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Opening New Windows to the Universe

IIB matrix model: Emergence of spacetime and matter

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1. Introduction

This conference has a truly wonderful title:

Opening New Windows to the Universe

Our approach:

not hand-waving but calculating

Specifically, we will consider a particular **algebraic equation**, as the solution of that equation may contain information about

the emergence of spacetime and the birth of the Universe.

These results, once established, would "resolve" the Big Bang singularity of the standard Friedmann cosmology.

2. Background

We start from the IIB matrix model of Kawai and collaborators [1, 2], which reproduces the structure of the light-cone string field theory of type-IIB superstrings.

The IIB matrix model has a **finite number** of $N \times N$ traceless Hermitian matrices: ten bosonic matrices A^μ and eight fermionic Majorana–Weyl matrices Ψ_α.

The partition function Z of the IIB matrix model is defined by a "path" integral over A and Ψ with a weight factor $\exp[-S_{\text{bos}}(A) - S_{\text{ferm}}(\Psi,\, A)]$.
The fermionic matrices Ψ can be integrated out exactly (Gaussian integrals) and then give the Pfaffian $\mathcal{P}(A)$.
Numerical results have been presented in Ref. [3, 4, 5].

For strings of bosonic observables, the expectation values are defined by the same A-integral as Z, that is, involving the exponential weight factor with the bosonic action, $\exp[-S_{\text{bos}}(A)]$ and the Pfaffian $\mathcal{P}(A)$.

2. Background

As first realized by Witten [6, 7], these expectation values, at large N, can also be obtained by inserting the matrices $\widehat{A}^{\,\mu}$ of the so-called **master field** directly into the observables, without need of integration.

Recently, we have suggested [8] that precisely the master-field matrices $\widehat{A}^{\,\mu}$ of the IIB matrix model may contain the information of an emergent classical spacetime (the heuristics is clarified in a recent review [9]).

Assuming that the matrices $\widehat{A}^{\,\mu}$ of the IIB-matrix-model master field are known and that they are approximately band-diagonal (as suggested by the numerical results [3, 4, 5]), it is relatively easy [8] to extract a discrete set of spacetime points $\{\widehat{x}_k^{\,\mu}\}$ and an interpolating metric $g_{\mu\nu}(x)$.

But, instead of assuming the matrices $\widehat{A}^{\,\mu}$, we want to **calculate** them. And, for that, we need an equation . . .

3. Bosonic master-field equation

GOOD NEWS:

the master-field equation has already been established, nearly 40 years ago, by Greensite and Halpern [7], who write in the first line of their abstract:

"We derive an exact algebraic (master) equation for the euclidean master field of any large-N matrix theory, including quantum chromodynamics."

Now, "any" means "any" and we may as well consider the large-N IIB matrix theory [1, 2].

3.1 Algebraic equation

Building on the work by Greensite and Halpern [7], we then have the IIB-matrix-model bosonic master field in "quenched" form [8]:

$$\widehat{A}^{\,\mu}_{\,kl} = e^{i\,\widehat{p}_k\,\tau_{\text{eq}}}\;\widehat{a}^{\,\mu}_{\,kl}\;e^{-i\,\widehat{p}_l\,\tau_{\text{eq}}}\,. \tag{1a}$$

The $\widehat{p}_k$ are random constants (see below) and the dimensionless time τ_{eq} must have a sufficiently large value in order to represent an equilibrium situation (τ is the fictitious Langevin time of the stochastic-quantization procedure).

The τ-independent matrix $\widehat{a}^{\,\mu}$ on the right-hand side of (1a) solves the following algebraic equation [8]:

$$i\left(\widehat{p}_k - \widehat{p}_l\right)\widehat{a}^{\,\mu}_{\,kl} \;=\; -\frac{\partial\, S_{\text{eff}}}{\partial\, \widehat{a}_{\mu\;lk}} + \widehat{\eta}^{\,\mu}_{\,kl}\,, \tag{1b}$$

in terms of the master momenta $\widehat{p}_k$ (real uniform random numbers) and the master-noise matrices $\widehat{\eta}^{\,\mu}_{\,kl}$ (real Gaussian random numbers).

3.1 Algebraic equation

Specifically, the IIB-matrix-model algebraic equation (1b) for D traceless Hermitian matrices $\widehat{a}^{\,\mu}$ of dimension $N \times N$ reads:

$$
\begin{aligned}
i\left(\widehat{p}_k - \widehat{p}_l\right)\widehat{a}^{\,\mu}_{\;kl} &= -\Big[\widehat{a}^{\,\nu},\,\big[\widehat{a}^{\,\nu},\,\widehat{a}^{\,\mu}\big]\Big]_{kl} + F\,\frac{1}{\mathcal{P}(\widehat{a})}\,\frac{\partial\,\mathcal{P}(\widehat{a})}{\partial\,\widehat{a}^{\,\mu}_{\;lk}} + \widehat{\eta}^{\,\mu}_{\;kl}\,, && \text{(2a)}\\
\mathcal{P}(\widehat{a}) &= \text{homogeneous polynomial of degree } K\,, && \text{(2b)}\\
K &\equiv \big(D-2\big)\,\big(N^2-1\big)\,, && \text{(2c)}\\
(D,\,F) &= (10,\,1), \quad N \gg 1\,, && \text{(2d)}
\end{aligned}
$$

with matrix indices k, l running over $\{1,\,\ldots\,,\,N\}$ and directional indices μ, ν running over $\{1,\,\ldots\,,\,D\}$, while ν in (2a) is implicitly summed over.

