{\psi^0_R}(N_e)^c) + \mathrm{h.c.}$$

For the dark sector in the basis $((\psi^0_R)^c, \psi^0_L, (N_1)^c)^T$ as :

$$\mathcal{M} = \begin{pmatrix} 0 & M & m_D \\ M & 0 & m_D \\ m_D & m_D & M_1 \end{pmatrix}$$

Diagonalisation by a unitary matrix
$\mathcal{U}(\theta) = U_{13}(\theta_{13} = \theta).U_{23}(\theta_{23} = 0).U_{12}(\theta_{12} = \frac{\pi}{4})$

$$\mathcal{U} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\pi/2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}}\cos\theta & \frac{1}{\sqrt{2}}\cos\theta & \sin\theta \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}}\sin\theta & -\frac{1}{\sqrt{2}}\sin\theta & \cos\theta \end{pmatrix}$$

Dark States and and Parameters

The emerging physical states: $\chi_i = \frac{\chi_{iL} + (\chi_{iL})^c}{\sqrt{2}}$

The diagonalisation requires:

$$\tan 2\theta = \frac{2\sqrt{2}\ m_D}{M - M_1}$$

$$\chi_{1L} = \frac{\cos\theta}{\sqrt{2}}(\psi_L^0 + (\psi_R^0)^c) + \sin\theta (N_1)^c,$$
$$\chi_{2L} = \frac{i}{\sqrt{2}}(\psi_L^0 - (\psi_R^0)^c),$$
$$\chi_{3L} = -\frac{\sin\theta}{\sqrt{2}}(\psi_L^0 + (\psi_R^0)^c) + \cos\theta (N_1)^c\,.$$

Mass Eigen Values:

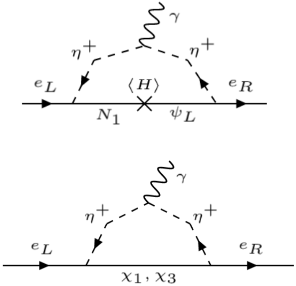
$$m_{\chi_1} = M\cos^2\theta + M_1\sin^2\theta + m_D\sin 2\theta,$$
$$m_{\chi_2} = M,$$
$$m_{\chi_3} = M_1\cos^2\theta + M\sin^2\theta - m_D\sin 2\theta\,.$$

Dark Parameters:

$$Y_\psi \approx \frac{\Delta M\ \sin 2\theta}{2v},$$
$$M \approx m_{\chi_1}\cos^2\theta + m_{\chi_3}\sin^2\theta,$$
$$M_1 \approx m_{\chi_3}\cos^2\theta + m_{\chi_1}\sin^2\theta;$$

Electron $(g-2)$ in Extended Model

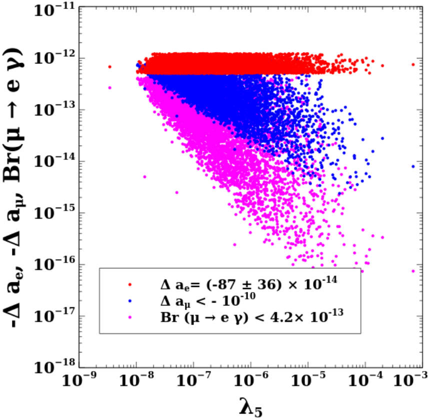
$$
\begin{aligned}
(\psi_R^0)^c &= \frac{\cos\theta}{\sqrt{2}}\chi_{1L} - \frac{1}{\sqrt{2}}\chi_{2L} - \frac{\sin\theta}{\sqrt{2}}\chi_{3L} \\
\psi_L^0 &= \frac{\cos\theta}{\sqrt{2}}\chi_{1L} + \frac{1}{\sqrt{2}}\chi_{2L} - \frac{\sin\theta}{\sqrt{2}}\chi_{3L} \\
(N_1)^c &= \sin\theta\ \chi_{1L} + \cos\theta\ \chi_{3L}
\end{aligned}
$$

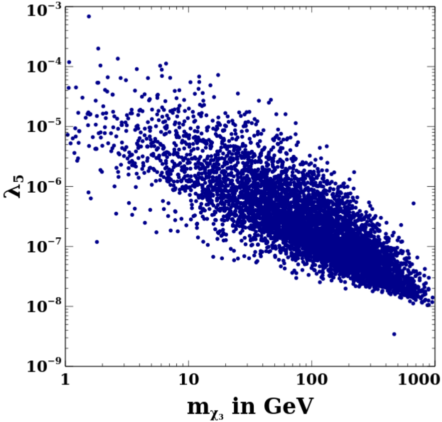


$$
\Delta a_e = -\frac{m_e}{8\pi^2 M_{\eta^+}^2}\frac{\sin\theta\cos\theta}{\sqrt{2}}\mathrm{Re}(h_{1e}Y_{\psi e}^*) \times \left[m_{\chi_1} f_{LR}\left(\frac{m_{\chi_1}^2}{M_{\eta^+}^2}\right) - m_{\chi_3} f_{LR}\left(\frac{m_{\chi_3}^2}{M_{\eta^+}^2}\right)\right]
$$