As mentioned before, the $\widehat{p}_k$ are fixed uniform random numbers and the $\widehat{\eta}^{\,\mu}_{\;kl}$ fixed Gaussian random numbers.

3.2 Simplified algebraic equation

The algebraic equation (2a) is formidable and it makes sense to first consider the simplified equation obtained by setting $F = 0$:

$$i\left(\widehat{p}_k - \widehat{p}_l\right)\widehat{a}^{\;\mu}_{\;kl} \;=\; -\Big[\widehat{a}^{\;\nu},\,\left[\widehat{a}^{\;\nu},\,\widehat{a}^{\;\mu}\right]\Big]_{kl} + \widehat{\eta}^{\;\mu}_{\;kl}\,. \tag{3}$$

The matrices $\widehat{a}^{\;\mu}$ are $N \times N$ traceless Hermitian matrices and the number of variables is

$$N_{\text{var}} = D\left(N^2 - 1\right). \tag{4}$$

The simplified equation (3) is essentially a **cubic polynomial**.

3.2 Simplified algebraic equation

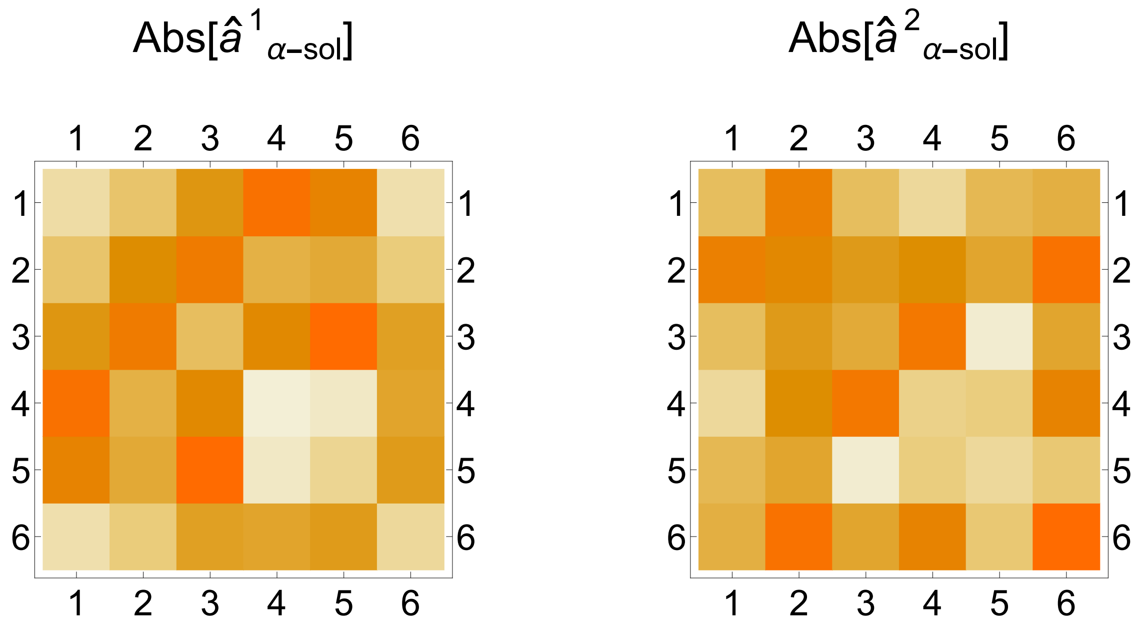
It appears impossible to obtain a general analytic solution of (3) in terms of the master constants $\widehat{p}_k$ and $\widehat{\eta}\,^{\mu}_{kl}$. Instead, we will try to get solutions for an explicit choice for the random master constants.

For $(D,\,N)=(2,\,6)$, consider the simplified equation (3) for 70 real variables, with a particular realization, the “alpha-realization,” of the pseudorandom numbers entering the equation (details in Ref. [10]).

Other realizations give similar results.

3.2 Simplified algebraic equation

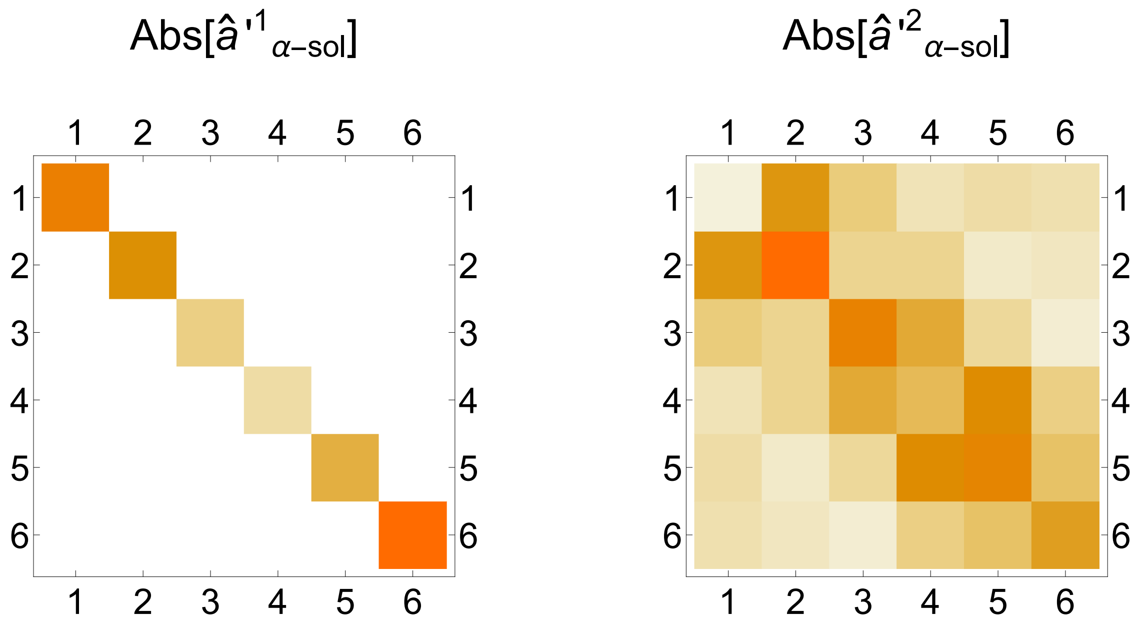
A particular solution [10] of the $(D, N) = (2, 6)$ simplified equation (3), with the alpha-realization of pseudorandom constants, is given by $\widehat{a}^{\,1}_{\alpha\text{-sol}}$ and $\widehat{a}^{\,2}_{\alpha\text{-sol}}$. Consider the absolute values of the matrix entries and get the following density plots:



$\Rightarrow$ no obvious band-diagonal structure.

3.2 Simplified algebraic equation

Now, change the basis, in order to diagonalize and order the $\mu = 1$ matrix. This gives the matrices $\widehat{a}\,'^{\,1}_{\alpha\text{-sol}}$ and $\widehat{a}\,'^{\,2}_{\alpha\text{-sol}}$, with density plots:



$\Rightarrow$ diagonal/band-diagonal structure, a highly nontrivial result!

$\rightsquigarrow$ But what are the effects of **dynamical fermions** on this structure?

3.3 Full algebraic equation

We now look for solutions of the **full** bosonic master-field equation (2a), with $F = 1$ to include the dynamic fermions, but, first, with rather small values of D and N.

We have an algebraic solution [11] for the case

$$\{D,\, N,\, F\} = \{3,\, 3,\, 1\}\,. \tag{5}$$

For larger values of D and N, we can use a numerical approach [12].

The present status of these calculations is summarized on the next slide.

3.3 Full algebraic equation

Table 1: Numerical solutions [12] of the full ($F = 1$) bosonic master-field equation (2a). The number of variables is given by $N_{\text{var}} = D\,(N^2 - 1)$ and the order of the Pfaffian by $K = (D-2)\,(N^2-1)$.

	N_{var}	K	status
$(D,\,N) = (3,\,3)$	24	8	done ($\sim 1/2$ hr) [a]
$(D,\,N) = (10,\,3)$	80	64	done (~ 76 hrs)
$(D,\,N) = (10,\,4)$	150 [b]	120	approximate solution ($\gtrsim 90$ days)

[a] previous algebraic results reproduced

[b] complex variables in the solution, as the Pfaffian $\mathcal{P}(\widehat{a})$ is complex

4. Conclusions

It is conceivable that a **new physics phase** gives rise to classical spacetime, gravity, and matter, as described by our current theories (General Relativity and the Standard Model).

For an explicit calculation, we have considered the **IIB matrix model**, which has been proposed as a nonperturbative formulation of type-IIB superstring theory (M-theory).

The crucial insight is that the emergent classical spacetime may reside in the **large-N master field** $\widehat{A}^{\mu}$ of the IIB matrix model.
The emergence of matter may also be driven by the large-N master field (possibly, its anti-Hermitian part) or by suitable perturbations around the master field. The precise scenario for the matter is not yet clear.

We have now started to **solve** the full bosonic master-field equation of the IIB matrix model: first results are in, but the road ahead is long and arduous ...

5. References

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