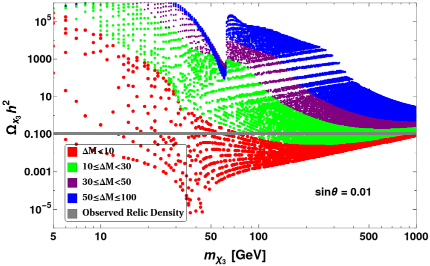
$$
f_{LR}(x) = \frac{1 - x^2 + 2x\log x}{2(1-x)^3}
$$

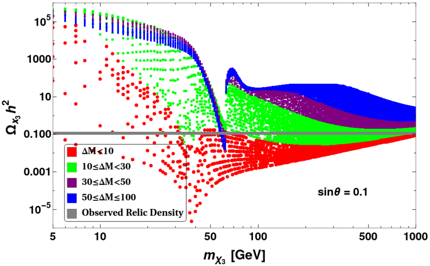
Common Parameter Space

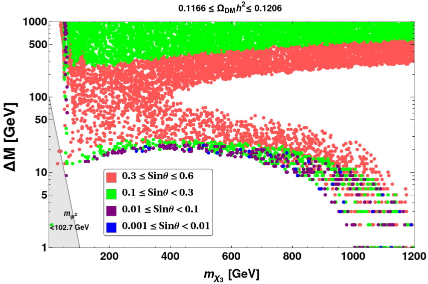


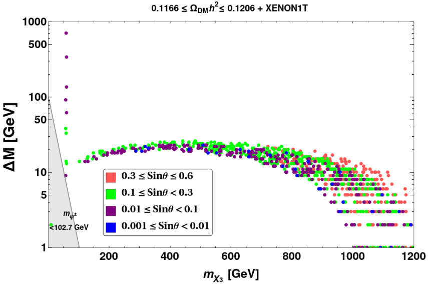


Dark Matter Phenomenology







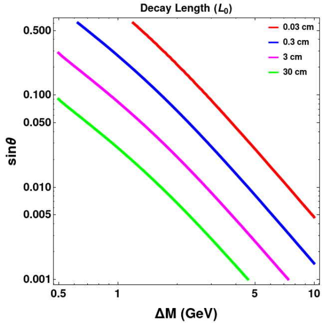


Collider Signatures

$$\Gamma_{\psi^\pm \to \chi_3 \pi^\pm} \approx \frac{G_F^2}{\pi}(f_\pi \cos\theta_c)^2 \sin^2\theta \Delta M^3 \sqrt{1 - \frac{m_{\pi^\pm}^2}{\Delta M^2}}$$

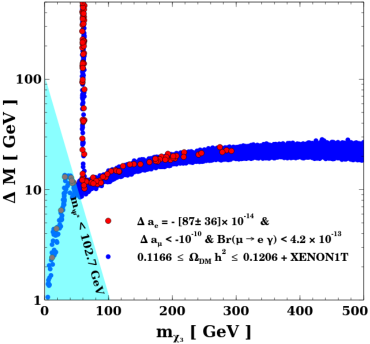
$$\Gamma_{\psi^\pm \to \chi_3 \ell^\pm \nu_l} \approx \frac{G_F^2}{15\pi^3} \sin^2\theta \Delta M^5 \sqrt{1 - \frac{m_l^2}{\Delta M^2}}$$

- Opposite sign dilepton + missing energy ($\ell^+\ell^- + \not{E}_T$)
- Three leptons + missing energy ($\ell\ell\ell + \not{E}_T$)
- Four leptons + missing energy ($\ell\ell\ell\ell + \not{E}_T$)
- Single lepton with jets ($\ell^\pm + jj + \not{E}_T$)
- **Displaced vertex signature of $\psi^\pm$**



Summary

- Both dark sector phenomenology and the flavour observables are deeply coupled.
- Being in agreement with all relevant bounds, the model remains predictive at CLFV, DM direct detection as well as collider.
- In addition to the singlet-doublet parameter space sensitive to both high and low energy experiments like the LHC, MEG (or $(g-2)$) respectively, the existence of light $L_\mu - L_\tau$ gauge boson at sub-GeV scale also remains sensitive at low energy experiments like NA62 at CERN, offering a variety of complementary probes.



Thank You !!!

Neutral Fermion Mass Matrix

Neutral fermion mass matrix for the dark sector in the basis $((\psi_R^0)^c, \psi_L^0, (N_e)^c, (N_\mu)^c, (N_\tau)^c)^T$ as :

$$\mathcal{M} = \begin{pmatrix} 0 & M & \frac{Y_\psi v}{\sqrt{2}} & 0 & 0 \\ M & 0 & \frac{Y_\psi v}{\sqrt{2}} & 0 & 0 \\ \frac{Y_\psi v}{\sqrt{2}} & \frac{Y_\psi v}{\sqrt{2}} & M_{ee} & \frac{Y_{e\mu} v_1}{\sqrt{2}} & \frac{Y_{e\tau} v_1}{\sqrt{2}} \\ 0 & 0 & \frac{Y_{e\mu} v_1}{\sqrt{2}} & \frac{Y_\mu v_2}{\sqrt{2}} & M_{\mu\tau} \\ 0 & 0 & \frac{Y_{e\tau} v_1}{\sqrt{2}} & M_{\mu\tau} & \frac{Y_\tau v_2}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \mathrm{M} & \mathrm{M_D} \\ \mathrm{M_D^T} & \mathrm{M_R} \end{pmatrix} \quad (1)$$

$$\mathrm{M} = \begin{pmatrix} 0 & M \\ M & 0 \end{pmatrix}, \; \mathrm{M_D} = \begin{pmatrix} \frac{Y_\psi v}{\sqrt{2}} & 0 & 0 \\ \frac{Y_\psi v}{\sqrt{2}} & 0 & 0 \end{pmatrix} \; \mathrm{M_R} = \begin{pmatrix} M_{ee} & \frac{Y_{e\mu} v_1}{\sqrt{2}} & \frac{Y_{e\tau} v_1}{\sqrt{2}} \\ \frac{Y_{e\mu} v_1}{\sqrt{2}} & \frac{Y_\mu v_2}{\sqrt{2}} & M_{\mu\tau} \\ \frac{Y_{e\tau} v_1}{\sqrt{2}} & M_{\mu\tau} & \frac{Y_\tau v_2}{\sqrt{2}} \end{pmatrix}.$$

Neutral Fermion Mass Matrix

$$
\begin{aligned}
N_e &= c_{12}c_{13}N_1 + (-c_{23}s_{12} - c_{12}s_{13}s_{23})N_2 \\
&+ (-c_{12}c_{23}s_{13} + s_{12}s_{23})N_3 \\
N_\mu &= s_{12}c_{13}N_1 + (c_{12}c_{23} - s_{12}s_{23}s_{13})N_2 \\
&+ (-s_{12}c_{23}s_{13} + c_{12}s_{23})N_3 \\
N_\tau &= s_{13}N_1 + c_{13}s_{23}N_2 + c_{13}c_{23}N_3
\end{aligned} \tag{2}
$$

Thus the neutral fermion mass matrix relevant for singlet-doublet DM phenomenology can be written in the basis $((\psi_R^0)^c, \psi_L^0, (N_1)^c)^T$ as :

$$
\mathcal{M} = \begin{pmatrix} 0 & M & c_{12}c_{13}\frac{Y_\psi v}{\sqrt{2}} \\ M & 0 & c_{12}c_{13}\frac{Y_\psi v}{\sqrt{2}} \\ c_{12}c_{13}\frac{Y_\psi v}{\sqrt{2}} & c_{12}c_{13}\frac{Y_\psi v}{\sqrt{2}} & c_{12}^2 c_{13}^2 M_1' \end{pmatrix} = \begin{pmatrix} 0 & M & m_D \\ M & 0 & m_D \\ m_D & m_D & M_1 \end{pmatrix}.
$$

Where $M_1 = c_{12}^2 c_{13}^2 M_1'$ and $m_D = c_{12}c_{13}\frac{Y_\psi v}{\sqrt{2}} = c_{12}c_{13}m_D'$

DM-SM Interaction

$$
\begin{aligned}
\mathcal{L}_{int} &= \overline{\Psi} i\gamma^{\mu}[-i\frac{g}{2}\tau.W_{\mu} - ig'\frac{Y}{2}B_{\mu}]\Psi \\
&+ \overline{N_{R_i}} i\gamma^{\mu}(-ig_{\mu\tau}Y_{\mu\tau}(Z_{\mu\tau})_{\mu})N_{R_i} \\
&= \left(\frac{e}{2\sin\theta_W\cos\theta_W}\right)\overline{\psi^0}\gamma^{\mu}Z_{\mu}\psi^0 \\
&+ \frac{e}{\sqrt{2}\sin\theta_W}(\overline{\psi^0}\gamma^{\mu}W_{\mu}^{+}\psi^{-} + \psi^{+}\gamma^{\mu}W_{\mu}^{-}\psi^0) \\
&- e\ \psi^{+}\gamma^{\mu}A_{\mu}\psi^{-} \\
&- \left(\frac{e\cos 2\theta_W}{2\sin\theta_W\cos\theta_W}\right)\psi^{+}\gamma^{\mu}Z_{\mu}\psi^{-} \\
&+ Y_{\psi}\Psi\tilde{H}(N_e + N_e^c).
\end{aligned} \tag{3}
$$

where $g = \frac{e}{\sin\theta_W}$ and $g' = \frac{e}{\cos\theta_W}$ with e being the electromagnetic coupling constant, θ_W being the Weinberg angle and $g_{\mu\tau}$ is the $U(1)_{L_\mu - L_\tau}$ coupling constant.